Computer algebra independent integration tests

4-Trig-functions/4.2-Cosine/4.2.3.1-a+b-cos-^m-c+d-cos-^n-A+B-cos-

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [644]. This is test number [92].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (644)	% 0.00 (0)
Mathematica	% 98.45 (634)	% 1.55 (10)
Maple	% 98.45 (634)	% 1.55 (10)
Maxima	% 29.35 (189)	% 70.65 (455)
Fricas	% 49.22 (317)	% 50.78 (327)
Sympy	% 9.78 (63)	% 90.22 (581)
Giac	% 30.43 (196)	% 69.57 (448)
Mupad	% 35.87 (231)	% 64.13 (413)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

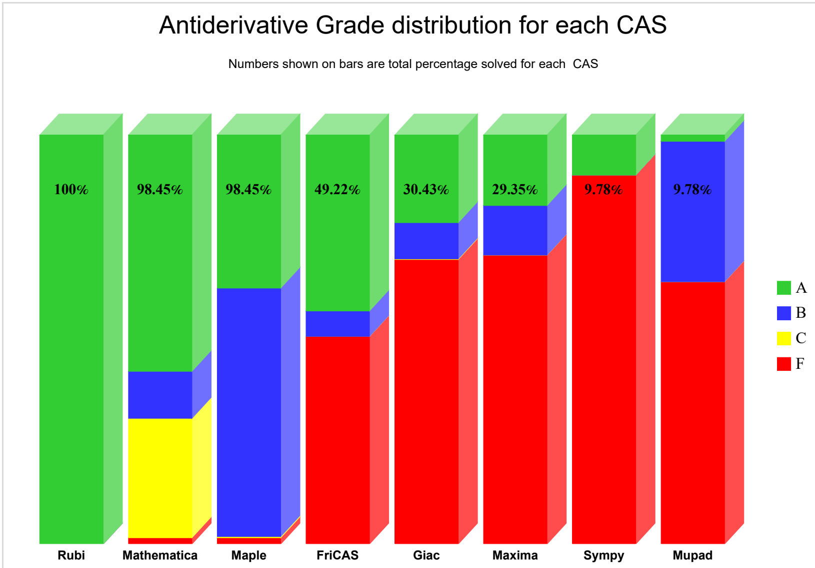
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

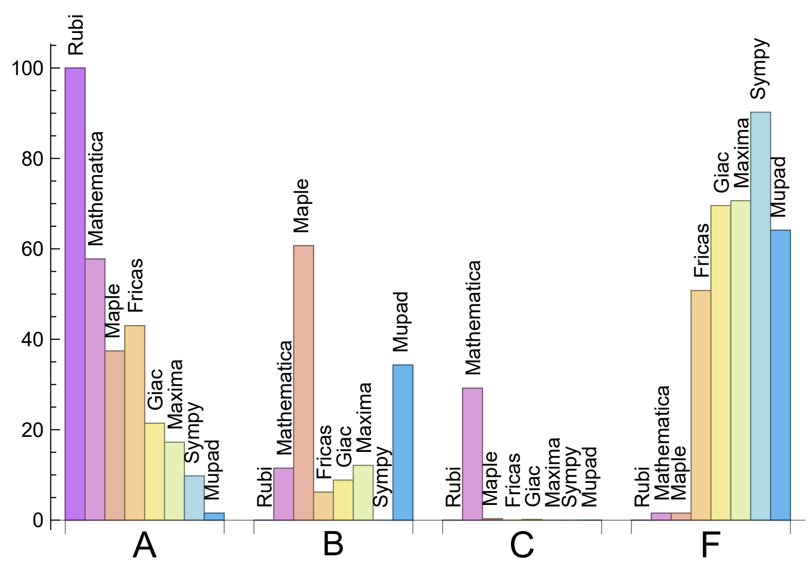
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	57.76	11.49	29.19	1.55
Maple	37.42	60.71	0.31	1.55
Maxima	17.24	12.11	0.00	70.65
Fricas	43.01	6.21	0.00	50.78
Sympy	9.78	0.00	0.00	90.22
Giac	21.43	8.85	0.16	69.57
Mupad	1.55	34.32	0.00	64.13

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	10	80.00 %	20.00 %	0.00 %
Maple	10	100.00 %	0.00 %	0.00 %
Maxima	455	71.87 %	14.51 %	13.63 %
Fricas	327	79.20 %	20.80 %	0.00 %
Sympy	581	29.78 %	70.22 %	0.00 %
Giac	448	73.88 %	25.22 %	0.89 %
Mupad	413	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

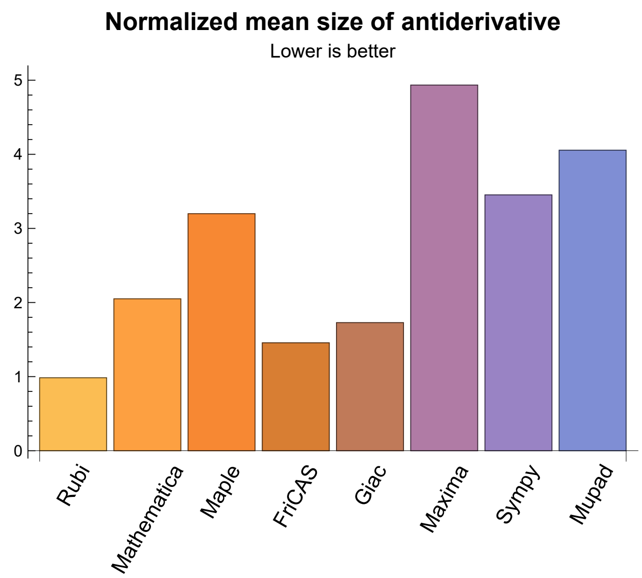
1.3 Performance

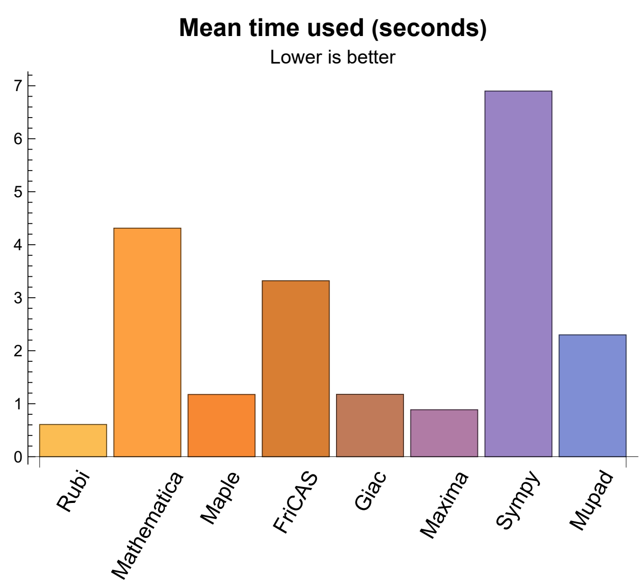
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.61	222.34	0.98	187.50	1.00
Mathematica	4.31	520.33	2.05	221.00	1.09
Maple	1.17	887.71	3.20	423.50	2.51
Maxima	0.89	822.32	4.93	230.00	1.59
Fricas	3.32	248.45	1.46	164.00	1.05
Sympy	6.90	452.87	3.45	264.00	2.50
Giac	1.18	260.45	1.73	181.50	1.38
Mupad	2.30	881.70	4.06	202.00	1.43

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{449, 455, 456, 457, 458, 635, 641, 642, 643, 644}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 151, 152, 157, 158, 159, 165, 194, 195, 196, 200, 201, 318, 332, 338, 339, 340, 395, 399, 401, 402, 403, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 421, 424, 426, 430, 431, 432, 434, 435, 436, 454, 492, 522, 524, 529, 531, 532, 546, 547, 549, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 590, 591, 592, 593, 595, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 612, 614, 615, 616, 620, 621, 622, 624, 625, 626, 627, 628, 629, 630, 633, 640}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

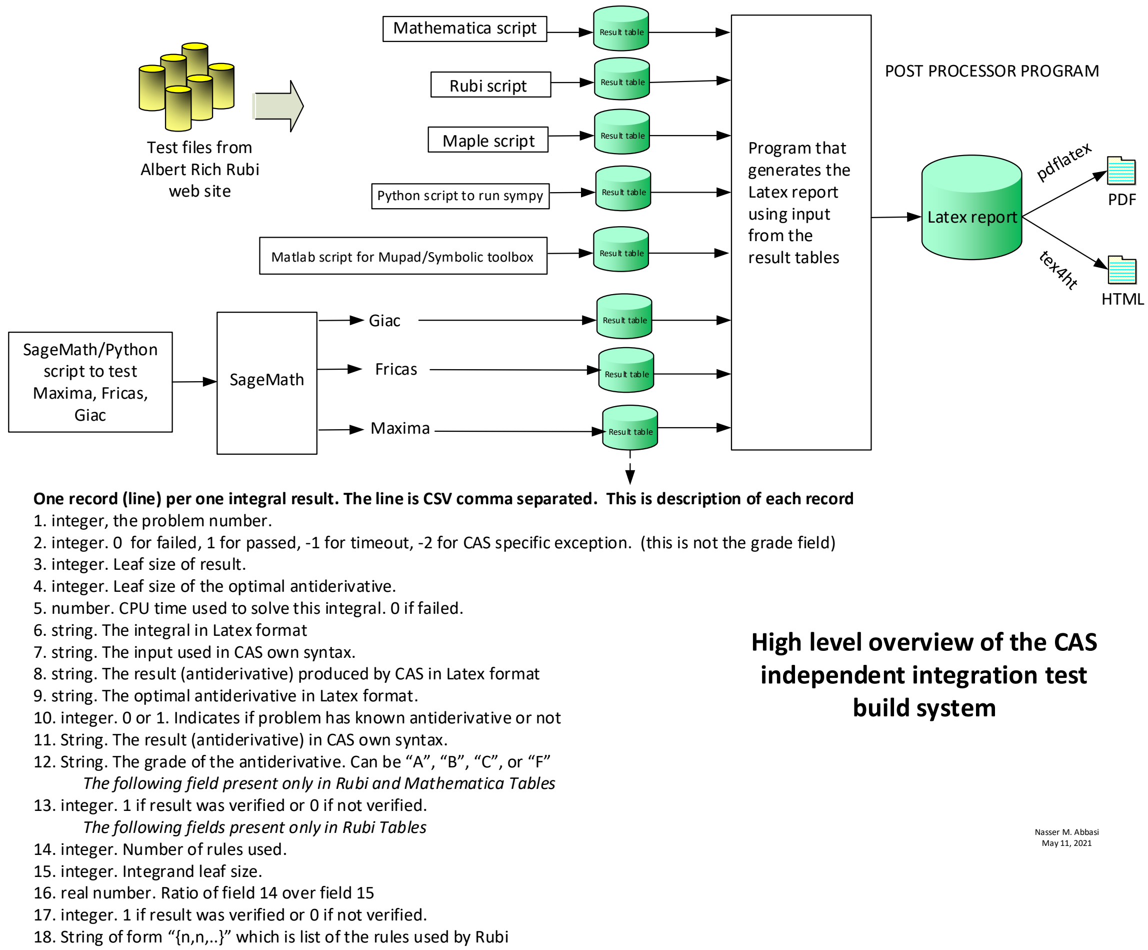
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 24, 26, 27, 28, 29, 30, 31, 35, 36, 37, 49, 51, 60, 61, 62, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 193, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 304, 305, 306, 311, 312, 313, 319, 320, 321, 322, 323, 326, 327, 328, 329, 333, 334, 335, 336, 337, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 397, 398, 399, 422, 423, 424, 438, 439, 440, 449, 450, 451, 452, 453, 455, 456, 457, 458, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 525, 528, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 591, 592, 593, 594, 599, 601, 613, 615, 616, 617, 621, 622, 628, 631, 632, 633, 635, 636, 637, 638, 639, 641, 642, 643, 644 }

B grade: { 16, 17, 23, 25, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 65, 66, 67, 72, 73, 236, 246, 256, 257, 273, 274, 283, 369, 393, 454, 573, 574, 575, 576, 590, 595, 596, 597, 598, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 614, 619, 620, 624, 625, 626, 627, 630, 640 }

C grade: { 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 301, 302, 303, 307, 308, 309, 310, 314, 315, 316, 317, 318, 324, 325, 330, 331, 332, 338, 339, 340, 344, 395, 396, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 520, 521, 522, 524, 526, 527, 529, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 618, 623, 629, 634 }

F grade: { 441, 442, 443, 444, 445, 446, 447, 448, 523, 530 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 41, 42, 43, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 82, 83, 84, 85, 91, 92, 93, 100, 101, 102, 107, 130, 131, 137, 138, 139, 145, 146, 147, 148, 149, 151, 158, 159, 170, 171, 172, 173, 178, 179, 180, 181, 188, 189, 190, 192, 193, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 253, 254, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 300, 322, 323, 329, 330, 342, 343, 344, 367, 368, 387, 390, 391, 422, 437, 438, 439, 449, 455, 456, 457, 458, 461, 464, 467, 468, 469, 474, 475, 477, 478, 479, 480, 481, 483, 484, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 504, 507, 508, 509, 510, 511, 512, 513, 517, 518, 519, 525, 526, 527, 528, 534, 535, 552, 555, 567, 568, 569, 570, 585, 617, 632, 633, 634, 635, 641, 642, 643, 644 }

B grade: { 38, 39, 40, 44, 45, 46, 47, 54, 55, 78, 79, 80, 81, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 134, 135, 136, 140, 141, 142, 143, 144, 150, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 174, 175, 176, 177, 182, 183, 184, 185, 186, 187, 191, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 250, 251, 252, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 289, 295, 296, 297, 298, 299, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 388, 389, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 440, 441, 442, 443, 444, 445, 446, 447, 448, 459, 460, 462, 463, 465, 466, 470, 471, 472, 473, 476, 482, 487, 497, 505, 506, 514, 515, 516, 520, 521, 522, 523, 524, 529, 530, 531, 532, 533, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631 }

C grade: { 341, 386 }

F grade: { 450, 451, 452, 453, 454, 636, 637, 638, 639, 640 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 50, 51, 52, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 82, 83, 84, 85, 86, 91, 92, 93, 94, 104, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 449, 455, 456, 457, 458, 635, 641, 642, 643, 644 }

B grade: { 6, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 56, 63, 64, 79, 80, 81, 87, 88, 95, 97, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 189, 190, 219, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 518 }

C grade: { }

F grade: { 89, 90, 96, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 182, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 516, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 107, 108, 109, 110, 114, 115, 116, 117, 122, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 257, 258, 261, 281, 282, 283, 284, 286, 287, 288, 289, 290, 291, 292, 293, 295, 449, 455, 456, 457, 458, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 635, 641, 642, 643, 644 }

B grade: { 6, 53, 72, 78, 79, 103, 104, 105, 106, 111, 112, 113, 118, 119, 120, 121, 219, 255, 256, 259, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 285, 294 }

C grade: { }

F grade: { 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 279, 280, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 10, 11, 12, 13, 19, 20, 21, 28, 29, 30, 38, 39, 40, 41, 42, 47, 48, 49, 50, 51, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 215, 216, 217, 223, 224, 225, 231, 232, 233, 240, 241, 242, 252, 253, 281, 282, 283, 284, 285, 286, 288, 456, 457, 458, 642, 643, 644 }

B grade: { }

C grade: { }

F grade: { 5, 6, 7, 8, 9, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 52, 53, 54, 55, 62, 63, 64, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 218, 219, 220, 221, 222, 226, 227, 228, 229, 230, 234, 235, 236, 237, 238, 239, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 287, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 82, 83, 84, 85, 91, 92, 93, 100, 101, 102, 103, 107, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 215, 216, 217, 223, 224, 225, 231, 232, 233, 235, 240, 241, 242, 244, 251, 252, 254, 255, 258, 260, 261, 262, 264, 281, 282, 283, 286, 287, 288, 289, 290, 292, 293, 294, 449, 456, 457, 458, 635, 641, 642, 643, 644 }

B grade: { 5, 6, 7, 15, 78, 104, 105, 106, 112, 113, 121, 218, 219, 220, 221, 222, 226, 227, 228, 229, 230, 234, 236, 237, 238, 239, 243, 245, 246, 247, 248, 249, 250, 253, 256, 257, 259, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 285, 291, 295 }

C grade: { 284 }

F grade: { 79, 80, 81, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 114, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 455, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

2.1.8 Mupad

A grade: { 449, 455, 456, 457, 458, 635, 641, 642, 643, 644 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 102, 103, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 171, 172, 173, 179, 180, 181, 188, 189, 190, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 321, 322, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 493, 494, 495, 496, 501, 502, 503, 504, 510, 511, 512, 513 }

C grade: { }

F grade: { 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 182, 183, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 497, 498, 499, 500, 505, 506, 507, 508, 509, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	77	128	124	88	333	112	236
normalized size	1	1.00	0.62	1.02	0.99	0.70	2.66	0.90	1.89
time (sec)	N/A	0.167	0.275	0.066	0.761	1.083	2.159	0.662	1.546

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	107	101	74	252	89	212
normalized size	1	1.00	0.77	1.10	1.04	0.76	2.60	0.92	2.19
time (sec)	N/A	0.149	0.259	0.065	0.372	0.977	1.049	0.909	1.228

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	65	85	79	56	168	68	84
normalized size	1	1.00	0.84	1.10	1.03	0.73	2.18	0.88	1.09
time (sec)	N/A	0.078	0.194	0.060	0.584	1.080	0.533	0.363	0.234

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	57	55	38	94	45	50
normalized size	1	1.00	0.94	1.21	1.17	0.81	2.00	0.96	1.06
time (sec)	N/A	0.021	0.103	0.062	0.382	1.059	0.249	0.497	0.194

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	46	56	47	51	0	79	100
normalized size	1	1.00	1.44	1.75	1.47	1.59	0.00	2.47	3.12
time (sec)	N/A	0.091	0.027	0.110	0.574	0.937	0.000	0.804	0.283

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	43	65	73	79	0	84	100
normalized size	1	1.00	1.34	2.03	2.28	2.47	0.00	2.62	3.12
time (sec)	N/A	0.103	0.021	0.120	0.614	0.625	0.000	0.348	0.307

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	86	95	89	0	124	94
normalized size	1	1.00	1.34	1.54	1.70	1.59	0.00	2.21	1.68
time (sec)	N/A	0.137	0.028	0.136	0.374	0.788	0.000	0.468	0.834

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	56	128	127	105	0	154	126
normalized size	1	1.00	0.65	1.49	1.48	1.22	0.00	1.79	1.47
time (sec)	N/A	0.154	0.352	0.166	0.371	0.732	0.000	2.500	2.067

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	77	171	163	127	0	188	166
normalized size	1	1.00	0.73	1.61	1.54	1.20	0.00	1.77	1.57
time (sec)	N/A	0.165	0.419	0.160	0.459	0.679	0.000	1.686	2.665

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	134	217	216	130	600	166	315
normalized size	1	1.00	0.70	1.14	1.13	0.68	3.14	0.87	1.65
time (sec)	N/A	0.310	0.649	0.078	0.646	0.556	4.339	0.414	1.585

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	108	186	178	110	459	137	277
normalized size	1	1.00	0.68	1.16	1.11	0.69	2.87	0.86	1.73
time (sec)	N/A	0.278	0.449	0.076	0.359	0.681	2.580	0.700	1.500

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	86	154	144	90	338	110	134
normalized size	1	1.00	0.67	1.19	1.12	0.70	2.62	0.85	1.04
time (sec)	N/A	0.176	0.372	0.055	0.412	0.753	1.229	1.067	0.291

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	61	116	110	70	199	85	98
normalized size	1	1.00	0.65	1.23	1.17	0.74	2.12	0.90	1.04
time (sec)	N/A	0.059	0.205	0.065	0.377	0.913	0.637	0.327	0.233

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	96	108	94	79	0	145	141
normalized size	1	1.00	1.17	1.32	1.15	0.96	0.00	1.77	1.72
time (sec)	N/A	0.193	0.204	0.119	0.356	0.889	0.000	0.429	0.337

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	143	107	105	108	0	155	161
normalized size	1	1.00	1.93	1.45	1.42	1.46	0.00	2.09	2.18
time (sec)	N/A	0.210	0.369	0.138	0.424	0.665	0.000	1.057	0.320

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	277	113	142	119	0	154	162
normalized size	1	1.00	3.15	1.28	1.61	1.35	0.00	1.75	1.84
time (sec)	N/A	0.219	1.357	0.146	0.536	0.847	0.000	0.498	0.299

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	451	141	174	125	0	178	145
normalized size	1	1.00	3.99	1.25	1.54	1.11	0.00	1.58	1.28
time (sec)	N/A	0.270	6.355	0.147	0.693	0.603	0.000	0.498	2.082

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	262	187	230	145	0	212	183
normalized size	1	1.00	1.82	1.30	1.60	1.01	0.00	1.47	1.27
time (sec)	N/A	0.304	1.255	0.165	0.695	0.626	0.000	0.870	2.682

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	134	266	262	130	695	166	315
normalized size	1	1.00	0.67	1.32	1.30	0.65	3.46	0.83	1.57
time (sec)	N/A	0.432	0.590	0.072	0.553	0.832	4.834	0.357	1.606

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	108	223	213	110	530	136	277
normalized size	1	1.00	0.70	1.45	1.38	0.71	3.44	0.88	1.80
time (sec)	N/A	0.229	0.457	0.062	0.741	0.868	2.813	0.410	1.505

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	86	176	167	90	371	112	134
normalized size	1	1.00	0.74	1.52	1.44	0.78	3.20	0.97	1.16
time (sec)	N/A	0.098	0.333	0.060	0.443	0.807	1.325	0.358	0.272

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	153	141	102	0	180	178
normalized size	1	1.00	1.02	1.38	1.27	0.92	0.00	1.62	1.60
time (sec)	N/A	0.304	0.282	0.136	0.518	0.811	0.000	0.452	0.421

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	272	145	140	127	0	192	197
normalized size	1	1.00	2.47	1.32	1.27	1.15	0.00	1.75	1.79
time (sec)	N/A	0.308	1.853	0.146	0.630	0.713	0.000	0.919	0.371

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	208	144	165	137	0	192	207
normalized size	1	1.00	1.82	1.26	1.45	1.20	0.00	1.68	1.82
time (sec)	N/A	0.338	2.005	0.161	0.796	0.627	0.000	0.612	0.372

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	786	158	212	141	0	189	209
normalized size	1	1.00	6.29	1.26	1.70	1.13	0.00	1.51	1.67
time (sec)	N/A	0.337	6.406	0.160	0.689	0.651	0.000	0.411	0.332

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	273	188	269	145	0	212	185
normalized size	1	1.00	1.77	1.22	1.75	0.94	0.00	1.38	1.20
time (sec)	N/A	0.418	1.386	0.177	0.353	0.765	0.000	0.759	2.708

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	294	234	337	165	0	246	224
normalized size	1	1.00	1.59	1.26	1.82	0.89	0.00	1.33	1.21
time (sec)	N/A	0.447	1.604	0.172	0.676	0.732	0.000	1.717	2.816

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	156	358	356	150	960	193	353
normalized size	1	1.00	0.65	1.49	1.48	0.62	3.98	0.80	1.46
time (sec)	N/A	0.593	0.870	0.069	0.457	0.877	8.008	0.531	1.637

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	134	306	297	130	765	166	316
normalized size	1	1.00	0.72	1.65	1.61	0.70	4.14	0.90	1.71
time (sec)	N/A	0.304	0.555	0.071	0.600	0.700	4.831	0.997	1.616

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	108	248	236	110	544	139	278
normalized size	1	1.00	0.72	1.65	1.57	0.73	3.63	0.93	1.85
time (sec)	N/A	0.139	0.377	0.059	0.843	0.621	3.024	1.391	1.558

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	138	199	198	118	0	214	188
normalized size	1	1.00	0.91	1.32	1.31	0.78	0.00	1.42	1.25
time (sec)	N/A	0.413	0.402	0.145	0.472	0.649	0.000	1.065	0.673

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	312	190	187	150	0	226	242
normalized size	1	1.00	2.08	1.27	1.25	1.00	0.00	1.51	1.61
time (sec)	N/A	0.453	1.706	0.157	0.402	0.954	0.000	0.893	0.421

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	343	182	199	156	0	230	243
normalized size	1	1.00	2.12	1.12	1.23	0.96	0.00	1.42	1.50
time (sec)	N/A	0.476	4.674	0.172	0.580	0.670	0.000	1.941	0.402

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	380	189	235	159	0	227	254
normalized size	1	1.00	2.30	1.15	1.42	0.96	0.00	1.38	1.54
time (sec)	N/A	0.514	6.219	0.176	0.594	0.937	0.000	0.494	0.407

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	326	204	307	157	0	223	255
normalized size	1	1.00	1.88	1.18	1.77	0.91	0.00	1.29	1.47
time (sec)	N/A	0.523	2.025	0.203	0.415	0.655	0.000	0.677	0.378

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	306	234	376	165	0	246	224
normalized size	1	1.00	1.55	1.18	1.90	0.83	0.00	1.24	1.13
time (sec)	N/A	0.587	1.753	0.199	0.697	1.242	0.000	0.459	2.791

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	358	280	464	185	0	280	262
normalized size	1	1.00	1.56	1.22	2.03	0.81	0.00	1.22	1.14
time (sec)	N/A	0.650	2.354	0.221	0.400	0.730	0.000	0.543	2.839

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	311	351	394	120	1794	181	170
normalized size	1	1.00	2.03	2.29	2.58	0.78	11.73	1.18	1.11
time (sec)	N/A	0.207	0.704	0.089	0.557	0.618	7.462	0.850	0.379

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	249	281	310	98	1161	151	138
normalized size	1	1.00	2.04	2.30	2.54	0.80	9.52	1.24	1.13
time (sec)	N/A	0.172	0.641	0.097	0.703	0.649	4.507	0.493	1.357

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	99	197	211	225	83	665	124	107
normalized size	1	1.10	2.19	2.34	2.50	0.92	7.39	1.38	1.19
time (sec)	N/A	0.123	0.505	0.096	0.769	0.559	3.232	1.061	0.490

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	126	108	143	61	264	78	65
normalized size	1	1.00	2.33	2.00	2.65	1.13	4.89	1.44	1.20
time (sec)	N/A	0.139	0.282	0.096	0.688	0.659	1.707	0.912	0.265

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	72	56	73	43	49	43	30
normalized size	1	1.00	2.12	1.65	2.15	1.26	1.44	1.26	0.88
time (sec)	N/A	0.050	0.149	0.065	0.681	0.711	0.861	0.737	0.198

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	109	78	99	74	0	71	42
normalized size	1	1.00	2.48	1.77	2.25	1.68	0.00	1.61	0.95
time (sec)	N/A	0.078	0.273	0.129	0.820	0.653	0.000	1.040	0.216

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	201	163	196	127	0	110	78
normalized size	1	1.00	2.91	2.36	2.84	1.84	0.00	1.59	1.13
time (sec)	N/A	0.156	1.341	0.144	0.446	0.567	0.000	0.349	0.290

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	289	252	282	156	0	157	119
normalized size	1	1.00	2.70	2.36	2.64	1.46	0.00	1.47	1.11
time (sec)	N/A	0.169	3.636	0.160	0.391	0.657	0.000	0.366	0.373

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	490	340	368	168	0	182	152
normalized size	1	1.00	3.74	2.60	2.81	1.28	0.00	1.39	1.16
time (sec)	N/A	0.180	4.606	0.171	0.474	0.552	0.000	0.396	0.639

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	369	322	372	154	1425	192	189
normalized size	1	1.00	2.17	1.89	2.19	0.91	8.38	1.13	1.11
time (sec)	N/A	0.322	0.703	0.101	0.713	1.032	10.807	0.733	0.335

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	315	252	283	138	843	164	152
normalized size	1	1.00	2.14	1.71	1.93	0.94	5.73	1.12	1.03
time (sec)	N/A	0.341	0.844	0.102	0.768	0.785	6.980	0.544	0.288

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	137	149	191	117	411	119	105
normalized size	1	1.00	1.38	1.51	1.93	1.18	4.15	1.20	1.06
time (sec)	N/A	0.276	0.765	0.095	0.757	0.753	4.164	1.604	0.259

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	153	97	120	91	105	86	65
normalized size	1	1.00	2.19	1.39	1.71	1.30	1.50	1.23	0.93
time (sec)	N/A	0.155	0.373	0.084	0.626	0.805	2.334	0.677	0.218

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	76	60	93	58	94	60	45
normalized size	1	1.00	1.17	0.92	1.43	0.89	1.45	0.92	0.69
time (sec)	N/A	0.054	0.194	0.067	0.367	0.767	1.742	1.925	0.189

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	170	119	145	131	0	113	74
normalized size	1	1.00	2.15	1.51	1.84	1.66	0.00	1.43	0.94
time (sec)	N/A	0.180	0.541	0.139	0.494	0.805	0.000	0.435	0.229

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	264	205	244	207	0	155	123
normalized size	1	1.00	2.47	1.92	2.28	1.93	0.00	1.45	1.15
time (sec)	N/A	0.295	1.853	0.148	0.533	0.754	0.000	0.450	0.280

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	496	294	336	228	0	198	165
normalized size	1	1.00	3.26	1.93	2.21	1.50	0.00	1.30	1.09
time (sec)	N/A	0.314	3.430	0.178	0.501	0.871	0.000	1.295	0.301

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	609	382	425	247	0	226	203
normalized size	1	1.00	3.40	2.13	2.37	1.38	0.00	1.26	1.13
time (sec)	N/A	0.365	5.600	0.176	0.617	1.584	0.000	0.972	0.341

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	491	362	412	205	1584	228	238
normalized size	1	1.00	2.25	1.66	1.89	0.94	7.27	1.05	1.09
time (sec)	N/A	0.515	1.086	0.094	0.740	0.654	25.372	1.904	0.333

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	435	292	322	190	966	200	203
normalized size	1	1.00	2.25	1.51	1.67	0.98	5.01	1.04	1.05
time (sec)	N/A	0.468	0.923	0.103	1.004	0.587	15.908	1.941	0.267

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	361	189	231	165	496	155	152
normalized size	1	1.00	2.46	1.29	1.57	1.12	3.37	1.05	1.03
time (sec)	N/A	0.457	0.963	0.097	0.700	0.624	9.802	0.358	0.263

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	241	137	160	137	148	120	134
normalized size	1	1.00	2.08	1.18	1.38	1.18	1.28	1.03	1.16
time (sec)	N/A	0.321	0.624	0.083	0.539	0.883	5.798	0.504	0.384

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	135	64	115	93	117	75	66
normalized size	1	1.00	1.32	0.63	1.13	0.91	1.15	0.74	0.65
time (sec)	N/A	0.188	0.363	0.082	0.807	0.759	3.722	1.900	0.208

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	96	64	115	93	114	75	66
normalized size	1	1.00	0.94	0.63	1.13	0.91	1.12	0.74	0.65
time (sec)	N/A	0.079	0.293	0.071	0.464	0.539	2.537	0.448	0.195

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	197	159	187	185	0	148	130
normalized size	1	1.00	1.68	1.36	1.60	1.58	0.00	1.26	1.11
time (sec)	N/A	0.311	1.039	0.157	0.570	0.725	0.000	4.309	0.246

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	482	245	286	272	0	190	168
normalized size	1	1.00	3.32	1.69	1.97	1.88	0.00	1.31	1.16
time (sec)	N/A	0.469	3.279	0.165	0.485	0.664	0.000	0.440	0.280

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	610	334	377	295	0	233	216
normalized size	1	1.00	3.11	1.70	1.92	1.51	0.00	1.19	1.10
time (sec)	N/A	0.541	5.224	0.182	0.561	0.822	0.000	2.213	0.277

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	555	332	364	238	1085	233	259
normalized size	1	1.00	2.42	1.45	1.59	1.04	4.74	1.02	1.13
time (sec)	N/A	0.672	1.424	0.094	0.776	0.775	33.041	0.519	0.307

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	481	229	271	213	578	188	201
normalized size	1	1.00	2.60	1.24	1.46	1.15	3.12	1.02	1.09
time (sec)	N/A	0.679	0.978	0.096	0.656	0.754	21.682	0.791	0.388

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	329	177	201	180	192	155	162
normalized size	1	1.00	2.14	1.15	1.31	1.17	1.25	1.01	1.05
time (sec)	N/A	0.498	0.829	0.084	0.760	0.907	13.270	0.401	0.345

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	193	90	175	124	182	117	86
normalized size	1	1.00	1.42	0.66	1.29	0.91	1.34	0.86	0.63
time (sec)	N/A	0.348	0.500	0.079	0.362	0.744	9.237	0.624	0.247

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	163	88	174	124	178	117	84
normalized size	1	1.00	1.18	0.64	1.26	0.90	1.29	0.85	0.61
time (sec)	N/A	0.215	0.426	0.087	0.394	0.621	6.689	0.585	0.248

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	109	88	175	125	177	117	87
normalized size	1	1.00	0.79	0.64	1.27	0.91	1.28	0.85	0.63
time (sec)	N/A	0.138	0.373	0.068	0.454	0.538	4.875	0.928	0.243

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	239	199	228	236	0	182	199
normalized size	1	1.00	1.63	1.35	1.55	1.61	0.00	1.24	1.35
time (sec)	N/A	0.466	1.594	0.150	0.416	0.783	0.000	0.971	0.361

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	595	285	326	337	0	224	236
normalized size	1	1.00	3.40	1.63	1.86	1.93	0.00	1.28	1.35
time (sec)	N/A	0.672	5.726	0.164	0.651	0.957	0.000	0.411	0.280

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	798	374	419	360	0	267	273
normalized size	1	1.00	3.44	1.61	1.81	1.55	0.00	1.15	1.18
time (sec)	N/A	0.688	6.509	0.186	0.618	0.588	0.000	1.451	0.287

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	103	121	145	99	0	194	-1
normalized size	1	1.00	0.55	0.65	0.78	0.53	0.00	1.04	-0.01
time (sec)	N/A	0.304	0.705	0.402	1.035	0.609	0.000	0.527	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	80	102	118	82	0	165	-1
normalized size	1	1.00	0.56	0.71	0.82	0.57	0.00	1.15	-0.01
time (sec)	N/A	0.265	0.370	0.439	0.802	0.629	0.000	1.444	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	64	83	88	64	0	113	-1
normalized size	1	1.00	0.63	0.82	0.87	0.63	0.00	1.12	-0.01
time (sec)	N/A	0.202	0.211	0.335	1.248	0.803	0.000	2.889	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	46	62	57	47	0	83	-1
normalized size	1	1.00	0.74	1.00	0.92	0.76	0.00	1.34	-0.02
time (sec)	N/A	0.059	0.087	0.379	1.286	0.672	0.000	0.373	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	210	21	127	0	1884	-1
normalized size	1	1.00	1.00	3.18	0.32	1.92	0.00	28.55	-0.02
time (sec)	N/A	0.138	0.099	1.158	0.608	0.662	0.000	15.589	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	85	642	710	153	0	0	-1
normalized size	1	1.00	1.25	9.44	10.44	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.222	1.219	1.388	0.801	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	101	991	3352	178	0	0	-1
normalized size	1	1.00	0.86	8.47	28.65	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.883	1.328	7.760	0.777	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	129	1311	5021	197	0	0	-1
normalized size	1	1.00	0.81	8.19	31.38	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.293	2.014	1.475	7.421	0.775	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	125	142	185	125	0	250	-1
normalized size	1	1.00	0.53	0.61	0.79	0.53	0.00	1.07	-0.00
time (sec)	N/A	0.529	1.073	0.401	0.855	1.154	0.000	2.996	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	103	123	154	107	0	245	-1
normalized size	1	1.00	0.54	0.65	0.81	0.57	0.00	1.30	-0.01
time (sec)	N/A	0.446	0.599	0.415	0.791	0.589	0.000	1.842	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	81	104	123	88	0	164	-1
normalized size	1	1.00	0.59	0.75	0.89	0.64	0.00	1.19	-0.01
time (sec)	N/A	0.250	0.400	0.452	0.933	0.832	0.000	0.496	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	65	85	93	69	0	161	-1
normalized size	1	1.00	0.64	0.84	0.92	0.68	0.00	1.59	-0.01
time (sec)	N/A	0.087	0.202	0.340	0.880	0.604	0.000	0.540	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	85	272	39	149	0	0	-1
normalized size	1	1.00	0.81	2.59	0.37	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.265	0.218	1.125	1.270	0.627	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	98	696	1315	172	0	0	-1
normalized size	1	1.00	0.95	6.76	12.77	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.284	0.342	1.171	0.995	0.656	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	109	991	3339	182	0	0	-1
normalized size	1	1.00	0.92	8.33	28.06	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.325	0.583	1.223	1.217	0.621	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	132	1310	0	202	0	0	-1
normalized size	1	1.00	0.80	7.99	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.400	1.010	1.360	0.000	0.594	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	151	1631	0	220	0	0	-1
normalized size	1	1.00	0.72	7.80	0.00	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.485	1.603	1.409	0.000	0.863	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	127	142	207	137	0	319	-1
normalized size	1	1.00	0.54	0.60	0.87	0.58	0.00	1.35	-0.00
time (sec)	N/A	0.648	1.133	0.339	1.982	0.672	0.000	0.928	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	105	123	172	116	0	225	-1
normalized size	1	1.00	0.60	0.70	0.98	0.66	0.00	1.29	-0.01
time (sec)	N/A	0.280	0.753	0.467	1.019	0.809	0.000	0.861	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	83	104	139	95	0	225	-1
normalized size	1	1.00	0.60	0.75	1.01	0.69	0.00	1.63	-0.01
time (sec)	N/A	0.109	0.356	0.331	1.054	0.472	0.000	1.390	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	104	311	61	177	0	0	-1
normalized size	1	1.00	0.73	2.19	0.43	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.414	0.423	1.278	0.868	0.441	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	120	756	8114	202	0	0	-1
normalized size	1	1.00	0.83	5.25	56.35	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.447	0.563	1.210	2.004	1.207	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	126	1016	0	204	0	0	-1
normalized size	1	1.00	0.81	6.51	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.474	0.675	1.192	0.000	0.608	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	131	1310	7994	212	0	0	-1
normalized size	1	1.00	0.80	7.99	48.74	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.526	1.113	1.457	8.018	0.616	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	152	1630	0	232	0	0	-1
normalized size	1	1.00	0.73	7.80	0.00	1.11	0.00	0.00	-0.00
time (sec)	N/A	0.609	1.799	1.500	0.000	0.718	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	176	1951	0	252	0	0	-1
normalized size	1	1.00	0.69	7.68	0.00	0.99	0.00	0.00	-0.00
time (sec)	N/A	0.713	2.249	1.620	0.000	1.118	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	111	281	0	184	0	181	-1
normalized size	1	1.00	0.55	1.39	0.00	0.91	0.00	0.90	-0.00
time (sec)	N/A	0.578	0.722	0.830	0.000	0.605	0.000	1.842	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	94	240	0	166	0	158	-1
normalized size	1	1.00	0.59	1.51	0.00	1.04	0.00	0.99	-0.01
time (sec)	N/A	0.384	0.365	0.732	0.000	0.686	0.000	1.975	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	78	194	0	149	0	113	160
normalized size	1	1.00	0.66	1.64	0.00	1.26	0.00	0.96	1.36
time (sec)	N/A	0.210	0.173	0.673	0.000	0.634	0.000	1.558	0.379

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	60	160	0	135	0	88	112
normalized size	1	1.00	0.77	2.05	0.00	1.73	0.00	1.13	1.44
time (sec)	N/A	0.071	0.074	0.676	0.000	0.759	0.000	2.922	0.350

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	72	268	91	171	0	168	-1
normalized size	1	1.00	0.79	2.95	1.00	1.88	0.00	1.85	-0.01
time (sec)	N/A	0.166	0.084	1.405	1.129	0.679	0.000	2.080	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	95	810	0	259	0	321	-1
normalized size	1	1.00	0.80	6.81	0.00	2.18	0.00	2.70	-0.01
time (sec)	N/A	0.308	0.363	1.457	0.000	0.930	0.000	3.891	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	114	1240	0	284	0	535	-1
normalized size	1	1.00	0.69	7.52	0.00	1.72	0.00	3.24	-0.01
time (sec)	N/A	0.480	0.845	1.543	0.000	0.814	0.000	3.635	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	167	448	0	241	0	254	-1
normalized size	1	1.00	0.64	1.72	0.00	0.92	0.00	0.97	-0.00
time (sec)	N/A	0.786	1.166	0.801	0.000	0.960	0.000	2.399	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	142	407	0	224	0	202	-1
normalized size	1	1.00	0.66	1.88	0.00	1.04	0.00	0.94	-0.00
time (sec)	N/A	0.595	0.943	0.782	0.000	0.897	0.000	3.100	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	97	327	0	205	0	168	-1
normalized size	1	1.00	0.57	1.91	0.00	1.20	0.00	0.98	-0.01
time (sec)	N/A	0.420	0.767	0.796	0.000	0.720	0.000	1.931	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	104	256	0	189	0	131	-1
normalized size	1	1.00	0.88	2.17	0.00	1.60	0.00	1.11	-0.01
time (sec)	N/A	0.223	0.424	0.742	0.000	0.603	0.000	1.703	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	63	220	0	172	0	101	-1
normalized size	1	1.00	0.72	2.53	0.00	1.98	0.00	1.16	-0.01
time (sec)	N/A	0.077	0.202	0.720	0.000	0.844	0.000	1.302	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	131	374	0	281	0	214	-1
normalized size	1	1.00	1.03	2.94	0.00	2.21	0.00	1.69	-0.01
time (sec)	N/A	0.315	0.710	1.537	0.000	0.850	0.000	2.864	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	141	1051	0	339	0	373	-1
normalized size	1	1.00	0.83	6.18	0.00	1.99	0.00	2.19	-0.01
time (sec)	N/A	0.521	1.165	1.570	0.000	1.311	0.000	3.227	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	205	1540	0	361	0	0	-1
normalized size	1	1.00	0.93	6.97	0.00	1.63	0.00	0.00	-0.00
time (sec)	N/A	0.705	1.629	1.679	0.000	0.733	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	139	467	0	270	0	257	-1
normalized size	1	1.00	0.53	1.79	0.00	1.03	0.00	0.98	-0.00
time (sec)	N/A	0.799	1.643	0.820	0.000	0.797	0.000	2.685	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	117	397	0	254	0	204	-1
normalized size	1	1.00	0.54	1.84	0.00	1.18	0.00	0.94	-0.00
time (sec)	N/A	0.611	1.110	0.838	0.000	0.683	0.000	4.721	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	100	327	0	237	0	181	-1
normalized size	1	1.00	0.59	1.93	0.00	1.40	0.00	1.07	-0.01
time (sec)	N/A	0.420	0.779	0.862	0.000	0.736	0.000	2.606	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	87	292	0	223	0	134	-1
normalized size	1	1.00	0.69	2.32	0.00	1.77	0.00	1.06	-0.01
time (sec)	N/A	0.230	0.619	0.678	0.000	0.772	0.000	1.306	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	80	292	0	223	0	134	-1
normalized size	1	1.00	0.63	2.32	0.00	1.77	0.00	1.06	-0.01
time (sec)	N/A	0.103	0.517	0.751	0.000	0.543	0.000	3.840	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	126	445	0	339	0	250	-1
normalized size	1	1.00	0.77	2.71	0.00	2.07	0.00	1.52	-0.01
time (sec)	N/A	0.466	1.652	1.582	0.000	0.811	0.000	3.502	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	142	1122	0	404	0	409	-1
normalized size	1	1.00	0.69	5.42	0.00	1.95	0.00	1.98	-0.00
time (sec)	N/A	0.715	3.471	1.713	0.000	1.478	0.000	3.949	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	178	1610	0	428	0	0	-1
normalized size	1	1.00	0.67	6.10	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	0.923	6.202	1.989	0.000	0.665	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	914	411	0	0	0	0	177
normalized size	1	1.00	5.75	2.58	0.00	0.00	0.00	0.00	1.11
time (sec)	N/A	0.199	6.351	1.066	0.000	0.803	0.000	0.000	1.085

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	872	383	0	0	0	0	166
normalized size	1	1.00	6.61	2.90	0.00	0.00	0.00	0.00	1.26
time (sec)	N/A	0.175	6.274	0.980	0.000	1.138	0.000	0.000	0.609

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	830	355	0	0	0	0	128
normalized size	1	1.00	8.22	3.51	0.00	0.00	0.00	0.00	1.27
time (sec)	N/A	0.158	6.259	1.024	0.000	0.894	0.000	0.000	0.522

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	784	321	0	0	0	0	79
normalized size	1	1.00	11.20	4.59	0.00	0.00	0.00	0.00	1.13
time (sec)	N/A	0.144	6.273	1.244	0.000	0.917	0.000	0.000	0.526

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	783	240	0	0	0	0	90
normalized size	1	1.00	11.86	3.64	0.00	0.00	0.00	0.00	1.36
time (sec)	N/A	0.148	6.312	1.089	0.000	0.705	0.000	0.000	0.964

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	813	426	0	0	0	0	150
normalized size	1	1.00	8.56	4.48	0.00	0.00	0.00	0.00	1.58
time (sec)	N/A	0.169	6.361	2.330	0.000	0.837	0.000	0.000	1.300

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	865	661	0	0	0	0	177
normalized size	1	1.00	6.55	5.01	0.00	0.00	0.00	0.00	1.34
time (sec)	N/A	0.177	6.428	3.029	0.000	0.884	0.000	0.000	1.606

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	944	413	0	0	0	0	266
normalized size	1	1.00	4.87	2.13	0.00	0.00	0.00	0.00	1.37
time (sec)	N/A	0.319	6.298	1.103	0.000	0.714	0.000	0.000	1.071

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	898	385	0	0	0	0	231
normalized size	1	1.00	5.58	2.39	0.00	0.00	0.00	0.00	1.43
time (sec)	N/A	0.289	6.266	1.107	0.000	1.103	0.000	0.000	1.012

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	852	357	0	0	0	0	153
normalized size	1	1.00	6.76	2.83	0.00	0.00	0.00	0.00	1.21
time (sec)	N/A	0.274	6.302	0.989	0.000	0.842	0.000	0.000	1.004

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	623	388	0	0	0	0	134
normalized size	1	1.00	5.28	3.29	0.00	0.00	0.00	0.00	1.14
time (sec)	N/A	0.269	6.370	1.008	0.000	0.845	0.000	0.000	1.136

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	624	513	0	0	0	0	196
normalized size	1	1.00	5.20	4.28	0.00	0.00	0.00	0.00	1.63
time (sec)	N/A	0.280	6.448	1.232	0.000	0.614	0.000	0.000	1.692

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	883	741	0	0	0	0	229
normalized size	1	1.00	5.55	4.66	0.00	0.00	0.00	0.00	1.44
time (sec)	N/A	0.306	6.533	3.343	0.000	1.022	0.000	0.000	1.974

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	925	851	0	0	0	0	235
normalized size	1	1.00	4.77	4.39	0.00	0.00	0.00	0.00	1.21
time (sec)	N/A	0.337	6.628	4.040	0.000	0.877	0.000	0.000	2.299

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	990	441	0	0	0	0	360
normalized size	1	1.00	4.18	1.86	0.00	0.00	0.00	0.00	1.52
time (sec)	N/A	0.479	6.323	1.113	0.000	1.048	0.000	0.000	1.310

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	944	413	0	0	0	0	323
normalized size	1	1.00	4.63	2.02	0.00	0.00	0.00	0.00	1.58
time (sec)	N/A	0.447	6.297	1.059	0.000	0.934	0.000	0.000	1.074

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	898	385	0	0	0	0	255
normalized size	1	1.00	5.25	2.25	0.00	0.00	0.00	0.00	1.49
time (sec)	N/A	0.427	6.358	1.018	0.000	0.981	0.000	0.000	0.998

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	888	519	0	0	0	0	229
normalized size	1	1.00	5.25	3.07	0.00	0.00	0.00	0.00	1.36
time (sec)	N/A	0.434	6.458	1.185	0.000	0.858	0.000	0.000	1.043

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	879	654	0	0	0	0	251
normalized size	1	1.00	5.46	4.06	0.00	0.00	0.00	0.00	1.56
time (sec)	N/A	0.430	6.536	1.271	0.000	0.733	0.000	0.000	1.628

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	890	916	0	0	0	0	287
normalized size	1	1.00	5.20	5.36	0.00	0.00	0.00	0.00	1.68
time (sec)	N/A	0.462	6.618	3.388	0.000	0.825	0.000	0.000	2.503

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	925	929	0	0	0	0	307
normalized size	1	1.00	4.53	4.55	0.00	0.00	0.00	0.00	1.50
time (sec)	N/A	0.491	6.654	4.328	0.000	0.852	0.000	0.000	2.666

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	967	1178	0	0	0	0	552
normalized size	1	1.00	4.08	4.97	0.00	0.00	0.00	0.00	2.33
time (sec)	N/A	0.521	6.712	4.922	0.000	0.680	0.000	0.000	3.031

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	1182	281	0	0	0	0	-1
normalized size	1	1.00	7.58	1.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	6.623	1.159	0.000	0.936	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	1129	262	0	0	0	0	-1
normalized size	1	1.00	9.18	2.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	6.555	1.072	0.000	0.839	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	1098	244	0	0	0	0	-1
normalized size	1	1.00	12.92	2.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	6.455	1.033	0.000	0.783	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1094	243	0	0	0	0	-1
normalized size	1	1.00	13.18	2.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	6.485	1.148	0.000	0.841	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	1130	319	0	0	0	0	-1
normalized size	1	1.00	9.50	2.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	6.708	2.357	0.000	0.650	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	1167	493	0	0	0	0	-1
normalized size	1	1.00	7.63	3.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.195	7.089	3.156	0.000	0.891	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	1262	465	0	0	0	0	-1
normalized size	1	1.00	6.22	2.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.407	6.859	1.046	0.000	0.900	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	1218	435	0	0	0	0	-1
normalized size	1	1.00	7.34	2.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.389	6.758	1.142	0.000	1.164	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	1184	421	0	0	0	0	-1
normalized size	1	1.00	8.71	3.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.314	6.657	1.171	0.000	1.214	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	815	350	0	0	0	0	-1
normalized size	1	1.00	6.74	2.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.277	6.520	1.308	0.000	1.086	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	815	350	0	0	0	0	-1
normalized size	1	1.00	6.74	2.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.342	6.548	1.101	0.000	0.849	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	1217	494	0	0	0	0	-1
normalized size	1	1.00	7.24	2.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.359	6.812	1.346	0.000	0.997	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	1258	750	0	0	0	0	-1
normalized size	1	1.00	6.26	3.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	7.427	3.954	0.000	0.954	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	1346	493	0	0	0	0	-1
normalized size	1	1.00	5.30	1.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.548	7.157	1.418	0.000	1.220	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	1306	465	0	0	0	0	-1
normalized size	1	1.00	5.96	2.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.518	6.988	1.128	0.000	0.943	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	1273	451	0	0	0	0	-1
normalized size	1	1.00	6.77	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.478	6.896	1.249	0.000	0.948	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	1265	451	0	0	0	0	-1
normalized size	1	1.00	7.03	2.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.470	6.817	1.158	0.000	1.183	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	1264	451	0	0	0	0	-1
normalized size	1	1.00	7.10	2.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.464	6.717	1.275	0.000	1.079	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	1265	451	0	0	0	0	-1
normalized size	1	1.00	6.95	2.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.480	6.801	1.227	0.000	0.758	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	1305	685	0	0	0	0	-1
normalized size	1	1.00	5.90	3.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.519	7.113	1.662	0.000	1.160	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	1346	876	0	0	0	0	-1
normalized size	1	1.00	5.30	3.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.603	7.809	1.676	0.000	1.011	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	135	428	8220	151	0	0	-1
normalized size	1	1.00	0.61	1.94	37.19	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.350	1.030	0.319	2.976	0.998	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	118	356	2981	134	0	0	-1
normalized size	1	1.00	0.67	2.02	16.94	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.298	0.546	0.392	1.788	0.996	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	100	284	1851	117	0	0	-1
normalized size	1	1.00	0.76	2.17	14.13	0.89	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.299	0.281	1.371	1.026	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	83	164	939	97	0	0	-1
normalized size	1	1.00	1.06	2.10	12.04	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.159	0.253	1.463	1.010	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	86	109	245	109	0	0	-1
normalized size	1	1.00	1.13	1.43	3.22	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.170	0.252	0.994	1.071	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	57	62	289	67	0	0	112
normalized size	1	1.00	0.67	0.73	3.40	0.79	0.00	0.00	1.32
time (sec)	N/A	0.163	0.152	0.222	0.958	0.784	0.000	0.000	1.557

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	78	86	428	86	0	0	194
normalized size	1	1.00	0.60	0.66	3.29	0.66	0.00	0.00	1.49
time (sec)	N/A	0.218	0.253	0.235	0.726	1.023	0.000	0.000	3.253

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	102	108	522	104	0	0	479
normalized size	1	1.00	0.58	0.62	2.98	0.59	0.00	0.00	2.74
time (sec)	N/A	0.286	0.396	0.237	1.037	0.963	0.000	0.000	6.226

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	136	429	8904	162	0	0	-1
normalized size	1	1.00	0.60	1.89	39.22	0.71	0.00	0.00	-0.00
time (sec)	N/A	0.504	1.122	0.246	3.180	1.166	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	119	357	3023	144	0	0	-1
normalized size	1	1.00	0.66	1.98	16.79	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.412	0.685	0.348	2.088	1.348	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	101	283	1884	125	0	0	-1
normalized size	1	1.00	0.76	2.13	14.17	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.330	0.384	0.307	1.678	1.241	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	107	300	1801	135	0	0	-1
normalized size	1	1.00	0.85	2.38	14.29	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.331	0.331	0.283	1.601	1.151	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	106	211	1124	133	0	0	-1
normalized size	1	1.00	0.85	1.69	8.99	1.06	0.00	0.00	-0.01
time (sec)	N/A	0.315	0.386	0.289	0.961	0.877	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	80	87	344	88	0	0	195
normalized size	1	1.00	0.60	0.65	2.57	0.66	0.00	0.00	1.46
time (sec)	N/A	0.340	0.327	0.227	0.870	1.355	0.000	0.000	3.094

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	102	109	481	107	0	0	236
normalized size	1	1.00	0.56	0.60	2.66	0.59	0.00	0.00	1.30
time (sec)	N/A	0.431	0.538	0.235	0.955	0.689	0.000	0.000	6.720

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	124	131	573	126	0	0	289
normalized size	1	1.00	0.54	0.57	2.51	0.55	0.00	0.00	1.27
time (sec)	N/A	0.505	0.691	0.270	1.176	0.851	0.000	0.000	7.023

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	159	503	0	194	0	0	-1
normalized size	1	1.00	0.58	1.84	0.00	0.71	0.00	0.00	-0.00
time (sec)	N/A	0.709	1.978	0.260	0.000	1.196	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	137	431	9415	174	0	0	-1
normalized size	1	1.00	0.60	1.90	41.48	0.77	0.00	0.00	-0.00
time (sec)	N/A	0.713	1.247	0.204	2.895	1.283	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	121	357	3071	154	0	0	-1
normalized size	1	1.00	0.67	1.98	17.06	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.547	0.778	0.336	2.095	1.134	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	126	336	2080	164	0	0	-1
normalized size	1	1.00	0.71	1.89	11.69	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.552	0.686	0.314	1.887	0.878	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	130	484	2370	169	0	0	-1
normalized size	1	1.00	0.75	2.80	13.70	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.533	0.713	0.279	1.676	1.012	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	130	306	1548	161	0	0	-1
normalized size	1	1.00	0.76	1.78	9.00	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.506	0.783	0.388	1.180	0.988	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	104	111	396	114	0	0	551
normalized size	1	1.00	0.57	0.61	2.19	0.63	0.00	0.00	3.04
time (sec)	N/A	0.552	0.631	0.229	1.308	0.989	0.000	0.000	6.864

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	126	133	533	135	0	0	647
normalized size	1	1.00	0.55	0.58	2.34	0.59	0.00	0.00	2.84
time (sec)	N/A	0.703	0.858	0.278	0.964	1.035	0.000	0.000	8.236

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	147	155	626	156	0	0	773
normalized size	1	1.00	0.53	0.56	2.28	0.57	0.00	0.00	2.81
time (sec)	N/A	0.714	0.986	0.310	0.991	0.947	0.000	0.000	7.322

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	348	346	0	184	0	0	-1
normalized size	1	1.00	1.83	1.82	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.595	1.971	0.319	0.000	8.758	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	222	216	0	168	0	0	-1
normalized size	1	1.00	1.57	1.53	0.00	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.397	1.275	0.262	0.000	4.159	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	82	149	0	96	0	0	-1
normalized size	1	1.00	0.82	1.49	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.242	0.146	0.230	0.000	3.879	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	203	230	0	143	0	0	-1
normalized size	1	1.00	2.05	2.32	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.192	1.636	0.260	0.000	1.178	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	627	383	0	163	0	0	-1
normalized size	1	1.00	4.42	2.70	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.336	6.815	0.337	0.000	0.753	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	1728	519	0	180	0	0	-1
normalized size	1	1.00	9.24	2.78	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.609	7.817	0.357	0.000	1.166	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	362	379	0	237	0	0	-1
normalized size	1	1.00	1.84	1.92	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.636	2.202	0.296	0.000	15.060	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	226	298	0	203	0	0	-1
normalized size	1	1.00	1.56	2.06	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.403	1.907	0.277	0.000	10.313	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	212	246	0	164	0	0	-1
normalized size	1	1.00	1.98	2.30	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.216	1.155	0.265	0.000	0.923	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	423	299	0	201	0	0	-1
normalized size	1	1.00	2.71	1.92	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.384	3.869	0.284	0.000	1.055	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	1054	443	0	221	0	0	-1
normalized size	1	1.00	5.19	2.18	0.00	1.09	0.00	0.00	-0.00
time (sec)	N/A	0.555	6.806	0.343	0.000	0.984	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	376	647	0	302	0	0	-1
normalized size	1	1.00	1.53	2.63	0.00	1.23	0.00	0.00	-0.00
time (sec)	N/A	0.837	3.533	0.335	0.000	28.556	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	246	515	0	267	0	0	-1
normalized size	1	1.00	1.27	2.65	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.582	2.173	0.307	0.000	24.442	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	198	413	0	215	0	0	-1
normalized size	1	1.00	1.29	2.68	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.377	1.512	0.321	0.000	0.888	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	200	413	0	217	0	0	-1
normalized size	1	1.00	1.28	2.65	0.00	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.439	1.473	0.274	0.000	1.132	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	217	443	0	248	0	0	-1
normalized size	1	1.00	1.07	2.18	0.00	1.22	0.00	0.00	-0.00
time (sec)	N/A	0.567	2.771	0.296	0.000	0.840	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	239	571	0	270	0	0	-1
normalized size	1	1.00	0.96	2.28	0.00	1.08	0.00	0.00	-0.00
time (sec)	N/A	0.752	3.584	0.237	0.000	1.012	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	396	887	0	368	0	0	-1
normalized size	1	1.00	1.35	3.03	0.00	1.26	0.00	0.00	-0.00
time (sec)	N/A	1.044	5.817	0.385	0.000	43.470	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	266	703	0	327	0	0	-1
normalized size	1	1.00	1.10	2.92	0.00	1.36	0.00	0.00	-0.00
time (sec)	N/A	0.765	3.235	0.372	0.000	24.593	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	217	549	0	266	0	0	-1
normalized size	1	1.00	1.08	2.73	0.00	1.32	0.00	0.00	-0.00
time (sec)	N/A	0.587	2.357	0.339	0.000	0.603	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	215	549	0	264	0	0	-1
normalized size	1	1.00	1.07	2.73	0.00	1.31	0.00	0.00	-0.00
time (sec)	N/A	0.579	2.144	0.336	0.000	1.746	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	216	549	0	266	0	0	-1
normalized size	1	1.00	1.06	2.70	0.00	1.31	0.00	0.00	-0.00
time (sec)	N/A	0.591	2.155	0.326	0.000	1.054	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	240	581	0	298	0	0	-1
normalized size	1	1.00	0.96	2.32	0.00	1.19	0.00	0.00	-0.00
time (sec)	N/A	0.804	2.925	0.375	0.000	0.818	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	262	715	0	319	0	0	-1
normalized size	1	1.00	0.88	2.41	0.00	1.07	0.00	0.00	-0.00
time (sec)	N/A	1.032	5.405	0.243	0.000	0.826	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	91	107	101	81	252	89	117
normalized size	1	1.00	0.87	1.02	0.96	0.77	2.40	0.85	1.11
time (sec)	N/A	0.170	0.223	0.054	1.372	1.001	1.044	0.413	0.474

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	75	85	79	60	168	68	84
normalized size	1	1.00	0.89	1.01	0.94	0.71	2.00	0.81	1.00
time (sec)	N/A	0.090	0.162	0.051	0.305	1.034	0.514	0.313	0.397

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	51	57	55	42	94	45	50
normalized size	1	1.00	0.98	1.10	1.06	0.81	1.81	0.87	0.96
time (sec)	N/A	0.023	0.085	0.046	0.605	0.795	0.248	0.303	0.356

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	46	56	47	54	0	79	100
normalized size	1	1.00	1.31	1.60	1.34	1.54	0.00	2.26	2.86
time (sec)	N/A	0.105	0.027	0.090	0.305	1.233	0.000	0.669	0.479

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	65	73	85	0	84	114
normalized size	1	1.00	1.23	1.86	2.09	2.43	0.00	2.40	3.26
time (sec)	N/A	0.114	0.014	0.103	0.307	0.620	0.000	0.431	0.484

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	75	86	95	96	0	151	104
normalized size	1	1.00	1.23	1.41	1.56	1.57	0.00	2.48	1.70
time (sec)	N/A	0.148	0.020	0.113	0.466	0.527	0.000	0.459	1.273

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	67	128	127	115	0	210	145
normalized size	1	1.00	0.72	1.38	1.37	1.24	0.00	2.26	1.56
time (sec)	N/A	0.163	0.282	0.117	0.473	0.994	0.000	0.407	2.574

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	85	171	163	136	0	304	194
normalized size	1	1.00	0.75	1.50	1.43	1.19	0.00	2.67	1.70
time (sec)	N/A	0.179	0.620	0.140	1.299	0.652	0.000	0.512	3.863

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	146	184	176	142	459	156	307
normalized size	1	1.00	0.77	0.97	0.93	0.75	2.43	0.83	1.62
time (sec)	N/A	0.311	0.483	0.052	0.829	0.647	2.471	0.404	3.933

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	118	152	142	114	338	124	169
normalized size	1	1.00	0.69	0.89	0.84	0.67	1.99	0.73	0.99
time (sec)	N/A	0.234	0.455	0.049	0.531	0.748	1.187	0.376	0.512

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	90	114	108	85	199	93	115
normalized size	1	1.00	0.84	1.07	1.01	0.79	1.86	0.87	1.07
time (sec)	N/A	0.094	0.225	0.050	0.308	0.657	0.590	0.469	0.453

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	120	120	92	87	0	178	169
normalized size	1	1.00	1.40	1.40	1.07	1.01	0.00	2.07	1.97
time (sec)	N/A	0.176	0.233	0.099	1.213	0.726	0.000	0.450	0.693

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	109	104	103	117	0	152	169
normalized size	1	1.00	1.82	1.73	1.72	1.95	0.00	2.53	2.82
time (sec)	N/A	0.169	0.497	0.116	0.551	0.610	0.000	0.728	0.876

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	67	133	140	136	0	190	176
normalized size	1	1.00	0.84	1.66	1.75	1.70	0.00	2.38	2.20
time (sec)	N/A	0.200	0.275	0.117	0.409	0.614	0.000	0.517	0.978

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	92	174	172	150	0	294	227
normalized size	1	1.00	0.79	1.50	1.48	1.29	0.00	2.53	1.96
time (sec)	N/A	0.270	0.472	0.132	0.442	1.059	0.000	0.820	3.660

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	120	241	228	180	0	478	314
normalized size	1	1.00	0.77	1.54	1.46	1.15	0.00	3.06	2.01
time (sec)	N/A	0.293	0.724	0.134	0.578	0.915	0.000	0.546	3.868

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	289	270	266	211	721	230	352
normalized size	1	1.00	1.07	1.00	0.99	0.78	2.68	0.86	1.31
time (sec)	N/A	0.507	0.685	0.096	0.569	0.955	4.621	0.466	1.108

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	176	227	217	174	551	188	277
normalized size	1	1.00	0.72	0.93	0.89	0.72	2.27	0.77	1.14
time (sec)	N/A	0.333	0.718	0.051	0.319	0.697	2.758	0.512	0.778

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	140	180	171	136	386	148	202
normalized size	1	1.00	0.82	1.05	1.00	0.80	2.26	0.87	1.18
time (sec)	N/A	0.197	0.417	0.055	0.649	1.164	1.317	0.591	0.569

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	159	207	145	131	0	314	1924
normalized size	1	1.00	1.16	1.51	1.06	0.96	0.00	2.29	14.04
time (sec)	N/A	0.324	0.396	0.109	0.324	0.537	0.000	0.648	1.910

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	217	168	144	152	0	234	236
normalized size	1	1.00	1.66	1.28	1.10	1.16	0.00	1.79	1.80
time (sec)	N/A	0.332	0.680	0.127	0.319	0.965	0.000	1.292	1.351

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	277	172	169	167	0	239	249
normalized size	1	1.00	2.23	1.39	1.36	1.35	0.00	1.93	2.01
time (sec)	N/A	0.339	2.103	0.130	1.539	0.902	0.000	0.569	1.556

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	108	223	216	189	0	336	526
normalized size	1	1.00	0.74	1.54	1.49	1.30	0.00	2.32	3.63
time (sec)	N/A	0.346	0.593	0.141	0.648	0.521	0.000	0.441	1.948

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	140	290	273	211	0	586	395
normalized size	1	1.00	0.74	1.54	1.45	1.12	0.00	3.12	2.10
time (sec)	N/A	0.458	0.842	0.144	0.347	0.901	0.000	0.693	3.946

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	181	382	341	249	0	722	470
normalized size	1	1.00	0.77	1.62	1.44	1.06	0.00	3.06	1.99
time (sec)	N/A	0.489	3.260	0.152	0.512	0.859	0.000	0.887	3.894

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	408	368	366	289	1017	313	436
normalized size	1	1.00	1.11	1.01	1.00	0.79	2.78	0.86	1.19
time (sec)	N/A	0.838	0.878	0.061	0.388	1.603	8.196	0.520	2.635

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	333	316	307	243	811	263	403
normalized size	1	1.00	1.02	0.97	0.94	0.75	2.50	0.81	1.24
time (sec)	N/A	0.509	1.161	0.055	0.480	0.854	4.990	0.471	1.371

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	263	258	246	197	580	212	307
normalized size	1	1.00	1.09	1.07	1.02	0.82	2.41	0.88	1.27
time (sec)	N/A	0.338	0.656	0.054	0.579	1.136	2.833	0.507	0.878

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	210	319	208	183	0	603	369
normalized size	1	1.00	1.05	1.60	1.04	0.92	0.00	3.02	1.84
time (sec)	N/A	0.547	0.610	0.124	0.339	1.521	0.000	0.641	1.419

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	257	255	197	196	0	371	2522
normalized size	1	1.00	1.32	1.31	1.01	1.01	0.00	1.90	12.93
time (sec)	N/A	0.570	1.074	0.127	0.322	1.588	0.000	1.232	2.269

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	310	236	209	202	0	526	330
normalized size	1	1.00	1.48	1.13	1.00	0.97	0.00	2.52	1.58
time (sec)	N/A	0.615	1.847	0.147	1.209	0.698	0.000	0.689	2.314

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	415	262	245	219	0	387	636
normalized size	1	1.00	2.10	1.32	1.24	1.11	0.00	1.95	3.21
time (sec)	N/A	0.580	5.954	0.162	0.910	1.620	0.000	0.680	2.831

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	160	338	317	250	0	635	1969
normalized size	1	1.00	0.74	1.56	1.47	1.16	0.00	2.94	9.12
time (sec)	N/A	0.597	1.082	0.160	0.578	0.844	0.000	0.694	2.976

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	198	431	386	281	0	850	555
normalized size	1	1.00	0.74	1.61	1.45	1.05	0.00	3.18	2.08
time (sec)	N/A	0.723	4.249	0.150	0.593	0.657	0.000	0.544	3.879

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	244	550	474	327	0	1186	706
normalized size	1	1.00	0.75	1.70	1.46	1.01	0.00	3.66	2.18
time (sec)	N/A	0.802	2.774	0.161	0.440	1.100	0.000	0.932	3.751

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	152	641	0	541	0	360	4568
normalized size	1	1.00	0.85	3.60	0.00	3.04	0.00	2.02	25.66
time (sec)	N/A	0.493	0.468	0.087	0.000	1.106	0.000	0.488	5.088

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	121	367	0	426	0	227	3761
normalized size	1	1.00	0.90	2.74	0.00	3.18	0.00	1.69	28.07
time (sec)	N/A	0.287	0.313	0.077	0.000	1.148	0.000	0.492	3.998

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	85	172	0	322	3225	142	541
normalized size	1	1.00	0.96	1.93	0.00	3.62	36.24	1.60	6.08
time (sec)	N/A	0.174	0.210	0.076	0.000	0.944	118.447	0.449	1.117

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	113	0	242	524	296	344
normalized size	1	1.00	1.01	1.69	0.00	3.61	7.82	4.42	5.13
time (sec)	N/A	0.078	0.121	0.061	0.000	0.720	24.724	0.661	1.720

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	112	135	0	304	0	127	342
normalized size	1	1.00	1.47	1.78	0.00	4.00	0.00	1.67	4.50
time (sec)	N/A	0.115	0.163	0.122	0.000	1.911	0.000	0.467	1.600

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	129	228	0	460	0	175	675
normalized size	1	1.00	1.30	2.30	0.00	4.65	0.00	1.77	6.82
time (sec)	N/A	0.183	0.565	0.148	0.000	0.950	0.000	0.966	1.988

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	300	410	0	589	0	269	4051
normalized size	1	1.00	2.10	2.87	0.00	4.12	0.00	1.88	28.33
time (sec)	N/A	0.490	1.782	0.158	0.000	8.205	0.000	0.772	4.207

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	422	688	0	729	0	412	4696
normalized size	1	1.00	2.26	3.68	0.00	3.90	0.00	2.20	25.11
time (sec)	N/A	0.770	2.272	0.175	0.000	2.717	0.000	0.616	4.895

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	184	643	0	965	0	338	6744
normalized size	1	1.00	0.70	2.44	0.00	3.67	0.00	1.29	25.64
time (sec)	N/A	0.659	1.072	0.090	0.000	1.566	0.000	0.831	9.209

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	147	445	0	788	0	1116	3276
normalized size	1	1.00	0.95	2.87	0.00	5.08	0.00	7.20	21.14
time (sec)	N/A	0.440	0.846	0.095	0.000	1.255	0.000	3.219	5.170

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	119	320	0	552	0	199	3775
normalized size	1	1.00	0.98	2.62	0.00	4.52	0.00	1.63	30.94
time (sec)	N/A	0.241	0.552	0.081	0.000	0.731	0.000	0.440	7.786

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	97	234	0	379	0	159	113
normalized size	1	1.00	0.97	2.34	0.00	3.79	0.00	1.59	1.13
time (sec)	N/A	0.089	0.347	0.071	0.000	0.783	0.000	0.414	0.732

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	191	342	0	684	0	223	3763
normalized size	1	1.00	1.44	2.57	0.00	5.14	0.00	1.68	28.29
time (sec)	N/A	0.282	0.628	0.143	0.000	7.687	0.000	1.489	7.811

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	240	502	0	1088	0	404	5464
normalized size	1	1.00	1.27	2.66	0.00	5.76	0.00	2.14	28.91
time (sec)	N/A	0.673	1.947	0.177	0.000	21.474	0.000	0.800	8.516

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	438	690	0	1329	0	378	6692
normalized size	1	1.00	1.62	2.56	0.00	4.92	0.00	1.40	24.79
time (sec)	N/A	0.975	6.268	0.192	0.000	33.864	0.000	1.231	9.279

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	734	1504	0	1812	0	2712	10598
normalized size	1	1.00	1.84	3.78	0.00	4.55	0.00	6.81	26.63
time (sec)	N/A	1.724	3.605	0.097	0.000	1.227	0.000	2.317	12.006

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	232	1301	0	1561	0	543	5542
normalized size	1	1.00	0.83	4.65	0.00	5.58	0.00	1.94	19.79
time (sec)	N/A	1.222	2.162	0.094	0.000	0.869	0.000	1.177	7.663

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	204	1023	0	1152	0	455	6923
normalized size	1	1.00	0.97	4.85	0.00	5.46	0.00	2.16	32.81
time (sec)	N/A	0.564	1.359	0.096	0.000	0.893	0.000	1.212	9.949

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	172	886	0	740	0	391	248
normalized size	1	1.00	0.96	4.92	0.00	4.11	0.00	2.17	1.38
time (sec)	N/A	0.290	0.856	0.079	0.000	0.775	0.000	0.706	3.742

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	157	886	0	742	0	390	248
normalized size	1	1.00	0.96	5.40	0.00	4.52	0.00	2.38	1.51
time (sec)	N/A	0.188	0.667	0.070	0.000	0.676	0.000	0.786	3.544

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	269	1045	0	1400	0	481	6913
normalized size	1	1.00	1.26	4.88	0.00	6.54	0.00	2.25	32.30
time (sec)	N/A	0.706	1.333	0.156	0.000	37.506	0.000	1.766	9.627

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	352	1358	0	2100	0	574	9312
normalized size	1	1.00	1.18	4.54	0.00	7.02	0.00	1.92	31.14
time (sec)	N/A	1.764	5.971	0.183	0.000	76.797	0.000	6.994	12.905

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	507	1551	0	2416	0	1395	10547
normalized size	1	1.00	1.26	3.86	0.00	6.01	0.00	3.47	26.24
time (sec)	N/A	2.236	2.956	0.198	0.000	121.638	0.000	1.921	12.558

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	1278	2787	0	2567	0	966	7823
normalized size	1	1.00	3.12	6.81	0.00	6.28	0.00	2.36	19.13
time (sec)	N/A	5.175	6.646	0.095	0.000	1.404	0.000	3.431	12.515

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	717	2158	0	1857	0	813	9733
normalized size	1	1.00	2.38	7.17	0.00	6.17	0.00	2.70	32.34
time (sec)	N/A	1.207	3.305	0.096	0.000	1.308	0.000	2.192	12.575

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	251	1726	0	1220	0	689	440
normalized size	1	1.00	0.92	6.30	0.00	4.45	0.00	2.51	1.61
time (sec)	N/A	0.636	1.325	0.085	0.000	0.873	0.000	1.031	4.147

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	252	1883	0	1232	0	722	451
normalized size	1	1.00	0.96	7.16	0.00	4.68	0.00	2.75	1.71
time (sec)	N/A	0.530	1.159	0.082	0.000	1.444	0.000	1.892	4.032

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	227	1727	0	1228	0	691	440
normalized size	1	1.00	0.96	7.29	0.00	5.18	0.00	2.92	1.86
time (sec)	N/A	0.478	2.371	0.077	0.000	1.133	0.000	1.217	4.002

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	368	2180	0	2269	0	837	9727
normalized size	1	1.00	1.22	7.24	0.00	7.54	0.00	2.78	32.32
time (sec)	N/A	1.509	1.733	0.214	0.000	136.531	0.000	2.401	12.810

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	549	2844	0	0	0	996	13119
normalized size	1	1.00	1.31	6.77	0.00	0.00	0.00	2.37	31.24
time (sec)	N/A	6.221	3.270	0.211	0.000	0.000	0.000	6.388	18.114

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	547	547	781	3042	0	0	0	1090	14398
normalized size	1	1.00	1.43	5.56	0.00	0.00	0.00	1.99	26.32
time (sec)	N/A	7.304	5.288	0.255	0.000	0.000	0.000	1.830	13.936

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	0	25	56	25	24
normalized size	1	1.00	1.00	0.82	0.00	0.89	2.00	0.89	0.86
time (sec)	N/A	0.016	0.008	0.085	0.000	0.686	1.260	0.421	0.479

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	28	0	24	68	33	50
normalized size	1	1.00	0.89	1.04	0.00	0.89	2.52	1.22	1.85
time (sec)	N/A	0.015	0.020	0.084	0.000	0.844	0.886	0.375	0.867

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	12	0	11	31	11	11
normalized size	1	1.00	2.09	1.09	0.00	1.00	2.82	1.00	1.00
time (sec)	N/A	0.009	0.008	0.063	0.000	0.932	0.602	0.349	0.472

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	3	2	10	3
normalized size	1	1.00	1.00	1.33	0.00	1.00	0.67	3.33	1.00
time (sec)	N/A	0.001	0.000	0.000	0.000	0.511	0.127	0.334	0.446

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	20	0	31	39	47	16
normalized size	1	1.00	1.00	1.67	0.00	2.58	3.25	3.92	1.33
time (sec)	N/A	0.007	0.003	0.073	0.000	1.092	3.815	0.449	0.487

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	19	32	11	30
normalized size	1	1.00	1.00	1.09	0.00	1.73	2.91	1.00	2.73
time (sec)	N/A	0.012	0.005	0.081	0.000	0.693	3.097	0.419	0.474

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	40	0	64	0	52	73
normalized size	1	1.00	1.00	1.11	0.00	1.78	0.00	1.44	2.03
time (sec)	N/A	0.020	0.008	0.092	0.000	0.655	0.000	0.587	0.855

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	25	0	32	42	25	39
normalized size	1	1.00	0.86	0.89	0.00	1.14	1.50	0.89	1.39
time (sec)	N/A	0.016	0.040	0.103	0.000	1.147	17.647	0.574	0.520

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	98	229	0	350	0	185	173
normalized size	1	1.00	0.86	2.01	0.00	3.07	0.00	1.62	1.52
time (sec)	N/A	0.217	0.251	0.098	0.000	0.587	0.000	0.425	1.167

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	105	0	281	0	128	193
normalized size	1	1.00	0.92	1.33	0.00	3.56	0.00	1.62	2.44
time (sec)	N/A	0.132	0.138	0.089	0.000	0.645	0.000	0.460	0.894

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	69	0	231	0	245	101
normalized size	1	1.00	0.97	1.13	0.00	3.79	0.00	4.02	1.66
time (sec)	N/A	0.067	0.080	0.080	0.000	0.869	0.000	0.647	0.799

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	45	0	177	0	78	44
normalized size	1	1.00	0.98	0.90	0.00	3.54	0.00	1.56	0.88
time (sec)	N/A	0.035	0.038	0.048	0.000	0.718	0.000	0.522	0.504

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	103	91	0	292	0	122	101
normalized size	1	1.00	1.47	1.30	0.00	4.17	0.00	1.74	1.44
time (sec)	N/A	0.081	0.081	0.093	0.000	0.941	0.000	0.501	0.758

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	116	139	0	398	0	155	326
normalized size	1	1.00	1.32	1.58	0.00	4.52	0.00	1.76	3.70
time (sec)	N/A	0.145	0.374	0.110	0.000	1.095	0.000	0.791	1.058

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	239	273	0	487	0	221	1099
normalized size	1	1.00	1.94	2.22	0.00	3.96	0.00	1.80	8.93
time (sec)	N/A	0.346	1.053	0.133	0.000	0.892	0.000	0.746	1.826

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	292	1635	0	0	0	0	-1
normalized size	1	1.00	0.76	4.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.785	1.572	1.582	0.000	0.586	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	232	1305	0	0	0	0	-1
normalized size	1	1.00	0.77	4.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.542	1.010	1.641	0.000	0.700	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	179	993	0	0	0	0	-1
normalized size	1	1.00	0.77	4.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.408	0.891	1.644	0.000	0.715	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	146	600	0	0	0	0	-1
normalized size	1	1.00	0.85	3.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.594	1.648	0.000	0.901	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	107	247	0	0	0	0	-1
normalized size	1	1.00	0.60	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.360	2.401	1.212	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	372	746	0	0	0	0	-1
normalized size	1	1.00	1.75	3.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.607	10.516	2.254	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	420	1290	0	0	0	0	-1
normalized size	1	1.00	1.44	4.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.955	4.256	3.455	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	635	2213	0	0	0	0	-1
normalized size	1	1.00	1.68	5.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.339	6.547	4.468	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	291	1635	0	0	0	0	-1
normalized size	1	1.00	0.77	4.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.733	1.534	1.979	0.000	1.044	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	233	1305	0	0	0	0	-1
normalized size	1	1.00	0.78	4.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.528	1.072	1.581	0.000	0.884	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	203	993	0	0	0	0	-1
normalized size	1	1.00	0.90	4.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.784	1.646	0.000	0.738	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	406	738	0	0	0	0	-1
normalized size	1	1.00	1.72	3.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.708	2.583	1.517	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	398	1167	0	0	0	0	-1
normalized size	1	1.00	1.72	5.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.686	2.536	1.595	0.000	8.694	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	422	1403	0	0	0	0	-1
normalized size	1	1.00	1.43	4.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.058	4.953	3.674	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	634	2327	0	0	0	0	-1
normalized size	1	1.00	1.69	6.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.450	6.711	4.807	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	357	1983	0	0	0	0	-1
normalized size	1	1.00	0.77	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.931	2.119	1.879	0.000	1.109	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	291	1635	0	0	0	0	-1
normalized size	1	1.00	0.78	4.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.796	1.600	1.807	0.000	1.369	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	254	1305	0	0	0	0	-1
normalized size	1	1.00	0.88	4.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.516	1.085	1.470	0.000	1.193	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	453	1067	0	0	0	0	-1
normalized size	1	1.00	1.55	3.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.014	2.912	1.467	0.000	3.345	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	442	1563	0	0	0	0	-1
normalized size	1	1.00	1.49	5.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.109	3.937	1.793	0.000	6.422	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	451	1742	0	0	0	0	-1
normalized size	1	1.00	1.43	5.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.055	5.875	3.890	0.000	8.149	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	486	2438	0	0	0	0	-1
normalized size	1	1.00	1.29	6.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.433	6.052	4.607	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	465	465	729	3548	0	0	0	0	-1
normalized size	1	1.00	1.57	7.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.845	6.770	6.827	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	230	1305	0	0	0	0	-1
normalized size	1	1.00	0.72	4.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.619	1.058	1.646	0.000	0.962	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	180	993	0	0	0	0	-1
normalized size	1	1.00	0.73	4.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.430	0.907	1.732	0.000	0.555	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	154	671	0	0	0	0	199
normalized size	1	1.00	0.84	3.67	0.00	0.00	0.00	0.00	1.09
time (sec)	N/A	0.292	0.691	1.749	0.000	0.658	0.000	0.000	0.800

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	93	249	0	0	0	0	135
normalized size	1	1.00	0.72	1.92	0.00	0.00	0.00	0.00	1.04
time (sec)	N/A	0.126	3.294	1.221	0.000	0.634	0.000	0.000	0.885

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	81	194	0	0	0	0	-1
normalized size	1	1.00	0.69	1.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.313	0.203	1.186	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	320	639	0	0	0	0	-1
normalized size	1	1.00	1.48	2.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.655	6.525	1.941	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	420	1182	0	0	0	0	-1
normalized size	1	1.00	1.40	3.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.955	6.021	3.275	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	304	1308	0	0	0	0	-1
normalized size	1	1.00	0.79	3.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.727	1.865	5.105	0.000	1.203	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	189	954	0	0	0	0	-1
normalized size	1	1.00	0.72	3.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.486	1.509	4.671	0.000	1.622	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	170	515	0	0	0	0	-1
normalized size	1	1.00	0.83	2.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	0.821	4.191	0.000	1.195	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	151	428	0	0	0	0	-1
normalized size	1	1.00	0.82	2.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.565	3.451	0.000	1.163	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	460	429	0	0	0	0	-1
normalized size	1	1.00	2.42	2.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.508	3.980	3.072	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	482	908	0	0	0	0	-1
normalized size	1	1.00	1.59	3.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.991	5.738	4.302	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	678	1564	0	0	0	0	-1
normalized size	1	1.00	1.70	3.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.431	6.936	5.079	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	372	1746	0	0	0	0	-1
normalized size	1	1.00	0.68	3.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.187	4.434	8.744	0.000	1.635	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	334	1389	0	0	0	0	-1
normalized size	1	1.00	0.81	3.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.805	2.885	7.258	0.000	1.471	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	274	950	0	0	0	0	-1
normalized size	1	1.00	0.83	2.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	2.330	5.832	0.000	1.050	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	224	860	0	0	0	0	-1
normalized size	1	1.00	0.73	2.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.472	2.026	5.282	0.000	0.783	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	193	750	0	0	0	0	-1
normalized size	1	1.00	0.70	2.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	1.648	5.005	0.000	0.627	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	743	854	0	0	0	0	-1
normalized size	1	1.00	2.13	2.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.099	6.812	5.595	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	750	1341	0	0	0	0	-1
normalized size	1	1.00	1.72	3.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.481	7.263	7.934	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	820	2000	0	0	0	0	-1
normalized size	1	1.00	1.54	3.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.934	7.951	9.707	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	76	0	0	0	0	-1
normalized size	1	1.00	1.00	1.31	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.050	0.130	0.000	0.419	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	167	0	0	0	0	-1
normalized size	1	1.00	1.00	2.83	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.146	0.079	1.132	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	84	218	0	0	0	0	-1
normalized size	1	1.00	0.78	2.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.214	1.711	0.000	2.139	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	403	377	0	0	0	0	-1
normalized size	1	1.00	2.25	2.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.424	4.868	1.808	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	125	451	0	0	0	0	177
normalized size	1	1.00	0.74	2.65	0.00	0.00	0.00	0.00	1.04
time (sec)	N/A	0.207	1.293	1.472	0.000	0.898	0.000	0.000	1.349

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	103	413	0	0	0	0	166
normalized size	1	1.00	0.74	2.95	0.00	0.00	0.00	0.00	1.19
time (sec)	N/A	0.184	0.861	1.388	0.000	0.527	0.000	0.000	1.156

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	86	371	0	0	0	0	128
normalized size	1	1.00	0.80	3.44	0.00	0.00	0.00	0.00	1.19
time (sec)	N/A	0.168	0.414	1.240	0.000	0.755	0.000	0.000	1.011

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	326	0	0	0	0	85
normalized size	1	1.00	0.89	4.35	0.00	0.00	0.00	0.00	1.13
time (sec)	N/A	0.146	0.231	1.284	0.000	0.731	0.000	0.000	0.993

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	244	0	0	0	0	96
normalized size	1	1.00	0.90	3.44	0.00	0.00	0.00	0.00	1.35
time (sec)	N/A	0.154	0.355	1.413	0.000	0.697	0.000	0.000	1.440

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	107	428	0	0	0	0	150
normalized size	1	1.00	1.04	4.16	0.00	0.00	0.00	0.00	1.46
time (sec)	N/A	0.170	0.477	3.239	0.000	0.444	0.000	0.000	1.965

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	134	663	0	0	0	0	177
normalized size	1	1.00	0.96	4.74	0.00	0.00	0.00	0.00	1.26
time (sec)	N/A	0.188	0.832	4.016	0.000	0.607	0.000	0.000	2.394

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	196	666	0	0	0	0	275
normalized size	1	1.00	0.74	2.52	0.00	0.00	0.00	0.00	1.04
time (sec)	N/A	0.381	1.769	1.289	0.000	0.496	0.000	0.000	1.535

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	167	610	0	0	0	0	264
normalized size	1	1.00	0.75	2.74	0.00	0.00	0.00	0.00	1.18
time (sec)	N/A	0.330	1.412	1.306	0.000	0.631	0.000	0.000	1.350

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	139	548	0	0	0	0	229
normalized size	1	1.00	0.76	3.01	0.00	0.00	0.00	0.00	1.26
time (sec)	N/A	0.316	1.127	1.441	0.000	0.568	0.000	0.000	1.342

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	106	487	0	0	0	0	177
normalized size	1	1.00	0.76	3.48	0.00	0.00	0.00	0.00	1.26
time (sec)	N/A	0.266	0.600	1.189	0.000	0.482	0.000	0.000	1.340

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	404	0	0	0	0	158
normalized size	1	1.00	0.84	3.34	0.00	0.00	0.00	0.00	1.31
time (sec)	N/A	0.246	0.641	1.391	0.000	0.512	0.000	0.000	1.570

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	105	677	0	0	0	0	194
normalized size	1	1.00	0.83	5.37	0.00	0.00	0.00	0.00	1.54
time (sec)	N/A	0.305	1.182	3.017	0.000	0.455	0.000	0.000	2.288

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	175	750	0	0	0	0	227
normalized size	1	1.00	1.02	4.36	0.00	0.00	0.00	0.00	1.32
time (sec)	N/A	0.351	1.111	4.042	0.000	0.445	0.000	0.000	2.616

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	235	825	0	0	0	0	364
normalized size	1	1.00	0.77	2.70	0.00	0.00	0.00	0.00	1.19
time (sec)	N/A	0.545	1.992	1.404	0.000	0.759	0.000	0.000	1.738

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	197	745	0	0	0	0	328
normalized size	1	1.00	0.77	2.92	0.00	0.00	0.00	0.00	1.29
time (sec)	N/A	0.496	1.229	1.575	0.000	0.613	0.000	0.000	1.539

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	158	664	0	0	0	0	275
normalized size	1	1.00	0.77	3.24	0.00	0.00	0.00	0.00	1.34
time (sec)	N/A	0.478	1.304	1.500	0.000	0.693	0.000	0.000	1.432

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	150	867	0	0	0	0	248
normalized size	1	1.00	0.74	4.29	0.00	0.00	0.00	0.00	1.23
time (sec)	N/A	0.464	1.153	1.632	0.000	0.806	0.000	0.000	1.458

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	165	1212	0	0	0	0	255
normalized size	1	1.00	0.86	6.31	0.00	0.00	0.00	0.00	1.33
time (sec)	N/A	0.465	1.114	3.795	0.000	0.732	0.000	0.000	2.339

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	176	997	0	0	0	0	291
normalized size	1	1.00	0.86	4.89	0.00	0.00	0.00	0.00	1.43
time (sec)	N/A	0.482	2.212	4.332	0.000	0.588	0.000	0.000	3.587

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	260	1074	0	0	0	0	-1
normalized size	1	1.00	1.43	5.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.820	2.469	1.637	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	207	786	0	0	0	0	-1
normalized size	1	1.00	1.51	5.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.513	1.438	1.640	0.000	114.015	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	128	295	0	0	0	0	-1
normalized size	1	1.00	1.44	3.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.910	1.414	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	58	217	0	0	0	0	-1
normalized size	1	1.00	0.95	3.56	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.144	0.208	1.365	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	206	327	0	0	0	0	-1
normalized size	1	1.00	2.40	3.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.309	2.482	2.674	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	260	468	0	0	0	0	-1
normalized size	1	1.00	1.73	3.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.770	2.289	3.722	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	318	1066	0	0	0	0	-1
normalized size	1	1.00	1.05	3.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.933	3.242	4.849	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	280	849	0	0	0	0	-1
normalized size	1	1.00	1.25	3.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.627	2.736	3.957	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	260	808	0	0	0	0	-1
normalized size	1	1.00	1.31	4.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.540	2.378	3.532	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	274	721	0	0	0	0	-1
normalized size	1	1.00	1.37	3.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.618	2.677	3.361	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	316	883	0	0	0	0	-1
normalized size	1	1.00	1.23	3.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.923	4.219	4.253	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	427	1031	0	0	0	0	-1
normalized size	1	1.00	1.24	2.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.292	6.919	6.823	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	390	1977	0	0	0	0	-1
normalized size	1	1.00	1.06	5.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.013	5.059	6.777	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	360	1937	0	0	0	0	-1
normalized size	1	1.00	1.05	5.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.990	3.730	6.060	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	365	1850	0	0	0	0	-1
normalized size	1	1.00	1.08	5.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.919	4.550	5.815	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	383	1744	0	0	0	0	-1
normalized size	1	1.00	1.11	5.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.060	4.887	5.772	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	458	2002	0	0	0	0	-1
normalized size	1	1.00	1.09	4.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.474	5.507	7.374	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	523	523	570	2158	0	0	0	0	-1
normalized size	1	1.00	1.09	4.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.953	7.275	11.609	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	203	0	0	0	0	-1
normalized size	1	1.00	0.93	4.61	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.039	1.176	0.000	2.563	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	37	180	0	0	0	0	-1
normalized size	1	1.00	0.84	4.09	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.036	1.103	0.000	1.126	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	134	0	0	0	0	-1
normalized size	1	1.00	1.00	7.88	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.011	0.015	0.955	0.000	1.648	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	0	0	0	0	-1
normalized size	1	1.00	1.00	1.12	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.011	0.018	0.008	0.000	1.259	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	102	0	0	0	0	-1
normalized size	1	1.00	1.00	2.55	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.053	1.174	0.000	2.571	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	37	214	0	0	0	0	-1
normalized size	1	1.00	0.84	4.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.062	1.286	0.000	0.455	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	159	517	0	0	0	0	-1
normalized size	1	1.00	1.37	4.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.403	1.578	1.510	0.000	97.865	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	82	228	0	0	0	0	-1
normalized size	1	1.00	1.05	2.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.110	1.350	0.000	94.557	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	189	0	0	0	0	-1
normalized size	1	1.00	0.89	3.44	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.103	0.056	1.354	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	151	0	0	0	0	-1
normalized size	1	1.00	1.00	5.03	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.049	0.065	0.994	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	196	355	0	0	0	0	-1
normalized size	1	1.00	2.45	4.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.247	2.676	1.477	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	211	452	0	0	0	0	-1
normalized size	1	1.00	1.59	3.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.560	4.115	3.338	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	1224	2949	0	0	0	0	-1
normalized size	1	1.00	2.19	5.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.507	6.327	0.439	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	1175	2052	0	0	0	0	-1
normalized size	1	1.00	2.48	4.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.042	21.106	0.257	0.000	3.760	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	408	1693	0	0	0	0	-1
normalized size	1	1.00	1.06	4.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.713	11.361	0.407	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	273	1687	0	0	0	0	-1
normalized size	1	1.00	0.78	4.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.504	12.743	0.294	0.000	69.788	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	407	1727	0	0	0	0	-1
normalized size	1	1.00	1.43	6.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.505	13.501	0.227	0.000	1.030	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	1315	2481	0	0	0	0	-1
normalized size	1	1.00	3.76	7.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.825	6.371	0.332	0.000	1.799	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	1408	3428	0	0	0	0	-1
normalized size	1	1.00	3.25	7.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.176	6.465	0.440	0.000	1.026	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	670	670	1284	4048	0	0	0	0	-1
normalized size	1	1.00	1.92	6.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.096	6.413	0.631	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	566	566	1227	3139	0	0	0	0	-1
normalized size	1	1.00	2.17	5.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.663	6.321	0.394	0.000	176.209	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	472	472	1198	2430	0	0	0	0	-1
normalized size	1	1.00	2.54	5.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.147	6.349	0.393	0.000	7.272	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	1196	2185	0	0	0	0	-1
normalized size	1	1.00	2.66	4.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.176	6.350	0.237	0.000	3.310	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	1236	2318	0	0	0	0	-1
normalized size	1	1.00	2.95	5.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.858	6.365	0.396	0.000	2.511	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	1314	2666	0	0	0	0	-1
normalized size	1	1.00	3.72	7.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.924	6.446	0.325	0.000	0.912	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	1407	3413	0	0	0	0	-1
normalized size	1	1.00	3.25	7.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.346	6.544	0.533	0.000	0.889	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	1515	4392	0	0	0	0	-1
normalized size	1	1.00	2.90	8.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.885	6.630	0.709	0.000	0.937	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	779	779	1353	5164	0	0	0	0	-1
normalized size	1	1.00	1.74	6.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.083	6.534	0.971	0.000	9.117	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	664	664	1287	4238	0	0	0	0	-1
normalized size	1	1.00	1.94	6.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.206	6.419	0.584	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	564	564	1251	3512	0	0	0	0	-1
normalized size	1	1.00	2.22	6.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.698	6.512	0.550	0.000	106.740	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	547	547	1241	3270	0	0	0	0	-1
normalized size	1	1.00	2.27	5.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.665	6.490	0.307	0.000	4.090	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	1269	3204	0	0	0	0	-1
normalized size	1	1.00	2.37	5.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.669	6.511	0.319	0.000	2.229	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	1319	3274	0	0	0	0	-1
normalized size	1	1.00	2.68	6.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.247	6.568	0.376	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	1409	3628	0	0	0	0	-1
normalized size	1	1.00	3.25	8.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.372	6.634	0.453	0.000	1.042	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	1517	4392	0	0	0	0	-1
normalized size	1	1.00	2.91	8.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.939	6.729	0.543	0.000	0.533	0.000	0.000	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	622	622	1640	5373	0	0	0	0	-1
normalized size	1	1.00	2.64	8.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.625	6.848	1.428	0.000	0.498	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	1236	2346	0	0	0	0	-1
normalized size	1	1.00	2.96	5.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.950	19.452	0.509	0.000	39.858	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	1175	1871	0	0	0	0	-1
normalized size	1	1.00	2.45	3.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.076	12.433	0.383	0.000	2.606	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	4017	1002	0	0	0	0	-1
normalized size	1	1.00	9.41	2.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.088	17.355	0.493	0.000	1.842	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	144	197	0	0	0	0	-1
normalized size	1	1.00	0.63	0.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.274	1.481	0.283	0.000	1.370	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	299	935	0	0	0	0	-1
normalized size	1	1.00	1.30	4.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.316	13.001	0.273	0.000	0.730	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	416	1536	0	0	0	0	-1
normalized size	1	1.00	1.43	5.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.524	15.727	0.274	0.000	0.782	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	1319	2480	0	0	0	0	-1
normalized size	1	1.00	3.63	6.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.862	6.411	0.436	0.000	0.523	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	500	500	1234	2885	0	0	0	0	-1
normalized size	1	1.00	2.47	5.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.287	6.418	0.352	0.000	99.980	0.000	0.000	0.000

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	1012	2013	0	0	0	0	-1
normalized size	1	1.00	2.43	4.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.612	17.985	0.326	0.000	2.046	0.000	0.000	0.000

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	1223	1633	0	0	0	0	-1
normalized size	1	1.00	4.31	5.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.512	6.364	0.408	0.000	0.575	0.000	0.000	0.000

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	1281	2280	0	0	0	0	-1
normalized size	1	1.00	4.20	7.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.615	6.506	0.346	0.000	1.013	0.000	0.000	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	1357	3334	0	0	0	0	-1
normalized size	1	1.00	3.45	8.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.980	6.706	0.353	0.000	0.665	0.000	0.000	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	674	674	1396	8611	0	0	0	0	-1
normalized size	1	1.00	2.07	12.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.189	6.697	0.705	0.000	3.801	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	1342	5749	0	0	0	0	-1
normalized size	1	1.00	2.46	10.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.402	6.534	0.512	0.000	1.370	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	1335	4237	0	0	0	0	-1
normalized size	1	1.00	3.41	10.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.869	6.455	0.394	0.000	1.488	0.000	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	1384	5203	0	0	0	0	-1
normalized size	1	1.00	3.23	12.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.985	6.579	0.905	0.000	0.719	0.000	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	456	456	1431	6498	0	0	0	0	-1
normalized size	1	1.00	3.14	14.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.161	6.723	1.420	0.000	1.241	0.000	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	567	567	1499	8093	0	0	0	0	-1
normalized size	1	1.00	2.64	14.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.877	6.950	0.495	0.000	1.955	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	480	623	0	0	0	0	-1
normalized size	1	1.00	1.15	1.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.789	1.437	0.342	0.000	53.184	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	131	160	0	0	0	0	-1
normalized size	1	1.00	1.12	1.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.143	0.333	0.000	1.519	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	171	124	0	0	0	0	-1
normalized size	1	1.00	1.55	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.893	0.233	0.000	0.820	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	212	613	0	0	0	0	-1
normalized size	1	1.00	0.94	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.268	2.186	0.257	0.000	2.593	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	0	658	0	0	0	0	-1
normalized size	1	1.00	0.00	9.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	35.237	0.449	0.000	0.871	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	0	600	0	0	0	0	-1
normalized size	1	1.00	0.00	8.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	38.260	0.424	0.000	0.917	0.000	0.000	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	614	0	0	0	0	-1
normalized size	1	1.00	0.00	6.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	33.185	0.415	0.000	0.561	0.000	0.000	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	0	703	0	0	0	0	-1
normalized size	1	1.00	0.00	7.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	30.158	0.390	0.000	0.984	0.000	0.000	0.000

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	0	665	0	0	0	0	-1
normalized size	1	1.00	0.00	9.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	37.987	0.423	0.000	1.602	0.000	0.000	0.000

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	0	663	0	0	0	0	-1
normalized size	1	1.00	0.00	8.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	38.379	0.420	0.000	1.172	0.000	0.000	0.000

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	0	714	0	0	0	0	-1
normalized size	1	1.00	0.00	7.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	41.063	0.329	0.000	0.952	0.000	0.000	0.000

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	0	740	0	0	0	0	-1
normalized size	1	1.00	0.00	7.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	30.579	0.335	0.000	1.490	0.000	0.000	0.000

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.082	7.983	3.390	0.000	1.768	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	595	595	487	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.984	6.202	2.882	0.000	0.864	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	269	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.053	2.815	3.002	0.000	0.977	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	217	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.541	1.704	1.884	0.000	0.914	0.000	0.000	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	151	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	0.339	1.993	0.000	0.675	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	10482	0	0	0	0	0	-1
normalized size	1	1.00	36.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	26.941	1.477	0.000	0.896	0.000	0.000	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.527	66.753	0.455	0.000	1.977	0.000	0.000	0.000

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.116	9.667	0.344	0.000	1.001	0.000	0.000	0.000

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.123	8.097	0.319	0.000	1.026	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.500	10.720	0.371	0.000	1.388	0.000	0.000	0.000

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	292	661	0	0	0	0	-1
normalized size	1	1.00	1.70	3.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	1.932	4.275	0.000	2.105	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	225	426	0	0	0	0	-1
normalized size	1	1.00	1.67	3.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	1.229	3.571	0.000	0.847	0.000	0.000	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	157	240	0	0	0	0	-1
normalized size	1	1.00	1.48	2.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	1.059	1.664	0.000	2.415	0.000	0.000	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	148	321	0	0	0	0	-1
normalized size	1	1.00	1.35	2.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	1.274	1.369	0.000	1.065	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	148	355	0	0	0	0	-1
normalized size	1	1.00	1.05	2.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	1.597	1.313	0.000	0.828	0.000	0.000	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	182	383	0	0	0	0	-1
normalized size	1	1.00	1.06	2.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	2.181	1.551	0.000	1.129	0.000	0.000	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	299	741	0	0	0	0	-1
normalized size	1	1.00	1.50	3.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.345	2.991	4.386	0.000	1.892	0.000	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	279	513	0	0	0	0	-1
normalized size	1	1.00	1.74	3.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.318	2.279	1.623	0.000	0.971	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	302	388	0	0	0	0	-1
normalized size	1	1.00	1.89	2.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.323	1.868	1.497	0.000	1.287	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	153	357	0	0	0	0	-1
normalized size	1	1.00	0.92	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.340	1.625	1.235	0.000	1.098	0.000	0.000	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	193	385	0	0	0	0	-1
normalized size	1	1.00	0.96	1.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	2.272	1.278	0.000	0.812	0.000	0.000	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	435	929	0	0	0	0	-1
normalized size	1	1.00	1.78	3.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.505	4.158	5.512	0.000	0.899	0.000	0.000	0.000

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	268	916	0	0	0	0	-1
normalized size	1	1.00	1.27	4.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.495	3.168	4.267	0.000	0.838	0.000	0.000	0.000

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	202	654	0	0	0	0	-1
normalized size	1	1.00	1.02	3.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.491	1.940	1.604	0.000	0.636	0.000	0.000	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	207	519	0	0	0	0	-1
normalized size	1	1.00	0.98	2.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.495	1.640	1.699	0.000	0.910	0.000	0.000	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	194	385	0	0	0	0	-1
normalized size	1	1.00	0.92	1.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.508	2.408	1.440	0.000	0.886	0.000	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	196	413	0	0	0	0	-1
normalized size	1	1.00	0.80	1.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.535	2.746	1.389	0.000	2.495	0.000	0.000	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	650	493	0	0	0	0	-1
normalized size	1	1.00	3.37	2.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.304	7.319	4.056	0.000	1.437	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	400	319	0	0	0	0	-1
normalized size	1	1.00	2.52	2.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.276	4.383	3.357	0.000	2.115	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	200	243	0	0	0	0	-1
normalized size	1	1.00	1.63	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.245	1.123	1.384	0.000	0.450	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	422	244	0	0	0	0	-1
normalized size	1	1.00	3.38	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.253	2.585	1.479	0.000	1.089	0.000	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	444	262	0	0	0	0	-1
normalized size	1	1.00	2.72	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.281	4.884	1.397	0.000	0.819	0.000	0.000	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	518	281	0	0	0	0	-1
normalized size	1	1.00	2.64	1.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	3.246	1.575	0.000	2.452	0.000	0.000	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	303	494	0	0	0	0	-1
normalized size	1	1.00	1.46	2.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.431	3.213	1.879	0.000	1.338	0.000	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	256	350	0	0	0	0	-1
normalized size	1	1.00	1.59	2.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.388	2.021	1.622	0.000	1.326	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	256	350	0	0	0	0	-1
normalized size	1	1.00	1.52	2.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.391	2.386	1.624	0.000	0.914	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	475	421	0	0	0	0	-1
normalized size	1	1.00	2.70	2.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.406	6.473	1.687	0.000	1.095	0.000	0.000	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	777	435	0	0	0	0	-1
normalized size	1	1.00	3.77	2.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	6.907	1.853	0.000	2.048	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	358	685	0	0	0	0	-1
normalized size	1	1.00	1.37	2.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.612	5.394	2.053	0.000	2.312	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	793	451	0	0	0	0	-1
normalized size	1	1.00	3.57	2.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.581	6.979	1.651	0.000	0.854	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	792	451	0	0	0	0	-1
normalized size	1	1.00	3.67	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.566	6.929	1.856	0.000	0.927	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	793	451	0	0	0	0	-1
normalized size	1	1.00	3.57	2.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.580	7.037	1.650	0.000	1.261	0.000	0.000	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	817	451	0	0	0	0	-1
normalized size	1	1.00	3.58	1.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.577	7.148	1.636	0.000	0.508	0.000	0.000	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	589	465	0	0	0	0	-1
normalized size	1	1.00	2.27	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.624	4.722	1.562	0.000	0.540	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	124	138	659	121	0	0	479
normalized size	1	1.00	0.56	0.63	3.00	0.55	0.00	0.00	2.18
time (sec)	N/A	0.486	0.577	0.417	0.761	0.675	0.000	0.000	5.605

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	102	116	568	104	0	0	441
normalized size	1	1.00	0.58	0.66	3.25	0.59	0.00	0.00	2.52
time (sec)	N/A	0.406	0.451	0.377	0.493	0.646	0.000	0.000	4.552

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	78	94	475	86	0	0	196
normalized size	1	1.00	0.60	0.72	3.65	0.66	0.00	0.00	1.51
time (sec)	N/A	0.334	0.280	0.350	0.506	0.631	0.000	0.000	2.689

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	57	70	380	65	0	0	114
normalized size	1	1.00	0.67	0.82	4.47	0.76	0.00	0.00	1.34
time (sec)	N/A	0.266	0.178	0.376	0.818	0.658	0.000	0.000	1.074

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	86	171	906	91	0	0	-1
normalized size	1	1.00	0.90	1.78	9.44	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.273	0.209	0.391	1.357	0.571	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	103	168	939	97	0	0	-1
normalized size	1	1.00	1.05	1.71	9.58	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.271	0.210	0.465	1.474	0.651	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	120	238	1851	127	0	0	-1
normalized size	1	1.00	0.79	1.58	12.26	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.338	0.388	0.412	0.817	0.579	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	138	308	2981	146	0	0	-1
normalized size	1	1.00	0.70	1.57	15.21	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.414	0.705	0.439	0.999	0.829	0.000	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	146	161	712	144	0	0	348
normalized size	1	1.00	0.53	0.59	2.59	0.52	0.00	0.00	1.27
time (sec)	N/A	0.723	0.782	0.412	0.517	0.652	0.000	0.000	5.142

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	124	139	619	126	0	0	316
normalized size	1	1.00	0.54	0.61	2.71	0.55	0.00	0.00	1.39
time (sec)	N/A	0.650	0.714	0.375	0.515	0.568	0.000	0.000	4.915

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	102	117	527	107	0	0	259
normalized size	1	1.00	0.56	0.65	2.91	0.59	0.00	0.00	1.43
time (sec)	N/A	0.561	0.562	0.357	0.510	0.515	0.000	0.000	4.804

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	80	95	436	88	0	0	197
normalized size	1	1.00	0.60	0.71	3.25	0.66	0.00	0.00	1.47
time (sec)	N/A	0.469	0.331	0.447	0.505	0.796	0.000	0.000	2.444

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	106	287	1462	130	0	0	-1
normalized size	1	1.00	0.73	1.98	10.08	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.451	0.402	0.470	0.705	0.614	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	107	308	1801	119	0	0	-1
normalized size	1	1.00	0.73	2.11	12.34	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.466	0.321	0.457	0.853	0.615	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	121	233	1884	133	0	0	-1
normalized size	1	1.00	0.79	1.52	12.31	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.455	0.453	0.387	0.856	0.604	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	141	309	3023	153	0	0	-1
normalized size	1	1.00	0.70	1.54	15.12	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.542	0.502	0.452	1.034	0.690	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	158	381	8901	171	0	0	-1
normalized size	1	1.00	0.64	1.54	36.04	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.644	0.789	0.359	1.474	0.691	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	171	185	763	176	0	0	789
normalized size	1	1.00	0.53	0.57	2.37	0.55	0.00	0.00	2.45
time (sec)	N/A	0.939	0.907	0.485	0.535	0.567	0.000	0.000	6.192

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	147	163	672	156	0	0	751
normalized size	1	1.00	0.53	0.59	2.44	0.57	0.00	0.00	2.73
time (sec)	N/A	0.848	1.274	0.462	0.519	0.691	0.000	0.000	5.810

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	126	141	579	135	0	0	617
normalized size	1	1.00	0.55	0.62	2.54	0.59	0.00	0.00	2.71
time (sec)	N/A	0.768	0.947	0.395	0.504	0.748	0.000	0.000	5.725

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	104	119	488	114	0	0	579
normalized size	1	1.00	0.57	0.66	2.70	0.63	0.00	0.00	3.20
time (sec)	N/A	0.676	0.695	0.354	0.509	0.572	0.000	0.000	4.995

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	130	389	1713	162	0	0	-1
normalized size	1	1.00	0.68	2.03	8.92	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.623	0.844	0.464	0.755	0.646	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	130	492	2780	166	0	0	-1
normalized size	1	1.00	0.67	2.55	14.40	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.654	0.736	0.426	0.877	0.584	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	126	344	0	147	0	0	-1
normalized size	1	1.00	0.64	1.74	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.664	0.777	0.441	0.000	0.617	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	141	305	3071	163	0	0	-1
normalized size	1	1.00	0.70	1.52	15.36	0.82	0.00	0.00	-0.00
time (sec)	N/A	0.650	0.977	0.424	3.043	0.802	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	159	383	9390	183	0	0	-1
normalized size	1	1.00	0.64	1.55	38.02	0.74	0.00	0.00	-0.00
time (sec)	N/A	0.752	0.979	0.375	2.310	0.927	0.000	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	181	455	0	203	0	0	-1
normalized size	1	1.00	0.62	1.55	0.00	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.866	1.437	0.347	0.000	0.960	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	272	793	0	198	0	0	-1
normalized size	1	1.00	0.92	2.69	0.00	0.67	0.00	0.00	-0.00
time (sec)	N/A	1.057	9.330	0.368	0.000	1.380	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	250	657	0	181	0	0	-1
normalized size	1	1.00	1.00	2.63	0.00	0.72	0.00	0.00	-0.00
time (sec)	N/A	0.838	6.839	0.495	0.000	0.740	0.000	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	1718	521	0	164	0	0	-1
normalized size	1	1.00	8.30	2.52	0.00	0.79	0.00	0.00	-0.00
time (sec)	N/A	0.647	7.799	0.427	0.000	0.553	0.000	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F(-2)	A	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	0	384	0	143	0	0	-1
normalized size	1	1.00	0.00	2.37	0.00	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.453	0.000	0.400	0.000	1.826	0.000	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	203	231	0	110	0	0	-1
normalized size	1	1.00	1.71	1.94	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.306	1.580	0.380	0.000	0.766	0.000	0.000	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	102	153	0	96	0	0	-1
normalized size	1	1.00	0.73	1.09	0.00	0.69	0.00	0.00	-0.01
time (sec)	N/A	0.350	0.211	0.385	0.000	3.095	0.000	0.000	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	467	232	0	168	0	0	-1
normalized size	1	1.00	2.58	1.28	0.00	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.513	1.368	0.397	0.000	7.176	0.000	0.000	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	412	300	0	194	0	0	-1
normalized size	1	1.00	1.79	1.30	0.00	0.84	0.00	0.00	-0.00
time (sec)	N/A	0.699	1.455	0.420	0.000	6.973	0.000	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	143	317	0	208	0	0	-1
normalized size	1	1.00	0.74	1.65	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.665	0.466	0.481	0.000	33.098	0.000	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	2966	731	0	237	0	0	-1
normalized size	1	1.00	9.36	2.31	0.00	0.75	0.00	0.00	-0.00
time (sec)	N/A	1.106	10.181	0.554	0.000	0.694	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	0	595	0	220	0	0	-1
normalized size	1	1.00	0.00	2.20	0.00	0.81	0.00	0.00	-0.00
time (sec)	N/A	0.887	0.000	0.562	0.000	2.016	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	981	457	0	197	0	0	-1
normalized size	1	1.00	4.40	2.05	0.00	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.702	6.832	0.439	0.000	0.604	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	443	312	0	163	0	0	-1
normalized size	1	1.00	2.52	1.77	0.00	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.519	4.487	0.408	0.000	0.756	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	196	235	0	144	0	0	-1
normalized size	1	1.00	1.54	1.85	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.337	1.643	0.366	0.000	0.714	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	243	288	0	203	0	0	-1
normalized size	1	1.00	1.31	1.56	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.544	1.624	0.383	0.000	8.003	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	836	370	0	246	0	0	-1
normalized size	1	1.00	3.53	1.56	0.00	1.04	0.00	0.00	-0.00
time (sec)	N/A	0.736	6.709	0.444	0.000	10.958	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	261	729	0	266	0	0	-1
normalized size	1	1.00	0.82	2.30	0.00	0.84	0.00	0.00	-0.00
time (sec)	N/A	1.125	8.464	0.520	0.000	0.607	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	243	585	0	246	0	0	-1
normalized size	1	1.00	0.90	2.17	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.929	3.791	0.513	0.000	0.873	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	219	457	0	210	0	0	-1
normalized size	1	1.00	0.98	2.05	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.729	2.303	0.426	0.000	0.743	0.000	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	216	375	0	207	0	0	-1
normalized size	1	1.00	1.23	2.13	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.526	1.843	0.392	0.000	0.716	0.000	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	213	375	0	205	0	0	-1
normalized size	1	1.00	1.22	2.16	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.511	1.823	0.468	0.000	0.568	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	264	476	0	277	0	0	-1
normalized size	1	1.00	1.13	2.03	0.00	1.18	0.00	0.00	-0.00
time (sec)	N/A	0.738	2.580	0.391	0.000	16.162	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	929	609	0	313	0	0	-1
normalized size	1	1.00	3.25	2.13	0.00	1.09	0.00	0.00	-0.00
time (sec)	N/A	0.981	7.215	0.434	0.000	29.035	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	267	729	0	295	0	0	-1
normalized size	1	1.00	0.84	2.30	0.00	0.93	0.00	0.00	-0.00
time (sec)	N/A	1.147	5.771	0.484	0.000	0.836	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	242	595	0	260	0	0	-1
normalized size	1	1.00	0.90	2.20	0.00	0.96	0.00	0.00	-0.00
time (sec)	N/A	0.950	3.277	0.419	0.000	1.337	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	228	512	0	257	0	0	-1
normalized size	1	1.00	1.02	2.30	0.00	1.15	0.00	0.00	-0.00
time (sec)	N/A	0.728	3.070	0.391	0.000	0.775	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	233	512	0	255	0	0	-1
normalized size	1	1.00	1.05	2.32	0.00	1.15	0.00	0.00	-0.00
time (sec)	N/A	0.725	2.950	0.386	0.000	2.388	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	488	512	0	257	0	0	-1
normalized size	1	1.00	2.21	2.32	0.00	1.16	0.00	0.00	-0.00
time (sec)	N/A	0.723	7.232	0.426	0.000	1.509	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	281	667	0	338	0	0	-1
normalized size	1	1.00	1.00	2.37	0.00	1.20	0.00	0.00	-0.00
time (sec)	N/A	0.919	3.964	0.414	0.000	27.169	0.000	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	1017	855	0	379	0	0	-1
normalized size	1	1.00	3.05	2.57	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	1.205	7.655	0.440	0.000	52.360	0.000	0.000	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	132	663	0	0	0	0	-1
normalized size	1	1.00	0.73	3.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.223	1.866	4.511	0.000	0.590	0.000	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	104	428	0	0	0	0	-1
normalized size	1	1.00	0.73	2.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.829	3.343	0.000	4.580	0.000	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	85	244	0	0	0	0	-1
normalized size	1	1.00	0.77	2.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.267	1.390	0.000	1.097	0.000	0.000	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	90	326	0	0	0	0	-1
normalized size	1	1.00	0.78	2.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.234	1.348	0.000	1.826	0.000	0.000	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	108	371	0	0	0	0	-1
normalized size	1	1.00	0.73	2.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.542	1.425	0.000	1.727	0.000	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	125	413	0	0	0	0	-1
normalized size	1	1.00	0.69	2.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.234	0.994	1.398	0.000	0.737	0.000	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	171	750	0	0	0	0	-1
normalized size	1	1.00	0.77	3.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.380	2.415	4.506	0.000	0.907	0.000	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	125	677	0	0	0	0	-1
normalized size	1	1.00	0.71	3.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.352	1.138	3.472	0.000	2.987	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	124	404	0	0	0	0	-1
normalized size	1	1.00	0.77	2.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.320	0.749	1.539	0.000	1.406	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	128	487	0	0	0	0	-1
normalized size	1	1.00	0.75	2.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.336	0.906	1.395	0.000	1.809	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	161	548	0	0	0	0	-1
normalized size	1	1.00	0.76	2.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.372	1.341	1.400	0.000	0.786	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	225	944	0	0	0	0	-1
normalized size	1	1.00	0.76	3.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.597	3.652	6.091	0.000	1.028	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	192	997	0	0	0	0	-1
normalized size	1	1.00	0.79	4.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.577	1.597	4.752	0.000	1.015	0.000	0.000	0.000

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	166	1212	0	0	0	0	-1
normalized size	1	1.00	0.69	5.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.566	1.941	4.128	0.000	1.027	0.000	0.000	0.000

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	172	867	0	0	0	0	-1
normalized size	1	1.00	0.73	3.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.533	1.436	1.663	0.000	0.839	0.000	0.000	0.000

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	180	664	0	0	0	0	-1
normalized size	1	1.00	0.73	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	1.306	1.457	0.000	1.440	0.000	0.000	0.000

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	219	745	0	0	0	0	-1
normalized size	1	1.00	0.74	2.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.579	1.832	1.595	0.000	0.958	0.000	0.000	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	225	468	0	0	0	0	-1
normalized size	1	1.00	1.07	2.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.808	3.426	4.125	0.000	0.000	0.000	0.000	0.000

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	125	327	0	0	0	0	-1
normalized size	1	1.00	0.99	2.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.459	1.285	2.865	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	76	217	0	0	0	0	-1
normalized size	1	1.00	0.75	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.283	0.536	1.499	0.000	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	220	295	0	0	0	0	-1
normalized size	1	1.00	1.48	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.346	6.359	1.586	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	278	786	0	0	0	0	-1
normalized size	1	1.00	1.41	3.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.560	6.660	1.567	0.000	160.199	0.000	0.000	0.000

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	735	1031	0	0	0	0	-1
normalized size	1	1.00	1.81	2.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.289	7.109	6.901	0.000	0.000	0.000	0.000	0.000

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	681	883	0	0	0	0	-1
normalized size	1	1.00	2.16	2.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.942	6.905	4.264	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	639	721	0	0	0	0	-1
normalized size	1	1.00	2.46	2.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.613	6.838	3.646	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	626	808	0	0	0	0	-1
normalized size	1	1.00	2.43	3.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.605	6.819	3.802	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	655	849	0	0	0	0	-1
normalized size	1	1.00	2.31	2.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.658	6.888	4.748	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	701	1066	0	0	0	0	-1
normalized size	1	1.00	1.93	2.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.962	6.992	5.323	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	844	2002	0	0	0	0	-1
normalized size	1	1.00	1.76	4.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.450	7.209	7.641	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	797	1744	0	0	0	0	-1
normalized size	1	1.00	1.97	4.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.036	7.108	6.011	0.000	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	784	1850	0	0	0	0	-1
normalized size	1	1.00	1.95	4.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.090	6.931	6.468	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	786	1937	0	0	0	0	-1
normalized size	1	1.00	1.96	4.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.955	6.925	6.605	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	820	1977	0	0	0	0	-1
normalized size	1	1.00	1.92	4.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.057	7.080	7.444	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	521	521	865	2195	0	0	0	0	-1
normalized size	1	1.00	1.66	4.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.529	7.315	8.182	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	47	214	0	0	0	0	-1
normalized size	1	1.00	0.73	3.34	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.071	1.301	0.000	1.075	0.000	0.000	0.000

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	46	102	0	0	0	0	-1
normalized size	1	1.00	0.77	1.70	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.046	1.507	0.000	0.981	0.000	0.000	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	134	0	0	0	0	-1
normalized size	1	1.00	1.00	3.62	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.030	1.068	0.000	1.437	0.000	0.000	0.000

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	134	0	0	0	0	-1
normalized size	1	1.00	1.00	3.62	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.037	0.873	0.000	0.647	0.000	0.000	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	50	180	0	0	0	0	-1
normalized size	1	1.00	0.78	2.81	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.046	1.426	0.000	0.885	0.000	0.000	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	203	0	0	0	0	-1
normalized size	1	1.00	0.88	3.17	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.073	1.225	0.000	1.367	0.000	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	3321	3435	0	0	0	0	-1
normalized size	1	1.00	7.02	7.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.438	23.759	0.621	0.000	1.636	0.000	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	423	2489	0	0	0	0	-1
normalized size	1	1.00	1.08	6.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.057	17.473	0.456	0.000	0.563	0.000	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	346	1735	0	0	0	0	-1
normalized size	1	1.00	1.07	5.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.675	14.451	0.395	0.000	0.689	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	635	1361	0	0	0	0	-1
normalized size	1	1.00	1.55	3.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.685	17.281	0.459	0.000	1.173	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	445	787	1369	0	0	0	0	-1
normalized size	1	1.00	1.77	3.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.903	17.547	0.492	0.000	1.473	0.000	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	1121	2054	0	0	0	0	-1
normalized size	1	1.00	2.10	3.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.265	18.619	0.398	0.000	2.272	0.000	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	620	620	1533	2956	0	0	0	0	-1
normalized size	1	1.00	2.47	4.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.740	14.649	0.468	0.000	0.000	0.000	0.000	0.000

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	3739	4400	0	0	0	0	-1
normalized size	1	1.00	6.65	7.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.078	25.987	0.720	0.000	2.835	0.000	0.000	0.000

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	3318	3421	0	0	0	0	-1
normalized size	1	1.00	7.01	7.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.529	23.895	0.513	0.000	1.058	0.000	0.000	0.000

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	427	2674	0	0	0	0	-1
normalized size	1	1.00	1.09	6.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.091	18.753	0.409	0.000	0.581	0.000	0.000	0.000

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	5981	2326	0	0	0	0	-1
normalized size	1	1.00	12.49	4.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.076	24.553	0.358	0.000	1.136	0.000	0.000	0.000

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	927	2196	0	0	0	0	-1
normalized size	1	1.00	1.82	4.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.380	16.718	0.365	0.000	1.709	0.000	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	1134	2432	0	0	0	0	-1
normalized size	1	1.00	2.13	4.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.367	18.563	0.368	0.000	72.047	0.000	0.000	0.000

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	626	626	1489	3141	0	0	0	0	-1
normalized size	1	1.00	2.38	5.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.973	19.347	0.483	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	1888	4056	0	0	0	0	-1
normalized size	1	1.00	2.59	5.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.441	21.400	0.615	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	662	662	4198	5381	0	0	0	0	-1
normalized size	1	1.00	6.34	8.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.911	27.243	0.938	0.000	1.063	0.000	0.000	0.000

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	3755	4400	0	0	0	0	-1
normalized size	1	1.00	6.68	7.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.072	26.364	0.703	0.000	1.167	0.000	0.000	0.000

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	3348	3636	0	0	0	0	-1
normalized size	1	1.00	7.06	7.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.501	24.672	0.535	0.000	1.015	0.000	0.000	0.000

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	7032	3282	0	0	0	0	-1
normalized size	1	1.00	12.72	5.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.473	25.574	0.449	0.000	27.550	0.000	0.000	0.000

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	7700	3212	0	0	0	0	-1
normalized size	1	1.00	12.92	5.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.895	26.068	0.411	0.000	65.794	0.000	0.000	0.000

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	607	607	1278	3278	0	0	0	0	-1
normalized size	1	1.00	2.11	5.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.879	19.516	0.435	0.000	2.095	0.000	0.000	0.000

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	1504	3514	0	0	0	0	-1
normalized size	1	1.00	2.41	5.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.946	19.599	0.474	0.000	94.330	0.000	0.000	0.000

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	724	724	1857	4240	0	0	0	0	-1
normalized size	1	1.00	2.56	5.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.518	20.039	0.620	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	839	839	703	5172	0	0	0	0	-1
normalized size	1	1.00	0.84	6.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.603	16.074	0.838	0.000	5.786	0.000	0.000	0.000

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	2987	2488	0	0	0	0	-1
normalized size	1	1.00	7.41	6.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.026	22.265	0.414	0.000	0.703	0.000	0.000	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	355	1544	0	0	0	0	-1
normalized size	1	1.00	1.08	4.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.664	15.822	0.472	0.000	0.861	0.000	0.000	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	279	812	0	0	0	0	-1
normalized size	1	1.00	1.03	3.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.457	14.298	0.398	0.000	0.443	0.000	0.000	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	157	199	0	0	0	0	-1
normalized size	1	1.00	0.59	0.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	2.584	0.376	0.000	1.050	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	1091	1004	0	0	0	0	-1
normalized size	1	1.00	2.24	2.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.265	18.426	0.459	0.000	1.267	0.000	0.000	0.000

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	539	539	1157	1878	0	0	0	0	-1
normalized size	1	1.00	2.15	3.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.253	19.727	0.390	0.000	1.835	0.000	0.000	0.000

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	3433	3343	0	0	0	0	-1
normalized size	1	1.00	7.93	7.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.159	24.414	0.402	0.000	0.558	0.000	0.000	0.000

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	433	2291	0	0	0	0	-1
normalized size	1	1.00	1.26	6.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.791	18.936	0.441	0.000	0.485	0.000	0.000	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	305	1636	0	0	0	0	-1
normalized size	1	1.00	0.94	5.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.685	13.781	0.454	0.000	0.541	0.000	0.000	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	1403	2016	0	0	0	0	-1
normalized size	1	1.00	2.95	4.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.775	13.976	0.424	0.000	0.866	0.000	0.000	0.000

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	1551	2890	0	0	0	0	-1
normalized size	1	1.00	2.77	5.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.509	19.559	0.382	0.000	53.904	0.000	0.000	0.000

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	607	607	4316	8101	0	0	0	0	-1
normalized size	1	1.00	7.11	13.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.203	27.144	0.601	0.000	0.612	0.000	0.000	0.000

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	3891	6506	0	0	0	0	-1
normalized size	1	1.00	7.84	13.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.383	26.751	0.630	0.000	0.581	0.000	0.000	0.000

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	3493	5202	0	0	0	0	-1
normalized size	1	1.00	7.45	11.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.190	24.525	0.503	0.000	0.641	0.000	0.000	0.000

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	528	4243	0	0	0	0	-1
normalized size	1	1.00	1.23	9.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.081	18.801	0.470	0.000	0.745	0.000	0.000	0.000

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	602	602	1994	5757	0	0	0	0	-1
normalized size	1	1.00	3.31	9.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.645	16.206	0.433	0.000	25.692	0.000	0.000	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	733	733	2318	8621	0	0	0	0	-1
normalized size	1	1.00	3.16	11.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.494	22.369	0.563	0.000	1.602	0.000	0.000	0.000

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	298	621	0	0	0	0	-1
normalized size	1	1.00	1.12	2.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.370	6.105	0.333	0.000	0.527	0.000	0.000	0.000

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	104	126	0	0	0	0	-1
normalized size	1	1.00	0.80	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.145	0.318	0.000	0.644	0.000	0.000	0.000

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	147	144	0	0	0	0	-1
normalized size	1	1.00	1.07	1.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.189	0.299	0.000	0.713	0.000	0.000	0.000

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	508	631	0	0	0	0	-1
normalized size	1	1.00	1.06	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.922	3.329	0.352	0.000	25.390	0.000	0.000	0.000

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.181	9.519	3.111	0.000	0.476	0.000	0.000	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	644	644	317	0	0	0	0	0	-1
normalized size	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.042	4.371	2.793	0.000	0.681	0.000	0.000	0.000

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	455	259	0	0	0	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.145	2.521	2.228	0.000	0.435	0.000	0.000	0.000

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	205	0	0	0	0	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.643	0.986	1.801	0.000	0.478	0.000	0.000	0.000

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	163	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	0.383	1.942	0.000	0.462	0.000	0.000	0.000

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	10630	0	0	0	0	0	-1
normalized size	1	1.00	35.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.576	26.444	1.270	0.000	0.461	0.000	0.000	0.000

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.721	61.594	0.457	0.000	0.495	0.000	0.000	0.000

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.239	13.199	0.421	0.000	0.463	0.000	0.000	0.000

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.242	8.875	0.409	0.000	0.485	0.000	0.000	0.000

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	213	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.686	11.212	0.387	0.000	0.462	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [317] had the largest ratio of [.3333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	6	1.00	29	0.207
2	A	7	6	1.00	29	0.207
3	A	3	3	1.00	27	0.111
4	A	1	1	1.00	21	0.048
5	A	4	4	1.00	27	0.148
6	A	4	4	1.00	29	0.138
7	A	6	6	1.00	29	0.207
8	A	7	7	1.00	29	0.241
9	A	7	6	1.00	29	0.207
10	A	9	7	1.00	31	0.226

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
11	A	8	7	1.00	31	0.226
12	A	4	4	1.00	29	0.138
13	A	2	2	1.00	23	0.087
14	A	5	5	1.00	29	0.172
15	A	5	5	1.00	31	0.161
16	A	5	5	1.00	31	0.161
17	A	7	7	1.00	31	0.226
18	A	8	8	1.00	31	0.258
19	A	9	7	1.00	31	0.226
20	A	10	8	1.00	29	0.276
21	A	8	6	1.00	23	0.261
22	A	6	5	1.00	29	0.172
23	A	6	6	1.00	31	0.194
24	A	6	5	1.00	31	0.161
25	A	6	5	1.00	31	0.161
26	A	8	7	1.00	31	0.226
27	A	9	8	1.00	31	0.258
28	A	10	7	1.00	31	0.226
29	A	13	8	1.00	29	0.276
30	A	11	6	1.00	23	0.261
31	A	7	5	1.00	29	0.172
32	A	7	6	1.00	31	0.194
33	A	7	6	1.00	31	0.194
34	A	7	5	1.00	31	0.161
35	A	7	5	1.00	31	0.161
36	A	9	7	1.00	31	0.226

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
37	A	10	8	1.00	31	0.258
38	A	7	5	1.00	31	0.161
39	A	6	5	1.00	31	0.161
40	A	2	2	1.10	31	0.065
41	A	5	5	1.00	29	0.172
42	A	2	2	1.00	23	0.087
43	A	3	3	1.00	29	0.103
44	A	5	5	1.00	31	0.161
45	A	6	6	1.00	31	0.194
46	A	6	5	1.00	31	0.161
47	A	7	5	1.00	31	0.161
48	A	3	2	1.00	31	0.065
49	A	6	6	1.00	31	0.194
50	A	4	4	1.00	29	0.138
51	A	2	2	1.00	23	0.087
52	A	4	3	1.00	29	0.103
53	A	6	5	1.00	31	0.161
54	A	7	6	1.00	31	0.194
55	A	7	5	1.00	31	0.161
56	A	8	5	1.00	31	0.161
57	A	4	2	1.00	31	0.065
58	A	7	6	1.00	31	0.194
59	A	5	5	1.00	31	0.161
60	A	4	4	1.00	29	0.138
61	A	3	3	1.00	23	0.130
62	A	5	3	1.00	29	0.103

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
63	A	7	5	1.00	31	0.161
64	A	8	6	1.00	31	0.194
65	A	5	2	1.00	31	0.065
66	A	8	6	1.00	31	0.194
67	A	6	5	1.00	31	0.161
68	A	5	5	1.00	31	0.161
69	A	5	5	1.00	29	0.172
70	A	4	3	1.00	23	0.130
71	A	6	3	1.00	29	0.103
72	A	8	5	1.00	31	0.161
73	A	9	6	1.00	31	0.194
74	A	5	5	1.00	33	0.152
75	A	4	4	1.00	33	0.121
76	A	4	4	1.00	31	0.129
77	A	2	2	1.00	25	0.080
78	A	3	3	1.00	31	0.097
79	A	3	3	1.00	33	0.091
80	A	4	4	1.00	33	0.121
81	A	5	4	1.00	33	0.121
82	A	6	6	1.00	33	0.182
83	A	5	5	1.00	33	0.152
84	A	5	5	1.00	31	0.161
85	A	3	3	1.00	25	0.120
86	A	4	4	1.00	31	0.129
87	A	4	4	1.00	33	0.121
88	A	4	4	1.00	33	0.121

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	5	5	1.00	33	0.152
90	A	6	5	1.00	33	0.152
91	A	6	5	1.00	33	0.152
92	A	6	5	1.00	31	0.161
93	A	4	3	1.00	25	0.120
94	A	5	4	1.00	31	0.129
95	A	5	5	1.00	33	0.152
96	A	5	4	1.00	33	0.121
97	A	5	4	1.00	33	0.121
98	A	6	5	1.00	33	0.152
99	A	7	5	1.00	33	0.152
100	A	7	6	1.00	33	0.182
101	A	6	6	1.00	33	0.182
102	A	5	5	1.00	31	0.161
103	A	3	3	1.00	25	0.120
104	A	5	4	1.00	31	0.129
105	A	6	5	1.00	33	0.152
106	A	7	5	1.00	33	0.152
107	A	8	7	1.00	33	0.212
108	A	7	7	1.00	33	0.212
109	A	6	6	1.00	33	0.182
110	A	5	5	1.00	31	0.161
111	A	3	3	1.00	25	0.120
112	A	6	5	1.00	31	0.161
113	A	7	6	1.00	33	0.182
114	A	8	6	1.00	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
115	A	8	7	1.00	33	0.212
116	A	7	6	1.00	33	0.182
117	A	6	6	1.00	33	0.182
118	A	5	5	1.00	31	0.161
119	A	4	4	1.00	25	0.160
120	A	7	5	1.00	31	0.161
121	A	8	6	1.00	33	0.182
122	A	9	6	1.00	33	0.182
123	A	8	6	1.00	31	0.194
124	A	7	6	1.00	31	0.194
125	A	6	6	1.00	31	0.194
126	A	5	5	1.00	31	0.161
127	A	5	5	1.00	31	0.161
128	A	6	6	1.00	31	0.194
129	A	7	6	1.00	31	0.194
130	A	8	7	1.00	33	0.212
131	A	7	7	1.00	33	0.212
132	A	6	6	1.00	33	0.182
133	A	6	6	1.00	33	0.182
134	A	6	6	1.00	33	0.182
135	A	7	7	1.00	33	0.212
136	A	8	7	1.00	33	0.212
137	A	9	7	1.00	33	0.212
138	A	8	7	1.00	33	0.212
139	A	7	6	1.00	33	0.182
140	A	7	7	1.00	33	0.212

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
141	A	7	6	1.00	33	0.182
142	A	7	6	1.00	33	0.182
143	A	8	7	1.00	33	0.212
144	A	9	7	1.00	33	0.212
145	A	6	5	1.00	33	0.152
146	A	5	5	1.00	33	0.152
147	A	4	4	1.00	33	0.121
148	A	4	4	1.00	33	0.121
149	A	5	5	1.00	33	0.152
150	A	6	5	1.00	33	0.152
151	A	7	5	1.00	33	0.152
152	A	6	5	1.00	33	0.152
153	A	5	4	1.00	33	0.121
154	A	5	5	1.00	33	0.152
155	A	5	4	1.00	33	0.121
156	A	6	5	1.00	33	0.152
157	A	7	5	1.00	33	0.152
158	A	8	5	1.00	33	0.152
159	A	7	5	1.00	33	0.152
160	A	6	4	1.00	33	0.121
161	A	6	5	1.00	33	0.152
162	A	6	5	1.00	33	0.152
163	A	6	4	1.00	33	0.121
164	A	7	5	1.00	33	0.152
165	A	8	5	1.00	33	0.152
166	A	6	4	1.00	35	0.114

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
167	A	5	4	1.00	35	0.114
168	A	4	4	1.00	35	0.114
169	A	3	3	1.00	35	0.086
170	A	3	3	1.00	35	0.086
171	A	2	2	1.00	35	0.057
172	A	3	3	1.00	35	0.086
173	A	4	3	1.00	35	0.086
174	A	6	5	1.00	35	0.143
175	A	5	5	1.00	35	0.143
176	A	4	4	1.00	35	0.114
177	A	4	4	1.00	35	0.114
178	A	4	4	1.00	35	0.114
179	A	3	3	1.00	35	0.086
180	A	4	4	1.00	35	0.114
181	A	5	4	1.00	35	0.114
182	A	7	5	1.00	35	0.143
183	A	6	5	1.00	35	0.143
184	A	5	4	1.00	35	0.114
185	A	5	5	1.00	35	0.143
186	A	5	4	1.00	35	0.114
187	A	5	4	1.00	35	0.114
188	A	4	3	1.00	35	0.086
189	A	5	4	1.00	35	0.114
190	A	6	4	1.00	35	0.114
191	A	7	6	1.00	35	0.171
192	A	6	6	1.00	35	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	5	5	1.00	35	0.143
194	A	4	4	1.00	35	0.114
195	A	5	4	1.00	35	0.114
196	A	6	4	1.00	35	0.114
197	A	7	7	1.00	35	0.200
198	A	6	6	1.00	35	0.171
199	A	4	4	1.00	35	0.114
200	A	5	5	1.00	35	0.143
201	A	6	5	1.00	35	0.143
202	A	8	7	1.00	35	0.200
203	A	7	6	1.00	35	0.171
204	A	5	5	1.00	35	0.143
205	A	5	4	1.00	35	0.114
206	A	6	5	1.00	35	0.143
207	A	7	5	1.00	35	0.143
208	A	9	7	1.00	35	0.200
209	A	8	6	1.00	35	0.171
210	A	6	5	1.00	35	0.143
211	A	6	5	1.00	35	0.143
212	A	6	4	1.00	35	0.114
213	A	7	5	1.00	35	0.143
214	A	8	5	1.00	35	0.143
215	A	7	6	1.00	29	0.207
216	A	3	3	1.00	27	0.111
217	A	1	1	1.00	21	0.048
218	A	4	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
219	A	4	4	1.00	29	0.138
220	A	6	6	1.00	29	0.207
221	A	7	7	1.00	29	0.241
222	A	7	6	1.00	29	0.207
223	A	7	6	1.00	31	0.194
224	A	4	4	1.00	29	0.138
225	A	2	2	1.00	23	0.087
226	A	4	4	1.00	29	0.138
227	A	4	4	1.00	31	0.129
228	A	4	4	1.00	31	0.129
229	A	6	6	1.00	31	0.194
230	A	7	7	1.00	31	0.226
231	A	8	7	1.00	31	0.226
232	A	5	4	1.00	29	0.138
233	A	3	2	1.00	23	0.087
234	A	5	5	1.00	29	0.172
235	A	5	5	1.00	31	0.161
236	A	5	5	1.00	31	0.161
237	A	5	5	1.00	31	0.161
238	A	7	7	1.00	31	0.226
239	A	8	8	1.00	31	0.258
240	A	9	8	1.00	31	0.258
241	A	6	4	1.00	29	0.138
242	A	4	2	1.00	23	0.087
243	A	6	6	1.00	29	0.207
244	A	6	6	1.00	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
245	A	6	6	1.00	31	0.194
246	A	6	6	1.00	31	0.194
247	A	6	6	1.00	31	0.194
248	A	8	8	1.00	31	0.258
249	A	9	9	1.00	31	0.290
250	A	6	6	1.00	31	0.194
251	A	5	5	1.00	31	0.161
252	A	6	6	1.00	29	0.207
253	A	3	3	1.00	23	0.130
254	A	4	4	1.00	29	0.138
255	A	6	6	1.00	31	0.194
256	A	6	6	1.00	31	0.194
257	A	7	6	1.00	31	0.194
258	A	6	6	1.00	31	0.194
259	A	5	5	1.00	31	0.161
260	A	5	5	1.00	29	0.172
261	A	4	4	1.00	23	0.174
262	A	5	5	1.00	29	0.172
263	A	6	6	1.00	31	0.194
264	A	7	6	1.00	31	0.194
265	A	7	7	1.00	31	0.226
266	A	6	6	1.00	31	0.194
267	A	5	5	1.00	31	0.161
268	A	6	6	1.00	29	0.207
269	A	5	4	1.00	23	0.174
270	A	6	6	1.00	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	7	6	1.00	31	0.194
272	A	8	6	1.00	31	0.194
273	A	7	7	1.00	31	0.226
274	A	6	6	1.00	31	0.194
275	A	6	6	1.00	31	0.194
276	A	7	6	1.00	29	0.207
277	A	6	4	1.00	23	0.174
278	A	7	6	1.00	29	0.207
279	A	8	6	1.00	31	0.194
280	A	9	6	1.00	31	0.194
281	A	3	2	1.00	34	0.059
282	A	3	3	1.00	34	0.088
283	A	2	2	1.00	32	0.062
284	A	2	2	1.00	26	0.077
285	A	2	2	1.00	32	0.062
286	A	3	3	1.00	34	0.088
287	A	3	3	1.00	34	0.088
288	A	3	2	1.00	34	0.059
289	A	6	6	1.00	34	0.176
290	A	6	6	1.00	34	0.176
291	A	4	4	1.00	32	0.125
292	A	3	3	1.00	26	0.115
293	A	5	5	1.00	32	0.156
294	A	7	7	1.00	34	0.206
295	A	7	7	1.00	34	0.206
296	A	9	9	1.00	33	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
297	A	8	8	1.00	33	0.242
298	A	8	8	1.00	31	0.258
299	A	6	6	1.00	25	0.240
300	A	8	8	1.00	31	0.258
301	A	9	9	1.00	33	0.273
302	A	10	10	1.00	33	0.303
303	A	11	10	1.00	33	0.303
304	A	9	8	1.00	33	0.242
305	A	9	8	1.00	31	0.258
306	A	7	6	1.00	25	0.240
307	A	9	9	1.00	31	0.290
308	A	9	9	1.00	33	0.273
309	A	10	10	1.00	33	0.303
310	A	11	10	1.00	33	0.303
311	A	10	8	1.00	33	0.242
312	A	10	8	1.00	31	0.258
313	A	8	6	1.00	25	0.240
314	A	10	10	1.00	31	0.323
315	A	10	10	1.00	33	0.303
316	A	10	10	1.00	33	0.303
317	A	11	11	1.00	33	0.333
318	A	12	11	1.00	33	0.333
319	A	8	8	1.00	33	0.242
320	A	7	7	1.00	33	0.212
321	A	7	7	1.00	31	0.226
322	A	5	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
323	A	5	5	1.00	31	0.161
324	A	9	9	1.00	33	0.273
325	A	10	10	1.00	33	0.303
326	A	8	8	1.00	33	0.242
327	A	7	7	1.00	33	0.212
328	A	7	7	1.00	31	0.226
329	A	6	6	1.00	25	0.240
330	A	7	7	1.00	31	0.226
331	A	10	10	1.00	33	0.303
332	A	11	10	1.00	33	0.303
333	A	9	9	1.00	33	0.273
334	A	8	8	1.00	33	0.242
335	A	7	7	1.00	33	0.212
336	A	8	8	1.00	31	0.258
337	A	7	6	1.00	25	0.240
338	A	10	10	1.00	31	0.323
339	A	11	11	1.00	33	0.333
340	A	12	10	1.00	33	0.303
341	A	3	3	1.00	28	0.107
342	A	3	3	1.00	34	0.088
343	A	5	4	1.00	28	0.143
344	A	8	8	1.00	34	0.235
345	A	8	6	1.00	31	0.194
346	A	7	6	1.00	31	0.194
347	A	6	6	1.00	31	0.194
348	A	5	5	1.00	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
349	A	5	5	1.00	31	0.161
350	A	6	6	1.00	31	0.194
351	A	7	6	1.00	31	0.194
352	A	8	6	1.00	33	0.182
353	A	7	6	1.00	33	0.182
354	A	6	6	1.00	33	0.182
355	A	5	5	1.00	33	0.152
356	A	5	5	1.00	33	0.152
357	A	5	5	1.00	33	0.152
358	A	6	6	1.00	33	0.182
359	A	8	7	1.00	33	0.212
360	A	7	7	1.00	33	0.212
361	A	6	6	1.00	33	0.182
362	A	6	6	1.00	33	0.182
363	A	6	6	1.00	33	0.182
364	A	6	6	1.00	33	0.182
365	A	7	7	1.00	33	0.212
366	A	6	6	1.00	33	0.182
367	A	5	5	1.00	33	0.152
368	A	3	3	1.00	33	0.091
369	A	5	5	1.00	33	0.152
370	A	7	7	1.00	33	0.212
371	A	7	7	1.00	33	0.212
372	A	6	6	1.00	33	0.182
373	A	6	6	1.00	33	0.182
374	A	6	6	1.00	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	7	7	1.00	33	0.212
376	A	8	7	1.00	33	0.212
377	A	7	7	1.00	33	0.212
378	A	7	7	1.00	33	0.212
379	A	7	7	1.00	33	0.212
380	A	7	7	1.00	33	0.212
381	A	8	7	1.00	33	0.212
382	A	9	7	1.00	33	0.212
383	A	3	3	1.00	36	0.083
384	A	3	3	1.00	36	0.083
385	A	2	2	1.00	36	0.056
386	A	2	2	1.00	36	0.056
387	A	3	3	1.00	36	0.083
388	A	3	3	1.00	36	0.083
389	A	7	7	1.00	36	0.194
390	A	6	6	1.00	36	0.167
391	A	4	4	1.00	36	0.111
392	A	2	2	1.00	36	0.056
393	A	6	6	1.00	36	0.167
394	A	8	8	1.00	36	0.222
395	A	8	8	1.00	35	0.229
396	A	7	7	1.00	35	0.200
397	A	6	6	1.00	35	0.171
398	A	5	5	1.00	35	0.143
399	A	4	4	1.00	35	0.114
400	A	5	5	1.00	35	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
401	A	6	5	1.00	35	0.143
402	A	9	8	1.00	35	0.229
403	A	8	8	1.00	35	0.229
404	A	7	7	1.00	35	0.200
405	A	7	7	1.00	35	0.200
406	A	6	6	1.00	35	0.171
407	A	5	5	1.00	35	0.143
408	A	6	5	1.00	35	0.143
409	A	7	5	1.00	35	0.143
410	A	10	8	1.00	35	0.229
411	A	9	8	1.00	35	0.229
412	A	8	8	1.00	35	0.229
413	A	8	8	1.00	35	0.229
414	A	8	8	1.00	35	0.229
415	A	7	7	1.00	35	0.200
416	A	6	6	1.00	35	0.171
417	A	7	6	1.00	35	0.171
418	A	8	6	1.00	35	0.171
419	A	6	6	1.00	43	0.140
420	A	7	7	1.00	35	0.200
421	A	7	7	1.00	35	0.200
422	A	3	3	1.00	35	0.086
423	A	3	3	1.00	35	0.086
424	A	4	4	1.00	35	0.114
425	A	5	5	1.00	35	0.143
426	A	7	7	1.00	35	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
427	A	6	6	1.00	35	0.171
428	A	4	4	1.00	35	0.114
429	A	4	4	1.00	35	0.114
430	A	5	5	1.00	35	0.143
431	A	8	8	1.00	35	0.229
432	A	7	7	1.00	35	0.200
433	A	5	5	1.00	35	0.143
434	A	5	5	1.00	35	0.143
435	A	5	5	1.00	35	0.143
436	A	6	5	1.00	35	0.143
437	A	9	9	1.00	38	0.237
438	A	2	2	1.00	38	0.053
439	A	2	2	1.00	38	0.053
440	A	4	4	1.00	38	0.105
441	A	1	1	1.00	33	0.030
442	A	1	1	1.00	33	0.030
443	A	2	2	1.00	33	0.061
444	A	2	2	1.00	33	0.061
445	A	1	1	1.00	33	0.030
446	A	1	1	1.00	33	0.030
447	A	2	2	1.00	33	0.061
448	A	2	2	1.00	33	0.061
449	A	0	0	0.00	0	0.000
450	A	7	6	1.00	33	0.182
451	A	6	5	1.00	33	0.152
452	A	5	4	1.00	33	0.121

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
453	A	5	4	1.00	31	0.129
454	A	7	5	1.00	33	0.152
455	A	0	0	0.00	0	0.000
456	A	0	0	0.00	0	0.000
457	A	0	0	0.00	0	0.000
458	A	0	0	0.00	0	0.000
459	A	9	7	1.00	31	0.226
460	A	8	7	1.00	31	0.226
461	A	7	6	1.00	31	0.194
462	A	7	6	1.00	31	0.194
463	A	8	7	1.00	31	0.226
464	A	9	7	1.00	31	0.226
465	A	9	8	1.00	33	0.242
466	A	8	7	1.00	33	0.212
467	A	8	7	1.00	33	0.212
468	A	8	7	1.00	33	0.212
469	A	9	8	1.00	33	0.242
470	A	10	8	1.00	33	0.242
471	A	9	7	1.00	33	0.212
472	A	9	8	1.00	33	0.242
473	A	9	7	1.00	33	0.212
474	A	9	7	1.00	33	0.212
475	A	10	8	1.00	33	0.242
476	A	9	7	1.00	33	0.212
477	A	8	7	1.00	33	0.212
478	A	7	6	1.00	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
479	A	7	6	1.00	33	0.182
480	A	8	7	1.00	33	0.212
481	A	9	7	1.00	33	0.212
482	A	9	7	1.00	33	0.212
483	A	8	6	1.00	33	0.182
484	A	8	7	1.00	33	0.212
485	A	8	6	1.00	33	0.182
486	A	9	7	1.00	33	0.212
487	A	10	7	1.00	33	0.212
488	A	9	6	1.00	33	0.182
489	A	9	7	1.00	33	0.212
490	A	9	7	1.00	33	0.212
491	A	9	6	1.00	33	0.182
492	A	10	7	1.00	33	0.212
493	A	6	4	1.00	35	0.114
494	A	5	4	1.00	35	0.114
495	A	4	4	1.00	35	0.114
496	A	3	3	1.00	35	0.086
497	A	4	4	1.00	35	0.114
498	A	4	4	1.00	35	0.114
499	A	5	5	1.00	35	0.143
500	A	6	5	1.00	35	0.143
501	A	7	5	1.00	35	0.143
502	A	6	5	1.00	35	0.143
503	A	5	5	1.00	35	0.143
504	A	4	4	1.00	35	0.114

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
505	A	5	5	1.00	35	0.143
506	A	5	5	1.00	35	0.143
507	A	5	5	1.00	35	0.143
508	A	6	6	1.00	35	0.171
509	A	7	6	1.00	35	0.171
510	A	8	5	1.00	35	0.143
511	A	7	5	1.00	35	0.143
512	A	6	5	1.00	35	0.143
513	A	5	4	1.00	35	0.114
514	A	6	5	1.00	35	0.143
515	A	6	5	1.00	35	0.143
516	A	6	6	1.00	35	0.171
517	A	6	5	1.00	35	0.143
518	A	7	6	1.00	35	0.171
519	A	8	6	1.00	35	0.171
520	A	9	5	1.00	35	0.143
521	A	8	5	1.00	35	0.143
522	A	7	5	1.00	35	0.143
523	A	6	5	1.00	35	0.143
524	A	5	5	1.00	35	0.143
525	A	6	6	1.00	35	0.171
526	A	7	7	1.00	35	0.200
527	A	8	7	1.00	35	0.200
528	A	7	7	1.00	54	0.130
529	A	9	6	1.00	35	0.171
530	A	8	6	1.00	35	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
531	A	7	6	1.00	35	0.171
532	A	6	6	1.00	35	0.171
533	A	5	5	1.00	35	0.143
534	A	7	7	1.00	35	0.200
535	A	8	8	1.00	35	0.229
536	A	9	6	1.00	35	0.171
537	A	8	6	1.00	35	0.171
538	A	7	6	1.00	35	0.171
539	A	6	5	1.00	35	0.143
540	A	6	6	1.00	35	0.171
541	A	8	7	1.00	35	0.200
542	A	9	8	1.00	35	0.229
543	A	9	6	1.00	35	0.171
544	A	8	6	1.00	35	0.171
545	A	7	5	1.00	35	0.143
546	A	7	6	1.00	35	0.171
547	A	7	6	1.00	35	0.171
548	A	9	7	1.00	35	0.200
549	A	10	8	1.00	35	0.229
550	A	9	7	1.00	31	0.226
551	A	8	7	1.00	31	0.226
552	A	7	6	1.00	31	0.194
553	A	7	6	1.00	31	0.194
554	A	8	7	1.00	31	0.226
555	A	9	7	1.00	31	0.226
556	A	9	8	1.00	33	0.242

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
557	A	8	7	1.00	33	0.212
558	A	8	7	1.00	33	0.212
559	A	8	7	1.00	33	0.212
560	A	9	8	1.00	33	0.242
561	A	10	9	1.00	33	0.273
562	A	9	8	1.00	33	0.242
563	A	9	8	1.00	33	0.242
564	A	9	8	1.00	33	0.242
565	A	9	8	1.00	33	0.242
566	A	10	9	1.00	33	0.273
567	A	11	10	1.00	33	0.303
568	A	8	8	1.00	33	0.242
569	A	6	6	1.00	33	0.182
570	A	8	8	1.00	33	0.242
571	A	10	9	1.00	33	0.273
572	A	12	10	1.00	33	0.303
573	A	11	10	1.00	33	0.303
574	A	10	9	1.00	33	0.273
575	A	10	9	1.00	33	0.273
576	A	10	9	1.00	33	0.273
577	A	11	10	1.00	33	0.303
578	A	12	11	1.00	33	0.333
579	A	11	10	1.00	33	0.303
580	A	11	10	1.00	33	0.303
581	A	11	10	1.00	33	0.303
582	A	11	10	1.00	33	0.303

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
583	A	12	11	1.00	33	0.333
584	A	4	4	1.00	36	0.111
585	A	4	4	1.00	36	0.111
586	A	3	3	1.00	36	0.083
587	A	3	3	1.00	36	0.083
588	A	4	4	1.00	36	0.111
589	A	4	4	1.00	36	0.111
590	A	7	6	1.00	35	0.171
591	A	6	6	1.00	35	0.171
592	A	5	5	1.00	35	0.143
593	A	6	6	1.00	35	0.171
594	A	7	7	1.00	35	0.200
595	A	8	8	1.00	35	0.229
596	A	9	9	1.00	35	0.257
597	A	8	6	1.00	35	0.171
598	A	7	6	1.00	35	0.171
599	A	6	6	1.00	35	0.171
600	A	7	7	1.00	35	0.200
601	A	8	8	1.00	35	0.229
602	A	8	8	1.00	35	0.229
603	A	9	9	1.00	35	0.257
604	A	10	9	1.00	35	0.257
605	A	9	7	1.00	35	0.200
606	A	8	7	1.00	35	0.200
607	A	7	7	1.00	35	0.200
608	A	8	8	1.00	35	0.229

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
609	A	9	9	1.00	35	0.257
610	A	9	9	1.00	35	0.257
611	A	9	9	1.00	35	0.257
612	A	10	9	1.00	35	0.257
613	A	11	9	1.00	35	0.257
614	A	6	6	1.00	35	0.171
615	A	5	5	1.00	35	0.143
616	A	4	4	1.00	35	0.114
617	A	4	4	1.00	35	0.114
618	A	8	8	1.00	35	0.229
619	A	8	8	1.00	35	0.229
620	A	6	6	1.00	35	0.171
621	A	5	5	1.00	35	0.143
622	A	5	5	1.00	35	0.143
623	A	7	7	1.00	35	0.200
624	A	8	8	1.00	35	0.229
625	A	7	6	1.00	35	0.171
626	A	6	6	1.00	35	0.171
627	A	6	6	1.00	35	0.171
628	A	6	6	1.00	35	0.171
629	A	8	8	1.00	35	0.229
630	A	9	9	1.00	35	0.257
631	A	5	5	1.00	38	0.132
632	A	3	3	1.00	38	0.079
633	A	3	3	1.00	38	0.079
634	A	10	10	1.00	38	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
635	A	0	0	0.00	0	0.000
636	A	10	8	1.00	33	0.242
637	A	9	7	1.00	33	0.212
638	A	8	6	1.00	33	0.182
639	A	7	5	1.00	31	0.161
640	A	10	8	1.00	33	0.242
641	A	0	0	0.00	0	0.000
642	A	0	0	0.00	0	0.000
643	A	0	0	0.00	0	0.000
644	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=125

$$-\frac{a(5A+4B)\sin^3(c+dx)}{15d}+\frac{a(5A+4B)\sin(c+dx)}{5d}+\frac{a(A+B)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a(A+B)\sin(c+dx)}{8d}$$

```
[Out] 3/8*a*(A+B)*x+1/5*a*(5*A+4*B)*sin(d*x+c)/d+3/8*a*(A+B)*cos(d*x+c)*sin(d*x+c
)/d+1/4*a*(A+B)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*a*B*cos(d*x+c)^4*sin(d*x+c)/d
-1/15*a*(5*A+4*B)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.17, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3023, 2748, 2633, 2635, 8}

$$-\frac{a(5A+4B)\sin^3(c+dx)}{15d}+\frac{a(5A+4B)\sin(c+dx)}{5d}+\frac{a(A+B)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a(A+B)\sin(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3*a*(A + B)*x)/8 + (a*(5*A + 4*B)*Sin[c + d*x])/(5*d) + (3*a*(A + B)*Cos[c
 + d*x]*Sin[c + d*x])/(8*d) + (a*(A + B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
 + (a*B*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(5*A + 4*B)*Sin[c + d*x]^3)
/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^3(c+dx)\left(aA+(aA+aB)\cos(c+dx)+aB\,c\right.$$
$$= \frac{aB\cos^4(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\int \cos^3(c+dx)(a(5$$
$$= \frac{aB\cos^4(c+dx)\sin(c+dx)}{5d} + (a(A+B))\int \cos^4(c$$
$$= \frac{a(A+B)\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{aB\cos^4(c+dx}{5d}$$
$$= \frac{a(5A+4B)\sin(c+dx)}{5d} + \frac{3a(A+B)\cos(c+dx)\sin}{8d}$$
$$= \frac{3}{8}a(A+B)x + \frac{a(5A+4B)\sin(c+dx)}{5d} + \frac{3a(A+B)}{}$$

Mathematica [A] time = 0.27, size = 77, normalized size = 0.62

$$\frac{a\left(-160(A+2B)\sin^3(c+dx)+480(A+B)\sin(c+dx)+15(A+B)(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+d\right.}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(480*(A + B)*Sin[c + d*x] - 160*(A + 2*B)*Sin[c + d*x]^3 + 96*B*Sin[c +
d*x]^5 + 15*(A + B)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))
)/(480*d)
```

fricas [A] time = 1.08, size = 88, normalized size = 0.70

$$\frac{45\,(A+B)adx+\left(24\,Ba\cos\left(dx+c\right)^4+30\,(A+B)a\cos\left(dx+c\right)^3+8\,(5\,A+4\,B)a\cos\left(dx+c\right)^2+45\,(A+B)a\,c\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/120*(45*(A + B)*a*d*x + (24*B*a*cos(d*x + c)^4 + 30*(A + B)*a*cos(d*x + c
)^3 + 8*(5*A + 4*B)*a*cos(d*x + c)^2 + 45*(A + B)*a*cos(d*x + c) + 16*(5*A
+ 4*B)*a)*sin(d*x + c))/d
```

giac [A] time = 0.66, size = 112, normalized size = 0.90

$$\frac{3}{8}\,(Aa+Ba)x+\frac{Ba\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{(Aa+Ba)\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{(4\,Aa+5\,Ba)\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{(Aa+Ba)\sin\left(2\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 3/8*(A*a + B*a)*x + 1/80*B*a*sin(5*d*x + 5*c)/d + 1/32*(A*a + B*a)*sin(4*d*
x + 4*c)/d + 1/48*(4*A*a + 5*B*a)*sin(3*d*x + 3*c)/d + 1/4*(A*a + B*a)*sin(
2*d*x + 2*c)/d + 1/8*(6*A*a + 5*B*a)*sin(d*x + c)/d
```

maple [A] time = 0.07, size = 128, normalized size = 1.02

$$\frac{\frac{aB\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+aA\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+aB\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c}{4}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a*B*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*A*(2+cos(d*x+c)^2)*sin(d*x+c)
)
```

maxima [A] time = 0.76, size = 124, normalized size = 0.99

$$-\frac{160\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa-15\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Aa-32\left(3\sin(\right.}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/480*(160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 15*(12*d*x + 12*c + sin
(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 32*(3*sin(d*x + c)^5 - 10*sin(d*x
 + c)^3 + 15*sin(d*x + c))*B*a - 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*s
in(2*d*x + 2*c))*B*a)/d
```

mupad [B] time = 1.55, size = 236, normalized size = 1.89

$$\frac{\left(\frac{3Aa}{4}+\frac{3Ba}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(\frac{29Aa}{6}+\frac{13Ba}{6}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{20Aa}{3}+\frac{116Ba}{15}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{35Aa}{6}+\frac{19Ba}{6}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((13*A*a)/4 + (13*B*a)/4) + tan(c/2 + (d*x)/2)^9*((3*A*
a)/4 + (3*B*a)/4) + tan(c/2 + (d*x)/2)^7*((29*A*a)/6 + (13*B*a)/6) + tan(c/
2 + (d*x)/2)^3*((35*A*a)/6 + (19*B*a)/6) + tan(c/2 + (d*x)/2)^5*((20*A*a)/3
 + (116*B*a)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10
*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)
) - (3*a*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(A + B))/(4*d) + (3*a*atan((3
*a*tan(c/2 + (d*x)/2)*(A + B))/(4*((3*A*a)/4 + (3*B*a)/4)))*(A + B))/(4*d)
```

sympy [A] time = 2.16, size = 333, normalized size = 2.66

$$\begin{cases} \frac{3Aax\sin^4(c+dx)}{8} + \frac{3Aax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3Aax\cos^4(c+dx)}{8} + \frac{3Aa\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2Aa\sin^3(c+dx)}{3d} + \frac{5Aa\sin(c+dx)\cos \\ x(A+B\cos(c))(a\cos(c)+a)\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a*x*sin(c + d*x)**4/8 + 3*A*a*x*sin(c + d*x)**2*cos(c + d*x)
**2/4 + 3*A*a*x*cos(c + d*x)**4/8 + 3*A*a*sin(c + d*x)**3*cos(c + d*x)/(8*d
) + 2*A*a*sin(c + d*x)**3/(3*d) + 5*A*a*sin(c + d*x)*cos(c + d*x)**3/(8*d)
+ A*a*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a*x*sin(c + d*x)**4/8 + 3*B*a*x*
sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B*a*x*cos(c + d*x)**4/8 + 8*B*a*sin(c
 + d*x)**5/(15*d) + 4*B*a*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a*sin
(c + d*x)**3*cos(c + d*x)/(8*d) + B*a*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*
a*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c
) + a)*cos(c)**3, True))
```

3.2 $\int \cos^2(c + dx)(a + a\cos(c + dx))(A + B\cos(c + dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a(A+B)\sin^3(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(4A+3B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+3B)+\frac{aB\sin(c+d}{}$$

```
[Out] 1/8*a*(4*A+3*B)*x+a*(A+B)*sin(d*x+c)/d+1/8*a*(4*A+3*B)*cos(d*x+c)*sin(d*x+c
)/d+1/4*a*B*cos(d*x+c)^3*sin(d*x+c)/d-1/3*a*(A+B)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.15, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a(A+B)\sin^3(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(4A+3B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+3B)+\frac{aB\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(4*A + 3*B)*x)/8 + (a*(A + B)*Sin[c + d*x])/d + (a*(4*A + 3*B)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(A +
 B)*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^2(c+dx)\left(aA+(aA+aB)\cos(c+dx)+aB\,c\right.$$

$$= \frac{aB\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^2(c+dx)(a(4$$

$$= \frac{aB\cos^3(c+dx)\sin(c+dx)}{4d}+(a(A+B))\int \cos^3(c$$

$$= \frac{a(4A+3B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{aB\cos^3(c+d}{4}$$

$$= \frac{1}{8}a(4A+3B)x+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(4A+3B}{}$$

Mathematica [A] time = 0.26, size = 75, normalized size = 0.77

$$\frac{a\left(-32(A+B)\sin^3(c+dx)+96(A+B)\sin(c+dx)+24(A+B)\sin(2(c+dx))+48Ac+48Adx+3B\sin(4(c\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(48*A*c + 36*B*c + 48*A*d*x + 36*B*d*x + 96*(A + B)*Sin[c + d*x] - 32*(A
 + B)*Sin[c + d*x]^3 + 24*(A + B)*Sin[2*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))
/(96*d)
```

fricas [A] time = 0.98, size = 74, normalized size = 0.76

$$\frac{3\,(4\,A+3\,B)adx+\left(6\,Ba\cos\left(dx+c\right)^{3}+8\,(A+B)a\cos\left(dx+c\right)^{2}+3\,(4\,A+3\,B)a\cos\left(dx+c\right)+16\,(A+B)a\right)\sin}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(4*A + 3*B)*a*d*x + (6*B*a*cos(d*x + c)^3 + 8*(A + B)*a*cos(d*x + c
)^2 + 3*(4*A + 3*B)*a*cos(d*x + c) + 16*(A + B)*a)*sin(d*x + c))/d
```

giac [A] time = 0.91, size = 89, normalized size = 0.92

$$\frac{1}{8}\,(4\,Aa+3\,Ba)x+\frac{Ba\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{(Aa+Ba)\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{(Aa+Ba)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{3\,(Aa+Ba)\sin\left(d\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/8*(4*A*a + 3*B*a)*x + 1/32*B*a*sin(4*d*x + 4*c)/d + 1/12*(A*a + B*a)*sin(
3*d*x + 3*c)/d + 1/4*(A*a + B*a)*sin(2*d*x + 2*c)/d + 3/4*(A*a + B*a)*sin(d
*x + c)/d
```

maple [A] time = 0.06, size = 107, normalized size = 1.10

$$\frac{aB\left(\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{aA\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}+\frac{aB\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}+aA\left(\frac{\cos(dx+c)\sin(d}{2}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a
*A*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a*B*(2+cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.37, size = 101, normalized size = 1.04

$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa-24\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa+32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 24*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a - 3*(12*d*x + 12*
c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a)/d
```

mupad [B] time = 1.23, size = 212, normalized size = 2.19

$$\frac{\left(A\,a+\frac{3\,B\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{7\,A\,a}{3}+\frac{49\,B\,a}{12}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{13\,A\,a}{3}+\frac{31\,B\,a}{12}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(3\,A\,a+\frac{13\,B\,a}{4}\right)\,\mathrm{ta}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(3*A*a + (13*B*a)/4) + tan(c/2 + (d*x)/2)^7*(A*a + (3*B
*a)/4) + tan(c/2 + (d*x)/2)^3*((13*A*a)/3 + (31*B*a)/12) + tan(c/2 + (d*x)/
2)^5*((7*A*a)/3 + (49*B*a)/12))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d
*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (a*atan((a
*tan(c/2 + (d*x)/2)*(4*A + 3*B))/(4*(A*a + (3*B*a)/4)))*(4*A + 3*B))/(4*d)
- (a*(4*A + 3*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4*d)
```

sympy [A] time = 1.05, size = 252, normalized size = 2.60

$$\begin{cases} \frac{Aax\sin^2(c+dx)}{2}+\frac{Aax\cos^2(c+dx)}{2}+\frac{2Aa\sin^3(c+dx)}{3d}+\frac{Aa\sin(c+dx)\cos^2(c+dx)}{d}+\frac{Aa\sin(c+dx)\cos(c+dx)}{2d}+\frac{3Bax\sin^4(c+dx)}{8}+\frac{3Bax}{} \\ x\left(A+B\cos(c)\right)\left(a\cos(c)+a\right)\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + 2*A*a*sin(c
+ d*x)**3/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**2/d + A*a*sin(c + d*x)*cos
(c + d*x)/(2*d) + 3*B*a*x*sin(c + d*x)**4/8 + 3*B*a*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 + 3*B*a*x*cos(c + d*x)**4/8 + 3*B*a*sin(c + d*x)**3*cos(c + d*
x)/(8*d) + 2*B*a*sin(c + d*x)**3/(3*d) + 5*B*a*sin(c + d*x)*cos(c + d*x)**3
/(8*d) + B*a*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(
a*cos(c) + a)*cos(c)**2, True))
```

3.3 $\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=77

$$\frac{a(3A+2B)\sin(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+B)+\frac{aB\sin(c+dx)\cos^2(c+dx)}{3d}$$

```
[Out] 1/2*a*(A+B)*x+1/3*a*(3*A+2*B)*sin(d*x+c)/d+1/2*a*(A+B)*cos(d*x+c)*sin(d*x+c
)/d+1/3*a*B*cos(d*x+c)^2*sin(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2968, 3023, 2734}

$$\frac{a(3A+2B)\sin(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+B)+\frac{aB\sin(c+dx)\cos^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(A + B)*x)/2 + (a*(3*A + 2*B)*Sin[c + d*x])/(3*d) + (a*(A + B)*Cos[c + d
*x]*Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos(c+dx)\left(aA+(aA+aB)\cos(c+dx)+aB\cos\right.$$
$$= \frac{aB\cos^2(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\int \cos(c+dx)(a(3A$$
$$= \frac{1}{2}a(A+B)x + \frac{a(3A+2B)\sin(c+dx)}{3d} + \frac{a(A+B)\mathrm{c}}{}$$

Mathematica [A] time = 0.19, size = 65, normalized size = 0.84

$$\frac{a(3(4A+3B)\sin(c+dx)+3(A+B)\sin(2(c+dx))+6Ac+6Adx+B\sin(3(c+dx))+6Bc+6Bdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(6*A*c + 6*B*c + 6*A*d*x + 6*B*d*x + 3*(4*A + 3*B)*Sin[c + d*x] + 3*(A +
 B)*Sin[2*(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

fricas [A] time = 1.08, size = 56, normalized size = 0.73

$$\frac{3\,(A+B)adx + \left(2\,Ba\cos\left(dx+c\right)^2 + 3\,(A+B)a\cos\left(dx+c\right) + 2\,(3\,A+2\,B)a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/6*(3*(A + B)*a*d*x + (2*B*a*cos(d*x + c)^2 + 3*(A + B)*a*cos(d*x + c) + 2
*(3*A + 2*B)*a)*sin(d*x + c))/d
```

giac [A] time = 0.36, size = 68, normalized size = 0.88

$$\frac{1}{2}\,(Aa+Ba)x + \frac{Ba\sin\left(3\,dx+3\,c\right)}{12\,d} + \frac{(Aa+Ba)\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{(4\,Aa+3\,Ba)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(A*a + B*a)*x + 1/12*B*a*sin(3*d*x + 3*c)/d + 1/4*(A*a + B*a)*sin(2*d*x
 + 2*c)/d + 1/4*(4*A*a + 3*B*a)*sin(d*x + c)/d
```

maple [A] time = 0.06, size = 85, normalized size = 1.10

$$\frac{\frac{aB\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3} + aA\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + aB\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + aA\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*B*(2+cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*A*sin(d*x+c))
```

maxima [A] time = 0.58, size = 79, normalized size = 1.03

$$\frac{3\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa - 4\left(\sin(dx+c)^3 - 3\,\sin(dx+c)\right)Ba + 3\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Ba + 12}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a - 4*(sin(d*x + c)^3 - 3*sin(d*
x + c))*B*a + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 12*A*a*sin(d*x + c))
/d
```

mupad [B] time = 0.23, size = 84, normalized size = 1.09

$$\frac{A\,a\,x}{2}+\frac{B\,a\,x}{2}+\frac{A\,a\,\sin(c+d\,x)}{d}+\frac{3\,B\,a\,\sin(c+d\,x)}{4\,d}+\frac{A\,a\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{B\,a\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{B\,a\,\sin(3\,c+3\,d}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] (A*a*x)/2 + (B*a*x)/2 + (A*a*sin(c + d*x))/d + (3*B*a*sin(c + d*x))/(4*d) +
 (A*a*sin(2*c + 2*d*x))/(4*d) + (B*a*sin(2*c + 2*d*x))/(4*d) + (B*a*sin(3*c
 + 3*d*x))/(12*d)
```

sympy [A] time = 0.53, size = 168, normalized size = 2.18

$$\begin{cases} \frac{Aax\sin^2(c+dx)}{2} + \frac{Aax\cos^2(c+dx)}{2} + \frac{Aa\sin(c+dx)\cos(c+dx)}{2d} + \frac{Aa\sin(c+dx)}{d} + \frac{Bax\sin^2(c+dx)}{2} + \frac{Bax\cos^2(c+dx)}{2} + \frac{2Ba\sin^3(c+dx)}{3d} + \\ x\,(A + B\cos(c))\,(a\cos(c) + a)\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
d*x)*cos(c + d*x)/(2*d) + A*a*sin(c + d*x)/d + B*a*x*sin(c + d*x)**2/2 + B*
a*x*cos(c + d*x)**2/2 + 2*B*a*sin(c + d*x)**3/(3*d) + B*a*sin(c + d*x)*cos(
c + d*x)**2/d + B*a*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A + B*c
os(c))*(a*cos(c) + a)*cos(c), True))
```

3.4 $\int (a + a\cos(c + dx))(A + B\cos(c + dx))\,dx$

Optimal. Leaf size=47

$$\frac{a(A+B)\sin(c+dx)}{d} + \frac{1}{2}ax(2A+B) + \frac{aB\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] 1/2*a*(2*A+B)*x+a*(A+B)*sin(d*x+c)/d+1/2*a*B*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2734}

$$\frac{a(A+B)\sin(c+dx)}{d} + \frac{1}{2}ax(2A+B) + \frac{aB\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(2*A + B)*x)/2 + (a*(A + B)*Sin[c + d*x])/d + (a*B*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a + a\cos(c+dx))(A + B\cos(c+dx))\,dx = \frac{1}{2}a(2A+B)x + \frac{a(A+B)\sin(c+dx)}{d} + \frac{aB\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.10, size = 44, normalized size = 0.94

$$\frac{a(4(A+B)\sin(c+dx) + 4Adx + B\sin(2(c+dx)) + 2Bc + 2Bdx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*(2*B*c + 4*A*d*x + 2*B*d*x + 4*(A + B)*Sin[c + d*x] + B*Sin[2*(c + d*x)]
))/(4*d)
```

fricas [A] time = 1.06, size = 38, normalized size = 0.81

$$\frac{(2\,A+B)adx+(Ba\cos\left(dx+c\right)+2\,(A+B)a)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((2*A + B)*a*d*x + (B*a*cos(d*x + c) + 2*(A + B)*a)*sin(d*x + c))/d
```

giac [A] time = 0.50, size = 45, normalized size = 0.96

$$\frac{1}{2}\,(2\,Aa+Ba)x+\frac{Ba\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{(Aa+Ba)\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a + B*a)*x + 1/4*B*a*sin(2*d*x + 2*c)/d + (A*a + B*a)*sin(d*x + c)
/d
```

maple [A] time = 0.06, size = 57, normalized size = 1.21

$$\frac{aB\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+aA\sin\left(dx+c\right)+aB\sin\left(dx+c\right)+aA\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(a*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*A*sin(d*x+c)+a*B*sin(d
*x+c)+a*A*(d*x+c))
```

maxima [A] time = 0.38, size = 55, normalized size = 1.17

$$\frac{4\,(dx+c)Aa+(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba+4\,Aa\sin\left(dx+c\right)+4\,Ba\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 4*A*a*sin(d*x
 + c) + 4*B*a*sin(d*x + c))/d
```

mupad [B] time = 0.19, size = 50, normalized size = 1.06

$$A\,a\,x+\frac{B\,a\,x}{2}+\frac{A\,a\,\sin\left(c+d\,x\right)}{d}+\frac{B\,a\,\sin\left(c+d\,x\right)}{d}+\frac{B\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] A*a*x + (B*a*x)/2 + (A*a*sin(c + d*x))/d + (B*a*sin(c + d*x))/d + (B*a*sin(
2*c + 2*d*x))/(4*d)
```

sympy [A] time = 0.25, size = 94, normalized size = 2.00

$$\begin{cases} Aax + \frac{Aa\sin{(c+dx)}}{d} + \frac{Bax\sin^2{(c+dx)}}{2} + \frac{Bax\cos^2{(c+dx)}}{2} + \frac{Ba\sin{(c+dx)}\cos{(c+dx)}}{2d} + \frac{Ba\sin{(c+dx)}}{d} & \text{for}\ d \neq 0 \\ x\left(A + B\cos{(c)}\right)\left(a\cos{(c)} + a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x + A*a*sin(c + d*x)/d + B*a*x*sin(c + d*x)**2/2 + B*a*x*cos
(c + d*x)**2/2 + B*a*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*sin(c + d*x)/d,
Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a), True))
```

3.5 $\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec(c + dx)\,dx$

Optimal. Leaf size=32

$$ax(A + B) + \frac{aA\tanh^{-1}(\sin(c + dx))}{d} + \frac{aB\sin(c + dx)}{d}$$

```
[Out] a*(A+B)*x+a*A*arctanh(sin(d*x+c))/d+a*B*sin(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2968, 3023, 2735, 3770}

$$ax(A + B) + \frac{aA\tanh^{-1}(\sin(c + dx))}{d} + \frac{aB\sin(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] a*(A + B)*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*Sin[c + d*x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx = \int \left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)\right)\mathrm{se}$$
$$= \frac{aB\sin(c+dx)}{d} + \int (aA+a(A+B)\cos(c+dx))\sec(c$$
$$= a(A+B)x + \frac{aB\sin(c+dx)}{d} + (aA)\int \sec(c+dx)\,dx$$
$$= a(A+B)x + \frac{aA\tanh^{-1}(\sin(c+dx))}{d} + \frac{aB\sin(c+dx)}{d}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.44

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d} + aAx + \frac{aB\sin(c)\cos(dx)}{d} + \frac{aB\cos(c)\sin(dx)}{d} + aBx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] a*A*x + a*B*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*Cos[d*x]*Sin[c])/d + (
a*B*Cos[c]*Sin[d*x])/d
```

fricas [A] time = 0.94, size = 51, normalized size = 1.59

$$\frac{2\,(A+B)adx + Aa\log\left(\sin\left(dx+c\right)+1\right) - Aa\log\left(-\sin\left(dx+c\right)+1\right) + 2\,Ba\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(A + B)*a*d*x + A*a*log(sin(d*x + c) + 1) - A*a*log(-sin(d*x + c) +
1) + 2*B*a*sin(d*x + c))/d
```

giac [B] time = 0.80, size = 79, normalized size = 2.47

$$\frac{Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + (Aa+Ba)(dx+c) + \frac{2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

```
[Out] (A*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - A*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (A*a + B*a)*(d*x + c) + 2*B*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

maple [A] time = 0.11, size = 56, normalized size = 1.75

$$aAx + aBx + \frac{aA\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d} + \frac{Aac}{d} + \frac{aB\sin\left(dx+c\right)}{d} + \frac{Bac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a*A*x+a*B*x+1/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a*c+a*B*sin(d*x+c)/d+1/
d*B*a*c
```

maxima [A] time = 0.57, size = 47, normalized size = 1.47

$$\frac{(dx+c)Aa + (dx+c)Ba + Aa\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right) + Ba\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima
")
```

```
[Out] ((d*x + c)*A*a + (d*x + c)*B*a + A*a*log(sec(d*x + c) + tan(d*x + c)) + B*a
*sin(d*x + c))/d
```

mupad [B] time = 0.28, size = 100, normalized size = 3.12

$$\frac{B\,a\,\sin\left(c+d\,x\right)}{d} + \frac{2\,A\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,A\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x),x)
```

```
[Out] (B*a*sin(c + d*x))/d + (2*A*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d + (2*A*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec\left(c+dx\right)dx+\int A\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos^{2}\left(c+dx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a*(Integral(A*sec(c + d*x), x) + Integral(A*cos(c + d*x)*sec(c + d*x), x) +
 Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(B*cos(c + d*x)**2*sec(
c + d*x), x))
```

3.6 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=32

$$\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+aBx$$

```
[Out] a*B*x+a*(A+B)*arctanh(sin(d*x+c))/d+a*A*tan(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3021, 2735, 3770}

$$\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+aBx$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] a*B*x + (a*(A + B)*ArcTanh[Sin[c + d*x]])/d + (a*A*Tan[c + d*x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int \left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)\right)\text{se}$$
$$= \frac{aA\tan(c+dx)}{d} + \int (a(A+B)+aB\cos(c+dx))\,\text{sec}($$
$$= aBx + \frac{aA\tan(c+dx)}{d} + (a(A+B))\int \sec(c+dx)\,dx$$
$$= aBx + \frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d} + \frac{aA\tan(c+dx}{d}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.34

$$\frac{aA\tan(c+dx)}{d} + \frac{aA\tanh^{-1}(\sin(c+dx))}{d} + \frac{aB\tanh^{-1}(\sin(c+dx))}{d} + aBx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] a*B*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*
A*Tan[c + d*x])/d
```

fricas [B] time = 0.62, size = 79, normalized size = 2.47

$$\frac{2\,Badx\cos(dx+c) + (A+B)a\cos(dx+c)\log(\sin(dx+c)+1) - (A+B)a\cos(dx+c)\log(-\sin(dx+c)+1)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fric
as")
```

```
[Out] 1/2*(2*B*a*d*x*cos(d*x + c) + (A + B)*a*cos(d*x + c)*log(sin(d*x + c) + 1)
- (A + B)*a*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*A*a*sin(d*x + c))/(d*co
s(d*x + c))
```

giac [B] time = 0.35, size = 84, normalized size = 2.62

$$\frac{(dx+c)Ba+(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac
")
```

```
[Out] ((d*x + c)*B*a + (A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + B*
a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

maple [A] time = 0.12, size = 65, normalized size = 2.03

$$aBx+\frac{aA\tan(dx+c)}{d}+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aB\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Bac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] a*B*x+a*A*tan(d*x+c)/d+1/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*B*ln(sec(d*x
+c)+tan(d*x+c))+1/d*B*a*c
```

maxima [B] time = 0.61, size = 73, normalized size = 2.28

$$\frac{2\,(dx+c)Ba+Aa\Big(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\Big)+Ba\Big(\log(\sin(dx+c)+1)-\log(\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(d*x + c)*B*a + A*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*A*a*tan(d*x + c))
/d
```

mupad [B] time = 0.31, size = 100, normalized size = 3.12

$$\frac{A\,a\tan(c+d\,x)}{d}+\frac{2\,A\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] (A*a*tan(c + d*x))/d + (2*A*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d + (2*B*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a*atanh(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^2(c+dx)\,dx+\int A\cos(c+dx)\sec^2(c+dx)\,dx+\int B\cos(c+dx)\sec^2(c+dx)\,dx+\int B\cos^2(c+d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**2, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**2
, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x
)**2*sec(c + d*x)**2, x))
```

3.7 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=56

$$\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 1/2*a*(A+2*B)*arctanh(sin(d*x+c))/d+a*(A+B)*tan(d*x+c)/d+1/2*a*A*sec(d*x+c)
*tan(d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*(A + 2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(A + B)*Tan[c + d*x])/d + (a
*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int \left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)\right)\text{se}$$

$$= \frac{aA\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (2a(A+B)+a(A$$

$$= \frac{aA\sec(c+dx)\tan(c+dx)}{2d}+(a(A+B))\int \sec^2(c+$$

$$= \frac{a(A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\sec(c+dx)\tan(}{2d}$$

$$= \frac{a(A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\tan(c+d}{d}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 1.34

$$\frac{aA\tan(c+dx)}{d}+\frac{aA\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+\frac{aB\tan(c+dx)}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/(2*d) + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Ta
n[c + d*x])/d + (a*B*Tan[c + d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

fricas [A] time = 0.79, size = 89, normalized size = 1.59

$$\frac{(A+2\,B)a\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(A+2\,B)a\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\left(A+B\right)a\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fric
as")
```

```
[Out] 1/4*((A + 2*B)*a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A + 2*B)*a*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 2*(2*(A + B)*a*cos(d*x + c) + A*a)*sin(d*x
 + c))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.47, size = 124, normalized size = 2.21

$$\frac{(Aa+2\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa+2\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ba\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac
")
```

```
[Out] 1/2*((A*a + 2*B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + 2*B*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*tan(1
/2*d*x + 1/2*c)^3 - 3*A*a*tan(1/2*d*x + 1/2*c) - 2*B*a*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.14, size = 86, normalized size = 1.54

$$\frac{aA\tan\left(dx+c\right)}{d}+\frac{aB\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{aA\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{aA\ln\left(\sec\left(dx+c\right)+\tan\left(dx\right.\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] a*A*tan(d*x+c)/d+1/d*a*B*ln(sec(d*x+c)+tan(d*x+c))+1/2*a*A*sec(d*x+c)*tan(d
*x+c)/d+1/2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*B*tan(d*x+c)
```

maxima [A] time = 0.37, size = 95, normalized size = 1.70

$$-\frac{Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-2\,Ba\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right.\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/4*(A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 2*B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 4*A*a*tan(d*x + c) - 4*B*a*tan(d*x + c))/d
```

mupad [B] time = 0.83, size = 94, normalized size = 1.68

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,A\,a+2\,B\,a\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(A\,a+2\,B\,a\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A+2\,B\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(3*A*a + 2*B*a) - tan(c/2 + (d*x)/2)^3*(A*a + 2*B*a))/(
d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (a*atanh(tan(c/2 +
 (d*x)/2))*(A + 2*B))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^3(c+dx)\,dx+\int A\cos(c+dx)\sec^3(c+dx)\,dx+\int B\cos(c+dx)\sec^3(c+dx)\,dx+\int B\cos^2(c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**3, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**3
, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(B*cos(c + d*x
)**2*sec(c + d*x)**3, x))
```

3.8 $\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^4(c + dx)\,dx$

Optimal. Leaf size=86

$$\frac{a(2A+3B)\tan(c+dx)}{3d}+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

```
[Out] 1/2*a*(A+B)*arctanh(sin(d*x+c))/d+1/3*a*(2*A+3*B)*tan(d*x+c)/d+1/2*a*(A+B)*
sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a(2A+3B)\tan(c+dx)}{3d}+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a*(A + B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*A + 3*B)*Tan[c + d*x])/(3*d
) + (a*(A + B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \int \left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)\right)\text{se}$$

$$= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}\int (3a(A+B)+a(2$$

$$= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d} + (a(A+B))\int \sec^3(c+$$

$$= \frac{a(A+B)\sec(c+dx)\tan(c+dx)}{2d} + \frac{aA\sec^2(c+dx)\,\text{t}}{3d}$$

$$= \frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a(2A+3B)\tan(c+}{3d}$$

Mathematica [A] time = 0.35, size = 56, normalized size = 0.65

$$\frac{a\left(3(A+B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3(A+B)\sec(c+dx)+6(A+B)+2A\tan^2(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a*(3*(A + B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*(A + B) + 3*(A + B)*S
ec[c + d*x] + 2*A*Tan[c + d*x]^2)))/(6*d)
```

fricas [A] time = 0.73, size = 105, normalized size = 1.22

$$\frac{3\,(A+B)a\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(A+B)a\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(2\,A+3\right.}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(A + B)*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(A + B)*a*cos(d*
x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(2*A + 3*B)*a*cos(d*x + c)^2 + 3*(A
+ B)*a*cos(d*x + c) + 2*A*a)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 2.50, size = 154, normalized size = 1.79

$$\frac{3\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ba\tan\left(\frac{1}{2}\right.\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac
")
```

```
[Out] 1/6*(3*(A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(A*a + B*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a*tan(1/2*d*x + 1/2*c)^5 + 3*B*a*tan
(1/2*d*x + 1/2*c)^5 - 4*A*a*tan(1/2*d*x + 1/2*c)^3 - 12*B*a*tan(1/2*d*x + 1
/2*c)^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 9*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2
*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.17, size = 128, normalized size = 1.49

$$\frac{aA\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{aA\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{aB\tan\left(dx+c\right)}{d}+\frac{2aA\tan\left(dx+c\right)}{3d}+\frac{aA\left(\sec^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 1/2*a*A*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*B
*tan(d*x+c)+2/3*a*A*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*a*
B*sec(d*x+c)*tan(d*x+c)+1/2/d*a*B*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.37, size = 127, normalized size = 1.48

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Aa-3\,Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ba\left(\cdots\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a - 3*A*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*a*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*B*a*tan(d*x + c))/d
```

mupad [B] time = 2.07, size = 126, normalized size = 1.47

$$\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A+B\right)}{d}-\frac{\left(A\,a+B\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{4\,A\,a}{3}-4\,B\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(3\,A\,a+3\,B\,a\right)\,\mathrm{ta}\cdots}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] (a*atanh(tan(c/2 + (d*x)/2))*(A + B))/d - (tan(c/2 + (d*x)/2)*(3*A*a + 3*B*
a) + tan(c/2 + (d*x)/2)^5*(A*a + B*a) - tan(c/2 + (d*x)/2)^3*((4*A*a)/3 + 4
*B*a))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x
)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^{4}\left(c+dx\right)dx+\int A\cos\left(c+dx\right)\sec^{4}\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec^{4}\left(c+dx\right)dx+\int B\cos^{2}\left(c+\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**4, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**4
, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**4, x) + Integral(B*cos(c + d*x
)**2*sec(c + d*x)**4, x))
```

3.9 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=106

$$\frac{a(A+B)\tan^3(c+dx)}{3d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3A+4B)\tan(c+dx)\sec(c+}{8d}$$

```
[Out] 1/8*a*(3*A+4*B)*arctanh(sin(d*x+c))/d+a*(A+B)*tan(d*x+c)/d+1/8*a*(3*A+4*B)*
sec(d*x+c)*tan(d*x+c)/d+1/4*a*A*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*(A+B)*tan(d
*x+c)^3/d
```

Rubi [A] time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3021, 2748, 3767, 3768, 3770}

$$\frac{a(A+B)\tan^3(c+dx)}{3d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(3A+4B)\tan(c+dx)\sec(c+}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a*(3*A + 4*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(A + B)*Tan[c + d*x])/d +
(a*(3*A + 4*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*A*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (a*(A + B)*Tan[c + d*x]^3)/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
```

```
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \int \left(aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)\right)$$

$$= \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (4a(A+B)+a($$

$$= \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+(a(A+B))\int \sec^4(c$$

$$= \frac{a(3A+4B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aA\sec^3(c+}{}$$

$$= \frac{a(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(A+B)\tan(c+}{d}$$

Mathematica [A] time = 0.42, size = 77, normalized size = 0.73

$$\frac{a\left(3(3A+4B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\left(8(A+B)(\cos(2(c+dx))+2)\sec(c+dx)+6A\sec\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a*(3*(3*A + 4*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(9*A + 12*B + 8*(A +
 B)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*A*Sec[c + d*x]^2)*Tan[c + d*x])
)/(24*d)
```

fricas [A] time = 0.68, size = 127, normalized size = 1.20

$$\frac{3\,(3\,A+4\,B)a\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,(3\,A+4\,B)a\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,(A\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(3*A + 4*B)*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A + 4*B)*
a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(A + B)*a*cos(d*x + c)^3 +
3*(3*A + 4*B)*a*cos(d*x + c)^2 + 8*(A + B)*a*cos(d*x + c) + 6*A*a)*sin(d*x
+ c))/(d*cos(d*x + c)^4)
```

giac [A] time = 1.69, size = 188, normalized size = 1.77

$$3\,(3\,Aa+4\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(3\,Aa+4\,Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac
")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a + 4*B
*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9*A*a*tan(1/2*d*x + 1/2*c)^7 +
12*B*a*tan(1/2*d*x + 1/2*c)^7 - 49*A*a*tan(1/2*d*x + 1/2*c)^5 - 28*B*a*tan(
1/2*d*x + 1/2*c)^5 + 31*A*a*tan(1/2*d*x + 1/2*c)^3 + 52*B*a*tan(1/2*d*x + 1
/2*c)^3 - 39*A*a*tan(1/2*d*x + 1/2*c) - 36*B*a*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.16, size = 171, normalized size = 1.61

$$\frac{2aA\tan\left(dx+c\right)}{3d}+\frac{aA\left(\sec^{2}\left(dx+c\right)\right)\tan\left(dx+c\right)}{3d}+\frac{aB\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{aB\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right.\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 2/3*a*A*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*a*B*sec(d*x+c)
*tan(d*x+c)+1/2/d*a*B*ln(sec(d*x+c)+tan(d*x+c))+1/4*a*A*sec(d*x+c)^3*tan(d*
x+c)/d+3/8*a*A*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+
2/3/d*a*B*tan(d*x+c)+1/3/d*a*B*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.46, size = 163, normalized size = 1.54

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa+16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba-3\,Aa\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*a - 3*A*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 2.66, size = 166, normalized size = 1.57

$$\frac{\left(-\frac{3\,A\,a}{4}-B\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{49\,A\,a}{12}+\frac{7\,B\,a}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{31\,A\,a}{12}-\frac{13\,B\,a}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{13\,A\,a}{4}+3\,B\,a\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((13*A*a)/4 + 3*B*a) - tan(c/2 + (d*x)/2)^7*((3*A*a)/4
+ B*a) - tan(c/2 + (d*x)/2)^3*((31*A*a)/12 + (13*B*a)/3) + tan(c/2 + (d*x)/
2)^5*((49*A*a)/12 + (7*B*a)/3))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d
*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (a*atanh(t
an(c/2 + (d*x)/2))*(3*A + 4*B))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.10 $\int \cos^3(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=191

$$-\frac{a^2(9A+8B)\sin^3(c+dx)}{15d}+\frac{a^2(9A+8B)\sin(c+dx)}{5d}+\frac{a^2(6A+7B)\sin(c+dx)\cos^4(c+dx)}{30d}+\frac{a^2(12A+11B)\mathrm{si}}{}$$

[Out] 1/16*a^2*(12*A+11*B)*x+1/5*a^2*(9*A+8*B)*sin(d*x+c)/d+1/16*a^2*(12*A+11*B)*cos(d*x+c)*sin(d*x+c)/d+1/24*a^2*(12*A+11*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/30*a^2*(6*A+7*B)*cos(d*x+c)^4*sin(d*x+c)/d+1/6*B*cos(d*x+c)^4*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d-1/15*a^2*(9*A+8*B)*sin(d*x+c)^3/d

Rubi [A] time = 0.31, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2976, 2968, 3023, 2748, 2633, 2635, 8}

$$-\frac{a^2(9A+8B)\sin^3(c+dx)}{15d}+\frac{a^2(9A+8B)\sin(c+dx)}{5d}+\frac{a^2(6A+7B)\sin(c+dx)\cos^4(c+dx)}{30d}+\frac{a^2(12A+11B)\mathrm{si}}{}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]

[Out] (a^2*(12*A + 11*B)*x)/16 + (a^2*(9*A + 8*B)*Sin[c + d*x])/(5*d) + (a^2*(12*A + 11*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(12*A + 11*B)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^2*(6*A + 7*B)*Cos[c + d*x]^4*Sin[c + d*x])/(30*d) + (B*Cos[c + d*x]^4*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(6*d) - (a^2*(9*A + 8*B)*Sin[c + d*x]^3)/(15*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{B\cos^4(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{6d} +$$
$$= \frac{B\cos^4(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{6d} +$$
$$= \frac{a^2(6A+7B)\cos^4(c+dx)\sin(c+dx)}{30d} + \frac{B\cos^4(c+$$
$$= \frac{a^2(6A+7B)\cos^4(c+dx)\sin(c+dx)}{30d} + \frac{B\cos^4(c+$$
$$= \frac{a^2(12A+11B)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a^2(6A+$$
$$= \frac{a^2(9A+8B)\sin(c+dx)}{5d} + \frac{a^2(12A+11B)\cos(c+}{16d}$$
$$= \frac{1}{16}a^2(12A+11B)x + \frac{a^2(9A+8B)\sin(c+dx)}{5d} + \frac{a}{}$$

Mathematica [A] time = 0.65, size = 134, normalized size = 0.70

$$\frac{a^2(120(11A+10B)\sin(c+dx)+15(32A+31B)\sin(2(c+dx))+180A\sin(3(c+dx))+60A\sin(4(c+dx))+1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(660*B*c + 720*A*d*x + 660*B*d*x + 120*(11*A + 10*B)*Sin[c + d*x] + 15
*(32*A + 31*B)*Sin[2*(c + d*x)] + 180*A*Sin[3*(c + d*x)] + 200*B*Sin[3*(c +
 d*x)] + 60*A*Sin[4*(c + d*x)] + 75*B*Sin[4*(c + d*x)] + 12*A*Sin[5*(c + d*
x)] + 24*B*Sin[5*(c + d*x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

fricas [A] time = 0.56, size = 130, normalized size = 0.68

$$\frac{15\,(12\,A+11\,B)a^2dx+\left(40\,Ba^2\cos\left(dx+c\right)^5+48\,(A+2\,B)a^2\cos\left(dx+c\right)^4+10\,(12\,A+11\,B)a^2\cos\left(dx+c\right)^3+}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(12*A + 11*B)*a^2*d*x + (40*B*a^2*cos(d*x + c)^5 + 48*(A + 2*B)*a
^2*cos(d*x + c)^4 + 10*(12*A + 11*B)*a^2*cos(d*x + c)^3 + 16*(9*A + 8*B)*a^
```

```
2*cos(d*x + c)^2 + 15*(12*A + 11*B)*a^2*cos(d*x + c) + 32*(9*A + 8*B)*a^2)*
sin(d*x + c))/d
```

giac [A] time = 0.41, size = 166, normalized size = 0.87

$$\frac{Ba^2\sin(6\,dx+6\,c)}{192\,d}+\frac{1}{16}\left(12\,Aa^2+11\,Ba^2\right)x+\frac{\left(Aa^2+2\,Ba^2\right)\sin(5\,dx+5\,c)}{80\,d}+\frac{\left(4\,Aa^2+5\,Ba^2\right)\sin(4\,dx+4\,c)}{64\,d}+\frac{(9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*B*a^2*sin(6*d*x + 6*c)/d + 1/16*(12*A*a^2 + 11*B*a^2)*x + 1/80*(A*a^2
 + 2*B*a^2)*sin(5*d*x + 5*c)/d + 1/64*(4*A*a^2 + 5*B*a^2)*sin(4*d*x + 4*c)/
d + 1/48*(9*A*a^2 + 10*B*a^2)*sin(3*d*x + 3*c)/d + 1/64*(32*A*a^2 + 31*B*a^
2)*sin(2*d*x + 2*c)/d + 1/8*(11*A*a^2 + 10*B*a^2)*sin(d*x + c)/d
```

maple [A] time = 0.08, size = 217, normalized size = 1.14

$$\frac{\frac{a^2A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+B\,a^2\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+2a^2A\left(\frac{\left(\cos^3(dx+c)\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+B*a^2*(1/6*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+2
*a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/5*B*a
^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+1/3*a^2*A*(2+cos(d*x+c)^2
)*sin(d*x+c)+B*a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/
8*c))
```

maxima [A] time = 0.65, size = 216, normalized size = 1.13

$$\frac{64\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^2-320\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2+60\,(12\,dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 -
320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 + 60*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 + 128*(3*sin(d*x + c)^5 - 10*sin(d*x +
c)^3 + 15*sin(d*x + c))*B*a^2 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*a^2 + 30*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2)/d
```

mupad [B] time = 1.59, size = 315, normalized size = 1.65

$$\frac{\left(\frac{3\,A\,a^2}{2}+\frac{11\,B\,a^2}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{17\,A\,a^2}{2}+\frac{187\,B\,a^2}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(\frac{107\,A\,a^2}{5}+\frac{331\,B\,a^2}{20}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+\left(\frac{117\,A}{5}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+20\,\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((13*A*a^2)/2 + (53*B*a^2)/8) + tan(c/2 + (d*x)/2)^11*(
(3*A*a^2)/2 + (11*B*a^2)/8) + tan(c/2 + (d*x)/2)^3*((31*A*a^2)/2 + (87*B*a^
2)/8) + tan(c/2 + (d*x)/2)^9*((17*A*a^2)/2 + (187*B*a^2)/24) + tan(c/2 + (d
*x)/2)^7*((107*A*a^2)/5 + (331*B*a^2)/20) + tan(c/2 + (d*x)/2)^5*((117*A*a^
2)/5 + (501*B*a^2)/20))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^
4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2
)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - (a^2*(12*A + 11*B)*(atan(tan(c/2 + (d*
x)/2)) - (d*x)/2))/(8*d) + (a^2*atan((a^2*tan(c/2 + (d*x)/2)*(12*A + 11*B))
/(8*((3*A*a^2)/2 + (11*B*a^2)/8)))*(12*A + 11*B))/(8*d)
```

sympy [A] time = 4.34, size = 600, normalized size = 3.14

$$\begin{cases} \frac{3Aa^2x\sin^4(c+dx)}{4}+\frac{3Aa^2x\sin^2(c+dx)\cos^2(c+dx)}{2}+\frac{3Aa^2x\cos^4(c+dx)}{4}+\frac{8Aa^2\sin^5(c+dx)}{15d}+\frac{4Aa^2\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{3Aa^2\sin^3(\ldots}{} \\ x\left(A+B\cos(c)\right)\left(a\cos(c)+a\right)^2\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**2*x*sin(c + d*x)**4/4 + 3*A*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/2 + 3*A*a**2*x*cos(c + d*x)**4/4 + 8*A*a**2*sin(c + d*x)**5/(15*d
) + 4*A*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*A*a**2*sin(c + d*x)*
*3*cos(c + d*x)/(4*d) + 2*A*a**2*sin(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x
)*cos(c + d*x)**4/d + 5*A*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + A*a**2*
sin(c + d*x)*cos(c + d*x)**2/d + 5*B*a**2*x*sin(c + d*x)**6/16 + 15*B*a**2*
```

```
x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*B*a**2*x*sin(c + d*x)**4/8 + 15*B*
a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*B*a**2*x*sin(c + d*x)**2*cos(
c + d*x)**2/4 + 5*B*a**2*x*cos(c + d*x)**6/16 + 3*B*a**2*x*cos(c + d*x)**4/
8 + 5*B*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 16*B*a**2*sin(c + d*x)**
5/(15*d) + 5*B*a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 8*B*a**2*sin(c
+ d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*
d) + 11*B*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 2*B*a**2*sin(c + d*x)*
cos(c + d*x)**4/d + 5*B*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)),
 (x*(A + B*cos(c))*(a*cos(c) + a)**2*cos(c)**3, True))
```

3.11 $\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=160

$$-\frac{a^2(10A+9B)\sin^3(c+dx)}{15d}+\frac{a^2(10A+9B)\sin(c+dx)}{5d}+\frac{a^2(5A+6B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{a^2(7A+6B)\mathrm{si}}{}$$

```
[Out] 1/8*a^2*(7*A+6*B)*x+1/5*a^2*(10*A+9*B)*sin(d*x+c)/d+1/8*a^2*(7*A+6*B)*cos(d
*x+c)*sin(d*x+c)/d+1/20*a^2*(5*A+6*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*B*cos(d
*x+c)^3*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d-1/15*a^2*(10*A+9*B)*sin(d*x+c)^3/
d
```

Rubi [A] time = 0.28, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a^2(10A+9B)\sin^3(c+dx)}{15d}+\frac{a^2(10A+9B)\sin(c+dx)}{5d}+\frac{a^2(5A+6B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{a^2(7A+6B)\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(7*A + 6*B)*x)/8 + (a^2*(10*A + 9*B)*Sin[c + d*x])/(5*d) + (a^2*(7*A +
 6*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(5*A + 6*B)*Cos[c + d*x]^3*Si
n[c + d*x])/(20*d) + (B*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x
])/(5*d) - (a^2*(10*A + 9*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{B\cos^3(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d} +$$
$$= \frac{B\cos^3(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d} +$$
$$= \frac{a^2(5A+6B)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{B\cos^3(c+$$
$$= \frac{a^2(5A+6B)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{B\cos^3(c+$$
$$= \frac{a^2(7A+6B)\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2(5A+6B}{}$$
$$= \frac{1}{8}a^2(7A+6B)x + \frac{a^2(10A+9B)\sin(c+dx)}{5d} + \frac{a^2(7}{}$$

Mathematica [A] time = 0.45, size = 108, normalized size = 0.68

$$\frac{a^2(60(12A+11B)\sin(c+dx)+240(A+B)\sin(2(c+dx))+80A\sin(3(c+dx))+15A\sin(4(c+dx))+420Ad}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(360*B*c + 420*A*d*x + 360*B*d*x + 60*(12*A + 11*B)*Sin[c + d*x] + 240
*(A + B)*Sin[2*(c + d*x)] + 80*A*Sin[3*(c + d*x)] + 90*B*Sin[3*(c + d*x)] +
 15*A*Sin[4*(c + d*x)] + 30*B*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)]))/(48
0*d)
```

fricas [A] time = 0.68, size = 110, normalized size = 0.69

$$\frac{15\,(7\,A+6\,B)a^2dx+\left(24\,Ba^2\cos\left(dx+c\right)^4+30\,(A+2\,B)a^2\cos\left(dx+c\right)^3+8\,(10\,A+9\,B)a^2\cos\left(dx+c\right)^2+15\,(}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(7*A + 6*B)*a^2*d*x + (24*B*a^2*cos(d*x + c)^4 + 30*(A + 2*B)*a^2
*cos(d*x + c)^3 + 8*(10*A + 9*B)*a^2*cos(d*x + c)^2 + 15*(7*A + 6*B)*a^2*co
s(d*x + c) + 16*(10*A + 9*B)*a^2)*sin(d*x + c))/d
```

giac [A] time = 0.70, size = 137, normalized size = 0.86

$$\frac{Ba^2\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{8}\left(7\,Aa^2+6\,Ba^2\right)x+\frac{\left(Aa^2+2\,Ba^2\right)\sin(4\,dx+4\,c)}{32\,d}+\frac{\left(8\,Aa^2+9\,Ba^2\right)\sin(3\,dx+3\,c)}{48\,d}+\frac{\left(Aa^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/80*B*a^2*sin(5*d*x + 5*c)/d + 1/8*(7*A*a^2 + 6*B*a^2)*x + 1/32*(A*a^2 + 2
*B*a^2)*sin(4*d*x + 4*c)/d + 1/48*(8*A*a^2 + 9*B*a^2)*sin(3*d*x + 3*c)/d +
1/2*(A*a^2 + B*a^2)*sin(2*d*x + 2*c)/d + 1/8*(12*A*a^2 + 11*B*a^2)*sin(d*x
+ c)/d
```

maple [A] time = 0.08, size = 186, normalized size = 1.16

$$\frac{a^2A\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{B\,a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+\frac{2a^2A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5
*B*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2/3*a^2*A*(2+cos(d*x+
c)^2)*sin(d*x+c)+2*B*a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*
d*x+3/8*c)+a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*B*a^2*(2+cos
(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.36, size = 178, normalized size = 1.11

$$-\frac{320\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Aa^2-15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c)\right)Aa^2-120\,(2\,dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/480*(320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 120*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*a^2 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*B*a^2 + 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 30*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2)/d
```

mupad [B] time = 1.50, size = 277, normalized size = 1.73

$$\frac{\left(\frac{7\,A\,a^2}{4}+\frac{3\,B\,a^2}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{49\,A\,a^2}{6}+7\,B\,a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{40\,A\,a^2}{3}+\frac{72\,B\,a^2}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{79\,A\,a^2}{6}+9\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((25*A*a^2)/4 + (13*B*a^2)/2) + tan(c/2 + (d*x)/2)^9*((
7*A*a^2)/4 + (3*B*a^2)/2) + tan(c/2 + (d*x)/2)^7*((49*A*a^2)/6 + 7*B*a^2) +
 tan(c/2 + (d*x)/2)^3*((79*A*a^2)/6 + 9*B*a^2) + tan(c/2 + (d*x)/2)^5*((40*
A*a^2)/3 + (72*B*a^2)/5))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2
)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)
^10 + 1)) - (a^2*(7*A + 6*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4*d) +
(a^2*atan((a^2*tan(c/2 + (d*x)/2)*(7*A + 6*B))/(4*((7*A*a^2)/4 + (3*B*a^2)/
2)))*(7*A + 6*B))/(4*d)
```

sympy [A] time = 2.58, size = 459, normalized size = 2.87

$$\begin{cases} \frac{3Aa^2x\sin^4(c+dx)}{8}+\frac{3Aa^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{Aa^2x\sin^2(c+dx)}{2}+\frac{3Aa^2x\cos^4(c+dx)}{8}+\frac{Aa^2x\cos^2(c+dx)}{2}+\frac{3Aa^2\sin^3(c+dx)\cos(c}{8d} \\ x\,(A+B\cos(c))\,(a\cos(c)+a)^2\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**2*x*sin(c + d*x)**4/8 + 3*A*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + A*a**2*x*sin(c + d*x)**2/2 + 3*A*a**2*x*cos(c + d*x)**4/8 + A
*a**2*x*cos(c + d*x)**2/2 + 3*A*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4
*A*a**2*sin(c + d*x)**3/(3*d) + 5*A*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d)
 + 2*A*a**2*sin(c + d*x)*cos(c + d*x)**2/d + A*a**2*sin(c + d*x)*cos(c + d*
x)/(2*d) + 3*B*a**2*x*sin(c + d*x)**4/4 + 3*B*a**2*x*sin(c + d*x)**2*cos(c
+ d*x)**2/2 + 3*B*a**2*x*cos(c + d*x)**4/4 + 8*B*a**2*sin(c + d*x)**5/(15*d
) + 4*B*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a**2*sin(c + d*x)*
*3*cos(c + d*x)/(4*d) + 2*B*a**2*sin(c + d*x)**3/(3*d) + B*a**2*sin(c + d*x
)*cos(c + d*x)**4/d + 5*B*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + B*a**2*
sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)
**2*cos(c)**2, True))
```

3.12 $\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a^2(8A+7B)\sin(c+dx)}{6d}+\frac{a^2(8A+7B)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}a^2x(8A+7B)+\frac{(4A-B)\sin(c+dx)(a\cos(c+a$$

```
[Out] 1/8*a^2*(8*A+7*B)*x+1/6*a^2*(8*A+7*B)*sin(d*x+c)/d+1/24*a^2*(8*A+7*B)*cos(d
*x+c)*sin(d*x+c)/d+1/12*(4*A-B)*(a+a*cos(d*x+c))^2*sin(d*x+c)/d+1/4*B*(a+a*
cos(d*x+c))^3*sin(d*x+c)/a/d
```

Rubi [A] time = 0.18, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3023, 2751, 2644}

$$\frac{a^2(8A+7B)\sin(c+dx)}{6d}+\frac{a^2(8A+7B)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}a^2x(8A+7B)+\frac{(4A-B)\sin(c+dx)(a\cos(c+a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(8*A + 7*B)*x)/8 + (a^2*(8*A + 7*B)*Sin[c + d*x])/(6*d) + (a^2*(8*A +
7*B)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*A - B)*(a + a*Cos[c + d*x])^2*
Sin[c + d*x])/(12*d) + (B*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^2\left(A\cos(c+dx)+B\cos^2(c+\right.$$

$$= \frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4ad} + \frac{\int (a+a\cos(c+}{}$$

$$= \frac{(4A-B)(a+a\cos(c+dx))^2\sin(c+dx)}{12d} + \frac{B(a+a}{}$$

$$= \frac{1}{8}a^2(8A+7B)x + \frac{a^2(8A+7B)\sin(c+dx)}{6d} + \frac{a^2(8A}{}$$

Mathematica [A] time = 0.37, size = 86, normalized size = 0.67

$$\frac{a^2(24(7A+6B)\sin(c+dx)+48(A+B)\sin(2(c+dx))+8A\sin(3(c+dx))+96Adx+16B\sin(3(c+dx))+3B}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(84*B*c + 96*A*d*x + 84*B*d*x + 24*(7*A + 6*B)*Sin[c + d*x] + 48*(A +
B)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 16*B*Sin[3*(c + d*x)] + 3*B*Si
n[4*(c + d*x)]))/(96*d)
```

fricas [A] time = 0.75, size = 90, normalized size = 0.70

$$\frac{3\,(8\,A+7\,B)a^2dx+\left(6\,Ba^2\cos\left(dx+c\right)^3+8\,(A+2\,B)a^2\cos\left(dx+c\right)^2+3\,(8\,A+7\,B)a^2\cos\left(dx+c\right)+8\,(5\,A+\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(8*A + 7*B)*a^2*d*x + (6*B*a^2*cos(d*x + c)^3 + 8*(A + 2*B)*a^2*cos
(d*x + c)^2 + 3*(8*A + 7*B)*a^2*cos(d*x + c) + 8*(5*A + 4*B)*a^2)*sin(d*x +
 c))/d
```

giac [A] time = 1.07, size = 110, normalized size = 0.85

$$\frac{Ba^2 \sin(4\,dx + 4\,c)}{32\,d} + \frac{1}{8}\left(8\,Aa^2 + 7\,Ba^2\right)x + \frac{\left(Aa^2 + 2\,Ba^2\right)\sin(3\,dx + 3\,c)}{12\,d} + \frac{\left(Aa^2 + Ba^2\right)\sin(2\,dx + 2\,c)}{2\,d} + \frac{\left(7\,Aa^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/32*B*a^2*sin(4*d*x + 4*c)/d + 1/8*(8*A*a^2 + 7*B*a^2)*x + 1/12*(A*a^2 + 2
*B*a^2)*sin(3*d*x + 3*c)/d + 1/2*(A*a^2 + B*a^2)*sin(2*d*x + 2*c)/d + 1/4*(
7*A*a^2 + 6*B*a^2)*sin(d*x + c)/d
```

maple [A] time = 0.06, size = 154, normalized size = 1.19

$$\frac{\frac{a^2 A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3} + B\,a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + 2a^2 A\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + \frac{2B\,a}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a^2*(1/4*(cos(d*x+c)^3+3/2*cos
(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d
*x+1/2*c)+2/3*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*A*sin(d*x+c)+B*a^2*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.41, size = 144, normalized size = 1.12

$$-\frac{32\left(\sin(dx + c)^3 - 3\,\sin(dx + c)\right)Aa^2 - 48\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Aa^2 + 64\left(\sin(dx + c)^3 - 3\,\sin(dx + c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 48*(2*d*x + 2*c + sin(2
*d*x + 2*c))*A*a^2 + 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 3*(12*d*x
```

```
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 - 24*(2*d*x + 2*c +
sin(2*d*x + 2*c))*B*a^2 - 96*A*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.29, size = 134, normalized size = 1.04

$$A\,a^2\,x+\frac{7\,B\,a^2\,x}{8}+\frac{7\,A\,a^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{3\,B\,a^2\,\sin\left(c+d\,x\right)}{2\,d}+\frac{A\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{A\,a^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{B\,a^2\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

```
[Out] A*a^2*x + (7*B*a^2*x)/8 + (7*A*a^2*sin(c + d*x))/(4*d) + (3*B*a^2*sin(c + d
*x))/(2*d) + (A*a^2*sin(2*c + 2*d*x))/(2*d) + (A*a^2*sin(3*c + 3*d*x))/(12*
d) + (B*a^2*sin(2*c + 2*d*x))/(2*d) + (B*a^2*sin(3*c + 3*d*x))/(6*d) + (B*a
^2*sin(4*c + 4*d*x))/(32*d)
```

sympy [A] time = 1.23, size = 338, normalized size = 2.62

$$\begin{cases} Aa^2x\sin^2\left(c+dx\right)+Aa^2x\cos^2\left(c+dx\right)+\frac{2Aa^2\sin^3\left(c+dx\right)}{3d}+\frac{Aa^2\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{Aa^2\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{Aa^2\sin}{d} \\ x\left(A+B\cos\left(c\right)\right)\left(a\cos\left(c\right)+a\right)^2\cos\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2 + A*a**2*x*cos(c + d*x)**2 + 2*A*a**2*s
in(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)*cos(c + d*x)**2/d + A*a**2*sin(c
 + d*x)*cos(c + d*x)/d + A*a**2*sin(c + d*x)/d + 3*B*a**2*x*sin(c + d*x)**4
/8 + 3*B*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + B*a**2*x*sin(c + d*x)**
2/2 + 3*B*a**2*x*cos(c + d*x)**4/8 + B*a**2*x*cos(c + d*x)**2/2 + 3*B*a**2*
sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4*B*a**2*sin(c + d*x)**3/(3*d) + 5*B*a
**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 2*B*a**2*sin(c + d*x)*cos(c + d*x)
**2/d + B*a**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A + B*cos(c)
)*(a*cos(c) + a)**2*cos(c), True))
```

3.13 $\int (a + a\cos(c + dx))^2 (A + B\cos(c + dx))\,dx$

Optimal. Leaf size=94

$$\frac{2a^2(3A + 2B)\sin(c + dx)}{3d} + \frac{a^2(3A + 2B)\sin(c + dx)\cos(c + dx)}{6d} + \frac{1}{2}a^2x(3A+2B) + \frac{B\sin(c + dx)(a\cos(c + dx) + a}{3d}$$

```
[Out] 1/2*a^2*(3*A+2*B)*x+2/3*a^2*(3*A+2*B)*sin(d*x+c)/d+1/6*a^2*(3*A+2*B)*cos(d*
x+c)*sin(d*x+c)/d+1/3*B*(a+a*cos(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2751, 2644}

$$\frac{2a^2(3A + 2B)\sin(c + dx)}{3d} + \frac{a^2(3A + 2B)\sin(c + dx)\cos(c + dx)}{6d} + \frac{1}{2}a^2x(3A+2B) + \frac{B\sin(c + dx)(a\cos(c + dx) + a}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(3*A + 2*B)*x)/2 + (2*a^2*(3*A + 2*B)*Sin[c + d*x])/(3*d) + (a^2*(3*A
+ 2*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (B*(a + a*Cos[c + d*x])^2*Sin[c +
 d*x])/(3*d)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (a + a\cos(c + dx))^2(A + B\cos(c + dx))\,dx &= \frac{B(a + a\cos(c + dx))^2\sin(c + dx)}{3d} + \frac{1}{3}(3A + 2B)\int (a + a\cos(c +\\ &= \frac{1}{2}a^2(3A + 2B)x + \frac{2a^2(3A + 2B)\sin(c + dx)}{3d} + \frac{a^2(3A + 2B)\cos(c}{6}\end{aligned}$$

Mathematica [A] time = 0.20, size = 61, normalized size = 0.65

$$\frac{a^2(3(8A+7B)\sin(c+dx)+3(A+2B)\sin(2(c+dx))+18Adx+B\sin(3(c+dx))+12Bdx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*(18*A*d*x + 12*B*d*x + 3*(8*A + 7*B)*Sin[c + d*x] + 3*(A + 2*B)*Sin[2*
(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

fricas [A] time = 0.91, size = 70, normalized size = 0.74

$$\frac{3\,(3\,A+2\,B)a^2dx+\left(2\,Ba^2\cos\left(dx+c\right)^2+3\,(A+2\,B)a^2\cos\left(dx+c\right)+2\,(6\,A+5\,B)a^2\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(3*A + 2*B)*a^2*d*x + (2*B*a^2*cos(d*x + c)^2 + 3*(A + 2*B)*a^2*cos(
d*x + c) + 2*(6*A + 5*B)*a^2)*sin(d*x + c))/d
```

giac [A] time = 0.33, size = 85, normalized size = 0.90

$$\frac{Ba^2\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{1}{2}\left(3\,Aa^2+2\,Ba^2\right)x+\frac{\left(Aa^2+2\,Ba^2\right)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{\left(8\,Aa^2+7\,Ba^2\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*B*a^2*sin(3*d*x + 3*c)/d + 1/2*(3*A*a^2 + 2*B*a^2)*x + 1/4*(A*a^2 + 2*
B*a^2)*sin(2*d*x + 2*c)/d + 1/4*(8*A*a^2 + 7*B*a^2)*sin(d*x + c)/d
```

maple [A] time = 0.06, size = 116, normalized size = 1.23

$$\frac{\frac{B\,a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a^2A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2B\,a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2A\sin\left(dx+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/3*B*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+2*B*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a^2*A*s
in(d*x+c)+B*a^2*sin(d*x+c)+a^2*A*(d*x+c))
```

maxima [A] time = 0.38, size = 110, normalized size = 1.17

$$\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2+12\,(dx+c)Aa^2-4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^2+6\,(2\,dx+2\,c+\sin}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 12*(d*x + c)*A*a^2 - 4*(si
n(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 + 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*B
*a^2 + 24*A*a^2*sin(d*x + c) + 12*B*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.23, size = 98, normalized size = 1.04

$$\frac{3\,A\,a^2\,x}{2}+B\,a^2\,x+\frac{2\,A\,a^2\,\sin(c+d\,x)}{d}+\frac{7\,B\,a^2\,\sin(c+d\,x)}{4\,d}+\frac{A\,a^2\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{B\,a^2\,\sin(2\,c+2\,d\,x)}{2\,d}+\frac{B\,a^2\,\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (3*A*a^2*x)/2 + B*a^2*x + (2*A*a^2*sin(c + d*x))/d + (7*B*a^2*sin(c + d*x))
/(4*d) + (A*a^2*sin(2*c + 2*d*x))/(4*d) + (B*a^2*sin(2*c + 2*d*x))/(2*d) +
(B*a^2*sin(3*c + 3*d*x))/(12*d)
```

sympy [A] time = 0.64, size = 199, normalized size = 2.12

$$\begin{cases}\frac{Aa^2x\sin^2(c+dx)}{2}+\frac{Aa^2x\cos^2(c+dx)}{2}+Aa^2x+\frac{Aa^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2Aa^2\sin(c+dx)}{d}+Ba^2x\sin^2(c+dx)+Ba^2x\cos^2(c \\ x\,(A+B\cos(c))\,(a\cos(c)+a)^2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**2/2 + A*a**2
*x + A*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*a**2*sin(c + d*x)/d + B*a
**2*x*sin(c + d*x)**2 + B*a**2*x*cos(c + d*x)**2 + 2*B*a**2*sin(c + d*x)**3
/(3*d) + B*a**2*sin(c + d*x)*cos(c + d*x)**2/d + B*a**2*sin(c + d*x)*cos(c
+ d*x)/d + B*a**2*sin(c + d*x)/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) +
a)**2, True))
```

3.14 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=82

$$\frac{a^2(2A+3B)\sin(c+dx)}{2d}+\frac{1}{2}a^2x(4A+3B)+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

```
[Out] 1/2*a^2*(4*A+3*B)*x+a^2*A*arctanh(sin(d*x+c))/d+1/2*a^2*(2*A+3*B)*sin(d*x+c
)/d+1/2*B*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d
```

Rubi [A] time = 0.19, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2976, 2968, 3023, 2735, 3770}

$$\frac{a^2(2A+3B)\sin(c+dx)}{2d}+\frac{1}{2}a^2x(4A+3B)+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^2*(4*A + 3*B)*x)/2 + (a^2*A*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*A + 3*B)*
Sin[c + d*x])/(2*d) + (B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
```

```
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{2d}+\frac{1}{2}\int (a+a\cos($$

$$= \frac{B\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{2d}+\frac{1}{2}\int \left(2a^2A+(\right.$$

$$= \frac{a^2(2A+3B)\sin(c+dx)}{2d}+\frac{B\left(a^2+a^2\cos(c+dx)\right)\sin}{2d}$$

$$= \frac{1}{2}a^2(4A+3B)x+\frac{a^2(2A+3B)\sin(c+dx)}{2d}+\frac{B\left(a^2+\right.}{}$$

$$= \frac{1}{2}a^2(4A+3B)x+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2(2A}{}$$

Mathematica [A] time = 0.20, size = 96, normalized size = 1.17

$$\frac{a^2\left(4(A+2B)\sin(c+dx)-4A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+d\right.\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^2*(8*A*d*x + 6*B*d*x - 4*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*
A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*(A + 2*B)*Sin[c + d*x] + B*S
in[2*(c + d*x)]))/(4*d)
```

fricas [A] time = 0.89, size = 79, normalized size = 0.96

$$\frac{(4\,A+3\,B)a^2dx+Aa^2\log\left(\sin\left(dx+c\right)+1\right)-Aa^2\log\left(-\sin\left(dx+c\right)+1\right)+\left(Ba^2\cos\left(dx+c\right)+2\,(A+2\,B)a^2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/2*((4*A + 3*B)*a^2*d*x + A*a^2*log(sin(d*x + c) + 1) - A*a^2*log(-sin(d*x
 + c) + 1) + (B*a^2*cos(d*x + c) + 2*(A + 2*B)*a^2)*sin(d*x + c))/d
```

giac [A] time = 0.43, size = 145, normalized size = 1.77

$$\frac{2\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\left(4\,Aa^2+3\,Ba^2\right)(dx+c)+\frac{2\left(2\,Aa^2\tan\left(\frac{1}{2}\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/2*(2*A*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*A*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (4*A*a^2 + 3*B*a^2)*(d*x + c) + 2*(2*A*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 3*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*a^2*tan(1/2*d*x + 1/2*c)
+ 5*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.12, size = 108, normalized size = 1.32

$$\frac{a^2A\sin\left(dx+c\right)}{d}+\frac{B\,a^2\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d}+\frac{3a^2Bx}{2}+\frac{3B\,a^2c}{2d}+2a^2Ax+\frac{2A\,a^2c}{d}+\frac{2B\,a^2\sin\left(dx+c\right)}{d}+\frac{a^2A\ln(\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/d*a^2*A*sin(d*x+c)+1/2/d*B*a^2*cos(d*x+c)*sin(d*x+c)+3/2*a^2*B*x+3/2/d*B*
a^2*c+2*a^2*A*x+2/d*A*a^2*c+2/d*B*a^2*sin(d*x+c)+1/d*a^2*A*ln(sec(d*x+c)+ta
n(d*x+c))
```

maxima [A] time = 0.36, size = 94, normalized size = 1.15

$$\frac{8\,(dx+c)Aa^2+(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2+4\,(dx+c)Ba^2+4\,Aa^2\log(\sec(dx+c)+\tan(dx+c))+4\,A}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(8*(d*x + c)*A*a^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 + 4*(d*x +
c)*B*a^2 + 4*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 4*A*a^2*sin(d*x + c)
+ 8*B*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.34, size = 141, normalized size = 1.72

$$\frac{A\,a^2\,\sin(c+d\,x)}{d}+\frac{2\,B\,a^2\,\sin(c+d\,x)}{d}+\frac{4\,A\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x),x)
```

```
[Out] (A*a^2*sin(c + d*x))/d + (2*B*a^2*sin(c + d*x))/d + (4*A*a^2*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)))/d + (3*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
+ (B*a^2*sin(2*c + 2*d*x))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sec(c+dx)\,dx+\int 2A\cos(c+dx)\sec(c+dx)\,dx+\int A\cos^2(c+dx)\sec(c+dx)\,dx+\int B\cos(c+d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x), x) + Integral(2*A*cos(c + d*x)*sec(c + d*x),
 x) + Integral(A*cos(c + d*x)**2*sec(c + d*x), x) + Integral(B*cos(c + d*x)
*sec(c + d*x), x) + Integral(2*B*cos(c + d*x)**2*sec(c + d*x), x) + Integra
l(B*cos(c + d*x)**3*sec(c + d*x), x))
```

3.15 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=74

$$-\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{a^2(2A+B)\tanh^{-1}(\sin(c+dx))}{d}+a^2x(A+2B)+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

```
[Out] a^2*(A+2*B)*x+a^2*(2*A+B)*arctanh(sin(d*x+c))/d-a^2*(A-B)*sin(d*x+c)/d+A*(a
^2+a^2*cos(d*x+c))*tan(d*x+c)/d
```

Rubi [A] time = 0.21, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2975, 2968, 3023, 2735, 3770}

$$-\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{a^2(2A+B)\tanh^{-1}(\sin(c+dx))}{d}+a^2x(A+2B)+\frac{A\tan(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] a^2*(A + 2*B)*x + (a^2*(2*A + B)*ArcTanh[Sin[c + d*x]])/d - (a^2*(A - B)*Si
n[c + d*x])/d + (A*(a^2 + a^2*Cos[c + d*x])*Tan[c + d*x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
```

```
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{A\left(a^2+a^2\cos(c+dx)\right)\tan(c+dx)}{d}+\int (a+a\cos($$
$$= \frac{A\left(a^2+a^2\cos(c+dx)\right)\tan(c+dx)}{d}+\int \left(a^2(2A+$$
$$= -\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{A\left(a^2+a^2\cos(c+dx)\right)\text{ta}}{d}$$
$$= a^2(A+2B)x-\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{A\left(a^2+a^2\text{c}\right.}{}$$
$$= a^2(A+2B)x+\frac{a^2(2A+B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2}{}$$

Mathematica [A] time = 0.37, size = 143, normalized size = 1.93

$$\frac{a^2\left(A\tan(c+dx)-2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^2*(A*c + 2*B*c + A*d*x + 2*B*d*x - 2*A*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*A*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + B*S
in[c + d*x] + A*Tan[c + d*x]))/d
```

fricas [A] time = 0.67, size = 108, normalized size = 1.46

$$\frac{2\,(A+2\,B)a^2dx\cos\left(dx+c\right)+(2\,A+B)a^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-(2\,A+B)a^2\cos\left(dx+c\right)\log\left(-\mathrm{s}\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*(2*(A + 2*B)*a^2*d*x*cos(d*x + c) + (2*A + B)*a^2*cos(d*x + c)*log(sin(
d*x + c) + 1) - (2*A + B)*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(B*a^
2*cos(d*x + c) + A*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 1.06, size = 155, normalized size = 2.09

$$\frac{\left(Aa^2+2\,Ba^2\right)(dx+c)+\left(2\,Aa^2+Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Aa^2+Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] ((A*a^2 + 2*B*a^2)*(d*x + c) + (2*A*a^2 + B*a^2)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (2*A*a^2 + B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a^2*
tan(1/2*d*x + 1/2*c)^3 - B*a^2*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*x +
 1/2*c) + B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

maple [A] time = 0.14, size = 107, normalized size = 1.45

$$a^2Ax+2a^2Bx+\frac{a^2A\tan\left(dx+c\right)}{d}+\frac{2a^2A\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{A\,a^2c}{d}+\frac{B\,a^2\sin\left(dx+c\right)}{d}+\frac{B\,a^2\ln\left(\sec\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] a^2*A*x+2*a^2*B*x+a^2*A*tan(d*x+c)/d+2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/
d*A*a^2*c+1/d*B*a^2*sin(d*x+c)+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*B*a^
2*c
```

maxima [A] time = 0.42, size = 105, normalized size = 1.42

$$\frac{2\,(dx+c)Aa^2+4\,(dx+c)Ba^2+2\,Aa^2\Big(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\Big)+Ba^2\Big(\log\left(\sin\left(dx+c\right)+1\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*(2*(d*x + c)*A*a^2 + 4*(d*x + c)*B*a^2 + 2*A*a^2*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*B*a^2*sin(d*x + c) + 2*A*a^2*tan(d*x + c))/d
```

mupad [B] time = 0.32, size = 161, normalized size = 2.18

$$\frac{B\,a^2\,\sin\left(c+d\,x\right)}{d}+\frac{2\,A\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^2\,\mathrm{atanh}\left(\frac{\sin}{\cos}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] (B*a^2*sin(c + d*x))/d + (2*A*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d + (4*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*B*a^2
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^2*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^2*sin(c + d*x))/(d*cos(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sec^2\left(c+dx\right)dx+\int 2A\cos\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int A\cos^2\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int B\cos\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**2, x) + Integral(2*A*cos(c + d*x)*sec(c + d*
x)**2, x) + Integral(A*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(B*cos
(c + d*x)*sec(c + d*x)**2, x) + Integral(2*B*cos(c + d*x)**2*sec(c + d*x)**
2, x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)**2, x))
```

3.16 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{a^2(3A+2B)\tan(c+dx)}{2d}+\frac{a^2(3A+4B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}+a$$

```
[Out] a^2*B*x+1/2*a^2*(3*A+4*B)*arctanh(sin(d*x+c))/d+1/2*a^2*(3*A+2*B)*tan(d*x+c
)/d+1/2*A*(a^2+a^2*cos(d*x+c))*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.22, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2975, 2968, 3021, 2735, 3770}

$$\frac{a^2(3A+2B)\tan(c+dx)}{2d}+\frac{a^2(3A+4B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{A\tan(c+dx)\sec(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{2d}+a$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] a^2*B*x + (a^2*(3*A + 4*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(3*A + 2*B)*
Tan[c + d*x])/(2*d) + (A*(a^2 + a^2*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x]
)/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
```

```
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}$$

$$=\frac{A\left(a^2+a^2\cos(c+dx)\right)\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}$$

$$=\frac{a^2(3A+2B)\tan(c+dx)}{2d}+\frac{A\left(a^2+a^2\cos(c+dx)\right)}{2d}$$

$$=a^2Bx+\frac{a^2(3A+2B)\tan(c+dx)}{2d}+\frac{A\left(a^2+a^2\cos(c\right.}{}$$

$$=a^2Bx+\frac{a^2(3A+4B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(3A+}{}$$

Mathematica [B] time = 1.36, size = 277, normalized size = 3.15

$$\frac{1}{16}a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{4(2A+B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{}{d\left(\sin\left(\frac{c}{2}\right)+c\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(4*B*x - (2*(3*A + 4*B)*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(3*A + 4*B)*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]])/d + A/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
4*(2*A + B)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])) - A/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(2*A +
B)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]))))/16
```

fricas [A] time = 0.85, size = 119, normalized size = 1.35

$$\frac{4\,Ba^2dx\cos\left(dx+c\right)^2+\left(3\,A+4\,B\right)a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(3\,A+4\,B\right)a^2\cos\left(dx+c\right)^2\log\left(-\sin\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*B*a^2*d*x*cos(d*x + c)^2 + (3*A + 4*B)*a^2*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - (3*A + 4*B)*a^2*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
(2*A + B)*a^2*cos(d*x + c) + A*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.50, size = 154, normalized size = 1.75

$$\frac{2\left(dx+c\right)Ba^2+\left(3\,Aa^2+4\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(3\,Aa^2+4\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2(}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(d*x + c)*B*a^2 + (3*A*a^2 + 4*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - (3*A*a^2 + 4*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a^2*
tan(1/2*d*x + 1/2*c)^3 + 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*A*a^2*tan(1/2*d
*x + 1/2*c) - 2*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)
/d
```

maple [A] time = 0.15, size = 113, normalized size = 1.28

$$\frac{3a^2A\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+a^2Bx+\frac{B\,a^2c}{d}+\frac{2a^2A\tan\left(dx+c\right)}{d}+\frac{2B\,a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 3/2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+a^2*B*x+1/d*B*a^2*c+2*a^2*A*tan(d*x+c
)/d+2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2*a^2*A*sec(d*x+c)*tan(d*x+c)/d+1
/d*B*a^2*tan(d*x+c)
```

maxima [A] time = 0.54, size = 142, normalized size = 1.61

$$\frac{4\,(dx+c)Ba^2 - Aa^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) + 2\,Aa^2\left(\log\left(\sin\left(dx+c\right)+1\right) - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*(d*x + c)*B*a^2 - A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*A*a^2*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 4*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 8*A*a^2*tan(d*x + c) + 4*B*a^2*tan(d*x + c))/d
```

mupad [B] time = 0.30, size = 162, normalized size = 1.84

$$\frac{3\,A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,B\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,a^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{A\,a^2\,\sin\left(c+\right.}{2\,d\cos\left(c+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^3,x)
```

```
[Out] (3*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^2*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*B*a^2*atanh(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)))/d + (2*A*a^2*sin(c + d*x))/(d*cos(c + d*x)) + (A*a^2*
sin(c + d*x))/(2*d*cos(c + d*x)^2) + (B*a^2*sin(c + d*x))/(d*cos(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sec^3\left(c+dx\right)dx + \int 2A\cos\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int A\cos^2\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int B\cos\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**3, x) + Integral(2*A*cos(c + d*x)*sec(c + d*
x)**3, x) + Integral(A*cos(c + d*x)**2*sec(c + d*x)**3, x) + Integral(B*cos
(c + d*x)*sec(c + d*x)**3, x) + Integral(2*B*cos(c + d*x)**2*sec(c + d*x)**
3, x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)**3, x))
```

3.17 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=113

$$\frac{a^2(5A+6B)\tan(c+dx)}{3d}+\frac{a^2(2A+3B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(4A+3B)\tan(c+dx)\sec(c+dx)}{6d}+\frac{A\tan(c+dx}{}$$

```
[Out] 1/2*a^2*(2*A+3*B)*arctanh(sin(d*x+c))/d+1/3*a^2*(5*A+6*B)*tan(d*x+c)/d+1/6*
a^2*(4*A+3*B)*sec(d*x+c)*tan(d*x+c)/d+1/3*A*(a^2+a^2*cos(d*x+c))*sec(d*x+c)
^2*tan(d*x+c)/d
```

Rubi [A] time = 0.27, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a^2(5A+6B)\tan(c+dx)}{3d}+\frac{a^2(2A+3B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(4A+3B)\tan(c+dx)\sec(c+dx)}{6d}+\frac{A\tan(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a^2*(2*A + 3*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(5*A + 6*B)*Tan[c + d*
x])/(3*d) + (a^2*(4*A + 3*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (A*(a^2 + a
^2*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}$$

$$= \frac{A\left(a^2+a^2\cos(c+dx)\right)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}$$

$$= \frac{a^2(4A+3B)\sec(c+dx)\tan(c+dx)}{6d} + \frac{A\left(a^2+a^2\mathrm{co}\right.}{}$$

$$= \frac{a^2(4A+3B)\sec(c+dx)\tan(c+dx)}{6d} + \frac{A\left(a^2+a^2\mathrm{co}\right.}{}$$

$$= \frac{a^2(2A+3B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2(4A+3B)\sec}{}$$

$$= \frac{a^2(2A+3B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2(5A+6B)\tan}{3d}$$

Mathematica [B] time = 6.36, size = 451, normalized size = 3.99

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{4(5A+6B)\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{4(5A+6B)\sin\left(\frac{dx}{2}\right)}{\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right.\right.}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-6*(2*A + 3*B)*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] + 6*(2*A + 3*B)*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + (2*A*Sin[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^3) + ((7*A + 3*B)*Cos[c/2] - (5*A + 3*B)*Sin[c/2])/((Cos[c/
2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(5*A + 6*B)*Si
n[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) +
 (2*A*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^3) - ((7*A + 3*B)*Cos[c/2] + (5*A + 3*B)*Sin[c/2])/((Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(5*A + 6*B)*Sin[(d*x)/2]
)/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(48*d)
```

fricas [A] time = 0.60, size = 125, normalized size = 1.11

$$\frac{3\,(2\,A+3\,B)a^2\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(2\,A+3\,B)a^2\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(5\right.}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*(2*A + 3*B)*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*A + 3*B
)*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(5*A + 6*B)*a^2*cos(d*x
+ c)^2 + 3*(2*A + B)*a^2*cos(d*x + c) + 2*A*a^2)*sin(d*x + c))/(d*cos(d*x +
 c)^3)
```

giac [A] time = 0.50, size = 178, normalized size = 1.58

$$\frac{3\left(2\,Aa^2+3\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,Aa^2+3\,Ba^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(2*A*a^2 + 3*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*A*a^2
+ 3*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^2*tan(1/2*d*x + 1/
2*c)^5 + 9*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 16*A*a^2*tan(1/2*d*x + 1/2*c)^3 -
 24*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 18*A*a^2*tan(1/2*d*x + 1/2*c) + 15*B*a^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.15, size = 141, normalized size = 1.25

$$\frac{5a^2A\tan(dx+c)}{3d}+\frac{3B\,a^2\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{a^2A\sec(dx+c)\tan(dx+c)}{d}+\frac{a^2A\ln(\sec(dx+c)+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 5/3*a^2*A*tan(d*x+c)/d+3/2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*A*sec(d*x+
c)*tan(d*x+c)/d+1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+2/d*B*a^2*tan(d*x+c)+1/
3*a^2*A*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*B*a^2*sec(d*x+c)*tan(d*x+c)
```

maxima [A] time = 0.69, size = 174, normalized size = 1.54

$$\frac{4\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^2-6\,Aa^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-3\,B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 - 6*A*a^2*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*
a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 6*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
12*A*a^2*tan(d*x + c) + 24*B*a^2*tan(d*x + c))/d
```

mupad [B] time = 2.08, size = 145, normalized size = 1.28

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A+\frac{3\,B}{2}\right)}{d}-\frac{\left(2\,A\,a^2+3\,B\,a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{16\,A\,a^2}{3}-8\,B\,a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(6\,A}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^4,x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2))*(A + (3*B)/2))/d - (tan(c/2 + (d*x)/2)*(6*
A*a^2 + 5*B*a^2) + tan(c/2 + (d*x)/2)^5*(2*A*a^2 + 3*B*a^2) - tan(c/2 + (d*
x)/2)^3*((16*A*a^2)/3 + 8*B*a^2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 +
(d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.18 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{a^2(4A+5B)\tan(c+dx)}{3d}+\frac{a^2(7A+8B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(5A+4B)\tan(c+dx)\sec^2(c+dx)}{12d}+\frac{a^2(7A+8$$

```
[Out] 1/8*a^2*(7*A+8*B)*arctanh(sin(d*x+c))/d+1/3*a^2*(4*A+5*B)*tan(d*x+c)/d+1/8*
a^2*(7*A+8*B)*sec(d*x+c)*tan(d*x+c)/d+1/12*a^2*(5*A+4*B)*sec(d*x+c)^2*tan(d
*x+c)/d+1/4*A*(a^2+a^2*cos(d*x+c))*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] time = 0.30, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^2(4A+5B)\tan(c+dx)}{3d}+\frac{a^2(7A+8B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(5A+4B)\tan(c+dx)\sec^2(c+dx)}{12d}+\frac{a^2(7A+8$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a^2*(7*A + 8*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(4*A + 5*B)*Tan[c + d*
x])/(3*d) + (a^2*(7*A + 8*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(5*A +
 4*B)*Sec[c + d*x]^2*Tan[c + d*x])/(12*d) + (A*(a^2 + a^2*Cos[c + d*x])*Sec
[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c + dx))^2(A + B\cos(c + dx))\sec^5(c + dx)\,dx = \frac{A\left(a^2 + a^2\cos(c + dx)\right)\sec^3(c + dx)\tan(c + dx)}{4d} +$$
$$= \frac{A\left(a^2 + a^2\cos(c + dx)\right)\sec^3(c + dx)\tan(c + dx)}{4d} +$$
$$= \frac{a^2(5A + 4B)\sec^2(c + dx)\tan(c + dx)}{12d} + \frac{A\left(a^2 + a^2\right.}{}$$
$$= \frac{a^2(5A + 4B)\sec^2(c + dx)\tan(c + dx)}{12d} + \frac{A\left(a^2 + a^2\right.}{}$$
$$= \frac{a^2(7A + 8B)\sec(c + dx)\tan(c + dx)}{8d} + \frac{a^2(5A + 4B}{}$$
$$= \frac{a^2(7A + 8B)\tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^2(4A + 5B)\mathrm{ta}}{3d}$$

Mathematica [A] time = 1.26, size = 262, normalized size = 1.82

$$-\frac{a^2(\cos(c + dx) + 1)^2\sec^4\left(\frac{1}{2}(c + dx)\right)\sec^4(c + dx)\left(24(7A + 8B)\cos^4(c + dx)\left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] -1/768*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^4*(24*(7*A
 + 8*B)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-24*(4*A + 5*B)*Sin[c] + 3*(15*
A + 8*B)*Sin[d*x] + 45*A*Sin[2*c + d*x] + 24*B*Sin[2*c + d*x] + 128*A*Sin[c
 + 2*d*x] + 136*B*Sin[c + 2*d*x] - 24*B*Sin[3*c + 2*d*x] + 21*A*Sin[2*c + 3
*d*x] + 24*B*Sin[2*c + 3*d*x] + 21*A*Sin[4*c + 3*d*x] + 24*B*Sin[4*c + 3*d*
x] + 32*A*Sin[3*c + 4*d*x] + 40*B*Sin[3*c + 4*d*x])))/d
```

fricas [A] time = 0.63, size = 145, normalized size = 1.01

$$\frac{3\,(7\,A + 8\,B)a^2\cos\left(dx + c\right)^4\log\left(\sin\left(dx + c\right) + 1\right) - 3\,(7\,A + 8\,B)a^2\cos\left(dx + c\right)^4\log\left(-\sin\left(dx + c\right) + 1\right) + 2\left(8\right.}{48\,d\cos\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(7*A + 8*B)*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(7*A + 8*B
)*a^2*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(4*A + 5*B)*a^2*cos(d*x
+ c)^3 + 3*(7*A + 8*B)*a^2*cos(d*x + c)^2 + 8*(2*A + B)*a^2*cos(d*x + c) +
6*A*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.87, size = 212, normalized size = 1.47

$$\frac{3\left(7\,Aa^{2}+8\,Ba^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(7\,Aa^{2}+8\,Ba^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(7*A*a^2 + 8*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(7*A*a^2
 + 8*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(21*A*a^2*tan(1/2*d*x +
1/2*c)^7 + 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*A*a^2*tan(1/2*d*x + 1/2*c)^
5 - 88*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 83*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 136
*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*A*a^2*tan(1/2*d*x + 1/2*c) - 72*B*a^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.16, size = 187, normalized size = 1.30

$$\frac{7a^{2}A\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{7a^{2}A\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{5B\,a^{2}\tan\left(dx+c\right)}{3d}+\frac{4a^{2}A\tan\left(dx+c\right)}{3d}+\frac{2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 7/8*a^2*A*sec(d*x+c)*tan(d*x+c)/d+7/8/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+5/3
/d*B*a^2*tan(d*x+c)+4/3*a^2*A*tan(d*x+c)/d+2/3*a^2*A*sec(d*x+c)^2*tan(d*x+c
)/d+1/d*B*a^2*sec(d*x+c)*tan(d*x+c)+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/4
*a^2*A*sec(d*x+c)^3*tan(d*x+c)/d+1/3/d*B*a^2*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.69, size = 230, normalized size = 1.60

$$\frac{32\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{2}+16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{2}-3\,Aa^{2}\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^2 - 3*A*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 12*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 24*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*B*a^2*tan(d*x +
 c))/d
```

mupad [**B**] time = 2.68, size = 183, normalized size = 1.27

$$\frac{\left(-\frac{7\,A\,a^2}{4}-2\,B\,a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{77\,A\,a^2}{12}+\frac{22\,B\,a^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{83\,A\,a^2}{12}-\frac{34\,B\,a^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{25\,A\,a^2}{4}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((25*A*a^2)/4 + 6*B*a^2) - tan(c/2 + (d*x)/2)^7*((7*A*a
^2)/4 + 2*B*a^2) + tan(c/2 + (d*x)/2)^5*((77*A*a^2)/12 + (22*B*a^2)/3) - ta
n(c/2 + (d*x)/2)^3*((83*A*a^2)/12 + (34*B*a^2)/3))/(d*(6*tan(c/2 + (d*x)/2)
^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8
 + 1)) + (2*a^2*atanh(tan(c/2 + (d*x)/2))*((7*A)/8 + B))/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.19 $\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=201

$$-\frac{a^3(19A+17B)\sin^3(c+dx)}{15d}+\frac{a^3(19A+17B)\sin(c+dx)}{5d}+\frac{a^3(22A+21B)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{(3A+4B)\mathrm{s}}{}$$

```
[Out] 1/16*a^3*(26*A+23*B)*x+1/5*a^3*(19*A+17*B)*sin(d*x+c)/d+1/16*a^3*(26*A+23*B
)*cos(d*x+c)*sin(d*x+c)/d+1/40*a^3*(22*A+21*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/
6*a*B*cos(d*x+c)^3*(a+a*cos(d*x+c))^2*sin(d*x+c)/d+1/15*(3*A+4*B)*cos(d*x+c
)^3*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d-1/15*a^3*(19*A+17*B)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.43, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a^3(19A+17B)\sin^3(c+dx)}{15d}+\frac{a^3(19A+17B)\sin(c+dx)}{5d}+\frac{a^3(22A+21B)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{(3A+4B)\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^3*(26*A + 23*B)*x)/16 + (a^3*(19*A + 17*B)*Sin[c + d*x])/(5*d) + (a^3*(2
6*A + 23*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(22*A + 21*B)*Cos[c +
d*x]^3*Sin[c + d*x])/(40*d) + (a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2*Si
n[c + d*x])/(6*d) + ((3*A + 4*B)*Cos[c + d*x]^3*(a^3 + a^3*Cos[c + d*x])*Si
n[c + d*x])/(15*d) - (a^3*(19*A + 17*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{6d} + \frac{1}{6}$$
$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{6d} + \frac{(3}{}$$
$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{6d} + \frac{(3}{}$$
$$= \frac{a^3(22A+21B)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{aB\cos^3(c}{}$$
$$= \frac{a^3(22A+21B)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{aB\cos^3(c}{}$$
$$= \frac{a^3(26A+23B)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^3(22A+}{}$$
$$= \frac{1}{16}a^3(26A+23B)x + \frac{a^3(19A+17B)\sin(c+dx)}{5d} +$$

Mathematica [A] time = 0.59, size = 134, normalized size = 0.67

$$\frac{a^3(120(23A+21B)\sin(c+dx)+15(64A+63B)\sin(2(c+dx))+340A\sin(3(c+dx))+90A\sin(4(c+dx))+12}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^3*(1380*B*c + 1560*A*d*x + 1380*B*d*x + 120*(23*A + 21*B)*Sin[c + d*x] +
 15*(64*A + 63*B)*Sin[2*(c + d*x)] + 340*A*Sin[3*(c + d*x)] + 380*B*Sin[3*(
c + d*x)] + 90*A*Sin[4*(c + d*x)] + 135*B*Sin[4*(c + d*x)] + 12*A*Sin[5*(c
+ d*x)] + 36*B*Sin[5*(c + d*x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

fricas [A] time = 0.83, size = 130, normalized size = 0.65

$$\frac{15\,(26\,A+23\,B)a^3dx+\left(40\,Ba^3\cos\left(dx+c\right)^5+48\,(A+3\,B)a^3\cos\left(dx+c\right)^4+10\,(18\,A+23\,B)a^3\cos\left(dx+c\right)^3+\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(26*A + 23*B)*a^3*d*x + (40*B*a^3*cos(d*x + c)^5 + 48*(A + 3*B)*a
^3*cos(d*x + c)^4 + 10*(18*A + 23*B)*a^3*cos(d*x + c)^3 + 16*(19*A + 17*B)*
```

```
a^3*cos(d*x + c)^2 + 15*(26*A + 23*B)*a^3*cos(d*x + c) + 32*(19*A + 17*B)*a
^3)*sin(d*x + c))/d
```

giac [A] time = 0.36, size = 166, normalized size = 0.83

$$\frac{Ba^3\sin(6\,dx+6\,c)}{192\,d}+\frac{1}{16}\left(26\,Aa^3+23\,Ba^3\right)x+\frac{\left(Aa^3+3\,Ba^3\right)\sin(5\,dx+5\,c)}{80\,d}+\frac{3\left(2\,Aa^3+3\,Ba^3\right)\sin(4\,dx+4\,c)}{64\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*B*a^3*sin(6*d*x + 6*c)/d + 1/16*(26*A*a^3 + 23*B*a^3)*x + 1/80*(A*a^3
 + 3*B*a^3)*sin(5*d*x + 5*c)/d + 3/64*(2*A*a^3 + 3*B*a^3)*sin(4*d*x + 4*c)/
d + 1/48*(17*A*a^3 + 19*B*a^3)*sin(3*d*x + 3*c)/d + 1/64*(64*A*a^3 + 63*B*a
^3)*sin(2*d*x + 2*c)/d + 1/8*(23*A*a^3 + 21*B*a^3)*sin(d*x + c)/d
```

maple [A] time = 0.07, size = 266, normalized size = 1.32

$$\frac{\frac{A\,a^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a^3B\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+3A\,a^3\left(\frac{\left(\cos^3(dx+\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(1/6*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+3
*A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3/5*a^3
*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^3*(2+cos(d*x+c)^2)*si
n(d*x+c)+3*a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*
c)+A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^3*B*(2+cos(d*x+c)^
2)*sin(d*x+c))
```

maxima [A] time = 0.55, size = 262, normalized size = 1.30

$$\frac{64\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c)\right)Aa^3-960\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Aa^3+90\,(12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 -
960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 90*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*
A*a^3 + 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^3
- 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d
*x + 2*c))*B*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 90*(12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3)/d
```

mupad [B] time = 1.61, size = 315, normalized size = 1.57

$$\frac{\left(\frac{13\,A\,a^3}{4}+\frac{23\,B\,a^3}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{221\,A\,a^3}{12}+\frac{391\,B\,a^3}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{429\,A\,a^3}{10}+\frac{759\,B\,a^3}{20}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{499\,A}{10}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+20\,\tan\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((51*A*a^3)/4 + (105*B*a^3)/8) + tan(c/2 + (d*x)/2)^11*
((13*A*a^3)/4 + (23*B*a^3)/8) + tan(c/2 + (d*x)/2)^3*((419*A*a^3)/12 + (211
*B*a^3)/8) + tan(c/2 + (d*x)/2)^9*((221*A*a^3)/12 + (391*B*a^3)/24) + tan(c
/2 + (d*x)/2)^7*((429*A*a^3)/10 + (759*B*a^3)/20) + tan(c/2 + (d*x)/2)^5*((
499*A*a^3)/10 + (969*B*a^3)/20))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 +
(d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2
+ (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - (a^3*(26*A + 23*B)*(atan(tan(
c/2 + (d*x)/2)) - (d*x)/2))/(8*d) + (a^3*atan((a^3*tan(c/2 + (d*x)/2)*(26*A
 + 23*B))/(8*((13*A*a^3)/4 + (23*B*a^3)/8)))*(26*A + 23*B))/(8*d)
```

sympy [A] time = 4.83, size = 695, normalized size = 3.46

$$\begin{cases} \frac{9Aa^3x\sin^4(c+dx)}{8}+\frac{9Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{Aa^3x\sin^2(c+dx)}{2}+\frac{9Aa^3x\cos^4(c+dx)}{8}+\frac{Aa^3x\cos^2(c+dx)}{2}+\frac{8Aa^3\sin^5(c+dx)}{15d}+\frac{4Aa}{} \\ x\,(A+B\cos(c))\,(a\cos(c)+a)^3\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((9*A*a**3*x*sin(c + d*x)**4/8 + 9*A*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + A*a**3*x*sin(c + d*x)**2/2 + 9*A*a**3*x*cos(c + d*x)**4/8 + A
*a**3*x*cos(c + d*x)**2/2 + 8*A*a**3*sin(c + d*x)**5/(15*d) + 4*A*a**3*sin(
c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(
8*d) + 2*A*a**3*sin(c + d*x)**3/d + A*a**3*sin(c + d*x)*cos(c + d*x)**4/d +
```

```
 15*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*A*a**3*sin(c + d*x)*cos(c
 + d*x)**2/d + A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 5*B*a**3*x*sin(c +
d*x)**6/16 + 15*B*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*B*a**3*x*si
n(c + d*x)**4/8 + 15*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 9*B*a**3
*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*B*a**3*x*cos(c + d*x)**6/16 + 9*B*
a**3*x*cos(c + d*x)**4/8 + 5*B*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 8
*B*a**3*sin(c + d*x)**5/(5*d) + 5*B*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6
*d) + 4*B*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d + 9*B*a**3*sin(c + d*x)**3
*cos(c + d*x)/(8*d) + 2*B*a**3*sin(c + d*x)**3/(3*d) + 11*B*a**3*sin(c + d*
x)*cos(c + d*x)**5/(16*d) + 3*B*a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*B*
a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**3*sin(c + d*x)*cos(c + d*x)*
*2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**3*cos(c)**2, True))
```

3.20 $\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=154

$$-\frac{a^3(15A+13B)\sin^3(c+dx)}{60d}+\frac{a^3(15A+13B)\sin(c+dx)}{5d}+\frac{3a^3(15A+13B)\sin(c+dx)\cos(c+dx)}{40d}+\frac{1}{8}a^3x(15A+$$

```
[Out] 1/8*a^3*(15*A+13*B)*x+1/5*a^3*(15*A+13*B)*sin(d*x+c)/d+3/40*a^3*(15*A+13*B)
*cos(d*x+c)*sin(d*x+c)/d+1/20*(5*A-B)*(a+a*cos(d*x+c))^3*sin(d*x+c)/d+1/5*B
*(a+a*cos(d*x+c))^4*sin(d*x+c)/a/d-1/60*a^3*(15*A+13*B)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.23, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2968, 3023, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{a^3(15A+13B)\sin^3(c+dx)}{60d}+\frac{a^3(15A+13B)\sin(c+dx)}{5d}+\frac{3a^3(15A+13B)\sin(c+dx)\cos(c+dx)}{40d}+\frac{1}{8}a^3x(15A+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^3*(15*A + 13*B)*x)/8 + (a^3*(15*A + 13*B)*Sin[c + d*x])/(5*d) + (3*a^3*(
15*A + 13*B)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + ((5*A - B)*(a + a*Cos[c +
d*x])^3*Sin[c + d*x])/(20*d) + (B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*a
*d) - (a^3*(15*A + 13*B)*Sin[c + d*x]^3)/(60*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^3\left(A\cos(c+dx)+B\cos^2(c+dx\right.$$

$$= \frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5ad} + \frac{\int (a+a\cos(c+\right.}{}$$

$$= \frac{(5A-B)(a+a\cos(c+dx))^3\sin(c+dx)}{20d} + \frac{B(a+a\,\mathrm{c}}{}$$

$$= \frac{(5A-B)(a+a\cos(c+dx))^3\sin(c+dx)}{20d} + \frac{B(a+a\,\mathrm{c}}{}$$

$$= \frac{1}{20}a^3(15A+13B)x + \frac{(5A-B)(a+a\cos(c+dx))^3\,\mathrm{si}}{20d}$$

$$= \frac{1}{20}a^3(15A+13B)x + \frac{3a^3(15A+13B)\sin(c+dx)}{20d} + $$

$$= \frac{1}{8}a^3(15A+13B)x + \frac{a^3(15A+13B)\sin(c+dx)}{5d} + \frac{3a}{}$$

Mathematica [A] time = 0.46, size = 108, normalized size = 0.70

$$\frac{a^3(60(26A+23B)\sin(c+dx)+480(A+B)\sin(2(c+dx))+120A\sin(3(c+dx))+15A\sin(4(c+dx))+900Ad}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^3*(780*B*c + 900*A*d*x + 780*B*d*x + 60*(26*A + 23*B)*Sin[c + d*x] + 480
*(A + B)*Sin[2*(c + d*x)] + 120*A*Sin[3*(c + d*x)] + 170*B*Sin[3*(c + d*x)]
 + 15*A*Sin[4*(c + d*x)] + 45*B*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)]))/(
480*d)
```

fricas [A] time = 0.87, size = 110, normalized size = 0.71

$$\frac{15\,(15\,A+13\,B)a^3dx+\left(24\,Ba^3\cos\left(dx+c\right)^4+30\,(A+3\,B)a^3\cos\left(dx+c\right)^3+8\,(15\,A+19\,B)a^3\cos\left(dx+c\right)^2+1\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/120*(15*(15*A + 13*B)*a^3*d*x + (24*B*a^3*cos(d*x + c)^4 + 30*(A + 3*B)*a
^3*cos(d*x + c)^3 + 8*(15*A + 19*B)*a^3*cos(d*x + c)^2 + 15*(15*A + 13*B)*a
^3*cos(d*x + c) + 8*(45*A + 38*B)*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.41, size = 136, normalized size = 0.88

$$\frac{Ba^3\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{8}\left(15\,Aa^3+13\,Ba^3\right)x+\frac{\left(Aa^3+3\,Ba^3\right)\sin(4\,dx+4\,c)}{32\,d}+\frac{\left(12\,Aa^3+17\,Ba^3\right)\sin(3\,dx+3\,c)}{48\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/80*B*a^3*sin(5*d*x + 5*c)/d + 1/8*(15*A*a^3 + 13*B*a^3)*x + 1/32*(A*a^3 +
 3*B*a^3)*sin(4*d*x + 4*c)/d + 1/48*(12*A*a^3 + 17*B*a^3)*sin(3*d*x + 3*c)/
d + (A*a^3 + B*a^3)*sin(2*d*x + 2*c)/d + 1/8*(26*A*a^3 + 23*B*a^3)*sin(d*x
+ c)/d
```

maple [A] time = 0.06, size = 223, normalized size = 1.45

$$\frac{A\,a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^3B\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+A\,a^3\left(2+\cos^2(dx+c)\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5
*a^3*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^3*(2+cos(d*x+c)^2
)*sin(d*x+c)+3*a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+
3/8*c)+3*A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*B*(2+cos(d*x+c
)^2)*sin(d*x+c)+A*a^3*sin(d*x+c)+a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1
/2*c))
```

maxima [A] time = 0.74, size = 213, normalized size = 1.38

$$-\frac{480\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Aa^3-15\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^3-360\,(2\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/480*(480*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 360*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*a^3 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
```

```
*B*a^3 + 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 45*(12*d*x + 12*c +
sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 - 120*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*B*a^3 - 480*A*a^3*sin(d*x + c))/d
```

mupad [B] time = 1.50, size = 277, normalized size = 1.80

$$\frac{\left(\frac{15\,A\,a^3}{4}+\frac{13\,B\,a^3}{4}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{35\,A\,a^3}{2}+\frac{91\,B\,a^3}{6}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(32\,A\,a^3+\frac{416\,B\,a^3}{15}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{61\,A\,a^3}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((49*A*a^3)/4 + (51*B*a^3)/4) + tan(c/2 + (d*x)/2)^9*((
15*A*a^3)/4 + (13*B*a^3)/4) + tan(c/2 + (d*x)/2)^7*((35*A*a^3)/2 + (91*B*a^
3)/6) + tan(c/2 + (d*x)/2)^3*((61*A*a^3)/2 + (133*B*a^3)/6) + tan(c/2 + (d*
x)/2)^5*(32*A*a^3 + (416*B*a^3)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/
2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2
 + (d*x)/2)^10 + 1)) - (a^3*(15*A + 13*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)
/2))/(4*d) + (a^3*atan((a^3*tan(c/2 + (d*x)/2)*(15*A + 13*B))/(4*((15*A*a^3
)/4 + (13*B*a^3)/4)))*(15*A + 13*B))/(4*d)
```

sympy [A] time = 2.81, size = 530, normalized size = 3.44

$$\begin{cases}\frac{3Aa^3x\sin^4(c+dx)}{8}+\frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3Aa^3x\sin^2(c+dx)}{2}+\frac{3Aa^3x\cos^4(c+dx)}{8}+\frac{3Aa^3x\cos^2(c+dx)}{2}+\frac{3Aa^3\sin^3(c+dx)\cos(c}{8d}\\ x\left(A+B\cos(c)\right)\left(a\cos(c)+a\right)^3\cos(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + 3*A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**4/8 +
 3*A*a**3*x*cos(c + d*x)**2/2 + 3*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d)
 + 2*A*a**3*sin(c + d*x)**3/d + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d)
 + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a**3*sin(c + d*x)*cos(c +
d*x)/(2*d) + A*a**3*sin(c + d*x)/d + 9*B*a**3*x*sin(c + d*x)**4/8 + 9*B*a**
3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + B*a**3*x*sin(c + d*x)**2/2 + 9*B*a*
*3*x*cos(c + d*x)**4/8 + B*a**3*x*cos(c + d*x)**2/2 + 8*B*a**3*sin(c + d*x)
**5/(15*d) + 4*B*a**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*B*a**3*sin(
c + d*x)**3*cos(c + d*x)/(8*d) + 2*B*a**3*sin(c + d*x)**3/d + B*a**3*sin(c
```

```
+ d*x)*cos(c + d*x)**4/d + 15*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3
*B*a**3*sin(c + d*x)*cos(c + d*x)**2/d + B*a**3*sin(c + d*x)*cos(c + d*x)/(
2*d), Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**3*cos(c), True))
```

3.21 $\int (a + a\cos(c + dx))^3 (A + B\cos(c + dx))\, dx$

Optimal. Leaf size=116

$$-\frac{a^3(4A+3B)\sin^3(c+dx)}{12d}+\frac{a^3(4A+3B)\sin(c+dx)}{d}+\frac{3a^3(4A+3B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3x(4A+3B)+\frac{B}{}$$

```
[Out] 5/8*a^3*(4*A+3*B)*x+a^3*(4*A+3*B)*sin(d*x+c)/d+3/8*a^3*(4*A+3*B)*cos(d*x+c)
*sin(d*x+c)/d+1/4*B*(a+a*cos(d*x+c))^3*sin(d*x+c)/d-1/12*a^3*(4*A+3*B)*sin(
d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{a^3(4A+3B)\sin^3(c+dx)}{12d}+\frac{a^3(4A+3B)\sin(c+dx)}{d}+\frac{3a^3(4A+3B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3x(4A+3B)+\frac{B}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (5*a^3*(4*A + 3*B)*x)/8 + (a^3*(4*A + 3*B)*Sin[c + d*x])/d + (3*a^3*(4*A +
3*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (B*(a + a*Cos[c + d*x])^3*Sin[c + d
*x])/(4*d) - (a^3*(4*A + 3*B)*Sin[c + d*x]^3)/(12*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx &= \frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}(4A+3B)\int (a+a\cos(c\\ &= \frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}(4A+3B)\int \left(a^3+3a^3\text{co}\right.\\ &= \frac{1}{4}a^3(4A+3B)x+\frac{B(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}\left(a^3(4A\right.\\ &= \frac{1}{4}a^3(4A+3B)x+\frac{3a^3(4A+3B)\sin(c+dx)}{4d}+\frac{3a^3(4A+3B)\,\text{co}}{}\\ &= \frac{5}{8}a^3(4A+3B)x+\frac{a^3(4A+3B)\sin(c+dx)}{d}+\frac{3a^3(4A+3B)\cos}{}\end{aligned}$$

Mathematica [A] time = 0.33, size = 86, normalized size = 0.74

$$\frac{a^3(24(15A+13B)\sin(c+dx)+24(3A+4B)\sin(2(c+dx))+8A\sin(3(c+dx))+240Adx+24B\sin(3(c+dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^3*(240*A*d*x + 180*B*d*x + 24*(15*A + 13*B)*Sin[c + d*x] + 24*(3*A + 4*B
)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 24*B*Sin[3*(c + d*x)] + 3*B*Sin
[4*(c + d*x)]))/(96*d)
```

fricas [A] time = 0.81, size = 90, normalized size = 0.78

$$\frac{15\,(4\,A+3\,B)a^3dx+\left(6\,Ba^3\cos\left(dx+c\right)^3+8\,(A+3\,B)a^3\cos\left(dx+c\right)^2+9\,(4\,A+5\,B)a^3\cos\left(dx+c\right)+8\,(11\,A+\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(15*(4*A + 3*B)*a^3*d*x + (6*B*a^3*cos(d*x + c)^3 + 8*(A + 3*B)*a^3*co
s(d*x + c)^2 + 9*(4*A + 5*B)*a^3*cos(d*x + c) + 8*(11*A + 9*B)*a^3)*sin(d*x
 + c))/d
```

giac [A] time = 0.36, size = 112, normalized size = 0.97

$$\frac{Ba^3\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{5}{8}\left(4\,Aa^3+3\,Ba^3\right)x+\frac{\left(Aa^3+3\,Ba^3\right)\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{\left(3\,Aa^3+4\,Ba^3\right)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{(15\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*B*a^3*sin(4*d*x + 4*c)/d + 5/8*(4*A*a^3 + 3*B*a^3)*x + 1/12*(A*a^3 + 3
*B*a^3)*sin(3*d*x + 3*c)/d + 1/4*(3*A*a^3 + 4*B*a^3)*sin(2*d*x + 2*c)/d + 1
/4*(15*A*a^3 + 13*B*a^3)*sin(d*x + c)/d
```

maple [A] time = 0.06, size = 176, normalized size = 1.52

$$\frac{a^3B\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{A\,a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a^3B\left(2+\cos^2\left(dx+c\right)\right)\sin\left(dx+c\right)+3.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+3*A*a^
3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^3*B*(1/2*cos(d*x+c)*sin(d*x
+c)+1/2*d*x+1/2*c)+3*A*a^3*sin(d*x+c)+a^3*B*sin(d*x+c)+A*a^3*(d*x+c))
```

maxima [A] time = 0.44, size = 167, normalized size = 1.44

$$-\frac{32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Aa^3-72\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa^3-96\,(dx+c)Aa^3+96\left(\sin\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 72*(2*d*x + 2*c + sin(2
*d*x + 2*c))*A*a^3 - 96*(d*x + c)*A*a^3 + 96*(sin(d*x + c)^3 - 3*sin(d*x +
c))*B*a^3 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3
 - 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 - 288*A*a^3*sin(d*x + c) - 96*
B*a^3*sin(d*x + c))/d
```

mupad [B] time = 0.27, size = 134, normalized size = 1.16

$$\frac{5\,A\,a^3\,x}{2}+\frac{15\,B\,a^3\,x}{8}+\frac{15\,A\,a^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{13\,B\,a^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{3\,A\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,a^3\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (5*A*a^3*x)/2 + (15*B*a^3*x)/8 + (15*A*a^3*sin(c + d*x))/(4*d) + (13*B*a^3*
sin(c + d*x))/(4*d) + (3*A*a^3*sin(2*c + 2*d*x))/(4*d) + (A*a^3*sin(3*c + 3
*d*x))/(12*d) + (B*a^3*sin(2*c + 2*d*x))/d + (B*a^3*sin(3*c + 3*d*x))/(4*d)
 + (B*a^3*sin(4*c + 4*d*x))/(32*d)
```

sympy [A] time = 1.32, size = 371, normalized size = 3.20

$$\begin{cases} \frac{3Aa^3x\sin^2{(c+dx)}}{2} + \frac{3Aa^3x\cos^2{(c+dx)}}{2} + Aa^3x + \frac{2Aa^3\sin^3{(c+dx)}}{3d} + \frac{Aa^3\sin{(c+dx)}\cos^2{(c+dx)}}{d} + \frac{3Aa^3\sin{(c+dx)}\cos{(c+dx)}}{2d} + \frac{3Aa^3 s}{} \\ x\left(A+B\cos{(c)}\right)\left(a\cos{(c)}+a\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**2/2 + A*
a**3*x + 2*A*a**3*sin(c + d*x)**3/(3*d) + A*a**3*sin(c + d*x)*cos(c + d*x)*
*2/d + 3*A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*A*a**3*sin(c + d*x)/d +
 3*B*a**3*x*sin(c + d*x)**4/8 + 3*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*B*a**3*x*sin(c + d*x)**2/2 + 3*B*a**3*x*cos(c + d*x)**4/8 + 3*B*a**3*
x*cos(c + d*x)**2/2 + 3*B*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*B*a**
3*sin(c + d*x)**3/d + 5*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*B*a**
3*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a**3*sin(c + d*x)*cos(c + d*x)/(2*d)
 + B*a**3*sin(c + d*x)/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)**3, T
rue))
```

3.22 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=111

$$\frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{(3A+5B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}+\frac{1}{2}a^3x(7A+5B)+\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+$$

```
[Out] 1/2*a^3*(7*A+5*B)*x+a^3*A*arctanh(sin(d*x+c))/d+5/2*a^3*(A+B)*sin(d*x+c)/d+
1/3*a*B*(a+a*cos(d*x+c))^2*sin(d*x+c)/d+1/6*(3*A+5*B)*(a^3+a^3*cos(d*x+c))*
sin(d*x+c)/d
```

Rubi [A] time = 0.30, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2976, 2968, 3023, 2735, 3770}

$$\frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{(3A+5B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{6d}+\frac{1}{2}a^3x(7A+5B)+\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^3*(7*A + 5*B)*x)/2 + (a^3*A*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(A + B)*Si
n[c + d*x])/(2*d) + (a*B*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d) + ((3*A
 + 5*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(6*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
```

```
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{aB(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (a+a\cos$$

$$=\frac{aB(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{(3A+5B)\left(a^3\right.}{}$$

$$=\frac{aB(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{(3A+5B)\left(a^3\right.}{}$$

$$=\frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{aB(a+a\cos(c+dx))^2\sin}{3d}$$

$$=\frac{1}{2}a^3(7A+5B)x+\frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{aB(a+}{}$$

$$=\frac{1}{2}a^3(7A+5B)x+\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^3(A}{}$$

Mathematica [A] time = 0.28, size = 113, normalized size = 1.02

$$\frac{a^3\left(9(4A+5B)\sin(c+dx)+3(A+3B)\sin(2(c+dx))-12A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12A\log\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^3*(42*A*d*x + 30*B*d*x - 12*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 12*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*(4*A + 5*B)*Sin[c + d*x]
 + 3*(A + 3*B)*Sin[2*(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d)
```

fricas [A] time = 0.81, size = 102, normalized size = 0.92

$$\frac{3\,(7\,A+5\,B)a^3dx+3\,Aa^3\log\left(\sin\left(dx+c\right)+1\right)-3\,Aa^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,Ba^3\cos\left(dx+c\right)^2+3\,(A+3}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(7*A + 5*B)*a^3*d*x + 3*A*a^3*log(sin(d*x + c) + 1) - 3*A*a^3*log(-s
in(d*x + c) + 1) + (2*B*a^3*cos(d*x + c)^2 + 3*(A + 3*B)*a^3*cos(d*x + c) +
 2*(9*A + 11*B)*a^3)*sin(d*x + c))/d
```

giac [A] time = 0.45, size = 180, normalized size = 1.62

$$\frac{6\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(7\,Aa^3+5\,Ba^3\right)(dx+c)+\frac{2\left(15\,Aa^3\tan\left(\frac{1}{2}\right.\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/6*(6*A*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*A*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(7*A*a^3 + 5*B*a^3)*(d*x + c) + 2*(15*A*a^3*tan(1/2*d
*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^3*tan(1/2*d*x + 1/
2*c)^3 + 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*A*a^3*tan(1/2*d*x + 1/2*c) +
33*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.14, size = 153, normalized size = 1.38

$$\frac{A\,a^3\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d}+\frac{7A\,a^3x}{2}+\frac{7A\,a^3c}{2d}+\frac{B\sin\left(dx+c\right)\left(\cos^2\left(dx+c\right)\right)a^3}{3d}+\frac{11a^3B\sin\left(dx+c\right)}{3d}+\frac{3a^3A\sin(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/2/d*A*a^3*cos(d*x+c)*sin(d*x+c)+7/2*A*a^3*x+7/2/d*A*a^3*c+1/3/d*B*sin(d*x
+c)*cos(d*x+c)^2*a^3+11/3*a^3*B*sin(d*x+c)/d+3*a^3*A*sin(d*x+c)/d+3/2/d*a^3
*B*cos(d*x+c)*sin(d*x+c)+5/2*a^3*B*x+5/2/d*a^3*B*c+1/d*A*a^3*ln(sec(d*x+c)+
tan(d*x+c))
```

maxima [A] time = 0.52, size = 141, normalized size = 1.27

$$\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^3+36\,(dx+c)Aa^3-4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^3+9\,(2\,dx+2\,c+\sin\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 36*(d*x + c)*A*a^3 - 4*(si
n(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*B
*a^3 + 12*(d*x + c)*B*a^3 + 12*A*a^3*log(sec(d*x + c) + tan(d*x + c)) + 36*
A*a^3*sin(d*x + c) + 36*B*a^3*sin(d*x + c))/d
```

mupad [B] time = 0.42, size = 178, normalized size = 1.60

$$\frac{3\,A\,a^3\,\sin(c+d\,x)}{d}+\frac{15\,B\,a^3\,\sin(c+d\,x)}{4\,d}+\frac{7\,A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{5\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\cdots\right.}{\cos\left(\cdots\right.}\right)}{d}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x),x)
```

```
[Out] (3*A*a^3*sin(c + d*x))/d + (15*B*a^3*sin(c + d*x))/(4*d) + (7*A*a^3*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*A*a^3*atanh(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)))/d + (5*B*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/d + (A*a^3*sin(2*c + 2*d*x))/(4*d) + (3*B*a^3*sin(2*c + 2*d*x))/(4*d)
+ (B*a^3*sin(3*c + 3*d*x))/(12*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\sec(c+dx)\,dx+\int 3A\cos(c+dx)\sec(c+dx)\,dx+\int 3A\cos^2(c+dx)\sec(c+dx)\,dx+\int A\cos^3(c\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x), x) + Integral(3*A*cos(c + d*x)*sec(c + d*x),
 x) + Integral(3*A*cos(c + d*x)**2*sec(c + d*x), x) + Integral(A*cos(c + d*
x)**3*sec(c + d*x), x) + Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integra
l(3*B*cos(c + d*x)**2*sec(c + d*x), x) + Integral(3*B*cos(c + d*x)**3*sec(c
 + d*x), x) + Integral(B*cos(c + d*x)**4*sec(c + d*x), x))
```

3.23 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=110

$$\frac{a^3(3A+B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{1}{2}a^3x(6A+7B)+\frac{5a^3B\sin(c+dx)}{2d}$$

```
[Out] 1/2*a^3*(6*A+7*B)*x+a^3*(3*A+B)*arctanh(sin(d*x+c))/d+5/2*a^3*B*sin(d*x+c)/
d-1/2*(2*A-B)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d+a*A*(a+a*cos(d*x+c))^2*tan(
d*x+c)/d
```

Rubi [A] time = 0.31, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2975, 2976, 2968, 3023, 2735, 3770}

$$\frac{a^3(3A+B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(2A-B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{2d}+\frac{1}{2}a^3x(6A+7B)+\frac{5a^3B\sin(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^3*(6*A + 7*B)*x)/2 + (a^3*(3*A + B)*ArcTanh[Sin[c + d*x]])/d + (5*a^3*B*
Sin[c + d*x])/(2*d) - ((2*A - B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(2*
d) + (a*A*(a + a*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^2\tan(c+dx)}{d} + \int (a+a\cos($$
$$= -\frac{(2A-B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{2d} + \frac{aA($$
$$= -\frac{(2A-B)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{2d} + \frac{aA($$
$$= \frac{5a^3B\sin(c+dx)}{2d} - \frac{(2A-B)\left(a^3+a^3\cos(c+dx)\right)}{2d}$$
$$= \frac{1}{2}a^3(6A+7B)x + \frac{5a^3B\sin(c+dx)}{2d} - \frac{(2A-B)\left(a^3\right.}{}$$
$$= \frac{1}{2}a^3(6A+7B)x + \frac{a^3(3A+B)\tanh^{-1}(\sin(c+dx))}{d}$$

Mathematica [B] time = 1.85, size = 272, normalized size = 2.47

$$\frac{1}{32}a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{4(A+3B)\sin(c)\cos(dx)}{d} + \frac{4(A+3B)\cos(c)\sin(dx)}{d} - \frac{4(3A+B)\log\left(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(2*(6*A + 7*B)*x - (4*(3*A + B
)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (4*(3*A + B)*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]])/d + (4*(A + 3*B)*Cos[d*x]*Sin[c])/d + (B*Cos[2*
d*x]*Sin[2*c])/d + (4*(A + 3*B)*Cos[c]*Sin[d*x])/d + (B*Cos[2*c]*Sin[2*d*x]
)/d + (4*A*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])) + (4*A*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]))))/32
```

fricas [A] time = 0.71, size = 127, normalized size = 1.15

$$\frac{(6A+7B)a^3dx\cos(dx+c) + (3A+B)a^3\cos(dx+c)\log(\sin(dx+c)+1) - (3A+B)a^3\cos(dx+c)\log(-s}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*((6*A + 7*B)*a^3*d*x*cos(d*x + c) + (3*A + B)*a^3*cos(d*x + c)*log(sin(
d*x + c) + 1) - (3*A + B)*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + (B*a^3*
cos(d*x + c)^2 + 2*(A + 3*B)*a^3*cos(d*x + c) + 2*A*a^3)*sin(d*x + c))/(d*c
os(d*x + c))
```

giac [A] time = 0.92, size = 192, normalized size = 1.75

$$-\frac{\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\left(6\,Aa^3+7\,Ba^3\right)(dx+c)-2\left(3\,Aa^3+Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+2\left(3\,Aa^3+Ba^3\right)\log}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (6*A*a^3
+ 7*B*a^3)*(d*x + c) - 2*(3*A*a^3 + B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) + 2*(3*A*a^3 + B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*A*a^3*ta
n(1/2*d*x + 1/2*c)^3 + 5*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 2*A*a^3*tan(1/2*d*x
 + 1/2*c) + 7*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.15, size = 145, normalized size = 1.32

$$\frac{a^3A\sin(dx+c)}{d}+\frac{a^3B\cos(dx+c)\sin(dx+c)}{2d}+\frac{7a^3Bx}{2}+\frac{7a^3Bc}{2d}+3A\,a^3x+\frac{3A\,a^3c}{d}+\frac{3a^3B\sin(dx+c)}{d}+\frac{3A\,a^3\ln(\mathrm{se}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] a^3*A*sin(d*x+c)/d+1/2/d*a^3*B*cos(d*x+c)*sin(d*x+c)+7/2*a^3*B*x+7/2/d*a^3*
B*c+3*A*a^3*x+3/d*A*a^3*c+3*a^3*B*sin(d*x+c)/d+3/d*A*a^3*ln(sec(d*x+c)+tan(
d*x+c))+1/d*A*a^3*tan(d*x+c)+1/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.63, size = 140, normalized size = 1.27

$$\frac{12\,(dx+c)Aa^3+(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^3+12\,(dx+c)Ba^3+6\,Aa^3\left(\log(\sin(dx+c)+1)-\log(\sin(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/4*(12*(d*x + c)*A*a^3 + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 + 12*(d*x
+ c)*B*a^3 + 6*A*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*B*
a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*A*a^3*sin(d*x + c)
+ 12*B*a^3*sin(d*x + c) + 4*A*a^3*tan(d*x + c))/d
```

mupad [B] time = 0.37, size = 197, normalized size = 1.79

$$\frac{A\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{3\,B\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{6\,A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{7\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\right.}{\cos\left(\frac{c}{2}+\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] (A*a^3*sin(c + d*x))/d + (3*B*a^3*sin(c + d*x))/d + (6*A*a^3*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*A*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)))/d + (7*B*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
+ (2*B*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^3*sin(c +
 d*x))/(d*cos(c + d*x)) + (B*a^3*cos(c + d*x)*sin(c + d*x))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\sec^2\left(c+dx\right)dx+\int 3A\cos\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int 3A\cos^2\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int A\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x)**2, x) + Integral(3*A*cos(c + d*x)*sec(c + d*
x)**2, x) + Integral(3*A*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(A*c
os(c + d*x)**3*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**
2, x) + Integral(3*B*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(3*B*cos
(c + d*x)**3*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x)**4*sec(c + d*x)*
*2, x))
```

3.24 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=114

$$\frac{a^3(7A+6B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2A+B)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{d}+a^3x(A+3B)-\frac{5a^3A\sin(c+dx)}{2d}+$$

```
[Out] a^3*(A+3*B)*x+1/2*a^3*(7*A+6*B)*arctanh(sin(d*x+c))/d-5/2*a^3*A*sin(d*x+c)/
d+(2*A+B)*(a^3+a^3*cos(d*x+c))*tan(d*x+c)/d+1/2*a*A*(a+a*cos(d*x+c))^2*sec(
d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.34, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2975, 2968, 3023, 2735, 3770}

$$\frac{a^3(7A+6B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2A+B)\tan(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{d}+a^3x(A+3B)-\frac{5a^3A\sin(c+dx)}{2d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] a^3*(A + 3*B)*x + (a^3*(7*A + 6*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*A*
Sin[c + d*x])/(2*d) + ((2*A + B)*(a^3 + a^3*Cos[c + d*x])*Tan[c + d*x])/d +
 (a*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}$$

$$= \frac{(2A+B)\left(a^3+a^3\cos(c+dx)\right)\tan(c+dx)}{d}+\frac{aA(a}{}$$

$$= \frac{(2A+B)\left(a^3+a^3\cos(c+dx)\right)\tan(c+dx)}{d}+\frac{aA(a}{}$$

$$= -\frac{5a^3A\sin(c+dx)}{2d}+\frac{(2A+B)\left(a^3+a^3\cos(c+dx\right.}{d}$$

$$= a^3(A+3B)x-\frac{5a^3A\sin(c+dx)}{2d}+\frac{(2A+B)\left(a^3+\right.}{}$$

$$= a^3(A+3B)x+\frac{a^3(7A+6B)\tanh^{-1}(\sin(c+dx))}{2d}-$$

Mathematica [A] time = 2.00, size = 208, normalized size = 1.82

$$\frac{a^3\left(4(3A+B)\tan(c+dx)+\frac{A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{A}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}-14A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^3*(4*A*c + 12*B*c + 4*A*d*x + 12*B*d*x - 14*A*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - 12*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 14*A*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*B*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + A/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - A/(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^2 + 4*B*Sin[c + d*x] + 4*(3*A + B)*Tan[c + d*x]))/(4*d)
```

fricas [A] time = 0.63, size = 137, normalized size = 1.20

$$\frac{4\,(A+3\,B)a^3dx\cos\left(dx+c\right)^2+(7\,A+6\,B)a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(7\,A+6\,B)a^3\cos\left(dx+c\right)^2\log}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*(A + 3*B)*a^3*d*x*cos(d*x + c)^2 + (7*A + 6*B)*a^3*cos(d*x + c)^2*lo
g(sin(d*x + c) + 1) - (7*A + 6*B)*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
 + 2*(2*B*a^3*cos(d*x + c)^2 + 2*(3*A + B)*a^3*cos(d*x + c) + A*a^3)*sin(d*
x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.61, size = 192, normalized size = 1.68

$$\frac{\frac{4\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+2\left(Aa^3+3\,Ba^3\right)(dx+c)+\left(7\,Aa^3+6\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(7\,Aa^3+6\,Ba^3\right)\log\left(\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*B*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(A*a^3 +
 3*B*a^3)*(d*x + c) + (7*A*a^3 + 6*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
```

```
) - (7*A*a^3 + 6*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*A*a^3*tan
(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*A*a^3*tan(1/2*d*x
+ 1/2*c) - 2*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.16, size = 144, normalized size = 1.26

$$A\,a^3x+\frac{A\,a^3c}{d}+\frac{a^3B\sin(dx+c)}{d}+\frac{7A\,a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+3a^3Bx+\frac{3a^3Bc}{d}+\frac{3A\,a^3\tan(dx+c)}{d}+\frac{3a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] A*a^3*x+1/d*A*a^3*c+a^3*B*sin(d*x+c)/d+7/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c)
)+3*a^3*B*x+3/d*a^3*B*c+3/d*A*a^3*tan(d*x+c)+3/d*a^3*B*ln(sec(d*x+c)+tan(d*
x+c))+1/2/d*A*a^3*sec(d*x+c)*tan(d*x+c)+1/d*a^3*B*tan(d*x+c)
```

maxima [A] time = 0.80, size = 165, normalized size = 1.45

$$\frac{4(dx+c)Aa^3+12(dx+c)Ba^3-Aa^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6Aa^3\left(\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a^3 + 12*(d*x + c)*B*a^3 - A*a^3*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*A*a^3*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*B*a^3*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 4*B*a^3*sin(d*x + c) + 12*A*a^3*tan(d*x + c)
 + 4*B*a^3*tan(d*x + c))/d
```

mupad [B] time = 0.37, size = 207, normalized size = 1.82

$$\frac{B\,a^3\,\sin(c+d\,x)}{d}+\frac{2\,A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{7\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,B\,a^3\,\mathrm{atanh}\left(\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^3,x)
```

```
[Out] (B*a^3*sin(c + d*x))/d + (2*A*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d + (7*A*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*B*a^3
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*B*a^3*atanh(sin(c/2 +
```

```
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*A*a^3*sin(c + d*x))/(d*cos(c + d*x)) +
 (A*a^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (B*a^3*sin(c + d*x))/(d*cos(c
+ d*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.25 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=125

$$\frac{5a^3(A+B)\tan(c+dx)}{2d}+\frac{a^3(5A+7B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(5A+3B)\tan(c+dx)\sec(c+dx)\left(a^3\cos(c+dx)\right.}{6d}$$

```
[Out] a^3*B*x+1/2*a^3*(5*A+7*B)*arctanh(sin(d*x+c))/d+5/2*a^3*(A+B)*tan(d*x+c)/d+
1/6*(5*A+3*B)*(a^3+a^3*cos(d*x+c))*sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*(a+a*cos
(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.34, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2975, 2968, 3021, 2735, 3770}

$$\frac{5a^3(A+B)\tan(c+dx)}{2d}+\frac{a^3(5A+7B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(5A+3B)\tan(c+dx)\sec(c+dx)\left(a^3\cos(c+dx)\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] a^3*B*x + (a^3*(5*A + 7*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*(A + B)*Ta
n[c + d*x])/(2*d) + ((5*A + 3*B)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]*Tan[
c + d*x])/(6*d) + (a*A*(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/
(3*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}$$

$$= \frac{(5A+3B)\left(a^3+a^3\cos(c+dx)\right)\sec(c+dx)\tan(c+}{6d}$$

$$= \frac{(5A+3B)\left(a^3+a^3\cos(c+dx)\right)\sec(c+dx)\tan(c+}{6d}$$

$$= \frac{5a^3(A+B)\tan(c+dx)}{2d} + \frac{(5A+3B)\left(a^3+a^3\cos(c\right.}{}$$

$$= a^3Bx + \frac{5a^3(A+B)\tan(c+dx)}{2d} + \frac{(5A+3B)\left(a^3+a\right.}{}$$

$$= a^3Bx + \frac{a^3(5A+7B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{5a^3(A+}{}$$

Mathematica [B] time = 6.41, size = 786, normalized size = 6.29

$$\frac{\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(11A\sin\left(\frac{dx}{2}\right)+9B\sin\left(\frac{dx}{2}\right)\right)}{24d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(11A\sin\left(\frac{dx}{2}\right.\right.}{24d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (B*x*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6)/8 + ((-5*A - 7*B)*(a + a*
Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x
)/2]^6)/(16*d) + ((5*A + 7*B)*(a + a*Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6)/(16*d) + (A*(a + a*Cos[c + d*x
])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(48*d*(Cos[c/2] - Sin[c/2])*(Cos[c/
2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (
d*x)/2]^6*(10*A*Cos[c/2] + 3*B*Cos[c/2] - 8*A*Sin[c/2] - 3*B*Sin[c/2]))/(96
*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + ((a
 + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(11*A*Sin[(d*x)/2] + 9*B*Sin[(d*x
)/2]))/(24*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]
)) + (A*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(48*d*(Co
s[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + ((a + a*C
os[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-10*A*Cos[c/2] - 3*B*Cos[c/2] - 8*A*Si
n[c/2] - 3*B*Sin[c/2]))/(96*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + S
in[c/2 + (d*x)/2])^2) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(11*A*
Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/(24*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d
*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 0.65, size = 141, normalized size = 1.13

$$\frac{12\,Ba^3dx\cos(dx+c)^3+3\,(5\,A+7\,B)a^3\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,(5\,A+7\,B)a^3\cos(dx+c)^3\log(}{12\,d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(12*B*a^3*d*x*cos(d*x + c)^3 + 3*(5*A + 7*B)*a^3*cos(d*x + c)^3*log(si
n(d*x + c) + 1) - 3*(5*A + 7*B)*a^3*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
 2*(2*(11*A + 9*B)*a^3*cos(d*x + c)^2 + 3*(3*A + B)*a^3*cos(d*x + c) + 2*A*
a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 0.41, size = 189, normalized size = 1.51

$$\frac{6\,(dx+c)Ba^3+3\left(5\,Aa^3+7\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(5\,Aa^3+7\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\ \cdots}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(d*x + c)*B*a^3 + 3*(5*A*a^3 + 7*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3*(5*A*a^3 + 7*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*A
*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 36*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 33*A*a^3*tan(1/2*d
*x + 1/2*c) + 21*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3
)/d
```

maple [A] time = 0.16, size = 158, normalized size = 1.26

$$\frac{5A\,a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+a^3Bx+\frac{a^3Bc}{d}+\frac{11A\,a^3\tan\left(dx+c\right)}{3d}+\frac{7a^3B\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 5/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*B*x+1/d*a^3*B*c+11/3/d*A*a^3*tan(
d*x+c)+7/2/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*A*a^3*sec(d*x+c)*tan(d*x
+c)+3/d*a^3*B*tan(d*x+c)+1/3/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^3*B*se
c(d*x+c)*tan(d*x+c)
```

maxima [A] time = 0.69, size = 212, normalized size = 1.70

$$\frac{4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa^3+12\,(dx+c)Ba^3-9\,Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx\cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 12*(d*x + c)*B*a^3 - 9*A*
a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
```

```
d*x + c) - 1)) - 3*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 6*A*a^3*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1)) + 18*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)) + 36*A*a^3*tan(d*x + c) + 36*B*a^3*tan(d*x + c))/d
```

mupad [B] time = 0.33, size = 209, normalized size = 1.67

$$\frac{5\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{7\,B\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{11\,A\,a^3\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)}+\frac{3\,A\,a^3\,\sin}{2\,d\cos(c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] (5*A*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^3*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (7*B*a^3*atanh(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)))/d + (11*A*a^3*sin(c + d*x))/(3*d*cos(c + d*x)) + (3*A
*a^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (A*a^3*sin(c + d*x))/(3*d*cos(c +
 d*x)^3) + (3*B*a^3*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^3*sin(c + d*x))/(
2*d*cos(c + d*x)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.26 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=154

$$\frac{a^3(9A+11B)\tan(c+dx)}{3d}+\frac{5a^3(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(27A+28B)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(3A+2}{}$$

```
[Out] 5/8*a^3*(3*A+4*B)*arctanh(sin(d*x+c))/d+1/3*a^3*(9*A+11*B)*tan(d*x+c)/d+1/2
4*a^3*(27*A+28*B)*sec(d*x+c)*tan(d*x+c)/d+1/6*(3*A+2*B)*(a^3+a^3*cos(d*x+c)
)*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a*A*(a+a*cos(d*x+c))^2*sec(d*x+c)^3*tan(d*x
+c)/d
```

Rubi [A] time = 0.42, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a^3(9A+11B)\tan(c+dx)}{3d}+\frac{5a^3(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(27A+28B)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(3A+2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (5*a^3*(3*A + 4*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(9*A + 11*B)*Tan[c +
 d*x])/(3*d) + (a^3*(27*A + 28*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((3*A
 + 2*B)*(a^3 + a^3*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a*A*
(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}$$
$$= \frac{(3A+2B)\left(a^3+a^3\cos(c+dx)\right)\sec^2(c+dx)\tan(c+}{6d}$$
$$= \frac{(3A+2B)\left(a^3+a^3\cos(c+dx)\right)\sec^2(c+dx)\tan(c+}{6d}$$
$$= \frac{a^3(27A+28B)\sec(c+dx)\tan(c+dx)}{24d} + \frac{(3A+2B)}{}$$
$$= \frac{a^3(27A+28B)\sec(c+dx)\tan(c+dx)}{24d} + \frac{(3A+2B)}{}$$
$$= \frac{5a^3(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(27A+28B)}{}$$
$$= \frac{5a^3(3A+4B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(9A+11B)\,t}{3d}$$

Mathematica [A] time = 1.39, size = 273, normalized size = 1.77

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(120(3A+4B)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] -1/1536*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^4*(120*(3
*A + 4*B)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-24*(9*A + 11*B)*Sin[c] + (69
*A + 36*B)*Sin[d*x] + 69*A*Sin[2*c + d*x] + 36*B*Sin[2*c + d*x] + 264*A*Sin
[c + 2*d*x] + 280*B*Sin[c + 2*d*x] - 24*A*Sin[3*c + 2*d*x] - 72*B*Sin[3*c +
 2*d*x] + 45*A*Sin[2*c + 3*d*x] + 36*B*Sin[2*c + 3*d*x] + 45*A*Sin[4*c + 3*
d*x] + 36*B*Sin[4*c + 3*d*x] + 72*A*Sin[3*c + 4*d*x] + 88*B*Sin[3*c + 4*d*x
])))/d
```

fricas [A] time = 0.76, size = 145, normalized size = 0.94

$$\frac{15\,(3\,A+4\,B)a^3\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-15\,(3\,A+4\,B)a^3\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\right.}{48\,d\cos\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(15*(3*A + 4*B)*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*(3*A + 4
*B)*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(9*A + 11*B)*a^3*cos(d
*x + c)^3 + 9*(5*A + 4*B)*a^3*cos(d*x + c)^2 + 8*(3*A + B)*a^3*cos(d*x + c)
 + 6*A*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.76, size = 212, normalized size = 1.38

$$15\left(3\,Aa^{3}+4\,Ba^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(3\,Aa^{3}+4\,Ba^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(45\,Aa^{3}\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(15*(3*A*a^3 + 4*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*A*a
^3 + 4*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*A*a^3*tan(1/2*d*x
+ 1/2*c)^7 + 60*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 165*A*a^3*tan(1/2*d*x + 1/2*
c)^5 - 220*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 219*A*a^3*tan(1/2*d*x + 1/2*c)^3
+ 292*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 147*A*a^3*tan(1/2*d*x + 1/2*c) - 132*B
*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.18, size = 188, normalized size = 1.22

$$\frac{3A\,a^{3}\tan\left(dx+c\right)}{d}+\frac{5a^{3}B\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}+\frac{15A\,a^{3}\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{15A\,a^{3}\ln\left(\sec\left(dx+\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 3/d*A*a^3*tan(d*x+c)+5/2/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+15/8/d*A*a^3*sec
(d*x+c)*tan(d*x+c)+15/8/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+11/3/d*a^3*B*tan(
d*x+c)+1/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+3/2/d*a^3*B*sec(d*x+c)*tan(d*x+c)+
1/4/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+1/3/d*a^3*B*tan(d*x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.35, size = 269, normalized size = 1.75

$$48\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{3}+16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{3}-3\,Aa^{3}\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(48*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^3 - 3*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 36*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 36*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*B*a^3*(log(sin(
d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*A*a^3*tan(d*x + c) + 144*B*a^3*
tan(d*x + c))/d
```

mupad [B] time = 2.71, size = 185, normalized size = 1.20

$$\frac{\left(-\frac{15\,A\,a^3}{4}-5\,B\,a^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{55\,A\,a^3}{4}+\frac{55\,B\,a^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{73\,A\,a^3}{4}-\frac{73\,B\,a^3}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{49\,A\,a^3}{4}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((49*A*a^3)/4 + 11*B*a^3) - tan(c/2 + (d*x)/2)^7*((15*A
*a^3)/4 + 5*B*a^3) + tan(c/2 + (d*x)/2)^5*((55*A*a^3)/4 + (55*B*a^3)/3) - t
an(c/2 + (d*x)/2)^3*((73*A*a^3)/4 + (73*B*a^3)/3))/(d*(6*tan(c/2 + (d*x)/2)
^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8
 + 1)) + (5*a^3*atanh(tan(c/2 + (d*x)/2))*(3*A + 4*B))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.27 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=185

$$\frac{a^3(38A+45B)\tan(c+dx)}{15d}+\frac{a^3(13A+15B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(43A+45B)\tan(c+dx)\sec^2(c+dx)}{60d}+\frac{a^3(}{}$$

```
[Out] 1/8*a^3*(13*A+15*B)*arctanh(sin(d*x+c))/d+1/15*a^3*(38*A+45*B)*tan(d*x+c)/d
+1/8*a^3*(13*A+15*B)*sec(d*x+c)*tan(d*x+c)/d+1/60*a^3*(43*A+45*B)*sec(d*x+c
)^2*tan(d*x+c)/d+1/20*(7*A+5*B)*(a^3+a^3*cos(d*x+c))*sec(d*x+c)^3*tan(d*x+c
)/d+1/5*a*A*(a+a*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d
```

Rubi [A] time = 0.45, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^3(38A+45B)\tan(c+dx)}{15d}+\frac{a^3(13A+15B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(43A+45B)\tan(c+dx)\sec^2(c+dx)}{60d}+\frac{a^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (a^3*(13*A + 15*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(38*A + 45*B)*Tan[c
+ d*x])/(15*d) + (a^3*(13*A + 15*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3
*(43*A + 45*B)*Sec[c + d*x]^2*Tan[c + d*x])/(60*d) + ((7*A + 5*B)*(a^3 + a^
3*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*A*(a + a*Cos[c + d
*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^3 (A + B\cos(c+dx)) \sec^6(c+dx)\, dx = \frac{aA(a+a\cos(c+dx))^2 \sec^4(c+dx)\tan(c+dx)}{5d} +$$
$$= \frac{(7A+5B)\left(a^3 + a^3\cos(c+dx)\right)\sec^3(c+dx)\tan(c}{20d}$$
$$= \frac{(7A+5B)\left(a^3 + a^3\cos(c+dx)\right)\sec^3(c+dx)\tan(c}{20d}$$
$$= \frac{a^3(43A+45B)\sec^2(c+dx)\tan(c+dx)}{60d} + \frac{(7A+5}{}$$
$$= \frac{a^3(43A+45B)\sec^2(c+dx)\tan(c+dx)}{60d} + \frac{(7A+5}{}$$
$$= \frac{a^3(13A+15B)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^3(43A+}{}$$
$$= \frac{a^3(13A+15B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(38A+45}{1}$$

Mathematica [A] time = 1.60, size = 294, normalized size = 1.59

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(240(13A+15B)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] -1/15360*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^5*(240*(
13*A + 15*B)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(29*A + 30*B)*Sin[d*x]
- 240*(3*A + 5*B)*Sin[2*c + d*x] + 750*A*Sin[c + 2*d*x] + 570*B*Sin[c + 2*d
*x] + 750*A*Sin[3*c + 2*d*x] + 570*B*Sin[3*c + 2*d*x] + 1520*A*Sin[2*c + 3*
d*x] + 1680*B*Sin[2*c + 3*d*x] - 120*B*Sin[4*c + 3*d*x] + 195*A*Sin[3*c + 4
*d*x] + 225*B*Sin[3*c + 4*d*x] + 195*A*Sin[5*c + 4*d*x] + 225*B*Sin[5*c + 4
*d*x] + 304*A*Sin[4*c + 5*d*x] + 360*B*Sin[4*c + 5*d*x])))/d
```

fricas [A] time = 0.73, size = 165, normalized size = 0.89

$$\frac{15\,(13\,A + 15\,B)a^3\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right) - 15\,(13\,A+15\,B)a^3\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(13*A + 15*B)*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(13*A
 + 15*B)*a^3*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(38*A + 45*B)*a^3
*cos(d*x + c)^4 + 15*(13*A + 15*B)*a^3*cos(d*x + c)^3 + 8*(19*A + 15*B)*a^3
*cos(d*x + c)^2 + 30*(3*A + B)*a^3*cos(d*x + c) + 24*A*a^3)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

giac [A] time = 1.72, size = 246, normalized size = 1.33

$$\frac{15\left(13\,Aa^3+15\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(13\,Aa^3+15\,Ba^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(195\,Aa^3\tan\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(13*A*a^3 + 15*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(13
*A*a^3 + 15*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(195*A*a^3*tan(1/
2*d*x + 1/2*c)^9 + 225*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*A*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 1050*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*A*a^3*tan(1/2*d*x +
1/2*c)^5 + 1920*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*A*a^3*tan(1/2*d*x + 1/2
*c)^3 - 1830*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*A*a^3*tan(1/2*d*x + 1/2*c)
+ 735*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 0.17, size = 234, normalized size = 1.26

$$\frac{13A\,a^3\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{13A\,a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{3a^3B\tan\left(dx+c\right)}{d}+\frac{38A\,a^3\tan\left(dx+c\right)}{15d}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 13/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)+13/8/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+3
/d*a^3*B*tan(d*x+c)+38/15/d*A*a^3*tan(d*x+c)+19/15/d*A*a^3*tan(d*x+c)*sec(d
*x+c)^2+15/8/d*a^3*B*sec(d*x+c)*tan(d*x+c)+15/8/d*a^3*B*ln(sec(d*x+c)+tan(d
*x+c))+3/4/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+1/d*a^3*B*tan(d*x+c)*sec(d*x+c)^
2+1/5/d*A*a^3*tan(d*x+c)*sec(d*x+c)^4+1/4/d*a^3*B*tan(d*x+c)*sec(d*x+c)^3
```

maxima [A] time = 0.68, size = 337, normalized size = 1.82

$$\frac{16\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)Aa^3+240\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa^3+240\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
240*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^3 - 45*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 15*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 60*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) - 180*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - l
og(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*B*a^3*tan(d*x + c))/d
```

mupad [B] time = 2.82, size = 224, normalized size = 1.21

$$\frac{a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(13\,A+15\,B)}{4\,d}-\frac{\left(\frac{13\,A\,a^3}{4}+\frac{15\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{91\,A\,a^3}{6}-\frac{35\,B\,a^3}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\cdots\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^6,x)
```

```
[Out] (a^3*atanh(tan(c/2 + (d*x)/2))*(13*A + 15*B))/(4*d) - (tan(c/2 + (d*x)/2)*(
(51*A*a^3)/4 + (49*B*a^3)/4) + tan(c/2 + (d*x)/2)^9*((13*A*a^3)/4 + (15*B*a
^3)/4) - tan(c/2 + (d*x)/2)^7*((91*A*a^3)/6 + (35*B*a^3)/2) - tan(c/2 + (d*
x)/2)^3*((133*A*a^3)/6 + (61*B*a^3)/2) + tan(c/2 + (d*x)/2)^5*((416*A*a^3)/
15 + 32*B*a^3))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*t
an(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.28 $\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=241

$$-\frac{a^4(252A+227B)\sin^3(c+dx)}{105d}+\frac{a^4(252A+227B)\sin(c+dx)}{35d}+\frac{a^4(301A+276B)\sin(c+dx)\cos^3(c+dx)}{280d}+\frac{7(A}{}$$

```
[Out] 1/16*a^4*(49*A+44*B)*x+1/35*a^4*(252*A+227*B)*sin(d*x+c)/d+1/16*a^4*(49*A+4
4*B)*cos(d*x+c)*sin(d*x+c)/d+1/280*a^4*(301*A+276*B)*cos(d*x+c)^3*sin(d*x+c
)/d+1/7*a*B*cos(d*x+c)^3*(a+a*cos(d*x+c))^3*sin(d*x+c)/d+1/42*(7*A+10*B)*co
s(d*x+c)^3*(a^2+a^2*cos(d*x+c))^2*sin(d*x+c)/d+7/15*(A+B)*cos(d*x+c)^3*(a^4
+a^4*cos(d*x+c))*sin(d*x+c)/d-1/105*a^4*(252*A+227*B)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.59, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2976, 2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{a^4(252A+227B)\sin^3(c+dx)}{105d}+\frac{a^4(252A+227B)\sin(c+dx)}{35d}+\frac{a^4(301A+276B)\sin(c+dx)\cos^3(c+dx)}{280d}+\frac{(7A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^4*(49*A + 44*B)*x)/16 + (a^4*(252*A + 227*B)*Sin[c + d*x])/(35*d) + (a^4
*(49*A + 44*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^4*(301*A + 276*B)*Cos
[c + d*x]^3*Sin[c + d*x])/(280*d) + (a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x]
)^3*Sin[c + d*x])/(7*d) + ((7*A + 10*B)*Cos[c + d*x]^3*(a^2 + a^2*Cos[c + d
*x])^2*Sin[c + d*x])/(42*d) + (7*(A + B)*Cos[c + d*x]^3*(a^4 + a^4*Cos[c +
d*x])*Sin[c + d*x])/(15*d) - (a^4*(252*A + 227*B)*Sin[c + d*x]^3)/(105*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{7d} + \frac{1}{7}$$

$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{7d} + \frac{(7}{}$$

$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{7d} + \frac{(7}{}$$

$$= \frac{aB\cos^3(c+dx)(a+a\cos(c+dx))^3\sin(c+dx)}{7d} + \frac{(7}{}$$

$$= \frac{a^4(301A+276B)\cos^3(c+dx)\sin(c+dx)}{280d} + \frac{aB\cos}{}$$

$$= \frac{a^4(301A+276B)\cos^3(c+dx)\sin(c+dx)}{280d} + \frac{aB\cos}{}$$

$$= \frac{a^4(49A+44B)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^4(301A+}{}$$

$$= \frac{1}{16}a^4(49A+44B)x + \frac{a^4(252A+227B)\sin(c+dx)}{35d}$$

Mathematica [A] time = 0.87, size = 156, normalized size = 0.65

$$\frac{a^4(105(352A+323B)\sin(c+dx)+105(127A+124B)\sin(2(c+dx))+5040A\sin(3(c+dx))+1575A\sin(4(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^4*(18480*B*c + 20580*A*d*x + 18480*B*d*x + 105*(352*A + 323*B)*Sin[c + d
*x] + 105*(127*A + 124*B)*Sin[2*(c + d*x)] + 5040*A*Sin[3*(c + d*x)] + 5495
*B*Sin[3*(c + d*x)] + 1575*A*Sin[4*(c + d*x)] + 2100*B*Sin[4*(c + d*x)] + 3
36*A*Sin[5*(c + d*x)] + 651*B*Sin[5*(c + d*x)] + 35*A*Sin[6*(c + d*x)] + 14
0*B*Sin[6*(c + d*x)] + 15*B*Sin[7*(c + d*x)]))/(6720*d)
```

fricas [A] time = 0.88, size = 150, normalized size = 0.62

$$\frac{105\,(49\,A+44\,B)a^4dx + \left(240\,Ba^4\cos\left(dx+c\right)^6 + 280\,(A+4\,B)a^4\cos\left(dx+c\right)^5 + 192\,(7\,A+12\,B)a^4\cos\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1680*(105*(49*A + 44*B)*a^4*d*x + (240*B*a^4*cos(d*x + c)^6 + 280*(A + 4*
B)*a^4*cos(d*x + c)^5 + 192*(7*A + 12*B)*a^4*cos(d*x + c)^4 + 70*(41*A + 44
*B)*a^4*cos(d*x + c)^3 + 16*(252*A + 227*B)*a^4*cos(d*x + c)^2 + 105*(49*A
+ 44*B)*a^4*cos(d*x + c) + 32*(252*A + 227*B)*a^4)*sin(d*x + c))/d
```

giac [A] time = 0.53, size = 193, normalized size = 0.80

$$\frac{Ba^4 \sin(7\,dx + 7\,c)}{448\,d} + \frac{1}{16}\left(49\,Aa^4 + 44\,Ba^4\right)x + \frac{\left(Aa^4 + 4\,Ba^4\right)\sin(6\,dx + 6\,c)}{192\,d} + \frac{\left(16\,Aa^4 + 31\,Ba^4\right)\sin(5\,dx + 5\,c)}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/448*B*a^4*sin(7*d*x + 7*c)/d + 1/16*(49*A*a^4 + 44*B*a^4)*x + 1/192*(A*a^
4 + 4*B*a^4)*sin(6*d*x + 6*c)/d + 1/320*(16*A*a^4 + 31*B*a^4)*sin(5*d*x + 5
*c)/d + 5/64*(3*A*a^4 + 4*B*a^4)*sin(4*d*x + 4*c)/d + 1/192*(144*A*a^4 + 15
7*B*a^4)*sin(3*d*x + 3*c)/d + 1/64*(127*A*a^4 + 124*B*a^4)*sin(2*d*x + 2*c)
/d + 1/64*(352*A*a^4 + 323*B*a^4)*sin(d*x + c)/d
```

maple [A] time = 0.07, size = 358, normalized size = 1.49

$$A\,a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{a^4B\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}+\frac{4A\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/7*a^4*B*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+
c)^2)*sin(d*x+c)+4/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4
*a^4*B*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)+6*A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+
3/8*c)+6/5*a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4/3*A*a^4*(
2+cos(d*x+c)^2)*sin(d*x+c)+4*a^4*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d
*x+c)+3/8*d*x+3/8*c)+A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^
4*B*(2+cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.46, size = 356, normalized size = 1.48

$$1792\left(3\,\sin(dx+c)^5 - 10\,\sin(dx+c)^3 + 15\,\sin(dx+c)\right)Aa^4 - 35\left(4\,\sin(2\,dx+2\,c)^3 - 60\,dx - 60\,c - 9\,\sin(4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/6720*(1792*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4
 - 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2
*d*x + 2*c))*A*a^4 - 8960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 1260*(1
2*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 1680*(2*d*x +
 2*c + sin(2*d*x + 2*c))*A*a^4 - 192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5
+ 35*sin(d*x + c)^3 - 35*sin(d*x + c))*B*a^4 + 2688*(3*sin(d*x + c)^5 - 10*
sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^4 - 140*(4*sin(2*d*x + 2*c)^3 - 60*d*
x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*a^4 - 2240*(sin(d*x
+ c)^3 - 3*sin(d*x + c))*B*a^4 + 840*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*
sin(2*d*x + 2*c))*B*a^4)/d
```

mupad [B] time = 1.64, size = 353, normalized size = 1.46

$$\frac{\left(\frac{49\,A\,a^4}{8}+\frac{11\,B\,a^4}{2}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\left(\frac{245\,A\,a^4}{6}+\frac{110\,B\,a^4}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{13867\,A\,a^4}{120}+\frac{3113\,B\,a^4}{30}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(\frac{8}{}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((207*A*a^4)/8 + (53*B*a^4)/2) + tan(c/2 + (d*x)/2)^13*
((49*A*a^4)/8 + (11*B*a^4)/2) + tan(c/2 + (d*x)/2)^11*((245*A*a^4)/6 + (110
*B*a^4)/3) + tan(c/2 + (d*x)/2)^3*((523*A*a^4)/6 + 70*B*a^4) + tan(c/2 + (d
*x)/2)^7*((896*A*a^4)/5 + (5632*B*a^4)/35) + tan(c/2 + (d*x)/2)^9*((13867*A
*a^4)/120 + (3113*B*a^4)/30) + tan(c/2 + (d*x)/2)^5*((19157*A*a^4)/120 + (1
501*B*a^4)/10))/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*t
an(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 +
7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1)) - (a^4*(49*A + 44*B)*
(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(8*d) + (a^4*atan((a^4*tan(c/2 + (d*x
)/2)*(49*A + 44*B))/(8*((49*A*a^4)/8 + (11*B*a^4)/2)))*(49*A + 44*B))/(8*d)
```

sympy [A] time = 8.01, size = 960, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((5*A*a**4*x*sin(c + d*x)**6/16 + 15*A*a**4*x*sin(c + d*x)**4*cos(
c + d*x)**2/16 + 9*A*a**4*x*sin(c + d*x)**4/4 + 15*A*a**4*x*sin(c + d*x)**2
*cos(c + d*x)**4/16 + 9*A*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + A*a**4
*x*sin(c + d*x)**2/2 + 5*A*a**4*x*cos(c + d*x)**6/16 + 9*A*a**4*x*cos(c + d
*x)**4/4 + A*a**4*x*cos(c + d*x)**2/2 + 5*A*a**4*sin(c + d*x)**5*cos(c + d*
x)/(16*d) + 32*A*a**4*sin(c + d*x)**5/(15*d) + 5*A*a**4*sin(c + d*x)**3*cos
(c + d*x)**3/(6*d) + 16*A*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*A*
a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*A*a**4*sin(c + d*x)**3/(3*d) +
11*A*a**4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 4*A*a**4*sin(c + d*x)*cos(c
 + d*x)**4/d + 15*A*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 4*A*a**4*sin(
c + d*x)*cos(c + d*x)**2/d + A*a**4*sin(c + d*x)*cos(c + d*x)/(2*d) + 5*B*a
**4*x*sin(c + d*x)**6/4 + 15*B*a**4*x*sin(c + d*x)**4*cos(c + d*x)**2/4 + 3
*B*a**4*x*sin(c + d*x)**4/2 + 15*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/4
 + 3*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2 + 5*B*a**4*x*cos(c + d*x)**6/
4 + 3*B*a**4*x*cos(c + d*x)**4/2 + 16*B*a**4*sin(c + d*x)**7/(35*d) + 8*B*a
**4*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 5*B*a**4*sin(c + d*x)**5*cos(c
+ d*x)/(4*d) + 16*B*a**4*sin(c + d*x)**5/(5*d) + 2*B*a**4*sin(c + d*x)**3*c
os(c + d*x)**4/d + 10*B*a**4*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) + 8*B*a*
*4*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*B*a**4*sin(c + d*x)**3*cos(c + d*x
)/(2*d) + 2*B*a**4*sin(c + d*x)**3/(3*d) + B*a**4*sin(c + d*x)*cos(c + d*x)
**6/d + 11*B*a**4*sin(c + d*x)*cos(c + d*x)**5/(4*d) + 6*B*a**4*sin(c + d*x
)*cos(c + d*x)**4/d + 5*B*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*d) + B*a**4*
sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) + a)
**4*cos(c)**2, True))
```

3.29 $\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=185

$$-\frac{2a^4(8A+7B)\sin^3(c+dx)}{15d}+\frac{4a^4(8A+7B)\sin(c+dx)}{5d}+\frac{a^4(8A+7B)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{27a^4(8A+7B)\text{s}}{}$$

```
[Out] 7/16*a^4*(8*A+7*B)*x+4/5*a^4*(8*A+7*B)*sin(d*x+c)/d+27/80*a^4*(8*A+7*B)*cos
(d*x+c)*sin(d*x+c)/d+1/40*a^4*(8*A+7*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/30*(6*A
-B)*(a+a*cos(d*x+c))^4*sin(d*x+c)/d+1/6*B*(a+a*cos(d*x+c))^5*sin(d*x+c)/a/d
-2/15*a^4*(8*A+7*B)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.30, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2968, 3023, 2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{2a^4(8A+7B)\sin^3(c+dx)}{15d}+\frac{4a^4(8A+7B)\sin(c+dx)}{5d}+\frac{a^4(8A+7B)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{27a^4(8A+7B)\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (7*a^4*(8*A + 7*B)*x)/16 + (4*a^4*(8*A + 7*B)*Sin[c + d*x])/(5*d) + (27*a^4
*(8*A + 7*B)*Cos[c + d*x]*Sin[c + d*x])/(80*d) + (a^4*(8*A + 7*B)*Cos[c + d
*x]^3*Sin[c + d*x])/(40*d) + ((6*A - B)*(a + a*Cos[c + d*x])^4*Sin[c + d*x]
)/(30*d) + (B*(a + a*Cos[c + d*x])^5*Sin[c + d*x])/(6*a*d) - (2*a^4*(8*A +
7*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^4\left(A\cos(c+dx)+B\cos^2(c+dx\right.$$
$$= \frac{B(a+a\cos(c+dx))^5\sin(c+dx)}{6ad} + \frac{\int (a+a\cos(c+}{}$$
$$= \frac{(6A-B)(a+a\cos(c+dx))^4\sin(c+dx)}{30d} + \frac{B(a+a\,\mathrm{c}}{}$$
$$= \frac{(6A-B)(a+a\cos(c+dx))^4\sin(c+dx)}{30d} + \frac{B(a+a\,\mathrm{c}}{}$$
$$= \frac{1}{10}a^4(8A+7B)x + \frac{(6A-B)(a+a\cos(c+dx))^4\sin(}{30d}$$
$$= \frac{1}{10}a^4(8A+7B)x + \frac{2a^4(8A+7B)\sin(c+dx)}{5d} + \frac{3a^4(}{}$$
$$= \frac{2}{5}a^4(8A+7B)x + \frac{4a^4(8A+7B)\sin(c+dx)}{5d} + \frac{27a^4(}{}$$
$$= \frac{7}{16}a^4(8A+7B)x + \frac{4a^4(8A+7B)\sin(c+dx)}{5d} + \frac{27a^4}{}$$

Mathematica [A] time = 0.56, size = 134, normalized size = 0.72

$$\frac{a^4(120(49A+44B)\sin(c+dx)+15(128A+127B)\sin(2(c+dx))+580A\sin(3(c+dx))+120A\sin(4(c+dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^4*(2940*B*c + 3360*A*d*x + 2940*B*d*x + 120*(49*A + 44*B)*Sin[c + d*x] +
 15*(128*A + 127*B)*Sin[2*(c + d*x)] + 580*A*Sin[3*(c + d*x)] + 720*B*Sin[3
*(c + d*x)] + 120*A*Sin[4*(c + d*x)] + 225*B*Sin[4*(c + d*x)] + 12*A*Sin[5*
(c + d*x)] + 48*B*Sin[5*(c + d*x)] + 5*B*Sin[6*(c + d*x)]))/(960*d)
```

fricas [A] time = 0.70, size = 130, normalized size = 0.70

$$\frac{105\,(8\,A+7\,B)a^4dx+\left(40\,Ba^4\cos(dx+c)^5+48\,(A+4\,B)a^4\cos(dx+c)^4+10\,(24\,A+41\,B)a^4\cos(dx+c)^3+3\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/240*(105*(8*A + 7*B)*a^4*d*x + (40*B*a^4*cos(d*x + c)^5 + 48*(A + 4*B)*a^
4*cos(d*x + c)^4 + 10*(24*A + 41*B)*a^4*cos(d*x + c)^3 + 32*(17*A + 18*B)*a
^4*cos(d*x + c)^2 + 105*(8*A + 7*B)*a^4*cos(d*x + c) + 16*(83*A + 72*B)*a^4
)*sin(d*x + c))/d
```

giac [A] time = 1.00, size = 166, normalized size = 0.90

$$\frac{Ba^4\sin(6\,dx+6\,c)}{192\,d}+\frac{7}{16}\left(8\,Aa^4+7\,Ba^4\right)x+\frac{\left(Aa^4+4\,Ba^4\right)\sin(5\,dx+5\,c)}{80\,d}+\frac{\left(8\,Aa^4+15\,Ba^4\right)\sin(4\,dx+4\,c)}{64\,d}+\frac{(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/192*B*a^4*sin(6*d*x + 6*c)/d + 7/16*(8*A*a^4 + 7*B*a^4)*x + 1/80*(A*a^4 +
 4*B*a^4)*sin(5*d*x + 5*c)/d + 1/64*(8*A*a^4 + 15*B*a^4)*sin(4*d*x + 4*c)/d
 + 1/48*(29*A*a^4 + 36*B*a^4)*sin(3*d*x + 3*c)/d + 1/64*(128*A*a^4 + 127*B*
a^4)*sin(2*d*x + 2*c)/d + 1/8*(49*A*a^4 + 44*B*a^4)*sin(d*x + c)/d
```

maple [A] time = 0.07, size = 306, normalized size = 1.65

$$\frac{\frac{A\,a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a^4B\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+4A\,a^4\left(\left(\cos^3(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^4*B*(1/6*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+4
*A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/5*a^4
*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*A*a^4*(2+cos(d*x+c)^2)*
sin(d*x+c)+6*a^4*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/
8*c)+4*A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4/3*a^4*B*(2+cos(d*x
+c)^2)*sin(d*x+c)+A*a^4*sin(d*x+c)+a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

maxima [A] time = 0.60, size = 297, normalized size = 1.61

$$\frac{64\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c)\right)Aa^4-1920\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Aa^4+120\,(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 -
1920*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 120*(12*d*x + 12*c + sin(4*d
*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 960*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*A*a^4 + 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^
4 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2
*d*x + 2*c))*B*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 180*(12
*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 240*(2*d*x + 2
*c + sin(2*d*x + 2*c))*B*a^4 + 960*A*a^4*sin(d*x + c))/d
```

mupad [B] time = 1.62, size = 316, normalized size = 1.71

$$\frac{\left(7\,A\,a^4+\frac{49\,B\,a^4}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{119\,A\,a^4}{3}+\frac{833\,B\,a^4}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{462\,A\,a^4}{5}+\frac{1617\,B\,a^4}{20}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{562\,\ldots}{5}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+20\,\tan\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(25*A*a^4 + (207*B*a^4)/8) + tan(c/2 + (d*x)/2)^11*(7*A
*a^4 + (49*B*a^4)/8) + tan(c/2 + (d*x)/2)^9*((119*A*a^4)/3 + (833*B*a^4)/24
) + tan(c/2 + (d*x)/2)^3*((233*A*a^4)/3 + (1471*B*a^4)/24) + tan(c/2 + (d*x
)/2)^7*((462*A*a^4)/5 + (1617*B*a^4)/20) + tan(c/2 + (d*x)/2)^5*((562*A*a^4
)/5 + (1967*B*a^4)/20))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^
4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2
)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - (7*a^4*(8*A + 7*B)*(atan(tan(c/2 + (d*
x)/2)) - (d*x)/2))/(8*d) + (7*a^4*atan((7*a^4*tan(c/2 + (d*x)/2)*(8*A + 7*B
))/(8*(7*A*a^4 + (49*B*a^4)/8)))*(8*A + 7*B))/(8*d)
```

sympy [A] time = 4.83, size = 765, normalized size = 4.14

$$\begin{cases} \frac{3Aa^4x\sin^4(c+dx)}{2}+3Aa^4x\sin^2(c+dx)\cos^2(c+dx)+2Aa^4x\sin^2(c+dx)+\frac{3Aa^4x\cos^4(c+dx)}{2}+2Aa^4x\cos^2(c+dx)\ldots \\ x\left(A+B\cos(c)\right)\left(a\cos(c)+a\right)^4\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**4*x*sin(c + d*x)**4/2 + 3*A*a**4*x*sin(c + d*x)**2*cos(c
+ d*x)**2 + 2*A*a**4*x*sin(c + d*x)**2 + 3*A*a**4*x*cos(c + d*x)**4/2 + 2*A
```

```
*a**4*x*cos(c + d*x)**2 + 8*A*a**4*sin(c + d*x)**5/(15*d) + 4*A*a**4*sin(c
+ d*x)**3*cos(c + d*x)**2/(3*d) + 3*A*a**4*sin(c + d*x)**3*cos(c + d*x)/(2*
d) + 4*A*a**4*sin(c + d*x)**3/d + A*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 5
*A*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 6*A*a**4*sin(c + d*x)*cos(c +
d*x)**2/d + 2*A*a**4*sin(c + d*x)*cos(c + d*x)/d + A*a**4*sin(c + d*x)/d +
5*B*a**4*x*sin(c + d*x)**6/16 + 15*B*a**4*x*sin(c + d*x)**4*cos(c + d*x)**2
/16 + 9*B*a**4*x*sin(c + d*x)**4/4 + 15*B*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**4/16 + 9*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + B*a**4*x*sin(c +
d*x)**2/2 + 5*B*a**4*x*cos(c + d*x)**6/16 + 9*B*a**4*x*cos(c + d*x)**4/4 +
B*a**4*x*cos(c + d*x)**2/2 + 5*B*a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) +
 32*B*a**4*sin(c + d*x)**5/(15*d) + 5*B*a**4*sin(c + d*x)**3*cos(c + d*x)**
3/(6*d) + 16*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*B*a**4*sin(c
+ d*x)**3*cos(c + d*x)/(4*d) + 8*B*a**4*sin(c + d*x)**3/(3*d) + 11*B*a**4*s
in(c + d*x)*cos(c + d*x)**5/(16*d) + 4*B*a**4*sin(c + d*x)*cos(c + d*x)**4/
d + 15*B*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 4*B*a**4*sin(c + d*x)*co
s(c + d*x)**2/d + B*a**4*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(A
+ B*cos(c))*(a*cos(c) + a)**4*cos(c), True))
```

3.30 $\int (a + a\cos(c + dx))^4 (A + B\cos(c + dx))\,dx$

Optimal. Leaf size=150

$$-\frac{4a^4(5A+4B)\sin^3(c+dx)}{15d}+\frac{8a^4(5A+4B)\sin(c+dx)}{5d}+\frac{a^4(5A+4B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{27a^4(5A+4B)\,\mathrm{s}}{}$$

```
[Out] 7/8*a^4*(5*A+4*B)*x+8/5*a^4*(5*A+4*B)*sin(d*x+c)/d+27/40*a^4*(5*A+4*B)*cos(
d*x+c)*sin(d*x+c)/d+1/20*a^4*(5*A+4*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*B*(a+a
*cos(d*x+c))^4*sin(d*x+c)/d-4/15*a^4*(5*A+4*B)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.14, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{4a^4(5A+4B)\sin^3(c+dx)}{15d}+\frac{8a^4(5A+4B)\sin(c+dx)}{5d}+\frac{a^4(5A+4B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{27a^4(5A+4B)\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (7*a^4*(5*A + 4*B)*x)/8 + (8*a^4*(5*A + 4*B)*Sin[c + d*x])/(5*d) + (27*a^4*
(5*A + 4*B)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + (a^4*(5*A + 4*B)*Cos[c + d*
x]^3*Sin[c + d*x])/(20*d) + (B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d) -
 (4*a^4*(5*A + 4*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5d} + \frac{1}{5}(5A+4B)\int (a+a\cos(c$$

$$= \frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5d} + \frac{1}{5}(5A+4B)\int \left(a^4+4a^4\mathrm{co}\right.$$

$$= \frac{1}{5}a^4(5A+4B)x + \frac{B(a+a\cos(c+dx))^4\sin(c+dx)}{5d} + \frac{1}{5}\left(a^4(5A\right.$$

$$= \frac{1}{5}a^4(5A+4B)x + \frac{4a^4(5A+4B)\sin(c+dx)}{5d} + \frac{3a^4(5A+4B)\mathrm{co}}{}$$

$$= \frac{4}{5}a^4(5A+4B)x + \frac{8a^4(5A+4B)\sin(c+dx)}{5d} + \frac{27a^4(5A+4B)\mathrm{c}}{}$$

$$= \frac{7}{8}a^4(5A+4B)x + \frac{8a^4(5A+4B)\sin(c+dx)}{5d} + \frac{27a^4(5A+4B)\mathrm{c}}{}$$

Mathematica [A] time = 0.38, size = 108, normalized size = 0.72

$$\frac{a^4(420(8A+7B)\sin(c+dx)+120(7A+8B)\sin(2(c+dx))+160A\sin(3(c+dx))+15A\sin(4(c+dx))+2100}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^4*(2100*A*d*x + 1680*B*d*x + 420*(8*A + 7*B)*Sin[c + d*x] + 120*(7*A + 8
*B)*Sin[2*(c + d*x)] + 160*A*Sin[3*(c + d*x)] + 290*B*Sin[3*(c + d*x)] + 15
*A*Sin[4*(c + d*x)] + 60*B*Sin[4*(c + d*x)] + 6*B*Sin[5*(c + d*x)]))/(480*d
)
```

fricas [A] time = 0.62, size = 110, normalized size = 0.73

$$\frac{105\,(5\,A+4\,B)a^4dx+\left(24\,Ba^4\cos\left(dx+c\right)^4+30\,(A+4\,B)a^4\cos\left(dx+c\right)^3+16\,(10\,A+17\,B)a^4\cos\left(dx+c\right)^2+1\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(105*(5*A + 4*B)*a^4*d*x + (24*B*a^4*cos(d*x + c)^4 + 30*(A + 4*B)*a^
4*cos(d*x + c)^3 + 16*(10*A + 17*B)*a^4*cos(d*x + c)^2 + 15*(27*A + 28*B)*a
^4*cos(d*x + c) + 8*(100*A + 83*B)*a^4)*sin(d*x + c))/d
```

giac [A] time = 1.39, size = 139, normalized size = 0.93

$$\frac{Ba^4\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{7}{8}\left(5\,Aa^4+4\,Ba^4\right)x+\frac{\left(Aa^4+4\,Ba^4\right)\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{\left(16\,Aa^4+29\,Ba^4\right)\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{(7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/80*B*a^4*sin(5*d*x + 5*c)/d + 7/8*(5*A*a^4 + 4*B*a^4)*x + 1/32*(A*a^4 + 4
*B*a^4)*sin(4*d*x + 4*c)/d + 1/48*(16*A*a^4 + 29*B*a^4)*sin(3*d*x + 3*c)/d
+ 1/4*(7*A*a^4 + 8*B*a^4)*sin(2*d*x + 2*c)/d + 7/8*(8*A*a^4 + 7*B*a^4)*sin(
d*x + c)/d
```

maple [A] time = 0.06, size = 248, normalized size = 1.65

$$\frac{\frac{a^4B\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+A\,a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+4a^4B\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin}{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a^4*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*a^4*B*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*A*a^4*(2+cos(d*x+c)^2)*
sin(d*x+c)+2*a^4*B*(2+cos(d*x+c)^2)*sin(d*x+c)+6*A*a^4*(1/2*cos(d*x+c)*sin(
```

```
d*x+c)+1/2*d*x+1/2*c)+4*a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*A
*a^4*sin(d*x+c)+a^4*B*sin(d*x+c)+A*a^4*(d*x+c))
```

maxima [**A**] time = 0.84, size = 236, normalized size = 1.57

$$-\frac{640\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4-15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c)\right)Aa^4-720\,(2\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/480*(640*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 720*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*A*a^4 - 480*(d*x + c)*A*a^4 - 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c
)^3 + 15*sin(d*x + c))*B*a^4 + 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4
- 60*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 480*(2
*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 1920*A*a^4*sin(d*x + c) - 480*B*a^4*
sin(d*x + c))/d
```

mupad [**B**] time = 1.56, size = 278, normalized size = 1.85

$$\frac{\left(\frac{35\,A\,a^4}{4}+7\,B\,a^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{245\,A\,a^4}{6}+\frac{98\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{224\,A\,a^4}{3}+\frac{896\,B\,a^4}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{395\,A}{6}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((93*A*a^4)/4 + 25*B*a^4) + tan(c/2 + (d*x)/2)^9*((35*A
*a^4)/4 + 7*B*a^4) + tan(c/2 + (d*x)/2)^7*((245*A*a^4)/6 + (98*B*a^4)/3) +
tan(c/2 + (d*x)/2)^3*((395*A*a^4)/6 + (158*B*a^4)/3) + tan(c/2 + (d*x)/2)^5
*((224*A*a^4)/3 + (896*B*a^4)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2
+ (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 +
 (d*x)/2)^10 + 1)) - (7*a^4*(5*A + 4*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2
))/(4*d) + (7*a^4*atan((7*a^4*tan(c/2 + (d*x)/2)*(5*A + 4*B))/(4*((35*A*a^4
)/4 + 7*B*a^4)))*(5*A + 4*B))/(4*d)
```

sympy [**A**] time = 3.02, size = 544, normalized size = 3.63

$$\begin{cases} \frac{3Aa^4x\sin^4(c+dx)}{8}+\frac{3Aa^4x\sin^2(c+dx)\cos^2(c+dx)}{4}+3Aa^4x\sin^2(c+dx)+\frac{3Aa^4x\cos^4(c+dx)}{8}+3Aa^4x\cos^2(c+dx)+Aa^4x \\ x\left(A+B\cos(c)\right)\left(a\cos(c)+a\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**4*x*sin(c + d*x)**4/8 + 3*A*a**4*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + 3*A*a**4*x*sin(c + d*x)**2 + 3*A*a**4*x*cos(c + d*x)**4/8 + 3
*A*a**4*x*cos(c + d*x)**2 + A*a**4*x + 3*A*a**4*sin(c + d*x)**3*cos(c + d*x
)/(8*d) + 8*A*a**4*sin(c + d*x)**3/(3*d) + 5*A*a**4*sin(c + d*x)*cos(c + d*
x)**3/(8*d) + 4*A*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a**4*sin(c + d*
x)*cos(c + d*x)/d + 4*A*a**4*sin(c + d*x)/d + 3*B*a**4*x*sin(c + d*x)**4/2
+ 3*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2 + 2*B*a**4*x*sin(c + d*x)**2 +
 3*B*a**4*x*cos(c + d*x)**4/2 + 2*B*a**4*x*cos(c + d*x)**2 + 8*B*a**4*sin(c
 + d*x)**5/(15*d) + 4*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*B*a*
*4*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 4*B*a**4*sin(c + d*x)**3/d + B*a**4
*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*a**4*sin(c + d*x)*cos(c + d*x)**3/(2*
d) + 6*B*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 2*B*a**4*sin(c + d*x)*cos(c
+ d*x)/d + B*a**4*sin(c + d*x)/d, Ne(d, 0)), (x*(A + B*cos(c))*(a*cos(c) +
a)**4, True))
```

3.31 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=151

$$\frac{5a^4(8A+7B)\sin(c+dx)}{8d}+\frac{(32A+35B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{24d}+\frac{1}{8}a^4x(48A+35B)+\frac{a^4A\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] 1/8*a^4*(48*A+35*B)*x+a^4*A*arctanh(sin(d*x+c))/d+5/8*a^4*(8*A+7*B)*sin(d*x
+c)/d+1/4*a*B*(a+a*cos(d*x+c))^3*sin(d*x+c)/d+1/12*(4*A+7*B)*(a^2+a^2*cos(d
*x+c))^2*sin(d*x+c)/d+1/24*(32*A+35*B)*(a^4+a^4*cos(d*x+c))*sin(d*x+c)/d
```

Rubi [A] time = 0.41, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2976, 2968, 3023, 2735, 3770}

$$\frac{5a^4(8A+7B)\sin(c+dx)}{8d}+\frac{(4A+7B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{12d}+\frac{(32A+35B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^4*(48*A + 35*B)*x)/8 + (a^4*A*ArcTanh[Sin[c + d*x]])/d + (5*a^4*(8*A + 7
*B)*Sin[c + d*x])/(8*d) + (a*B*(a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) +
 ((4*A + 7*B)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x])/(12*d) + ((32*A + 35
*B)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(24*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx=\frac{aB(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}\int (a+a\cos(\ldots$$

$$=\frac{aB(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{(4A+7B)\left(a^2+\ldots\right.}{\ldots}$$

$$=\frac{aB(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{(4A+7B)\left(a^2+\ldots\right.}{\ldots}$$

$$=\frac{aB(a+a\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{(4A+7B)\left(a^2+\ldots\right.}{\ldots}$$

$$=\frac{5a^4(8A+7B)\sin(c+dx)}{8d}+\frac{aB(a+a\cos(c+dx))^3\,\mathrm{si}\ldots}{4d}$$

$$=\frac{1}{8}a^4(48A+35B)x+\frac{5a^4(8A+7B)\sin(c+dx)}{8d}+\frac{aB(\ldots}{\ldots}$$

$$=\frac{1}{8}a^4(48A+35B)x+\frac{a^4A\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^4(\ldots}{\ldots}$$

Mathematica [A] time = 0.40, size = 138, normalized size = 0.91

$$a^4\left(24(27A+28B)\sin(c+dx)+24(4A+7B)\sin(2(c+dx))+8A\sin(3(c+dx))-96A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^4*(576*A*d*x + 420*B*d*x - 96*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 96*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24*(27*A + 28*B)*Sin[c +
 d*x] + 24*(4*A + 7*B)*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 32*B*Sin[3
*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d)
```

fricas [A] time = 0.65, size = 118, normalized size = 0.78

$$\frac{3\,(48\,A+35\,B)a^4dx+12\,Aa^4\log\left(\sin\left(dx+c\right)+1\right)-12\,Aa^4\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,Ba^4\cos\left(dx+c\right)^3+8\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(48*A + 35*B)*a^4*d*x + 12*A*a^4*log(sin(d*x + c) + 1) - 12*A*a^4*l
og(-sin(d*x + c) + 1) + (6*B*a^4*cos(d*x + c)^3 + 8*(A + 4*B)*a^4*cos(d*x +
 c)^2 + 3*(16*A + 27*B)*a^4*cos(d*x + c) + 160*(A + B)*a^4)*sin(d*x + c))/d
```

giac [A] time = 1.07, size = 214, normalized size = 1.42

$$24\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(48\,Aa^4+35\,Ba^4\right)(dx+c)+\frac{2\left(120\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/24*(24*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*A*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(48*A*a^4 + 35*B*a^4)*(d*x + c) + 2*(120*A*a^4*tan
(1/2*d*x + 1/2*c)^7 + 105*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 424*A*a^4*tan(1/2*
d*x + 1/2*c)^5 + 385*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 520*A*a^4*tan(1/2*d*x +
 1/2*c)^3 + 511*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 216*A*a^4*tan(1/2*d*x + 1/2*
c) + 279*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.14, size = 199, normalized size = 1.32

$$\frac{A\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)a^{4}}{3d}+\frac{20A\,a^{4}\sin\left(dx+c\right)}{3d}+\frac{a^{4}B\sin\left(dx+c\right)\left(\cos^{3}\left(dx+c\right)\right)}{4d}+\frac{27a^{4}B\cos\left(dx+c\right)\sin\left(\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3/d*A*a^4*sin(d*x+c)+1/4/d*a^4*B*sin
(d*x+c)*cos(d*x+c)^3+27/8/d*a^4*B*cos(d*x+c)*sin(d*x+c)+35/8*a^4*B*x+35/8/d
*a^4*B*c+2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+6*A*a^4*x+6/d*A*a^4*c+4/3/d*B*sin(
d*x+c)*cos(d*x+c)^2*a^4+20/3/d*a^4*B*sin(d*x+c)+1/d*A*a^4*ln(sec(d*x+c)+tan
(d*x+c))
```

maxima [A] time = 0.47, size = 198, normalized size = 1.31

$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Aa^{4}-96\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^{4}-384\left(dx+c\right)Aa^{4}+128\left(\sin\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 96*(2*d*x + 2*c + sin(2
*d*x + 2*c))*A*a^4 - 384*(d*x + c)*A*a^4 + 128*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*B*a^4 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a
^4 - 144*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 96*(d*x + c)*B*a^4 - 96*A
*a^4*log(sec(d*x + c) + tan(d*x + c)) - 576*A*a^4*sin(d*x + c) - 384*B*a^4*
sin(d*x + c))/d
```

mupad [B] time = 0.67, size = 188, normalized size = 1.25

$$\frac{144\,A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+24\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+105\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+12\,A\,a^4\,\sin\left(2\,c+2\,d\,x\right)+A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x),x)
```

```
[Out] (144*A*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 24*A*a^4*atanh(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 105*B*a^4*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)) + 12*A*a^4*sin(2*c + 2*d*x) + A*a^4*sin(3*c + 3*d*x) + 21*
```

```
B*a^4*sin(2*c + 2*d*x) + 4*B*a^4*sin(3*c + 3*d*x) + (3*B*a^4*sin(4*c + 4*d*
x))/8 + 81*A*a^4*sin(c + d*x) + 84*B*a^4*sin(c + d*x))/(12*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int A\sec(c+dx)\,dx+\int 4A\cos(c+dx)\sec(c+dx)\,dx+\int 6A\cos^2(c+dx)\sec(c+dx)\,dx+\int 4A\cos^3($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a**4*(Integral(A*sec(c + d*x), x) + Integral(4*A*cos(c + d*x)*sec(c + d*x),
 x) + Integral(6*A*cos(c + d*x)**2*sec(c + d*x), x) + Integral(4*A*cos(c +
d*x)**3*sec(c + d*x), x) + Integral(A*cos(c + d*x)**4*sec(c + d*x), x) + In
tegral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(4*B*cos(c + d*x)**2*sec(c
 + d*x), x) + Integral(6*B*cos(c + d*x)**3*sec(c + d*x), x) + Integral(4*B*
cos(c + d*x)**4*sec(c + d*x), x) + Integral(B*cos(c + d*x)**5*sec(c + d*x),
 x))
```

3.32 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=150

$$\frac{5a^4(A+2B)\sin(c+dx)}{2d}+\frac{a^4(4A+B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(3A-8B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{6d}+\frac{1}{2}a^4x(1$$

```
[Out] 1/2*a^4*(13*A+12*B)*x+a^4*(4*A+B)*arctanh(sin(d*x+c))/d+5/2*a^4*(A+2*B)*sin
(d*x+c)/d-1/3*(3*A-B)*(a^2+a^2*cos(d*x+c))^2*sin(d*x+c)/d-1/6*(3*A-8*B)*(a^
4+a^4*cos(d*x+c))*sin(d*x+c)/d+a*A*(a+a*cos(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A] time = 0.45, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2975, 2976, 2968, 3023, 2735, 3770}

$$\frac{5a^4(A+2B)\sin(c+dx)}{2d}+\frac{a^4(4A+B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(3A-B)\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)^2}{3d}-\frac{(3A-8}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^4*(13*A + 12*B)*x)/2 + (a^4*(4*A + B)*ArcTanh[Sin[c + d*x]])/d + (5*a^4*
(A + 2*B)*Sin[c + d*x])/(2*d) - ((3*A - B)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c
 + d*x])/(3*d) - ((3*A - 8*B)*(a^4 + a^4*Cos[c + d*x])*Sin[c + d*x])/(6*d)
+ (a*A*(a + a*Cos[c + d*x])^3*Tan[c + d*x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\tan(c+dx)}{d} + \int (a+a\cos(c$$

$$= -\frac{(3A-B)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{3d} + \frac{aA($$

$$= -\frac{(3A-B)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{3d} - \frac{(3A$$

$$= -\frac{(3A-B)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{3d} - \frac{(3A$$

$$= \frac{5a^4(A+2B)\sin(c+dx)}{2d} - \frac{(3A-B)\left(a^2+a^2\cos(c+\right.}{3d}$$

$$= \frac{1}{2}a^4(13A+12B)x + \frac{5a^4(A+2B)\sin(c+dx)}{2d} - \frac{(3A}{}$$

$$= \frac{1}{2}a^4(13A+12B)x + \frac{a^4(4A+B)\tanh^{-1}(\sin(c+dx))}{d}$$

Mathematica [B] time = 1.71, size = 312, normalized size = 2.08

$$\frac{1}{192}a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{3(16A+27B)\sin(c)\cos(dx)}{d} + \frac{3(A+4B)\sin(2c)\cos(2dx)}{d} + \frac{3(16A+2}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(78*A*x + 72*B*x - (12*(4*A +
B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (12*(4*A + B)*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]])/d + (3*(16*A + 27*B)*Cos[d*x]*Sin[c])/d + (3*
(A + 4*B)*Cos[2*d*x]*Sin[2*c])/d + (B*Cos[3*d*x]*Sin[3*c])/d + (3*(16*A + 2
7*B)*Cos[c]*Sin[d*x])/d + (3*(A + 4*B)*Cos[2*c]*Sin[2*d*x])/d + (B*Cos[3*c]
*Sin[3*d*x])/d + (12*A*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])) + (12*A*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))))/192
```

fricas [A] time = 0.95, size = 150, normalized size = 1.00

$$\frac{3(13A+12B)a^4dx\cos(dx+c) + 3(4A+B)a^4\cos(dx+c)\log(\sin(dx+c)+1) - 3(4A+B)a^4\cos(dx+c)\log}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(13*A + 12*B)*a^4*d*x*cos(d*x + c) + 3*(4*A + B)*a^4*cos(d*x + c)*lo
g(sin(d*x + c) + 1) - 3*(4*A + B)*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) +
 (2*B*a^4*cos(d*x + c)^3 + 3*(A + 4*B)*a^4*cos(d*x + c)^2 + 8*(3*A + 5*B)*a
^4*cos(d*x + c) + 6*A*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.89, size = 226, normalized size = 1.51

$$-\frac{\frac{12\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-3\left(13\,Aa^4+12\,Ba^4\right)(dx+c)-6\left(4\,Aa^4+Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+6\left(4\,Aa^4+Ba^4\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(13*A*
a^4 + 12*B*a^4)*(d*x + c) - 6*(4*A*a^4 + B*a^4)*log(abs(tan(1/2*d*x + 1/2*c
) + 1)) + 6*(4*A*a^4 + B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(21*A*
a^4*tan(1/2*d*x + 1/2*c)^5 + 30*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^4*tan
(1/2*d*x + 1/2*c)^3 + 76*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 27*A*a^4*tan(1/2*d*
x + 1/2*c) + 54*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)
/d
```

maple [A] time = 0.16, size = 190, normalized size = 1.27

$$\frac{A\,a^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{13A\,a^4x}{2}+\frac{13A\,a^4c}{2d}+\frac{B\sin(dx+c)\left(\cos^2(dx+c)\right)a^4}{3d}+\frac{20a^4B\sin(dx+c)}{3d}+\frac{4A\,a^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] 1/2/d*A*a^4*cos(d*x+c)*sin(d*x+c)+13/2*A*a^4*x+13/2/d*A*a^4*c+1/3/d*B*sin(d
*x+c)*cos(d*x+c)^2*a^4+20/3/d*a^4*B*sin(d*x+c)+4/d*A*a^4*sin(d*x+c)+2/d*a^4
*B*cos(d*x+c)*sin(d*x+c)+6*a^4*B*x+6/d*a^4*B*c+4/d*A*a^4*ln(sec(d*x+c)+tan(
d*x+c))+1/d*A*a^4*tan(d*x+c)+1/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.40, size = 187, normalized size = 1.25

$$\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^4+72\,(dx+c)Aa^4-4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^4+12\,(2\,dx+2\,c+\text{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 72*(d*x + c)*A*a^4 - 4*(si
n(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 12*(2*d*x + 2*c + sin(2*d*x + 2*c))*
B*a^4 + 48*(d*x + c)*B*a^4 + 24*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x
+ c) - 1)) + 6*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*A
*a^4*sin(d*x + c) + 72*B*a^4*sin(d*x + c) + 12*A*a^4*tan(d*x + c))/d
```

mupad [B] time = 0.42, size = 242, normalized size = 1.61

$$\frac{4\,A\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{20\,B\,a^4\,\sin\left(c+d\,x\right)}{3\,d}+\frac{13\,A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{8\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{12\,B\,a^4\,\mathrm{atan}\left(\frac{\sin}{\cos}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^2,x)
```

```
[Out] (4*A*a^4*sin(c + d*x))/d + (20*B*a^4*sin(c + d*x))/(3*d) + (13*A*a^4*atan(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*A*a^4*atanh(sin(c/2 + (d*x)/2
)/cos(c/2 + (d*x)/2)))/d + (12*B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/d + (2*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^
4*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^4*cos(c + d*x)^2*sin(c + d*x))/(3*d
) + (A*a^4*cos(c + d*x)*sin(c + d*x))/(2*d) + (2*B*a^4*cos(c + d*x)*sin(c +
 d*x))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.33 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=162

$$-\frac{5a^4(A-B)\sin(c+dx)}{2d}+\frac{a^4(13A+8B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(6A+B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{2d}+\frac{1}{2}a^4x$$

```
[Out] 1/2*a^4*(8*A+13*B)*x+1/2*a^4*(13*A+8*B)*arctanh(sin(d*x+c))/d-5/2*a^4*(A-B)
*sin(d*x+c)/d-1/2*(6*A+B)*(a^4+a^4*cos(d*x+c))*sin(d*x+c)/d+1/2*(5*A+2*B)*(
a^2+a^2*cos(d*x+c))^2*tan(d*x+c)/d+1/2*a*A*(a+a*cos(d*x+c))^3*sec(d*x+c)*ta
n(d*x+c)/d
```

Rubi [A] time = 0.48, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2975, 2976, 2968, 3023, 2735, 3770}

$$-\frac{5a^4(A-B)\sin(c+dx)}{2d}+\frac{a^4(13A+8B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(6A+B)\sin(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{2d}+\frac{(5A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^4*(8*A + 13*B)*x)/2 + (a^4*(13*A + 8*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (
5*a^4*(A - B)*Sin[c + d*x])/(2*d) - ((6*A + B)*(a^4 + a^4*Cos[c + d*x])*Sin
[c + d*x])/(2*d) + ((5*A + 2*B)*(a^2 + a^2*Cos[c + d*x])^2*Tan[c + d*x])/(2
*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}$$

$$= \frac{(5A+2B)\left(a^2+a^2\cos(c+dx)\right)^2\tan(c+dx)}{2d} + \frac{aA}{}$$

$$= -\frac{(6A+B)\left(a^4+a^4\cos(c+dx)\right)\sin(c+dx)}{2d} + \frac{(5A}{}$$

$$= -\frac{(6A+B)\left(a^4+a^4\cos(c+dx)\right)\sin(c+dx)}{2d} + \frac{(5A}{}$$

$$= -\frac{5a^4(A-B)\sin(c+dx)}{2d} - \frac{(6A+B)\left(a^4+a^4\cos(c\right.}{2d}$$

$$= \frac{1}{2}a^4(8A+13B)x - \frac{5a^4(A-B)\sin(c+dx)}{2d} - \frac{(6A+}{}$$

$$= \frac{1}{2}a^4(8A+13B)x + \frac{a^4(13A+8B)\tanh^{-1}(\sin(c+d}{2d}$$

Mathematica [B] time = 4.67, size = 343, normalized size = 2.12

$$\frac{1}{64}a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{4(A+4B)\sin(c)\cos(dx)}{d} + \frac{4(A+4B)\cos(c)\sin(dx)}{d} + \frac{}{d\left(\cos\left(\frac{c}{2}\right)-\sin\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(2*(8*A + 13*B)*x - (2*(13*A +
 8*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (2*(13*A + 8*B)*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (4*(A + 4*B)*Cos[d*x]*Sin[c])/d + (B
*Cos[2*d*x]*Sin[2*c])/d + (4*(A + 4*B)*Cos[c]*Sin[d*x])/d + (B*Cos[2*c]*Sin
[2*d*x])/d + A/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(4*A + B)*S
in[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
) - A/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(4*A + B)*Sin[(d*x)/
2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/64
```

fricas [A] time = 0.67, size = 156, normalized size = 0.96

$$\frac{2\,(8\,A+13\,B)a^4dx\cos\left(dx+c\right)^2+(13\,A+8\,B)a^4\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(13\,A+8\,B)a^4\cos\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(2*(8*A + 13*B)*a^4*d*x*cos(d*x + c)^2 + (13*A + 8*B)*a^4*cos(d*x + c)^
2*log(sin(d*x + c) + 1) - (13*A + 8*B)*a^4*cos(d*x + c)^2*log(-sin(d*x + c)
 + 1) + 2*(B*a^4*cos(d*x + c)^3 + 2*(A + 4*B)*a^4*cos(d*x + c)^2 + 2*(4*A +
 B)*a^4*cos(d*x + c) + A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 1.94, size = 230, normalized size = 1.42

$$\left(8\,Aa^4+13\,Ba^4\right)(dx+c)+\left(13\,Aa^4+8\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(13\,Aa^4+8\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*((8*A*a^4 + 13*B*a^4)*(d*x + c) + (13*A*a^4 + 8*B*a^4)*log(abs(tan(1/2*
d*x + 1/2*c) + 1)) - (13*A*a^4 + 8*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*(5*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 5*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 7*
A*a^4*tan(1/2*d*x + 1/2*c)^5 + 7*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^4*tan
(1/2*d*x + 1/2*c)^3 + 9*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 11*A*a^4*tan(1/2*d*x
 + 1/2*c) - 11*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/
d
```

maple [A] time = 0.17, size = 182, normalized size = 1.12

$$\frac{A\,a^4\sin(dx+c)}{d}+\frac{a^4B\cos(dx+c)\sin(dx+c)}{2d}+\frac{13a^4Bx}{2}+\frac{13a^4Bc}{2d}+4A\,a^4x+\frac{4A\,a^4c}{d}+\frac{4a^4B\sin(dx+c)}{d}+\frac{13A\,a^4\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/d*A*a^4*sin(d*x+c)+1/2/d*a^4*B*cos(d*x+c)*sin(d*x+c)+13/2*a^4*B*x+13/2/d*
a^4*B*c+4*A*a^4*x+4/d*A*a^4*c+4/d*a^4*B*sin(d*x+c)+13/2/d*A*a^4*ln(sec(d*x+
c)+tan(d*x+c))+4/d*A*a^4*tan(d*x+c)+4/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+1/2
/d*A*a^4*sec(d*x+c)*tan(d*x+c)+1/d*a^4*B*tan(d*x+c)
```

maxima [A] time = 0.58, size = 199, normalized size = 1.23

$$16\,(dx+c)Aa^4+(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^4+24\,(dx+c)Ba^4-Aa^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(16*(d*x + c)*A*a^4 + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 + 24*(d*x
+ c)*B*a^4 - A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)
+ 1) + log(sin(d*x + c) - 1)) + 12*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) + 8*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4
*A*a^4*sin(d*x + c) + 16*B*a^4*sin(d*x + c) + 16*A*a^4*tan(d*x + c) + 4*B*a
^4*tan(d*x + c))/d
```

mupad [B] time = 0.40, size = 243, normalized size = 1.50

$$\frac{A a^4 \sin(c+dx)}{d}+\frac{4 B a^4 \sin(c+dx)}{d}+\frac{8 A a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{13 A a^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{13 B a^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^3,x)
```

```
[Out] (A*a^4*sin(c + d*x))/d + (4*B*a^4*sin(c + d*x))/d + (8*A*a^4*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (13*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2)))/d + (13*B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d + (8*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*A*a^4*sin
(c + d*x))/(d*cos(c + d*x)) + (A*a^4*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (
B*a^4*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^4*cos(c + d*x)*sin(c + d*x))/(2
*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.34 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=165

$$-\frac{5a^4(2A+B)\sin(c+dx)}{2d}+\frac{a^4(12A+13B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(11A+9B)\tan(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{3d}+a$$

```
[Out] a^4*(A+4*B)*x+1/2*a^4*(12*A+13*B)*arctanh(sin(d*x+c))/d-5/2*a^4*(2*A+B)*sin
(d*x+c)/d+1/3*(11*A+9*B)*(a^4+a^4*cos(d*x+c))*tan(d*x+c)/d+1/2*(2*A+B)*(a^2
+a^2*cos(d*x+c))^2*sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*(a+a*cos(d*x+c))^3*sec(d
*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.51, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2975, 2968, 3023, 2735, 3770}

$$-\frac{5a^4(2A+B)\sin(c+dx)}{2d}+\frac{a^4(12A+13B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(11A+9B)\tan(c+dx)\left(a^4\cos(c+dx)+a^4\right)}{3d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] a^4*(A + 4*B)*x + (a^4*(12*A + 13*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^4*
(2*A + B)*Sin[c + d*x])/(2*d) + ((11*A + 9*B)*(a^4 + a^4*Cos[c + d*x])*Tan[
c + d*x])/(3*d) + ((2*A + B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c
+ d*x])/(2*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3
*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}$$
$$= \frac{(2A+B)\left(a^2+a^2\cos(c+dx)\right)^2\sec(c+dx)\tan(c+}{2d}$$
$$= \frac{(11A+9B)\left(a^4+a^4\cos(c+dx)\right)\tan(c+dx)}{3d} + \frac{(2A}{}$$
$$= \frac{(11A+9B)\left(a^4+a^4\cos(c+dx)\right)\tan(c+dx)}{3d} + \frac{(2A}{}$$
$$= -\frac{5a^4(2A+B)\sin(c+dx)}{2d} + \frac{(11A+9B)\left(a^4+a^4\cos\right.}{3d}$$
$$= a^4(A+4B)x - \frac{5a^4(2A+B)\sin(c+dx)}{2d} + \frac{(11A+9}{}$$
$$= a^4(A+4B)x + \frac{a^4(12A+13B)\tanh^{-1}(\sin(c+dx))}{2d}$$

Mathematica [B] time = 6.22, size = 380, normalized size = 2.30

$$a^4\left(\frac{(A+4B)(c+dx)}{d} + \frac{-13A-3B}{12d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{4\left(5A\sin\left(\frac{1}{2}(c+dx)\right)+3B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] a^4*(((A + 4*B)*(c + d*x))/d + ((-12*A - 13*B)*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]])/(2*d) + ((12*A + 13*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]])/(2*d) + (13*A + 3*B)/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
A*Sin[(c + d*x)/2])/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (A*Sin[
(c + d*x)/2])/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (-13*A - 3*B)
/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(5*A*Sin[(c + d*x)/2]
+ 3*B*Sin[(c + d*x)/2]))/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*(
5*A*Sin[(c + d*x)/2] + 3*B*Sin[(c + d*x)/2]))/(3*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])) + (B*Sin[c + d*x])/d)
```

fricas [A] time = 0.94, size = 159, normalized size = 0.96

$$\frac{12\,(A+4\,B)a^4dx\cos\left(dx+c\right)^3 + 3\,(12\,A+13\,B)a^4\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right) - 3\,(12\,A+13\,B)a^4\cos\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(12*(A + 4*B)*a^4*d*x*cos(d*x + c)^3 + 3*(12*A + 13*B)*a^4*cos(d*x + c
)^3*log(sin(d*x + c) + 1) - 3*(12*A + 13*B)*a^4*cos(d*x + c)^3*log(-sin(d*x
 + c) + 1) + 2*(6*B*a^4*cos(d*x + c)^3 + 8*(5*A + 3*B)*a^4*cos(d*x + c)^2 +
 3*(4*A + B)*a^4*cos(d*x + c) + 2*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [A] time = 0.49, size = 227, normalized size = 1.38

$$\frac{12\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+6\left(Aa^4+4\,Ba^4\right)(dx+c)+3\left(12\,Aa^4+13\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(12\,Aa^4+13\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(12*B*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(A*a^4
+ 4*B*a^4)*(d*x + c) + 3*(12*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3*(12*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30
*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 21*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 76*A*a^4*
tan(1/2*d*x + 1/2*c)^3 - 48*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 54*A*a^4*tan(1/2
*d*x + 1/2*c) + 27*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)
^3)/d
```

maple [A] time = 0.18, size = 189, normalized size = 1.15

$$A\,a^4x+\frac{A\,a^4c}{d}+\frac{a^4B\sin(dx+c)}{d}+\frac{6A\,a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+4a^4Bx+\frac{4a^4Bc}{d}+\frac{20A\,a^4\tan(dx+c)}{3d}+\frac{13}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] A*a^4*x+1/d*A*a^4*c+1/d*a^4*B*sin(d*x+c)+6/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c)
)+4*a^4*B*x+4/d*a^4*B*c+20/3/d*A*a^4*tan(d*x+c)+13/2/d*a^4*B*ln(sec(d*x+c)+
tan(d*x+c))+2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+4/d*a^4*B*tan(d*x+c)+1/3/d*A*a^
4*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^4*B*sec(d*x+c)*tan(d*x+c)
```

maxima [A] time = 0.59, size = 235, normalized size = 1.42

$$4\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^4+12\,(dx+c)Aa^4+48\,(dx+c)Ba^4-12\,Aa^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 12*(d*x + c)*A*a^4 + 48*(
d*x + c)*B*a^4 - 12*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*a^4*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*A*a^4*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*B*a^4*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + 12*B*a^4*sin(d*x + c) + 72*A*a^4*tan(d*x + c) +
 48*B*a^4*tan(d*x + c))/d
```

mupad [B] time = 0.41, size = 254, normalized size = 1.54

$$\frac{B\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{2\,A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{12\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{8\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{13\,B\,a^4\,\mathrm{atanh}\left(\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^4,x)
```

```
[Out] (B*a^4*sin(c + d*x))/d + (2*A*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d + (12*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*B*a^
4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (13*B*a^4*atanh(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (20*A*a^4*sin(c + d*x))/(3*d*cos(c + d*
x)) + (2*A*a^4*sin(c + d*x))/(d*cos(c + d*x)^2) + (A*a^4*sin(c + d*x))/(3*d
*cos(c + d*x)^3) + (4*B*a^4*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^4*sin(c +
 d*x))/(2*d*cos(c + d*x)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.35 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=173

$$\frac{5a^4(7A+8B)\tan(c+dx)}{8d}+\frac{a^4(35A+48B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(35A+32B)\tan(c+dx)\sec(c+dx)\left(a^4\cos(c\right.}{24d}$$

```
[Out] a^4*B*x+1/8*a^4*(35*A+48*B)*arctanh(sin(d*x+c))/d+5/8*a^4*(7*A+8*B)*tan(d*x
+c)/d+1/24*(35*A+32*B)*(a^4+a^4*cos(d*x+c))*sec(d*x+c)*tan(d*x+c)/d+1/12*(7
*A+4*B)*(a^2+a^2*cos(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a*A*(a+a*cos(d
*x+c))^3*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] time = 0.52, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2975, 2968, 3021, 2735, 3770}

$$\frac{5a^4(7A+8B)\tan(c+dx)}{8d}+\frac{a^4(35A+48B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(7A+4B)\tan(c+dx)\sec^2(c+dx)\left(a^2\cos(c+\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] a^4*B*x + (a^4*(35*A + 48*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^4*(7*A + 8
*B)*Tan[c + d*x])/(8*d) + ((35*A + 32*B)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d
*x]*Tan[c + d*x])/(24*d) + ((7*A + 4*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c +
d*x]^2*Tan[c + d*x])/(12*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3*Ta
n[c + d*x])/(4*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\sec^3(c+dx)\tan(c+dx)}{4d} +$$

$$= \frac{(7A+4B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^2(c+dx)\tan(}{12d}$$

$$= \frac{(35A+32B)\left(a^4+a^4\cos(c+dx)\right)\sec(c+dx)\tan(}{24d}$$

$$= \frac{(35A+32B)\left(a^4+a^4\cos(c+dx)\right)\sec(c+dx)\tan(}{24d}$$

$$= \frac{5a^4(7A+8B)\tan(c+dx)}{8d} + \frac{(35A+32B)\left(a^4+a^4\right.}{}$$

$$= a^4Bx + \frac{5a^4(7A+8B)\tan(c+dx)}{8d} + \frac{(35A+32B)\left(\right.}{}$$

$$= a^4Bx + \frac{a^4(35A+48B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{5a^4(}{}$$

Mathematica [A] time = 2.03, size = 326, normalized size = 1.88

$$\frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4\left(\sec(c)(105A\sin(2c+dx)+544A\sin(c+2dx)-96A\sin(3c+2dx)+81\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(-24*(35*A + 48*B)*Cos[c + d*x
]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]]) + Sec[c]*(72*B*d*x*Cos[c] + 48*B*d*x*Cos[c + 2*d*x] + 48*B*d
*x*Cos[3*c + 2*d*x] + 12*B*d*x*Cos[3*c + 4*d*x] + 12*B*d*x*Cos[5*c + 4*d*x]
 - 480*A*Sin[c] - 480*B*Sin[c] + 105*A*Sin[d*x] + 48*B*Sin[d*x] + 105*A*Sin
[2*c + d*x] + 48*B*Sin[2*c + d*x] + 544*A*Sin[c + 2*d*x] + 496*B*Sin[c + 2*
d*x] - 96*A*Sin[3*c + 2*d*x] - 144*B*Sin[3*c + 2*d*x] + 81*A*Sin[2*c + 3*d*
x] + 48*B*Sin[2*c + 3*d*x] + 81*A*Sin[4*c + 3*d*x] + 48*B*Sin[4*c + 3*d*x]
+ 160*A*Sin[3*c + 4*d*x] + 160*B*Sin[3*c + 4*d*x])))/(3072*d)
```

fricas [A] time = 0.66, size = 157, normalized size = 0.91

$$\frac{48\,Ba^4dx\cos\left(dx+c\right)^4+3\,(35\,A+48\,B)a^4\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(35\,A+48\,B)a^4\cos\left(dx+c\right)^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(48*B*a^4*d*x*cos(d*x + c)^4 + 3*(35*A + 48*B)*a^4*cos(d*x + c)^4*log(
sin(d*x + c) + 1) - 3*(35*A + 48*B)*a^4*cos(d*x + c)^4*log(-sin(d*x + c) +
1) + 2*(160*(A + B)*a^4*cos(d*x + c)^3 + 3*(27*A + 16*B)*a^4*cos(d*x + c)^2
 + 8*(4*A + B)*a^4*cos(d*x + c) + 6*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [A] time = 0.68, size = 223, normalized size = 1.29

$$\frac{24\,(dx+c)Ba^4+3\left(35\,Aa^4+48\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(35\,Aa^4+48\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(24*(d*x + c)*B*a^4 + 3*(35*A*a^4 + 48*B*a^4)*log(abs(tan(1/2*d*x + 1/
2*c) + 1)) - 3*(35*A*a^4 + 48*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2
*(105*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 385
*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 424*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 511*A*a^
4*tan(1/2*d*x + 1/2*c)^3 + 520*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*A*a^4*tan
(1/2*d*x + 1/2*c) - 216*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 - 1)^4)/d
```

maple [A] time = 0.20, size = 204, normalized size = 1.18

$$\frac{35A\,a^4\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+a^4Bx+\frac{a^4Bc}{d}+\frac{20A\,a^4\tan\left(dx+c\right)}{3d}+\frac{6a^4B\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 35/8/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*B*x+1/d*a^4*B*c+20/3/d*A*a^4*tan
(d*x+c)+6/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+27/8/d*A*a^4*sec(d*x+c)*tan(d*x
+c)+20/3/d*a^4*B*tan(d*x+c)+4/3/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+2/d*a^4*B*s
ec(d*x+c)*tan(d*x+c)+1/4/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+1/3/d*a^4*B*tan(d*
x+c)*sec(d*x+c)^2
```

maxima [A] time = 0.42, size = 307, normalized size = 1.77

$$\frac{64\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Aa^{4}+16\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Ba^{4}+48\left(dx+c\right)Ba^{4}-3\,Aa^{4}\left(\frac{2\left(3\sin\right.}{\sin(d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^4 + 48*(d*x + c)*B*a^4 - 3*A*a^4*(2*(3*sin(d*x + c)^3 - 5*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 72*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 48*B*a^4*(2*sin(d*x
 + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))
 + 24*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 96*B*a^4*(log
(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 192*A*a^4*tan(d*x + c) + 288*
B*a^4*tan(d*x + c))/d
```

mupad [B] time = 0.38, size = 255, normalized size = 1.47

$$\frac{35\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}+\frac{2\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{12\,B\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{20\,A\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)}+\frac{27\,A\,a^4}{8\,d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^5,x)
```

```
[Out] (35*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*d) + (2*B*a^4*at
an(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (12*B*a^4*atanh(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2)))/d + (20*A*a^4*sin(c + d*x))/(3*d*cos(c + d*x))
+ (27*A*a^4*sin(c + d*x))/(8*d*cos(c + d*x)^2) + (4*A*a^4*sin(c + d*x))/(3*
d*cos(c + d*x)^3) + (A*a^4*sin(c + d*x))/(4*d*cos(c + d*x)^4) + (20*B*a^4*s
in(c + d*x))/(3*d*cos(c + d*x)) + (2*B*a^4*sin(c + d*x))/(d*cos(c + d*x)^2)
 + (B*a^4*sin(c + d*x))/(3*d*cos(c + d*x)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.36 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=198

$$\frac{a^4(83A+100B)\tan(c+dx)}{15d}+\frac{7a^4(4A+5B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4(244A+275B)\tan(c+dx)\sec(c+dx)}{120d}+$$

```
[Out] 7/8*a^4*(4*A+5*B)*arctanh(sin(d*x+c))/d+1/15*a^4*(83*A+100*B)*tan(d*x+c)/d+
1/120*a^4*(244*A+275*B)*sec(d*x+c)*tan(d*x+c)/d+1/30*(26*A+25*B)*(a^4+a^4*c
os(d*x+c))*sec(d*x+c)^2*tan(d*x+c)/d+1/20*(8*A+5*B)*(a^2+a^2*cos(d*x+c))^2*
sec(d*x+c)^3*tan(d*x+c)/d+1/5*a*A*(a+a*cos(d*x+c))^3*sec(d*x+c)^4*tan(d*x+c
)/d
```

Rubi [A] time = 0.59, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2975, 2968, 3021, 2748, 3767, 8, 3770}

$$\frac{a^4(83A+100B)\tan(c+dx)}{15d}+\frac{7a^4(4A+5B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4(244A+275B)\tan(c+dx)\sec(c+dx)}{120d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (7*a^4*(4*A + 5*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^4*(83*A + 100*B)*Tan[c
 + d*x])/(15*d) + (a^4*(244*A + 275*B)*Sec[c + d*x]*Tan[c + d*x])/(120*d) +
 ((26*A + 25*B)*(a^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(30*d
) + ((8*A + 5*B)*(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(2
0*d) + (a*A*(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{1}{5}$$

$$= \frac{(8A+5B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^3(c+dx)\tan(c}{20d}$$

$$= \frac{(26A+25B)\left(a^4+a^4\cos(c+dx)\right)\sec^2(c+dx)\tan(}{30d}$$

$$= \frac{(26A+25B)\left(a^4+a^4\cos(c+dx)\right)\sec^2(c+dx)\tan(}{30d}$$

$$= \frac{a^4(244A+275B)\sec(c+dx)\tan(c+dx)}{120d} + \frac{(26A+}{}$$

$$= \frac{a^4(244A+275B)\sec(c+dx)\tan(c+dx)}{120d} + \frac{(26A+}{}$$

$$= \frac{7a^4(4A+5B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^4(244A+275}{}$$

$$= \frac{7a^4(4A+5B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^4(83A+100}{15}$$

Mathematica [A] time = 1.75, size = 306, normalized size = 1.55

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(1680(4A+5B)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] -1/30720*(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^5*(1680*
(4*A + 5*B)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(59*A + 64*B)*Sin[d*x] -
 960*(2*A + 3*B)*Sin[2*c + d*x] + 1320*A*Sin[c + 2*d*x] + 930*B*Sin[c + 2*d
*x] + 1320*A*Sin[3*c + 2*d*x] + 930*B*Sin[3*c + 2*d*x] + 3200*A*Sin[2*c + 3
*d*x] + 3520*B*Sin[2*c + 3*d*x] - 120*A*Sin[4*c + 3*d*x] - 480*B*Sin[4*c +
3*d*x] + 420*A*Sin[3*c + 4*d*x] + 405*B*Sin[3*c + 4*d*x] + 420*A*Sin[5*c +
4*d*x] + 405*B*Sin[5*c + 4*d*x] + 664*A*Sin[4*c + 5*d*x] + 800*B*Sin[4*c +
5*d*x])))/d
```

fricas [A] time = 1.24, size = 165, normalized size = 0.83

$$\frac{105\,(4\,A+5\,B)a^4\cos(dx+c)^5\log(\sin(dx+c)+1)-105\,(4\,A+5\,B)a^4\cos(dx+c)^5\log(-\sin(dx+c)+1)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fr
icas")
```

```
[Out] 1/240*(105*(4*A + 5*B)*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*(4*A
+ 5*B)*a^4*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(83*A + 100*B)*a^4*
cos(d*x + c)^4 + 15*(28*A + 27*B)*a^4*cos(d*x + c)^3 + 16*(17*A + 10*B)*a^4
*cos(d*x + c)^2 + 30*(4*A + B)*a^4*cos(d*x + c) + 24*A*a^4)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

giac [A] time = 0.46, size = 246, normalized size = 1.24

$$\frac{105\left(4\,Aa^4+5\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\left(4\,Aa^4+5\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(420\,Aa^4\tan\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="gi
ac")
```

```
[Out] 1/120*(105*(4*A*a^4 + 5*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(4*
A*a^4 + 5*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(420*A*a^4*tan(1/2*
d*x + 1/2*c)^9 + 525*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 1960*A*a^4*tan(1/2*d*x
+ 1/2*c)^7 - 2450*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 3584*A*a^4*tan(1/2*d*x + 1
/2*c)^5 + 4480*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 3160*A*a^4*tan(1/2*d*x + 1/2*
c)^3 - 3950*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 1500*A*a^4*tan(1/2*d*x + 1/2*c)
+ 1395*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 0.20, size = 234, normalized size = 1.18

$$\frac{83A\,a^4\tan(dx+c)}{15d}+\frac{35a^4B\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{7A\,a^4\sec(dx+c)\tan(dx+c)}{2d}+\frac{7A\,a^4\ln(\sec(dx\cdots}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 83/15/d*A*a^4*tan(d*x+c)+35/8/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+7/2/d*A*a^4
*sec(d*x+c)*tan(d*x+c)+7/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3/d*a^4*B*t
an(d*x+c)+34/15/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+27/8/d*a^4*B*sec(d*x+c)*tan
(d*x+c)+1/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+4/3/d*a^4*B*tan(d*x+c)*sec(d*x+c)
^2+1/5/d*A*a^4*tan(d*x+c)*sec(d*x+c)^4+1/4/d*a^4*B*tan(d*x+c)*sec(d*x+c)^3
```

maxima [B] time = 0.70, size = 376, normalized size = 1.90

$$\frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^4+480\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^4+320\left(\text{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
480*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 320*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^4 - 60*A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 15*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 240*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
 log(sin(d*x + c) - 1)) - 360*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) -
log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120*B*a^4*(log(sin(d*x + c
) + 1) - log(sin(d*x + c) - 1)) + 240*A*a^4*tan(d*x + c) + 960*B*a^4*tan(d*
x + c))/d
```

mupad [B] time = 2.79, size = 224, normalized size = 1.13

$$\frac{7\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,A+5\,B\right)}{4\,d}-\frac{\left(7\,A\,a^4+\frac{35\,B\,a^4}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{98\,A\,a^4}{3}-\frac{245\,B\,a^4}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{8}{}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^6,x)
```

```
[Out] (7*a^4*atanh(tan(c/2 + (d*x)/2))*(4*A + 5*B))/(4*d) - (tan(c/2 + (d*x)/2)*(
25*A*a^4 + (93*B*a^4)/4) + tan(c/2 + (d*x)/2)^9*(7*A*a^4 + (35*B*a^4)/4) -
tan(c/2 + (d*x)/2)^7*((98*A*a^4)/3 + (245*B*a^4)/6) - tan(c/2 + (d*x)/2)^3*
((158*A*a^4)/3 + (395*B*a^4)/6) + tan(c/2 + (d*x)/2)^5*((896*A*a^4)/15 + (2
24*B*a^4)/3))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan
(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
[Out] Timed out
```

3.37 $\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$

Optimal. Leaf size=229

$$\frac{a^4(72A+83B)\tan(c+dx)}{15d}+\frac{7a^4(7A+8B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(159A+176B)\tan(c+dx)\sec^2(c+dx)}{120d}+\frac{7a^4}{}$$

```
[Out] 7/16*a^4*(7*A+8*B)*arctanh(sin(d*x+c))/d+1/15*a^4*(72*A+83*B)*tan(d*x+c)/d+
7/16*a^4*(7*A+8*B)*sec(d*x+c)*tan(d*x+c)/d+1/120*a^4*(159*A+176*B)*sec(d*x+
c)^2*tan(d*x+c)/d+1/120*(73*A+72*B)*(a^4+a^4*cos(d*x+c))*sec(d*x+c)^3*tan(d
*x+c)/d+1/10*(3*A+2*B)*(a^2+a^2*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d+1/6
*a*A*(a+a*cos(d*x+c))^3*sec(d*x+c)^5*tan(d*x+c)/d
```

Rubi [A] time = 0.65, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2975, 2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{a^4(72A+83B)\tan(c+dx)}{15d}+\frac{7a^4(7A+8B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(159A+176B)\tan(c+dx)\sec^2(c+dx)}{120d}+\frac{7a^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

```
[Out] (7*a^4*(7*A + 8*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^4*(72*A + 83*B)*Tan[c
 + d*x])/(15*d) + (7*a^4*(7*A + 8*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a
^4*(159*A + 176*B)*Sec[c + d*x]^2*Tan[c + d*x])/(120*d) + ((73*A + 72*B)*(a
^4 + a^4*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + ((3*A + 2*B)*
(a^2 + a^2*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(10*d) + (a*A*(a +
a*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^3\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{1}{6}$$

$$= \frac{(3A+2B)\left(a^2+a^2\cos(c+dx)\right)^2\sec^4(c+dx)\tan(c}{10d}$$

$$= \frac{(73A+72B)\left(a^4+a^4\cos(c+dx)\right)\sec^3(c+dx)\tan(}{120d}$$

$$= \frac{(73A+72B)\left(a^4+a^4\cos(c+dx)\right)\sec^3(c+dx)\tan(}{120d}$$

$$= \frac{a^4(159A+176B)\sec^2(c+dx)\tan(c+dx)}{120d} + \frac{(73A+}{}$$

$$= \frac{a^4(159A+176B)\sec^2(c+dx)\tan(c+dx)}{120d} + \frac{(73A+}{}$$

$$= \frac{7a^4(7A+8B)\sec(c+dx)\tan(c+dx)}{16d} + \frac{a^4(159A+}{}$$

$$= \frac{7a^4(7A+8B)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{a^4(72A+83B}{15}$$

Mathematica [A] time = 2.35, size = 358, normalized size = 1.56

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(3360(7A+8B)\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

```
[Out] -1/122880*(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^6*(3360
*(7*A + 8*B)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-160*(72*A + 83*B)*Sin[c]
+ 30*(125*A + 88*B)*Sin[d*x] + 3750*A*Sin[2*c + d*x] + 2640*B*Sin[2*c + d*x
] + 15360*A*Sin[c + 2*d*x] + 15840*B*Sin[c + 2*d*x] - 1920*A*Sin[3*c + 2*d*
x] - 4080*B*Sin[3*c + 2*d*x] + 3845*A*Sin[2*c + 3*d*x] + 3480*B*Sin[2*c + 3
*d*x] + 3845*A*Sin[4*c + 3*d*x] + 3480*B*Sin[4*c + 3*d*x] + 6912*A*Sin[3*c
+ 4*d*x] + 7728*B*Sin[3*c + 4*d*x] - 240*B*Sin[5*c + 4*d*x] + 735*A*Sin[4*c
 + 5*d*x] + 840*B*Sin[4*c + 5*d*x] + 735*A*Sin[6*c + 5*d*x] + 840*B*Sin[6*c
 + 5*d*x] + 1152*A*Sin[5*c + 6*d*x] + 1328*B*Sin[5*c + 6*d*x])))/d
```

fricas [A] time = 0.73, size = 185, normalized size = 0.81

$$\frac{105\,(7\,A+8\,B)a^4\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-105\,(7\,A+8\,B)a^4\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="fr
icas")
```

```
[Out] 1/480*(105*(7*A + 8*B)*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 105*(7*A
+ 8*B)*a^4*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(72*A + 83*B)*a^4*
cos(d*x + c)^5 + 105*(7*A + 8*B)*a^4*cos(d*x + c)^4 + 32*(18*A + 17*B)*a^4*
cos(d*x + c)^3 + 10*(41*A + 24*B)*a^4*cos(d*x + c)^2 + 48*(4*A + B)*a^4*cos
(d*x + c) + 40*A*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

giac [A] time = 0.54, size = 280, normalized size = 1.22

$$\frac{105\left(7\,Aa^4+8\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\left(7\,Aa^4+8\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(735\,Aa^4\tan\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="gi
ac")
```

```
[Out] 1/240*(105*(7*A*a^4 + 8*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(7*
A*a^4 + 8*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(735*A*a^4*tan(1/2*
d*x + 1/2*c)^11 + 840*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*A*a^4*tan(1/2*d*
x + 1/2*c)^9 - 4760*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 9702*A*a^4*tan(1/2*d*x +
 1/2*c)^7 + 11088*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*A*a^4*tan(1/2*d*x +
1/2*c)^5 - 13488*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 7355*A*a^4*tan(1/2*d*x + 1/
2*c)^3 + 9320*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*A*a^4*tan(1/2*d*x + 1/2*c
) - 3000*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

maple [A] time = 0.22, size = 280, normalized size = 1.22

$$\frac{49A\,a^4\sec\left(dx+c\right)\tan\left(dx+c\right)}{16d}+\frac{49A\,a^4\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{16d}+\frac{83a^4B\tan\left(dx+c\right)}{15d}+\frac{24A\,a^4\tan\left(dx+\right.}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x)
```

```
[Out] 49/16/d*A*a^4*sec(d*x+c)*tan(d*x+c)+49/16/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))
+83/15/d*a^4*B*tan(d*x+c)+24/5/d*A*a^4*tan(d*x+c)+12/5/d*A*a^4*tan(d*x+c)*s
ec(d*x+c)^2+7/2/d*a^4*B*sec(d*x+c)*tan(d*x+c)+7/2/d*a^4*B*ln(sec(d*x+c)+tan
(d*x+c))+41/24/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+34/15/d*a^4*B*tan(d*x+c)*sec
(d*x+c)^2+4/5/d*A*a^4*tan(d*x+c)*sec(d*x+c)^4+1/d*a^4*B*tan(d*x+c)*sec(d*x+
c)^3+1/6/d*A*a^4*tan(d*x+c)*sec(d*x+c)^5+1/5/d*a^4*B*tan(d*x+c)*sec(d*x+c)^
4
```

maxima [B] time = 0.40, size = 464, normalized size = 2.03

$$\frac{128\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)Aa^{4}+640\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Aa^{4}+32\left(3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="ma
xima")
```

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 32*(3*tan(d*x + c)^5 + 10*ta
n(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 + 960*(tan(d*x + c)^3 + 3*tan(d*x + c
))*B*a^4 - 5*A*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x +
 c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(si
n(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 180*A*a^4*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
 + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*B*a^4*(2*(3*sin(d*x + c)^3 - 5*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 120*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 480*B*a^4*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1
)) + 480*B*a^4*tan(d*x + c))/d
```

mupad [B] time = 2.84, size = 262, normalized size = 1.14

$$\frac{\left(-\frac{49\,A\,a^4}{8}-7\,B\,a^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{833\,A\,a^4}{24}+\frac{119\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(-\frac{1617\,A\,a^4}{20}-\frac{462\,B\,a^4}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+\left(\frac{1}{}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\,t\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^4)/cos(c + d*x)^7,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((207*A*a^4)/8 + 25*B*a^4) - tan(c/2 + (d*x)/2)^11*((49
*A*a^4)/8 + 7*B*a^4) + tan(c/2 + (d*x)/2)^9*((833*A*a^4)/24 + (119*B*a^4)/3
```

```
) - tan(c/2 + (d*x)/2)^3*((1471*A*a^4)/24 + (233*B*a^4)/3) - tan(c/2 + (d*x
)/2)^7*((1617*A*a^4)/20 + (462*B*a^4)/5) + tan(c/2 + (d*x)/2)^5*((1967*A*a^
4)/20 + (562*B*a^4)/5))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^
2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2
)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (7*a^4*atanh(tan(c/2 + (d*x)/2))*(7*A
+ 8*B))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

3.38 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=153

$$-\frac{4(A-B)\sin^3(c+dx)}{3ad}+\frac{4(A-B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(4A-5B)\sin(c+dx)\cos^3(c+$$

```
[Out] -3/8*(4*A-5*B)*x/a+4*(A-B)*sin(d*x+c)/a/d-3/8*(4*A-5*B)*cos(d*x+c)*sin(d*x+
c)/a/d-1/4*(4*A-5*B)*cos(d*x+c)^3*sin(d*x+c)/a/d+(A-B)*cos(d*x+c)^4*sin(d*x
+c)/d/(a+a*cos(d*x+c))-4/3*(A-B)*sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.21, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2748, 2633, 2635, 8}

$$-\frac{4(A-B)\sin^3(c+dx)}{3ad}+\frac{4(A-B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(4A-5B)\sin(c+dx)\cos^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-3*(4*A - 5*B)*x)/(8*a) + (4*(A - B)*Sin[c + d*x])/(a*d) - (3*(4*A - 5*B)*
Cos[c + d*x]*Sin[c + d*x])/(8*a*d) - ((4*A - 5*B)*Cos[c + d*x]^3*Sin[c + d*
x])/(4*a*d) + ((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])
) - (4*(A - B)*Sin[c + d*x]^3)/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^3(c+dx)(4a(A-B)-a(4A}{a^2}$$
$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(4A-5B)\int \cos^4(c+dx)\,dx}{a} + \frac{(4(}{}$$
$$= -\frac{(4A-5B)\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{(A-B)\cos^4(c+dx)\sin(c+d}{d(a+a\cos(c+dx))}$$
$$= \frac{4(A-B)\sin(c+dx)}{ad} - \frac{3(4A-5B)\cos(c+dx)\sin(c+dx)}{8ad} - \frac{(4A-5}{}$$
$$= -\frac{3(4A-5B)x}{8a} + \frac{4(A-B)\sin(c+dx)}{ad} - \frac{3(4A-5B)\cos(c+dx)\sin(c}{8ad}$$

Mathematica [B] time = 0.70, size = 311, normalized size = 2.03

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-72dx(4A-5B)\cos\left(c+\frac{dx}{2}\right)-72dx(4A-5B)\cos\left(\frac{dx}{2}\right)+168A\sin\left(c+\frac{dx}{2}\right)+144A\,\mathrm{si}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-72*(4*A - 5*B)*d*x*Cos[(d*x)/2] - 72*(4*A - 5*
B)*d*x*Cos[c + (d*x)/2] + 552*A*Sin[(d*x)/2] - 552*B*Sin[(d*x)/2] + 168*A*S
```

```
in[c + (d*x)/2] - 168*B*Sin[c + (d*x)/2] + 144*A*Sin[c + (3*d*x)/2] - 120*B
*Sin[c + (3*d*x)/2] + 144*A*Sin[2*c + (3*d*x)/2] - 120*B*Sin[2*c + (3*d*x)/
2] - 16*A*Sin[2*c + (5*d*x)/2] + 40*B*Sin[2*c + (5*d*x)/2] - 16*A*Sin[3*c +
 (5*d*x)/2] + 40*B*Sin[3*c + (5*d*x)/2] + 8*A*Sin[3*c + (7*d*x)/2] - 5*B*Si
n[3*c + (7*d*x)/2] + 8*A*Sin[4*c + (7*d*x)/2] - 5*B*Sin[4*c + (7*d*x)/2] +
3*B*Sin[4*c + (9*d*x)/2] + 3*B*Sin[5*c + (9*d*x)/2]))/(192*a*d*(1 + Cos[c +
 d*x]))
```

fricas [A] time = 0.62, size = 120, normalized size = 0.78

$$-\frac{9\,(4\,A-5\,B)dx\cos\left(dx+c\right)+9\,(4\,A-5\,B)dx-\left(6\,B\cos\left(dx+c\right)^{4}+2\,(4\,A-B)\cos\left(dx+c\right)^{3}-(4\,A-13\,B)\cos\right.}{24\,(ad\cos\left(dx+c\right)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/24*(9*(4*A - 5*B)*d*x*cos(d*x + c) + 9*(4*A - 5*B)*d*x - (6*B*cos(d*x +
c)^4 + 2*(4*A - B)*cos(d*x + c)^3 - (4*A - 13*B)*cos(d*x + c)^2 + (28*A - 1
9*B)*cos(d*x + c) + 64*A - 64*B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 0.85, size = 181, normalized size = 1.18

$$-\frac{\frac{9\,(dx+c)(4\,A-5\,B)}{a}-\frac{24\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}-\frac{2\left(60\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-75\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+124\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-115\,B\tan\left(\right.\right.}{\left(\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/24*(9*(d*x + c)*(4*A - 5*B)/a - 24*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d
*x + 1/2*c))/a - 2*(60*A*tan(1/2*d*x + 1/2*c)^7 - 75*B*tan(1/2*d*x + 1/2*c)
^7 + 124*A*tan(1/2*d*x + 1/2*c)^5 - 115*B*tan(1/2*d*x + 1/2*c)^5 + 100*A*ta
n(1/2*d*x + 1/2*c)^3 - 109*B*tan(1/2*d*x + 1/2*c)^3 + 36*A*tan(1/2*d*x + 1/
2*c) - 21*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

maple [B] time = 0.09, size = 351, normalized size = 2.29

$$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{25\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{4ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{5\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{115\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{12ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)-25/4/a/d/(1+tan(1/2*d
*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*B+5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(
1/2*d*x+1/2*c)^7*A-115/12/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)
^5*B+31/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*A-109/12/a/d/
(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*B+25/3/a/d/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*A-7/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*B*tan(
1/2*d*x+1/2*c)+3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*A*tan(1/2*d*x+1/2*c)-3/a/d*
arctan(tan(1/2*d*x+1/2*c))*A+15/4/a/d*arctan(tan(1/2*d*x+1/2*c))*B
```

maxima [B] time = 0.56, size = 394, normalized size = 2.58

$$-\frac{B\left(\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{115\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-4A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\cdots}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\cdots}\cdots\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/12*(B*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 4*A*((9*sin(d*x +
 c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 3*sin(d*x + c)/(a*
(cos(d*x + c) + 1))))/d
```

mupad [B] time = 0.38, size = 170, normalized size = 1.11

$$\frac{15Bx}{8a}-\frac{3Ax}{2a}+\frac{7A\sin(c+dx)}{4ad}-\frac{7B\sin(c+dx)}{4ad}-\frac{A\sin(2c+2dx)}{4ad}+\frac{A\sin(3c+3dx)}{12ad}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad}+\frac{B\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

```
[Out] (15*B*x)/(8*a) - (3*A*x)/(2*a) + (7*A*sin(c + d*x))/(4*a*d) - (7*B*sin(c +
d*x))/(4*a*d) - (A*sin(2*c + 2*d*x))/(4*a*d) + (A*sin(3*c + 3*d*x))/(12*a*d
) + (A*tan(c/2 + (d*x)/2))/(a*d) + (B*sin(2*c + 2*d*x))/(2*a*d) - (B*sin(3*
c + 3*d*x))/(12*a*d) + (B*sin(4*c + 4*d*x))/(32*a*d) - (B*tan(c/2 + (d*x)/2
))/(a*d)
```

sympy [A] time = 7.46, size = 1794, normalized size = 11.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((-36*A*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a
*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x
/2)**2 + 24*a*d) - 144*A*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**
8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c
/2 + d*x/2)**2 + 24*a*d) - 216*A*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 +
d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a
*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 144*A*d*x*tan(c/2 + d*x/2)**2/(24*a*d*ta
n(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**
4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 36*A*d*x/(24*a*d*tan(c/2 + d*x/2
)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*ta
n(c/2 + d*x/2)**2 + 24*a*d) + 24*A*tan(c/2 + d*x/2)**9/(24*a*d*tan(c/2 + d*
x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d
*tan(c/2 + d*x/2)**2 + 24*a*d) + 216*A*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2
+ d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96
*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 392*A*tan(c/2 + d*x/2)**5/(24*a*d*tan(
c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4
+ 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 296*A*tan(c/2 + d*x/2)**3/(24*a*d*
tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)
**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 96*A*tan(c/2 + d*x/2)/(24*a*d*
tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)
**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 45*B*d*x*tan(c/2 + d*x/2)**8/(
24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 +
 d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 180*B*d*x*tan(c/2 + d*x
/2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*t
an(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 270*B*d*x*tan(c
/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 1
44*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 180*B*d
*x*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2
)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) +
 45*B*d*x/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*
d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 24*B*tan(c/2
 + d*x/2)**9/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144
```

```
*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 246*B*tan
(c/2 + d*x/2)**7/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 +
 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 374*B
*tan(c/2 + d*x/2)**5/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)*
*6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 3
14*B*tan(c/2 + d*x/2)**3/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x
/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d)
 - 66*B*tan(c/2 + d*x/2)/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x
/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d)
, Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**4/(a*cos(c) + a), True))
```

3.39 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{(3A-4B)\sin^3(c+dx)}{3ad}-\frac{(3A-4B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3(A-B)\sin(c+dx)\cos(c+dx)}{2ad}$$

```
[Out] 3/2*(A-B)*x/a-(3*A-4*B)*sin(d*x+c)/a/d+3/2*(A-B)*cos(d*x+c)*sin(d*x+c)/a/d+
(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))+1/3*(3*A-4*B)*sin(d*x+c)^3
/a/d
```

Rubi [A] time = 0.17, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2748, 2635, 8, 2633}

$$\frac{(3A-4B)\sin^3(c+dx)}{3ad}-\frac{(3A-4B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3(A-B)\sin(c+dx)\cos(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*(A - B)*x)/(2*a) - ((3*A - 4*B)*Sin[c + d*x])/(a*d) + (3*(A - B)*Cos[c +
 d*x]*Sin[c + d*x])/(2*a*d) + ((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(d*(a +
 a*Cos[c + d*x])) + ((3*A - 4*B)*Sin[c + d*x]^3)/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^2(c+dx)(3a(A-B)-a(3A}{a^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(3A-4B)\int \cos^3(c+dx)\,dx}{a} + \frac{(3(}{}$$

$$= \frac{3(A-B)\cos(c+dx)\sin(c+dx)}{2ad} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} +$$

$$= \frac{3(A-B)x}{2a} - \frac{(3A-4B)\sin(c+dx)}{ad} + \frac{3(A-B)\cos(c+dx)\sin(c+dx}{2ad}$$

Mathematica [B] time = 0.64, size = 249, normalized size = 2.04

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(36dx(A-B)\cos\left(c+\frac{dx}{2}\right)+36dx(A-B)\cos\left(\frac{dx}{2}\right)-12A\sin\left(c+\frac{dx}{2}\right)-9A\sin\left(c+\frac{3dx}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(A - B)*d*x*Cos[(d*x)/2] + 36*(A - B)*d*x*Co
s[c + (d*x)/2] - 60*A*Sin[(d*x)/2] + 69*B*Sin[(d*x)/2] - 12*A*Sin[c + (d*x)
/2] + 21*B*Sin[c + (d*x)/2] - 9*A*Sin[c + (3*d*x)/2] + 18*B*Sin[c + (3*d*x)
/2] - 9*A*Sin[2*c + (3*d*x)/2] + 18*B*Sin[2*c + (3*d*x)/2] + 3*A*Sin[2*c +
```

```
(5*d*x)/2] - 2*B*Sin[2*c + (5*d*x)/2] + 3*A*Sin[3*c + (5*d*x)/2] - 2*B*Sin[
3*c + (5*d*x)/2] + B*Sin[3*c + (7*d*x)/2] + B*Sin[4*c + (7*d*x)/2]))/(24*a*
d*(1 + Cos[c + d*x]))
```

fricas [A] time = 0.65, size = 98, normalized size = 0.80

$$\frac{9\,(A-B)dx\cos\left(dx+c\right)+9\,(A-B)dx+\left(2\,B\cos\left(dx+c\right)^{3}+(3\,A-B)\cos\left(dx+c\right)^{2}-(3\,A-7\,B)\cos\left(dx+c\right)-\right.}{6\,(ad\cos\left(dx+c\right)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/6*(9*(A - B)*d*x*cos(d*x + c) + 9*(A - B)*d*x + (2*B*cos(d*x + c)^3 + (3*
A - B)*cos(d*x + c)^2 - (3*A - 7*B)*cos(d*x + c) - 12*A + 16*B)*sin(d*x + c
))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 0.49, size = 151, normalized size = 1.24

$$\frac{\frac{9\,(dx+c)(A-B)}{a}-\frac{6\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a}-\frac{2\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-16\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(9*(d*x + c)*(A - B)/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/
2*c))/a - 2*(9*A*tan(1/2*d*x + 1/2*c)^5 - 15*B*tan(1/2*d*x + 1/2*c)^5 + 12*
A*tan(1/2*d*x + 1/2*c)^3 - 16*B*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x +
1/2*c) - 9*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

maple [B] time = 0.10, size = 281, normalized size = 2.30

$$-\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{5\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{16\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{3ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1/2*d*x+1/2*c)-3/a/d/(1+tan(1/2*d*x
+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/
2*d*x+1/2*c)^5*B+16/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*B
-4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A-1/a/d/(1+tan(1/2*d
*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)+3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(
1/2*d*x+1/2*c)+3/a/d*arctan(tan(1/2*d*x+1/2*c))*A-3/a/d*arctan(tan(1/2*d*x+
1/2*c))*B
```

maxima [B] time = 0.70, size = 310, normalized size = 2.54

$$\frac{B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3A\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3a}{\cdots}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*(B*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 3*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*A*((sin(d*x + c)/(cos(d*x + c)
 + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

mupad [B] time = 1.36, size = 138, normalized size = 1.13

$$\frac{3x(A-B)}{2a}-\frac{(3A-5B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(4A-\frac{16B}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(A-3B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(\cdots}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

```
[Out] (3*x*(A - B))/(2*a) - (tan(c/2 + (d*x)/2)^5*(3*A - 5*B) + tan(c/2 + (d*x)/2
)^3*(4*A - (16*B)/3) + tan(c/2 + (d*x)/2)*(A - 3*B))/(d*(a + 3*a*tan(c/2 +
(d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 + a*tan(c/2 + (d*x)/2)^6)) - (tan(c/2
 + (d*x)/2)*(A - B))/(a*d)
```

sympy [A] time = 4.51, size = 1161, normalized size = 9.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((9*A*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*
tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 27*A*d*x*tan(c/
2 + d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*
a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 27*A*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(
c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
 6*a*d) + 9*A*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 +
 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 6*A*tan(c/2 + d*x/2)**7/(6*a*d*tan(c
/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
6*a*d) - 36*A*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c
/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 42*A*tan(c/2 + d*x/2
)**3/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c
/2 + d*x/2)**2 + 6*a*d) - 12*A*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6
+ 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 9*B*d*
x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)*
*4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*B*d*x*tan(c/2 + d*x/2)**4/(6*
a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x
/2)**2 + 6*a*d) - 27*B*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*x/2)**6 +
 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 9*B*d*x
/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 +
 d*x/2)**2 + 6*a*d) + 6*B*tan(c/2 + d*x/2)**7/(6*a*d*tan(c/2 + d*x/2)**6 +
18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 48*B*tan
(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 +
18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 50*B*tan(c/2 + d*x/2)**3/(6*a*d*tan(c
/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 +
6*a*d) + 24*B*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2
+ d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*(A + B*cos
(c))*cos(c)**3/(a*cos(c) + a), True))
```

3.40 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=90

$$\frac{(A-B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)}{ad(\cos(c+dx)+1)}-\frac{x(A-B)}{a}+\frac{B\sin(c+dx)\cos(c+dx)}{2ad}+\frac{Bx}{2a}$$

```
[Out] -(A-B)*x/a+1/2*B*x/a+(A-B)*sin(d*x+c)/a/d+1/2*B*cos(d*x+c)*sin(d*x+c)/a/d+(
A-B)*sin(d*x+c)/a/d/(1+cos(d*x+c))
```

Rubi [A] time = 0.12, antiderivative size = 99, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2977, 2734}

$$\frac{2(A-B)\sin(c+dx)}{ad}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(2A-3B)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x(2A-3B)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] -((2*A - 3*B)*x)/(2*a) + (2*(A - B)*Sin[c + d*x])/(a*d) - ((2*A - 3*B)*Cos[
c + d*x]*Sin[c + d*x])/(2*a*d) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(
a + a*Cos[c + d*x]))
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos(c+dx)(2a(A-B)-a(2A-3\cdots}{a^2}$$
$$= -\frac{(2A-3B)x}{2a} + \frac{2(A-B)\sin(c+dx)}{ad} - \frac{(2A-3B)\cos(c+dx)\sin(c+d\cdots}{2ad}$$

Mathematica [B] time = 0.50, size = 197, normalized size = 2.19

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-4dx(2A-3B)\cos\left(c+\frac{dx}{2}\right)-4dx(2A-3B)\cos\left(\frac{dx}{2}\right)+4A\sin\left(c+\frac{dx}{2}\right)+4A\sin\left(c+\frac{3\cdots}{2}\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-4*(2*A - 3*B)*d*x*Cos[(d*x)/2] - 4*(2*A - 3*B)
*d*x*Cos[c + (d*x)/2] + 20*A*Sin[(d*x)/2] - 20*B*Sin[(d*x)/2] + 4*A*Sin[c +
 (d*x)/2] - 4*B*Sin[c + (d*x)/2] + 4*A*Sin[c + (3*d*x)/2] - 3*B*Sin[c + (3*
d*x)/2] + 4*A*Sin[2*c + (3*d*x)/2] - 3*B*Sin[2*c + (3*d*x)/2] + B*Sin[2*c +
 (5*d*x)/2] + B*Sin[3*c + (5*d*x)/2]))/(8*a*d*(1 + Cos[c + d*x]))
```

fricas [A] time = 0.56, size = 83, normalized size = 0.92

$$-\frac{(2A-3B)dx\cos(dx+c)+(2A-3B)dx-\left(B\cos(dx+c)^2+(2A-B)\cos(dx+c)+4A-4B\right)\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2*((2*A - 3*B)*d*x*cos(d*x + c) + (2*A - 3*B)*d*x - (B*cos(d*x + c)^2 +
(2*A - B)*cos(d*x + c) + 4*A - 4*B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 1.06, size = 124, normalized size = 1.38

$$-\frac{\frac{(dx+c)(2A-3B)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}-\frac{2\left(2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2*((d*x + c)*(2*A - 3*B)/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x +
 1/2*c))/a - 2*(2*A*tan(1/2*d*x + 1/2*c)^3 - 3*B*tan(1/2*d*x + 1/2*c)^3 + 2
*A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
+ 1)^2*a))/d
```

maple [B] time = 0.10, size = 211, normalized size = 2.34

$$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{3\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)-3/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B+2/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2
*d*x+1/2*c)^3*A-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)+2/a/d
/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)-2/a/d*arctan(tan(1/2*d*x+1
/2*c))*A+3/a/d*arctan(tan(1/2*d*x+1/2*c))*B
```

maxima [B] time = 0.77, size = 225, normalized size = 2.50

$$-\frac{B\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a+\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(B*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1))) + A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a -
 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c)
+ 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

mupad [B] time = 0.49, size = 107, normalized size = 1.19

$$\frac{(2A-3B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(2A-B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}-\frac{x(2A-3B)}{2a}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(A-B)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(2*A - 3*B) + tan(c/2 + (d*x)/2)*(2*A - B))/(d*(a + 2
*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4)) - (x*(2*A - 3*B))/(2*a)
+ (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

sympy [A] time = 3.23, size = 665, normalized size = 7.39

$$\begin{cases} -\frac{2Adx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}-\frac{4Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}-\frac{2Adx}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}+\frac{}{2ad\tan^4\left(\frac{c}{2}\right.} \\ \frac{x(A+B\cos(c))\cos^2(c)}{a\cos(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((-2*A*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*
tan(c/2 + d*x/2)**2 + 2*a*d) - 4*A*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 +
 d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*A*d*x/(2*a*d*tan(c/2 +
d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 2*A*tan(c/2 + d*x/2)**5/(2
*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 8*A*tan(c/2
 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d
) + 6*A*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2
)**2 + 2*a*d) + 3*B*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*
a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 6*B*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c
/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 3*B*d*x/(2*a*d*tan(c/
2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*B*tan(c/2 + d*x/2)**
5/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 10*B*ta
n(c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 +
2*a*d) - 4*B*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 +
d*x/2)**2 + 2*a*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**2/(a*cos(c) + a),
True))
```

3.41 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=54

$$-\frac{(A-B)\sin(c+dx)}{ad(\cos(c+dx)+1)}+\frac{x(A-B)}{a}+\frac{B\sin(c+dx)}{ad}$$

```
[Out] (A-B)*x/a+B*sin(d*x+c)/a/d-(A-B)*sin(d*x+c)/a/d/(1+cos(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2968, 3023, 12, 2735, 2648}

$$-\frac{(A-B)\sin(c+dx)}{ad(\cos(c+dx)+1)}+\frac{x(A-B)}{a}+\frac{B\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((A - B)*x)/a + (B*Sin[c + d*x])/(a*d) - ((A - B)*Sin[c + d*x])/(a*d*(1 + C
os[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx &= \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx\\
&= \frac{B\sin(c+dx)}{ad}+\frac{\int \frac{a(A-B)\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{a}\\
&= \frac{B\sin(c+dx)}{ad}+(A-B)\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx\\
&= \frac{(A-B)x}{a}+\frac{B\sin(c+dx)}{ad}+(-A+B)\int \frac{1}{a+a\cos(c+dx)}\,dx\\
&= \frac{(A-B)x}{a}+\frac{B\sin(c+dx)}{ad}-\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}
\end{aligned}$$

Mathematica [B] time = 0.28, size = 126, normalized size = 2.33

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(2dx(A-B)\cos\left(c+\frac{dx}{2}\right)+2dx(A-B)\cos\left(\frac{dx}{2}\right)-4A\sin\left(\frac{dx}{2}\right)+B\sin\left(c+\frac{dx}{2}\right)+B\sin\left(c\right.\right.}{2ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(2*(A - B)*d*x*Cos[(d*x)/2] + 2*(A - B)*d*x*Cos[
c + (d*x)/2] - 4*A*Sin[(d*x)/2] + 5*B*Sin[(d*x)/2] + B*Sin[c + (d*x)/2] + B
*Sin[c + (3*d*x)/2] + B*Sin[2*c + (3*d*x)/2]))/(2*a*d*(1 + Cos[c + d*x]))
```

fricas [A] time = 0.66, size = 61, normalized size = 1.13

$$\frac{(A-B)dx\cos(dx+c)+(A-B)dx+(B\cos(dx+c)-A+2B)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] ((A - B)*d*x*cos(d*x + c) + (A - B)*d*x + (B*cos(d*x + c) - A + 2*B)*sin(d*
x + c))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 0.91, size = 78, normalized size = 1.44

$$\frac{\frac{(dx+c)(A-B)}{a} - \frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} + \frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*(A - B)/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a
+ 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

maple [A] time = 0.10, size = 108, normalized size = 2.00

$$-\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{ad}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1/2*d*x+1/2*c)+2/a/d*B*tan(1/2*d*x+
1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2/a/d*arctan(tan(1/2*d*x+1/2*c))*A-2/a/d*ar
ctan(tan(1/2*d*x+1/2*c))*B
```

maxima [B] time = 0.69, size = 143, normalized size = 2.65

$$-\frac{B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -(B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(
cos(d*x + c) + 1))) - A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(
d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

mupad [B] time = 0.27, size = 65, normalized size = 1.20

$$\frac{x\,(A-B)}{a}+\frac{2\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

```
[Out] (x*(A - B))/a + (2*B*tan(c/2 + (d*x)/2))/(d*(a + a*tan(c/2 + (d*x)/2)^2)) -
 (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

sympy [A] time = 1.71, size = 264, normalized size = 4.89

$$\begin{cases} \frac{Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}+\frac{Adx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{Bdx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}-\frac{Bdx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad}+\frac{B\ldots}{ad\,\mathrm{ta}\ldots} \\ \frac{x(A+B\cos(c))\cos(c)}{a\cos(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) + A*d*
x/(a*d*tan(c/2 + d*x/2)**2 + a*d) - A*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*
x/2)**2 + a*d) - A*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d) - B*d*x
*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) - B*d*x/(a*d*tan(c/2 +
 d*x/2)**2 + a*d) + B*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**2 + a*d) +
 3*B*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*(A + B
*cos(c))*cos(c)/(a*cos(c) + a), True))
```

3.42 $\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}+\frac{Bx}{a}$$

```
[Out] B*x/a+(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2735, 2648}

$$\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}+\frac{Bx}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (B*x)/a + ((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{Bx}{a}-(-A+B)\int \frac{1}{a+a\cos(c+dx)}\,dx\\ &= \frac{Bx}{a}+\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.15, size = 72, normalized size = 2.12

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A-B)\sin\left(\frac{dx}{2}\right)+Bdx\cos\left(c+\frac{dx}{2}\right)+Bdx\cos\left(\frac{dx}{2}\right)\right)}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(B*d*x*Cos[(d*x)/2] + B*d*x*Cos[c + (d*x)/2] + 2
*(A - B)*Sin[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

fricas [A] time = 0.71, size = 43, normalized size = 1.26

$$\frac{Bdx\cos\left(dx+c\right)+Bdx+(A-B)\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] (B*d*x*cos(d*x + c) + B*d*x + (A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d
)
```

giac [A] time = 0.74, size = 43, normalized size = 1.26

$$\frac{\frac{(dx+c)B}{a}+\frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*B/a + (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d
```

maple [A] time = 0.06, size = 56, normalized size = 1.65

$$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{ad}-\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)+2/a/d*arctan(tan(1/2*d*x+1/2*c))*B-1/a/d*B*tan(1
/2*d*x+1/2*c)
```

maxima [B] time = 0.68, size = 73, normalized size = 2.15

$$\frac{B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{A\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] (B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x
+ c) + 1))) + A*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.20, size = 30, normalized size = 0.88

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a}+\frac{B\,d\,x}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x)),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(A - B))/a + (B*d*x)/a)/d
```

sympy [A] time = 0.86, size = 49, normalized size = 1.44

$$\begin{cases} \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad}+\frac{Bx}{a}-\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad} & \text{for}\ d\neq 0 \\ \frac{x(A+B\cos(c))}{a\cos(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)/(a*d) + B*x/a - B*tan(c/2 + d*x/2)/(a*d), Ne(
d, 0)), (x*(A + B*cos(c))/(a*cos(c) + a), True))
```

3.43 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] A*arctanh(sin(d*x+c))/a/d-(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2978, 12, 3770}

$$\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a*d) - ((A - B)*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int aA\sec(c+dx)\,dx}{a^2}\\
&= -\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{A\int\sec(c+dx)\,dx}{a}\\
&= \frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))}
\end{aligned}
$$

Mathematica [B] time = 0.27, size = 109, normalized size = 2.48

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((B-A)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+A\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\right.\right.\right.}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(A*Cos[(c + d*x)/2]*(-Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (-A + B)*Sec[c/2]*Sin
[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

fricas [A] time = 0.65, size = 74, normalized size = 1.68

$$\frac{(A\cos(dx+c)+A)\log(\sin(dx+c)+1)-(A\cos(dx+c)+A)\log(-\sin(dx+c)+1)-2(A-B)\sin(dx+c}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/2*((A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - (A*cos(d*x + c) + A)*log(
-sin(d*x + c) + 1) - 2*(A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 1.04, size = 71, normalized size = 1.61

$$\frac{\frac{A\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] (A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - A*log(abs(tan(1/2*d*x + 1/2*c) -
1))/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d
```

maple [A] time = 0.13, size = 78, normalized size = 1.77

$$-\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}+\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)-1/a/d*A*ln(tan
(1/2*d*x+1/2*c)-1)+1/a/d*B*tan(1/2*d*x+1/2*c)
```

maxima [B] time = 0.82, size = 99, normalized size = 2.25

$$\frac{A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{B\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] (A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
+ c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) + B*sin(d*x + c)/(a
*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 0.22, size = 42, normalized size = 0.95

$$\frac{2\,A\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A-B\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))),x)
```

```
[Out] (2*A*atanh(tan(c/2 + (d*x)/2)))/(a*d) - (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec(c+dx)}{\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*s
ec(c + d*x)/(cos(c + d*x) + 1), x))/a
```

3.44 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{(2A-B)\tan(c+dx)}{ad}-\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] -(A-B)*arctanh(sin(d*x+c))/a/d+(2*A-B)*tan(d*x+c)/a/d-(A-B)*tan(d*x+c)/d/(a
+a*cos(d*x+c))
```

Rubi [A] time = 0.16, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2748, 3767, 8, 3770}

$$\frac{(2A-B)\tan(c+dx)}{ad}-\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(((A - B)*ArcTanh[Sin[c + d*x]])/(a*d)) + ((2*A - B)*Tan[c + d*x])/(a*d) -
 ((A - B)*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int (a(2A-B)-a(A-B)\cos(c+dx))\sec^2(c+}{a^2}$$
$$= -\frac{(A-B)\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{(A-B)\int \sec(c+dx)\,dx}{a} + \frac{(2A-B)\int \sec^2(}{a}$$
$$= -\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad} - \frac{(A-B)\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{(2A-B)\,\mathrm{Sub}}{}$$
$$= -\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad} + \frac{(2A-B)\tan(c+dx)}{ad} - \frac{(A-B)\tan(}{d(a+a\cos(}$$

Mathematica [B] time = 1.34, size = 201, normalized size = 2.91

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.\right.}{ad(\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((A -
 B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]]) + (A*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
))))/(a*d*(1 + Cos[c + d*x]))
```

fricas [A] time = 0.57, size = 127, normalized size = 1.84

$$-\frac{\left((A-B)\cos(dx+c)^2+(A-B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-\left((A-B)\cos(dx+c)^2+(A-B)\cos(dx\right.}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2*(((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(sin(d*x + c) + 1)
 - ((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) -
 2*((2*A - B)*cos(d*x + c) + A)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos
(d*x + c))
```

giac [A] time = 0.35, size = 110, normalized size = 1.59

$$-\frac{\frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -((A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (A - B)*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a +
2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

maple [B] time = 0.14, size = 163, normalized size = 2.36

$$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{A}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)B}{ad}-\frac{}{ad\left(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)-1/a/d*A/(tan(1/2*d*x+
1/2*c)-1)+1/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B
-1/a/d*A/(tan(1/2*d*x+1/2*c)+1)-1/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)+1/a/d*ln(t
an(1/2*d*x+1/2*c)+1)*B
```

maxima [B] time = 0.45, size = 196, normalized size = 2.84

$$-\frac{A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - B*(log(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

mupad [B] time = 0.29, size = 78, normalized size = 1.13

$$\frac{2\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(A-B)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))),x)
```

```
[Out] (2*A*tan(c/2 + (d*x)/2))/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c/
2 + (d*x)/2))*(A - B))/(a*d) + (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^2(c+dx)}{\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^2(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)*sec(c + d*x)**2/(cos(c + d*x) + 1), x))/a
```

3.45 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=107

$$-\frac{2(A-B)\tan(c+dx)}{ad}+\frac{(3A-2B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(3A-2B)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx}$$

```
[Out] 1/2*(3*A-2*B)*arctanh(sin(d*x+c))/a/d-2*(A-B)*tan(d*x+c)/a/d+1/2*(3*A-2*B)*
sec(d*x+c)*tan(d*x+c)/a/d-(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))
```

Rubi [A] time = 0.17, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{2(A-B)\tan(c+dx)}{ad}+\frac{(3A-2B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(3A-2B)\tan(c+dx)\sec(c+dx)}{2ad}-\frac{(A-B)\tan(c+dx)}{d(a\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((3*A - 2*B)*ArcTanh[Sin[c + d*x]])/(2*a*d) - (2*(A - B)*Tan[c + d*x])/(a*d
) + ((3*A - 2*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A - B)*Sec[c + d*x]
*Tan[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int (a(3A-2B)-2a(A-B)\cos(c}{a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{(3A-2B)\int \sec^3(c+dx)\,dx}{a} - \frac{(2}{}$$

$$= \frac{(3A-2B)\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}$$

$$= \frac{(3A-2B)\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{2(A-B)\tan(c+dx)}{ad} + \frac{(3A-2B)\,s}{}$$

Mathematica [B] time = 3.64, size = 289, normalized size = 2.70

$$\cos\left(\frac{1}{2}(c+dx)\right)\left(4(B-A)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4(A-B)\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right.\right.}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(4*(-A + B)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((-6
*A + 4*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*A*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] - 4*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + A/(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - A/(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2 - (4*(A - B)*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)))))/(2*a*d*(1 + Cos[c + d*x]))
```

fricas [A] time = 0.66, size = 156, normalized size = 1.46

$$\frac{\left((3A-2B)\cos(dx+c)^3+(3A-2B)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-\left((3A-2B)\cos(dx+c)^3+(3A-\right.}{4\left(ad\cos(dx+c)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/4*(((3*A - 2*B)*cos(d*x + c)^3 + (3*A - 2*B)*cos(d*x + c)^2)*log(sin(d*x
+ c) + 1) - ((3*A - 2*B)*cos(d*x + c)^3 + (3*A - 2*B)*cos(d*x + c)^2)*log(-
sin(d*x + c) + 1) - 2*(4*(A - B)*cos(d*x + c)^2 + (A - 2*B)*cos(d*x + c) -
A)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

giac [A] time = 0.37, size = 157, normalized size = 1.47

$$\frac{\frac{(3A-2B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{(3A-2B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2B\tan\right.}{}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((3*A - 2*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (3*A - 2*B)*log(abs
(tan(1/2*d*x + 1/2*c) - 1))/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x +
 1/2*c))/a + 2*(3*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 - A
*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
- 1)^2*a))/d
```

maple [B] time = 0.16, size = 252, normalized size = 2.36

$$-\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{A}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3A}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{B}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3A}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/a/d*A*tan(1/2*d*x+1/2*c)+1/a/d*B*tan(1/2*d*x+1/2*c)+1/2/a/d*A/(tan(1/2*d
*x+1/2*c)-1)^2+3/2/a/d*A/(tan(1/2*d*x+1/2*c)-1)-1/a/d/(tan(1/2*d*x+1/2*c)-1
)*B-3/2/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)+1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B-1/2
/a/d*A/(tan(1/2*d*x+1/2*c)+1)^2+3/2/a/d*A/(tan(1/2*d*x+1/2*c)+1)-1/a/d/(tan
(1/2*d*x+1/2*c)+1)*B+3/2/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)-1/a/d*ln(tan(1/2*d*
x+1/2*c)+1)*B
```

maxima [B] time = 0.39, size = 282, normalized size = 2.64

$$-\frac{A\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+2B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\cdots}{\cdots}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(A*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) +
 1))) + 2*B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/
(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

mupad [B] time = 0.37, size = 119, normalized size = 1.11

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\,\left(3\,A-2\,B\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A-2\,B\right)}{d\,\left(a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+a\right)}+\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,A}{2}-B\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A-B\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(3*A - 2*B) - tan(c/2 + (d*x)/2)*(A - 2*B))/(d*(a - 2
*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4)) + (2*atanh(tan(c/2 + (d*
x)/2))*((3*A)/2 - B))/(a*d) - (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^3(c+dx)}{\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec^3(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)*sec(c + d*x)**3/(cos(c + d*x) + 1), x))/a
```

3.46 $\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{(4A-3B)\tan^3(c+dx)}{3ad}+\frac{(4A-3B)\tan(c+dx)}{ad}-\frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{3(A-B)\tan(c+dx)\sec(c+d$$

```
[Out] -3/2*(A-B)*arctanh(sin(d*x+c))/a/d+(4*A-3*B)*tan(d*x+c)/a/d-3/2*(A-B)*sec(d
*x+c)*tan(d*x+c)/a/d-(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))+1/3*(
4*A-3*B)*tan(d*x+c)^3/a/d
```

Rubi [A] time = 0.18, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2748, 3767, 3768, 3770}

$$\frac{(4A-3B)\tan^3(c+dx)}{3ad}+\frac{(4A-3B)\tan(c+dx)}{ad}-\frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{3(A-B)\tan(c+dx)\sec(c+d$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-3*(A - B)*ArcTanh[Sin[c + d*x]])/(2*a*d) + ((4*A - 3*B)*Tan[c + d*x])/(a*
d) - (3*(A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) - ((A - B)*Sec[c + d*x]^
2*Tan[c + d*x])/(d*(a + a*Cos[c + d*x])) + ((4*A - 3*B)*Tan[c + d*x]^3)/(3*
a*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int (a(4A-3B)-3a(A-B)\cos(c-}{a^2}$$
$$= -\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} + \frac{(4A-3B)\int \sec^4(c+dx)\,dx}{a} - (3($$
$$= -\frac{3(A-B)\sec(c+dx)\tan(c+dx)}{2ad} - \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}$$
$$= -\frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad} + \frac{(4A-3B)\tan(c+dx)}{ad} - \frac{3(A-B)\,\mathrm{se}}{}$$

Mathematica [B] time = 4.61, size = 490, normalized size = 3.74

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sec(c)\sec^3(c+dx)\left(6(A+B)\sin\left(\frac{dx}{2}\right)+3(13A-9B)\sin\left(\frac{3dx}{2}\right)-24A\sin\left(c-\frac{dx}{2}\right)-6A\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(144*(A - B)*Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sec[c
```

```
]*Sec[c + d*x]^3*(6*(A + B)*Sin[(d*x)/2] + 3*(13*A - 9*B)*Sin[(3*d*x)/2] -
24*A*Sin[c - (d*x)/2] + 12*B*Sin[c - (d*x)/2] - 6*A*Sin[c + (d*x)/2] + 6*B*
Sin[c + (d*x)/2] - 24*A*Sin[2*c + (d*x)/2] + 24*B*Sin[2*c + (d*x)/2] + 21*A
*Sin[c + (3*d*x)/2] - 9*B*Sin[c + (3*d*x)/2] + 9*A*Sin[2*c + (3*d*x)/2] - 9
*B*Sin[2*c + (3*d*x)/2] - 9*A*Sin[3*c + (3*d*x)/2] + 9*B*Sin[3*c + (3*d*x)/
2] + 7*A*Sin[c + (5*d*x)/2] - 3*B*Sin[c + (5*d*x)/2] + A*Sin[2*c + (5*d*x)/
2] + 3*B*Sin[2*c + (5*d*x)/2] - 3*A*Sin[3*c + (5*d*x)/2] + 3*B*Sin[3*c + (5
*d*x)/2] - 9*A*Sin[4*c + (5*d*x)/2] + 9*B*Sin[4*c + (5*d*x)/2] + 16*A*Sin[2
*c + (7*d*x)/2] - 12*B*Sin[2*c + (7*d*x)/2] + 10*A*Sin[3*c + (7*d*x)/2] - 6
*B*Sin[3*c + (7*d*x)/2] + 6*A*Sin[4*c + (7*d*x)/2] - 6*B*Sin[4*c + (7*d*x)/
2])))/(48*a*d*(1 + Cos[c + d*x]))
```

fricas [A] time = 0.55, size = 168, normalized size = 1.28

$$-\frac{9\left((A-B)\cos(dx+c)^4+(A-B)\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-9\left((A-B)\cos(dx+c)^4+(A-B)\mathrm{co}\cdots\right.}{12\left(ad\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/12*(9*((A - B)*cos(d*x + c)^4 + (A - B)*cos(d*x + c)^3)*log(sin(d*x + c)
 + 1) - 9*((A - B)*cos(d*x + c)^4 + (A - B)*cos(d*x + c)^3)*log(-sin(d*x +
c) + 1) - 2*(4*(4*A - 3*B)*cos(d*x + c)^3 + (7*A - 3*B)*cos(d*x + c)^2 - (A
 - 3*B)*cos(d*x + c) + 2*A)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x
 + c)^3)
```

giac [A] time = 0.40, size = 182, normalized size = 1.39

$$-\frac{\frac{9(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{9(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{6\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(15A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9B\,\mathrm{t}\cdots\right.}{\cdots}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(9*(A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*(A - B)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1))/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/
2*c))/a + 2*(15*A*tan(1/2*d*x + 1/2*c)^5 - 9*B*tan(1/2*d*x + 1/2*c)^5 - 16*
A*tan(1/2*d*x + 1/2*c)^3 + 12*B*tan(1/2*d*x + 1/2*c)^3 + 9*A*tan(1/2*d*x +
1/2*c) - 3*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

maple [B] time = 0.17, size = 340, normalized size = 2.60

$$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{A}{3ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{B}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{A}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/a/d*A*tan(1/2*d*x+1/2*c)-1/a/d*B*tan(1/2*d*x+1/2*c)-1/3/a/d*A/(tan(1/2*d*
x+1/2*c)-1)^3+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2*B-1/a/d*A/(tan(1/2*d*x+1/2*c
)-1)^2+3/2/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)-3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*
B-5/2/a/d*A/(tan(1/2*d*x+1/2*c)-1)+3/2/a/d/(tan(1/2*d*x+1/2*c)-1)*B-1/3/a/d
*A/(tan(1/2*d*x+1/2*c)+1)^3+1/a/d*A/(tan(1/2*d*x+1/2*c)+1)^2-1/2/a/d/(tan(1
/2*d*x+1/2*c)+1)^2*B-3/2/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)+3/2/a/d*ln(tan(1/2*
d*x+1/2*c)+1)*B-5/2/a/d*A/(tan(1/2*d*x+1/2*c)+1)+3/2/a/d/(tan(1/2*d*x+1/2*c
)+1)*B
```

maxima [B] time = 0.47, size = 368, normalized size = 2.81

$$\frac{A\left(\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a-\frac{3a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3B\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\right.}{a-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(A*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(
d*x + c) + 1))) - 3*B*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)/a + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(
cos(d*x + c) + 1))))/d
```

mupad [B] time = 0.64, size = 152, normalized size = 1.16

$$\frac{(5A-3B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(4B-\frac{16A}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(3A-B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)}-\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(A-}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^4*(a + a*cos(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(5*A - 3*B) - tan(c/2 + (d*x)/2)^3*((16*A)/3 - 4*B) +
 tan(c/2 + (d*x)/2)*(3*A - B))/(d*(a - 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c
/2 + (d*x)/2)^4 - a*tan(c/2 + (d*x)/2)^6)) - (3*atanh(tan(c/2 + (d*x)/2))*(
A - B))/(a*d) + (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^4(c+dx)}{\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^4(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**4/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(cos(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)*sec(c + d*x)**4/(cos(c + d*x) + 1), x))/a
```

3.47 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=170

$$\frac{4(2A-3B)\sin^3(c+dx)}{3a^2d}-\frac{4(2A-3B)\sin(c+dx)}{a^2d}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(7A-10B)\sin(c+dx)}{2a^2d}$$

```
[Out] 1/2*(7*A-10*B)*x/a^2-4*(2*A-3*B)*sin(d*x+c)/a^2/d+1/2*(7*A-10*B)*cos(d*x+c)
*sin(d*x+c)/a^2/d+1/3*(7*A-10*B)*cos(d*x+c)^3*sin(d*x+c)/a^2/d/(1+cos(d*x+c
))+1/3*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+4/3*(2*A-3*B)*sin
(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.32, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2748, 2635, 8, 2633}

$$\frac{4(2A-3B)\sin^3(c+dx)}{3a^2d}-\frac{4(2A-3B)\sin(c+dx)}{a^2d}+\frac{(7A-10B)\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{(7A-10B)\sin(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((7*A - 10*B)*x)/(2*a^2) - (4*(2*A - 3*B)*Sin[c + d*x])/(a^2*d) + ((7*A - 1
0*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + ((7*A - 10*B)*Cos[c + d*x]^3*Si
n[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^4*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (4*(2*A - 3*B)*Sin[c + d*x]^3)/(3*a^2*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^3(c+dx)(4a(A-B)-3a(A-2B)\cos(c+dx)}{a+a\cos(c+dx)}}{3a^2}$$

$$= \frac{(7A-10B)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^4(c+dx)\sin(c+d}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{(7A-10B)\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^4(c+dx)\sin(c+d}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{(7A-10B)\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{(7A-10B)\cos^3(c+dx)\sin(c+}{3a^2d(1+\cos(c+dx))}$$

$$= \frac{(7A-10B)x}{2a^2} - \frac{4(2A-3B)\sin(c+dx)}{a^2d} + \frac{(7A-10B)\cos(c+dx)\sin(}{2a^2d}$$

Mathematica [B] time = 0.70, size = 369, normalized size = 2.17

$$\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(36dx(7A-10B)\cos\left(c+\frac{dx}{2}\right)+36dx(7A-10B)\cos\left(\frac{dx}{2}\right)+147A\sin\left(c+\frac{dx}{2}\right)-239A\,s\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(7*A - 10*B)*d*x*Cos[(d*x)/2] + 36*(7*A - 10
*B)*d*x*Cos[c + (d*x)/2] + 84*A*d*x*Cos[c + (3*d*x)/2] - 120*B*d*x*Cos[c +
(3*d*x)/2] + 84*A*d*x*Cos[2*c + (3*d*x)/2] - 120*B*d*x*Cos[2*c + (3*d*x)/2]
 - 381*A*Sin[(d*x)/2] + 516*B*Sin[(d*x)/2] + 147*A*Sin[c + (d*x)/2] - 156*B
*Sin[c + (d*x)/2] - 239*A*Sin[c + (3*d*x)/2] + 342*B*Sin[c + (3*d*x)/2] - 6
3*A*Sin[2*c + (3*d*x)/2] + 118*B*Sin[2*c + (3*d*x)/2] - 15*A*Sin[2*c + (5*d
*x)/2] + 30*B*Sin[2*c + (5*d*x)/2] - 15*A*Sin[3*c + (5*d*x)/2] + 30*B*Sin[3
*c + (5*d*x)/2] + 3*A*Sin[3*c + (7*d*x)/2] - 3*B*Sin[3*c + (7*d*x)/2] + 3*A
*Sin[4*c + (7*d*x)/2] - 3*B*Sin[4*c + (7*d*x)/2] + B*Sin[4*c + (9*d*x)/2] +
 B*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 1.03, size = 154, normalized size = 0.91

$$\frac{3\,(7\,A-10\,B)dx\cos\left(dx+c\right)^{2}+6\,(7\,A-10\,B)dx\cos\left(dx+c\right)+3\,(7\,A-10\,B)dx+\left(2\,B\cos\left(dx+c\right)^{4}+(3\,A-2\,B}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(7*A - 10*B)*d*x*cos(d*x + c)^2 + 6*(7*A - 10*B)*d*x*cos(d*x + c) +
3*(7*A - 10*B)*d*x + (2*B*cos(d*x + c)^4 + (3*A - 2*B)*cos(d*x + c)^3 - 6*(
A - 2*B)*cos(d*x + c)^2 - (43*A - 66*B)*cos(d*x + c) - 32*A + 48*B)*sin(d*x
 + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.73, size = 192, normalized size = 1.13

$$\frac{\frac{3\,(dx+c)(7\,A-10\,B)}{a^{2}}-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-30\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+24\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-40\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-18\,B\tan\left(\frac{1}{2}\,dx+\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a^{2}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(d*x + c)*(7*A - 10*B)/a^2 - 2*(15*A*tan(1/2*d*x + 1/2*c)^5 - 30*B*t
an(1/2*d*x + 1/2*c)^5 + 24*A*tan(1/2*d*x + 1/2*c)^3 - 40*B*tan(1/2*d*x + 1/
2*c)^3 + 9*A*tan(1/2*d*x + 1/2*c) - 18*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*
x + 1/2*c)^2 + 1)^3*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*
x + 1/2*c)^3 - 21*A*a^4*tan(1/2*d*x + 1/2*c) + 27*B*a^4*tan(1/2*d*x + 1/2*c
))/a^6)/d
```

maple [B] time = 0.10, size = 322, normalized size = 1.89

$$\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{6d\,a^2}-\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d\,a^2}-\frac{7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}+\frac{9B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{5\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{10\left(\tan\right.}{d\,a^2\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3-7/2/d/a^2
*A*tan(1/2*d*x+1/2*c)+9/2/d/a^2*B*tan(1/2*d*x+1/2*c)-5/d/a^2/(1+tan(1/2*d*x
+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+10/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan
(1/2*d*x+1/2*c)^5*B-8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3
*A+40/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1/2*d*x+1/2*c)^3-3/d/a^2/(1+
tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)+6/d/a^2/(1+tan(1/2*d*x+1/2*c)^
2)^3*B*tan(1/2*d*x+1/2*c)+7/d/a^2*arctan(tan(1/2*d*x+1/2*c))*A-10/d/a^2*arc
tan(tan(1/2*d*x+1/2*c))*B
```

maxima [B] time = 0.71, size = 372, normalized size = 2.19

$$\frac{B\left(\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-A\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}\cdots\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(B*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^2) - A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x +
c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a^2))/d
```

mupad [B] time = 0.33, size = 189, normalized size = 1.11

$$\frac{x\,(7\,A-10\,B)}{2\,a^2}-\frac{(5\,A-10\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(8\,A-\frac{40\,B}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(3\,A-6\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] (x*(7*A - 10*B))/(2*a^2) - (tan(c/2 + (d*x)/2)^5*(5*A - 10*B) + tan(c/2 + (
d*x)/2)^3*(8*A - (40*B)/3) + tan(c/2 + (d*x)/2)*(3*A - 6*B))/(d*(3*a^2*tan(
c/2 + (d*x)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 +
a^2)) - (tan(c/2 + (d*x)/2)*((2*(A - B))/a^2 + (3*A - 5*B)/(2*a^2)))/d + (t
an(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d)
```

sympy [A] time = 10.81, size = 1425, normalized size = 8.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((21*A*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*
a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 63
*A*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/
2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 63*A*d*x*tan(c/
2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4
 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 21*A*d*x/(6*a**2*d*tan(c/2 +
 d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2
+ 6*a**2*d) + A*tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2
*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 18*A*t
an(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/
2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*A*tan(c/2 + d*x/2)**
5/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d
*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 110*A*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(
c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2
)**2 + 6*a**2*d) - 39*A*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18
*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 3
0*B*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c
/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*B*d*x*tan(c
/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**
4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*B*d*x*tan(c/2 + d*x/2)**
2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d
```

```
*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 30*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**6 +
 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d)
- B*tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 +
 d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 24*B*tan(c/2 + d*x
/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a
**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 138*B*tan(c/2 + d*x/2)**5/(6*a**2*d
*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 +
d*x/2)**2 + 6*a**2*d) + 160*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2
)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a*
*2*d) + 63*B*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan
(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x
*(A + B*cos(c))*cos(c)**4/(a*cos(c) + a)**2, True))
```

3.48 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=147

$$\frac{2(5A-8B)\sin(c+dx)}{3a^2d}+\frac{(5A-8B)\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{(4A-7B)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{x(4A-7B)}{2a^2}+\frac{(A$$

```
[Out] -1/2*(4*A-7*B)*x/a^2+2/3*(5*A-8*B)*sin(d*x+c)/a^2/d-1/2*(4*A-7*B)*cos(d*x+c
)*sin(d*x+c)/a^2/d+1/3*(5*A-8*B)*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(1+cos(d*x+c
))+1/3*(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Rubi [A] time = 0.34, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2977, 2734}

$$\frac{2(5A-8B)\sin(c+dx)}{3a^2d}+\frac{(5A-8B)\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{(4A-7B)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{x(4A-7B)}{2a^2}+\frac{(A$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -((4*A - 7*B)*x)/(2*a^2) + (2*(5*A - 8*B)*Sin[c + d*x])/(3*a^2*d) - ((4*A -
 7*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + ((5*A - 8*B)*Cos[c + d*x]^2*Si
n[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^3*Sin[c +
d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)(3a(A-B)-a(2A-5B)\cos(c+dx)}{a+a\cos(c+dx)}}{3a^2}$$

$$= \frac{(5A-8B)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{(4A-7B)x}{2a^2} + \frac{2(5A-8B)\sin(c+dx)}{3a^2d} - \frac{(4A-7B)\cos(c+dx)\sin(c}{2a^2d}$$

Mathematica [B] time = 0.84, size = 315, normalized size = 2.14

$$\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-36dx(4A-7B)\cos\left(c+\frac{dx}{2}\right)-36dx(4A-7B)\cos\left(\frac{dx}{2}\right)-120A\sin\left(c+\frac{dx}{2}\right)+164A\,\mathrm{si}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-36*(4*A - 7*B)*d*x*Cos[(d*x)/2] - 36*(4*A - 7*
B)*d*x*Cos[c + (d*x)/2] - 48*A*d*x*Cos[c + (3*d*x)/2] + 84*B*d*x*Cos[c + (3
*d*x)/2] - 48*A*d*x*Cos[2*c + (3*d*x)/2] + 84*B*d*x*Cos[2*c + (3*d*x)/2] +
264*A*Sin[(d*x)/2] - 381*B*Sin[(d*x)/2] - 120*A*Sin[c + (d*x)/2] + 147*B*Si
n[c + (d*x)/2] + 164*A*Sin[c + (3*d*x)/2] - 239*B*Sin[c + (3*d*x)/2] + 36*A
*Sin[2*c + (3*d*x)/2] - 63*B*Sin[2*c + (3*d*x)/2] + 12*A*Sin[2*c + (5*d*x)/
2] - 15*B*Sin[2*c + (5*d*x)/2] + 12*A*Sin[3*c + (5*d*x)/2] - 15*B*Sin[3*c +
 (5*d*x)/2] + 3*B*Sin[3*c + (7*d*x)/2] + 3*B*Sin[4*c + (7*d*x)/2]))/(48*a^2
*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 0.79, size = 138, normalized size = 0.94

$$-\frac{3\,(4\,A-7\,B)dx\cos\,(dx+c)^2+6\,(4\,A-7\,B)dx\cos\,(dx+c)+3\,(4\,A-7\,B)dx-\left(3\,B\cos\,(dx+c)^3+6\,(A-B)\right.}{6\left(a^2d\cos\,(dx+c)^2+2\,a^2d\cos\,(dx+c)+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*(4*A - 7*B)*d*x*cos(d*x + c)^2 + 6*(4*A - 7*B)*d*x*cos(d*x + c) + 3
*(4*A - 7*B)*d*x - (3*B*cos(d*x + c)^3 + 6*(A - B)*cos(d*x + c)^2 + (28*A -
 43*B)*cos(d*x + c) + 20*A - 32*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*
a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.54, size = 164, normalized size = 1.12

$$-\frac{\frac{3\,(dx+c)(4\,A-7\,B)}{a^2}-\frac{6\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(d*x + c)*(4*A - 7*B)/a^2 - 6*(2*A*tan(1/2*d*x + 1/2*c)^3 - 5*B*tan
(1/2*d*x + 1/2*c)^3 + 2*A*tan(1/2*d*x + 1/2*c) - 3*B*tan(1/2*d*x + 1/2*c))/
((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^
4*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^4*tan(1/2*d*x + 1/2*c) + 21*B*a^4*tan(1/2
*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.10, size = 252, normalized size = 1.71

$$-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{6d\,a^2}+\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d\,a^2}+\frac{5A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{7B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{5B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2\left(\tan\right.}{d\,a^2\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3+5/2/d/a^
2*A*tan(1/2*d*x+1/2*c)-7/2/d/a^2*B*tan(1/2*d*x+1/2*c)-5/d/a^2/(1+tan(1/2*d*
x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)^3+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan
(1/2*d*x+1/2*c)^3*A-3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)
+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)-4/d/a^2*arctan(tan
(1/2*d*x+1/2*c))*A+7/d/a^2*arctan(tan(1/2*d*x+1/2*c))*B
```

maxima [B] time = 0.77, size = 283, normalized size = 1.93

$$-\frac{B\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\right.}{a^2}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2) - A*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*si
n(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) +
 1))))/d
```

mupad [B] time = 0.29, size = 152, normalized size = 1.03

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3(A-B)}{2a^2}+\frac{2A-4B}{2a^2}\right)}{d}-\frac{x(4A-7B)}{2a^2}+\frac{(2A-5B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(2A-3B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^2\right)}-\frac{\tan\left(\frac{c}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((3*(A - B))/(2*a^2) + (2*A - 4*B)/(2*a^2)))/d - (x*(4*
A - 7*B))/(2*a^2) + (tan(c/2 + (d*x)/2)^3*(2*A - 5*B) + tan(c/2 + (d*x)/2)*
(2*A - 3*B))/(d*(2*a^2*tan(c/2 + (d*x)/2)^2 + a^2*tan(c/2 + (d*x)/2)^4 + a^
2)) - (tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d)
```

sympy [A] time = 6.98, size = 843, normalized size = 5.73

$$\begin{cases} -\frac{12Adx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}-\frac{24Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}-\frac{12Adx}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} \\ \frac{x(A+B\cos(c))\cos^3(c)}{(a\cos(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-12*A*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12
*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 24*A*d*x*tan(c/2 + d*x/2)**2/(6*a
**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*
A*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**
```

```
2*d) - A*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(
c/2 + d*x/2)**2 + 6*a**2*d) + 13*A*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 +
d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 41*A*tan(c/2 + d*x/
2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**
2*d) + 27*A*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(
c/2 + d*x/2)**2 + 6*a**2*d) + 21*B*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/
2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 42*B*d*x*tan(c/
2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2
 + 6*a**2*d) + 21*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 6*a**2*d) + B*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**
4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 19*B*tan(c/2 + d*x/2)**5/(6
*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 7
1*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 6*a**2*d) - 39*B*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**
4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*(A + B*cos(c))
*cos(c)**3/(a*cos(c) + a)**2, True))
```

3.49 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=99

$$-\frac{(A-4B)\sin(c+dx)}{3a^2d}-\frac{(A-2B)\sin(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{x(A-2B)}{a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (A-2*B)*x/a^2-1/3*(A-4*B)*sin(d*x+c)/a^2/d-(A-2*B)*sin(d*x+c)/a^2/d/(1+cos(
d*x+c))+1/3*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Rubi [A] time = 0.28, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2977, 2968, 3023, 12, 2735, 2648}

$$-\frac{(A-4B)\sin(c+dx)}{3a^2d}-\frac{(A-2B)\sin(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{x(A-2B)}{a^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((A - 2*B)*x)/a^2 - ((A - 4*B)*Sin[c + d*x])/(3*a^2*d) - ((A - 2*B)*Sin[c +
 d*x])/(a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(
3*d*(a + a*Cos[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos(c+dx)(2a(A-B)-a(A-4B)\cos(c+dx))}{a+a\cos(c+dx)}}{3a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{2a(A-B)\cos(c+dx)-a(A-4B)\cos^2(c+dx)}{a+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{(A-4B)\sin(c+dx)}{3a^2d} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{3a^2(A-2}{a+a\,c}}{}$$

$$= -\frac{(A-4B)\sin(c+dx)}{3a^2d} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(A-2B)}{}$$

$$= \frac{(A-2B)x}{a^2} - \frac{(A-4B)\sin(c+dx)}{3a^2d} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{(A-2B)x}{a^2} - \frac{(A-4B)\sin(c+dx)}{3a^2d} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

Mathematica [A] time = 0.77, size = 137, normalized size = 1.38

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(6\cos^3\left(\frac{1}{2}(c+dx)\right)(dx(A-2B)+B\sin(c+dx))+(A-B)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(A-B)\right.}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] - 2*(5*A - 8*B)*Cos[(c +
 d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*((A - 2*B)*d*x + B*
Sin[c + d*x]) + (A - B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d
*x])^2)
```

fricas [A] time = 0.75, size = 117, normalized size = 1.18

$$\frac{3(A-2B)dx\cos(dx+c)^2 + 6(A-2B)dx\cos(dx+c) + 3(A-2B)dx + \left(3B\cos(dx+c)^2 - (5A-14B)\cos\right.}{3\left(a^2d\cos(dx+c)^2 + 2a^2d\cos(dx+c) + a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/3*(3*(A - 2*B)*d*x*cos(d*x + c)^2 + 6*(A - 2*B)*d*x*cos(d*x + c) + 3*(A -
 2*B)*d*x + (3*B*cos(d*x + c)^2 - (5*A - 14*B)*cos(d*x + c) - 4*A + 10*B)*s
in(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 1.60, size = 119, normalized size = 1.20

$$\frac{\frac{6\,(dx+c)(A-2\,B)}{a^2}+\frac{12\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(d*x + c)*(A - 2*B)/a^2 + 12*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x +
1/2*c)^2 + 1)*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/
2*c)^3 - 9*A*a^4*tan(1/2*d*x + 1/2*c) + 15*B*a^4*tan(1/2*d*x + 1/2*c))/a^6)
/d
```

maple [A] time = 0.10, size = 149, normalized size = 1.51

$$\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{6d\,a^2}-\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d\,a^2}-\frac{3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}+\frac{5B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}+\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2\arctan\left(\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2
*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*B*tan(1/2*d*x+1/2*c)+2/d/a^2*B*tan(1/2*d*x+
1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))*A-4/d/a^
2*arctan(tan(1/2*d*x+1/2*c))*B
```

maxima [B] time = 0.76, size = 191, normalized size = 1.93

$$\frac{B\left(\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\,\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-A\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\,\arctan\left(\frac{\mathrm{si}}{\mathrm{cos}}\right.}{a^2}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(B*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x +
 c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) -
 A*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2))/d
```

mupad [B] time = 0.26, size = 105, normalized size = 1.06

$$\frac{x\,(A-2\,B)}{a^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{a^2}+\frac{A-3\,B}{2\,a^2}\right)}{d}+\frac{2\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A-B)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] (x*(A - 2*B))/a^2 - (tan(c/2 + (d*x)/2)*((A - B)/a^2 + (A - 3*B)/(2*a^2)))/
d + (2*B*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 + a^2)) + (tan(c/
2 + (d*x)/2)^3*(A - B))/(6*a^2*d)
```

sympy [A] time = 4.16, size = 411, normalized size = 4.15

$$\begin{cases} \frac{6Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}+\frac{6Adx}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}+\frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}-\frac{8A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}-\frac{9A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d}-\frac{1}{6a^2} \\ \frac{x(A+B\cos(c))\cos^2(c)}{(a\cos(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((6*A*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a*
*2*d) + 6*A*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + A*tan(c/2 + d*x
/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 8*A*tan(c/2 + d*x/2)**3/
(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 9*A*tan(c/2 + d*x/2)/(6*a**2*d*
tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*B*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*ta
n(c/2 + d*x/2)**2 + 6*a**2*d) - 12*B*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*
a**2*d) - B*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) +
 14*B*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 27*B*
tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*(
A + B*cos(c))*cos(c)**2/(a*cos(c) + a)**2, True))
```

3.50 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$\frac{(2A-5B)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{Bx}{a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] B*x/a^2+1/3*(2*A-5*B)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*sin(d*x+c)/
d/(a+a*cos(d*x+c))^2
```

Rubi [A] time = 0.16, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3019, 2735, 2648}

$$\frac{(2A-5B)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{Bx}{a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (B*x)/a^2 + ((2*A - 5*B)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A -
 B)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx &= \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx \\
&= -\frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{-2a(A-B)-3aB\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2} \\
&= \frac{Bx}{a^2} - \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(2A-5B)\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a} \\
&= \frac{Bx}{a^2} - \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(2A-5B)\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}
\end{aligned}
$$

Mathematica [B] time = 0.37, size = 153, normalized size = 2.19

$$
\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-6A\sin\left(c+\frac{dx}{2}\right)+4A\sin\left(c+\frac{3dx}{2}\right)+6A\sin\left(\frac{dx}{2}\right)+12B\sin\left(c+\frac{dx}{2}\right)-10B\sin\left(c+\right.\right.}{24a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*B*d*x*Cos[(d*x)/2] + 9*B*d*x*Cos[c + (d*x)/
2] + 3*B*d*x*Cos[c + (3*d*x)/2] + 3*B*d*x*Cos[2*c + (3*d*x)/2] + 6*A*Sin[(d
*x)/2] - 18*B*Sin[(d*x)/2] - 6*A*Sin[c + (d*x)/2] + 12*B*Sin[c + (d*x)/2] +
 4*A*Sin[c + (3*d*x)/2] - 10*B*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

fricas [A] time = 0.81, size = 91, normalized size = 1.30

$$
\frac{3\,Bdx\cos\left(dx+c\right)^2+6\,Bdx\cos\left(dx+c\right)+3\,Bdx+\left((2\,A-5\,B)\cos\left(dx+c\right)+A-4\,B\right)\sin\left(dx+c\right)}{3\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*(3*B*d*x*cos(d*x + c)^2 + 6*B*d*x*cos(d*x + c) + 3*B*d*x + ((2*A - 5*B)
*cos(d*x + c) + A - 4*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

giac [A] time = 0.68, size = 86, normalized size = 1.23

$$\frac{\frac{6\,(dx+c)B}{a^2} - \frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 3\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 9\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(6*(d*x + c)*B/a^2 - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x
+ 1/2*c)^3 - 3*A*a^4*tan(1/2*d*x + 1/2*c) + 9*B*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

maple [A] time = 0.08, size = 97, normalized size = 1.39

$$-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{6d\,a^2}+\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d\,a^2}+\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{3B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3+1/2/d/a^
2*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*B*tan(1/2*d*x+1/2*c)+2/d/a^2*arctan(tan(1/
2*d*x+1/2*c))*B
```

maxima [A] time = 0.63, size = 120, normalized size = 1.71

$$-\frac{B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(B*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) - A*(3*sin(d*
x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

mupad [B] time = 0.22, size = 65, normalized size = 0.93

$$\frac{3\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\,B\,d\,x}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] (3*A*tan(c/2 + (d*x)/2) - 9*B*tan(c/2 + (d*x)/2) - A*tan(c/2 + (d*x)/2)^3 +
 B*tan(c/2 + (d*x)/2)^3 + 6*B*d*x)/(6*a^2*d)
```

sympy [A] time = 2.33, size = 105, normalized size = 1.50

$$\begin{cases} -\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}+\frac{Bx}{a^2}+\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}-\frac{3B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0 \\ \frac{x(A+B\cos(c))\cos(c)}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**3/(6*a**2*d) + A*tan(c/2 + d*x/2)/(2*a**2*d
) + B*x/a**2 + B*tan(c/2 + d*x/2)**3/(6*a**2*d) - 3*B*tan(c/2 + d*x/2)/(2*a
**2*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)/(a*cos(c) + a)**2, True))
```

3.51 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{(A+2B)\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}+\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] 1/3*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+1/3*(A+2*B)*sin(d*x+c)/d/(a^2+a^2
*cos(d*x+c))
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2750, 2648}

$$\frac{(A+2B)\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}+\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((A - B)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((A + 2*B)*Sin[c + d*
x])/(3*d*(a^2 + a^2*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(A+2B)\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}$$
$$= \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(A+2B)\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}$$

Mathematica [A] time = 0.19, size = 76, normalized size = 1.17

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left((A+2B)\sin\left(c+\frac{3dx}{2}\right)+3(A+B)\sin\left(\frac{dx}{2}\right)-3B\sin\left(c+\frac{dx}{2}\right)\right)}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(3*(A + B)*Sin[(d*x)/2] - 3*B*Sin[c + (d*x)/2] +
 (A + 2*B)*Sin[c + (3*d*x)/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 0.77, size = 58, normalized size = 0.89

$$\frac{((A+2B)\cos(dx+c)+2A+B)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((A + 2*B)*cos(d*x + c) + 2*A + B)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 +
 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 1.93, size = 60, normalized size = 0.92

$$\frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{6a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(A*tan(1/2*d*x + 1/2*c)^3 - B*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x
+ 1/2*c) + 3*B*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

maple [A] time = 0.07, size = 60, normalized size = 0.92

$$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}-\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3*A-1/3*B*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*
d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.37, size = 93, normalized size = 1.43

$$\frac{\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}+\frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(A*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 + B*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/a^2)/d
```

mupad [B] time = 0.19, size = 45, normalized size = 0.69

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(A-B\right)}{6\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A+B\right)}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d) + (tan(c/2 + (d*x)/2)*(A + B))/(2*
a^2*d)
```

sympy [A] time = 1.74, size = 94, normalized size = 1.45

$$\begin{cases} \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}-\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0 \\ \frac{x(A+B\cos(c))}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**3/(6*a**2*d) + A*tan(c/2 + d*x/2)/(2*a**2*d)
 - B*tan(c/2 + d*x/2)**3/(6*a**2*d) + B*tan(c/2 + d*x/2)/(2*a**2*d), Ne(d,
0)), (x*(A + B*cos(c))/(a*cos(c) + a)**2, True))
```

3.52 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{(4A-B)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] A*arctanh(sin(d*x+c))/a^2/d-1/3*(4*A-B)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3
*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Rubi [A] time = 0.18, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2978, 12, 3770}

$$-\frac{(4A-B)\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^2*d) - ((4*A - B)*Sin[c + d*x])/(3*a^2*d*(1 +
Cos[c + d*x])) - ((A - B)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(3aA-a(A-B)\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= -\frac{(4A-B)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int 3a^2A\sec(c+dx}{3a^4}$$
$$= -\frac{(4A-B)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{A\int \sec(c+dx)\,dx}{a^2}$$
$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(4A-B)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d(a+a\cos(c+dx)}$$

Mathematica [B] time = 0.54, size = 170, normalized size = 2.15

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+2(4A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\cos}{3a^2d(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(6*A*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (A - B)*Sec[c/2]*
Sin[(d*x)/2] + 2*(4*A - B)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + (A -
B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 0.80, size = 131, normalized size = 1.66

$$\frac{3\left(A\cos(dx+c)^2+2A\cos(dx+c)+A\right)\log(\sin(dx+c)+1)-3\left(A\cos(dx+c)^2+2A\cos(dx+c)+A\right)\log}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - 3*
(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*log(-sin(d*x + c) + 1) - 2*((4*A
- B)*cos(d*x + c) + 5*A - 2*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.44, size = 113, normalized size = 1.43

$$\frac{\frac{6\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{6\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}-\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(6*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*A*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2
*c)^3 + 9*A*a^4*tan(1/2*d*x + 1/2*c) - 3*B*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

maple [A] time = 0.14, size = 119, normalized size = 1.51

$$-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{6d\,a^2}+\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d\,a^2}-\frac{3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}+\frac{B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3-3/2/d/a^
2*A*tan(1/2*d*x+1/2*c)+1/2/d/a^2*B*tan(1/2*d*x+1/2*c)-1/d/a^2*A*ln(tan(1/2*
d*x+1/2*c)-1)+1/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.49, size = 145, normalized size = 1.84

$$-\frac{A\left(\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-\frac{B\left(\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*(3*sin(d*x + c)/(cos(d*x + c) + 1)
+ sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

mupad [B] time = 0.23, size = 74, normalized size = 0.94

$$\frac{2\,A\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(A-B\right)}{6\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A}{a^2}+\frac{A-B}{2\,a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] (2*A*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (tan(c/2 + (d*x)/2)^3*(A - B))/(6
*a^2*d) - (tan(c/2 + (d*x)/2)*(A/a^2 + (A - B)/(2*a^2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec\left(c+dx\right)}{\cos^2\left(c+dx\right)+2\cos\left(c+dx\right)+1}\,dx+\int \frac{B\cos\left(c+dx\right)\sec\left(c+dx\right)}{\cos^2\left(c+dx\right)+2\cos\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + Integ
ral(B*cos(c + d*x)*sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x))
/a**2
```

3.53 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=107

$$\frac{2(5A-2B)\tan(c+dx)}{3a^2d}-\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(2A-B)\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -(2*A-B)*arctanh(sin(d*x+c))/a^2/d+2/3*(5*A-2*B)*tan(d*x+c)/a^2/d-(2*A-B)*t
an(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Rubi [A] time = 0.30, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2748, 3767, 8, 3770}

$$\frac{2(5A-2B)\tan(c+dx)}{3a^2d}-\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(2A-B)\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{(A-B)\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -(((2*A - B)*ArcTanh[Sin[c + d*x]])/(a^2*d)) + (2*(5*A - 2*B)*Tan[c + d*x])
/(3*a^2*d) - ((2*A - B)*Tan[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - ((A - B)
*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(a(4A-B)-2a(A-B)\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= -\frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \left(2a^2(5A-2B) - \right.}{}$$
$$= -\frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(2(5A-2B))\int \sec}{3a^2}$$
$$= -\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(2A-B)\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\tan}{3d(a+a\cos}$$
$$= -\frac{(2A-B)\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{2(5A-2B)\tan(c+dx)}{3a^2d} - \frac{(2A-B)}{a^2d(1+\cos}$$

Mathematica [B] time = 1.85, size = 264, normalized size = 2.47

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+(A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left((2A-B)\left(\log\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] + 2*(7*A - 4*B)*Cos[(c +
 d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*((2*A - B)*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
```

```
]) + (A*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (A - B)
*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [B] time = 0.75, size = 207, normalized size = 1.93

$$-\frac{3\left((2A-B)\cos(dx+c)^3+2(2A-B)\cos(dx+c)^2+(2A-B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-3\left((2A-\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*((2*A - B)*cos(d*x + c)^3 + 2*(2*A - B)*cos(d*x + c)^2 + (2*A - B)*
cos(d*x + c))*log(sin(d*x + c) + 1) - 3*((2*A - B)*cos(d*x + c)^3 + 2*(2*A
- B)*cos(d*x + c)^2 + (2*A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2
*(5*A - 2*B)*cos(d*x + c)^2 + (14*A - 5*B)*cos(d*x + c) + 3*A)*sin(d*x + c)
)/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

giac [A] time = 0.45, size = 155, normalized size = 1.45

$$-\frac{\frac{6(2A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{6(2A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{12A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}-\frac{Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+}{a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(6*(2*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*(2*A - B)*log(
abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 12*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*
x + 1/2*c)^2 - 1)*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x
+ 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x + 1/2*c) - 9*B*a^4*tan(1/2*d*x + 1/2*c))/
a^6)/d
```

maple [A] time = 0.15, size = 205, normalized size = 1.92

$$\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{6d\,a^2}-\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d\,a^2}+\frac{5A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{3B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}+\frac{2A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^2}-\frac{\ln\left(\tan\left(\frac{d}{2}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2
*A*tan(1/2*d*x+1/2*c)-3/2/d/a^2*B*tan(1/2*d*x+1/2*c)+2/d/a^2*A*ln(tan(1/2*d
*x+1/2*c)-1)-1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^2*A/(tan(1/2*d*x+1/2*
c)-1)-2/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B
-1/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.53, size = 244, normalized size = 2.28

$$\frac{A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{s}{(co}}{a^2}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(A*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) - B*((9*sin(d*x + c)/(cos
(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/a^2))/d
```

mupad [B] time = 0.28, size = 123, normalized size = 1.15

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{a^2}+\frac{3\,A-B}{2\,a^2}\right)}{d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(A-B\right)}{6\,a^2\,d}-\frac{2\,A\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-a^2\right)}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,A\right.}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((A - B)/a^2 + (3*A - B)/(2*a^2)))/d + (tan(c/2 + (d*x)
/2)^3*(A - B))/(6*a^2*d) - (2*A*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)
/2)^2 - a^2)) - (2*atanh(tan(c/2 + (d*x)/2))*(2*A - B))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^2(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^2(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + In
tegral(B*cos(c + d*x)*sec(c + d*x)**2/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1
), x))/a**2
```

3.54 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$-\frac{2(8A-5B)\tan(c+dx)}{3a^2d}+\frac{(7A-4B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7A-4B)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{(8A-5B)\tan(c}{3a^2d(\cos(}$$

```
[Out] 1/2*(7*A-4*B)*arctanh(sin(d*x+c))/a^2/d-2/3*(8*A-5*B)*tan(d*x+c)/a^2/d+1/2*
(7*A-4*B)*sec(d*x+c)*tan(d*x+c)/a^2/d-1/3*(8*A-5*B)*sec(d*x+c)*tan(d*x+c)/a
^2/d/(1+cos(d*x+c))-1/3*(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^2
```

Rubi [A] time = 0.31, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{2(8A-5B)\tan(c+dx)}{3a^2d}+\frac{(7A-4B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7A-4B)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{(8A-5B)\tan(c}{3a^2d(\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((7*A - 4*B)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (2*(8*A - 5*B)*Tan[c + d*x]
)/(3*a^2*d) + ((7*A - 4*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - ((8*A - 5
*B)*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B)*Sec[
c + d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(a(5A-2B)-3a(A-B)\cos(c+dx))\sec^3(c+dx}{a+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{(8A-5B)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{(8A-5B)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{(7A-4B)\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{(8A-5B)\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}$$

$$= \frac{(7A-4B)\tanh^{-1}(\sin(c+dx))}{2a^2d} - \frac{2(8A-5B)\tan(c+dx)}{3a^2d} + \frac{(7A-4B)}{}$$

Mathematica [B] time = 3.43, size = 496, normalized size = 3.26

$$-\frac{96(7A-4B)\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -1/48*(96*(7*A - 4*B)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[
c/2]*Sec[c]*Sec[c + d*x]^2*(-14*(A - B)*Sin[(d*x)/2] + (97*A - 64*B)*Sin[(3
*d*x)/2] - 126*A*Sin[c - (d*x)/2] + 84*B*Sin[c - (d*x)/2] + 42*A*Sin[c + (d
*x)/2] - 42*B*Sin[c + (d*x)/2] - 98*A*Sin[2*c + (d*x)/2] + 56*B*Sin[2*c + (
d*x)/2] - 3*A*Sin[c + (3*d*x)/2] + 6*B*Sin[c + (3*d*x)/2] + 37*A*Sin[2*c +
(3*d*x)/2] - 34*B*Sin[2*c + (3*d*x)/2] - 63*A*Sin[3*c + (3*d*x)/2] + 36*B*S
in[3*c + (3*d*x)/2] + 75*A*Sin[c + (5*d*x)/2] - 48*B*Sin[c + (5*d*x)/2] + 1
5*A*Sin[2*c + (5*d*x)/2] - 6*B*Sin[2*c + (5*d*x)/2] + 39*A*Sin[3*c + (5*d*x
)/2] - 30*B*Sin[3*c + (5*d*x)/2] - 21*A*Sin[4*c + (5*d*x)/2] + 12*B*Sin[4*c
 + (5*d*x)/2] + 32*A*Sin[2*c + (7*d*x)/2] - 20*B*Sin[2*c + (7*d*x)/2] + 12*
A*Sin[3*c + (7*d*x)/2] - 6*B*Sin[3*c + (7*d*x)/2] + 20*A*Sin[4*c + (7*d*x)/
2] - 14*B*Sin[4*c + (7*d*x)/2]))/(a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 0.87, size = 228, normalized size = 1.50

$$\frac{3\left((7A-4B)\cos(dx+c)^4+2(7A-4B)\cos(dx+c)^3+(7A-4B)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-3\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*((7*A - 4*B)*cos(d*x + c)^4 + 2*(7*A - 4*B)*cos(d*x + c)^3 + (7*A -
 4*B)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((7*A - 4*B)*cos(d*x + c)^4
 + 2*(7*A - 4*B)*cos(d*x + c)^3 + (7*A - 4*B)*cos(d*x + c)^2)*log(-sin(d*x
+ c) + 1) - 2*(4*(8*A - 5*B)*cos(d*x + c)^3 + (43*A - 28*B)*cos(d*x + c)^2
+ 6*(A - B)*cos(d*x + c) - 3*A)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2
*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

giac [A] time = 1.29, size = 198, normalized size = 1.30

$$\frac{\frac{3(7A-4B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{3(7A-4B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{6\left(5A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2B\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(7*A - 4*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(7*A - 4*B)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 6*(5*A*tan(1/2*d*x + 1/2*c)^3 - 2*B
*tan(1/2*d*x + 1/2*c)^3 - 3*A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 -
B*a^4*tan(1/2*d*x + 1/2*c)^3 + 21*A*a^4*tan(1/2*d*x + 1/2*c) - 15*B*a^4*tan
(1/2*d*x + 1/2*c))/a^6)/d
```

maple [B] time = 0.18, size = 294, normalized size = 1.93

$$-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{6d\,a^2}+\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d\,a^2}-\frac{7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}+\frac{5B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{7A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d\,a^2}+\frac{2\ln\left(\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3-7/2/d/a^
2*A*tan(1/2*d*x+1/2*c)+5/2/d/a^2*B*tan(1/2*d*x+1/2*c)-7/2/d/a^2*A*ln(tan(1/
2*d*x+1/2*c)-1)+2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*B+5/2/d/a^2*A/(tan(1/2*d*x
+1/2*c)-1)-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B+1/2/d/a^2*A/(tan(1/2*d*x+1/2*c)
-1)^2+5/2/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*B+7
/2/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)-2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B-1/2/
d/a^2*A/(tan(1/2*d*x+1/2*c)+1)^2
```

maxima [B] time = 0.50, size = 336, normalized size = 2.21

$$-\frac{A\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin}{(\cos}}{a^2}\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*((15*sin(d
*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*
log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1))))/d
```

mupad [B] time = 0.30, size = 165, normalized size = 1.09

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(5\,A-2\,B\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,A-2\,B\right)}{d\,\left(a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,\left(A-B\right)}{2\,a^2}+\frac{4\,A-2\,B}{2\,a^2}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(A-B\right)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(5*A - 2*B) - tan(c/2 + (d*x)/2)*(3*A - 2*B))/(d*(a^2
*tan(c/2 + (d*x)/2)^4 - 2*a^2*tan(c/2 + (d*x)/2)^2 + a^2)) - (tan(c/2 + (d*
x)/2)*((3*(A - B))/(2*a^2) + (4*A - 2*B)/(2*a^2)))/d - (tan(c/2 + (d*x)/2)^
3*(A - B))/(6*a^2*d) + (atanh(tan(c/2 + (d*x)/2))*(7*A - 4*B))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^3(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^3(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + In
tegral(B*cos(c + d*x)*sec(c + d*x)**3/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1
), x))/a**2
```

3.55 $\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$\frac{4(3A-2B)\tan^3(c+dx)}{3a^2d}+\frac{4(3A-2B)\tan(c+dx)}{a^2d}-\frac{(10A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{(10A-7B)\tan(c+dx)\sec}{2a^2d}$$

```
[Out] -1/2*(10*A-7*B)*arctanh(sin(d*x+c))/a^2/d+4*(3*A-2*B)*tan(d*x+c)/a^2/d-1/2*
(10*A-7*B)*sec(d*x+c)*tan(d*x+c)/a^2/d-1/3*(10*A-7*B)*sec(d*x+c)^2*tan(d*x+
c)/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c)
)^2+4/3*(3*A-2*B)*tan(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.36, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2748, 3767, 3768, 3770}

$$\frac{4(3A-2B)\tan^3(c+dx)}{3a^2d}+\frac{4(3A-2B)\tan(c+dx)}{a^2d}-\frac{(10A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{(10A-7B)\tan(c+dx)\sec}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -((10*A - 7*B)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + (4*(3*A - 2*B)*Tan[c + d*
x])/(a^2*d) - ((10*A - 7*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) - ((10*A -
 7*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - ((A - B)*
Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + (4*(3*A - 2*B)*
Tan[c + d*x]^3)/(3*a^2*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{(3a(2A-B)-4a(A-B)\cos(c+dx))\sec^4(c}{a+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{(10A-7B)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\sec^2(c+dx)\tan(c+}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{(10A-7B)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\sec^2(c+dx)\tan(c+}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{(10A-7B)\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{(10A-7B)\sec^2(c+dx)\tan(c}{3a^2d(1+\cos(c+dx))}$$

$$= -\frac{(10A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{4(3A-2B)\tan(c+dx)}{a^2d}-\frac{(10A-}{}$$

Mathematica [B] time = 5.60, size = 609, normalized size = 3.40

$$\frac{192(10A-7B)\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (192*(10*A - 7*B)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]
*Sec[c]*Sec[c + d*x]^3*((-6*A + 45*B)*Sin[(d*x)/2] + (310*A - 201*B)*Sin[(3
*d*x)/2] - 306*A*Sin[c - (d*x)/2] + 195*B*Sin[c - (d*x)/2] + 42*A*Sin[c + (
d*x)/2] - 51*B*Sin[c + (d*x)/2] - 270*A*Sin[2*c + (d*x)/2] + 189*B*Sin[2*c
+ (d*x)/2] + 50*A*Sin[c + (3*d*x)/2] - B*Sin[c + (3*d*x)/2] + 90*A*Sin[2*c
+ (3*d*x)/2] - 81*B*Sin[2*c + (3*d*x)/2] - 170*A*Sin[3*c + (3*d*x)/2] + 119
*B*Sin[3*c + (3*d*x)/2] + 198*A*Sin[c + (5*d*x)/2] - 129*B*Sin[c + (5*d*x)/
2] + 42*A*Sin[2*c + (5*d*x)/2] - 9*B*Sin[2*c + (5*d*x)/2] + 66*A*Sin[3*c +
(5*d*x)/2] - 57*B*Sin[3*c + (5*d*x)/2] - 90*A*Sin[4*c + (5*d*x)/2] + 63*B*S
in[4*c + (5*d*x)/2] + 114*A*Sin[2*c + (7*d*x)/2] - 75*B*Sin[2*c + (7*d*x)/2
] + 36*A*Sin[3*c + (7*d*x)/2] - 15*B*Sin[3*c + (7*d*x)/2] + 48*A*Sin[4*c +
(7*d*x)/2] - 39*B*Sin[4*c + (7*d*x)/2] - 30*A*Sin[5*c + (7*d*x)/2] + 21*B*S
in[5*c + (7*d*x)/2] + 48*A*Sin[3*c + (9*d*x)/2] - 32*B*Sin[3*c + (9*d*x)/2]
 + 22*A*Sin[4*c + (9*d*x)/2] - 12*B*Sin[4*c + (9*d*x)/2] + 26*A*Sin[5*c + (
9*d*x)/2] - 20*B*Sin[5*c + (9*d*x)/2]))/(96*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 1.58, size = 247, normalized size = 1.38

$$-\frac{3\left((10\,A-7\,B)\cos\left(dx+c\right)^{5}+2\,(10\,A-7\,B)\cos\left(dx+c\right)^{4}+(10\,A-7\,B)\cos\left(dx+c\right)^{3}\right)\log\left(\sin\left(dx+c\right)+1\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/12*(3*((10*A - 7*B)*cos(d*x + c)^5 + 2*(10*A - 7*B)*cos(d*x + c)^4 + (10
*A - 7*B)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((10*A - 7*B)*cos(d*x +
 c)^5 + 2*(10*A - 7*B)*cos(d*x + c)^4 + (10*A - 7*B)*cos(d*x + c)^3)*log(-s
in(d*x + c) + 1) - 2*(16*(3*A - 2*B)*cos(d*x + c)^4 + (66*A - 43*B)*cos(d*x
 + c)^3 + 6*(2*A - B)*cos(d*x + c)^2 - (2*A - 3*B)*cos(d*x + c) + 2*A)*sin(
d*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x +
c)^3)
```

giac [A] time = 0.97, size = 226, normalized size = 1.26

$$-\frac{\frac{3\,(10\,A-7\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{2}}-\frac{3\,(10\,A-7\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{2}}+\frac{2\left(30\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-40\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(\tan\left(\frac{1}{2}\,dx+\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(10*A - 7*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(10*A - 7*B
)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(30*A*tan(1/2*d*x + 1/2*c)^5 -
 15*B*tan(1/2*d*x + 1/2*c)^5 - 40*A*tan(1/2*d*x + 1/2*c)^3 + 24*B*tan(1/2*d
*x + 1/2*c)^3 + 18*A*tan(1/2*d*x + 1/2*c) - 9*B*tan(1/2*d*x + 1/2*c))/((tan
(1/2*d*x + 1/2*c)^2 - 1)^3*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan
(1/2*d*x + 1/2*c)^3 + 27*A*a^4*tan(1/2*d*x + 1/2*c) - 21*B*a^4*tan(1/2*d*x
+ 1/2*c))/a^6)/d
```

maple [B] time = 0.18, size = 382, normalized size = 2.13

$$\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{6d\,a^2}-\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d\,a^2}+\frac{9A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{7B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}-\frac{3A}{2d\,a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{}{2d\,a^2\left(\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3*A-1/6/d/a^2*B*tan(1/2*d*x+1/2*c)^3+9/2/d/a^2
*A*tan(1/2*d*x+1/2*c)-7/2/d/a^2*B*tan(1/2*d*x+1/2*c)-3/2/d/a^2*A/(tan(1/2*d
*x+1/2*c)-1)^2+1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2*B+5/d/a^2*A*ln(tan(1/2*d*
x+1/2*c)-1)-7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*B-5/d/a^2*A/(tan(1/2*d*x+1/2
*c)-1)+5/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B-1/3/d/a^2*A/(tan(1/2*d*x+1/2*c)-1
)^3-5/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)+7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B
+3/2/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)^2-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2*B-
5/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)+5/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)*B-1/3/d/a
^2*A/(tan(1/2*d*x+1/2*c)+1)^3
```

maxima [B] time = 0.62, size = 425, normalized size = 2.37

$$\frac{A\left(\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2-\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-B\left(\frac{6}{a^2}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(A*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a^
```

```
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 30*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^2 + 30*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - B*
(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3
/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^
2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2))/d
```

mupad [B] time = 0.34, size = 203, normalized size = 1.13

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{2\,(A-B)}{a^2}+\frac{5\,A-3\,B}{2\,a^2}\right)}{d}-\frac{(10\,A-5\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(8\,B-\frac{40\,A}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(6\,A-3\,B)\,\tan\left(\frac{c}{2}+\right.}{d\,\left(a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^4*(a + a*cos(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((2*(A - B))/a^2 + (5*A - 3*B)/(2*a^2)))/d - (tan(c/2 +
 (d*x)/2)^5*(10*A - 5*B) - tan(c/2 + (d*x)/2)^3*((40*A)/3 - 8*B) + tan(c/2
+ (d*x)/2)*(6*A - 3*B))/(d*(3*a^2*tan(c/2 + (d*x)/2)^2 - 3*a^2*tan(c/2 + (d
*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 - a^2)) + (tan(c/2 + (d*x)/2)^3*(A - B)
)/(6*a^2*d) - (atanh(tan(c/2 + (d*x)/2))*(10*A - 7*B))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^4(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^4(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**4/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) + In
tegral(B*cos(c + d*x)*sec(c + d*x)**4/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1
), x))/a**2
```

3.56 $\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=218

$$\frac{4(19A-34B)\sin^3(c+dx)}{15a^3d}-\frac{4(19A-34B)\sin(c+dx)}{5a^3d}+\frac{(13A-23B)\sin(c+dx)\cos^3(c+dx)}{3d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(13A-23B)\sin}{2}$$

```
[Out] 1/2*(13*A-23*B)*x/a^3-4/5*(19*A-34*B)*sin(d*x+c)/a^3/d+1/2*(13*A-23*B)*cos(
d*x+c)*sin(d*x+c)/a^3/d+1/5*(A-B)*cos(d*x+c)^5*sin(d*x+c)/d/(a+a*cos(d*x+c)
)^3+1/15*(8*A-13*B)*cos(d*x+c)^4*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+1/3*(13*
A-23*B)*cos(d*x+c)^3*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))+4/15*(19*A-34*B)*sin
(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.52, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2748, 2635, 8, 2633}

$$\frac{4(19A-34B)\sin^3(c+dx)}{15a^3d}-\frac{4(19A-34B)\sin(c+dx)}{5a^3d}+\frac{(13A-23B)\sin(c+dx)\cos^3(c+dx)}{3d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(13A-23B)\sin}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((13*A - 23*B)*x)/(2*a^3) - (4*(19*A - 34*B)*Sin[c + d*x])/(5*a^3*d) + ((13
*A - 23*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) + ((A - B)*Cos[c + d*x]^5*S
in[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((8*A - 13*B)*Cos[c + d*x]^4*Si
n[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((13*A - 23*B)*Cos[c + d*x]^3
*Sin[c + d*x])/(3*d*(a^3 + a^3*Cos[c + d*x])) + (4*(19*A - 34*B)*Sin[c + d*
x]^3)/(15*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^4(c+dx)(5a(A-B)-a(3A-8B)\cos(c+dx))}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(8A-13B)\cos^4(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(8A-13B)\cos^4(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(8A-13B)\cos^4(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(13A-23B)\cos(c+dx)\sin(c+dx)}{2a^3d} + \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}$$

$$= \frac{(13A-23B)x}{2a^3} - \frac{4(19A-34B)\sin(c+dx)}{5a^3d} + \frac{(13A-23B)\cos(c+dx)\sin(c+dx)}{2a^3d}$$

Mathematica [B] time = 1.09, size = 491, normalized size = 2.25

$$\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(600dx(13A-23B)\cos\left(c+\frac{dx}{2}\right)+600dx(13A-23B)\cos\left(\frac{dx}{2}\right)+7560A\sin\left(c+\frac{dx}{2}\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(600*(13*A - 23*B)*d*x*Cos[(d*x)/2] + 600*(13*A
- 23*B)*d*x*Cos[c + (d*x)/2] + 3900*A*d*x*Cos[c + (3*d*x)/2] - 6900*B*d*x*C
os[c + (3*d*x)/2] + 3900*A*d*x*Cos[2*c + (3*d*x)/2] - 6900*B*d*x*Cos[2*c +
(3*d*x)/2] + 780*A*d*x*Cos[2*c + (5*d*x)/2] - 1380*B*d*x*Cos[2*c + (5*d*x)/
2] + 780*A*d*x*Cos[3*c + (5*d*x)/2] - 1380*B*d*x*Cos[3*c + (5*d*x)/2] - 127
60*A*Sin[(d*x)/2] + 20410*B*Sin[(d*x)/2] + 7560*A*Sin[c + (d*x)/2] - 11110*
B*Sin[c + (d*x)/2] - 9230*A*Sin[c + (3*d*x)/2] + 15380*B*Sin[c + (3*d*x)/2]
 + 930*A*Sin[2*c + (3*d*x)/2] - 380*B*Sin[2*c + (3*d*x)/2] - 2782*A*Sin[2*c
 + (5*d*x)/2] + 4777*B*Sin[2*c + (5*d*x)/2] - 750*A*Sin[3*c + (5*d*x)/2] +
1625*B*Sin[3*c + (5*d*x)/2] - 105*A*Sin[3*c + (7*d*x)/2] + 230*B*Sin[3*c +
(7*d*x)/2] - 105*A*Sin[4*c + (7*d*x)/2] + 230*B*Sin[4*c + (7*d*x)/2] + 15*A
*Sin[4*c + (9*d*x)/2] - 20*B*Sin[4*c + (9*d*x)/2] + 15*A*Sin[5*c + (9*d*x)/
2] - 20*B*Sin[5*c + (9*d*x)/2] + 5*B*Sin[5*c + (11*d*x)/2] + 5*B*Sin[6*c +
(11*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 0.65, size = 205, normalized size = 0.94

$$15\,(13\,A-23\,B)dx\cos\left(dx+c\right)^{3}+45\,(13\,A-23\,B)dx\cos\left(dx+c\right)^{2}+45\,(13\,A-23\,B)dx\cos\left(dx+c\right)+15\,(13\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*(13*A - 23*B)*d*x*cos(d*x + c)^3 + 45*(13*A - 23*B)*d*x*cos(d*x +
c)^2 + 45*(13*A - 23*B)*d*x*cos(d*x + c) + 15*(13*A - 23*B)*d*x + (10*B*cos
(d*x + c)^5 + 15*(A - B)*cos(d*x + c)^4 - 5*(9*A - 19*B)*cos(d*x + c)^3 - (
479*A - 869*B)*cos(d*x + c)^2 - 3*(239*A - 429*B)*cos(d*x + c) - 304*A + 54
4*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d
*cos(d*x + c) + a^3*d)
```

giac [A] time = 1.90, size = 228, normalized size = 1.05

$$\frac{30\,(dx+c)(13\,A-23\,B)}{a^{3}}-\frac{20\left(21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-51\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+36\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-76\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-33\,B\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] 1/60*(30*(d*x + c)*(13*A - 23*B)/a^3 - 20*(21*A*tan(1/2*d*x + 1/2*c)^5 - 51*B*tan(1/2*d*x + 1/2*c)^5 + 36*A*tan(1/2*d*x + 1/2*c)^3 - 76*B*tan(1/2*d*x + 1/2*c)^3 + 15*A*tan(1/2*d*x + 1/2*c) - 33*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 50*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2*d*x + 1/2*c) - 735*B*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d

maple [A] time = 0.09, size = 362, normalized size = 1.66

$$-\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^3}+\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^3}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3d\,a^3}-\frac{5B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d\,a^3}-\frac{31A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}+\frac{49B\tan(}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)

[Out] -1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5+2/3/d/a^3*tan(1/2*d*x+1/2*c)^3*A-5/6/d/a^3*B*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*A*tan(1/2*d*x+1/2*c)+49/4/d/a^3*B*tan(1/2*d*x+1/2*c)-7/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)^5+17/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1/2*d*x+1/2*c)^5-12/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*A+76/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1/2*d*x+1/2*c)^3-5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*A*tan(1/2*d*x+1/2*c)+11/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*B*tan(1/2*d*x+1/2*c)+13/d/a^3*arctan(tan(1/2*d*x+1/2*c))*A-23/d/a^3*arctan(tan(1/2*d*x+1/2*c))*B

maxima [B] time = 0.74, size = 412, normalized size = 1.89

$$\frac{B\left(\frac{20\left(\frac{33\sin(dx+c)}{\cos(dx+c)+1}+\frac{76\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{51\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3+\frac{3a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{735\sin(dx+c)}{\cos(dx+c)+1}-\frac{50\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{1380\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-A\left(\frac{60\left(\frac{5\sin(d}{\cos(dx+}\right.}{a^3+\frac{2a^3\sin(}{(\cos(dx+}}\right.}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] 1/60*(B*(20*(33*sin(d*x + c)/(cos(d*x + c) + 1) + 76*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 51*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3 + 3*a^3*sin(d*

```
x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (735*sin(d*x + c)/(cos(d*x + c)
 + 1) - 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5)/a^3 - 1380*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*(6
0*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 780
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

mupad [**B**] time = 0.33, size = 238, normalized size = 1.09

$$\frac{x\,(13\,A-23\,B)}{2\,a^3}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{5\,(A-B)}{2\,a^3}+\frac{4\,A-6\,B}{a^3}+\frac{5\,A-15\,B}{4\,a^3}\right)}{d}-\frac{(7\,A-17\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(12\,A-\frac{76\,B}{3}\right)\,\tan\left(\frac{c}{2}\right.}{d\,\left(a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (x*(13*A - 23*B))/(2*a^3) - (tan(c/2 + (d*x)/2)*((5*(A - B))/(2*a^3) + (4*A
 - 6*B)/a^3 + (5*A - 15*B)/(4*a^3)))/d - (tan(c/2 + (d*x)/2)^5*(7*A - 17*B)
 + tan(c/2 + (d*x)/2)^3*(12*A - (76*B)/3) + tan(c/2 + (d*x)/2)*(5*A - 11*B)
)/(d*(3*a^3*tan(c/2 + (d*x)/2)^2 + 3*a^3*tan(c/2 + (d*x)/2)^4 + a^3*tan(c/2
 + (d*x)/2)^6 + a^3)) + (tan(c/2 + (d*x)/2)^3*((A - B)/(3*a^3) + (4*A - 6*B
)/(12*a^3)))/d - (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d)
```

sympy [**A**] time = 25.37, size = 1584, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((390*A*d*x*tan(c/2 + d*x/2)**6/(60*a**3*d*tan(c/2 + d*x/2)**6 + 1
80*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
 + 1170*A*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3
*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 1170
*A*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(
c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 390*A*d*x/(
60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d
*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3*A*tan(c/2 + d*x/2)**11/(60*a**3*d*tan
(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*
x/2)**2 + 60*a**3*d) + 31*A*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)
```

```
**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*
a**3*d) - 354*A*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a*
*3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 16
98*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/
2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 2075*A*tan(c/
2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)*
*4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 765*A*tan(c/2 + d*x/2)/(
60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d
*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 690*B*d*x*tan(c/2 + d*x/2)**6/(60*a**3*
d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2
 + d*x/2)**2 + 60*a**3*d) - 2070*B*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c
/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/
2)**2 + 60*a**3*d) - 2070*B*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*
x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 +
 60*a**3*d) - 690*B*d*x/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2
 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 3*B*tan(c/2 +
d*x/2)**11/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4
+ 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 41*B*tan(c/2 + d*x/2)**9/(6
0*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*
tan(c/2 + d*x/2)**2 + 60*a**3*d) + 594*B*tan(c/2 + d*x/2)**7/(60*a**3*d*tan
(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*
x/2)**2 + 60*a**3*d) + 3078*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/
2)**6 + 180*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 6
0*a**3*d) + 3675*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180
*a**3*d*tan(c/2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) +
 1395*B*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**6 + 180*a**3*d*tan(c/
2 + d*x/2)**4 + 180*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d), Ne(d, 0)), (x*
(A + B*cos(c))*cos(c)**5/(a*cos(c) + a)**3, True))
```

3.57 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3} dx$

Optimal. Leaf size=193

$$\frac{8(9A-19B)\sin(c+dx)}{15a^3d}+\frac{4(9A-19B)\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A-13B)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{x(6A-1}{2a^3}$$

```
[Out] -1/2*(6*A-13*B)*x/a^3+8/15*(9*A-19*B)*sin(d*x+c)/a^3/d-1/2*(6*A-13*B)*cos(d
*x+c)*sin(d*x+c)/a^3/d+1/5*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))
^3+1/15*(6*A-11*B)*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+4/15*(9*A
-19*B)*cos(d*x+c)^2*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.47, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2977, 2734}

$$\frac{8(9A-19B)\sin(c+dx)}{15a^3d}+\frac{4(9A-19B)\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A-13B)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{x(6A-1}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -((6*A - 13*B)*x)/(2*a^3) + (8*(9*A - 19*B)*Sin[c + d*x])/(15*a^3*d) - ((6*
A - 13*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) + ((A - B)*Cos[c + d*x]^4*Si
n[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((6*A - 11*B)*Cos[c + d*x]^3*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + (4*(9*A - 19*B)*Cos[c + d*x]^2
*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
```

```
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^3(c+dx)(4a(A-B)-a(2A-7B)\cos(c+dx))}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(6A-11B)\cos^3(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(6A-11B)\cos^3(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(6A-13B)x}{2a^3} + \frac{8(9A-19B)\sin(c+dx)}{15a^3d} - \frac{(6A-13B)\cos(c+dx)\sin}{2a^3d}$$

Mathematica [B] time = 0.92, size = 435, normalized size = 2.25

$$\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-600dx(6A-13B)\cos\left(c+\frac{dx}{2}\right)-600dx(6A-13B)\cos\left(\frac{dx}{2}\right)-4500A\sin\left(c+\frac{dx}{2}\right)+486\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-600*(6*A - 13*B)*d*x*Cos[(d*x)/2] - 600*(6*A -
 13*B)*d*x*Cos[c + (d*x)/2] - 1800*A*d*x*Cos[c + (3*d*x)/2] + 3900*B*d*x*Co
s[c + (3*d*x)/2] - 1800*A*d*x*Cos[2*c + (3*d*x)/2] + 3900*B*d*x*Cos[2*c + (
3*d*x)/2] - 360*A*d*x*Cos[2*c + (5*d*x)/2] + 780*B*d*x*Cos[2*c + (5*d*x)/2]
 - 360*A*d*x*Cos[3*c + (5*d*x)/2] + 780*B*d*x*Cos[3*c + (5*d*x)/2] + 7020*A
*Sin[(d*x)/2] - 12760*B*Sin[(d*x)/2] - 4500*A*Sin[c + (d*x)/2] + 7560*B*Sin
[c + (d*x)/2] + 4860*A*Sin[c + (3*d*x)/2] - 9230*B*Sin[c + (3*d*x)/2] - 900
*A*Sin[2*c + (3*d*x)/2] + 930*B*Sin[2*c + (3*d*x)/2] + 1452*A*Sin[2*c + (5*
d*x)/2] - 2782*B*Sin[2*c + (5*d*x)/2] + 300*A*Sin[3*c + (5*d*x)/2] - 750*B*
Sin[3*c + (5*d*x)/2] + 60*A*Sin[3*c + (7*d*x)/2] - 105*B*Sin[3*c + (7*d*x)/
2] + 60*A*Sin[4*c + (7*d*x)/2] - 105*B*Sin[4*c + (7*d*x)/2] + 15*B*Sin[4*c
+ (9*d*x)/2] + 15*B*Sin[5*c + (9*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 0.59, size = 190, normalized size = 0.98

$$-\frac{15\,(6\,A-13\,B)dx\cos\left(dx+c\right)^{3}+45\,(6\,A-13\,B)dx\cos\left(dx+c\right)^{2}+45\,(6\,A-13\,B)dx\cos\left(dx+c\right)+15\,(6\,A-}{30\left(a^{3}d\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*(6*A - 13*B)*d*x*cos(d*x + c)^3 + 45*(6*A - 13*B)*d*x*cos(d*x + c
)^2 + 45*(6*A - 13*B)*d*x*cos(d*x + c) + 15*(6*A - 13*B)*d*x - (15*B*cos(d*
x + c)^4 + 15*(2*A - 3*B)*cos(d*x + c)^3 + (234*A - 479*B)*cos(d*x + c)^2 +
 3*(114*A - 239*B)*cos(d*x + c) + 144*A - 304*B)*sin(d*x + c))/(a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 1.94, size = 200, normalized size = 1.04

$$-\frac{\frac{30\,(dx+c)(6\,A-13\,B)}{a^{3}}-\frac{60\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-7\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{3}}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ba^{12}\,\mathrm{t}}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(30*(d*x + c)*(6*A - 13*B)/a^3 - 60*(2*A*tan(1/2*d*x + 1/2*c)^3 - 7*B
*tan(1/2*d*x + 1/2*c)^3 + 2*A*tan(1/2*d*x + 1/2*c) - 5*B*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5
 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 40*
B*a^12*tan(1/2*d*x + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 465*B*a^1
2*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.10, size = 292, normalized size = 1.51

$$\frac{A\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^{3}}-\frac{B\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^{3}}-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{2d\,a^{3}}+\frac{2B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\,a^{3}}+\frac{17A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}-\frac{31B\tan\left(\frac{dx}{2}\right.}{4d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5-1/2/d/a
^3*tan(1/2*d*x+1/2*c)^3*A+2/3/d/a^3*B*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*A*tan
(1/2*d*x+1/2*c)-31/4/d/a^3*B*tan(1/2*d*x+1/2*c)+2/d/a^3/(1+tan(1/2*d*x+1/2*
c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-7/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2
*d*x+1/2*c)^3+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)-5/d/a
^3/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)-6/d/a^3*arctan(tan(1/2*d
*x+1/2*c))*A+13/d/a^3*arctan(tan(1/2*d*x+1/2*c))*B
```

maxima [A] time = 1.00, size = 322, normalized size = 1.67

$$-\frac{B\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-3A\left(\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos}\right.}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(B*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - 3*A*(40*sin(d*x
+ c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) +
 (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^3))/d
```

mupad [B] time = 0.27, size = 203, normalized size = 1.05

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3(A-B)}{2a^3}+\frac{3(3A-5B)}{4a^3}+\frac{2A-10B}{4a^3}\right)}{d}-\frac{x(6A-13B)}{2a^3}+\frac{(2A-7B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(2A-5B)\tan\left(\frac{c}{2}+\right.}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((3*(A - B))/(2*a^3) + (3*(3*A - 5*B))/(4*a^3) + (2*A -
 10*B)/(4*a^3)))/d - (x*(6*A - 13*B))/(2*a^3) + (tan(c/2 + (d*x)/2)^3*(2*A
- 7*B) + tan(c/2 + (d*x)/2)*(2*A - 5*B))/(d*(2*a^3*tan(c/2 + (d*x)/2)^2 + a
^3*tan(c/2 + (d*x)/2)^4 + a^3)) - (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^3) +
(3*A - 5*B)/(12*a^3)))/d + (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d)
```

sympy [A] time = 15.91, size = 966, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-180*A*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 +
120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 360*A*d*x*tan(c/2 + d*x/2)**2
/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*
d) - 180*A*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)
**2 + 60*a**3*d) + 3*A*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4 +
 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 24*A*tan(c/2 + d*x/2)**7/(60
*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) +
 198*A*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(
c/2 + d*x/2)**2 + 60*a**3*d) + 600*A*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2
 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 375*A*tan(c/2
+ d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 +
60*a**3*d) + 390*B*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 +
 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 780*B*d*x*tan(c/2 + d*x/2)**
2/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3
*d) + 390*B*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2
)**2 + 60*a**3*d) - 3*B*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4
+ 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 34*B*tan(c/2 + d*x/2)**7/(6
0*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d)
- 388*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan
(c/2 + d*x/2)**2 + 60*a**3*d) - 1310*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c
/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 765*B*tan(c/
2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2
+ 60*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**4/(a*cos(c) + a)**3, Tru
e))
```

3.58 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=147

$$-\frac{(7A-27B)\sin(c+dx)}{15a^3d}-\frac{(A-3B)\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{x(A-3B)}{a^3}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(4A-9B)\sin(}{15ad(a\,\mathrm{co}}$$

```
[Out] (A-3*B)*x/a^3-1/15*(7*A-27*B)*sin(d*x+c)/a^3/d+1/5*(A-B)*cos(d*x+c)^3*sin(d
*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(4*A-9*B)*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*
cos(d*x+c))^2-(A-3*B)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.46, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2977, 2968, 3023, 12, 2735, 2648}

$$-\frac{(7A-27B)\sin(c+dx)}{15a^3d}-\frac{(A-3B)\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{x(A-3B)}{a^3}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{(4A-9B)\sin(}{15ad(a\,\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((A - 3*B)*x)/a^3 - ((7*A - 27*B)*Sin[c + d*x])/(15*a^3*d) + ((A - B)*Cos[c
 + d*x]^3*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((4*A - 9*B)*Cos[c +
 d*x]^2*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((A - 3*B)*Sin[c +
d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^2(c+dx)(3a(A-B)-a(A-6B)\cos(c+dx))}{(a+a\cos(c+dx))^2}\,a}{5a^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A-9B)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A-9B)\cos^2(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(7A-27B)\sin(c+dx)}{15a^3d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A-9}{15a}$$

$$= -\frac{(7A-27B)\sin(c+dx)}{15a^3d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A-9}{15a}$$

$$= \frac{(A-3B)x}{a^3} - \frac{(7A-27B)\sin(c+dx)}{15a^3d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx}{5d(a+a\cos(c+dx))^3}$$

$$= \frac{(A-3B)x}{a^3} - \frac{(7A-27B)\sin(c+dx)}{15a^3d} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx}{5d(a+a\cos(c+dx))^3}$$

Mathematica [B] time = 0.96, size = 361, normalized size = 2.46

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(300dx(A-3B)\cos\left(c+\frac{dx}{2}\right)+300dx(A-3B)\cos\left(\frac{dx}{2}\right)+540A\sin\left(c+\frac{dx}{2}\right)-460A\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(300*(A - 3*B)*d*x*Cos[(d*x)/2] + 300*(A - 3*B)*
d*x*Cos[c + (d*x)/2] + 150*A*d*x*Cos[c + (3*d*x)/2] - 450*B*d*x*Cos[c + (3*
d*x)/2] + 150*A*d*x*Cos[2*c + (3*d*x)/2] - 450*B*d*x*Cos[2*c + (3*d*x)/2] +
 30*A*d*x*Cos[2*c + (5*d*x)/2] - 90*B*d*x*Cos[2*c + (5*d*x)/2] + 30*A*d*x*C
os[3*c + (5*d*x)/2] - 90*B*d*x*Cos[3*c + (5*d*x)/2] - 740*A*Sin[(d*x)/2] +
1755*B*Sin[(d*x)/2] + 540*A*Sin[c + (d*x)/2] - 1125*B*Sin[c + (d*x)/2] - 46
0*A*Sin[c + (3*d*x)/2] + 1215*B*Sin[c + (3*d*x)/2] + 180*A*Sin[2*c + (3*d*x
)/2] - 225*B*Sin[2*c + (3*d*x)/2] - 128*A*Sin[2*c + (5*d*x)/2] + 363*B*Sin[
2*c + (5*d*x)/2] + 75*B*Sin[3*c + (5*d*x)/2] + 15*B*Sin[3*c + (7*d*x)/2] +
15*B*Sin[4*c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 0.62, size = 165, normalized size = 1.12

$$\frac{15\,(A-3\,B)dx\cos\left(dx+c\right)^{3}+45\,(A-3\,B)dx\cos\left(dx+c\right)^{2}+45\,(A-3\,B)dx\cos\left(dx+c\right)+15\,(A-3\,B)dx+\left(15\right.}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*(A - 3*B)*d*x*cos(d*x + c)^3 + 45*(A - 3*B)*d*x*cos(d*x + c)^2 + 4
5*(A - 3*B)*d*x*cos(d*x + c) + 15*(A - 3*B)*d*x + (15*B*cos(d*x + c)^3 - (3
2*A - 117*B)*cos(d*x + c)^2 - 3*(17*A - 57*B)*cos(d*x + c) - 22*A + 72*B)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

giac [A] time = 0.36, size = 155, normalized size = 1.05

$$\frac{\frac{60\,(dx+c)(A-3\,B)}{a^{3}}+\frac{120\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{3}}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-20\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+30\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*(d*x + c)*(A - 3*B)/a^3 + 120*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x
 + 1/2*c)^2 + 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2
*d*x + 1/2*c)^5 - 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 30*B*a^12*tan(1/2*d*x
+ 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 255*B*a^12*tan(1/2*d*x + 1/2
*c))/a^15)/d
```

maple [A] time = 0.10, size = 189, normalized size = 1.29

$$-\frac{A\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^{3}}+\frac{B\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^{3}}+\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3d\,a^{3}}-\frac{B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{3}}-\frac{7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}+\frac{17B\tan\left(\frac{dx}{2}\right.}{4d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5+1/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*A-1/2/d/a^3*B*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*A*tan
```

```
(1/2*d*x+1/2*c)+17/4/d/a^3*B*tan(1/2*d*x+1/2*c)+2/d/a^3*B*tan(1/2*d*x+1/2*c
)/(1+tan(1/2*d*x+1/2*c)^2)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))*A-6/d/a^3*arc
tan(tan(1/2*d*x+1/2*c))*B
```

maxima [A] time = 0.70, size = 231, normalized size = 1.57

$$\frac{3\,B\left(\frac{40\,\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-A\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\,\sin(dx+c)}{(\cos(dx+c)+1}}{a^3}\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*B*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*((105*sin(d*x + c)/(cos(d*x
 + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

mupad [B] time = 0.26, size = 152, normalized size = 1.03

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{6\,a^3}+\frac{2\,A-4\,B}{12\,a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,(A-B)}{4\,a^3}-\frac{3\,B}{2\,a^3}+\frac{2\,A-4\,B}{2\,a^3}\right)}{d}+\frac{x\,\left(A-3\,B\right)}{a^3}+\frac{2\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((A - B)/(6*a^3) + (2*A - 4*B)/(12*a^3)))/d - (tan(c/
2 + (d*x)/2)*((3*(A - B))/(4*a^3) - (3*B)/(2*a^3) + (2*A - 4*B)/(2*a^3)))/d
 + (x*(A - 3*B))/a^3 + (2*B*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d*x)/2)^
2 + a^3)) - (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d)
```

sympy [A] time = 9.80, size = 496, normalized size = 3.37

$$\begin{cases} \frac{60Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d}+\frac{60Adx}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d}-\frac{3A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d}+\frac{17A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a^3d}-\frac{85A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{60a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+60a} \\ \frac{x(A+B\cos(c))\cos^3(c)}{(a\cos(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((60*A*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60
*a**3*d) + 60*A*d*x/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3*A*tan(c
/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 17*A*tan(c/2 +
 d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 85*A*tan(c/2 + d*x
/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 105*A*tan(c/2 + d*x/2)
/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*B*d*x*tan(c/2 + d*x/2)**
2/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 180*B*d*x/(60*a**3*d*tan(c/
2 + d*x/2)**2 + 60*a**3*d) + 3*B*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d
*x/2)**2 + 60*a**3*d) - 27*B*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2
)**2 + 60*a**3*d) + 225*B*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**
2 + 60*a**3*d) + 375*B*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**2 + 60
*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**3/(a*cos(c) + a)**3, True))
```

3.59 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=116

$$\frac{(4A-29B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{Bx}{a^3}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(2A-7B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

```
[Out] B*x/a^3+1/5*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-1/15*(2*A-7*
B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+1/15*(4*A-29*B)*sin(d*x+c)/d/(a^3+a^3*
cos(d*x+c))
```

Rubi [A] time = 0.32, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2968, 3019, 2735, 2648}

$$\frac{(4A-29B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{Bx}{a^3}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(2A-7B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (B*x)/a^3 + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])
^3) - ((2*A - 7*B)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((4*A -
29*B)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos(c+dx)(2a(A-B)+5aB\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{2a(A-B)\cos(c+dx)+5aB\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-7B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \int \frac{-2a^2(}{}$$

$$= \frac{Bx}{a^3} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-7B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \cdots$$

$$= \frac{Bx}{a^3} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-7B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \cdots$$

Mathematica [B] time = 0.62, size = 241, normalized size = 2.08

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(-60A\sin\left(c+\frac{dx}{2}\right)+40A\sin\left(c+\frac{3dx}{2}\right)-30A\sin\left(2c+\frac{3dx}{2}\right)+14A\sin\left(2c+\frac{5dx}{2}\right)+8\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(150*B*d*x*Cos[(d*x)/2] + 150*B*d*x*Cos[c + (d
*x)/2] + 75*B*d*x*Cos[c + (3*d*x)/2] + 75*B*d*x*Cos[2*c + (3*d*x)/2] + 15*B
*d*x*Cos[2*c + (5*d*x)/2] + 15*B*d*x*Cos[3*c + (5*d*x)/2] + 80*A*Sin[(d*x)/
2] - 370*B*Sin[(d*x)/2] - 60*A*Sin[c + (d*x)/2] + 270*B*Sin[c + (d*x)/2] +
40*A*Sin[c + (3*d*x)/2] - 230*B*Sin[c + (3*d*x)/2] - 30*A*Sin[2*c + (3*d*x)
/2] + 90*B*Sin[2*c + (3*d*x)/2] + 14*A*Sin[2*c + (5*d*x)/2] - 64*B*Sin[2*c
+ (5*d*x)/2]))/(480*a^3*d)
```

fricas [A] time = 0.88, size = 137, normalized size = 1.18

$$\frac{15\,Bdx\cos\left(dx+c\right)^{3}+45\,Bdx\cos\left(dx+c\right)^{2}+45\,Bdx\cos\left(dx+c\right)+15\,Bdx+\left(\left(7\,A-32\,B\right)\cos\left(dx+c\right)^{2}+3\,(2\,A}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*B*d*x*cos(d*x + c)^3 + 45*B*d*x*cos(d*x + c)^2 + 45*B*d*x*cos(d*x
+ c) + 15*B*d*x + ((7*A - 32*B)*cos(d*x + c)^2 + 3*(2*A - 17*B)*cos(d*x + c
) + 2*A - 22*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 0.50, size = 120, normalized size = 1.03

$$\frac{\frac{60\,(dx+c)B}{a^{3}}+\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-10\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+20\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-105\,Ba^{12}\,\text{ta}}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*(d*x + c)*B/a^3 + (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(
1/2*d*x + 1/2*c)^5 - 10*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 20*B*a^12*tan(1/2*d
*x + 1/2*c)^3 + 15*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x + 1
/2*c))/a^15)/d
```

maple [A] time = 0.08, size = 137, normalized size = 1.18

$$\frac{A\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^{3}}-\frac{B\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^{3}}-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{6d\,a^{3}}+\frac{B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\,a^{3}}+\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}-\frac{7B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5-1/6/d/a
^3*tan(1/2*d*x+1/2*c)^3*A+1/3/d/a^3*B*tan(1/2*d*x+1/2*c)^3+1/4/d/a^3*A*tan(
1/2*d*x+1/2*c)-7/4/d/a^3*B*tan(1/2*d*x+1/2*c)+2/d/a^3*arctan(tan(1/2*d*x+1/
2*c))*B
```

maxima [A] time = 0.54, size = 160, normalized size = 1.38

$$-\frac{B\left(\frac{\frac{105\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{20\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{3\,\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}}{a^{3}}-\frac{120\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a^{3}}\right)-\frac{A\left(\frac{15\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{10\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{3\,\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}\right)}{a^{3}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(B*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3) - A*(15*sin(d*x + c)/(cos(d*x + c) + 1) -
 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) +
1)^5)/a^3)/d
```

mupad [B] time = 0.38, size = 134, normalized size = 1.16

$$\frac{B\,x}{a^3}-\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}-\frac{B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}\right)-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{7\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)-\frac{A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20}+\frac{B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20}}{a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (B*x)/a^3 - (cos(c/2 + (d*x)/2)^2*((A*sin(c/2 + (d*x)/2)^3)/6 - (B*sin(c/2
+ (d*x)/2)^3)/3) - cos(c/2 + (d*x)/2)^4*((A*sin(c/2 + (d*x)/2))/4 - (7*B*si
n(c/2 + (d*x)/2))/4) - (A*sin(c/2 + (d*x)/2)^5)/20 + (B*sin(c/2 + (d*x)/2)^
5)/20)/(a^3*d*cos(c/2 + (d*x)/2)^5)
```

sympy [A] time = 5.80, size = 148, normalized size = 1.28

$$\begin{cases} \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} - \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} + \frac{Bx}{a^3} - \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d} - \frac{7B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x(A+B\cos(c))\cos^2(c)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**5/(20*a**3*d) - A*tan(c/2 + d*x/2)**3/(6*a**
3*d) + A*tan(c/2 + d*x/2)/(4*a**3*d) + B*x/a**3 - B*tan(c/2 + d*x/2)**5/(20
*a**3*d) + B*tan(c/2 + d*x/2)**3/(3*a**3*d) - 7*B*tan(c/2 + d*x/2)/(4*a**3*
d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**2/(a*cos(c) + a)**3, True))
```

3.60 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(3A+7B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(3A-8B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] -1/5*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(3*A-8*B)*sin(d*x+c)/a/d/(a
+a*cos(d*x+c))^2+1/15*(3*A+7*B)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.19, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3019, 2750, 2648}

$$\frac{(3A+7B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(3A-8B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -((A - B)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((3*A - 8*B)*Sin[c +
 d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((3*A + 7*B)*Sin[c + d*x])/(15*d*(
a^3 + a^3*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{-3a(A-B)-5aB\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(3A+7B)\int \frac{}{a+a\,c}}{15a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{(3A+7B)\sin(c}{15d\left(a^3+a^3\cos(c\right.}$$

Mathematica [A] time = 0.36, size = 135, normalized size = 1.32

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-15(A+2B)\sin\left(c+\frac{dx}{2}\right)+5(3A+8B)\sin\left(\frac{dx}{2}\right)+15A\sin\left(c+\frac{3dx}{2}\right)+3A\sin\left(2c+\frac{5dx}{2}\right.\right.}{30a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(3*A + 8*B)*Sin[(d*x)/2] - 15*(A + 2*B)*Sin[c
 + (d*x)/2] + 15*A*Sin[c + (3*d*x)/2] + 20*B*Sin[c + (3*d*x)/2] - 15*B*Sin[
2*c + (3*d*x)/2] + 3*A*Sin[2*c + (5*d*x)/2] + 7*B*Sin[2*c + (5*d*x)/2]))/(3
0*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 0.76, size = 93, normalized size = 0.91

$$\frac{\left((3A+7B)\cos(dx+c)^2+3(3A+2B)\cos(dx+c)+3A+2B\right)\sin(dx+c)}{15\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*((3*A + 7*B)*cos(d*x + c)^2 + 3*(3*A + 2*B)*cos(d*x + c) + 3*A + 2*B)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

giac [A] time = 1.90, size = 75, normalized size = 0.74

$$-\frac{3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+10\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,B\tan\left(\frac{1}{2}\,dx\right.}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] -1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 10*B*tan(1
/2*d*x + 1/2*c)^3 - 15*A*tan(1/2*d*x + 1/2*c) - 15*B*tan(1/2*d*x + 1/2*c))/
(a^3*d)
```

maple [A] time = 0.08, size = 64, normalized size = 0.63

$$\frac{\frac{(-A+B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*(-A+B)*tan(1/2*d*x+1/2*c)^5-2/3*B*tan(1/2*d*x+1/2*c)^3+A*tan
(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.81, size = 115, normalized size = 1.13

$$\frac{\frac{B\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a^{3}}+\frac{3\,A\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a^{3}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(B*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*A*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

mupad [B] time = 0.21, size = 66, normalized size = 0.65

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(15\,A+15\,B-3\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-10\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}{60\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(15*A + 15*B - 3*A*tan(c/2 + (d*x)/2)^4 - 10*B*tan(c/2
+ (d*x)/2)^2 + 3*B*tan(c/2 + (d*x)/2)^4))/(60*a^3*d)
```

sympy [A] time = 3.72, size = 117, normalized size = 1.15

$$\begin{cases} -\frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d}+\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}-\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for}\ d\neq 0 \\ \frac{x(A+B\cos(c))\cos(c)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**5/(20*a**3*d) + A*tan(c/2 + d*x/2)/(4*a**3*
d) + B*tan(c/2 + d*x/2)**5/(20*a**3*d) - B*tan(c/2 + d*x/2)**3/(6*a**3*d) +
 B*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)/(a*cos(
c) + a)**3, True))
```

3.61 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(2A+3B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(2A+3B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] 1/5*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(2*A+3*B)*sin(d*x+c)/a/d/(a+
a*cos(d*x+c))^2+1/15*(2*A+3*B)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2750, 2650, 2648}

$$\frac{(2A+3B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(2A+3B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((A - B)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + ((2*A + 3*B)*Sin[c +
d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + ((2*A + 3*B)*Sin[c + d*x])/(15*d*(a
^3 + a^3*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx &= \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(2A+3B)\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{5a}\\ &= \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(2A+3B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{(2A+3B)\int \frac{1}{a+a\cos(c+dx)}\,dx}{15a^2}\\ &= \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(2A+3B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{(2A+3B)\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.29, size = 96, normalized size = 0.94

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left((2A+3B)\left(5\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)\right)+5(4A+3B)\sin\left(\frac{dx}{2}\right)-15B\sin\left(c+\frac{dx}{2}\right)\right)}{30a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(4*A + 3*B)*Sin[(d*x)/2] - 15*B*Sin[c + (d*x)
/2] + (2*A + 3*B)*(5*Sin[c + (3*d*x)/2] + Sin[2*c + (5*d*x)/2])))/(30*a^3*d
*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 0.54, size = 93, normalized size = 0.91

$$\frac{\left((2\,A+3\,B)\cos\left(dx+c\right)^2+3\,(2\,A+3\,B)\cos\left(dx+c\right)+7\,A+3\,B\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*((2*A + 3*B)*cos(d*x + c)^2 + 3*(2*A + 3*B)*cos(d*x + c) + 7*A + 3*B)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

giac [A] time = 0.45, size = 75, normalized size = 0.74

$$\frac{3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,B\tan\left(\frac{1}{2}\,dx+\right.}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 + 10*A*tan(1/
2*d*x + 1/2*c)^3 + 15*A*tan(1/2*d*x + 1/2*c) + 15*B*tan(1/2*d*x + 1/2*c))/(
a^3*d)
```

maple [A] time = 0.07, size = 64, normalized size = 0.63

$$\frac{\frac{(A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*(A-B)*tan(1/2*d*x+1/2*c)^5+2/3*tan(1/2*d*x+1/2*c)^3*A+A*tan(
1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.46, size = 115, normalized size = 1.13

$$\frac{\frac{A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3B\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(A*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*B*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

mupad [B] time = 0.20, size = 66, normalized size = 0.65

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(15\,A+15\,B+10\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-3\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}{60\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(15*A + 15*B + 10*A*tan(c/2 + (d*x)/2)^2 + 3*A*tan(c/2
+ (d*x)/2)^4 - 3*B*tan(c/2 + (d*x)/2)^4))/(60*a^3*d)
```

sympy [A] time = 2.54, size = 114, normalized size = 1.12

$$\begin{cases} \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^3d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} - \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x(A+B\cos(c))}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**5/(20*a**3*d) + A*tan(c/2 + d*x/2)**3/(6*a**
3*d) + A*tan(c/2 + d*x/2)/(4*a**3*d) - B*tan(c/2 + d*x/2)**5/(20*a**3*d) +
B*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*(A + B*cos(c))/(a*cos(c) + a)*
*3, True))
```

3.62 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=117

$$-\frac{2(11A-B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(7A-2B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] A*arctanh(sin(d*x+c))/a^3/d-1/5*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-1/15*
(7*A-2*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2-2/15*(11*A-B)*sin(d*x+c)/d/(a^3
+a^3*cos(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2978, 12, 3770}

$$-\frac{2(11A-B)\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(7A-2B)\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((A - B)*Sin[c + d*x])/(5*d*(a + a*Cos[
c + d*x])^3) - ((7*A - 2*B)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) -
 (2*(11*A - B)*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(5aA-2a(A-B)\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-2B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{(15a^2A-a^2(7A-2B)}{a+a\,\mathrm{co}}}{1}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-2B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{2(11A-B)\sin(}{15d\left(a^3+a^3\cos(\right.}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-2B)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{2(11A-B)\sin(}{15d\left(a^3+a^3\cos(\right.}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(A-B)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(7A-2B)\sin(c+d}{15ad(a+a\cos(c+d}$$

Mathematica [A] time = 1.04, size = 197, normalized size = 1.68

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-5(29A-4B)\sin\left(\frac{dx}{2}\right)+75A\sin\left(c+\frac{dx}{2}\right)-95A\sin\left(c+\frac{3dx}{2}\right)+15A\sin\left(2c+\frac{3dx}{2}\right)-22\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-240*A*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-5*(29*A
 - 4*B)*Sin[(d*x)/2] + 75*A*Sin[c + (d*x)/2] - 95*A*Sin[c + (3*d*x)/2] + 10
*B*Sin[c + (3*d*x)/2] + 15*A*Sin[2*c + (3*d*x)/2] - 22*A*Sin[2*c + (5*d*x)/
2] + 2*B*Sin[2*c + (5*d*x)/2]))/(30*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 0.72, size = 185, normalized size = 1.58

$$\frac{15\left(A\cos(dx+c)^3+3A\cos(dx+c)^2+3A\cos(dx+c)+A\right)\log(\sin(dx+c)+1)-15\left(A\cos(dx+c)^3+3A\right.}{30\left(a^3d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/30*(15*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*log
(sin(d*x + c) + 1) - 15*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*
x + c) + A)*log(-sin(d*x + c) + 1) - 2*(2*(11*A - B)*cos(d*x + c)^2 + 3*(17
*A - 2*B)*cos(d*x + c) + 32*A - 7*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 4.31, size = 148, normalized size = 1.26

$$\frac{\frac{60\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-10\,Ba^{12}}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] 1/60*(60*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*A*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2*
d*x + 1/2*c)^5 + 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 10*B*a^12*tan(1/2*d*x +
 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 15*B*a^12*tan(1/2*d*x + 1/2*c
))/a^15)/d
```

maple [A] time = 0.16, size = 159, normalized size = 1.36

$$\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^3}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3d\,a^3}+\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d\,a^3}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^3}-\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*A*ln(tan(1/2*d*x+1/2*c)+1)-1/3/d/a^3*tan(1/2*d*x+1/2*c)^3*A+1/6/d/a
^3*B*tan(1/2*d*x+1/2*c)^3-1/d/a^3*A*ln(tan(1/2*d*x+1/2*c)-1)-1/20/d/a^3*A*t
an(1/2*d*x+1/2*c)^5+1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5-7/4/d/a^3*A*tan(1/2*d
*x+1/2*c)+1/4/d/a^3*B*tan(1/2*d*x+1/2*c)
```

maxima [A] time = 0.57, size = 187, normalized size = 1.60

$$-\frac{A\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-\frac{B\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx}{(\cos(dx+c}\right.}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] -1/60*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^3) - B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

mupad [B] time = 0.25, size = 130, normalized size = 1.11

$$\frac{2\,A\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{4\,a^3}+\frac{3\,A+B}{4\,a^3}+\frac{3\,A-B}{4\,a^3}\right)}{d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(A-B\right)}{20\,a^3\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{A-B}{12\,a^3}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] (2*A*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - (tan(c/2 + (d*x)/2)*((A - B)/(4*a
^3) + (3*A + B)/(4*a^3) + (3*A - B)/(4*a^3)))/d - (tan(c/2 + (d*x)/2)^5*(A
- B))/(20*a^3*d) - (tan(c/2 + (d*x)/2)^3*((A - B)/(12*a^3) + (3*A - B)/(12*
a^3)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d
*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)/(cos(c + d*x)**3 + 3*co
s(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

3.63 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$\frac{2(36A-11B)\tan(c+dx)}{15a^3d}-\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(3A-B)\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(9A-4B)\tan(c+dx)}{15ad(a\cos(c+dx)+a)^2}-5$$

```
[Out] -(3*A-B)*arctanh(sin(d*x+c))/a^3/d+2/15*(36*A-11*B)*tan(d*x+c)/a^3/d-1/5*(A
-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^3-1/15*(9*A-4*B)*tan(d*x+c)/a/d/(a+a*cos(
d*x+c))^2-(3*A-B)*tan(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.47, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2748, 3767, 8, 3770}

$$\frac{2(36A-11B)\tan(c+dx)}{15a^3d}-\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(3A-B)\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(9A-4B)\tan(c+dx)}{15ad(a\cos(c+dx)+a)^2}-5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -(((3*A - B)*ArcTanh[Sin[c + d*x]])/(a^3*d)) + (2*(36*A - 11*B)*Tan[c + d*x
])/(15*a^3*d) - ((A - B)*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((9*A
 - 4*B)*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - ((3*A - B)*Tan[c +
d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{(a(6A-B)-3a(A-B)\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(9A-4B)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{\int \frac{\left(a^2(27A-7B)-2a^2\right.}{a}$$

$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(9A-4B)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2}-\frac{(3A-B)\tan(c}{d\left(a^3+a^3\cos(c\right.}$$

$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(9A-4B)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2}-\frac{(3A-B)\tan(c}{d\left(a^3+a^3\cos(c\right.}$$

$$= -\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(A-B)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(9A-4B)}{15ad(a+a}$$

$$= -\frac{(3A-B)\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{2(36A-11B)\tan(c+dx)}{15a^3d}-\frac{(A-B}{5d(a+a}$$

Mathematica [B] time = 3.28, size = 482, normalized size = 3.32

$$\frac{960(3A-B)\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (960*(3*A - B)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Se
c[c]*Sec[c + d*x]*(-5*(51*A - 32*B)*Sin[(d*x)/2] + (567*A - 167*B)*Sin[(3*d
*x)/2] - 600*A*Sin[c - (d*x)/2] + 170*B*Sin[c - (d*x)/2] + 375*A*Sin[c + (d
*x)/2] - 170*B*Sin[c + (d*x)/2] - 480*A*Sin[2*c + (d*x)/2] + 160*B*Sin[2*c
+ (d*x)/2] - 60*A*Sin[c + (3*d*x)/2] + 75*B*Sin[c + (3*d*x)/2] + 402*A*Sin[
2*c + (3*d*x)/2] - 167*B*Sin[2*c + (3*d*x)/2] - 225*A*Sin[3*c + (3*d*x)/2]
+ 75*B*Sin[3*c + (3*d*x)/2] + 315*A*Sin[c + (5*d*x)/2] - 95*B*Sin[c + (5*d*
x)/2] + 30*A*Sin[2*c + (5*d*x)/2] + 15*B*Sin[2*c + (5*d*x)/2] + 240*A*Sin[3
*c + (5*d*x)/2] - 95*B*Sin[3*c + (5*d*x)/2] - 45*A*Sin[4*c + (5*d*x)/2] + 1
5*B*Sin[4*c + (5*d*x)/2] + 72*A*Sin[2*c + (7*d*x)/2] - 22*B*Sin[2*c + (7*d*
x)/2] + 15*A*Sin[3*c + (7*d*x)/2] + 57*A*Sin[4*c + (7*d*x)/2] - 22*B*Sin[4*
c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 0.66, size = 272, normalized size = 1.88

$$-\frac{15\left((3A-B)\cos(dx+c)^4+3(3A-B)\cos(dx+c)^3+3(3A-B)\cos(dx+c)^2+(3A-B)\cos(dx+c)\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*((3*A - B)*cos(d*x + c)^4 + 3*(3*A - B)*cos(d*x + c)^3 + 3*(3*A -
 B)*cos(d*x + c)^2 + (3*A - B)*cos(d*x + c))*log(sin(d*x + c) + 1) - 15*((3
*A - B)*cos(d*x + c)^4 + 3*(3*A - B)*cos(d*x + c)^3 + 3*(3*A - B)*cos(d*x +
 c)^2 + (3*A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(36*A - 11*B)
*cos(d*x + c)^3 + 3*(57*A - 17*B)*cos(d*x + c)^2 + (117*A - 32*B)*cos(d*x +
 c) + 15*A)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

giac [A] time = 0.44, size = 190, normalized size = 1.31

$$-\frac{\frac{60(3A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{60(3A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{120A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^3}-\frac{3Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3Ba^{12}\tan\left(\frac{1}{2}d\right.}{}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(60*(3*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*(3*A - B)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 120*A*tan(1/2*d*x + 1/2*c)/((tan(1/
2*d*x + 1/2*c)^2 - 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*ta
n(1/2*d*x + 1/2*c)^5 + 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 20*B*a^12*tan(1/2
*d*x + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x
+ 1/2*c))/a^15)/d
```

maple [A] time = 0.16, size = 245, normalized size = 1.69

$$\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^3}-\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^3}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{2d\,a^3}-\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\,a^3}+\frac{17A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}-\frac{7B\tan\left(\frac{dx}{2}+\right.}{4d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5-1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5+1/2/d/a
^3*tan(1/2*d*x+1/2*c)^3*A-1/3/d/a^3*B*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*A*tan
(1/2*d*x+1/2*c)-7/4/d/a^3*B*tan(1/2*d*x+1/2*c)+3/d/a^3*A*ln(tan(1/2*d*x+1/2
*c)-1)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)-
3/d/a^3*A*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B-1/d/a
^3*A/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.48, size = 286, normalized size = 1.97

$$\frac{3\,A\left(\frac{40\,\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-B\left(\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*A*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^3) - B*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 6
0*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^3))/d
```

mupad [B] time = 0.28, size = 168, normalized size = 1.16

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{6\,a^3}+\frac{4\,A-2\,B}{12\,a^3}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,A}{2\,a^3}+\frac{3\,(A-B)}{4\,a^3}+\frac{4\,A-2\,B}{2\,a^3}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A-B)}{20\,a^3\,d}-\frac{2\,A\tan\left(\frac{c}{2}\right.}{d\,\left(a^3\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((A - B)/(6*a^3) + (4*A - 2*B)/(12*a^3)))/d + (tan(c/
2 + (d*x)/2)*((3*A)/(2*a^3) + (3*(A - B))/(4*a^3) + (4*A - 2*B)/(2*a^3)))/d
 + (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d) - (2*A*tan(c/2 + (d*x)/2))/(d*
(a^3*tan(c/2 + (d*x)/2)^2 - a^3)) - (2*atanh(tan(c/2 + (d*x)/2))*(3*A - B))
/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^2(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^2(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c
+ d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)**2/(cos(c + d*x)**3
+ 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

3.64 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=196

$$-\frac{8(19A-9B)\tan(c+dx)}{15a^3d}+\frac{(13A-6B)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{(13A-6B)\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{4(19A-9B)\,\mathrm{t}}{15d\left(a^3\,\mathrm{c}\right.}$$

```
[Out] 1/2*(13*A-6*B)*arctanh(sin(d*x+c))/a^3/d-8/15*(19*A-9*B)*tan(d*x+c)/a^3/d+1
/2*(13*A-6*B)*sec(d*x+c)*tan(d*x+c)/a^3/d-1/5*(A-B)*sec(d*x+c)*tan(d*x+c)/d
/(a+a*cos(d*x+c))^3-1/15*(11*A-6*B)*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+
c))^2-4/15*(19*A-9*B)*sec(d*x+c)*tan(d*x+c)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.54, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{8(19A-9B)\tan(c+dx)}{15a^3d}+\frac{(13A-6B)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{(13A-6B)\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{4(19A-9B)\,\mathrm{t}}{15d\left(a^3\,\mathrm{c}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((13*A - 6*B)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (8*(19*A - 9*B)*Tan[c + d*
x])/(15*a^3*d) + ((13*A - 6*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((A -
 B)*Sec[c + d*x]*Tan[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - ((11*A - 6*B)
*Sec[c + d*x]*Tan[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (4*(19*A - 9*
B)*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(a(7A-2B)-4a(A-B)\cos(c+dx))\sec^3(c+dx}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(11A-6B)\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(11A-6B)\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(11A-6B)\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(13A-6B)\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3}$$

$$= \frac{(13A-6B)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{8(19A-9B)\tan(c+dx)}{15a^3d} + \frac{(13A-6}{}$$

Mathematica [B] time = 5.22, size = 610, normalized size = 3.11

$$-\frac{1920(13A-6B)\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -1/480*(1920*(13*A - 6*B)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*
Sec[c/2]*Sec[c]*Sec[c + d*x]^2*((-1235*A + 870*B)*Sin[(d*x)/2] + 5*(761*A -
 366*B)*Sin[(3*d*x)/2] - 4329*A*Sin[c - (d*x)/2] + 2094*B*Sin[c - (d*x)/2]
+ 1989*A*Sin[c + (d*x)/2] - 1314*B*Sin[c + (d*x)/2] - 3575*A*Sin[2*c + (d*x
)/2] + 1650*B*Sin[2*c + (d*x)/2] - 475*A*Sin[c + (3*d*x)/2] + 450*B*Sin[c +
 (3*d*x)/2] + 2005*A*Sin[2*c + (3*d*x)/2] - 1230*B*Sin[2*c + (3*d*x)/2] - 2
275*A*Sin[3*c + (3*d*x)/2] + 1050*B*Sin[3*c + (3*d*x)/2] + 2673*A*Sin[c + (
5*d*x)/2] - 1278*B*Sin[c + (5*d*x)/2] + 105*A*Sin[2*c + (5*d*x)/2] + 90*B*S
in[2*c + (5*d*x)/2] + 1593*A*Sin[3*c + (5*d*x)/2] - 918*B*Sin[3*c + (5*d*x)
/2] - 975*A*Sin[4*c + (5*d*x)/2] + 450*B*Sin[4*c + (5*d*x)/2] + 1325*A*Sin[
2*c + (7*d*x)/2] - 630*B*Sin[2*c + (7*d*x)/2] + 255*A*Sin[3*c + (7*d*x)/2]
- 60*B*Sin[3*c + (7*d*x)/2] + 875*A*Sin[4*c + (7*d*x)/2] - 480*B*Sin[4*c +
(7*d*x)/2] - 195*A*Sin[5*c + (7*d*x)/2] + 90*B*Sin[5*c + (7*d*x)/2] + 304*A
```

```
*Sin[3*c + (9*d*x)/2] - 144*B*Sin[3*c + (9*d*x)/2] + 90*A*Sin[4*c + (9*d*x)
/2] - 30*B*Sin[4*c + (9*d*x)/2] + 214*A*Sin[5*c + (9*d*x)/2] - 114*B*Sin[5*
c + (9*d*x)/2]))/(a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 0.82, size = 295, normalized size = 1.51

$$\frac{15 \left((13 A - 6 B) \cos (dx + c)^5 + 3 (13 A - 6 B) \cos (dx + c)^4 + 3 (13 A - 6 B) \cos (dx + c)^3 + (13 A - 6 B) \cos (dx \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*((13*A - 6*B)*cos(d*x + c)^5 + 3*(13*A - 6*B)*cos(d*x + c)^4 + 3*(
13*A - 6*B)*cos(d*x + c)^3 + (13*A - 6*B)*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1) - 15*((13*A - 6*B)*cos(d*x + c)^5 + 3*(13*A - 6*B)*cos(d*x + c)^4 + 3*
(13*A - 6*B)*cos(d*x + c)^3 + (13*A - 6*B)*cos(d*x + c)^2)*log(-sin(d*x + c
) + 1) - 2*(16*(19*A - 9*B)*cos(d*x + c)^4 + 3*(239*A - 114*B)*cos(d*x + c)
^3 + (479*A - 234*B)*cos(d*x + c)^2 + 15*(3*A - 2*B)*cos(d*x + c) - 15*A)*s
in(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d
*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

giac [A] time = 2.21, size = 233, normalized size = 1.19

$$\frac{\frac{30 (13 A-6 B) \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)}{a^3} - \frac{30 (13 A-6 B) \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right|\right)}{a^3} + \frac{60 \left(7 A \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3 -2 B \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3 -5 A \tan\left(\frac{1}{2} dx+\frac{1}{2} c \cdots\right.\right.}{\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 -1\right)^2 a^3} \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(30*(13*A - 6*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(13*A - 6
*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 60*(7*A*tan(1/2*d*x + 1/2*c)^3
 - 2*B*tan(1/2*d*x + 1/2*c)^3 - 5*A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*d*x
+ 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1/
2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^12*tan(1/2*d*x + 1/2*c)^3
 - 30*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2*d*x + 1/2*c) - 255
*B*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

maple [A] time = 0.18, size = 334, normalized size = 1.70

$$-\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^3}+\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^3}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{3d\,a^3}+\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^3}-\frac{31A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}+\frac{17B\tan\left(\frac{d}{2}\right.}{4d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*A*tan(1/2*d*x+1/2*c)^5+1/20/d/a^3*B*tan(1/2*d*x+1/2*c)^5-2/3/d/
a^3*tan(1/2*d*x+1/2*c)^3*A+1/2/d/a^3*B*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*A*ta
n(1/2*d*x+1/2*c)+17/4/d/a^3*B*tan(1/2*d*x+1/2*c)-13/2/d/a^3*A*ln(tan(1/2*d*
x+1/2*c)-1)+3/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B+7/2/d/a^3*A/(tan(1/2*d*x+1/2
*c)-1)-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*B+1/2/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)^
2+7/2/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*B+13/2/
d/a^3*A*ln(tan(1/2*d*x+1/2*c)+1)-3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B-1/2/d/a
^3*A/(tan(1/2*d*x+1/2*c)+1)^2
```

maxima [B] time = 0.56, size = 377, normalized size = 1.92

$$-\frac{A\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}\right)-3}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(A*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3) - 3*B*(40*sin(d*x + c)/((a^3 - a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(
d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(co
s(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 +
60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3))/d
```

mupad [B] time = 0.28, size = 216, normalized size = 1.10

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(7\,A-2\,B\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,A-2\,B\right)}{d\,\left(a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,\left(A-B\right)}{2\,a^3}+\frac{3\,\left(5\,A-3\,B\right)}{4\,a^3}+\frac{10\,A-2\,B}{4\,a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(7*A - 2*B) - tan(c/2 + (d*x)/2)*(5*A - 2*B))/(d*(a^3
*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2)^2 + a^3)) - (tan(c/2 + (d*
x)/2)*((3*(A - B))/(2*a^3) + (3*(5*A - 3*B))/(4*a^3) + (10*A - 2*B)/(4*a^3)
))/d - (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^3) + (5*A - 3*B)/(12*a^3)))/d -
(tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d) + (atanh(tan(c/2 + (d*x)/2))*(13*
A - 6*B))/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^3(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx+\int \frac{B\cos(c+dx)\sec^3(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c
+ d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)**3/(cos(c + d*x)**3
+ 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

3.65 $\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=229

$$\frac{8(83A-216B)\sin(c+dx)}{105a^4d}+\frac{(52A-129B)\sin(c+dx)\cos^3(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{4(83A-216B)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)}-(8$$

```
[Out] -1/2*(8*A-21*B)*x/a^4+8/105*(83*A-216*B)*sin(d*x+c)/a^4/d-1/2*(8*A-21*B)*co
s(d*x+c)*sin(d*x+c)/a^4/d+1/105*(52*A-129*B)*cos(d*x+c)^3*sin(d*x+c)/a^4/d/
(1+cos(d*x+c))^2+4/105*(83*A-216*B)*cos(d*x+c)^2*sin(d*x+c)/a^4/d/(1+cos(d*
x+c))+1/7*(A-B)*cos(d*x+c)^5*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+1/5*(A-2*B)*co
s(d*x+c)^4*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.67, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2977, 2734}

$$\frac{8(83A-216B)\sin(c+dx)}{105a^4d}+\frac{(52A-129B)\sin(c+dx)\cos^3(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{4(83A-216B)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)}-(8$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] -((8*A - 21*B)*x)/(2*a^4) + (8*(83*A - 216*B)*Sin[c + d*x])/(105*a^4*d) - (
(8*A - 21*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) + ((52*A - 129*B)*Cos[c +
 d*x]^3*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + (4*(83*A - 216*B)*
Cos[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[
c + d*x]^5*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((A - 2*B)*Cos[c +
d*x]^4*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
```

```
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^4(c+dx)(5a(A-B)-a(2A-9B)\cos(c+dx)}{(a+a\cos(c+dx))^3}}{7a^2}$$

$$= \frac{(A-B)\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(A-2B)\cos^4(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^3}$$

$$= \frac{(52A-129B)\cos^3(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^5(c+dx)\sin(c+}{7d(a+a\cos(c+dx))^4}$$

$$= \frac{(52A-129B)\cos^3(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^5(c+dx)\sin(c+}{7d(a+a\cos(c+dx))^4}$$

$$= -\frac{(8A-21B)x}{2a^4} + \frac{8(83A-216B)\sin(c+dx)}{105a^4d} - \frac{(8A-21B)\cos(c+dx}{2a^4d}$$

Mathematica [B] time = 1.42, size = 555, normalized size = 2.42

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-14700dx(8A-21B)\cos\left(c+\frac{dx}{2}\right)-14700dx(8A-21B)\cos\left(\frac{dx}{2}\right)-184520A\sin\left(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-14700*(8*A - 21*B)*d*x*Cos[(d*x)/2] - 14700*(8
*A - 21*B)*d*x*Cos[c + (d*x)/2] - 70560*A*d*x*Cos[c + (3*d*x)/2] + 185220*B
*d*x*Cos[c + (3*d*x)/2] - 70560*A*d*x*Cos[2*c + (3*d*x)/2] + 185220*B*d*x*C
os[2*c + (3*d*x)/2] - 23520*A*d*x*Cos[2*c + (5*d*x)/2] + 61740*B*d*x*Cos[2*
c + (5*d*x)/2] - 23520*A*d*x*Cos[3*c + (5*d*x)/2] + 61740*B*d*x*Cos[3*c + (
5*d*x)/2] - 3360*A*d*x*Cos[3*c + (7*d*x)/2] + 8820*B*d*x*Cos[3*c + (7*d*x)/
2] - 3360*A*d*x*Cos[4*c + (7*d*x)/2] + 8820*B*d*x*Cos[4*c + (7*d*x)/2] + 24
3320*A*Sin[(d*x)/2] - 539490*B*Sin[(d*x)/2] - 184520*A*Sin[c + (d*x)/2] + 3
86190*B*Sin[c + (d*x)/2] + 184464*A*Sin[c + (3*d*x)/2] - 422478*B*Sin[c + (
3*d*x)/2] - 72240*A*Sin[2*c + (3*d*x)/2] + 132930*B*Sin[2*c + (3*d*x)/2] +
```

```
77168*A*Sin[2*c + (5*d*x)/2] - 181461*B*Sin[2*c + (5*d*x)/2] - 8400*A*Sin[3
*c + (5*d*x)/2] + 3675*B*Sin[3*c + (5*d*x)/2] + 15164*A*Sin[3*c + (7*d*x)/2
] - 36003*B*Sin[3*c + (7*d*x)/2] + 2940*A*Sin[4*c + (7*d*x)/2] - 9555*B*Sin
[4*c + (7*d*x)/2] + 420*A*Sin[4*c + (9*d*x)/2] - 945*B*Sin[4*c + (9*d*x)/2]
 + 420*A*Sin[5*c + (9*d*x)/2] - 945*B*Sin[5*c + (9*d*x)/2] + 105*B*Sin[5*c
+ (11*d*x)/2] + 105*B*Sin[6*c + (11*d*x)/2]))/(6720*a^4*d*(1 + Cos[c + d*x]
)^4)
```

fricas [A] time = 0.78, size = 238, normalized size = 1.04

$$-\frac{105\,(8\,A-21\,B)dx\cos\left(dx+c\right)^4+420\,(8\,A-21\,B)dx\cos\left(dx+c\right)^3+630\,(8\,A-21\,B)dx\cos\left(dx+c\right)^2+420\,(8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] -1/210*(105*(8*A - 21*B)*d*x*cos(d*x + c)^4 + 420*(8*A - 21*B)*d*x*cos(d*x
+ c)^3 + 630*(8*A - 21*B)*d*x*cos(d*x + c)^2 + 420*(8*A - 21*B)*d*x*cos(d*x
 + c) + 105*(8*A - 21*B)*d*x - (105*B*cos(d*x + c)^5 + 210*(A - 2*B)*cos(d*
x + c)^4 + 4*(592*A - 1509*B)*cos(d*x + c)^3 + 4*(1318*A - 3411*B)*cos(d*x
+ c)^2 + (4472*A - 11619*B)*cos(d*x + c) + 1328*A - 3456*B)*sin(d*x + c))/(
a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.52, size = 233, normalized size = 1.02

$$-\frac{\frac{420\,(dx+c)(8\,A-21\,B)}{a^4}-\frac{840\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(420*(d*x + c)*(8*A - 21*B)/a^4 - 840*(2*A*tan(1/2*d*x + 1/2*c)^3 -
9*B*tan(1/2*d*x + 1/2*c)^3 + 2*A*tan(1/2*d*x + 1/2*c) - 7*B*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*
c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*A*a^24*tan(1/2*d*x + 1/2*c)^5
 + 189*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)^3 -
1365*B*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*A*a^24*tan(1/2*d*x + 1/2*c) + 116
55*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

maple [A] time = 0.09, size = 332, normalized size = 1.45

$$-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{56d\,a^{4}}+\frac{B\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56d\,a^{4}}+\frac{7A\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^{4}}-\frac{9B\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^{4}}-\frac{23\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{24d\,a^{4}}+\frac{13B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7+7/40/d
/a^4*A*tan(1/2*d*x+1/2*c)^5-9/40/d/a^4*B*tan(1/2*d*x+1/2*c)^5-23/24/d/a^4*t
an(1/2*d*x+1/2*c)^3*A+13/8/d/a^4*B*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*A*tan(1/
2*d*x+1/2*c)-111/8/d/a^4*B*tan(1/2*d*x+1/2*c)+2/d/a^4/(1+tan(1/2*d*x+1/2*c)
^2)^2*tan(1/2*d*x+1/2*c)^3*A-9/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d
*x+1/2*c)^3+2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*A*tan(1/2*d*x+1/2*c)-7/d/a^4
/(1+tan(1/2*d*x+1/2*c)^2)^2*B*tan(1/2*d*x+1/2*c)-8/d/a^4*arctan(tan(1/2*d*x
+1/2*c))*A+21/d/a^4*arctan(tan(1/2*d*x+1/2*c))*B
```

maxima [A] time = 0.78, size = 364, normalized size = 1.59

$$-\frac{3B\left(\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{4}+\frac{2a^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{4}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{63\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{5\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a^{4}}-\frac{5880\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{4}}\right)-A\left(\frac{}{\left(a^{4}+\right.}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(3*B*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
- 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4) - A*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*
x + c) + 1) - 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4))/d
```

mupad [B] time = 0.31, size = 259, normalized size = 1.13

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{5\,(A-B)}{4\,a^4}-\frac{5\,B}{2\,a^4}+\frac{3\,(4\,A-6\,B)}{4\,a^4}+\frac{3\,(5\,A-15\,B)}{8\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{A-B}{4\,a^4}+\frac{4\,A-6\,B}{8\,a^4}+\frac{5\,A-15\,B}{24\,a^4}\right)}{d}-\frac{x\,(8\,A-21\,B)}{2\,a^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((5*(A - B))/(4*a^4) - (5*B)/(2*a^4) + (3*(4*A - 6*B))/
(4*a^4) + (3*(5*A - 15*B))/(8*a^4)))/d - (tan(c/2 + (d*x)/2)^3*((A - B)/(4*
a^4) + (4*A - 6*B)/(8*a^4) + (5*A - 15*B)/(24*a^4)))/d - (x*(8*A - 21*B))/(
2*a^4) + (tan(c/2 + (d*x)/2)^3*(2*A - 9*B) + tan(c/2 + (d*x)/2)*(2*A - 7*B)
)/(d*(2*a^4*tan(c/2 + (d*x)/2)^2 + a^4*tan(c/2 + (d*x)/2)^4 + a^4)) + (tan(
c/2 + (d*x)/2)^5*((3*(A - B))/(40*a^4) + (4*A - 6*B)/(40*a^4)))/d - (tan(c/
2 + (d*x)/2)^7*(A - B))/(56*a^4*d)
```

sympy [A] time = 33.04, size = 1085, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-3360*A*d*x*tan(c/2 + d*x/2)**4/(840*a**4*d*tan(c/2 + d*x/2)**4
+ 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 6720*A*d*x*tan(c/2 + d*x/
2)**2/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 8
40*a**4*d) - 3360*A*d*x/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c
/2 + d*x/2)**2 + 840*a**4*d) - 15*A*tan(c/2 + d*x/2)**11/(840*a**4*d*tan(c/
2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 117*A*tan(c
/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/
2)**2 + 840*a**4*d) - 526*A*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2
)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 3682*A*tan(c/2 + d*x
/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 +
840*a**4*d) + 11165*A*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**4 +
 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*A*tan(c/2 + d*x/2)/(8
40*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*
d) + 8820*B*d*x*tan(c/2 + d*x/2)**4/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*
a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 17640*B*d*x*tan(c/2 + d*x/2)**2/
(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**
4*d) + 8820*B*d*x/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d
*x/2)**2 + 840*a**4*d) + 15*B*tan(c/2 + d*x/2)**11/(840*a**4*d*tan(c/2 + d*
x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 159*B*tan(c/2 + d
```

```
*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2
+ 840*a**4*d) + 1002*B*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**4
+ 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 9114*B*tan(c/2 + d*x/2)**
5/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a
**4*d) - 29505*B*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**4 + 1680
*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 17535*B*tan(c/2 + d*x/2)/(840*a
**4*d*tan(c/2 + d*x/2)**4 + 1680*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d),
Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**5/(a*cos(c) + a)**4, True))
```

3.66 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=185

$$-\frac{(55A-244B)\sin(c+dx)}{105a^4d}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(A-4B)\sin(c+dx)}{a^4d(\cos(c+dx)+1)}+\frac{x(A-4B)}{a^4}+\frac{(A-B)\,\mathrm{s}\cdots}{7d(a\cdots}$$

```
[Out] (A-4*B)*x/a^4-1/105*(55*A-244*B)*sin(d*x+c)/a^4/d+1/105*(25*A-88*B)*cos(d*x
+c)^2*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2-(A-4*B)*sin(d*x+c)/a^4/d/(1+cos(d*x
+c))+1/7*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+1/35*(5*A-12*B)
*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.68, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2977, 2968, 3023, 12, 2735, 2648}

$$-\frac{(55A-244B)\sin(c+dx)}{105a^4d}+\frac{(25A-88B)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(A-4B)\sin(c+dx)}{a^4d(\cos(c+dx)+1)}+\frac{x(A-4B)}{a^4}+\frac{(A-B)\,\mathrm{s}\cdots}{7d(a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] ((A - 4*B)*x)/a^4 - ((55*A - 244*B)*Sin[c + d*x])/(105*a^4*d) + ((25*A - 88
*B)*Cos[c + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) - ((A - 4
*B)*Sin[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^4*Sin[
c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((5*A - 12*B)*Cos[c + d*x]^3*Sin[c
 + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^3(c+dx)(4a(A-B)-a(A-8B)\cos(c+dx))}{(a+a\cos(c+dx))^3}\,d}{7a^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(5A-12B)\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}$$

$$= \frac{(25A-88B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}$$

$$= \frac{(25A-88B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4}$$

$$= -\frac{(55A-244B)\sin(c+dx)}{105a^4d} + \frac{(25A-88B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} +$$

$$= -\frac{(55A-244B)\sin(c+dx)}{105a^4d} + \frac{(25A-88B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} +$$

$$= \frac{(A-4B)x}{a^4} - \frac{(55A-244B)\sin(c+dx)}{105a^4d} + \frac{(25A-88B)\cos^2(c+dx)\sin}{105a^4d(1+\cos(c+dx)}$$

$$= \frac{(A-4B)x}{a^4} - \frac{(55A-244B)\sin(c+dx)}{105a^4d} + \frac{(25A-88B)\cos^2(c+dx)\sin}{105a^4d(1+\cos(c+dx)}$$

Mathematica [B] time = 0.98, size = 481, normalized size = 2.60

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(7350dx(A-4B)\cos\left(c+\frac{dx}{2}\right)+7350dx(A-4B)\cos\left(\frac{dx}{2}\right)+16520A\sin\left(c+\frac{dx}{2}\right)-14280\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(7350*(A - 4*B)*d*x*Cos[(d*x)/2] + 7350*(A - 4*B
)*d*x*Cos[c + (d*x)/2] + 4410*A*d*x*Cos[c + (3*d*x)/2] - 17640*B*d*x*Cos[c
+ (3*d*x)/2] + 4410*A*d*x*Cos[2*c + (3*d*x)/2] - 17640*B*d*x*Cos[2*c + (3*d
*x)/2] + 1470*A*d*x*Cos[2*c + (5*d*x)/2] - 5880*B*d*x*Cos[2*c + (5*d*x)/2]
+ 1470*A*d*x*Cos[3*c + (5*d*x)/2] - 5880*B*d*x*Cos[3*c + (5*d*x)/2] + 210*A
*d*x*Cos[3*c + (7*d*x)/2] - 840*B*d*x*Cos[3*c + (7*d*x)/2] + 210*A*d*x*Cos[
4*c + (7*d*x)/2] - 840*B*d*x*Cos[4*c + (7*d*x)/2] - 19880*A*Sin[(d*x)/2] +
60830*B*Sin[(d*x)/2] + 16520*A*Sin[c + (d*x)/2] - 46130*B*Sin[c + (d*x)/2]
```

```
- 14280*A*Sin[c + (3*d*x)/2] + 46116*B*Sin[c + (3*d*x)/2] + 7560*A*Sin[2*c
+ (3*d*x)/2] - 18060*B*Sin[2*c + (3*d*x)/2] - 5600*A*Sin[2*c + (5*d*x)/2] +
 19292*B*Sin[2*c + (5*d*x)/2] + 1680*A*Sin[3*c + (5*d*x)/2] - 2100*B*Sin[3*
c + (5*d*x)/2] - 1040*A*Sin[3*c + (7*d*x)/2] + 3791*B*Sin[3*c + (7*d*x)/2]
+ 735*B*Sin[4*c + (7*d*x)/2] + 105*B*Sin[4*c + (9*d*x)/2] + 105*B*Sin[5*c +
 (9*d*x)/2]))/(1680*a^4*d*(1 + Cos[c + d*x])^4)
```

fricas [A] time = 0.75, size = 213, normalized size = 1.15

$$\frac{105\,(A-4\,B)dx\cos\left(dx+c\right)^4+420\,(A-4\,B)dx\cos\left(dx+c\right)^3+630\,(A-4\,B)dx\cos\left(dx+c\right)^2+420\,(A-4\,B)d}{105\left(a^4d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*(105*(A - 4*B)*d*x*cos(d*x + c)^4 + 420*(A - 4*B)*d*x*cos(d*x + c)^3
+ 630*(A - 4*B)*d*x*cos(d*x + c)^2 + 420*(A - 4*B)*d*x*cos(d*x + c) + 105*(
A - 4*B)*d*x + (105*B*cos(d*x + c)^4 - 4*(65*A - 296*B)*cos(d*x + c)^3 - 4*
(155*A - 659*B)*cos(d*x + c)^2 - (535*A - 2236*B)*cos(d*x + c) - 160*A + 66
4*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d
*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.79, size = 188, normalized size = 1.02

$$\frac{\frac{840\,(dx+c)(A-4\,B)}{a^4}+\frac{1680\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+147\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\right.}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(840*(d*x + c)*(A - 4*B)/a^4 + 1680*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*
d*x + 1/2*c)^2 + 1)*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*ta
n(1/2*d*x + 1/2*c)^7 - 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 147*B*a^24*tan(1
/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 805*B*a^24*tan(1/2*
d*x + 1/2*c)^3 - 1575*A*a^24*tan(1/2*d*x + 1/2*c) + 5145*B*a^24*tan(1/2*d*x
 + 1/2*c))/a^28)/d
```

maple [A] time = 0.10, size = 229, normalized size = 1.24

$$\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{56d\,a^4}-\frac{B\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56d\,a^4}-\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}+\frac{7B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^4}+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{24d\,a^4}-\frac{23B\left(\tan\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7-1/8/d/a
^4*A*tan(1/2*d*x+1/2*c)^5+7/40/d/a^4*B*tan(1/2*d*x+1/2*c)^5+11/24/d/a^4*tan
(1/2*d*x+1/2*c)^3*A-23/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*A*tan(1/2
*d*x+1/2*c)+49/8/d/a^4*B*tan(1/2*d*x+1/2*c)+2/d/a^4*B*tan(1/2*d*x+1/2*c)/(1
+tan(1/2*d*x+1/2*c)^2)+2/d/a^4*arctan(tan(1/2*d*x+1/2*c))*A-8/d/a^4*arctan(
tan(1/2*d*x+1/2*c))*B
```

maxima [A] time = 0.66, size = 271, normalized size = 1.46

$$\frac{B\left(\frac{1680\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-5A\left(\frac{315\sin(\ldots}{\cos(dx+\ldots}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(B*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a^4) - 5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*s
in(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^4))/d
```

mupad [B] time = 0.39, size = 201, normalized size = 1.09

$$\frac{A\,d\,x-4\,B\,d\,x}{a^4\,d}-\frac{\left(\frac{52\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}-\frac{764\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{105}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{143\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{105}-\frac{16\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

```
[Out] (A*d*x - 4*B*d*x)/(a^4*d) - ((B*sin(c/2 + (d*x)/2))/56 - (A*sin(c/2 + (d*x)
/2))/56 + cos(c/2 + (d*x)/2)^2*((5*A*sin(c/2 + (d*x)/2))/28 - (8*B*sin(c/2
+ (d*x)/2))/35) - cos(c/2 + (d*x)/2)^4*((16*A*sin(c/2 + (d*x)/2))/21 - (143
*B*sin(c/2 + (d*x)/2))/105) + cos(c/2 + (d*x)/2)^6*((52*A*sin(c/2 + (d*x)/2
))/21 - (764*B*sin(c/2 + (d*x)/2))/105))/(a^4*d*cos(c/2 + (d*x)/2)^7) + (2*
B*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))/(a^4*d)
```

sympy [A] time = 21.68, size = 578, normalized size = 3.12

$$\begin{cases} \frac{840Adx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{840a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^4d} + \frac{840Adx}{840a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^4d} + \frac{15A\tan^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{840a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^4d} - \frac{90A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{840a^4d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+840a^4d} + \frac{280A\tan \\ \frac{x(A+B\cos(c))\cos^4(c)}{(a\cos(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((840*A*d*x*tan(c/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 +
840*a**4*d) + 840*A*d*x/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 15*
A*tan(c/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 90*A*
tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 280*A*t
an(c/2 + d*x/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 1190*A*t
an(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 1575*A*t
an(c/2 + d*x/2)/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 3360*B*d*x*
tan(c/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 3360*B*
d*x/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 15*B*tan(c/2 + d*x/2)**
9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 132*B*tan(c/2 + d*x/2)**7
/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 658*B*tan(c/2 + d*x/2)**5/
(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 4340*B*tan(c/2 + d*x/2)**3/
(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*B*tan(c/2 + d*x/2)/(84
0*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d), Ne(d, 0)), (x*(A + B*cos(c))*co
s(c)**4/(a*cos(c) + a)**4, True))
```

3.67 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=154

$$\frac{(12A-215B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(6A-55B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{Bx}{a^4}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{(3A-10B)\sin(c+}{35ad(a\cos(c}$$

```
[Out] B*x/a^4-1/105*(6*A-55*B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2+1/105*(12*A-215*
B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c))+1/7*(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a
*cos(d*x+c))^4+1/35*(3*A-10*B)*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))
^3
```

Rubi [A] time = 0.50, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2968, 3019, 2735, 2648}

$$\frac{(12A-215B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(6A-55B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{Bx}{a^4}+\frac{(A-B)\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}+\frac{(3A-10B)\sin(c+}{35ad(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (B*x)/a^4 - ((6*A - 55*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) +
((12*A - 215*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos
[c + d*x]^3*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((3*A - 10*B)*Cos[
c + d*x]^2*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos^2(c+dx)(3a(A-B)+7aB\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A-10B)\cos^2(c+dx)\sin(c+d}{35ad(a+a\cos(c+dx))^3}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A-10B)\cos^2(c+dx)\sin(c+d}{35ad(a+a\cos(c+dx))^3}$$

$$= -\frac{(6A-55B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A}{}$$

$$= \frac{Bx}{a^4} - \frac{(6A-55B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} +$$

$$= \frac{Bx}{a^4} - \frac{(6A-55B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} +$$

Mathematica [B] time = 0.83, size = 329, normalized size = 2.14

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(-1260A\sin\left(c+\frac{dx}{2}\right)+882A\sin\left(c+\frac{3dx}{2}\right)-630A\sin\left(2c+\frac{3dx}{2}\right)+294A\sin\left(2c+\frac{5dx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(3675*B*d*x*Cos[(d*x)/2] + 3675*B*d*x*Cos[c +
(d*x)/2] + 2205*B*d*x*Cos[c + (3*d*x)/2] + 2205*B*d*x*Cos[2*c + (3*d*x)/2]
+ 735*B*d*x*Cos[2*c + (5*d*x)/2] + 735*B*d*x*Cos[3*c + (5*d*x)/2] + 105*B*d
*x*Cos[3*c + (7*d*x)/2] + 105*B*d*x*Cos[4*c + (7*d*x)/2] + 1260*A*Sin[(d*x)
/2] - 9940*B*Sin[(d*x)/2] - 1260*A*Sin[c + (d*x)/2] + 8260*B*Sin[c + (d*x)/
2] + 882*A*Sin[c + (3*d*x)/2] - 7140*B*Sin[c + (3*d*x)/2] - 630*A*Sin[2*c +
 (3*d*x)/2] + 3780*B*Sin[2*c + (3*d*x)/2] + 294*A*Sin[2*c + (5*d*x)/2] - 28
00*B*Sin[2*c + (5*d*x)/2] - 210*A*Sin[3*c + (5*d*x)/2] + 840*B*Sin[3*c + (5
*d*x)/2] + 72*A*Sin[3*c + (7*d*x)/2] - 520*B*Sin[3*c + (7*d*x)/2]))/(13440*
a^4*d)
```

fricas [A] time = 0.91, size = 180, normalized size = 1.17

$$\frac{105\,Bdx\cos\left(dx+c\right)^4+420\,Bdx\cos\left(dx+c\right)^3+630\,Bdx\cos\left(dx+c\right)^2+420\,Bdx\cos\left(dx+c\right)+105\,Bdx+\left(4\,(9\right.}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*(105*B*d*x*cos(d*x + c)^4 + 420*B*d*x*cos(d*x + c)^3 + 630*B*d*x*cos(
d*x + c)^2 + 420*B*d*x*cos(d*x + c) + 105*B*d*x + (4*(9*A - 65*B)*cos(d*x +
 c)^3 + (39*A - 620*B)*cos(d*x + c)^2 + (24*A - 535*B)*cos(d*x + c) + 6*A -
 160*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^
4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [A] time = 0.40, size = 155, normalized size = 1.01

$$\frac{\frac{840\,(dx+c)B}{a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-63\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+105\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-385}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(840*(d*x + c)*B/a^4 - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*
tan(1/2*d*x + 1/2*c)^7 - 63*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 105*B*a^24*tan(
1/2*d*x + 1/2*c)^5 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*tan(1/2
*d*x + 1/2*c)^3 - 105*A*a^24*tan(1/2*d*x + 1/2*c) + 1575*B*a^24*tan(1/2*d*x
 + 1/2*c))/a^28)/d
```

maple [A] time = 0.08, size = 177, normalized size = 1.15

$$-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{56d\,a^4}+\frac{B\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56d\,a^4}+\frac{3A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^4}-\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{8d\,a^4}+\frac{11B\left(\tan\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7+3/40/d
/a^4*A*tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*B*tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*tan(
1/2*d*x+1/2*c)^3*A+11/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3+1/8/d/a^4*A*tan(1/2*d
*x+1/2*c)-15/8/d/a^4*B*tan(1/2*d*x+1/2*c)+2/d/a^4*arctan(tan(1/2*d*x+1/2*c)
)*B
```

maxima [A] time = 0.76, size = 201, normalized size = 1.31

$$-\frac{5\,B\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-\frac{3\,A\left(\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\right.}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(5*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^
4) - 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4)/d
```

mupad [B] time = 0.34, size = 162, normalized size = 1.05

$$\frac{B\,x}{a^4}+\frac{\left(\frac{12\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}-\frac{52\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{16\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}-\frac{23\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{70}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{9\,A\,\sin\left(\frac{c}{2}\right.}{70}\right.}{a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

```
[Out] (B*x)/a^4 + ((B*sin(c/2 + (d*x)/2))/56 - (A*sin(c/2 + (d*x)/2))/56 + cos(c/
2 + (d*x)/2)^2*((9*A*sin(c/2 + (d*x)/2))/70 - (5*B*sin(c/2 + (d*x)/2))/28)
+ cos(c/2 + (d*x)/2)^6*((12*A*sin(c/2 + (d*x)/2))/35 - (52*B*sin(c/2 + (d*x
)/2))/21) - cos(c/2 + (d*x)/2)^4*((23*A*sin(c/2 + (d*x)/2))/70 - (16*B*sin(
c/2 + (d*x)/2))/21))/(a^4*d*cos(c/2 + (d*x)/2)^7)
```

sympy [A] time = 13.27, size = 192, normalized size = 1.25

$$\begin{cases} -\frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} + \frac{3A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{Bx}{a^4} + \frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{11B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} - \cdots \\ \frac{x(A+B\cos(c))\cos^3(c)}{(a\cos(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*A*tan(c/2 + d*x/2)**5/(40
*a**4*d) - A*tan(c/2 + d*x/2)**3/(8*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
 + B*x/a**4 + B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/(8*
a**4*d) + 11*B*tan(c/2 + d*x/2)**3/(24*a**4*d) - 15*B*tan(c/2 + d*x/2)/(8*a
**4*d), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**3/(a*cos(c) + a)**4, True))
```

3.68 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=136

$$\frac{(13A+36B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{2(A+27B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{(A-8B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)}$$

```
[Out] -2/105*(A+27*B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2+1/105*(13*A+36*B)*sin(d*x
+c)/a^4/d/(1+cos(d*x+c))+1/7*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c
))^4-1/35*(A-8*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.35, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2977, 2968, 3019, 2750, 2648}

$$\frac{(13A+36B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{2(A+27B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{(A-8B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-2*(A + 27*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + ((13*A + 36
*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^2*
Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((A - 8*B)*Sin[c + d*x])/(35*a
*d*(a + a*Cos[c + d*x])^3)
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{\cos(c+dx)(2a(A-B)+a(A+6B)\cos(c+dx))}{(a+a\cos(c+dx))^3}\,d}{7a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{2a(A-B)\cos(c+dx)+a(A+6B)\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,d}{7a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(A-8B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{\int \frac{-3a^2(A}{}}{}$$

$$= -\frac{2(A+27B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(A}{35ad(}$$

$$= -\frac{2(A+27B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(A}{35ad(}$$

Mathematica [A] time = 0.50, size = 193, normalized size = 1.42

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-35(5A+18B)\sin\left(c+\frac{dx}{2}\right)+70(4A+9B)\sin\left(\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)-105A\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(70*(4*A + 9*B)*Sin[(d*x)/2] - 35*(5*A + 18*B)*S
in[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 441*B*Sin[c + (3*d*x)/2] - 105
*A*Sin[2*c + (3*d*x)/2] - 315*B*Sin[2*c + (3*d*x)/2] + 91*A*Sin[2*c + (5*d*
x)/2] + 147*B*Sin[2*c + (5*d*x)/2] - 105*B*Sin[3*c + (5*d*x)/2] + 13*A*Sin[
3*c + (7*d*x)/2] + 36*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c + d*x]
)^4)
```

fricas [A] time = 0.74, size = 124, normalized size = 0.91

$$\frac{\left((13\,A+36\,B)\cos\left(dx+c\right)^{3}+13\,(4\,A+3\,B)\cos\left(dx+c\right)^{2}+8\,(4\,A+3\,B)\cos\left(dx+c\right)+8\,A+6\,B\right)\sin\left(dx+c\right)}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*((13*A + 36*B)*cos(d*x + c)^3 + 13*(4*A + 3*B)*cos(d*x + c)^2 + 8*(4*
A + 3*B)*cos(d*x + c) + 8*A + 6*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a
^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d
)
```

giac [A] time = 0.62, size = 117, normalized size = 0.86

$$\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+63\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,A\tan\left(\frac{1}{2}\,c\right.}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 - 21*A*tan
(1/2*d*x + 1/2*c)^5 + 63*B*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2*
c)^3 - 105*B*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/2*c) + 105*B*ta
n(1/2*d*x + 1/2*c))/(a^4*d)
```

maple [A] time = 0.08, size = 90, normalized size = 0.66

$$\frac{\frac{(A-B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(-A+3B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(-A-3B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(A-B)*tan(1/2*d*x+1/2*c)^7+1/5*(-A+3*B)*tan(1/2*d*x+1/2*c)^5
+1/3*(-A-3*B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c
))
```

maxima [A] time = 0.36, size = 175, normalized size = 1.29

$$\frac{\frac{A\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{21\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{a^{4}}+\frac{3B\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{21\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{5\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{a^{4}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

mupad [B] time = 0.25, size = 86, normalized size = 0.63

$$-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\,(A+3\,B)}{24\,a^{4}}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}\,(A-3\,B)}{40\,a^{4}}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\,(A-B)}{56\,a^{4}}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A+B)}{8\,a^{4}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

```
[Out] -((tan(c/2 + (d*x)/2)^3*(A + 3*B))/(24*a^4) + (tan(c/2 + (d*x)/2)^5*(A - 3*
B))/(40*a^4) - (tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4) - (tan(c/2 + (d*x)/2
)*(A + B))/(8*a^4))/d
```

sympy [A] time = 9.24, size = 182, normalized size = 1.34

$$\begin{cases} \frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} - \frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d} + \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} - \frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} + \frac{3B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} \\ \frac{x(A+B\cos(c))\cos^2(c)}{(a\cos(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**7/(56*a**4*d) - A*tan(c/2 + d*x/2)**5/(40*a*
*4*d) - A*tan(c/2 + d*x/2)**3/(24*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d) -
 B*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*B*tan(c/2 + d*x/2)**5/(40*a**4*d) -
B*tan(c/2 + d*x/2)**3/(8*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0))
, (x*(A + B*cos(c))*cos(c)**2/(a*cos(c) + a)**4, True))
```

3.69 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{(8A+13B)\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{(8A+13B)\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{(4A-11B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

```
[Out] -1/7*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+1/35*(4*A-11*B)*sin(d*x+c)/a/d/(
a+a*cos(d*x+c))^3+1/105*(8*A+13*B)*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+1/10
5*(8*A+13*B)*sin(d*x+c)/d/(a^4+a^4*cos(d*x+c))
```

Rubi [A] time = 0.21, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2968, 3019, 2750, 2650, 2648}

$$\frac{(8A+13B)\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{(8A+13B)\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{(4A-11B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] -((A - B)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((4*A - 11*B)*Sin[c
+ d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + ((8*A + 13*B)*Sin[c + d*x])/(105*
d*(a^2 + a^2*Cos[c + d*x])^2) + ((8*A + 13*B)*Sin[c + d*x])/(105*d*(a^4 + a
^4*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{-4a(A-B)-7aB\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(4A-11B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(8A+13B)\int \frac{\ }{(a}}{35}$$

$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(4A-11B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(8A+13B)\,\mathrm{si}}{105d\left(a^2+a^2\,\mathrm{co}\right.}$$

$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(4A-11B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(8A+13B)\,\mathrm{si}}{105d\left(a^2+a^2\,\mathrm{co}\right.}$$

Mathematica [A] time = 0.43, size = 163, normalized size = 1.18

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-35(4A+5B)\sin\left(c+\frac{dx}{2}\right)+140(A+2B)\sin\left(\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)+56A\sin\left(2c\right.\right.}{420a^4d(\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(140*(A + 2*B)*Sin[(d*x)/2] - 35*(4*A + 5*B)*Sin
[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3*d*x)/2] - 105*B
*Sin[2*c + (3*d*x)/2] + 56*A*Sin[2*c + (5*d*x)/2] + 91*B*Sin[2*c + (5*d*x)/
2] + 8*A*Sin[3*c + (7*d*x)/2] + 13*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 +
 Cos[c + d*x])^4)
```

fricas [A] time = 0.62, size = 124, normalized size = 0.90

$$\frac{\left((8\,A+13\,B)\cos\left(dx+c\right)^{3}+4\,(8\,A+13\,B)\cos\left(dx+c\right)^{2}+4\,(13\,A+8\,B)\cos\left(dx+c\right)+13\,A+8\,B\right)\sin\left(dx+c\right)}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] 1/105*((8*A + 13*B)*cos(d*x + c)^3 + 4*(8*A + 13*B)*cos(d*x + c)^2 + 4*(13*
A + 8*B)*cos(d*x + c) + 13*A + 8*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

giac [A] time = 0.58, size = 117, normalized size = 0.85

$$-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,A\tan\left(\frac{1}{2}\,c\right.}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 21*A*ta
n(1/2*d*x + 1/2*c)^5 + 21*B*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2
*c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 - 105*A*tan(1/2*d*x + 1/2*c) - 105*B*ta
n(1/2*d*x + 1/2*c))/(a^4*d)
```

maple [A] time = 0.09, size = 88, normalized size = 0.64

$$\frac{\frac{(-A+B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(-A-B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(A-B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(-A+B)*tan(1/2*d*x+1/2*c)^7+1/5*(-A-B)*tan(1/2*d*x+1/2*c)^5+
1/3*(A-B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.39, size = 174, normalized size = 1.26

$$\frac{\frac{A\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{B\left(\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + B*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

mupad [B] time = 0.25, size = 84, normalized size = 0.61

$$-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A+B)}{40\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A-B)}{24\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(A-B)}{56\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A+B)}{8\,a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^4,x)
```

```
[Out] -((tan(c/2 + (d*x)/2)^5*(A + B))/(40*a^4) - (tan(c/2 + (d*x)/2)^3*(A - B))/
(24*a^4) + (tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4) - (tan(c/2 + (d*x)/2)*(A
 + B))/(8*a^4))/d
```

sympy [A] time = 6.69, size = 178, normalized size = 1.29

$$\begin{cases} -\frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d}-\frac{A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d}+\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}+\frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d}-\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d}-\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d}+\frac{B\tan\left(\frac{c}{2}+\right.}{8a^4d} \\ \frac{x(A+B\cos(c))\cos(c)}{(a\cos(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-A*tan(c/2 + d*x/2)**7/(56*a**4*d) - A*tan(c/2 + d*x/2)**5/(40*a
**4*d) + A*tan(c/2 + d*x/2)**3/(24*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
+ B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/(40*a**4*d) - B
*tan(c/2 + d*x/2)**3/(24*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0))
, (x*(A + B*cos(c))*cos(c)/(a*cos(c) + a)**4, True))
```

3.70 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{2(3A+4B)\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{2(3A+4B)\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{(3A+4B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

```
[Out] 1/7*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+1/35*(3*A+4*B)*sin(d*x+c)/a/d/(a+
a*cos(d*x+c))^3+2/105*(3*A+4*B)*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+2/105*(
3*A+4*B)*sin(d*x+c)/d/(a^4+a^4*cos(d*x+c))
```

Rubi [A] time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2750, 2650, 2648}

$$\frac{2(3A+4B)\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{2(3A+4B)\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{(3A+4B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{(A-B)\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] ((A - B)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) + ((3*A + 4*B)*Sin[c +
d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + (2*(3*A + 4*B)*Sin[c + d*x])/(105*d
*(a^2 + a^2*Cos[c + d*x])^2) + (2*(3*A + 4*B)*Sin[c + d*x])/(105*d*(a^4 + a
^4*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B)\int \frac{1}{(a+a\cos(c+dx))^3}\,dx}{7a}$$

$$= \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{(2(3A+4B))\int \frac{1}{(a+a\cos(c+dx))^2}\,d}{35a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{2(3A+4B)\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2} +$$

$$= \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{(3A+4B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{2(3A+4B)\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2} +$$

Mathematica [A] time = 0.37, size = 109, normalized size = 0.79

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left((3A+4B)\left(21\sin\left(c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{7dx}{2}\right)\right)+35(3A+2B)\sin\left(\frac{dx}{2}\right)\right.}{210a^4d(\cos(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(35*(3*A + 2*B)*Sin[(d*x)/2] - 70*B*Sin[c + (d*x
)/2] + (3*A + 4*B)*(21*Sin[c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2] + Sin[3*
c + (7*d*x)/2])))/(210*a^4*d*(1 + Cos[c + d*x])^4)
```

fricas [A] time = 0.54, size = 125, normalized size = 0.91

$$\frac{\left(2\,(3\,A+4\,B)\cos{(dx+c)}^3+8\,(3\,A+4\,B)\cos{(dx+c)}^2+13\,(3\,A+4\,B)\cos{(dx+c)}+36\,A+13\,B\right)\sin{(dx+c)}}{105\left(a^4d\cos{(dx+c)}^4+4\,a^4d\cos{(dx+c)}^3+6\,a^4d\cos{(dx+c)}^2+4\,a^4d\cos{(dx+c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*A + 4*B)*cos(d*x + c)^3 + 8*(3*A + 4*B)*cos(d*x + c)^2 + 13*(3*
A + 4*B)*cos(d*x + c) + 36*A + 13*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4
```

```
*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4
*d)
```

giac [A] time = 0.93, size = 117, normalized size = 0.85

$$\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+63\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+105\,A\tan\left(\frac{1}{2}\right.}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 63*A*tan
(1/2*d*x + 1/2*c)^5 - 21*B*tan(1/2*d*x + 1/2*c)^5 + 105*A*tan(1/2*d*x + 1/2
*c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/2*c) + 105*B*ta
n(1/2*d*x + 1/2*c))/(a^4*d)
```

maple [A] time = 0.07, size = 88, normalized size = 0.64

$$\frac{\frac{(A-B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(3A-B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(3A+B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*(A-B)*tan(1/2*d*x+1/2*c)^7+1/5*(3*A-B)*tan(1/2*d*x+1/2*c)^5+
1/3*(3*A+B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.45, size = 175, normalized size = 1.27

$$\frac{\frac{B\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{21\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{a^{4}}+\frac{3\,A\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{21\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{5\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{a^{4}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(B*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

mupad [B] time = 0.24, size = 87, normalized size = 0.63

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(3\,A+B)}{24\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(A-B)}{56\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A+B)}{8\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(3\,A-B)}{40\,a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^4,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(3*A + B))/(24*a^4) + (tan(c/2 + (d*x)/2)^7*(A - B))
/(56*a^4) + (tan(c/2 + (d*x)/2)*(A + B))/(8*a^4) + (tan(c/2 + (d*x)/2)^5*(3
*A - B))/(40*a^4))/d
```

sympy [A] time = 4.87, size = 177, normalized size = 1.28

$$\begin{cases} \frac{A\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d}+\frac{3A\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d}+\frac{A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}+\frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d}-\frac{B\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d}-\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d}+\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^4d}+\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} \\ \frac{x(A+B\cos(c))}{(a\cos(c)+a)^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((A*tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*A*tan(c/2 + d*x/2)**5/(40*
a**4*d) + A*tan(c/2 + d*x/2)**3/(8*a**4*d) + A*tan(c/2 + d*x/2)/(8*a**4*d)
- B*tan(c/2 + d*x/2)**7/(56*a**4*d) - B*tan(c/2 + d*x/2)**5/(40*a**4*d) + B
*tan(c/2 + d*x/2)**3/(24*a**4*d) + B*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0))
, (x*(A + B*cos(c))/(a*cos(c) + a)**4, True))
```

3.71 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=147

$$-\frac{2(80A-3B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(55A-6B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(10A-3B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{(A-}{7d(a}$$

```
[Out] A*arctanh(sin(d*x+c))/a^4/d-1/105*(55*A-6*B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c)
)^2-2/105*(80*A-3*B)*sin(d*x+c)/a^4/d/(1+cos(d*x+c))-1/7*(A-B)*sin(d*x+c)/d
/(a+a*cos(d*x+c))^4-1/35*(10*A-3*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.47, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2978, 12, 3770}

$$-\frac{2(80A-3B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)}-\frac{(55A-6B)\sin(c+dx)}{105a^4d(\cos(c+dx)+1)^2}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(10A-3B)\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{(A-}{7d(a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((55*A - 6*B)*Sin[c + d*x])/(105*a^4*d*
(1 + Cos[c + d*x])^2) - (2*(80*A - 3*B)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c
 + d*x])) - ((A - B)*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((10*A -
3*B)*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(7aA-3a(A-B)\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3B)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{(35a^2A-2a^2(10A-}{(a+a\,c}}{}$$

$$= -\frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3B)\sin(}{35ad(a+a\cos(c}$$

$$= -\frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3B)\sin(}{35ad(a+a\cos(c}$$

$$= -\frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(10A-3B)\sin(}{35ad(a+a\cos(c}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(55A-6B)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sin(c+a}{7d(a+a\cos(c+d}$$

Mathematica [A] time = 1.59, size = 239, normalized size = 1.63

$$\frac{\sec\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(-70(49A-3B)\sin\left(\frac{dx}{2}\right)+2170A\sin\left(c+\frac{dx}{2}\right)-2625A\sin\left(c+\frac{3dx}{2}\right)+735A\sin\left(2c+\frac{3}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-6720*A*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-70*(49
*A - 3*B)*Sin[(d*x)/2] + 2170*A*Sin[c + (d*x)/2] - 2625*A*Sin[c + (3*d*x)/2
] + 126*B*Sin[c + (3*d*x)/2] + 735*A*Sin[2*c + (3*d*x)/2] - 1015*A*Sin[2*c
+ (5*d*x)/2] + 42*B*Sin[2*c + (5*d*x)/2] + 105*A*Sin[3*c + (5*d*x)/2] - 160
*A*Sin[3*c + (7*d*x)/2] + 6*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c
+ d*x])^4)
```

fricas [A] time = 0.78, size = 236, normalized size = 1.61

$$\frac{105\left(A\cos(dx+c)^4+4\,A\cos(dx+c)^3+6\,A\cos(dx+c)^2+4\,A\cos(dx+c)+A\right)\log(\sin(dx+c)+1)-105}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] 1/210*(105*(A*cos(d*x + c)^4 + 4*A*cos(d*x + c)^3 + 6*A*cos(d*x + c)^2 + 4*
A*cos(d*x + c) + A)*log(sin(d*x + c) + 1) - 105*(A*cos(d*x + c)^4 + 4*A*cos
(d*x + c)^3 + 6*A*cos(d*x + c)^2 + 4*A*cos(d*x + c) + A)*log(-sin(d*x + c)
+ 1) - 2*(2*(80*A - 3*B)*cos(d*x + c)^3 + (535*A - 24*B)*cos(d*x + c)^2 + (
620*A - 39*B)*cos(d*x + c) + 260*A - 36*B)*sin(d*x + c))/(a^4*d*cos(d*x + c
)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c
) + a^4*d)
```

giac [A] time = 0.97, size = 182, normalized size = 1.24

$$\frac{\frac{840\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{840\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6}{}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] 1/840*(840*A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*A*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^4 - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan
(1/2*d*x + 1/2*c)^7 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 63*B*a^24*tan(1/2
*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 105*B*a^24*tan(1/2*d*
x + 1/2*c)^3 + 1575*A*a^24*tan(1/2*d*x + 1/2*c) - 105*B*a^24*tan(1/2*d*x +
1/2*c))/a^28)/d
```

maple [A] time = 0.15, size = 199, normalized size = 1.35

$$\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^4}-\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{24d\,a^4}+\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^4}-\frac{A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/d/a^4*A*ln(tan(1/2*d*x+1/2*c)+1)-11/24/d/a^4*tan(1/2*d*x+1/2*c)^3*A+1/8/d
/a^4*B*tan(1/2*d*x+1/2*c)^3-1/d/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)-1/8/d/a^4*A*
tan(1/2*d*x+1/2*c)^5+3/40/d/a^4*B*tan(1/2*d*x+1/2*c)^5-1/56/d/a^4*tan(1/2*d
*x+1/2*c)^7*A+1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7-15/8/d/a^4*A*tan(1/2*d*x+1/
2*c)+1/8/d/a^4*B*tan(1/2*d*x+1/2*c)
```

maxima [A] time = 0.42, size = 228, normalized size = 1.55

$$-\frac{5\,A\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)-3\,B\left(\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\mathrm{s}}{(\cos}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] -1/840*(5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
^4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - 3*B*(35*sin(d*x +
c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/
d
```

mupad [B] time = 0.36, size = 199, normalized size = 1.35

$$\frac{2\,A\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^4\,d}-\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{11\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}-\frac{B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8}-\frac{3\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{40}\right.}{a^4\,d\,\cos\left(\frac{c}{2}+\frac{d}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^4),x)
```

```
[Out] (2*A*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^4*d) - (cos(c/2 + (d*
x)/2)^4*((11*A*sin(c/2 + (d*x)/2)^3)/24 - (B*sin(c/2 + (d*x)/2)^3)/8) + cos
(c/2 + (d*x)/2)^2*((A*sin(c/2 + (d*x)/2)^5)/8 - (3*B*sin(c/2 + (d*x)/2)^5)/
40) + cos(c/2 + (d*x)/2)^6*((15*A*sin(c/2 + (d*x)/2))/8 - (B*sin(c/2 + (d*x
)/2))/8) + (A*sin(c/2 + (d*x)/2)^7)/56 - (B*sin(c/2 + (d*x)/2)^7)/56)/(a^4*
d*cos(c/2 + (d*x)/2)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.72 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=175

$$\frac{8(83A-20B)\tan(c+dx)}{105a^4d}-\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(4A-B)\tan(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{(88A-25B)\tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(12}{35a}$$

```
[Out] -(4*A-B)*arctanh(sin(d*x+c))/a^4/d+8/105*(83*A-20*B)*tan(d*x+c)/a^4/d-1/105
*(88*A-25*B)*tan(d*x+c)/a^4/d/(1+cos(d*x+c))^2-(4*A-B)*tan(d*x+c)/a^4/d/(1+
cos(d*x+c))-1/7*(A-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^4-1/35*(12*A-5*B)*tan(d
*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.67, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2748, 3767, 8, 3770}

$$\frac{8(83A-20B)\tan(c+dx)}{105a^4d}-\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(4A-B)\tan(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{(88A-25B)\tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{(12}{35a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] -(((4*A - B)*ArcTanh[Sin[c + d*x]])/(a^4*d)) + (8*(83*A - 20*B)*Tan[c + d*x
])/(105*a^4*d) - ((88*A - 25*B)*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])
^2) - ((4*A - B)*Tan[c + d*x])/(a^4*d*(1 + Cos[c + d*x])) - ((A - B)*Tan[c
+ d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((12*A - 5*B)*Tan[c + d*x])/(35*a*d*
(a + a*Cos[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
```

```
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(a(8A-B)-4a(A-B)\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5B)\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{\left(2a^2(26A-5B)-\right.}{}}{}$$

$$= -\frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5B)\,\mathrm{ta}}{35ad(a+a\cos}$$

$$= -\frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5B)\,\mathrm{ta}}{35ad(a+a\cos}$$

$$= -\frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(12A-5B)\,\mathrm{ta}}{35ad(a+a\cos}$$

$$= -\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(88A-25B)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-}{7d(a+}$$

$$= -\frac{(4A-B)\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{8(83A-20B)\tan(c+dx)}{105a^4d} - \frac{(88A}{105a^4}$$

Mathematica [B] time = 5.73, size = 595, normalized size = 3.40

$$\frac{26880(4A-B)\cos^8\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (26880*(4*A - B)*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*
Sec[c]*Sec[c + d*x]*(-245*(44*A - 17*B)*Sin[(d*x)/2] + 7*(2684*A - 635*B)*S
in[(3*d*x)/2] - 20524*A*Sin[c - (d*x)/2] + 4795*B*Sin[c - (d*x)/2] + 14644*
A*Sin[c + (d*x)/2] - 4795*B*Sin[c + (d*x)/2] - 16660*A*Sin[2*c + (d*x)/2] +
 4165*B*Sin[2*c + (d*x)/2] - 4690*A*Sin[c + (3*d*x)/2] + 2275*B*Sin[c + (3*
d*x)/2] + 14378*A*Sin[2*c + (3*d*x)/2] - 4445*B*Sin[2*c + (3*d*x)/2] - 9100
*A*Sin[3*c + (3*d*x)/2] + 2275*B*Sin[3*c + (3*d*x)/2] + 11668*A*Sin[c + (5*
d*x)/2] - 2785*B*Sin[c + (5*d*x)/2] - 630*A*Sin[2*c + (5*d*x)/2] + 735*B*Si
n[2*c + (5*d*x)/2] + 9358*A*Sin[3*c + (5*d*x)/2] - 2785*B*Sin[3*c + (5*d*x)
/2] - 2940*A*Sin[4*c + (5*d*x)/2] + 735*B*Sin[4*c + (5*d*x)/2] + 4228*A*Sin
[2*c + (7*d*x)/2] - 1015*B*Sin[2*c + (7*d*x)/2] + 315*A*Sin[3*c + (7*d*x)/2
] + 105*B*Sin[3*c + (7*d*x)/2] + 3493*A*Sin[4*c + (7*d*x)/2] - 1015*B*Sin[4
*c + (7*d*x)/2] - 420*A*Sin[5*c + (7*d*x)/2] + 105*B*Sin[5*c + (7*d*x)/2] +
 664*A*Sin[3*c + (9*d*x)/2] - 160*B*Sin[3*c + (9*d*x)/2] + 105*A*Sin[4*c +
(9*d*x)/2] + 559*A*Sin[5*c + (9*d*x)/2] - 160*B*Sin[5*c + (9*d*x)/2]))/(168
0*a^4*d*(1 + Cos[c + d*x])^4)
```

fricas [B] time = 0.96, size = 337, normalized size = 1.93

$$-\frac{105\left((4A-B)\cos(dx+c)^5+4(4A-B)\cos(dx+c)^4+6(4A-B)\cos(dx+c)^3+4(4A-B)\cos(dx+c)^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] -1/210*(105*((4*A - B)*cos(d*x + c)^5 + 4*(4*A - B)*cos(d*x + c)^4 + 6*(4*A
 - B)*cos(d*x + c)^3 + 4*(4*A - B)*cos(d*x + c)^2 + (4*A - B)*cos(d*x + c))
*log(sin(d*x + c) + 1) - 105*((4*A - B)*cos(d*x + c)^5 + 4*(4*A - B)*cos(d*
x + c)^4 + 6*(4*A - B)*cos(d*x + c)^3 + 4*(4*A - B)*cos(d*x + c)^2 + (4*A -
 B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(8*(83*A - 20*B)*cos(d*x + c)^
4 + (2236*A - 535*B)*cos(d*x + c)^3 + 4*(659*A - 155*B)*cos(d*x + c)^2 + 4*
(296*A - 65*B)*cos(d*x + c) + 105*A)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 +
```

```
4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 +
a^4*d*cos(d*x + c))
```

giac [A] time = 0.41, size = 224, normalized size = 1.28

$$-\frac{\frac{840\,(4\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{840\,(4\,A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{1680\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(840*(4*A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*(4*A - B
)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 1680*A*tan(1/2*d*x + 1/2*c)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)*a^4) - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a
^24*tan(1/2*d*x + 1/2*c)^7 + 147*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 105*B*a^24
*tan(1/2*d*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*ta
n(1/2*d*x + 1/2*c)^3 + 5145*A*a^24*tan(1/2*d*x + 1/2*c) - 1575*B*a^24*tan(1
/2*d*x + 1/2*c))/a^28)/d
```

maple [A] time = 0.16, size = 285, normalized size = 1.63

$$\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{56d\,a^4}-\frac{B\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56d\,a^4}+\frac{7A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^4}-\frac{B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^4}+\frac{23\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{24d\,a^4}-\frac{11B\,(\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A-1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7+7/40/d/
a^4*A*tan(1/2*d*x+1/2*c)^5-1/8/d/a^4*B*tan(1/2*d*x+1/2*c)^5+23/24/d/a^4*tan
(1/2*d*x+1/2*c)^3*A-11/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*A*tan(1/2
*d*x+1/2*c)-15/8/d/a^4*B*tan(1/2*d*x+1/2*c)+4/d/a^4*A*ln(tan(1/2*d*x+1/2*c)
-1)-1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)-4/d
/a^4*A*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*B-1/d/a^4*
A/(tan(1/2*d*x+1/2*c)+1)
```

maxima [A] time = 0.65, size = 326, normalized size = 1.86

$$\frac{A\left(\frac{1680\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{3360\log\left(\frac{\mathrm{s}}{\mathrm{co}}\right.}{a}\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(A*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - 5
*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 168*
log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))/d
```

mupad [B] time = 0.28, size = 236, normalized size = 1.35

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{8\,a^4}+\frac{5\,A-3\,B}{12\,a^4}+\frac{10\,A-2\,B}{24\,a^4}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{A-B}{20\,a^4}+\frac{5\,A-3\,B}{40\,a^4}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{2\,a^4}+\frac{3\,(5\,A-3\,B)}{8\,a^4}+\frac{10\,A\cdots}{4\,a\cdots}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^4),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((A - B)/(8*a^4) + (5*A - 3*B)/(12*a^4) + (10*A - 2*B
)/(24*a^4)))/d + (tan(c/2 + (d*x)/2)^5*((A - B)/(20*a^4) + (5*A - 3*B)/(40*
a^4)))/d + (tan(c/2 + (d*x)/2)*((A - B)/(2*a^4) + (3*(5*A - 3*B))/(8*a^4) +
 (10*A - 2*B)/(4*a^4) + (10*A + 2*B)/(8*a^4)))/d + (tan(c/2 + (d*x)/2)^7*(A
 - B))/(56*a^4*d) - (2*A*tan(c/2 + (d*x)/2))/(d*(a^4*tan(c/2 + (d*x)/2)^2 -
 a^4)) - (2*atanh(tan(c/2 + (d*x)/2))*(4*A - B))/(a^4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.73 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=232

$$-\frac{8(216A-83B)\tan(c+dx)}{105a^4d}+\frac{(21A-8B)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(21A-8B)\tan(c+dx)\sec(c+dx)}{2a^4d}-\frac{4(216A-}{10}$$

```
[Out] 1/2*(21*A-8*B)*arctanh(sin(d*x+c))/a^4/d-8/105*(216*A-83*B)*tan(d*x+c)/a^4/
d+1/2*(21*A-8*B)*sec(d*x+c)*tan(d*x+c)/a^4/d-1/105*(129*A-52*B)*sec(d*x+c)*
tan(d*x+c)/a^4/d/(1+cos(d*x+c))^2-4/105*(216*A-83*B)*sec(d*x+c)*tan(d*x+c)/
a^4/d/(1+cos(d*x+c))-1/7*(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^4-1
/5*(2*A-B)*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.69, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{8(216A-83B)\tan(c+dx)}{105a^4d}+\frac{(21A-8B)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(21A-8B)\tan(c+dx)\sec(c+dx)}{2a^4d}-\frac{4(216A-}{10}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] ((21*A - 8*B)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (8*(216*A - 83*B)*Tan[c +
d*x])/(105*a^4*d) + ((21*A - 8*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - ((
129*A - 52*B)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) -
 (4*(216*A - 83*B)*Sec[c + d*x]*Tan[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])
) - ((A - B)*Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - ((2*
A - B)*Sec[c + d*x]*Tan[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(a(9A-2B)-5a(A-B)\cos(c+dx))\sec^3(c+}{(a+a\cos(c+dx))^3}}{7a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{(2A-B)\sec(c+dx)\tan(c+dx)}{5ad(a+a\cos(c+dx))^3}$$

$$= -\frac{(129A-52B)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sec(c+dx)\tan(c+}{7d(a+a\cos(c+dx))^4}$$

$$= -\frac{(129A-52B)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sec(c+dx)\tan(c+}{7d(a+a\cos(c+dx))^4}$$

$$= -\frac{(129A-52B)\sec(c+dx)\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{(A-B)\sec(c+dx)\tan(c+}{7d(a+a\cos(c+dx))^4}$$

$$= \frac{(21A-8B)\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{(129A-52B)\sec(c+dx)\tan(c}{105a^4d(1+\cos(c+dx))^2}$$

$$= \frac{(21A-8B)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{8(216A-83B)\tan(c+dx)}{105a^4d} + \frac{(21A}{}$$

Mathematica [B] time = 6.51, size = 798, normalized size = 3.44

$$-\frac{8(21A-8B)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\cos^8\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(\cos(c+dx)a+a)^4} + \frac{8(21A-8B)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(\cos(c+dx)a+a)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-8*(21*A - 8*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2]])/(d*(a + a*Cos[c + d*x])^4) + (8*(21*A - 8*B)*Cos[c/2 + (d*x)/2]^8*
Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^4) +
(Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(73206*A*Sin[(d*x)/2] -
38668*B*Sin[(d*x)/2] - 166668*A*Sin[(3*d*x)/2] + 64384*B*Sin[(3*d*x)/2] + 1
83162*A*Sin[c - (d*x)/2] - 70896*B*Sin[c - (d*x)/2] - 100842*A*Sin[c + (d*x
)/2] + 50316*B*Sin[c + (d*x)/2] + 155526*A*Sin[2*c + (d*x)/2] - 59248*B*Sin
[2*c + (d*x)/2] + 37380*A*Sin[c + (3*d*x)/2] - 22820*B*Sin[c + (3*d*x)/2] -
 101148*A*Sin[2*c + (3*d*x)/2] + 48004*B*Sin[2*c + (3*d*x)/2] + 102900*A*Si
n[3*c + (3*d*x)/2] - 39200*B*Sin[3*c + (3*d*x)/2] - 119364*A*Sin[c + (5*d*x
)/2] + 46032*B*Sin[c + (5*d*x)/2] + 8820*A*Sin[2*c + (5*d*x)/2] - 8750*B*Si
```

```
n[2*c + (5*d*x)/2] - 78204*A*Sin[3*c + (5*d*x)/2] + 35742*B*Sin[3*c + (5*d*
x)/2] + 49980*A*Sin[4*c + (5*d*x)/2] - 19040*B*Sin[4*c + (5*d*x)/2] - 64053
*A*Sin[2*c + (7*d*x)/2] + 24664*B*Sin[2*c + (7*d*x)/2] - 3885*A*Sin[3*c + (
7*d*x)/2] - 1050*B*Sin[3*c + (7*d*x)/2] - 44733*A*Sin[4*c + (7*d*x)/2] + 19
834*B*Sin[4*c + (7*d*x)/2] + 15435*A*Sin[5*c + (7*d*x)/2] - 5880*B*Sin[5*c
+ (7*d*x)/2] - 21987*A*Sin[3*c + (9*d*x)/2] + 8456*B*Sin[3*c + (9*d*x)/2] -
 3675*A*Sin[4*c + (9*d*x)/2] + 630*B*Sin[4*c + (9*d*x)/2] - 16107*A*Sin[5*c
 + (9*d*x)/2] + 6986*B*Sin[5*c + (9*d*x)/2] + 2205*A*Sin[6*c + (9*d*x)/2] -
 840*B*Sin[6*c + (9*d*x)/2] - 3456*A*Sin[4*c + (11*d*x)/2] + 1328*B*Sin[4*c
 + (11*d*x)/2] - 840*A*Sin[5*c + (11*d*x)/2] + 210*B*Sin[5*c + (11*d*x)/2]
- 2616*A*Sin[6*c + (11*d*x)/2] + 1118*B*Sin[6*c + (11*d*x)/2]))/(6720*d*(a
+ a*Cos[c + d*x])^4)
```

fricas [A] time = 0.59, size = 360, normalized size = 1.55

$$\frac{105\left((21\,A-8\,B)\cos\left(dx+c\right)^{6}+4\,(21\,A-8\,B)\cos\left(dx+c\right)^{5}+6\,(21\,A-8\,B)\cos\left(dx+c\right)^{4}+4\,(21\,A-8\,B)\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/420*(105*((21*A - 8*B)*cos(d*x + c)^6 + 4*(21*A - 8*B)*cos(d*x + c)^5 + 6
*(21*A - 8*B)*cos(d*x + c)^4 + 4*(21*A - 8*B)*cos(d*x + c)^3 + (21*A - 8*B)
*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 105*((21*A - 8*B)*cos(d*x + c)^6 +
 4*(21*A - 8*B)*cos(d*x + c)^5 + 6*(21*A - 8*B)*cos(d*x + c)^4 + 4*(21*A -
8*B)*cos(d*x + c)^3 + (21*A - 8*B)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) -
 2*(16*(216*A - 83*B)*cos(d*x + c)^5 + (11619*A - 4472*B)*cos(d*x + c)^4 +
4*(3411*A - 1318*B)*cos(d*x + c)^3 + 4*(1509*A - 592*B)*cos(d*x + c)^2 + 21
0*(2*A - B)*cos(d*x + c) - 105*A)*sin(d*x + c))/(a^4*d*cos(d*x + c)^6 + 4*a
^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + a^4
*d*cos(d*x + c)^2)
```

giac [A] time = 1.45, size = 267, normalized size = 1.15

$$\frac{\frac{420\,(21\,A-8\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{4}}-\frac{420\,(21\,A-8\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{4}}+\frac{840\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-7\,A\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}a^{4}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(420*(21*A - 8*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 420*(21*A
- 8*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 840*(9*A*tan(1/2*d*x + 1/2*
c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 - 7*A*tan(1/2*d*x + 1/2*c) + 2*B*tan(1/2*
d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (15*A*a^24*tan(1/2*d*x
 + 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 + 189*A*a^24*tan(1/2*d*x + 1
/2*c)^5 - 147*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 1365*A*a^24*tan(1/2*d*x + 1/2
*c)^3 - 805*B*a^24*tan(1/2*d*x + 1/2*c)^3 + 11655*A*a^24*tan(1/2*d*x + 1/2*
c) - 5145*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

maple [A] time = 0.19, size = 374, normalized size = 1.61

$$-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{56d\,a^4}+\frac{B\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56d\,a^4}-\frac{9A\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^4}+\frac{7B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d\,a^4}-\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{8d\,a^4}+\frac{23B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7*A+1/56/d/a^4*B*tan(1/2*d*x+1/2*c)^7-9/40/d
/a^4*A*tan(1/2*d*x+1/2*c)^5+7/40/d/a^4*B*tan(1/2*d*x+1/2*c)^5-13/8/d/a^4*ta
n(1/2*d*x+1/2*c)^3*A+23/24/d/a^4*B*tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*A*tan(1
/2*d*x+1/2*c)+49/8/d/a^4*B*tan(1/2*d*x+1/2*c)-21/2/d/a^4*A*ln(tan(1/2*d*x+1
/2*c)-1)+4/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*B+9/2/d/a^4*A/(tan(1/2*d*x+1/2*c)
-1)-1/d/a^4/(tan(1/2*d*x+1/2*c)-1)*B+1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)^2+9
/2/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)-1/d/a^4/(tan(1/2*d*x+1/2*c)+1)*B+21/2/d/a
^4*A*ln(tan(1/2*d*x+1/2*c)+1)-4/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*B-1/2/d/a^4*
A/(tan(1/2*d*x+1/2*c)+1)^2
```

maxima [A] time = 0.62, size = 419, normalized size = 1.81

$$-\frac{3A\left(\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{2940\log(}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(3*A*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
+ 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
```

```
 + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) -
1)/a^4) - B*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d
*x + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))
/d
```

mupad [B] time = 0.29, size = 273, normalized size = 1.18

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(9\,A-2\,B\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(7\,A-2\,B\right)}{d\,\left(a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^4\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,A}{2\,a^4}+\frac{5\,\left(A-B\right)}{4\,a^4}+\frac{3\,\left(6\,A-4\,B\right)}{4\,a^4}+\frac{3\,\left(15\,A-5\,B\right)}{8\,a^4}\right)}{d}-\ \mathrm{tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^4),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(9*A - 2*B) - tan(c/2 + (d*x)/2)*(7*A - 2*B))/(d*(a^4
*tan(c/2 + (d*x)/2)^4 - 2*a^4*tan(c/2 + (d*x)/2)^2 + a^4)) - (tan(c/2 + (d*
x)/2)*((5*A)/(2*a^4) + (5*(A - B))/(4*a^4) + (3*(6*A - 4*B))/(4*a^4) + (3*(
15*A - 5*B))/(8*a^4)))/d - (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^4) + (6*A -
4*B)/(8*a^4) + (15*A - 5*B)/(24*a^4)))/d - (tan(c/2 + (d*x)/2)^5*((3*(A - B
))/(40*a^4) + (6*A - 4*B)/(40*a^4)))/d - (tan(c/2 + (d*x)/2)^7*(A - B))/(56
*a^4*d) + (atanh(tan(c/2 + (d*x)/2))*(21*A - 8*B))/(a^4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.74 $\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=187

$$\frac{2a(9A+8B)\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{4(9A+8B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105ad}-\frac{8(9A+8B)\sin(c+dx)\sqrt{\cdots}}{315d}$$

```
[Out] 4/105*(9*A+8*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+4/45*a*(9*A+8*B)*sin(
d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/63*a*(9*A+8*B)*cos(d*x+c)^3*sin(d*x+c)/d/
(a+a*cos(d*x+c))^(1/2)+2/9*a*B*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(
1/2)-8/315*(9*A+8*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.30, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2981, 2770, 2759, 2751, 2646}

$$\frac{2a(9A+8B)\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{4(9A+8B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105ad}-\frac{8(9A+8B)\sin(c+dx)\sqrt{\cdots}}{315d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*a*(9*A + 8*B)*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(9*A
+ 8*B)*Cos[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
B*Cos[c + d*x]^4*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (8*(9*A + 8
*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (4*(9*A + 8*B)*(a + a*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{2aB\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{1}{9}(9A+8B)\int \cos^3($$

$$= \frac{2a(9A+8B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\cos^4(c}{9d\sqrt{a+}}$$

$$= \frac{2a(9A+8B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\cos^4(c}{9d\sqrt{a+}}$$

$$= \frac{2a(9A+8B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\cos^4(c}{9d\sqrt{a+}}$$

$$= \frac{4a(9A+8B)\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} + \frac{2a(9A+8B)\cos^3(c+dx}{63d\sqrt{a+a\cos(c}}$$

Mathematica [A] time = 0.70, size = 103, normalized size = 0.55

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,(94(9A+8B)\cos(c+dx)+4(54A+83B)\cos(2(c+dx))+90A\cos(3(c+\cdots}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(1368*A + 1321*B + 94*(9*A + 8*B)*Cos[c + d*x]
+ 4*(54*A + 83*B)*Cos[2*(c + d*x)] + 90*A*Cos[3*(c + d*x)] + 80*B*Cos[3*(c
+ d*x)] + 35*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

fricas [A] time = 0.61, size = 99, normalized size = 0.53

$$\frac{2\left(35\,B\cos\left(dx+c\right)^{4}+5\,(9\,A+8\,B)\cos\left(dx+c\right)^{3}+6\,(9\,A+8\,B)\cos\left(dx+c\right)^{2}+8\,(9\,A+8\,B)\cos\left(dx+c\right)+14\cdots}{315\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/315*(35*B*cos(d*x + c)^4 + 5*(9*A + 8*B)*cos(d*x + c)^3 + 6*(9*A + 8*B)*c
os(d*x + c)^2 + 8*(9*A + 8*B)*cos(d*x + c) + 144*A + 128*B)*sqrt(a*cos(d*x
+ c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 0.53, size = 194, normalized size = 1.04

$$\frac{1}{2520}\sqrt{2}\left(\frac{35\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)}{d}+\frac{45\left(2\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\cdots\right.\right.\right.}{d}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*B*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c)/d + 45*
(2*A*sgn(cos(1/2*d*x + 1/2*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(7/2*d*x +
 7/2*c)/d + 126*(A*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*sgn(cos(1/2*d*x + 1/2*c)
))*sin(5/2*d*x + 5/2*c)/d + 210*(3*A*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*sgn(co
s(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c)/d + 1890*(A*sgn(cos(1/2*d*x + 1/2
*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.40, size = 121, normalized size = 0.65

$$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(560B\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-360A-1440B\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(756A+1512B\right)\left(\sin^4\right.\right.}{315\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 2/315*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(560*B*sin(1/2*d*x+1/2*c)^8+(
-360*A-1440*B)*sin(1/2*d*x+1/2*c)^6+(756*A+1512*B)*sin(1/2*d*x+1/2*c)^4+(-6
30*A-840*B)*sin(1/2*d*x+1/2*c)^2+315*A+315*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)
^2)^(1/2)/d
```

maxima [A] time = 1.03, size = 145, normalized size = 0.78

$$\frac{18\left(5\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+35\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+105\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+\left(3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/2520*(18*(5*sqrt(2)*sin(7/2*d*x + 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c)
 + 35*sqrt(2)*sin(3/2*d*x + 3/2*c) + 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*sq
rt(a) + (35*sqrt(2)*sin(9/2*d*x + 9/2*c) + 45*sqrt(2)*sin(7/2*d*x + 7/2*c)
+ 252*sqrt(2)*sin(5/2*d*x + 5/2*c) + 420*sqrt(2)*sin(3/2*d*x + 3/2*c) + 189
0*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.75 $\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=144

$$\frac{2(7A+6B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35ad}-\frac{4(7A+6B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(7A+6B)\sin(c+a}{15d\sqrt{a\cos(c+dx)+}}$$

```
[Out] 2/35*(7*A+6*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+2/15*a*(7*A+6*B)*sin(d
*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a*B*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d
*x+c))^(1/2)-4/105*(7*A+6*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.26, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2981, 2759, 2751, 2646}

$$\frac{2(7A+6B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35ad}-\frac{4(7A+6B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(7A+6B)\sin(c+a}{15d\sqrt{a\cos(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a*(7*A + 6*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*B*Cos
[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) - (4*(7*A + 6*B)*S
qrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A + 6*B)*(a + a*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(35*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{2aB\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}(7A+6B)\int \cos$$

$$= \frac{2aB\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2(7A+6B)(a+a\,c}{}$$

$$= \frac{2aB\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{4(7A+6B)\sqrt{a+a}}{}$$

$$= \frac{2a(7A+6B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\cos^3(c+dx)\sin(c}{7d\sqrt{a+a\cos(c+d}}$$

Mathematica [A] time = 0.37, size = 80, normalized size = 0.56

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,((112A+141B)\cos(c+dx)+6(7A+6B)\cos(2(c+dx))+266A+15B\cos}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(266*A + 228*B + (112*A + 141*B)*Cos[c + d*x] +
 6*(7*A + 6*B)*Cos[2*(c + d*x)] + 15*B*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/
(210*d)
```

fricas [A] time = 0.63, size = 82, normalized size = 0.57

$$\frac{2\left(15\,B\cos\left(dx+c\right)^{3}+3\,(7\,A+6\,B)\cos\left(dx+c\right)^{2}+4\,(7\,A+6\,B)\cos\left(dx+c\right)+56\,A+48\,B\right)\sqrt{a\cos\left(dx+c\right)+a}}{105\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/105*(15*B*cos(d*x + c)^3 + 3*(7*A + 6*B)*cos(d*x + c)^2 + 4*(7*A + 6*B)*c
os(d*x + c) + 56*A + 48*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x
 + c) + d)
```

giac [A] time = 1.44, size = 165, normalized size = 1.15

$$\frac{1}{420}\sqrt{2}\left(\frac{15\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}{d}+\frac{420\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}+\frac{315\,B\mathrm{sgn}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] 1/420*sqrt(2)*(15*B*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c)/d + 420*
A*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d + 315*B*sgn(cos(1/2*d*x
+ 1/2*c))*sin(1/2*d*x + 1/2*c)/d + 21*(2*A*sgn(cos(1/2*d*x + 1/2*c)) + B*sg
n(cos(1/2*d*x + 1/2*c)))*sin(5/2*d*x + 5/2*c)/d + 35*(2*A*sgn(cos(1/2*d*x +
 1/2*c)) + 3*B*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c)/d)*sqrt(a)
```

maple [A] time = 0.44, size = 102, normalized size = 0.71

$$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-120B\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(84A+252B)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-140A-210B)\left(\sin^{2}\left(\frac{d}{2}\right.\right.\right.}{105\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 2/105*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(-120*B*sin(1/2*d*x+1/2*c)^6+
(84*A+252*B)*sin(1/2*d*x+1/2*c)^4+(-140*A-210*B)*sin(1/2*d*x+1/2*c)^2+105*A
+105*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 0.80, size = 118, normalized size = 0.82

$$\frac{14\left(3\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+30\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+3\left(5\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/420*(14*(3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c)
+ 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 3*(5*sqrt(2)*sin(7/2*d*x + 7
/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c) + 35*sqrt(2)*sin(3/2*d*x + 3/2*c) +
105*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,\left(A+B\,\cos(c+d\,x)\right)\,\sqrt{a+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.76 $\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{2(5A-2B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a(5A+7B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

```
[Out] 2/5*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+2/15*a*(5*A+7*B)*sin(d*x+c)/d/(
a+a*cos(d*x+c))^(1/2)+2/15*(5*A-2*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2968, 3023, 2751, 2646}

$$\frac{2(5A-2B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a(5A+7B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a*(5*A + 7*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*(5*A -
2*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*B*(a + a*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(5*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \int \sqrt{a+a\cos(c+dx)}\,\left(A\cos(c+dx)+B\cos^2(c+\right.$$
$$= \frac{2B(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5ad} + \frac{2\int\sqrt{a+a\,\mathrm{co}}}{}$$
$$= \frac{2(5A-2B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2B(a}{}$$
$$= \frac{2a(5A+7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2(5A-2B)\sqrt{a+a\cos(c}}{15d}$$

Mathematica [A] time = 0.21, size = 64, normalized size = 0.63

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,(2(5A+4B)\cos(c+dx)+20A+3B\cos(2(c+dx))+19B)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(20*A + 19*B + 2*(5*A + 4*B)*Cos[c + d*x] + 3*B
*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d)
```

fricas [A] time = 0.80, size = 64, normalized size = 0.63

$$\frac{2\left(3\,B\cos(dx+c)^2+(5\,A+4\,B)\cos(dx+c)+10\,A+8\,B\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{15\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/15*(3*B*cos(d*x + c)^2 + (5*A + 4*B)*cos(d*x + c) + 10*A + 8*B)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 2.89, size = 113, normalized size = 1.12

$$\frac{1}{30}\sqrt{2}\left(\frac{3\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}{d}+\frac{5\left(2\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\sin}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/30*sqrt(2)*(3*B*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)/d + 5*(2*A
*sgn(cos(1/2*d*x + 1/2*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2
*c)/d + 30*(A*sgn(cos(1/2*d*x + 1/2*c)) + B*sgn(cos(1/2*d*x + 1/2*c)))*sin(
1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.34, size = 83, normalized size = 0.82

$$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(12B\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-10A-20B)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15A+15B\right)\sqrt{2}}{15\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 2/15*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(12*B*sin(1/2*d*x+1/2*c)^4+(-1
0*A-20*B)*sin(1/2*d*x+1/2*c)^2+15*A+15*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^
(1/2)/d
```

maxima [A] time = 1.25, size = 88, normalized size = 0.87

$$\frac{10\left(\sqrt{2}\,\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sqrt{2}\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+\left(3\,\sqrt{2}\,\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}\,\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+30\,\sqrt{2}\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/30*(10*(sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*A*
sqrt(a) + (3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c)
+ 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\left(A+B\cos\left(c+dx\right)\right)\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*cos(c + d*x), x)
```

3.77 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=62

$$\frac{2a(3A+B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

```
[Out] 2/3*a*(3*A+B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*B*sin(d*x+c)*(a+a*cos
(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2751, 2646}

$$\frac{2a(3A+B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a*(3*A + B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*B*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx &= \frac{2B\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{1}{3}(3A+B)\int \sqrt{a+a\cos(c\\ &= \frac{2a(3A+B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2B\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 46, normalized size = 0.74

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,(3A+B\cos(c+dx)+2B)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(3*A + 2*B + B*Cos[c + d*x])*Tan[(c + d*x)/2]
)/(3*d)
```

fricas [A] time = 0.67, size = 47, normalized size = 0.76

$$\frac{2\,(B\cos\left(dx+c\right)+3\,A+2\,B)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{3\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/3*(B*cos(d*x + c) + 3*A + 2*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*c
os(d*x + c) + d)
```

giac [A] time = 0.37, size = 83, normalized size = 1.34

$$\frac{1}{3}\sqrt{2}\left(\frac{B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}{d}+\frac{6\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}+\frac{3\,B\mathrm{sgn}\left(\cos\left(\frac{1}{2}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(B*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)/d + 6*A*sgn(c
os(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d + 3*B*sgn(cos(1/2*d*x + 1/2*c))
*sin(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.38, size = 62, normalized size = 1.00

$$\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2B\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A+B\right)\sqrt{2}}{3\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 2/3*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(2*B*cos(1/2*d*x+1/2*c)^2+3*A+B
)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.29, size = 57, normalized size = 0.92

$$\frac{6\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\left(\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)B\sqrt{a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(6*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*sin(3/2*d*x + 3/2*
c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (A+B\cos(c+d\,x))\sqrt{a+a\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\cos(c+dx)+1)}\,(A+B\cos(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x)), x)
```

3.78 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2*A*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+2*a*B*sin(
d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2981, 2773, 206}

$$\frac{2\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[a]*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
```

```
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + A\int \sqrt{a+a\cos(c+dx)}\,\sec($$

$$= \frac{2aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(2aA)\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\cdots\right.}{d}$$

$$= \frac{2\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.10, size = 66, normalized size = 1.00

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\,A\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*A*ArcTanh[Sqrt[2]*Sin
[(c + d*x)/2]] + 2*B*Sin[(c + d*x)/2]))/d
```

fricas [B] time = 0.66, size = 127, normalized size = 1.92

$$\frac{(A\cos(dx+c)+A)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,\sqrt{a\cos(dx+c)}\cdots}{2\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/2*((A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*
```

```
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*B*sin(d*
x + c))/(d*cos(d*x + c) + d)
```

giac [B] time = 15.59, size = 1884, normalized size = 28.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(a)*(sqrt(2)*(A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(
1/4*c)^6 - 6*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^5 + 3*A*sg
n(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^6 - 15*A*sgn(cos(1/2*d*x +
1/2*c))*tan(1/2*c)^3*tan(1/4*c)^4 + 18*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*
c)^2*tan(1/4*c)^5 - 3*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^6 +
 20*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^3 - 45*A*sgn(cos(1/
2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^4 + 18*A*sgn(cos(1/2*d*x + 1/2*c))*
tan(1/2*c)*tan(1/4*c)^5 - A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^6 + 15*A*s
gn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^2 - 60*A*sgn(cos(1/2*d*x +
 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^3 + 45*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2
*c)*tan(1/4*c)^4 - 6*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^5 - 6*A*sgn(cos
(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c) + 45*A*sgn(cos(1/2*d*x + 1/2*c))
*tan(1/2*c)^2*tan(1/4*c)^2 - 60*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(
1/4*c)^3 + 15*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^4 - A*sgn(cos(1/2*d*x
+ 1/2*c))*tan(1/2*c)^3 + 18*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/
4*c) - 45*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^2 + 20*A*sgn(co
s(1/2*d*x + 1/2*c))*tan(1/4*c)^3 - 3*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)
^2 + 18*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c) - 15*A*sgn(cos(1/
2*d*x + 1/2*c))*tan(1/4*c)^2 + 3*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c) - 6
*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c) + A*sgn(cos(1/2*d*x + 1/2*c)))*log(
abs(-2*tan(1/4*d*x + c)*tan(1/2*c)^3 + 6*tan(1/4*d*x + c)*tan(1/2*c)^2 - 2*
tan(1/2*c)^3 - 2*sqrt(2)*(tan(1/2*c)^2 + 1)^(3/2) + 6*tan(1/4*d*x + c)*tan(
1/2*c) - 6*tan(1/2*c)^2 - 2*tan(1/4*d*x + c) + 6*tan(1/2*c) + 2)/abs(-2*tan
(1/4*d*x + c)*tan(1/2*c)^3 + 6*tan(1/4*d*x + c)*tan(1/2*c)^2 - 2*tan(1/2*c)
^3 + 2*sqrt(2)*(tan(1/2*c)^2 + 1)^(3/2) + 6*tan(1/4*d*x + c)*tan(1/2*c) - 6
*tan(1/2*c)^2 - 2*tan(1/4*d*x + c) + 6*tan(1/2*c) + 2))/((tan(1/4*c)^6 + 3*
tan(1/4*c)^4 + 3*tan(1/4*c)^2 + 1)*(tan(1/2*c)^2 + 1)^(3/2)) + sqrt(2)*(A*s
gn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^6 + 6*A*sgn(cos(1/2*d*x +
1/2*c))*tan(1/2*c)^3*tan(1/4*c)^5 - 3*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c
)^2*tan(1/4*c)^6 - 15*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*tan(1/4*c)^4
 + 18*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^5 - 3*A*sgn(cos(1
/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^6 - 20*A*sgn(cos(1/2*d*x + 1/2*c))*t
an(1/2*c)^3*tan(1/4*c)^3 + 45*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(
1/4*c)^4 - 18*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^5 + A*sgn(c
```

os(1/2*d*x + 1/2*c))*tan(1/4*c)^6 + 15*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*
c)^3*tan(1/4*c)^2 - 60*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^
3 + 45*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^4 - 6*A*sgn(cos(1/
2*d*x + 1/2*c))*tan(1/4*c)^5 + 6*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3*t
an(1/4*c) - 45*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c)^2 + 60*A
*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)*tan(1/4*c)^3 - 15*A*sgn(cos(1/2*d*x +
 1/2*c))*tan(1/4*c)^4 - A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^3 + 18*A*sgn
(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2*tan(1/4*c) - 45*A*sgn(cos(1/2*d*x + 1/2
*c))*tan(1/2*c)*tan(1/4*c)^2 + 20*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^3
+ 3*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c)^2 - 18*A*sgn(cos(1/2*d*x + 1/2*c
))*tan(1/2*c)*tan(1/4*c) + 15*A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 3*
A*sgn(cos(1/2*d*x + 1/2*c))*tan(1/2*c) - 6*A*sgn(cos(1/2*d*x + 1/2*c))*tan(
1/4*c) - A*sgn(cos(1/2*d*x + 1/2*c)))*log(abs(-2*tan(1/4*d*x + c)*tan(1/2*c
)^3 - 6*tan(1/4*d*x + c)*tan(1/2*c)^2 + 2*tan(1/2*c)^3 - 2*sqrt(2)*(tan(1/2
*c)^2 + 1)^(3/2) + 6*tan(1/4*d*x + c)*tan(1/2*c) - 6*tan(1/2*c)^2 + 2*tan(1
/4*d*x + c) - 6*tan(1/2*c) + 2)/abs(-2*tan(1/4*d*x + c)*tan(1/2*c)^3 - 6*ta
n(1/4*d*x + c)*tan(1/2*c)^2 + 2*tan(1/2*c)^3 + 2*sqrt(2)*(tan(1/2*c)^2 + 1)
^(3/2) + 6*tan(1/4*d*x + c)*tan(1/2*c) - 6*tan(1/2*c)^2 + 2*tan(1/4*d*x + c
) - 6*tan(1/2*c) + 2))/((tan(1/4*c)^6 + 3*tan(1/4*c)^4 + 3*tan(1/4*c)^2 + 1
)*(tan(1/2*c)^2 + 1)^(3/2)) - 8*(B*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x +
c)*tan(1/4*c)^6 - 15*B*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c)*tan(1/4*c
)^4 + 6*B*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^5 + 15*B*sgn(cos(1/2*d*x + 1
/2*c))*tan(1/4*d*x + c)*tan(1/4*c)^2 - 20*B*sgn(cos(1/2*d*x + 1/2*c))*tan(1
/4*c)^3 - B*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + c) + 6*B*sgn(cos(1/2*d*
x + 1/2*c))*tan(1/4*c))/((tan(1/4*c)^6 + 3*tan(1/4*c)^4 + 3*tan(1/4*c)^2 +
1)*(tan(1/4*d*x + c)^2 + 1)))/d

maple [B] time = 1.16, size = 210, normalized size = 3.18

$$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(A\ln\left(\frac{4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+4a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)a+A\ln\left(-\frac{4\left(\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{-2\cos\left(\frac{dx}{2}\right.}\right.\right.}{\sqrt{a}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2*a))*a+2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/
2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 0.61, size = 21, normalized size = 0.32

$$\frac{2\sqrt{2}B\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
[Out] 2*sqrt(2)*B*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x),x)
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\cos(c+dx)+1)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sec(c + d*x), x)
```

3.79 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a}\,(A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aA\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (A+2*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+a*A*ta
n(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2980, 2773, 206}

$$\frac{\sqrt{a}\,(A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aA\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (Sqrt[a]*(A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]
)/d + (a*A*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
```

```
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{1}{2}(A+2B)\int \sqrt{a+a\cos(c$$

$$= \frac{aA\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(a(A+2B))\,\text{Subst}\left(\int \frac{1}{a-x^2}\right.}{d}$$

$$= \frac{\sqrt{a}\,(A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{aA\tan(c}{d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.22, size = 85, normalized size = 1.25

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\,(A+2B)\cos(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\sin\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*(A + 2*B
)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*A*Sin[(c + d*x)/2]))/(
2*d)
```

fricas [B] time = 0.80, size = 153, normalized size = 2.25

$$\frac{\left((A+2B)\cos(dx+c)^2+(A+2B)\cos(dx+c)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{4\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] 1/4*(((A + 2*B)*cos(d*x + c)^2 + (A + 2*B)*cos(d*x + c))*sqrt(a)*log((a*cos
(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(
```

```
d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sq
rt(a*cos(d*x + c) + a)*A*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.22, size = 642, normalized size = 9.44

$$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-2a\left(A\ln\left(-\frac{4\left(\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}-a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2a\right)}{-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)+A\ln\left(\frac{4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}{2\,\mathrm{co}}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(A*ln(-4/(-2*cos(1/
2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(
1/2)*cos(1/2*d*x+1/2*c)+2*a))+A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
+2*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+2*B*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^2+2*A*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)+A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+A*ln(
-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2*B*ln(4/(2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2
*d*x+1/2*c)+2*a))*a+2*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/a^
(1/2)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2
*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 1.39, size = 710, normalized size = 10.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*cos(3/2*d
*x + 3/2*c)*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2
*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*lo
g(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*
x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(
1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
 - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arct
an2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*
x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqr
t(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(
d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*sin(5/2*d*x + 5/2*c) + 4*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)
*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c))*A*sqrt(a)/(
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sec(c + d*x)**2, x
)
```

3.80 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=117

$$\frac{a(3A+4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\,(3A+4B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 1/4*(3*A+4*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+
1/4*a*(3*A+4*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a*A*sec(d*x+c)*tan(
d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2980, 2772, 2773, 206}

$$\frac{a(3A+4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\,(3A+4B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (Sqrt[a]*(3*A + 4*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) + (a*(3*A + 4*B)*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (
a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{4}(3A+4B)\int \sqrt{a+a$$

$$= \frac{a(3A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sec(c+dx)\tan(c+dx}{2d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a(3A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sec(c+dx)\tan(c+dx}{2d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{\sqrt{a}\,(3A+4B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{a(3A+4B)}{4d\sqrt{a+a\,c}}$$

Mathematica [A] time = 0.88, size = 101, normalized size = 0.86

$$\frac{\sqrt{a(\cos(c+dx)+1)}\left(6\tan\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)((3A+4B)\cos(c+dx)+2A)+3\sqrt{2}\,(3A+4B)\sec\left(\frac{1}{2}(c+dx\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(3*Sqrt[2]*(3*A + 4*B)*ArcTanh[Sqrt[2]*Sin[(c +
 d*x)/2]]*Sec[(c + d*x)/2] + 6*(2*A + (3*A + 4*B)*Cos[c + d*x])*Sec[c + d*x
]^2*Tan[(c + d*x)/2]))/(24*d)
```

fricas [A] time = 0.78, size = 178, normalized size = 1.52

$$\frac{\left((3\,A + 4\,B)\cos\left(dx + c\right)^3 + (3\,A + 4\,B)\cos\left(dx + c\right)^2\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3 - 7\,a\cos(dx+c)^2 - 4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c) -}{\cos(dx+c)^3 + \cos(dx+c)^2}\right.}{16\left(d\cos\left(dx + c\right)^3 + d\cos\left(dx + \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] 1/16*(((3*A + 4*B)*cos(d*x + c)^3 + (3*A + 4*B)*cos(d*x + c)^2)*sqrt(a)*log
((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
 + 4*((3*A + 4*B)*cos(d*x + c) + 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.33, size = 991, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*(3*A*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+3*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+4*B*ln(-4/(-2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^4-4*(3*A*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3*A*ln(-4/(-2*cos
```

```
(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))*a+4*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+
1/2*c)^2+10*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3*A*ln(-4/(-2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))*a+8*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4*B*ln(-4/(
-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(
1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x
+1/2*c)+2*a))*a)/a^(1/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 7.76, size = 3352, normalized size = 28.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] 1/16*((3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*
cos(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
```

```
)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 24*sqrt(2)
*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 8*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin
(2*d*x + 2*c) + 2*(6*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
 log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2))*cos(2*d*x + 2*c) + 6*sqrt(2)*sin(7/2*d*x + 7/2*c) + 2*sqrt(2)*s
in(5/2*d*x + 5/2*c) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) - 6*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2))*cos(4*d*x + 4*c) - 4*(2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)
*sin(1/2*d*x + 1/2*c) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*(3*(log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c) - 3*sqrt(2)*cos(7/2*d
*x + 7/2*c) - sqrt(2)*cos(5/2*d*x + 5/2*c) + sqrt(2)*cos(3/2*d*x + 3/2*c) +
 3*sqrt(2)*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) + 12*(2*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(7/2*d*x + 7/2*c) + 4*(2*sqrt(2)*cos(2*d*x + 2*c) + s
qrt(2))*sin(5/2*d*x + 5/2*c) + 8*(sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*
```

```
cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c) - 1
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2))*A*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
 + 2*c) + 1) - 4*(4*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 4*sqrt(
2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(2*d*x + 2*c)*sin(3
/2*d*x + 3/2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2
*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*sin(5/2*d*x + 5/2*c) + 4*(sqrt(2)*cos(2*d*x + 2*c)
^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c))
*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+dx))\sqrt{a+a\cos(c+dx)}}{\cos(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\left(A+B\cos\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sec(c + d*x)**3, x
)
```

3.81 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{a(5A+6B)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\,(5A+6B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(5A+6B)\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{aA\tan(c+\ldots}{3d\sqrt{a\,\text{co}\ldots}}$$

```
[Out] 1/8*(5*A+6*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+
1/8*a*(5*A+6*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/12*a*(5*A+6*B)*sec(d*
x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d/
(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2980, 2772, 2773, 206}

$$\frac{a(5A+6B)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\,(5A+6B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(5A+6B)\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{aA\tan(c+\ldots}{3d\sqrt{a\,\text{co}\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (Sqrt[a]*(5*A + 6*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(8*d) + (a*(5*A + 6*B)*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (
a*(5*A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) +
(a*A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{1}{6}(5A+6B)\int \sqrt{a \cdots}$$

$$= \frac{a(5A+6B)\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}}+\frac{aA\sec^2(c+ \cdots}{3d\sqrt{a+a \cdots}}$$

$$= \frac{a(5A+6B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}+\frac{a(5A+6B)\sec(c+dx)}{12d\sqrt{a+a\cos(c \cdots}}$$

$$= \frac{a(5A+6B)\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}}+\frac{a(5A+6B)\sec(c+dx)}{12d\sqrt{a+a\cos(c \cdots}}$$

$$= \frac{\sqrt{a}\,(5A+6B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d}+\frac{a(5A+6B \cdots}{8d\sqrt{a+a \cdots}}$$

Mathematica [A] time = 2.01, size = 129, normalized size = 0.81

$$\frac{\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\tan\left(\frac{1}{2}(c+dx)\right)(4(5A+6B)\cos(c+dx)+3(5A+6B)\cos(2(c+dx))+31A \cdots}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[c + d*x]^3*(3*Sqrt[2]*(5*A + 6*B)*ArcTanh[S
qrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3*Sec[(c + d*x)/2] + (31*A + 18*B + 4
*(5*A + 6*B)*Cos[c + d*x] + 3*(5*A + 6*B)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2
]))/(48*d)
```

fricas [A] time = 0.78, size = 197, normalized size = 1.23

$$\frac{3\left((5\,A+6\,B)\cos\left(dx+c\right)^{4}+(5\,A+6\,B)\cos\left(dx+c\right)^{3}\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{96\left(d\cos\left(dx+c\right)^{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] 1/96*(3*((5*A + 6*B)*cos(d*x + c)^4 + (5*A + 6*B)*cos(d*x + c)^3)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2
)) + 4*(3*(5*A + 6*B)*cos(d*x + c)^2 + 2*(5*A + 6*B)*cos(d*x + c) + 8*A)*sq
rt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.48, size = 1311, normalized size = 8.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(5*A*ln(-4/(-2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+5*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2
```

```
*c)+2*a))+6*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+6*B*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+12*(10*A*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+12*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)+15*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+15*A*ln(4
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+18*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2
*d*x+1/2*c)+2*a))*a+18*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin
(1/2*d*x+1/2*c)^4-2*(80*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+96
*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+45*A*ln(-4/(-2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))*a+45*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*a+54*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+54*B*ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+15*A*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))*a+15*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2*a))*a+66*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+18*B*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+18*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+
1/2*c)+2*a))*a+60*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(1/2)
/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/sin(1/2*
d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 7.42, size = 5021, normalized size = 31.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] -1/96*((120*(sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 3*sin(2*d*x + 2*c))*co
s(13/2*d*x + 13/2*c) - 8*(15*sin(11/2*d*x + 11/2*c) + 50*sin(9/2*d*x + 9/2*
c) + 42*sin(7/2*d*x + 7/2*c) + 3*sin(5/2*d*x + 5/2*c) - 5*sin(3/2*d*x + 3/2
*c))*cos(6*d*x + 6*c) + 360*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*cos(11/2*
d*x + 11/2*c) + 1200*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*cos(9/2*d*x + 9/
```

```
2*c) - 24*(42*sin(7/2*d*x + 7/2*c) + 3*sin(5/2*d*x + 5/2*c) - 5*sin(3/2*d*x
 + 3/2*c))*cos(4*d*x + 4*c) - 15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*co
s(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^
2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*
sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*c
os(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)
^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2
*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x +
6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 15*(sqrt(2)*cos(6*d*x + 6*c)^2
 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*si
n(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x
 + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x +
4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 2) + 15*(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^
2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*s
in(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2
)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2
))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 120*(cos(6*d*x
+ 6*c) + 3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(13/2*d*x + 13/2*c
) + 8*(15*cos(11/2*d*x + 11/2*c) + 50*cos(9/2*d*x + 9/2*c) + 42*cos(7/2*d*x
 + 7/2*c) + 3*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c))*sin(6*d*x + 6*
c) - 120*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(11/2*d*x + 11/2*
c) - 400*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(9/2*d*x + 9/2*c)
```

```
 + 24*(42*cos(7/2*d*x + 7/2*c) + 3*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3
/2*c))*sin(4*d*x + 4*c) - 336*(3*cos(2*d*x + 2*c) + 1)*sin(7/2*d*x + 7/2*c)
 - 24*(3*cos(2*d*x + 2*c) + 1)*sin(5/2*d*x + 5/2*c) + 1008*cos(7/2*d*x + 7/
2*c)*sin(2*d*x + 2*c) + 72*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 120*cos(
3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 120*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*
c) + 120*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
 1)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 40*sin(3/2*d*x + 3/2*c))
*A*sqrt(a)/(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*cos(4*d*x + 4*c)^2 + 9*s
qrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*sin(4*d*
x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*sin(2
*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(
4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)) - 6*(3*(log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 + 12*
(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x
 + 2*c)^2 + 3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2))*sin(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
```

```
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 24*sqrt(2)*cos(7/2*d*x + 7/2*c)*si
n(2*d*x + 2*c) - 8*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 2*(6*(lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x +
2*c) + 6*sqrt(2)*sin(7/2*d*x + 7/2*c) + 2*sqrt(2)*sin(5/2*d*x + 5/2*c) - 2*
sqrt(2)*sin(3/2*d*x + 3/2*c) - 6*sqrt(2)*sin(1/2*d*x + 1/2*c) + 3*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4
*c) - 4*(2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*sin(1/2*d*x + 1/2*c) -
3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*co
s(2*d*x + 2*c) + 4*(3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2))*sin(2*d*x + 2*c) - 3*sqrt(2)*cos(7/2*d*x + 7/2*c) - sqrt(2)*co
s(5/2*d*x + 5/2*c) + sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*cos(1/2*d*x +
 1/2*c))*sin(4*d*x + 4*c) + 12*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7
/2*d*x + 7/2*c) + 4*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*d*x + 5/
2*c) + 8*(sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*cos(1/2*d*x + 1/2*c))*si
n(2*d*x + 2*c) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c) - 12*sqrt(2)*sin(1/2*d*x +
1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
```

```
in(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2))*B*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+d\,x))\,\sqrt{a+a\cos(c+d\,x)}}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^4,x)
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^4, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
[Out] Timed out
```

3.82 $\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=234

$$\frac{2a^2(11A+12B)\sin(c+dx)\cos^4(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(187A+168B)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(187A+168B)\sin(c\cdots}{495d\sqrt{a\cos(c+dx)+\cdots}}$$

```
[Out] 4/1155*(187*A+168*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+4/495*a^2*(187*A+1
68*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/693*a^2*(187*A+168*B)*cos(d*x+c
)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/99*a^2*(11*A+12*B)*cos(d*x+c)^4*s
in(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-8/3465*a*(187*A+168*B)*sin(d*x+c)*(a+a*c
os(d*x+c))^(1/2)/d+2/11*a*B*cos(d*x+c)^4*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/
d
```

Rubi [A] time = 0.53, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2976, 2981, 2770, 2759, 2751, 2646}

$$\frac{2a^2(11A+12B)\sin(c+dx)\cos^4(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(187A+168B)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(187A+168B)\sin(c\cdots}{495d\sqrt{a\cos(c+dx)+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*a^2*(187*A + 168*B)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
a^2*(187*A + 168*B)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*a^2*(11*A + 12*B)*Cos[c + d*x]^4*Sin[c + d*x])/(99*d*Sqrt[a + a
*Cos[c + d*x]]) - (8*a*(187*A + 168*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x
])/(3465*d) + (2*a*B*Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/
(11*d) + (4*(187*A + 168*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(1155*
d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2aB\cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{11d} +$$

$$= \frac{2a^2(11A+12B)\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\,\text{co}}{}$$

$$= \frac{2a^2(187A+168B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(}{}$$

$$= \frac{2a^2(187A+168B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(}{}$$

$$= \frac{2a^2(187A+168B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(}{}$$

$$= \frac{4a^2(187A+168B)\sin(c+dx)}{495d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(187A+168B}{693d\sqrt{a}}$$

Mathematica [A] time = 1.07, size = 125, normalized size = 0.53

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,((35156A+34734B)\cos(c+dx)+8(1507A+1743B)\cos(2(c+dx))+37}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(59158*A + 55482*B + (35156*A + 34734*B)*Cos[
c + d*x] + 8*(1507*A + 1743*B)*Cos[2*(c + d*x)] + 3740*A*Cos[3*(c + d*x)] +
 4935*B*Cos[3*(c + d*x)] + 770*A*Cos[4*(c + d*x)] + 1470*B*Cos[4*(c + d*x)]
 + 315*B*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])/(27720*d)
```

fricas [A] time = 1.15, size = 125, normalized size = 0.53

$$\frac{2\left(315\,Ba\cos(dx+c)^5+35\,(11\,A+21\,B)a\cos(dx+c)^4+5\,(187\,A+168\,B)a\cos(dx+c)^3+6\,(187\,A+168\,B)a}{3465\,(d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/3465*(315*B*a*cos(d*x + c)^5 + 35*(11*A + 21*B)*a*cos(d*x + c)^4 + 5*(187
*A + 168*B)*a*cos(d*x + c)^3 + 6*(187*A + 168*B)*a*cos(d*x + c)^2 + 8*(187*
```

```
A + 168*B)*a*cos(d*x + c) + 16*(187*A + 168*B)*a)*sqrt(a*cos(d*x + c) + a)*
sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 3.00, size = 250, normalized size = 1.07

$$\frac{1}{55440}\sqrt{2}\left(\frac{315\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)}{d}+\frac{385\left(2\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+3\,Ba\mathrm{sgn}\left(\cos\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] 1/55440*sqrt(2)*(315*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(11/2*d*x + 11/2*c)/d
 + 385*(2*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*a*sgn(cos(1/2*d*x + 1/2*c)))*
sin(9/2*d*x + 9/2*c)/d + 495*(6*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 7*B*a*sgn(c
os(1/2*d*x + 1/2*c)))*sin(7/2*d*x + 7/2*c)/d + 693*(12*A*a*sgn(cos(1/2*d*x
+ 1/2*c)) + 13*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(5/2*d*x + 5/2*c)/d + 2310
*(10*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 9*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(3
/2*d*x + 3/2*c)/d + 6930*(12*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 11*B*a*sgn(cos
(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.40, size = 142, normalized size = 0.61

$$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-5040B\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(3080A+18480B)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-9900A-277\right.}{3465\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 4/3465*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(-5040*B*sin(1/2*d*x+1/2*c
)^10+(3080*A+18480*B)*sin(1/2*d*x+1/2*c)^8+(-9900*A-27720*B)*sin(1/2*d*x+1/
2*c)^6+(12474*A+22176*B)*sin(1/2*d*x+1/2*c)^4+(-8085*A-10395*B)*sin(1/2*d*x
+1/2*c)^2+3465*A+3465*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 0.85, size = 185, normalized size = 0.79

$$\frac{22\left(35\sqrt{2}\,a\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+135\sqrt{2}\,a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+378\sqrt{2}\,a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+1050\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/55440*(22*(35*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*sin(7/2*d*x
+ 7/2*c) + 378*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) + 3780*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 21*(15*sqrt(2)*
a*sin(11/2*d*x + 11/2*c) + 55*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 165*sqrt(2)*
a*sin(7/2*d*x + 7/2*c) + 429*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 990*sqrt(2)*a
*sin(3/2*d*x + 3/2*c) + 3630*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^3\,\left(A+B\,\cos(c+d\,x)\right)\,\left(a+a\,\cos(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.83 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=189

$$\frac{2a^2(9A+10B)\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(39A+34B)\sin(c+dx)}{45d\sqrt{a\cos(c+dx)+a}}+\frac{2(39A+34B)\sin(c+dx)(a\cos(c+dx)}{105d}$$

```
[Out] 2/105*(39*A+34*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/45*a^2*(39*A+34*B)*
sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/63*a^2*(9*A+10*B)*cos(d*x+c)^3*sin(d*
x+c)/d/(a+a*cos(d*x+c))^(1/2)-4/315*a*(39*A+34*B)*sin(d*x+c)*(a+a*cos(d*x+c
))^(1/2)/d+2/9*a*B*cos(d*x+c)^3*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.45, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2976, 2981, 2759, 2751, 2646}

$$\frac{2a^2(9A+10B)\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(39A+34B)\sin(c+dx)}{45d\sqrt{a\cos(c+dx)+a}}+\frac{2(39A+34B)\sin(c+dx)(a\cos(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a^2*(39*A + 34*B)*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(9*A + 10*B)*Cos[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]])
- (4*a*(39*A + 34*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*
B*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d) + (2*(39*A +
34*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2aB\cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{9d}$$

$$= \frac{2a^2(9A+10B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\,\mathrm{co}}{}$$

$$= \frac{2a^2(9A+10B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aB\,\mathrm{c}}{}$$

$$= \frac{2a^2(9A+10B)\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} - \frac{4a(39}{}$$

$$= \frac{2a^2(39A+34B)\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(9A+10B)\,\mathrm{co}}{63d\sqrt{a+}}$$

Mathematica [A] time = 0.60, size = 103, normalized size = 0.54

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,(2(759A+799B)\cos(c+dx)+(468A+548B)\cos(2(c+dx))+90A\cos}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(2964*A + 2689*B + 2*(759*A + 799*B)*Cos[c +
d*x] + (468*A + 548*B)*Cos[2*(c + d*x)] + 90*A*Cos[3*(c + d*x)] + 170*B*Cos
[3*(c + d*x)] + 35*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

fricas [A] time = 0.59, size = 107, normalized size = 0.57

$$\frac{2\left(35\,Ba\cos(dx+c)^4+5\,(9\,A+17\,B)a\cos(dx+c)^3+3\,(39\,A+34\,B)a\cos(dx+c)^2+4\,(39\,A+34\,B)a\cos(d}{315\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/315*(35*B*a*cos(d*x + c)^4 + 5*(9*A + 17*B)*a*cos(d*x + c)^3 + 3*(39*A +
34*B)*a*cos(d*x + c)^2 + 4*(39*A + 34*B)*a*cos(d*x + c) + 8*(39*A + 34*B)*a
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 1.84, size = 245, normalized size = 1.30

$$\frac{1}{2520}\sqrt{2}\left(\frac{35\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)}{d}+\frac{45\left(2\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+3\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\right.\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c)/d + 4
5*(2*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(7
/2*d*x + 7/2*c)/d + 378*(A*a*sgn(cos(1/2*d*x + 1/2*c)) + B*a*sgn(cos(1/2*d*
x + 1/2*c)))*sin(5/2*d*x + 5/2*c)/d + 1050*(A*a*sgn(cos(1/2*d*x + 1/2*c)) +
 B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c)/d + 630*(3*A*a*sgn(cos
(1/2*d*x + 1/2*c)) + 4*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c)/
d + 1260*(2*A*a*sgn(cos(1/2*d*x + 1/2*c)) + B*a*sgn(cos(1/2*d*x + 1/2*c)))*
sin(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.42, size = 123, normalized size = 0.65

$$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(280B\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-180A-900B\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(504A+1134B\right)\left(\sin^{4}\right.\right.}{315\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 4/315*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(280*B*sin(1/2*d*x+1/2*c)^8
+(-180*A-900*B)*sin(1/2*d*x+1/2*c)^6+(504*A+1134*B)*sin(1/2*d*x+1/2*c)^4+(-
525*A-735*B)*sin(1/2*d*x+1/2*c)^2+315*A+315*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c
)^2)^(1/2)/d
```

maxima [A] time = 0.79, size = 154, normalized size = 0.81

$$\frac{6\left(15\sqrt{2}\,a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+63\sqrt{2}\,a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+175\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+735\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/2520*(6*(15*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5
/2*c) + 175*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/
2*c))*A*sqrt(a) + (35*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*sin(7/
2*d*x + 7/2*c) + 378*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*sin(3/
2*d*x + 3/2*c) + 3780*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^2\ (A+B\ \cos(c+dx))\ (a+a\ \cos(c+dx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.84 $\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{8a^2(21A+19B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2(7A-2B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{2a(21A+19B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}$$

```
[Out] 2/35*(7*A-2*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(a+a*cos(d*x+c))^(
5/2)*sin(d*x+c)/a/d+8/105*a^2*(21*A+19*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/
2)+2/105*a*(21*A+19*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.25, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3023, 2751, 2647, 2646}

$$\frac{8a^2(21A+19B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2(7A-2B)\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{2a(21A+19B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*a^2*(21*A + 19*B)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
(21*A + 19*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A - 2*
B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*B*(a + a*Cos[c + d*
x])^(5/2)*Sin[c + d*x])/(7*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^{3/2}\left(A\cos(c+dx)+B\cos^2(c\right.$$

$$= \frac{2B(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7ad} + \frac{2\int(a+a\,c}{}$$

$$= \frac{2(7A-2B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2B}{}$$

$$= \frac{2a(21A+19B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{105d} + \frac{2}{}$$

$$= \frac{8a^2(21A+19B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a(21A+19B)\sqrt{a}}{}$$

Mathematica [A] time = 0.40, size = 81, normalized size = 0.59

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,((252A+253B)\cos(c+dx)+6(7A+13B)\cos(2(c+dx))+546A+15B}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(546*A + 494*B + (252*A + 253*B)*Cos[c + d*x]
 + 6*(7*A + 13*B)*Cos[2*(c + d*x)] + 15*B*Cos[3*(c + d*x)])*Tan[(c + d*x)/2
])/(210*d)
```

fricas [A] time = 0.83, size = 88, normalized size = 0.64

$$\frac{2\left(15\,Ba\cos\left(dx+c\right)^{3}+3\left(7\,A+13\,B\right)a\cos\left(dx+c\right)^{2}+\left(63\,A+52\,B\right)a\cos\left(dx+c\right)+2\left(63\,A+52\,B\right)a\right)\sqrt{a\cos\left(d\right.}}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/105*(15*B*a*cos(d*x + c)^3 + 3*(7*A + 13*B)*a*cos(d*x + c)^2 + (63*A + 52
*B)*a*cos(d*x + c) + 2*(63*A + 52*B)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/(d*cos(d*x + c) + d)
```

giac [A] time = 0.50, size = 164, normalized size = 1.19

$$\frac{1}{420}\sqrt{2}\left(\frac{15\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}{d}+\frac{21\left(2\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+3\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\right.\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/420*sqrt(2)*(15*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c)/d + 21
*(2*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(5/
2*d*x + 5/2*c)/d + 35*(6*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 5*B*a*sgn(cos(1/2*
d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c)/d + 105*(8*A*a*sgn(cos(1/2*d*x + 1/2*c)
) + 7*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.45, size = 104, normalized size = 0.75

$$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-60B\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(42A+168B\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-105A-175B\right)\left(\sin^{2}\left(\frac{d}{2}\right.\right.\right.}{105\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 4/105*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(-60*B*sin(1/2*d*x+1/2*c)^6
+(42*A+168*B)*sin(1/2*d*x+1/2*c)^4+(-105*A-175*B)*sin(1/2*d*x+1/2*c)^2+105*
A+105*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 0.93, size = 123, normalized size = 0.89

$$\frac{42\left(\sqrt{2}\,a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}\,+\left(15\,\sqrt{2}\,a\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/420*(42*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*c
) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + (15*sqrt(2)*a*sin(7/2*d*
x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 175*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.85 $\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\,dx$

Optimal. Leaf size=101

$$\frac{8a^2(5A + 3B)\sin(c + dx)}{15d\sqrt{a\cos(c + dx) + a}} + \frac{2a(5A + 3B)\sin(c + dx)\sqrt{a\cos(c + dx) + a}}{15d} + \frac{2B\sin(c + dx)(a\cos(c + dx) + a)^{3/2}}{5d}$$

```
[Out] 2/5*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+8/15*a^2*(5*A+3*B)*sin(d*x+c)/d/(
a+a*cos(d*x+c))^(1/2)+2/15*a*(5*A+3*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2751, 2647, 2646}

$$\frac{8a^2(5A + 3B)\sin(c + dx)}{15d\sqrt{a\cos(c + dx) + a}} + \frac{2a(5A + 3B)\sin(c + dx)\sqrt{a\cos(c + dx) + a}}{15d} + \frac{2B\sin(c + dx)(a\cos(c + dx) + a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*a^2*(5*A + 3*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(5*
A + 3*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*B*(a + a*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2B(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{1}{5}(5A+3B)\int (a+a\cos$$
$$= \frac{2a(5A+3B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15d}+\frac{2B(a+a\cos(c}{}$$
$$= \frac{8a^2(5A+3B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a(5A+3B)\sqrt{a+a\cos(c+dx)}\,\mathrm{s}}{15d}$$

Mathematica [A] time = 0.20, size = 65, normalized size = 0.64

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,(2(5A+9B)\cos(c+dx)+50A+3B\cos(2(c+dx))+39B)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(50*A + 39*B + 2*(5*A + 9*B)*Cos[c + d*x] + 3
*B*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d)
```

fricas [A] time = 0.60, size = 69, normalized size = 0.68

$$\frac{2\left(3\,Ba\cos\left(dx+c\right)^2+(5\,A+9\,B)a\cos\left(dx+c\right)+(25\,A+18\,B)a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/15*(3*B*a*cos(d*x + c)^2 + (5*A + 9*B)*a*cos(d*x + c) + (25*A + 18*B)*a)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 0.54, size = 161, normalized size = 1.59

$$\frac{1}{30}\sqrt{2}\left(\frac{3\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}{d}+\frac{5\left(2\,Aa\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+3\,Ba\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30*sqrt(2)*(3*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)/d + 5*(2
*A*a*sgn(cos(1/2*d*x + 1/2*c)) + 3*B*a*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d
*x + 3/2*c)/d + 30*(2*A*a*sgn(cos(1/2*d*x + 1/2*c)) + B*a*sgn(cos(1/2*d*x +
 1/2*c)))*sin(1/2*d*x + 1/2*c)/d + 30*(A*a*sgn(cos(1/2*d*x + 1/2*c)) + B*a*
sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.34, size = 85, normalized size = 0.84

$$\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(6B\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-5A-15B\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15A+15B\right)\sqrt{2}}{15\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 4/15*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(6*B*sin(1/2*d*x+1/2*c)^4+(-
5*A-15*B)*sin(1/2*d*x+1/2*c)^2+15*A+15*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^
(1/2)/d
```

maxima [A] time = 0.88, size = 93, normalized size = 0.92

$$\frac{10\left(\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}\;+3\left(\sqrt{2}\,a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(10*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c)
)*A*sqrt(a) + 3*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x +
 3/2*c) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\,\left(a+a\,\cos(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\cos\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)*(A + B*cos(c + d*x)), x)
```

3.86 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=105

$$\frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2(3A+4B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

```
[Out] 2*a^(3/2)*A*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2/3*a^2*(3
*A+4*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*a*B*sin(d*x+c)*(a+a*cos(d*x
+c))^(1/2)/d
```

Rubi [A] time = 0.27, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2976, 2981, 2773, 206}

$$\frac{2a^2(3A+4B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aB\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*a^(3/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^2*(3*A + 4*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*B*Sq
rt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2aB\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2}{3}\int\sqrt{a+a$$

$$=\frac{2a^2(3A+4B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2aB\sqrt{a+a\cos(c+dx}}{3d}$$

$$=\frac{2a^2(3A+4B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2aB\sqrt{a+a\cos(c+dx}}{3d}$$

$$=\frac{2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^2(3A+4B)\sin}{3d\sqrt{a+a\cos(c}}$$

Mathematica [A] time = 0.22, size = 85, normalized size = 0.81

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(3A+B\cos(c+dx)+5B)+3\sqrt{2}A\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}\right.\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*A*ArcTanh[Sqrt[2]
*Sin[(c + d*x)/2]] + 2*(3*A + 5*B + B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(3*d
)
```

fricas [A] time = 0.63, size = 149, normalized size = 1.42

$$\frac{3\left(Aa\cos\left(dx+c\right)+Aa\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(Ba\cos\left(dx\right.\right.}{6\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/6*(3*(A*a*cos(d*x + c) + A*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x
 + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c
) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(B*a*cos(d*x + c) + (3*A +
5*B)*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [B] time = 1.12, size = 272, normalized size = 2.59

$$\frac{\sqrt{a}\ \cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-4B\sqrt{a}\ \sqrt{2}\ \sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6A\sqrt{2}\ \sqrt{a\left(\sin^2\left(\frac{dx}{2}+\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/3*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*A*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2
```

```
*c)+2*a))*a+3*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+12*B*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x
+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.27, size = 39, normalized size = 0.37

$$\frac{\left(\sqrt{2}\, a \sin\left(\frac{3}{2}\, dx + \frac{3}{2}\, c\right) + 9\, \sqrt{2}\, a \sin\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)\right) B \sqrt{a}}{3\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*s
qrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\, \cos(c + d\, x))\, (a + a\, \cos(c + d\, x))^{3/2}}{\cos(c + d\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

3.87 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{a^{3/2}(3A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

```
[Out] a^(3/2)*(3*A+2*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d-a^2*
(A-2*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+a*A*(a+a*cos(d*x+c))^(1/2)*tan(
d*x+c)/d
```

Rubi [A] time = 0.28, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2975, 2981, 2773, 206}

$$-\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(3A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aA\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^(3/2)*(3*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d - (a^2*(A - 2*B)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (a*A*Sq
rt[a + a*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA\sqrt{a+a\cos(c+dx)}\,\tan(c+dx)}{d} + \int \sqrt{a+a\,\mathrm{c}$$

$$= -\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+a\cos(c+dx)}}{d}$$

$$= -\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+a\cos(c+dx)}}{d}$$

$$= \frac{a^{3/2}(3A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^2(A-2}{d\sqrt{a+a}}$$

Mathematica [A] time = 0.34, size = 98, normalized size = 0.95

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\,\left(2\sin\left(\frac{1}{2}(c+dx)\right)(A+2B\cos(c+dx))+\sqrt{2}\,(3A+2B)\cos(c}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*(3*A +
 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(A + 2*B*Cos[c + d
*x])*Sin[(c + d*x)/2]))/(2*d)
```

fricas [A] time = 0.66, size = 172, normalized size = 1.67

$$\frac{\left((3A+2B)a\cos(dx+c)^2+(3A+2B)a\cos(dx+c)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{4\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] 1/4*(((3*A + 2*B)*a*cos(d*x + c)^2 + (3*A + 2*B)*a*cos(d*x + c))*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(
a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)
) + 4*(2*B*a*cos(d*x + c) + A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*
cos(d*x + c)^2 + d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.17, size = 696, normalized size = 6.76

$$\frac{\sqrt{a}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\left(-6A\ln\left(\frac{4\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}+4a\sqrt{2}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8a}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)a-6A\ln\left(-\frac{4\left(\sqrt{2}\,\sqrt{a\left(\sin\right.\right.}}{}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-6*A*ln(4/(2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a
```

```
*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-6*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))*a-4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-4*B*ln(-4/(-
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-8*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))*a+3*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2*A*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(
1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x
+1/2*c)+2*a))*a+2*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4*B*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)
^(1/2)/d
```

maxima [B] time = 0.99, size = 1315, normalized size = 12.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 6*sqrt(2)*a*cos(5
/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 6*sq
rt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (2*sqrt(2)*a*sin(
3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c
)^2 - 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) -
```

```
 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
))*cos(2*d*x + 2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 2*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin
(7/2*d*x + 7/2*c) - 6*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/2*d*x
+ 5/2*c) + 2*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + sqrt(2)*a*cos(1/2*d*x + 1/
2*c))*sin(2*d*x + 2*c))*A*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{3/2}}{\cos(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^2,x)
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
[Out] Timed out
```

3.88 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{a^{3/2}(7A+12B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2(5A+4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+}}{2d}$$

```
[Out] 1/4*a^(3/2)*(7*A+12*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d
+1/4*a^2*(5*A+4*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a*A*sec(d*x+c)*(
a+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.33, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2975, 2980, 2773, 206}

$$\frac{a^2(5A+4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(7A+12B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{aA\tan(c+dx)\sec(c+dx)\sqrt{a\cos(c+dx)+}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^(3/2)*(7*A + 12*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(4*d) + (a^2*(5*A + 4*B)*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]])
+ (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA\sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\ldots$$

$$= \frac{a^2(5A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+a\cos(c+dx)}}{2}\ldots$$

$$= \frac{a^2(5A+4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+a\cos(c+dx)}}{2}\ldots$$

$$= \frac{a^{3/2}(7A+12B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{a^2(5A+\ldots}{4d\sqrt{a+\ldots}}$$

Mathematica [A] time = 0.58, size = 109, normalized size = 0.92

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)((7A+4B)\cos(c+dx)+2A)+\sqrt{2}\,(7A+1\ldots\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(Sqrt[2]*(7*A
 + 12*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*(2*A + (7*A +
 4*B)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

fricas [A] time = 0.62, size = 182, normalized size = 1.53

$$\frac{\left((7\,A+12\,B)a\cos\left(dx+c\right)^3+(7\,A+12\,B)a\cos\left(dx+c\right)^2\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(d}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{16\left(d\cos\left(dx+c\right)^3+d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] 1/16*(((7*A + 12*B)*a*cos(d*x + c)^3 + (7*A + 12*B)*a*cos(d*x + c)^2)*sqrt(
a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*
sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x +
c)^2)) + 4*((7*A + 4*B)*a*cos(d*x + c) + 2*A*a)*sqrt(a*cos(d*x + c) + a)*si
n(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.22, size = 991, normalized size = 8.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*(7*A*ln(
-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))+12*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+12*B*ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^
```

```
(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^4-4*(7*A*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)+7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+7*A*l
n(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+12*B*ln(-4/(-2*cos(1/2*d*x+1/2
*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(
1/2*d*x+1/2*c)+2*a))*a+12*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*
sin(1/2*d*x+1/2*c)^2+18*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+7*
A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+7*A*ln(-4/(-2*cos(1/2*d*x+1
/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*co
s(1/2*d*x+1/2*c)+2*a))*a+8*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
+12*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+12*B*ln(-4/(-2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/(2*cos(1/2*
d*x+1/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 1.22, size = 3339, normalized size = 28.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] -1/16*((12*a*cos(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 48*a*cos(2*d*x + 2*c
)^2*sin(3/2*d*x + 3/2*c) + 12*a*sin(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 4
8*a*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 160*a*cos(7/2*d*x + 7/2*c)*si
n(2*d*x + 2*c) + 168*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 72*a*cos(3/2
*d*x + 3/2*c)*sin(2*d*x + 2*c) - 24*a*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c)
 - 4*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 1
2*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 48*(
a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) + 4*(12*a*c
os(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 20*a*sin(7/2*d*x + 7/2*c) - 21*a*sin
(5/2*d*x + 5/2*c) - 3*a*sin(3/2*d*x + 3/2*c))*cos(4*d*x + 4*c) - 7*(sqrt(2)
*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*
x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*si
n(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x
+ 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x +
```

```
3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*
sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(
2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*
d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 2) - 7*(sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)
^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*
(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*lo
g(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(sqrt(2)*a*
cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + sqrt(2)*a*sin(4*d*x +
 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2
*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2
*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + sqrt(2)*a)*log(2*cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x +
 2*c) + a)*sin(13/2*d*x + 13/2*c) - 12*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x
+ 2*c) + a)*sin(11/2*d*x + 11/2*c) - 48*(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x
 + 2*c) + a)*sin(9/2*d*x + 9/2*c) + 4*(12*a*sin(2*d*x + 2*c)*sin(3/2*d*x +
3/2*c) + 20*a*cos(7/2*d*x + 7/2*c) + 21*a*cos(5/2*d*x + 5/2*c) + 9*a*cos(3/
2*d*x + 3/2*c))*sin(4*d*x + 4*c) - 80*(2*a*cos(2*d*x + 2*c) + a)*sin(7/2*d*
x + 7/2*c) - 84*(2*a*cos(2*d*x + 2*c) + a)*sin(5/2*d*x + 5/2*c) - 24*a*sin(
3/2*d*x + 3/2*c) - 4*(a*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin
(4*d*x + 4*c)^2 + 4*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2
*c)^2 + 2*(2*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c
) + a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 56*(a
*cos(4*d*x + 4*c)^2 + 4*a*cos(2*d*x + 2*c)^2 + a*sin(4*d*x + 4*c)^2 + 4*a*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*sin(2*d*x + 2*c)^2 + 2*(2*a*cos(2*d*
x + 2*c) + a)*cos(4*d*x + 4*c) + 4*a*cos(2*d*x + 2*c) + a)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(sqrt(2)*cos(4*d*x
+ 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sq
rt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*
(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d
*x + 2*c) + sqrt(2)) + 4*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c)
 + 6*sqrt(2)*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2
*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
```

```
rt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2
 + (2*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3
*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
- 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2))*sin(2*d*x + 2*c)^2 - 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*si
n(1/2*d*x + 1/2*c) - 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin(1/
2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 2*(sqrt(2)*a*cos(2*d*x + 2
*c) + sqrt(2)*a)*sin(7/2*d*x + 7/2*c) - 6*(sqrt(2)*a*cos(2*d*x + 2*c) + sqr
t(2)*a)*sin(5/2*d*x + 5/2*c) + 2*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + sqrt(2
)*a*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*B*sqrt(a)/(cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\cos(c + dx))\,(a + a\cos(c + dx))^{3/2}}{\cos(c + dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
[Out] Timed out
```

3.89 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=164

$$\frac{a^{3/2}(11A+14B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(11A+14B)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(7A+6B)\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{aA\,\mathrm{t}}{}$$

```
[Out] 1/8*a^(3/2)*(11*A+14*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/
d+1/8*a^2*(11*A+14*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/12*a^2*(7*A+6*B
)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a*A*sec(d*x+c)^2*(a+a*
cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.40, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2975, 2980, 2772, 2773, 206}

$$\frac{a^2(11A+14B)\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(11A+14B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(7A+6B)\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{aA\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a^(3/2)*(11*A + 14*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*d) + (a^2*(11*A + 14*B)*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]
]) + (a^2*(7*A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d
*x]]) + (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a + a\cos(c + dx))^{3/2}(A + B\cos(c + dx))\sec^4(c + dx)\,dx = \frac{aA\sqrt{a + a\cos(c + dx)}\ \sec^2(c + dx)\tan(c + dx)}{3d} + $$

$$= \frac{a^2(7A + 6B)\sec(c + dx)\tan(c + dx)}{12d\sqrt{a + a\cos(c + dx)}} + \frac{aA\sqrt{a + a\,c}}{}$$

$$= \frac{a^2(11A + 14B)\tan(c + dx)}{8d\sqrt{a + a\cos(c + dx)}} + \frac{a^2(7A + 6B)\sec(c +}{12d\sqrt{a + a\cos}}$$

$$= \frac{a^2(11A + 14B)\tan(c + dx)}{8d\sqrt{a + a\cos(c + dx)}} + \frac{a^2(7A + 6B)\sec(c +}{12d\sqrt{a + a\cos}}$$

$$= \frac{a^{3/2}(11A + 14B)\tanh^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a^2(11A}{8d\sqrt{a}}$$

Mathematica [A] time = 1.01, size = 132, normalized size = 0.80

$$\frac{a\sec\left(\frac{1}{2}(c + dx)\right)\sec^3(c + dx)\sqrt{a(\cos(c + dx) + 1)}\ \left(\sin\left(\frac{1}{2}(c + dx)\right)(4(11A + 6B)\cos(c + dx) + (33A + 42B)\cos(\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(3*Sqrt[2]*(1
1*A + 14*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (7*(7*A + 6*
B) + 4*(11*A + 6*B)*Cos[c + d*x] + (33*A + 42*B)*Cos[2*(c + d*x)])*Sin[(c +
 d*x)/2]))/(48*d)
```

fricas [A] time = 0.59, size = 202, normalized size = 1.23

$$\frac{3\left((11\,A + 14\,B)a\cos\left(dx + c\right)^4 + (11\,A + 14\,B)a\cos\left(dx + c\right)^3\right)\sqrt{a}\ \log\left(\frac{a\cos(dx+c)^3 - 7\,a\cos(dx+c)^2 - 4\,\sqrt{a\cos(dx+c)+a}\ \sqrt{a}\,(\mathrm{co}}{\cos(dx+c)^3 + \cos(dx+c)^2}\right.}{96\left(d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] 1/96*(3*((11*A + 14*B)*a*cos(d*x + c)^4 + (11*A + 14*B)*a*cos(d*x + c)^3)*s
qrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) +
```

```
 a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)) + 4*(3*(11*A + 14*B)*a*cos(d*x + c)^2 + 2*(11*A + 6*B)*a*cos(d*x
 + c) + 8*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^4 + d
*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.36, size = 1310, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 1/6*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(11*A*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+11*A*ln(-4/(-2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))+14*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+14*B*l
n(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+12*(22*A
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+28*B*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)+33*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a
+33*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+42*B*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))*a+42*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+
2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-352*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)-198*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-198*A*ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-384*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2)-252*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*
```

```
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-25
2*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+126
*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+33*A*ln(-4/(-2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))*a+33*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*a+108*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+42*B*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+42*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*
c)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
^3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\cos(c + d x))\,(a + a\cos(c + d x))^{3/2}}{\cos(c + d x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^4,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.90 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=209

$$\frac{a^{3/2}(75A+88B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(75A+88B)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(9A+8B)\tan(c+dx)\sec^2(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] 1/64*a^(3/2)*(75*A+88*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))
/d+1/64*a^2*(75*A+88*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/96*a^2*(75*A+
88*B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/24*a^2*(9*A+8*B)*sec
(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a*A*sec(d*x+c)^3*(a+a*cos
(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.48, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2975, 2980, 2772, 2773, 206}

$$\frac{a^2(75A+88B)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(75A+88B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(9A+8B)\tan(c+dx)\sec^2(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a^(3/2)*(75*A + 88*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(64*d) + (a^2*(75*A + 88*B)*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*
x]]) + (a^2*(75*A + 88*B)*Sec[c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*Cos[c
 + d*x]]) + (a^2*(9*A + 8*B)*Sec[c + d*x]^2*Tan[c + d*x])/(24*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d*x])
/(4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
```

```
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA\sqrt{a+a\cos(c+dx)}\,\sec^3(c+dx)\tan(c+dx)}{4d} +$$

$$= \frac{a^2(9A+8B)\sec^2(c+dx)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aA\sqrt{a+}}{}$$

$$= \frac{a^2(75A+88B)\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(9A}{}$$

$$= \frac{a^2(75A+88B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(75A+88B)\sec}{96d\sqrt{a+a}}$$

$$= \frac{a^2(75A+88B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(75A+88B)\sec}{96d\sqrt{a+a}}$$

$$= \frac{a^{3/2}(75A+88B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^2(75}{64d}$$

Mathematica [A] time = 1.60, size = 151, normalized size = 0.72

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((1155A+1048B)\cos(c+dx)+4(75A+8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^4*(6*Sqrt[2]*(7
5*A + 88*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (492*A + 352
*B + (1155*A + 1048*B)*Cos[c + d*x] + 4*(75*A + 88*B)*Cos[2*(c + d*x)] + 22
5*A*Cos[3*(c + d*x)] + 264*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*d)
```

fricas [A] time = 0.86, size = 220, normalized size = 1.05

$$\frac{3\left((75A+88B)a\cos(dx+c)^5+(75A+88B)a\cos(dx+c)^4\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\,\sqrt{a}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="fricas")
```

```
[Out] 1/768*(3*((75*A + 88*B)*a*cos(d*x + c)^5 + (75*A + 88*B)*a*cos(d*x + c)^4)*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c)
+ a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) + 4*(3*(75*A + 88*B)*a*cos(d*x + c)^3 + 2*(75*A + 88*B)*a*cos(d
*x + c)^2 + 8*(15*A + 8*B)*a*cos(d*x + c) + 48*A*a)*sqrt(a*cos(d*x + c) + a
)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.41, size = 1631, normalized size = 7.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/24*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*a*(75*A*
ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+75*A*ln(4/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))+88*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+88*B
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^8-48*(75*A
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+88*B*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)+150*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*a+150*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+176*B*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+176*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+8*(825*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)+968*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+675*A*l
n(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+675*A*ln(4/(2*cos(1/2*d*x+1/
```

```
2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))*a+792*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
*a+792*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^
4-4*(1095*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+1208*B*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+450*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^
(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*
x+1/2*c)+2*a))*a+450*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+528*B*
ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+528*B*ln(4/(2*cos(1/2*d*x+1
/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*co
s(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+1086*A*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)+225*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a
))*a+225*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1008*B*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+264*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+
1/2*c)+2*a))*a+264*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))^4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4/sin(1/2*d*x+1/2*
c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,\left(a+a\cos(c+d\,x)\right)^{3/2}}{\cos(c+d\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^5,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
[Out] Timed out
```

3.91 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=237

$$\frac{2a^3(209A+194B)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(803A+710B)\sin(c+dx)}{495d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(11A+14B)\sin(c+dx)\cos^3(}{99d}$$

```
[Out] 2/1155*a*(803*A+710*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/11*a*B*cos(d*x
+c)^3*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/495*a^3*(803*A+710*B)*sin(d*x+c
)/d/(a+a*cos(d*x+c))^(1/2)+2/693*a^3*(209*A+194*B)*cos(d*x+c)^3*sin(d*x+c)/
d/(a+a*cos(d*x+c))^(1/2)-4/3465*a^2*(803*A+710*B)*sin(d*x+c)*(a+a*cos(d*x+c
))^(1/2)/d+2/99*a^2*(11*A+14*B)*cos(d*x+c)^3*sin(d*x+c)*(a+a*cos(d*x+c))^(1
/2)/d
```

Rubi [A] time = 0.65, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2976, 2981, 2759, 2751, 2646}

$$\frac{2a^3(209A+194B)\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(11A+14B)\sin(c+dx)\cos^3(c+dx)\sqrt{a\cos(c+dx)+a}}{99d}+\frac{2a^3(}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a^3*(803*A + 710*B)*Sin[c + d*x])/(495*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
a^3*(209*A + 194*B)*Cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c +
d*x]]) - (4*a^2*(803*A + 710*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(346
5*d) + (2*a^2*(11*A + 14*B)*Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
 d*x])/(99*d) + (2*a*(803*A + 710*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x
])/(1155*d) + (2*a*B*Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x]
)/(11*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{2aB\cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx}{11d}$$

$$= \frac{2a^2(11A+14B)\cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\ \text{s}}{99d}$$

$$= \frac{2a^3(209A+194B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a}{}$$

$$= \frac{2a^3(209A+194B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2a}{}$$

$$= \frac{2a^3(209A+194B)\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} - \frac{4a}{}$$

$$= \frac{2a^3(803A+710B)\sin(c+dx)}{495d\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(209A+19}{693d\sqrt{}}$$

Mathematica [A] time = 1.13, size = 127, normalized size = 0.54

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,((68552A+69890B)\cos(c+dx)+16(1397A+1625B)\cos(2(c+dx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(124366*A + 114640*B + (68552*A + 69890*B)*
Cos[c + d*x] + 16*(1397*A + 1625*B)*Cos[2*(c + d*x)] + 5720*A*Cos[3*(c + d*
x)] + 8675*B*Cos[3*(c + d*x)] + 770*A*Cos[4*(c + d*x)] + 2240*B*Cos[4*(c +
d*x)] + 315*B*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])/(27720*d)
```

fricas [A] time = 0.67, size = 137, normalized size = 0.58

$$\frac{2\left(315\,Ba^2\cos(dx+c)^5+35\,(11\,A+32\,B)a^2\cos(dx+c)^4+5\,(286\,A+355\,B)a^2\cos(dx+c)^3+3\,(803\,A+71}{3465\,(d\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/3465*(315*B*a^2*cos(d*x + c)^5 + 35*(11*A + 32*B)*a^2*cos(d*x + c)^4 + 5*
(286*A + 355*B)*a^2*cos(d*x + c)^3 + 3*(803*A + 710*B)*a^2*cos(d*x + c)^2 +
```

```
4*(803*A + 710*B)*a^2*cos(d*x + c) + 8*(803*A + 710*B)*a^2)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 0.93, size = 319, normalized size = 1.35

$$\frac{1}{55440}\sqrt{2}\left(\frac{315\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)}{d}+\frac{385\left(2\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+5\,Ba^2\mathrm{sgn}\left(\mathrm{co}\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] 1/55440*sqrt(2)*(315*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(11/2*d*x + 11/2*c)
/d + 385*(2*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 5*B*a^2*sgn(cos(1/2*d*x + 1/2
*c)))*sin(9/2*d*x + 9/2*c)/d + 495*(10*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 13
*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(7/2*d*x + 7/2*c)/d + 693*(24*A*a^2*sg
n(cos(1/2*d*x + 1/2*c)) + 25*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(5/2*d*x +
 5/2*c)/d + 2310*(20*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 19*B*a^2*sgn(cos(1/2
*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c)/d + 6930*(14*A*a^2*sgn(cos(1/2*d*x + 1
/2*c)) + 15*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c)/d + 27720
*(3*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*si
n(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.34, size = 142, normalized size = 0.60

$$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-2520B\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(1540A+10780B)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-5940A-1881\right.}{3465\sqrt{a\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 8/3465*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(-2520*B*sin(1/2*d*x+1/2*c
)^10+(1540*A+10780*B)*sin(1/2*d*x+1/2*c)^8+(-5940*A-18810*B)*sin(1/2*d*x+1/
2*c)^6+(9009*A+17325*B)*sin(1/2*d*x+1/2*c)^4+(-6930*A-9240*B)*sin(1/2*d*x+1
/2*c)^2+3465*A+3465*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.98, size = 207, normalized size = 0.87

$$\frac{22\left(35\sqrt{2}\,a^2\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+225\sqrt{2}\,a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+756\sqrt{2}\,a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+2100\sqrt{2}\,a^2\sin\left(\frac{3}{2}\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/55440*(22*(35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)*a^2*sin(7/2*
d*x + 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqrt(2)*a^2*sin(
3/2*d*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 5*(63
*sqrt(2)*a^2*sin(11/2*d*x + 11/2*c) + 385*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c)
+ 1287*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 3465*sqrt(2)*a^2*sin(5/2*d*x + 5/
2*c) + 8778*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 31878*sqrt(2)*a^2*sin(1/2*d*
x + 1/2*c))*B*sqrt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^2\,(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.92 $\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=175

$$\frac{64a^3(15A+13B)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2(15A+13B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{2(9A-2B)\sin(c+dx)(a\cos(c$$

```
[Out] 2/105*a*(15*A+13*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/63*(9*A-2*B)*(a+a
*cos(d*x+c))^(5/2)*sin(d*x+c)/d+2/9*B*(a+a*cos(d*x+c))^(7/2)*sin(d*x+c)/a/d
+64/315*a^3*(15*A+13*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+16/315*a^2*(15*
A+13*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.28, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3023, 2751, 2647, 2646}

$$\frac{16a^2(15A+13B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{64a^3(15A+13B)\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2(9A-2B)\sin(c+dx)(a\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (64*a^3*(15*A + 13*B)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (16*
a^2*(15*A + 13*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (2*a*(15
*A + 13*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*d) + (2*(9*A - 2*B
)*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) + (2*B*(a + a*Cos[c + d*x
])^(7/2)*Sin[c + d*x])/(9*a*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (a+a\cos(c+dx))^{5/2}\left(A\cos(c+dx)+B\cos^2(c\right.$$

$$= \frac{2B(a+a\cos(c+dx))^{7/2}\sin(c+dx)}{9ad} + \frac{2\int(a+a\,\mathrm{c}}{}$$

$$= \frac{2(9A-2B)(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{63d} + \frac{2B}{}$$

$$= \frac{2a(15A+13B)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{105d} +$$

$$= \frac{16a^2(15A+13B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{315d} +$$

$$= \frac{64a^3(15A+13B)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2(15A+13B)}{}$$

Mathematica [A] time = 0.75, size = 105, normalized size = 0.60

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,((3030A+3116B)\cos(c+dx)+8(90A+127B)\cos(2(c+dx))+90A\,\mathrm{c}}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(6240*A + 5653*B + (3030*A + 3116*B)*Cos[c
+ d*x] + 8*(90*A + 127*B)*Cos[2*(c + d*x)] + 90*A*Cos[3*(c + d*x)] + 260*B*
Cos[3*(c + d*x)] + 35*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(1260*d)
```

fricas [A] time = 0.81, size = 116, normalized size = 0.66

$$\frac{2\left(35\,Ba^{2}\cos\left(dx+c\right)^{4}+5\,(9\,A+26\,B)a^{2}\cos\left(dx+c\right)^{3}+3\,(60\,A+73\,B)a^{2}\cos\left(dx+c\right)^{2}+(345\,A+292\,B)a^{2}\cos\right.}{315\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/315*(35*B*a^2*cos(d*x + c)^4 + 5*(9*A + 26*B)*a^2*cos(d*x + c)^3 + 3*(60*
A + 73*B)*a^2*cos(d*x + c)^2 + (345*A + 292*B)*a^2*cos(d*x + c) + 2*(345*A
+ 292*B)*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 0.86, size = 225, normalized size = 1.29

$$\frac{1}{2520}\sqrt{2}\left(\frac{35\,Ba^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)}{d}+\frac{45\left(2\,Aa^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+5\,Ba^{2}\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,d\right.\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/2520*sqrt(2)*(35*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c)/d +
 45*(2*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 5*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))
*sin(7/2*d*x + 7/2*c)/d + 126*(5*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 6*B*a^2*
sgn(cos(1/2*d*x + 1/2*c)))*sin(5/2*d*x + 5/2*c)/d + 210*(11*A*a^2*sgn(cos(1
/2*d*x + 1/2*c)) + 10*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c)
/d + 630*(15*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 13*B*a^2*sgn(cos(1/2*d*x + 1
/2*c)))*sin(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.47, size = 123, normalized size = 0.70

$$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{3}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(140B\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-90A-540B)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(315A+819B)\left(\sin^{4}\left(\frac{dx}{2}\right.\right.\right.}{315\sqrt{a\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 8/315*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(140*B*sin(1/2*d*x+1/2*c)^8
+(-90*A-540*B)*sin(1/2*d*x+1/2*c)^6+(315*A+819*B)*sin(1/2*d*x+1/2*c)^4+(-42
0*A-630*B)*sin(1/2*d*x+1/2*c)^2+315*A+315*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^
2)^(1/2)/d
```

maxima [A] time = 1.02, size = 172, normalized size = 0.98

$$\frac{30\left(3\sqrt{2}\,a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\,\sqrt{2}\,a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+77\,\sqrt{2}\,a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+315\,\sqrt{2}\,a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/2520*(30*(3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x
 + 5/2*c) + 77*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c))*A*sqrt(a) + (35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)
*a^2*sin(7/2*d*x + 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqr
t(2)*a^2*sin(3/2*d*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sq
rt(a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.93 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{64a^3(7A+5B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2(7A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(7A+5B)\sin(c+dx)(a\cos(c+d}{35d}$$

```
[Out] 2/35*a*(7*A+5*B)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(a+a*cos(d*x+c))
^(5/2)*sin(d*x+c)/d+64/105*a^3*(7*A+5*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2
)+16/105*a^2*(7*A+5*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2751, 2647, 2646}

$$\frac{64a^3(7A+5B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2(7A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}+\frac{2a(7A+5B)\sin(c+dx)(a\cos(c+d}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (64*a^3*(7*A + 5*B)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a^
2*(7*A + 5*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (2*a*(7*A +
5*B)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*B*(a + a*Cos[c +
d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\,dx = \frac{2B(a + a\cos(c + dx))^{5/2}\sin(c + dx)}{7d} + \frac{1}{7}(7A + 5B)\int (a + a\,\mathrm{co}$$
$$= \frac{2a(7A + 5B)(a + a\cos(c + dx))^{3/2}\sin(c + dx)}{35d} + \frac{2B(a + a\cos}{}$$
$$= \frac{16a^2(7A + 5B)\sqrt{a + a\cos(c + dx)}\,\sin(c + dx)}{105d} + \frac{2a(7A + 5B)}{}$$
$$= \frac{64a^3(7A + 5B)\sin(c + dx)}{105d\sqrt{a + a\cos(c + dx)}} + \frac{16a^2(7A + 5B)\sqrt{a + a\cos(c + d}}{105d}$$

Mathematica [A]   time = 0.36, size = 83, normalized size = 0.60

$$\frac{a^2\tan\left(\frac{1}{2}(c + dx)\right)\sqrt{a(\cos(c + dx) + 1)}\,((392A + 505B)\cos(c + dx) + 6(7A + 20B)\cos(2(c + dx)) + 1246A + 1}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(1246*A + 1040*B + (392*A + 505*B)*Cos[c +
d*x] + 6*(7*A + 20*B)*Cos[2*(c + d*x)] + 15*B*Cos[3*(c + d*x)])*Tan[(c + d*
x)/2])/(210*d)
```

fricas [A]   time = 0.47, size = 95, normalized size = 0.69

$$\frac{2\left(15\,Ba^2\cos(dx + c)^3 + 3\,(7\,A + 20\,B)a^2\cos(dx + c)^2 + (98\,A + 115\,B)a^2\cos(dx + c) + (301\,A + 230\,B)a^2\right)\sqrt{a}}{105\,(d\cos(dx + c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/105*(15*B*a^2*cos(d*x + c)^3 + 3*(7*A + 20*B)*a^2*cos(d*x + c)^2 + (98*A
+ 115*B)*a^2*cos(d*x + c) + (301*A + 230*B)*a^2)*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/(d*cos(d*x + c) + d)
```

giac [A] time = 1.39, size = 225, normalized size = 1.63

$$\frac{1}{420}\sqrt{2}\left(\frac{15\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}{d}+\frac{21\left(2\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+5\,Ba^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx\right.\right.\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*sqrt(2)*(15*B*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c)/d +
21*(2*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 5*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*
sin(5/2*d*x + 5/2*c)/d + 35*(10*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 11*B*a^2*
sgn(cos(1/2*d*x + 1/2*c)))*sin(3/2*d*x + 3/2*c)/d + 105*(8*A*a^2*sgn(cos(1/
2*d*x + 1/2*c)) + 7*B*a^2*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c)/d
 + 420*(3*A*a^2*sgn(cos(1/2*d*x + 1/2*c)) + 2*B*a^2*sgn(cos(1/2*d*x + 1/2*c
)))*sin(1/2*d*x + 1/2*c)/d)*sqrt(a)
```

maple [A] time = 0.33, size = 104, normalized size = 0.75

$$\frac{8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-30B\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(21A+105B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-70A-140B)\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}{105\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 8/105*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(-30*B*sin(1/2*d*x+1/2*c)^6
+(21*A+105*B)*sin(1/2*d*x+1/2*c)^4+(-70*A-140*B)*sin(1/2*d*x+1/2*c)^2+105*A
+105*B)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.05, size = 139, normalized size = 1.01

$$\frac{14\left(3\sqrt{2}\,a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\sqrt{2}\,a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\sqrt{2}\,a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A\sqrt{a}+5\left(3\sqrt{2}\,a^2\sin\left(\frac{7}{2}\,dx\right.\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*(14*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x
+ 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 5*(3*sqrt(2)*a
^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 77*sqrt(2)*
```

```
a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt(a))
/d
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\cos(c + dx))\,(a + a\cos(c + dx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
[Out] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.94 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=142

$$\frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^3(35A+32B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(5A+8B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2aB}{}$$

```
[Out] 2*a^(5/2)*A*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2/5*a*B*(a
+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/15*a^3*(35*A+32*B)*sin(d*x+c)/d/(a+a*co
s(d*x+c))^(1/2)+2/15*a^2*(5*A+8*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.41, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2976, 2981, 2773, 206}

$$\frac{2a^3(35A+32B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(5A+8B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aB}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*a^(5/2)*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d +
(2*a^3*(35*A + 32*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(5*A + 8*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*B*(a + a*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{2}{5}\int (a+a$$

$$=\frac{2a^2(5A+8B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15d}+\frac{2a$$

$$=\frac{2a^3(35A+32B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(5A+8B)\sqrt{a+$$

$$=\frac{2a^3(35A+32B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(5A+8B)\sqrt{a+$$

$$=\frac{2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^3(35A+32B)}{15d\sqrt{a+a\cos$$

Mathematica [A] time = 0.42, size = 104, normalized size = 0.73

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,\left(\sin\left(\frac{1}{2}(c+dx)\right)(2(5A+14B)\cos(c+dx)+80A+3B\cos(2(c+dx))+}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*A*ArcTanh[Sqrt
[2]*Sin[(c + d*x)/2]] + (80*A + 89*B + 2*(5*A + 14*B)*Cos[c + d*x] + 3*B*Co
s[2*(c + d*x)])*Sin[(c + d*x)/2]))/(15*d)
```

fricas [A] time = 0.44, size = 177, normalized size = 1.25

$$\frac{15\left(Aa^2\cos(dx+c)+Aa^2\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(3\,Ba^2\,\mathrm{co}\right.}{30\,(d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] 1/30*(15*(A*a^2*cos(d*x + c) + A*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*c
os(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d
*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*B*a^2*cos(d*x + c)
^2 + (5*A + 14*B)*a^2*cos(d*x + c) + (40*A + 43*B)*a^2)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [B] time = 1.28, size = 311, normalized size = 2.19

$$\frac{a^{\frac{3}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(24B\sqrt{a}\,\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/15*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*B*a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*a^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)*(A+4*B)*sin(1/2*d*x+1/2*c)^2+90*A*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+15*A*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*
d*x+1/2*c)+2*a))*a+15*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+120
*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)/(a*co
s(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 0.87, size = 61, normalized size = 0.43

$$\frac{\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)B\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt(a)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+a\,\cos(c+d\,x)\right)^{5/2}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

3.95 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{a^{5/2}(5A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(3A-2B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\cdots$$

```
[Out] a^(5/2)*(5*A+2*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+1/3*
a^3*(3*A+14*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-1/3*a^2*(3*A-2*B)*sin(d*
x+c)*(a+a*cos(d*x+c))^(1/2)/d+a*A*(a+a*cos(d*x+c))^(3/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.45, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2975, 2976, 2981, 2773, 206}

$$\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(3A-2B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{a^{5/2}(5A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^(5/2)*(5*A + 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/d + (a^3*(3*A + 14*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) - (a
^2*(3*A - 2*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (a*A*(a + a*C
os[c + d*x])^(3/2)*Tan[c + d*x])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^{3/2}\tan(c+dx)}{d} + \int (a+a\,\mathrm{co}$$

$$= -\frac{a^2(3A-2B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{aA}{}$$

$$= \frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(3A-2B)\sqrt{a+a\,\mathrm{co}}}{3a}$$

$$= \frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(3A-2B)\sqrt{a+a\,\mathrm{co}}}{3a}$$

$$= \frac{a^{5/2}(5A+2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a^3(3A+1}{3d\sqrt{a+}}$$

Mathematica [**A**] time = 0.56, size = 120, normalized size = 0.83

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2(3A+8B)\cos(c+dx)+3A+B\cos(2(c+\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(3*Sqrt[2]*(5
*A + 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*(3*A + B + 2*(
3*A + 8*B)*Cos[c + d*x] + B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d)
```

fricas [**A**] time = 1.21, size = 202, normalized size = 1.40

$$\frac{3\left((5A+2B)a^2\cos(dx+c)^2+(5A+2B)a^2\cos(dx+c)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{12\left(d\cos(dx+c)^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] 1/12*(3*((5*A + 2*B)*a^2*cos(d*x + c)^2 + (5*A + 2*B)*a^2*cos(d*x + c))*sqr
t(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a
)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)) + 4*(2*B*a^2*cos(d*x + c)^2 + 2*(3*A + 8*B)*a^2*cos(d*x + c) + 3*A
```

```
*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x
+ c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.21, size = 756, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] 1/3*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*B*a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+(-24*A*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-30*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a-30*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-80*B*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-12*B*ln(-4/(-2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))*a-12*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)
*sin(1/2*d*x+1/2*c)^2+18*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+1
5*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+15*A*ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))*a+36*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)+6*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+6*B*ln(4/(2*cos(1/
2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(
1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 2.00, size = 8114, normalized size = 56.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] -1/252*(1449*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^3*sin(2*d*x + 2*c) - 1260*sqr
t(2)*a^2*sin(1/2*d*x + 1/2*c)^3 - 1449*(sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt
(2)*a^2)*sin(5/2*d*x + 5/2*c)^3 + 21*(25*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin
(3/2*d*x + 3/2*c) + 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c)
- 60*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 5*(5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*
c) - 12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + (25*sqrt(2)*a^
2*cos(3/2*d*x + 3/2*c) + 198*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x +
2*c))*cos(5/2*d*x + 5/2*c)^2 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 25
*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*
sin(3/2*d*x + 3/2*c))*cos(2*d*x + 2*c)^2 + 21*(25*sqrt(2)*a^2*cos(2*d*x + 2
*c)^2*sin(3/2*d*x + 3/2*c) + 25*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x
+ 3/2*c) + 69*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 198*sqrt(
2)*a^2*sin(1/2*d*x + 1/2*c) + (25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 198*sq
rt(2)*a^2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 5*(5*sqrt(2)*a^2*cos(3/2
*d*x + 3/2*c) + 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sin(
5/2*d*x + 5/2*c)^2 - 21*(12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 25*(sqrt(2)*
a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*
x + 3/2*c))*sin(2*d*x + 2*c)^2 - 35*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^2*sin
(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c)*sin
(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) + 2*sqr
t(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + (sqrt
(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*
d*x + 2*c))*cos(13/2*d*x + 13/2*c) - 135*(sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^
2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1/2*c
)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c) +
2*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) +
(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*s
in(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) - 98*(sqrt(2)*a^2*cos(5/2*d*x + 5/2
*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x + 1
/2*c)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x + 2*c
) + 2*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c
) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^
2)*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) + 390*(sqrt(2)*a^2*cos(5/2*d*x +
5/2*c)^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*cos(1/2*d*x
+ 1/2*c)*sin(2*d*x + 2*c) + sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)^2*sin(2*d*x +
2*c) + 2*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/
2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*
c)^2)*sin(2*d*x + 2*c))*cos(7/2*d*x + 7/2*c) + 21*(50*sqrt(2)*a^2*cos(2*d*x
 + 2*c)^2*cos(1/2*d*x + 1/2*c)*sin(3/2*d*x + 3/2*c) + 50*sqrt(2)*a^2*cos(1/
2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) - 120*sqrt(2)*a^2*co
s(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 10*(5*sqrt(2)*a^2*cos(1/2*d*x + 1
/2*c)*sin(3/2*d*x + 3/2*c) - 12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*
```

```
x + 1/2*c))*cos(2*d*x + 2*c) + (50*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2
*d*x + 1/2*c) + 189*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 69*sqrt(2)*a^2*sin
(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c))*cos(5/2*d*x + 5/2*c) - 21*(60*sqrt(2
)*a^2*sin(1/2*d*x + 1/2*c)^3 - 25*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqr
t(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c) + 12*(5*sqrt(2)*a^2*c
os(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*a^2)*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*
c) - 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*co
s(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*c
os(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2
*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^
2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x
 + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2
*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2
*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x
 + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2
*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*
sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(
2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 315*(a^2*cos(1/
2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a
^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^
2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2
*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x
 + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*si
n(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/
2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*
d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5
/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d
*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*x
 + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c)
 + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a
^2*sin(1/2*d*x + 1/2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^
2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x
 + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d
*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/
2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2
)*sin(2*d*x + 2*c)^2 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2
*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d
```

```
*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1
/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*c
os(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x
 + 1/2*c) + 2*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x +
 1/2*c))*sin(5/2*d*x + 5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 2) + 315*(a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2
*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(5/2*d*x + 5/2*c)^2 + (a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*si
n(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (a^2*cos(2*d*x + 2*c)^2 + a^2*si
n(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c)^2 + (
a^2*cos(1/2*d*x + 1/2*c)^2 + a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2
 + 2*(a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + a^2*cos(1/2*d*x + 1/2*c
)*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + a^2*co
s(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(a^2*cos(1/2*d*x + 1/2*c)^2 +
a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(a^2*cos(2*d*x + 2*c)^2*si
n(1/2*d*x + 1/2*c) + a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*a^2*co
s(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x
 + 5/2*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2
*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*s
qrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) +
35*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2
 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (s
qrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt
(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1
/2*c))*cos(5/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)
*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x +
2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x +
 5/2*c))*sin(13/2*d*x + 13/2*c) + 135*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 +
 sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(
2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^
2)*sin(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1
/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (sqrt(2)*a
^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x +
 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*
sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(11/2*d*x + 11/2*c) + 7*(9*s
qrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 9*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 -
(5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 - 4*sq
rt(2)*a^2*cos(2*d*x + 2*c) - 9*sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 - 5*(sqr
t(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2
*d*x + 2*c)^2 - (5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 5*sqrt(2)*a^2*sin(2*d*x
 + 2*c)^2 - 4*sqrt(2)*a^2*cos(2*d*x + 2*c) - 9*sqrt(2)*a^2)*sin(5/2*d*x + 5
```

```
/2*c)^2 - 5*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x +
 1/2*c)^2)*sin(2*d*x + 2*c)^2 - 2*(5*sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2
*d*x + 1/2*c) + 5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 - 4*s
qrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) - 9*sqrt(2)*a^2*cos(1/2*d*
x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 4*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 +
sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) - 2*(5*sqrt(2)*a^2*cos
(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin
(1/2*d*x + 1/2*c) - 4*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) - 9
*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(9/2*d*x + 9/2*
c) - 390*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/
2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^
2 + (sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + 2
*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d
*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + s
qrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*
d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2
*d*x + 5/2*c))*sin(7/2*d*x + 7/2*c) - 21*(69*sqrt(2)*a^2*cos(1/2*d*x + 1/2*
c)^2 + 189*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + 69*(sqrt(2)*a^2*cos(2*d*x +
 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 - 2*(25*sqrt(2)*a^2*sin(3/2*d*x
 + 3/2*c)*sin(1/2*d*x + 1/2*c) - 6*sqrt(2)*a^2)*cos(2*d*x + 2*c)^2 - 2*(25*
sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) - 6*sqrt(2)*a^2)*sin(
2*d*x + 2*c)^2 + 12*sqrt(2)*a^2 + 138*(sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2
*d*x + 1/2*c) - sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)
*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + (69*sqrt(2)*a^2*cos(1/2*d
*x + 1/2*c)^2 - 50*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) +
189*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + 24*sqrt(2)*a^2)*cos(2*d*x + 2*c) -
 10*(5*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + 12*sqrt(2)*a
^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sin(5/2*d*x
 + 5/2*c) + 105*(12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^3 + 12*sqrt(2)*a^2*cos
(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + 5*(sqrt(2)*a^2*cos(1/2*d*x + 1/2
*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(3/2*d*x + 3/2*c))*sin(2*d*x
 + 2*c) - 252*(5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2)*sin(1/2*
d*x + 1/2*c) - 135*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/
2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x
+ 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*
c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*
c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*si
n(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*
x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*
x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/
2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*sq
rt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x +
 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqr
t(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*
x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*
```

```
x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*
a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(7/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) - 63*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2
+ sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sq
rt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2
)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^
2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(2)*a^2*cos(2*d*x + 2*c
)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqr
t(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sq
rt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(sqrt(2)*a^2*cos(2
*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(2
*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + sqrt(
2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(sqrt(2)*a^2*cos(1/2*
d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + 2*(
sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(2*d*x
 + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*sin(1/2*d*x
 + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c))*sin(5/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1260*(sqrt(2)*a^2*c
os(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2 + (sqrt(2)*a^2*c
os(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*
x + 2*c) + sqrt(2)*a^2)*cos(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1/2*d*x +
 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c)^2 + (sqrt(
2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*
cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^2 + (sqrt(2)*a^2*cos(1
/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2
+ 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(
1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)*cos(1/
2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2
*(sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*
cos(2*d*x + 2*c) + 2*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) +
 sqrt(2)*a^2*sin(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*a^2*cos(2*
d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sin(5/2
*d*x + 5/2*c))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
)*A*sqrt(a)/(((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)*cos(5/2*d*x + 5/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2)*cos(2*d*x + 2*c)^2 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)*sin(5/2*d*x + 5/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2*cos(1/2*d*x +
 1/2*c) + cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)*cos(
1/2*d*x + 1/2*c) + cos(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2)*cos(2*d*x + 2*c) + cos(1/2*d*x + 1
/2*c)^2 + 2*(cos(2*d*x + 2*c)^2*sin(1/2*d*x + 1/2*c) + sin(2*d*x + 2*c)^2*s
in(1/2*d*x + 1/2*c) + 2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + sin(1/2*d*x
 + 1/2*c))*sin(5/2*d*x + 5/2*c) + sin(1/2*d*x + 1/2*c)^2)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{5/2}}{\cos(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^2,x)
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
[Out] Timed out
```

3.96 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=156

$$\frac{a^{5/2}(19A+20B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(7A+4B)\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+$$

```
[Out] 1/4*a^(5/2)*(19*A+20*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/
d-1/4*a^3*(9*A-4*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*a*A*(a+a*cos(d*
x+c))^(3/2)*sec(d*x+c)*tan(d*x+c)/d+1/4*a^2*(7*A+4*B)*(a+a*cos(d*x+c))^(1/2
)*tan(d*x+c)/d
```

Rubi [A] time = 0.47, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2975, 2981, 2773, 206}

$$-\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(7A+4B)\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+\frac{a^{5/2}(19A+20B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^(5/2)*(19*A + 20*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(4*d) - (a^3*(9*A - 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]])
 + (a^2*(7*A + 4*B)*Sqrt[a + a*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a*A*(a
+ a*Cos[c + d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^{3/2}\sec(c+dx)\tan(c+dx)}{2d}$$

$$= \frac{a^2(7A+4B)\sqrt{a+a\cos(c+dx)}\,\tan(c+dx)}{4d} + \frac{aA}{}$$

$$= -\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(7A+4B)\sqrt{a+a}}{}$$

$$= -\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(7A+4B)\sqrt{a+a}}{}$$

$$= \frac{a^{5/2}(19A+20B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} - \frac{a^3(9A}{4d\sqrt{a}}$$

Mathematica [A] time = 0.68, size = 126, normalized size = 0.81

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\cos(c+dx)+1)}\,\left(2\sin\left(\frac{1}{2}(c+dx)\right)((11A+4B)\cos(c+dx)+2(A+2B\cos(}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(Sqrt[2]*(1
9*A + 20*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*((11*A + 4
*B)*Cos[c + d*x] + 2*(A + 2*B + 2*B*Cos[2*(c + d*x)]))*Sin[(c + d*x)/2]))/(
8*d)
```

fricas [A] time = 0.61, size = 204, normalized size = 1.31

$$\frac{\left((19\,A + 20\,B)a^2\cos\left(dx + c\right)^3 + (19\,A + 20\,B)a^2\cos\left(dx + c\right)^2\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3 - 7\,a\cos(dx+c)^2 - 4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(c}{\cos(dx+c)^3 + \cos(dx+c)^2}\right.}{16\left(d\cos\left(dx + c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] 1/16*(((19*A + 20*B)*a^2*cos(d*x + c)^3 + (19*A + 20*B)*a^2*cos(d*x + c)^2)
*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c)
 + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(
d*x + c)^2)) + 4*(8*B*a^2*cos(d*x + c)^2 + (11*A + 4*B)*a^2*cos(d*x + c) +
2*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d
*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.19, size = 1016, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*((76*A*ln(-4/
(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
```

```
1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a+64*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+80*B*
ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+80*B*ln(4/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-44*A*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)-76*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
*a-76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-80*B*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-80*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2*a))*a-80*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/
2*c)^2+26*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+19*A*ln(-4/(-2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+19*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))*a+24*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+20*B*ln(-4
/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^
(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+20*B*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*
d*x+1/2*c)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+d\,x))\,(a+a\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
[Out] Timed out
```

3.97 $\int (a+a\cos (c+dx))^{5/2}(A+B\cos (c+dx))\sec ^4(c+dx)\,dx$

Optimal. Leaf size=164

$$\frac{a^{5/2}(25A+38B)\tanh ^{-1}\left (\frac{\sqrt{a}\sin (c+dx)}{\sqrt{a\cos (c+dx)+a}}\right)}{8d}+\frac{a^3(49A+54B)\tan (c+dx)}{24d\sqrt{a\cos (c+dx)+a}}+\frac{a^2(3A+2B)\tan (c+dx)\sec (c+dx)\sqrt{a\,c}}{4d}$$

```
[Out] 1/8*a^(5/2)*(25*A+38*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/
d+1/3*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2*tan(d*x+c)/d+1/24*a^3*(49*A+5
4*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a^2*(3*A+2*B)*sec(d*x+c)*(a+a*
cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.53, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2975, 2980, 2773, 206}

$$\frac{a^3(49A+54B)\tan (c+dx)}{24d\sqrt{a\cos (c+dx)+a}}+\frac{a^{5/2}(25A+38B)\tanh ^{-1}\left (\frac{\sqrt{a}\sin (c+dx)}{\sqrt{a\cos (c+dx)+a}}\right)}{8d}+\frac{a^2(3A+2B)\tan (c+dx)\sec (c+dx)\sqrt{a\,c}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a^(5/2)*(25*A + 38*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*
x]]])/(8*d) + (a^3*(49*A + 54*B)*Tan[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x
]]) + (a^2*(3*A + 2*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/
(4*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\tan(c+dx)}{3d} +$$

$$= \frac{a^2(3A+2B)\sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\tan(c}{4d}$$

$$= \frac{a^3(49A+54B)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(3A+2B)\sqrt{a+a}}{}$$

$$= \frac{a^3(49A+54B)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(3A+2B)\sqrt{a+a}}{}$$

$$= \frac{a^{5/2}(25A+38B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a^3(49A}{24d\sqrt{}}$$

Mathematica [A] time = 1.11, size = 131, normalized size = 0.80

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(4(17A+6B)\cos(c+dx)+(75A+66B)\cos\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(3*Sqrt[2]*
(25*A + 38*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (91*A + 66
*B + 4*(17*A + 6*B)*Cos[c + d*x] + (75*A + 66*B)*Cos[2*(c + d*x)])*Sin[(c +
 d*x)/2]))/(48*d)
```

fricas [A] time = 0.62, size = 212, normalized size = 1.29

$$\frac{3\left((25\,A+38\,B)a^2\cos\left(dx+c\right)^4+(25\,A+38\,B)a^2\cos\left(dx+c\right)^3\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\cdots}{\cos(dx+c)^3+\cos(dx+\cdots}\right.}{96\left(d\,\mathrm{c}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] 1/96*(3*((25*A + 38*B)*a^2*cos(d*x + c)^4 + (25*A + 38*B)*a^2*cos(d*x + c)^
3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x +
c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + co
s(d*x + c)^2)) + 4*(3*(25*A + 22*B)*a^2*cos(d*x + c)^2 + 2*(17*A + 6*B)*a^2
*cos(d*x + c) + 8*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x
+ c)^4 + d*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.46, size = 1310, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 1/6*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(25*A*
ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
```

```
*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+25*A*ln(-4/(-2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))+38*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+38*B*l
n(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+12*(50*A
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+44*B*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)+75*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a
+75*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+114*B*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))*a+114*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-736*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)-450*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-450*A*ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^
(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-576*B*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)-684*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-
684*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+2
34*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+75*A*ln(-4/(-2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))*a+75*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*
a))*a+156*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+114*B*ln(-4/(-2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+114*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+
1/2*c)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)+2^(1
/2))^3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [B] time = 8.02, size = 7994, normalized size = 48.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] -1/96*((1530*a^2*cos(4*d*x + 4*c)^2*sin(3/2*d*x + 3/2*c) + 1530*a^2*cos(2*d
*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 1530*a^2*sin(4*d*x + 4*c)^2*sin(3/2*d*x
+ 3/2*c) + 1530*a^2*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 4176*a^2*cos(
```

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7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 2430*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x
 + 2*c) + 678*a^2*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 342*a^2*cos(2*d*x
 + 2*c)*sin(3/2*d*x + 3/2*c) + 10*(a^2*sin(9/2*d*x + 9/2*c) + 17*a^2*sin(3/
2*d*x + 3/2*c))*cos(6*d*x + 6*c)^2 + 10*(a^2*sin(9/2*d*x + 9/2*c) + 17*a^2*
sin(3/2*d*x + 3/2*c))*sin(6*d*x + 6*c)^2 - 56*a^2*sin(3/2*d*x + 3/2*c) + 10
*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2*c))*c
os(21/2*d*x + 21/2*c) - 30*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) +
 3*a^2*sin(2*d*x + 2*c))*cos(19/2*d*x + 19/2*c) - 48*(a^2*sin(6*d*x + 6*c)
+ 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2*c))*cos(17/2*d*x + 17/2*c) +
 80*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 3*a^2*sin(2*d*x + 2*c)
)*cos(15/2*d*x + 15/2*c) + 396*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*
c) + 3*a^2*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 6*(170*a^2*cos(4*d*x
+ 4*c)*sin(3/2*d*x + 3/2*c) + 170*a^2*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c)
 - 170*a^2*sin(11/2*d*x + 11/2*c) - 232*a^2*sin(7/2*d*x + 7/2*c) - 135*a^2*
sin(5/2*d*x + 5/2*c) + 19*a^2*sin(3/2*d*x + 3/2*c) + 10*(a^2*cos(4*d*x + 4*
c) + a^2*cos(2*d*x + 2*c) - 25*a^2)*sin(9/2*d*x + 9/2*c))*cos(6*d*x + 6*c)
+ 3060*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c)
 + 4560*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c)
+ 18*(170*a^2*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) - 232*a^2*sin(7/2*d*x +
 7/2*c) - 135*a^2*sin(5/2*d*x + 5/2*c) + 19*a^2*sin(3/2*d*x + 3/2*c))*cos(4
*d*x + 4*c) - 75*(sqrt(2)*a^2*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*cos(4*d*x
+ 4*c)^2 + 9*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(6*d*x + 6*c)^
2 + 9*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) + 9*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 6*sqrt(2)*a^2*cos(2*d*x +
 2*c) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*cos
(2*d*x + 2*c) + sqrt(2)*a^2)*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*a^2*cos(2*d*x
+ 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 6*(sqrt(2)*a^2*sin(4*d*x + 4*c) +
sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 75*(sqrt(2)*a^2*cos(6*d*x + 6*c)^2 + 9
*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2
)*a^2*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*a^
2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 6*
sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(4*d*x + 4
*c) + 3*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(6*d*x + 6*c) + 6*(3
*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 6*(sqrt(2)*
a^2*sin(4*d*x + 4*c) + sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(
2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(sqrt(2)*a^2
*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*a^2*cos(
2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*sin(4*d*x +
```

```
 4*c)^2 + 18*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sqrt(2)*a^2*
sin(2*d*x + 2*c)^2 + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(3*sq
rt(2)*a^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*
cos(6*d*x + 6*c) + 6*(3*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d
*x + 4*c) + 6*(sqrt(2)*a^2*sin(4*d*x + 4*c) + sqrt(2)*a^2*sin(2*d*x + 2*c))
*sin(6*d*x + 6*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 2) + 75*(sqrt(2)*a^2*cos(6*d*x + 6*c)^2 + 9*sqrt(2)*a^2*cos(4*d*x + 4*c)
^2 + 9*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(6*d*x + 6*c)^2 + 9*
sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 9*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 6*sqrt(2)*a^2*cos(2*d*x + 2*c)
+ sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*cos(2*d*x
 + 2*c) + sqrt(2)*a^2)*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*a^2*cos(2*d*x + 2*c)
 + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 6*(sqrt(2)*a^2*sin(4*d*x + 4*c) + sqrt(2
)*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 2) - 10*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4*d*x +
 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*sin(21/2*d*x + 21/2*c) + 30*(a^2*cos(
6*d*x + 6*c) + 3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*sin(1
9/2*d*x + 19/2*c) + 48*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4*d*x + 4*c) + 3*a
^2*cos(2*d*x + 2*c) + a^2)*sin(17/2*d*x + 17/2*c) - 80*(a^2*cos(6*d*x + 6*c
) + 3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*sin(15/2*d*x + 1
5/2*c) - 396*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d
*x + 2*c) + a^2)*sin(13/2*d*x + 13/2*c) + 2*(510*a^2*sin(4*d*x + 4*c)*sin(3
/2*d*x + 3/2*c) + 510*a^2*sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 510*a^2*c
os(11/2*d*x + 11/2*c) + 760*a^2*cos(9/2*d*x + 9/2*c) + 696*a^2*cos(7/2*d*x
+ 7/2*c) + 405*a^2*cos(5/2*d*x + 5/2*c) + 113*a^2*cos(3/2*d*x + 3/2*c) + 30
*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(9/2*d*x + 9/2*c))*sin(6*
d*x + 6*c) - 1020*(3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*s
in(11/2*d*x + 11/2*c) + 10*(9*a^2*cos(4*d*x + 4*c)^2 + 9*a^2*cos(2*d*x + 2*
c)^2 + 9*a^2*sin(4*d*x + 4*c)^2 + 18*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 9*a^2*sin(2*d*x + 2*c)^2 - 450*a^2*cos(2*d*x + 2*c) - 151*a^2 + 18*(a^2*c
os(2*d*x + 2*c) - 25*a^2)*cos(4*d*x + 4*c))*sin(9/2*d*x + 9/2*c) + 6*(510*a
^2*sin(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 696*a^2*cos(7/2*d*x + 7/2*c) + 4
05*a^2*cos(5/2*d*x + 5/2*c) + 113*a^2*cos(3/2*d*x + 3/2*c))*sin(4*d*x + 4*c
) - 1392*(3*a^2*cos(2*d*x + 2*c) + a^2)*sin(7/2*d*x + 7/2*c) - 810*(3*a^2*c
os(2*d*x + 2*c) + a^2)*sin(5/2*d*x + 5/2*c) - 30*(a^2*cos(6*d*x + 6*c)^2 +
9*a^2*cos(4*d*x + 4*c)^2 + 9*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(6*d*x + 6*c)^
2 + 9*a^2*sin(4*d*x + 4*c)^2 + 18*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9
*a^2*sin(2*d*x + 2*c)^2 + 6*a^2*cos(2*d*x + 2*c) + a^2 + 2*(3*a^2*cos(4*d*x
 + 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 6*(3*a^2*cos(2*d
```

```
*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 6*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c))*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 78*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(4*d*x + 4*c)^2 + 9*a^
2*cos(2*d*x + 2*c)^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(4*d*x + 4*c)^2 +
18*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a^2*sin(2*d*x + 2*c)^2 + 6*a^2
*cos(2*d*x + 2*c) + a^2 + 2*(3*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d*x + 2*c
) + a^2)*cos(6*d*x + 6*c) + 6*(3*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*
c) + 6*(a^2*sin(4*d*x + 4*c) + a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sin(
5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 600*(a^2*cos(6*d
*x + 6*c)^2 + 9*a^2*cos(4*d*x + 4*c)^2 + 9*a^2*cos(2*d*x + 2*c)^2 + a^2*sin
(6*d*x + 6*c)^2 + 9*a^2*sin(4*d*x + 4*c)^2 + 18*a^2*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 9*a^2*sin(2*d*x + 2*c)^2 + 6*a^2*cos(2*d*x + 2*c) + a^2 + 2*(3
*a^2*cos(4*d*x + 4*c) + 3*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 6*
(3*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 6*(a^2*sin(4*d*x + 4*c) +
 a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a)/(sqrt(2)*cos(6*d*x + 6*c)^2 + 9*sqrt(
2)*cos(4*d*x + 4*c)^2 + 9*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(6*d*x +
6*c)^2 + 9*sqrt(2)*sin(4*d*x + 4*c)^2 + 18*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 9*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(3*sqrt(2)*cos(4*d*x + 4*c) +
3*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 6*(3*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 6*(sqrt(2)*sin(4*d*x + 4*c) + sqr
t(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2)) + 6*(150*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 154*sqrt(
2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x +
 3/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - (3*sqrt(2)*a^2*sin(7/2*d*x
+ 7/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) - 17*sqrt(2)*a^2*sin(3/2*d*x
+ 3/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*
x + 4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sqrt(2)*a^2*sin(1/
2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)
```

```
^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*
sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
(3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) -
17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) +
 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
+ 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2
*c)^2 - 3*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c))*c
os(15/2*d*x + 15/2*c) - 5*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin
(2*d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 11*(sqrt(2)*a^2*sin(4*d*x + 4*c) +
2*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 45*(sqrt(2)*a^2*si
n(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) - (11
*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 99*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 3
8*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) - 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sqrt(2)*a^2*s
in(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*(4*sqrt(2)*a^2
```

```
*cos(2*d*x + 2*c) + 27*sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + (20*sqrt(2)*a^2*
cos(2*d*x + 2*c) + 87*sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c) -
 2*(11*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 99*sqrt(2)*a^2*sin(1/2*d*x + 1/2*
c) + 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*a^2*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*(sqrt(2)*a^2*cos(4*d*x + 4*c) +
 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/2*d*x + 15/2*c) + 5*(
sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2
)*sin(13/2*d*x + 13/2*c) - 11*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2
*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/2*d*x + 11/2*c) - 45*(sqrt(2)*a^2*c
os(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/2*d*x
 + 9/2*c) - (12*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 20*sqrt
(2)*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 75*sqrt(2)*a^2*cos(7/2*d*x
+ 7/2*c) - 77*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c) - 45*sqrt(2)*a^2*cos(3/2*d*x
 + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c) - 4*(17*sqrt(2)*a^2*sin(3/2
*d*x + 3/2*c) + 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin
(2*d*x + 2*c))*sin(4*d*x + 4*c) - 6*(2*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 2*s
qrt(2)*a^2*sin(2*d*x + 2*c)^2 + 27*sqrt(2)*a^2*cos(2*d*x + 2*c) + 13*sqrt(2
)*a^2)*sin(7/2*d*x + 7/2*c) - 2*(10*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 10*sqr
t(2)*a^2*sin(2*d*x + 2*c)^2 + 87*sqrt(2)*a^2*cos(2*d*x + 2*c) + 41*sqrt(2)*
a^2)*sin(5/2*d*x + 5/2*c) + 2*(45*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 11*sqr
t(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*B*sqrt(a)/(2*(2*cos(2*d*x
+ 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 +
sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*
c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\cos(c + dx))\,(a + a\cos(c + dx))^{5/2}}{\cos(c + dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^4,x)
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
[Out] Timed out
```

3.98 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=209

$$\frac{a^{5/2}(163A+200B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(163A+200B)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(95A+104B)\tan(c+dx)\sec(c+}{96d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 1/64*a^(5/2)*(163*A+200*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2
))/d+1/4*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3*tan(d*x+c)/d+1/64*a^3*(163
*A+200*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/96*a^3*(95*A+104*B)*sec(d*x
+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/24*a^2*(11*A+8*B)*sec(d*x+c)^2*(a
+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.61, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2975, 2980, 2772, 2773, 206}

$$\frac{a^3(163A+200B)\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(163A+200B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(11A+8B)\tan(c+dx)\sec^2(c+}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a^(5/2)*(163*A + 200*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(64*d) + (a^3*(163*A + 200*B)*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c
+ d*x]]) + (a^3*(95*A + 104*B)*Sec[c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*
Cos[c + d*x]]) + (a^2*(11*A + 8*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2*
Tan[c + d*x])/(24*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
```

```
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\tan(c+dx)}{4d}$$

$$= \frac{a^2(11A+8B)\sqrt{a+a\cos(c+dx)}\,\sec^2(c+dx)\tan}{24d}$$

$$= \frac{a^3(95A+104B)\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(11}{}$$

$$= \frac{a^3(163A+200B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(95A+104B)}{96d\sqrt{a}}$$

$$= \frac{a^3(163A+200B)\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(95A+104B)}{96d\sqrt{a}}$$

$$= \frac{a^{5/2}(163A+200B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^3(1}{6}$$

Mathematica [A] time = 1.80, size = 152, normalized size = 0.73

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)((2203A+2056B)\cos(c+dx)+(652A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^4*(6*Sqrt[2]*
(163*A + 200*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (844*A +
 544*B + (2203*A + 2056*B)*Cos[c + d*x] + (652*A + 544*B)*Cos[2*(c + d*x)]
+ 489*A*Cos[3*(c + d*x)] + 600*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*
d)
```

fricas [A] time = 0.72, size = 232, normalized size = 1.11

$$\frac{3\left(\left((163\,A+200\,B)a^2\cos(dx+c)^5+(163\,A+200\,B)a^2\cos(dx+c)^4\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+}}{\cos(dx+c)^3+c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="fricas")
```

```
[Out] 1/768*(3*((163*A + 200*B)*a^2*cos(d*x + c)^5 + (163*A + 200*B)*a^2*cos(d*x
+ c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d
*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)) + 4*(3*(163*A + 200*B)*a^2*cos(d*x + c)^3 + 2*(163*A +
136*B)*a^2*cos(d*x + c)^2 + 8*(23*A + 8*B)*a^2*cos(d*x + c) + 48*A*a^2)*sqr
t(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.50, size = 1630, normalized size = 7.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/24*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*a*(163*A
*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+163*A*ln(4/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))+200*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+2
00*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^8-48*(
163*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+200*B*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+326*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))*a+326*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+400*B*ln(-4/(
-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+400*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d
*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+8*(1793*A*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)+2072*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
+1467*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1467*A*ln(4/(2*cos(
```

```
1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1800*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1
/2*c)+2*a))*a+1800*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2
*d*x+1/2*c)^4+(-9212*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-3912*
A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-3912*A*ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))*a-9632*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)-4800*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-4800*B*ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/
2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+2094*A*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+489*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2
*d*x+1/2*c)+2*a))*a+489*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+187
2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+600*B*ln(-4/(-2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))*a+600*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2
*a))*a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4/s
in(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+a\,\cos(c+d\,x)\right)^{5/2}}{\cos(c+d\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^5,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.99 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=254

$$\frac{a^{5/2}(283A+326B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^3(283A+326B)\tan(c+dx)}{128d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(157A+170B)\tan(c+dx)\sec^2(c}{240d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 1/128*a^(5/2)*(283*A+326*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/
2))/d+1/5*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4*tan(d*x+c)/d+1/128*a^3*(2
83*A+326*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/192*a^3*(283*A+326*B)*sec
(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/240*a^3*(157*A+170*B)*sec(d*x
+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/40*a^2*(13*A+10*B)*sec(d*x+c)^3
*(a+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.71, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2975, 2980, 2772, 2773, 206}

$$\frac{a^3(283A+326B)\tan(c+dx)}{128d\sqrt{a\cos(c+dx)+a}}+\frac{a^{5/2}(283A+326B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^2(13A+10B)\tan(c+dx)\sec^3(c+}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (a^(5/2)*(283*A + 326*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c +
d*x]]])/(128*d) + (a^3*(283*A + 326*B)*Tan[c + d*x])/(128*d*Sqrt[a + a*Cos[
c + d*x]]) + (a^3*(283*A + 326*B)*Sec[c + d*x]*Tan[c + d*x])/(192*d*Sqrt[a
+ a*Cos[c + d*x]]) + (a^3*(157*A + 170*B)*Sec[c + d*x]^2*Tan[c + d*x])/(240
*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(13*A + 10*B)*Sqrt[a + a*Cos[c + d*x]]*
Sec[c + d*x]^3*Tan[c + d*x])/(40*d) + (a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c
 + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{aA(a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\tan(c+dx)}{5d}$$

$$= \frac{a^2(13A+10B)\sqrt{a+a\cos(c+dx)}\,\sec^3(c+dx)\,\mathrm{ta}}{40d}$$

$$= \frac{a^3(157A+170B)\sec^2(c+dx)\tan(c+dx)}{240d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(1}{}$$

$$= \frac{a^3(283A+326B)\sec(c+dx)\tan(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(15}{}$$

$$= \frac{a^3(283A+326B)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(283A+326B}{192d\sqrt{a}}$$

$$= \frac{a^3(283A+326B)\tan(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(283A+326B}{192d\sqrt{a}}$$

$$= \frac{a^{5/2}(283A+326B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{128d} + \frac{a^3(2}{1}$$

Mathematica [A] time = 2.25, size = 176, normalized size = 0.69

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)(36(781A+650B)\cos(c+dx)+4(6509A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^5*(60*Sqrt[2]
*(283*A + 326*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (24863*
A + 22030*B + 36*(781*A + 650*B)*Cos[c + d*x] + 4*(6509*A + 6730*B)*Cos[2*(
c + d*x)] + 5660*A*Cos[3*(c + d*x)] + 6520*B*Cos[3*(c + d*x)] + 4245*A*Cos[
4*(c + d*x)] + 4890*B*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(15360*d)
```

fricas [A] time = 1.12, size = 252, normalized size = 0.99

$$\frac{15\left((283\,A+326\,B)a^2\cos(dx+c)^6+(283\,A+326\,B)a^2\cos(dx+c)^5\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx}}{\cos(dx+c)^3+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm
="fricas")
```

```
[Out] 1/7680*(15*((283*A + 326*B)*a^2*cos(d*x + c)^6 + (283*A + 326*B)*a^2*cos(d*
x + c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos
(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)
^3 + cos(d*x + c)^2)) + 4*(15*(283*A + 326*B)*a^2*cos(d*x + c)^4 + 10*(283*
A + 326*B)*a^2*cos(d*x + c)^3 + 8*(283*A + 230*B)*a^2*cos(d*x + c)^2 + 48*(
29*A + 10*B)*a^2*cos(d*x + c) + 384*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x
 + c))/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.62, size = 1951, normalized size = 7.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 1/120*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-480*a*(28
3*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+283*A*ln(-4/(-2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*
cos(1/2*d*x+1/2*c)+2*a))+326*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+
326*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^10+
240*(566*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+652*B*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+1415*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1
/2*c)+2*a))*a+1415*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1630*B
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+1630*B*ln(-4/(-2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*
cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^8-80*(3962*A*2^(1/2)*(a*sin(
```

```
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4564*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)+4245*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4245*A*ln(-
4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4890*B*ln(4/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))*a+4890*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
a)*sin(1/2*d*x+1/2*c)^6+8*(36224*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)+40960*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+21225*A*ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+21225*A*ln(-4/(-2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))*a+24450*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a
+24450*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c
)^4-10*(12556*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+13400*B*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4245*A*ln(4/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/
2*d*x+1/2*c)+2*a))*a+4245*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a
+4890*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4890*B*ln(-4/(-2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+4245*A*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4245*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1
/2*c)+2*a))*a+22230*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+4890*B
*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4890*B*ln(-4/(-2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*
cos(1/2*d*x+1/2*c)+2*a))*a+20940*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^5/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^5/s
in(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{5/2}}{\cos(c+dx)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^6,x)
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
[Out] Timed out
```

3.100 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=202

$$\frac{2(7A-B)\sin(c+dx)\cos^2(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}-\frac{2(7A-31B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105ad}+\frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-$$

```
[Out] -(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/
2)/d/a^(1/2)+4/105*(49*A-37*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/35*(7*
A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*B*cos(d*x+c)^3*si
n(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-2/105*(7*A-31*B)*sin(d*x+c)*(a+a*cos(d*x+
c))^(1/2)/a/d
```

Rubi [A] time = 0.58, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2983, 2968, 3023, 2751, 2649, 206}

$$\frac{2(7A-B)\sin(c+dx)\cos^2(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}-\frac{2(7A-31B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105ad}+\frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (4*(49*A - 37*B)*Sin[c + d*x])/(105*d*Sqrt[a + a
*Cos[c + d*x]]) + (2*(7*A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(35*d*Sqrt[a +
a*Cos[c + d*x]]) + (2*B*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c
+ d*x]]) - (2*(7*A - 31*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2B\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\cos^2(c+dx)\left(3aB+\frac{1}{2}a(7A-B)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}dx}{7a}$$

$$= \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{4}{-}$$

$$= \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{4}{-}$$

$$= \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{2}{-}$$

$$= \frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B}{}$$

$$= \frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(7A-B)\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2B}{}$$

$$= -\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{4(49A-37B)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} +$$

Mathematica [A] time = 0.72, size = 111, normalized size = 0.55

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2\sin\left(\frac{1}{2}(c+dx)\right)((169B-28A)\cos(c+dx)+6(7A-B)\cos(2(c+dx))+406A+15B\cos(3(c}{210d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(-420*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + 2*(406*A - 178*
B + (-28*A + 169*B)*Cos[c + d*x] + 6*(7*A - B)*Cos[2*(c + d*x)] + 15*B*Cos[
3*(c + d*x)])*Sin[(c + d*x)/2]))/(210*d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 0.61, size = 184, normalized size = 0.91

$$\frac{4\left(15\,B\cos\left(dx+c\right)^{3}+3\,(7\,A-B)\cos\left(dx+c\right)^{2}-(7\,A-31\,B)\cos\left(dx+c\right)+91\,A-43\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\mathrm{s}}{210\,(ad\cos\left(dx+c\right)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/210*(4*(15*B*cos(d*x + c)^3 + 3*(7*A - B)*cos(d*x + c)^2 - (7*A - 31*B)*c
os(d*x + c) + 91*A - 43*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) - 105*sqrt
(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*s
qrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 1.84, size = 181, normalized size = 0.90

$$\frac{\frac{105\sqrt{2}(A-B)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{a}}+\frac{2\left(105\sqrt{2}Aa^3+\left(\left(\sqrt{2}\left(119Aa^3-92Ba^3\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+7\sqrt{2}\left(37Aa^3-16Ba^3\right)\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/105*(105*sqrt(2)*(A - B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a)))/sqrt(a) + 2*(105*sqrt(2)*A*a^3 + ((sqrt(2)*(11
9*A*a^3 - 92*B*a^3)*tan(1/2*d*x + 1/2*c)^2 + 7*sqrt(2)*(37*A*a^3 - 16*B*a^3
))*tan(1/2*d*x + 1/2*c)^2 + 35*sqrt(2)*(7*A*a^3 - 4*B*a^3))*tan(1/2*d*x + 1
/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2))/d
```

maple [A] time = 0.83, size = 281, normalized size = 1.39

$$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-240B\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+168\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/105*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-240*B*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+168*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(A+2*B)*sin(1/2*d*x+1/2*c)^4-140*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(A+2*B)*sin(1/2*d*x+1/2*c)^2-105*2^(1
```

```
/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A
+105*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)+a))*a*B+210*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/sin
(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.101 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=159

$$\frac{2(5A-B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15ad}-\frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\,\mathrm{si}}{5d}$$

```
[Out] (A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2
)/d/a^(1/2)-4/15*(5*A-7*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*B*cos(d*
x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/15*(5*A-B)*sin(d*x+c)*(a+a*cos
(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.38, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2983, 2968, 3023, 2751, 2649, 206}

$$\frac{2(5A-B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15ad}-\frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\,\mathrm{si}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(Sqrt[a]*d) - (4*(5*A - 7*B)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c
 + d*x]]) + (2*B*Cos[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]
) + (2*(5*A - B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
```

```
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{\cos(c+dx)\left(2aB+\frac{1}{2}a(5A-B)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}dx}{5a}$$

$$= \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{2aB\cos(c+dx)+\frac{1}{2}a(5A-B)\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx}{5a}$$

$$= \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2(5A-B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx}{15ad}$$

$$= -\frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2(5A-B)\sqrt{a}}{}$$

$$= -\frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2B\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2(5A-B)\sqrt{a}}{}$$

$$= \frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{4(5A-7B)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2B\,c}{5a}$$

Mathematica [A] time = 0.37, size = 94, normalized size = 0.59

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(15(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin\left(\frac{1}{2}(c+dx)\right)(2(5A-B)\cos(c+dx)-10A+3B\cos(}{15d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(15*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + (-10*A + 29*B +
 2*(5*A - B)*Cos[c + d*x] + 3*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(15*d*
Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 0.69, size = 166, normalized size = 1.04

$$\frac{4\left(3\,B\cos\left(dx+c\right)^2+(5\,A-B)\cos\left(dx+c\right)-5\,A+13\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)-\frac{15\,\sqrt{2}\,((A-B)a\cos(dx+c)+}{}}{30\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/30*(4*(3*B*cos(d*x + c)^2 + (5*A - B)*cos(d*x + c) - 5*A + 13*B)*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c) - 15*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B
)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(
a))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 1.98, size = 158, normalized size = 0.99

$$-\frac{\frac{15\left(\sqrt{2}A-\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{\sqrt{a}}-\frac{2\left(15\sqrt{2}Ba^{2}-\left(10\sqrt{2}Aa^{2}-20\sqrt{2}Ba^{2}+\left(10\sqrt{2}Aa^{2}-17\sqrt{2}Ba^{2}\right)\tan\left(\frac{1}{2}dx\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{5}{2}}}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/15*(15*(sqrt(2)*A - sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/sqrt(a) - 2*(15*sqrt(2)*B*a^2 - (10*sqr
t(2)*A*a^2 - 20*sqrt(2)*B*a^2 + (10*sqrt(2)*A*a^2 - 17*sqrt(2)*B*a^2)*tan(1
/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*
d*x + 1/2*c)^2 + a)^(5/2))/d
```

maple [A] time = 0.73, size = 240, normalized size = 1.51

$$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24B\sqrt{a}\,\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\sqrt{2}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{15a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/15*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*B*a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)*a^(1/2)*(A+B)*sin(1/2*d*x+1/2*c)^2+15*2^(1/2)*ln(4/cos
(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-15*2^(1/2)*
ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+30*
```

```
B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/sin(1/2*d*x+1/2*c
)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2\,\left(A+B\,\cos(c+d\,x)\right)}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.102 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$\frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

```
[Out] -(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/
2)/d/a^(1/2)+2/3*(3*A-2*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*B*sin(d*
x+c)*(a+a*cos(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.21, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3023, 2751, 2649, 206}

$$\frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)) + (2*(3*A - 2*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos
[c + d*x]]) + (2*B*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2B\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{2\int \frac{\frac{aB}{2}+\frac{1}{2}a(3A-2B)\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2B\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3ad} + (-A+B)$$

$$= \frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2B\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{(2(A-B}{}$$

$$= -\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2(3A-2B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2B}{}$$

Mathematica [A] time = 0.17, size = 78, normalized size = 0.66

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-3(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6A\sin\left(\frac{1}{2}(c+dx)\right)-4B\sin^3\left(\frac{1}{2}(c+dx)\right)\right)}{3d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(-3*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + 6*A*Sin[(c + d*
x)/2] - 4*B*Sin[(c + d*x)/2]^3))/(3*d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 0.63, size = 149, normalized size = 1.26

$$\frac{4\left(B\cos\left(dx+c\right)+3\,A-B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)-\frac{3\,\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\log\left(-\frac{\cos(dx+c)^{2}-\frac{2\,\sqrt{2}\,\sqrt{a\cos(dx\cdots}}{\cdots}}{\cos(dx+c)^{2}+\cdots}\right.}{\sqrt{a}}}{6\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/6*(4*(B*cos(d*x + c) + 3*A - B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c) - 3
*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 - 2*sqrt
(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(co
s(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 1.56, size = 113, normalized size = 0.96

$$\frac{\frac{3\,\sqrt{2}\,(A-B)\log\left(\left|-\sqrt{a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{a}}+\frac{2\left(\sqrt{2}\,(3\,Aa-2\,Ba)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,\sqrt{2}\,Aa\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] 1/3*(3*sqrt(2)*(A - B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a)))/sqrt(a) + 2*(sqrt(2)*(3*A*a - 2*B*a)*tan(1/2*d*x +
 1/2*c)^2 + 3*sqrt(2)*A*a)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 +
 a)^(3/2))/d
```

maple [A] time = 0.67, size = 194, normalized size = 1.64

$$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-4B\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6A\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}}{3a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+6*A*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)-3*A*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+a))*a+3*B*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/
2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [B] time = 0.38, size = 160, normalized size = 1.36

$$\frac{2\,A\,\left(2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)-\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\right)\,\sqrt{\frac{a+a\,\cos(c+d\,x)}{2\,a}}}{d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}+\frac{2\,B\,\sin\left(c+d\,x\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{3\,a\,d}-\frac{2\,B\,\left(4\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\right|}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*A*(2*ellipticE(c/2 + (d*x)/2, 1) - ellipticF(c/2 + (d*x)/2, 1))*((a + a*
cos(c + d*x))/(2*a))^(1/2))/(d*(a + a*cos(c + d*x))^(1/2)) + (2*B*sin(c + d
*x)*(a + a*cos(c + d*x))^(1/2))/(3*a*d) - (2*B*(4*a^2*ellipticE(c/2 + (d*x)
/2, 1) - 3*a^2*ellipticF(c/2 + (d*x)/2, 1))*((a + a*cos(c + d*x))/(2*a))^(1
/2))/(3*a^2*d*(a + a*cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\cos(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.103 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2
)/d/a^(1/2)+2*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2751, 2649, 206}

$$\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(Sqrt[a]*d) + (2*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + (A-B)\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(2(A-B))\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.77

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((A - B)*ArcTanh[Sin[(c + d*x)/2]] + 2*B*Sin[(c + d*x)/
2]))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [B] time = 0.76, size = 135, normalized size = 1.73

$$\frac{4\sqrt{a\cos(dx+c)+a}\,B\sin(dx+c) - \frac{\sqrt{2}\,((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{2\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(a*cos(d*x + c) + a)*B*sin(d*x + c) - sqrt(2)*((A - B)*a*cos(d*x
 + c) + (A - B)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a
)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 2.92, size = 88, normalized size = 1.13

$$\frac{\frac{2\sqrt{2}B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{\left(\sqrt{2}A-\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{a}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] (2*sqrt(2)*B*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) - (sqr
t(2)*A - sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a)))/sqrt(a))/d
```

maple [B] time = 0.68, size = 160, normalized size = 2.05

$$\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(A\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a+2B\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}-B\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\right)}{a^{\frac{3}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*ln(4/cos(1/2*d
*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a+2*B*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)*a^(1/2)-B*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(
1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 0.35, size = 112, normalized size = 1.44

$$\frac{A\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\cos\left(c+d\,x\right)\right)}{a}}+2\,B\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\cos\left(c+d\,x\right)\right)}{a}}-B\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,\left(a+a\,\cos\left(c+d\,x\right)\right)}{a}}}{d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (A*ellipticF(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2) + 2*B*ell
ipticE(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2) - B*ellipticF(c
/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2))/(d*(a + a*cos(c + d*x)
)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos{(c + dx)}}{\sqrt{a(\cos{(c + dx)} + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.104 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*A*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-(A-B)*arct
anh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2
)
```

Rubi [A] time = 0.17, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2985, 2649, 206, 2773}

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c
 + d*x]])])/(Sqrt[a]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\int \sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\,dx}{a} - (A-B)\int \frac{1}{\sqrt{a+a\cos(c+dx}}$$

$$= -\frac{(2A)\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{(2(A-B))\,\mathrm{Subst}\left(\int \frac{1}{2a-x}\right.}{d}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.79

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}\,A\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-2*((A - B)*ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*A*ArcTanh[Sqrt[2]*Sin[(c +
 d*x)/2]])*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [B] time = 0.68, size = 171, normalized size = 1.88

$$-\frac{\sqrt{2}\,(A-B)\sqrt{a}\,\log\left(-\frac{\cos(dx+c)^2-\frac{2\,\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{a}}-2\,\cos(dx+c)-3}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)-A\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(d}}{\cos(dx+c)^3+}\right.}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] -1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d
*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) - A*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
- 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a*d)
```

giac [B] time = 2.08, size = 168, normalized size = 1.85

$$\frac{\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{a}+\frac{2A\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{a}}-\cdots}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] 1/2*(sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a + 2*A*log(abs((sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/sqrt(
a) - 2*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 + a*(2*sqrt(2) - 3)))/sqrt(a))/d
```

maple [B] time = 1.40, size = 268, normalized size = 2.95

$$-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sqrt{2}\ \ln\left(\frac{4\sqrt{a}\ \sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)A-\sqrt{2}\ \ln\left(\frac{4\sqrt{a}\ \sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)B-A\ln\left(\cdots\right.\right.}{\sqrt{a}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\cdots\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln(4/cos(1/2*d*
x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*A-2^(1/2)*ln(4/cos(1/2
*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*B-A*ln(4/(2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))-A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
))/a^(1/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [A] time = 1.13, size = 91, normalized size = 1.00

$$\frac{\left(\sqrt{2}\,\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\,\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,d\right.\right.\right.}{2\,\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B/(sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.105 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=119

$$-\frac{(A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -(A-2*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)+(A-B)
*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)+A*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2984, 2985, 2649, 206, 2773}

$$-\frac{(A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{A\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -(((A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqr
t[a]*d)) + (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Cos[c + d*x]])])/(Sqrt[a]*d) + (A*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d
*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{\int \frac{\left(-\frac{1}{2}a(A-2B)+\frac{1}{2}aA\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}-\frac{(A-2B)\int \sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\,dx}{2a}$$

$$= \frac{A\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{(A-2B)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$

$$= -\frac{(A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx}{\sqrt{2}\,\sqrt{a+a\cos(c-}\right.}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.36, size = 95, normalized size = 0.80

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}\,(A-2B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\sin\left(\frac{1}{2}(c+\right.\right.}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(2*(A - B)*ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*(A - 2*B)*
ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*A*Sec[c + d*x]*Sin[(c + d*x)/2]))/(d*
Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [B] time = 0.93, size = 259, normalized size = 2.18

$$-\frac{\left((A-2\,B)\cos\left(dx+c\right)^{2}+(A-2\,B)\cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,(\cos(dx+c)-2)\sin(}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{4\left(ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/4*(((A - 2*B)*cos(d*x + c)^2 + (A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*co
s(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos
(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*s
qrt(a*cos(d*x + c) + a)*A*sin(d*x + c) + 2*sqrt(2)*((A - B)*a*cos(d*x + c)^
2 + (A - B)*a*cos(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*c
os(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

giac [B] time = 3.89, size = 321, normalized size = 2.70

$$-\frac{\frac{\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\left(\sqrt{a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}\right)}{a}+\frac{\left(A\sqrt{a}-2\,B\sqrt{a}\right)\log\left(\left|\left(\sqrt{a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a(2\right.\right.}{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/2*(sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a + (A*sqrt(a) - 2*B*sqrt(a))*log(abs
((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*
(2*sqrt(2) + 3)))/a - (A*sqrt(a) - 2*B*sqrt(a))*log(abs((sqrt(a)*tan(1/2*d*
x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a
- 4*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^2*A*sqrt(a) - A*a^(3/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

maple [B] time = 1.46, size = 810, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-2*2^(1/2)*ln(4/cos
(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*A+2*2^(1/2)*ln(
4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*B+A*ln(-4/
(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))-2*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))-2*B*ln(4/(2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a
*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*ln(4/cos(
1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-2*2^(1/2)*ln
(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+2*A*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-A*ln(4/(2*cos(1/2*d*x+1/2*c)+
2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*
d*x+1/2*c)+2*a))*a-A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2*B*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2*B*ln(-4/(-2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/
2*d*x+1/2*c)+2*a))*a)/a^(3/2)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/2*d*x
+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cos(c+d\,x)}{\cos(c+d\,x)^2\,\sqrt{a+a\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/sqrt(a*(cos(c + d*x) + 1)), x
)
```

3.106 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=165

$$-\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{(7A-4B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{A\tan(c+ }{2d\sqrt{a\cos }$$

```
[Out] 1/4*(7*A-4*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-
(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2
)/d/a^(1/2)-1/4*(A-4*B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/2*A*sec(d*x+c
)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2984, 2985, 2649, 206, 2773}

$$-\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{(7A-4B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{A\tan(c+ }{2d\sqrt{a\cos }$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((7*A - 4*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*S
qrt[a]*d) - (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(Sqrt[a]*d) - ((A - 4*B)*Tan[c + d*x])/(4*d*Sqrt[a +
a*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*
x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{1}{2}a(A-4B)+\frac{3}{2}aA\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx}{2a}$$

$$= -\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{4}a^2(7A-4B)-\right.}{}}{}$$

$$= -\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{(7A-4B)\int \sqrt{}}{}$$

$$= -\frac{(A-4B)\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{(7A-4B)\,\mathrm{Sub}}{}$$

$$= \frac{(7A-4B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}\,d} - \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx}{\sqrt{2}\sqrt{a+a\cos(c+}}\right.}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.84, size = 114, normalized size = 0.69

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(-8(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}(7A-4B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c\right.\right.}{4d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(-8*(A - B)*ArcTanh[Sin[(c + d*x)/2]] + Sqrt[2]*(7*A - 4*
B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(-A + 4*B + 2*A*Sec[c
 + d*x])*Sin[(c + d*x)/2]))/(4*d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [B] time = 0.81, size = 284, normalized size = 1.72

$$-\frac{\left((7A-4B)\cos(dx+c)^3+(7A-4B)\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/16*(((7*A - 4*B)*cos(d*x + c)^3 + (7*A - 4*B)*cos(d*x + c)^2)*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(
a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)
) + 4*((A - 4*B)*cos(d*x + c) - 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
+ 8*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*log(-(cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*c
os(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d
*x + c)^3 + a*d*cos(d*x + c)^2)
```

giac [B] time = 3.64, size = 535, normalized size = 3.24

$$\frac{\frac{4\sqrt{2}\left(A\sqrt{a}-B\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{a}+\frac{\left(7A\sqrt{a}-4B\sqrt{a}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a(2\right.\right.}{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/8*(4*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a + (7*A*sqrt(a) - 4*B*sqrt(a))*log(
abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
 a*(2*sqrt(2) + 3)))/a - (7*A*sqrt(a) - 4*B*sqrt(a))*log(abs((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)
))/a - 4*sqrt(2)*(17*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^6*A*sqrt(a) - 12*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(a) - 57*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^(3/2) + 76*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^(3/2) + 19*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^(5/2) - 36*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^(5/2)
 - 3*A*a^(7/2) + 4*B*a^(7/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

maple [B] time = 1.54, size = 1240, normalized size = 7.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*(8*2^(1/2)*ln(4
/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*A-8*2^(1/2)
*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*B-7*A*
ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))-7*A*ln(4/(2*cos(1/2*d*x+1/2*c
)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/
2*d*x+1/2*c)+2*a))+4*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+4*B*ln
(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^4-4*(8*2^(1/2
)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+A
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-8*2^(1/2)*ln(4/cos(1/2*d*x+
1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B-4*B*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-7*A*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+
2*a))*a-7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4*B*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A-8*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B-2*A*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)-7*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-7*A
*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-8*B*2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)+4*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+
4*B*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/a^(3/2)/(2*cos(1/2*d*x
+1/2*c)+2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*c
os(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,\sqrt{a+a\,\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/sqrt(a*(cos(c + d*x) + 1)), x
)
```

3.107 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=261

$$-\frac{(15A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(273A-397B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{210a^2d}+\frac{(A-B)\sin(c+dx)\mathrm{c}}{2d(a\cos(c+dx)}$$

```
[Out] 1/2*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-1/4*(15*A-19*B)*
arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^
(1/2)+1/105*(651*A-799*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/70*(63*A-
67*B)*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/14*(7*A-11*B)*co
s(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/210*(273*A-397*B)*sin(d*
x+c)*(a+a*cos(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.79, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2977, 2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{(273A-397B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{210a^2d}-\frac{(15A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)\mathrm{c}}{2d(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -((15*A - 19*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(2*d
*(a + a*Cos[c + d*x])^(3/2)) + ((651*A - 799*B)*Sin[c + d*x])/(105*a*d*Sqrt
[a + a*Cos[c + d*x]]) + ((63*A - 67*B)*Cos[c + d*x]^2*Sin[c + d*x])/(70*a*d
*Sqrt[a + a*Cos[c + d*x]]) - ((7*A - 11*B)*Cos[c + d*x]^3*Sin[c + d*x])/(14
*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((273*A - 397*B)*Sqrt[a + a*Cos[c + d*x]]*
Sin[c + d*x])/(210*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos^3(c+dx)\left(4a(A-B)-\frac{1}{2}a(7A-11B)\cos(c+\right.}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(7A-11B)\cos^3(c+dx)\sin(c+dx}{14ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(63A-67B)\cos^2(c+dx)\sin(c+}{70ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(63A-67B)\cos^2(c+dx)\sin(c+}{70ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(63A-67B)\cos^2(c+dx)\sin(c+}{70ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(651A-799B)\sin(c+dx)}{105ad\sqrt{a+a\cos(c+dx)}} + \frac{(63}{}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(651A-799B)\sin(c+dx)}{105ad\sqrt{a+a\cos(c+dx)}} + \frac{(63}{}$$

$$= -\frac{(15A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{(A-B)\cos^4(c+dx)\sin(c}{2d(a+a\cos(c+dx))^{3}}$$

Mathematica [A] time = 1.17, size = 167, normalized size = 0.64

$$\frac{105(15A-19B)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \frac{1}{2}\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(6(273A-277B}{105d\left(\sin^2\left(\frac{1}{2}(c\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (105*(15*A - 19*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - (Cos[(c +
 d*x)/2]^3*(1974*A - 2161*B + 6*(273*A - 277*B)*Cos[c + d*x] + (-84*A + 256
*B)*Cos[2*(c + d*x)] + 42*A*Cos[3*(c + d*x)] - 18*B*Cos[3*(c + d*x)] + 15*B
*Cos[4*(c + d*x)])*Sin[(c + d*x)/2])/2)/(105*d*(a*(1 + Cos[c + d*x]))^(3/2)
*(-1 + Sin[(c + d*x)/2]^2))
```

fricas [A] time = 0.96, size = 241, normalized size = 0.92

$$-\frac{105\sqrt{2}\left((15A-19B)\cos(dx+c)^2+2(15A-19B)\cos(dx+c)+15A-19B\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos\cdots}}{\cos(d\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/840*(105*sqrt(2)*((15*A - 19*B)*cos(d*x + c)^2 + 2*(15*A - 19*B)*cos(d*x
 + c) + 15*A - 19*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(
d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)) - 4*(60*B*cos(d*x + c)^4 + 12*(7*A - 3*B)*cos(d*x
+ c)^3 - 28*(3*A - 7*B)*cos(d*x + c)^2 + 12*(63*A - 67*B)*cos(d*x + c) + 10
29*A - 1201*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2
 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 2.40, size = 254, normalized size = 0.97

$$\frac{\frac{105\left(15\sqrt{2}A-19\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}+\left(\left(\left(\left(\frac{105\left(\sqrt{2}Aa^5-\sqrt{2}Ba^5\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3}+\frac{4\left(693\sqrt{2}Aa^5-877\sqrt{2}Ba^5\right)}{a^3}\right)\tan\left(\cdots\right.\right.\right.\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/420*(105*(15*sqrt(2)*A - 19*sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2
*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) + ((((105*(sqrt(2)*A*a^5
 - sqrt(2)*B*a^5)*tan(1/2*d*x + 1/2*c)^2/a^3 + 4*(693*sqrt(2)*A*a^5 - 877*s
qrt(2)*B*a^5)/a^3)*tan(1/2*d*x + 1/2*c)^2 + 14*(453*sqrt(2)*A*a^5 - 517*sqr
t(2)*B*a^5)/a^3)*tan(1/2*d*x + 1/2*c)^2 + 140*(39*sqrt(2)*A*a^5 - 47*sqrt(2
```

```
)*B*a^5)/a^3)*tan(1/2*d*x + 1/2*c)^2 + 1785*(sqrt(2)*A*a^5 - sqrt(2)*B*a^5)
/a^3)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(7/2))/d
```

maple [A] time = 0.80, size = 448, normalized size = 1.72

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(960B\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96\sqrt{2}\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}(7A+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/420/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(960*B*2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^8-96*2^(1/2)*a^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7*A+17*B)*sin(1/2*d*x+1/2*c)^6+224*2^(1/2)*
a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A+8*B)*sin(1/2*d*x+1/2*c)^4+35*2^
(1/2)*(45*A*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+a))*a-48*A*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-57*B*ln(4/cos(1/2*d*x+1/
2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a+16*B*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2-1575*2^(1/2)*ln(4/cos(1/2*d*x+1/2
*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*A+1995*2^(1/2)*ln(4/cos(1
/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*B+1785*A*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-1785*B*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2))/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1
/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.108 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=216

$$\frac{(11A-15B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(35A-39B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{30a^2d}+\frac{(A-B)\sin(c+dx)\cos^3(}{2d(a\cos(c+dx)+a)}$$

```
[Out] 1/2*(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/4*(11*A-15*B)*
arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^
(1/2)-1/15*(65*A-93*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/10*(5*A-9*B)
*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/30*(35*A-39*B)*sin(d*
x+c)*(a+a*cos(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.59, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2977, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{(35A-39B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{30a^2d}+\frac{(11A-15B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)\cos^3(}{2d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((11*A - 15*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(2*d*
(a + a*Cos[c + d*x])^(3/2)) - ((65*A - 93*B)*Sin[c + d*x])/(15*a*d*Sqrt[a +
 a*Cos[c + d*x]]) - ((5*A - 9*B)*Cos[c + d*x]^2*Sin[c + d*x])/(10*a*d*Sqrt[
a + a*Cos[c + d*x]]) + ((35*A - 39*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x]
)/(30*a^2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)\left(3a(A-B)-\frac{1}{2}a(5A-9B)\cos(c+a\right.}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(5A-9B)\cos^2(c+dx)\sin(c+dx}{10ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(5A-9B)\cos^2(c+dx)\sin(c+dx}{10ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(5A-9B)\cos^2(c+dx)\sin(c+dx}{10ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(65A-93B)\sin(c+dx)}{15ad\sqrt{a+a\cos(c+dx)}} - \frac{(5A-}{10}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(65A-93B)\sin(c+dx)}{15ad\sqrt{a+a\cos(c+dx)}} - \frac{(5A-}{10}$$

$$= \frac{(11A-15B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\cos^3(c+dx)\sin(c+}{2d(a+a\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.94, size = 142, normalized size = 0.66

$$\frac{\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(3(20A-39B)\cos(c+dx)+(6B-10A)\cos(2(c+dx))+85A-3B\cos(3(c+}{15d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)(a(\cos(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (-15*(11*A - 15*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + Cos[(c +
d*x)/2]^3*(85*A - 141*B + 3*(20*A - 39*B)*Cos[c + d*x] + (-10*A + 6*B)*Cos[
```

```
2*(c + d*x)] - 3*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2])/(15*d*(a*(1 + Cos[c
+ d*x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^2))
```

fricas [A] time = 0.90, size = 224, normalized size = 1.04

$$-\frac{15\sqrt{2}\left((11A-15B)\cos(dx+c)^2+2(11A-15B)\cos(dx+c)+11A-15B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(d}{\cos(dx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/120*(15*sqrt(2)*((11*A - 15*B)*cos(d*x + c)^2 + 2*(11*A - 15*B)*cos(d*x
+ c) + 11*A - 15*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d
*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2
 + 2*cos(d*x + c) + 1)) - 4*(12*B*cos(d*x + c)^3 + 4*(5*A - 3*B)*cos(d*x +
c)^2 - 12*(5*A - 9*B)*cos(d*x + c) - 95*A + 147*B)*sqrt(a*cos(d*x + c) + a)
*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 3.10, size = 202, normalized size = 0.94

$$-\frac{\frac{15\sqrt{2}(11A-15B)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\left(\frac{15\sqrt{2}(Aa^3-Ba^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2}+\frac{\sqrt{2}(245Aa^3-381Ba^3)}{a^2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\right.\right.\right.}{\left(a\tan\left(\frac{1}{2}d\right.\right.}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/60*(15*sqrt(2)*(11*A - 15*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) + (((15*sqrt(2)*(A*a^3 - B*a^3)*t
an(1/2*d*x + 1/2*c)^2/a^2 + sqrt(2)*(245*A*a^3 - 381*B*a^3)/a^2)*tan(1/2*d*
x + 1/2*c)^2 + 5*sqrt(2)*(73*A*a^3 - 105*B*a^3)/a^2)*tan(1/2*d*x + 1/2*c)^2
 + 15*sqrt(2)*(9*A*a^3 - 17*B*a^3)/a^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x
 + 1/2*c)^2 + a)^(5/2))/d
```

maple [B] time = 0.78, size = 407, normalized size = 1.88

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-96B\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+16\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\ (5A+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/60*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-96*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+16*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*a^(1/2)*(5*A+6*B)*sin(1/2*d*x+1/2*c)^4+5*2^(1/2)*(8*A*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)-33*A*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+a))*a-48*B*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+45*B*ln(4/
cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a)*sin(1/2*d
*x+1/2*c)^2+165*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+a))*a*A-225*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)+a))*a*B-135*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)+255*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1
/2*c)/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.109 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{(7A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(3A-7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{6a^2d}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 1/2*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-1/4*(7*A-11*B)*a
rctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(
1/2)+1/3*(9*A-13*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/6*(3*A-7*B)*sin
(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.42, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2977, 2968, 3023, 2751, 2649, 206}

$$-\frac{(3A-7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{6a^2d}-\frac{(7A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -((7*A - 11*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(2*d*
(a + a*Cos[c + d*x])^(3/2)) + ((9*A - 13*B)*Sin[c + d*x])/(3*a*d*Sqrt[a + a
*Cos[c + d*x]]) - ((3*A - 7*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(6*a^
2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(2a(A-B)-\frac{1}{2}a(3A-7B)\cos(c+dx\right.}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{2a(A-B)\cos(c+dx)-\frac{1}{2}a(3A-7B)\cos^2(c+d}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(3A-7B)\sqrt{a+a\cos(c+dx)}\,\sin(}{6a^2d}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(9A-13B)\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} - \frac{(3A-7}{}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(9A-13B)\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} - \frac{(3A-7}{}$$

$$= -\frac{(7A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{(A-B)\cos^2(c+dx)\sin(c+}{2d(a+a\cos(c+dx))^{3/2}}$$

Mathematica **[A]** time = 0.77, size = 97, normalized size = 0.57

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(12(A-B)\cos(c+dx)+15A+2B\cos(2(c+dx))-17B)-3(7A-11B)\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}}{6ad\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (-3*(7*A - 11*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (15*A - 17*B
+ 12*(A - B)*Cos[c + d*x] + 2*B*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(6*a*d*
Sqrt[a*(1 + Cos[c + d*x])])
```

fricas **[A]** time = 0.72, size = 205, normalized size = 1.20

$$-\frac{3\sqrt{2}\left((7A-11B)\cos(dx+c)^2+2(7A-11B)\cos(dx+c)+7A-11B\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)}}{\cos(dx+c)^2+}\right.}{24\left(a^2d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/24*(3*sqrt(2)*((7*A - 11*B)*cos(d*x + c)^2 + 2*(7*A - 11*B)*cos(d*x + c)
 + 7*A - 11*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x +
c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) - 4*(4*B*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) + 15*A
 - 19*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 1.93, size = 168, normalized size = 0.98

$$\frac{\frac{3\left(7\sqrt{2}A-11\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\left(\frac{3\left(\sqrt{2}Aa-\sqrt{2}Ba\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a}+\frac{2\left(15\sqrt{2}Aa-23\sqrt{2}Ba\right)}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+\cdots}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{3}{2}}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/12*(3*(7*sqrt(2)*A - 11*sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c)
+ sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) + ((3*(sqrt(2)*A*a - sqrt(2)
*B*a)*tan(1/2*d*x + 1/2*c)^2/a + 2*(15*sqrt(2)*A*a - 23*sqrt(2)*B*a)/a)*tan
(1/2*d*x + 1/2*c)^2 + 27*(sqrt(2)*A*a - sqrt(2)*B*a)/a)*tan(1/2*d*x + 1/2*c
)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2))/d
```

maple [B] time = 0.80, size = 327, normalized size = 1.91

$$\frac{\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(16B\sqrt{2}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-21A\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\left(\cos\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/12/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*B*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-21*A*ln(2*(2*a^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x
+1/2*c)^2*a+33*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/
```

```
2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+24*A*a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-40*B*a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+3*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)*a^(1/2)-3*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2
)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.110 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{(3A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{2B\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/4*(3*A-7*B)*arctanh(1/2*si
n(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+2*B*sin(
d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3019, 2751, 2649, 206}

$$\frac{(3A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{2B\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((3*A - 7*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*
x])^(3/2)) + (2*B*Sin[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
```

```
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{-\frac{3}{2}a(A-B)-2aB\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{(3A-7B)\int \frac{}{\sqrt{a+}}}{4a}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} - \frac{(3A-7B)\,\mathrm{Subst}}{}$$

$$= \frac{(3A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2}{ad\sqrt{}}$$

Mathematica [A] time = 0.42, size = 104, normalized size = 0.88

$$\frac{\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(A-4B\cos(c+dx)-5B)-(3A-7B)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx\right.\right.}{d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-((3*A - 7*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5) + Cos[(c + d*x
)/2]^3*(A - 5*B - 4*B*Cos[c + d*x])*Sin[(c + d*x)/2])/(d*(a*(1 + Cos[c + d*
x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^2))
```

fricas [A] time = 0.60, size = 189, normalized size = 1.60

$$-\frac{\sqrt{2}\left((3\,A-7\,B)\cos\left(dx+c\right)^2+2\,(3\,A-7\,B)\cos\left(dx+c\right)+3\,A-7\,B\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2+2\,\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}\,s}{\cos(dx+c)^2+2\,\cos(d}\right.}{8\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/8*(sqrt(2)*((3*A - 7*B)*cos(d*x + c)^2 + 2*(3*A - 7*B)*cos(d*x + c) + 3*
A - 7*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a
)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) - 4*(4*B*cos(d*x + c) - A + 5*B)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 1.70, size = 131, normalized size = 1.11

$$-\frac{\frac{\left(\frac{\sqrt{2}\left(Aa^2-Ba^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3}+\frac{\sqrt{2}\left(Aa^2-9\,Ba^2\right)}{a^3}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}+\frac{\sqrt{2}\,(3\,A-7\,B)\log\left(\left|-\sqrt{a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] -1/4*((sqrt(2)*(A*a^2 - B*a^2)*tan(1/2*d*x + 1/2*c)^2/a^3 + sqrt(2)*(A*a^2
- 9*B*a^2)/a^3)*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + s
qrt(2)*(3*A - 7*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a)))/a^(3/2))/d
```

maple [B] time = 0.74, size = 256, normalized size = 2.17

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(3A\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-7B\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\left(\right.}{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-7*B*ln(
2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2
)*cos(1/2*d*x+1/2*c)^2*a+8*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
*cos(1/2*d*x+1/2*c)^2-A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+B*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)/a^(5/2)/si
n(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\cos(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2), x
)
```

3.111 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/4*(A+3*B)*arctanh(1/2*sin(d
*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2750, 2649, 206}

$$\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((A + 3*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x]
)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx &= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{(A+3B)\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}\\ &= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}-\frac{(A+3B)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad}\\ &= \frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.20, size = 63, normalized size = 0.72

$$\frac{\frac{1}{2}(A-B)\sin(c+dx)+(A+3B)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((A + 3*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + ((A - B)*Sin[c +
d*x])/2)/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [B] time = 0.84, size = 172, normalized size = 1.98

$$\frac{\sqrt{2}\left((A+3B)\cos(dx+c)^2+2(A+3B)\cos(dx+c)+A+3B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*((A + 3*B)*cos(d*x + c)^2 + 2*(A + 3*B)*cos(d*x + c) + A + 3*B
)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)) + 4*sqrt(a*cos(d*x + c) + a)*(A - B)*sin(d*x + c))/(a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 1.30, size = 101, normalized size = 1.16

$$-\frac{\frac{\left(\sqrt{2}A+3\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\sqrt{2}Aa-\sqrt{2}Ba\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*((sqrt(2)*A + 3*sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a)*(sqrt(2)*A*a - sqrt(2)*B*a)*tan(1/2*d*x + 1/2*c)/a^3)/d
```

maple [B] time = 0.72, size = 220, normalized size = 2.53

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(A\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+3B\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\left(c\right.\right.}{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*ln(2*(2*a^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1
/2*c)^2*a+3*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d
*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)*a^(1/2)-B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/s
in(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cos\left(c+d\,x\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(a*(cos(c + d*x) + 1))**(3/2), x)
```

3.112 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{(5A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 2*A*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d-1/2*(A-B)*
sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-1/4*(5*A-B)*arctanh(1/2*sin(d*x+c)*a^(1
/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2985, 2649, 206, 2773}

$$-\frac{(5A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d)
- ((5*A - B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*
x])^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(2aA-\frac{1}{2}a(A-B)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{A\int \sqrt{a+a\cos(c+dx)}\,\sec(c+dx)\,dx}{a^2} - \frac{(5}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(2A)\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} +$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{(A}{2d(a}$$

Mathematica [A] time = 0.71, size = 131, normalized size = 1.03

$$\frac{(A-B)\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+(5A-B)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\sqrt{2}A\cos^5\left(\frac{1}{2}\right.}{d\left(\sin^2\left(\frac{1}{2}(c+dx)\right)-1\right)(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((5*A - B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - 4*Sqrt[2]*A*ArcTa
nh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (A - B)*Cos[(c + d*x)/2]^
3*Sin[(c + d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c + d*x)/2]^
2))
```

fricas [B] time = 0.85, size = 281, normalized size = 2.21

$$-\frac{\sqrt{2}\left((5A-B)\cos(dx+c)^2+2(5A-B)\cos(dx+c)+5A-B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(d}{\cos(dx+c)^2+2\cos(dx+c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/8*(sqrt(2)*((5*A - B)*cos(d*x + c)^2 + 2*(5*A - B)*cos(d*x + c) + 5*A -
B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1)) - 4*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x
 + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x
+ c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a
*cos(d*x + c) + a)*(A - B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*co
s(d*x + c) + a^2*d)
```

giac [B] time = 2.86, size = 214, normalized size = 1.69

$$\frac{\frac{\sqrt{2}\left(5A\sqrt{a}-B\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{a^2}+\frac{8A\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{a^{\frac{3}{2}}}-}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] 1/8*(sqrt(2)*(5*A*sqrt(a) - B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^2 + 8*A*log(abs((sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a
^(3/2) - 8*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(3/2) - 2*sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a)*(sqrt(2)*A*a - sqrt(2)*B*a)*tan(1/2*d*x + 1/2*c)/a^3)/d
```

maple [B] time = 1.54, size = 374, normalized size = 2.94

$$-\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(5A\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-B\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\left(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*ln(2*(2
*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*co
s(1/2*d*x+1/2*c)^2*a-B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/
cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-4*A*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^2*a-4*A*ln(-4*(a*2^(1/2)*cos(1/2
*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*
d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a+A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)-B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2
*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2)),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**(3/2),x)
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2), x
)
```

3.113 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{(3A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(9A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\,\mathrm{ta}}{2d(a\cos(c+}$$

```
[Out] -(3*A-2*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d+1/4
*(9*A-5*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a
^(3/2)/d*2^(1/2)-1/2*(A-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/2*(3*A-B)*
tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2978, 2984, 2985, 2649, 206, 2773}

$$-\frac{(3A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(9A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\,\mathrm{ta}}{2d(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -(((3*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a
^(3/2)*d)) + ((9*A - 5*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Tan[c + d*x])/(2*d*(a +
 a*Cos[c + d*x])^(3/2)) + ((3*A - B)*Tan[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c
+ d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(a(3A-B)-\frac{3}{2}a(A-B)\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-a^2(3A-2B)+\frac{1}{2}a}{\sqrt{}}}{}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{(9A-5B)\int \frac{}{\sqrt{a+}}}{4a}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A-B)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{(9A-5B)\,\mathrm{Subst}}{}$$

$$= -\frac{(3A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{(9A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+}}\right.}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [A] time = 1.16, size = 141, normalized size = 0.83

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(2(9A-5B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)(-2A\sec(c+dx)-3A+B)+4\sqrt{2}(3A-2B)\cos^2\left(\frac{1}{2}(c+dx)\right)\tan}{\sin^2\left(\frac{1}{2}(c+dx)\right)-1}\right.}{2d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(2*(9*A - 5*B)*ArcTanh[Sin[(c + d*x)/2]] + (4*Sqrt[2]*(
3*A - 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^2 + 2*(-3*A +
 B - 2*A*Sec[c + d*x])*Sin[(c + d*x)/2])/(-1 + Sin[(c + d*x)/2]^2)))/(2*d*(
a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [B] time = 1.31, size = 339, normalized size = 1.99

$$-\frac{\sqrt{2}\left((9A-5B)\cos(dx+c)^3 + 2(9A-5B)\cos(dx+c)^2 + (9A-5B)\cos(dx+c)\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/8*(sqrt(2)*((9*A - 5*B)*cos(d*x + c)^3 + 2*(9*A - 5*B)*cos(d*x + c)^2 +
(9*A - 5*B)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a
*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x
+ c)^2 + 2*cos(d*x + c) + 1)) + 2*((3*A - 2*B)*cos(d*x + c)^3 + 2*(3*A - 2*
B)*cos(d*x + c)^2 + (3*A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3
 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((3*A - B)*co
s(d*x + c) + 2*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c
)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

giac [B] time = 3.23, size = 373, normalized size = 2.19

$$-\frac{\frac{\sqrt{2}\left(9A\sqrt{a}-5B\sqrt{a}\right)\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\right)}{a^{2}}+\frac{4\left(3A\sqrt{a}-2B\sqrt{a}\right)\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right.\right.\right.}{a^{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/8*(sqrt(2)*(9*A*sqrt(a) - 5*B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^2 + 4*(3*A*sqrt(a) - 2*B*sqrt(a
))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^2 - a*(2*sqrt(2) + 3)))/a^2 - 4*(3*A*sqrt(a) - 2*B*sqrt(a))*log(abs((sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*s
qrt(2) - 3)))/a^2 - 16*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(a) - A*a^(3/2))/(((sqrt(a)*tan(1/2*d*x
+ 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*a) - 2*sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*A*a - sqrt(2)*B*a)*tan(1/2*d*x + 1/2*c)/a^
3)/d
```

maple [B] time = 1.57, size = 1051, normalized size = 6.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(18*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-10*B*l
n(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1
/2)*cos(1/2*d*x+1/2*c)^4*a-12*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
*cos(1/2*d*x+1/2*c)^4*a-12*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*c
os(1/2*d*x+1/2*c)^4*a+8*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1
/2*d*x+1/2*c)^4*a+8*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*
d*x+1/2*c)^4*a-9*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(
1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a+5*B*ln(2*(2*a^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^
2*a+6*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2
+6*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^2*a+6*A
*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a-2*B*a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-4*B*ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^2*a-4*B*ln(-4*(a*
2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2
*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a-A*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a
^(1/2))/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/
2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^2\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a*(cos(c + d*x) + 1))**(3/2)
, x)
```

3.114 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=221

$$\frac{(19A-12B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a\cos(c+dx)+a}}+\frac{(2A-B)\,\mathrm{ta}}{2ad\sqrt{a}}$$

```
[Out] 1/4*(19*A-12*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/
d-1/4*(13*A-9*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1
/2))/a^(3/2)/d*2^(1/2)-1/2*(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(
3/2)-1/4*(7*A-6*B)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/2*(2*A-B)*sec(d*
x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2978, 2984, 2985, 2649, 206, 2773}

$$\frac{(19A-12B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a\cos(c+dx)+a}}+\frac{(2A-B)\,\mathrm{ta}}{2ad\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((19*A - 12*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4
*a^(3/2)*d) - ((13*A - 9*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((7*A - 6*B)*Tan[c + d*x])/(4*
a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*d*(a
 + a*Cos[c + d*x])^(3/2)) + ((2*A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sq
rt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(2a(2A-B)-\frac{5}{2}a(A-B)\cos(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A-B)\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A-B)}{2ad}$$

$$= -\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A-B)}{2ad}$$

$$= -\frac{(7A-6B)\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(2A-B)}{2ad}$$

$$= \frac{(19A-12B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{3/2}d} - \frac{(13A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}$$

Mathematica [A] time = 1.63, size = 205, normalized size = 0.93

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(4\sin\left(\frac{1}{2}(c+dx)\right)((6A-8B)\cos(c+dx)+(7A-6B)\cos(2(c+dx))+3(A-2B))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (Cos[(c + d*x)/2]^3*Sec[c + d*x]^2*(4*(13*A - 9*B)*ArcTanh[Sin[(c + d*x)/2]
]*(Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2])^2 - 2*Sqrt[2]*(19*A - 12*B)*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2])^2
+ 4*(3*(A - 2*B) + (6*A - 8*B)*Cos[c + d*x] + (7*A - 6*B)*Cos[2*(c + d*x)])
*Sin[(c + d*x)/2]))/(16*d*(a*(1 + Cos[c + d*x]))^(3/2)*(-1 + Sin[(c + d*x)/
2]^2))
```

fricas [A] time = 0.73, size = 361, normalized size = 1.63

$$-\frac{2\sqrt{2}\left((13A-9B)\cos(dx+c)^4+2(13A-9B)\cos(dx+c)^3+(13A-9B)\cos(dx+c)^2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/16*(2*sqrt(2)*((13*A - 9*B)*cos(d*x + c)^4 + 2*(13*A - 9*B)*cos(d*x + c)
^3 + (13*A - 9*B)*cos(d*x + c)^2)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((19*A - 12*B)*cos(d*x + c)^4 + 2*(
19*A - 12*B)*cos(d*x + c)^3 + (19*A - 12*B)*cos(d*x + c)^2)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(c
os(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4
*((7*A - 6*B)*cos(d*x + c)^2 + (3*A - 4*B)*cos(d*x + c) - 2*A)*sqrt(a*cos(d
*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 +
 a^2*d*cos(d*x + c)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assum
es constant sign by intervals (correct if the argument is real):Check [abs(
cos((d*t_nostep+c)/2))]Discontinuities at zeroes of cos((d*t_nostep+c)/2) w
ere not checkedEvaluation time: 0.69Unable to divide, perhaps due to roundi
ng error%%%{%%{[%%%{%%{[23574053482485268906770432,0]:[1,0,-2]%%},[16]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[0]%%%} / %%%{%%%{%%{[604462909807314587353088,0
]:[1,0,-2]%%},[16]%%%},[0]%%%} Error: Bad Argument Value
```

maple [B] time = 1.68, size = 1540, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(104*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-72*B
*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^
(1/2)*cos(1/2*d*x+1/2*c)^6*a-76*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2
)))*cos(1/2*d*x+1/2*c)^6*a-76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
*cos(1/2*d*x+1/2*c)^6*a+48*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*c
os(1/2*d*x+1/2*c)^6*a+48*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(
1/2*d*x+1/2*c)^6*a-104*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a
)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+72*B*ln(2*(2*a^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+
1/2*c)^4*a+28*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+
1/2*c)^4+76*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*
c)^4*a+76*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^
4*a-24*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^
4-48*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a-
48*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^4*a+26*
A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2
^(1/2)*cos(1/2*d*x+1/2*c)^2*a-18*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^2*a-22*A*a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-19*A*ln(-4*(a*
2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2
*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a-19*A*ln(4/(2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^2*a+16*B*a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+12*B*ln(-4*(a*2^(1/2)
*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*
cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2*a+12*B*ln(4/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)
*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^2*a+2*A*2^(1/2)*(a*sin(1/2*d*x
```

```
+1/2*c)^2)^(1/2)*a^(1/2)-2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
)/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*
x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B \cos{\left(c + dx \right)}\right) \sec^{3}{\left(c + dx \right)}}{\left(a \left(\cos{\left(c + dx \right)} + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a*(cos(c + d*x) + 1))**(3/2)
, x)
```

3.115 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=261

$$\frac{(163A-283B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(475A-787B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{240a^3d}-\frac{(85A-157B)\sin(c+}{80a^2d\sqrt{a\cos(c}}$$

```
[Out] 1/4*(A-B)*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/16*(13*A-21*B)
*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+1/32*(163*A-283*B)*arct
anh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2
)-1/120*(985*A-1729*B)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)-1/80*(85*A-1
57*B)*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)+1/240*(475*A-787
*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 0.80, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2977, 2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{(85A-157B)\sin(c+dx)\cos^2(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(475A-787B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{240a^3d}-\frac{(985A-1729B)\sin(c}{120a^2d\sqrt{a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((163*A - 283*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^4*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) + ((13*A - 21*B)*Cos[c + d*x]^3*Sin[c + d*x]
)/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) - ((985*A - 1729*B)*Sin[c + d*x])/(12
0*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((85*A - 157*B)*Cos[c + d*x]^2*Sin[c +
d*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]]) + ((475*A - 787*B)*Sqrt[a + a*Cos
[c + d*x]]*Sin[c + d*x])/(240*a^3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^3(c+dx)\left(4a(A-B)-\frac{1}{2}a(5A-13B)\cos(c+d\right.}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^4(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(13A-21B)\cos^3(c+dx)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(163A-283B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{(A-B)\cos^4(c+dx)\sin(c}{4d(a+a\cos(c+dx))^{5/}}$$

Mathematica [A] time = 1.64, size = 139, normalized size = 0.53

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(-5(479A-887B)\cos(c+dx)+(832B-400A)\cos(2(c+dx))+40A\cos(3(c+dx))-1895A-}{240ad(a(\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (30*(163*A - 283*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-1895*A
 + 3491*B - 5*(479*A - 887*B)*Cos[c + d*x] + (-400*A + 832*B)*Cos[2*(c + d*
x)] + 40*A*Cos[3*(c + d*x)] - 40*B*Cos[3*(c + d*x)] + 12*B*Cos[4*(c + d*x)]
)*Tan[(c + d*x)/2])/(240*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 0.80, size = 270, normalized size = 1.03

$$-\frac{15\sqrt{2}\left((163\,A-283\,B)\cos\left(dx+c\right)^{3}+3\,(163\,A-283\,B)\cos\left(dx+c\right)^{2}+3\,(163\,A-283\,B)\cos\left(dx+c\right)+163\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/960*(15*sqrt(2)*((163*A - 283*B)*cos(d*x + c)^3 + 3*(163*A - 283*B)*cos(
d*x + c)^2 + 3*(163*A - 283*B)*cos(d*x + c) + 163*A - 283*B)*sqrt(a)*log(-(
a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c)
- 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(96*B*
cos(d*x + c)^4 + 160*(A - B)*cos(d*x + c)^3 - 32*(25*A - 49*B)*cos(d*x + c)
^2 - 5*(503*A - 911*B)*cos(d*x + c) - 1495*A + 2671*B)*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d
*cos(d*x + c) + a^3*d)
```

giac [A] time = 2.69, size = 257, normalized size = 0.98

$$-\frac{\frac{15\left(163\sqrt{2}A-283\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{5}{2}}}-\frac{\left(\left(\left(15\left(\frac{2\left(\sqrt{2}Aa^{2}-\sqrt{2}Ba^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{2}}-\frac{21\sqrt{2}Aa^{2}-29\sqrt{2}Ba^{2}}{a^{2}}\right)\tan\right.\right.\right.}{}}{480}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/480*(15*(163*sqrt(2)*A - 283*sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1
/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2) - (((15*(2*(sqrt(2)*A*
a^2 - sqrt(2)*B*a^2)*tan(1/2*d*x + 1/2*c)^2/a^2 - (21*sqrt(2)*A*a^2 - 29*sq
rt(2)*B*a^2)/a^2)*tan(1/2*d*x + 1/2*c)^2 - (3685*sqrt(2)*A*a^2 - 6733*sqrt(
2)*B*a^2)/a^2)*tan(1/2*d*x + 1/2*c)^2 - 5*(1133*sqrt(2)*A*a^2 - 1973*sqrt(2
```

```
)*B*a^2)/a^2)*tan(1/2*d*x + 1/2*c)^2 - 15*(155*sqrt(2)*A*a^2 - 291*sqrt(2)*
B*a^2)/a^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

maple [B] time = 0.82, size = 467, normalized size = 1.79

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(768B\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+640A\sqrt{2}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{a}\,\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/480/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(768*B*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^8+640*A*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-2176*B*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+2445*A*ln(2*(2*a^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/
2*d*x+1/2*c)^4*a-4245*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)
/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-2560*A*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+5248*B*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-435*A*a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+555*B*a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+30*A*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)-30*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2))/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.116 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{(75A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(39A-95B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{48a^3d}+\frac{(93A-197B)\sin(c+d}{24a^2d\sqrt{a\cos(c+dx)+}$$

```
[Out] 1/4*(A-B)*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/16*(9*A-17*B)*
cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)-1/32*(75*A-163*B)*arctan
h(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+
1/24*(93*A-197*B)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)-1/48*(39*A-95*B)*
sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/a^3/d
```

Rubi [A] time = 0.61, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2977, 2968, 3023, 2751, 2649, 206}

$$-\frac{(39A-95B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{48a^3d}+\frac{(93A-197B)\sin(c+dx)}{24a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(75A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right.}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] -((75*A - 163*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
 d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^3*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) + ((9*A - 17*B)*Cos[c + d*x]^2*Sin[c + d*x])
/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((93*A - 197*B)*Sin[c + d*x])/(24*a^
2*d*Sqrt[a + a*Cos[c + d*x]]) - ((39*A - 95*B)*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(48*a^3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^2(c+dx)\left(3a(A-B)-\frac{1}{2}a(3A-11B)\cos(c+dx\right.}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(9A-17B)\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(75A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\cos^3(c+dx)\sin(c}{4d(a+a\cos(c+dx))^{5/}}$$

Mathematica [A] time = 1.11, size = 117, normalized size = 0.54

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((255A-479B)\cos(c+dx)+16(3A-5B)\cos(2(c+dx))+195A+8B\cos(3(c+dx))-379B)-}{48ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (-6*(75*A - 163*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (195*A -
379*B + (255*A - 479*B)*Cos[c + d*x] + 16*(3*A - 5*B)*Cos[2*(c + d*x)] + 8*
B*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(48*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 0.68, size = 254, normalized size = 1.18

$$-\frac{3\sqrt{2}\left((75A-163B)\cos(dx+c)^3+3(75A-163B)\cos(dx+c)^2+3(75A-163B)\cos(dx+c)+75A-16\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/192*(3*sqrt(2)*((75*A - 163*B)*cos(d*x + c)^3 + 3*(75*A - 163*B)*cos(d*x
 + c)^2 + 3*(75*A - 163*B)*cos(d*x + c) + 75*A - 163*B)*sqrt(a)*log(-(a*cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a
*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*B*cos(d
*x + c)^3 + 32*(3*A - 5*B)*cos(d*x + c)^2 + (255*A - 503*B)*cos(d*x + c) +
147*A - 299*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 4.72, size = 204, normalized size = 0.94

$$-\frac{\frac{\left(\left(\left(3\left(\frac{2\sqrt{2}\left(Aa^5-Ba^5\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^6}-\frac{\sqrt{2}\left(15Aa^5-23Ba^5\right)}{a^6}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{4\sqrt{2}\left(75Aa^5-167Ba^5\right)}{a^6}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{3\sqrt{2}\left(83Aa^5-155Ba^5\right)}{a^6}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/96*(((3*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/2*d*x + 1/2*c)^2/a^6 - sqrt(2)*
(15*A*a^5 - 23*B*a^5)/a^6)*tan(1/2*d*x + 1/2*c)^2 - 4*sqrt(2)*(75*A*a^5 - 1
67*B*a^5)/a^6)*tan(1/2*d*x + 1/2*c)^2 - 3*sqrt(2)*(83*A*a^5 - 155*B*a^5)/a^
6)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 3*sqrt(2)*(7
5*A - 163*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a)))/a^(5/2))/d
```

maple [B] time = 0.84, size = 397, normalized size = 1.84

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(128B\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-225A\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/96/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(128*B*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6-225*A*ln(2*(2*a^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2
*d*x+1/2*c)^4*a+489*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/c
os(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+192*A*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-512*B*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+63*A*a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-87*B*a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-6*A*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*a^(1/2)+6*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(
7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.117 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=169

$$\frac{(19A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-9B)\sin(c+dx)}{4a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{(5A-13\ldots}{16ad(a\cos\ldots}$$

```
[Out] 1/4*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(5*A-13*B)*
sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+1/32*(19*A-75*B)*arctanh(1/2*sin(d*x+
c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*(A-9*B)*si
n(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2977, 2968, 3019, 2751, 2649, 206}

$$-\frac{(A-9B)\sin(c+dx)}{4a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(19A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{(5A-1\ldots}{16ad(a\,c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((19*A - 75*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d
*x]])]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(4*d
*(a + a*Cos[c + d*x])^(5/2)) - ((5*A - 13*B)*Sin[c + d*x])/(16*a*d*(a + a*C
os[c + d*x])^(3/2)) - ((A - 9*B)*Sin[c + d*x])/(4*a^2*d*Sqrt[a + a*Cos[c +
d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos(c+dx)\left(2a(A-B)-\frac{1}{2}a(A-9B)\cos(c+dx)\right.}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{2a(A-B)\cos(c+dx)-\frac{1}{2}a(A-9B)\cos^2(c+dx}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{-\frac{3}{4}}{}}{}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(A}{4a^2d}$$

$$= \frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(A}{4a^2d}$$

$$= \frac{(19A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{(A-B)\cos^2(c+dx)\sin(c+}{4d(a+a\cos(c+dx))^{5/2}}$$

Mathematica [A] time = 0.78, size = 100, normalized size = 0.59

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((85B-13A)\cos(c+dx)-9A+16B\cos(2(c+dx))+65B)+2(19A-75B)\cos^3\left(\frac{1}{2}(c+dx)\right)\text{ta}}{16ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (2*(19*A - 75*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-9*A + 65*
B + (-13*A + 85*B)*Cos[c + d*x] + 16*B*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/
(16*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 0.74, size = 237, normalized size = 1.40

$$-\frac{\sqrt{2}\left((19A-75B)\cos(dx+c)^3+3(19A-75B)\cos(dx+c)^2+3(19A-75B)\cos(dx+c)+19A-75B\right)\sqrt{}}{64\left(a^3d\,\text{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/64*(sqrt(2)*((19*A - 75*B)*cos(d*x + c)^3 + 3*(19*A - 75*B)*cos(d*x + c)
^2 + 3*(19*A - 75*B)*cos(d*x + c) + 19*A - 75*B)*sqrt(a)*log(-(a*cos(d*x +
c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*
x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(32*B*cos(d*x + c)
^2 - (13*A - 85*B)*cos(d*x + c) - 9*A + 49*B)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
+ c) + a^3*d)
```

giac [A] time = 2.61, size = 181, normalized size = 1.07

$$\frac{\frac{\left(\left(\frac{2\left(\sqrt{2}Aa^6-\sqrt{2}Ba^6\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^8}-\frac{9\sqrt{2}Aa^6-17\sqrt{2}Ba^6}{a^8}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{11\sqrt{2}Aa^6-83\sqrt{2}Ba^6}{a^8}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{\left(19\sqrt{2}A-75\sqrt{2}B\right)\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\right.\right.\right.}{a^{\frac{5}{2}}}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/32*(((2*(sqrt(2)*A*a^6 - sqrt(2)*B*a^6)*tan(1/2*d*x + 1/2*c)^2/a^8 - (9*s
qrt(2)*A*a^6 - 17*sqrt(2)*B*a^6)/a^8)*tan(1/2*d*x + 1/2*c)^2 - (11*sqrt(2)*
A*a^6 - 83*sqrt(2)*B*a^6)/a^8)*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a) - (19*sqrt(2)*A - 75*sqrt(2)*B)*log(abs(-sqrt(a)*tan(1/2*d*x +
1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

maple [B] time = 0.86, size = 327, normalized size = 1.93

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(19A\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-75B\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(19*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-75*B*
ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(
1/2)*cos(1/2*d*x+1/2*c)^4*a+64*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
```

```
1/2)*cos(1/2*d*x+1/2*c)^4-13*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)*cos(1/2*d*x+1/2*c)^2+21*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
*cos(1/2*d*x+1/2*c)^2+2*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*
B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/cos(1/2*d*x+1/2*c)^3/a^(7
/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.118 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(5A+19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5A-13B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] -1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/16*(5*A-13*B)*sin(d*x+c)/a
/d/(a+a*cos(d*x+c))^(3/2)+1/32*(5*A+19*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^
(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3019, 2750, 2649, 206}

$$\frac{(5A+19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5A-13B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((5*A + 19*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c +
d*x])^(5/2)) + ((5*A - 13*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/
2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{-\frac{5}{2}a(A-B)-4aB\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(5A+19B)\int}{3}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5A-13B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(5A+19B)\,\mathrm{S}}{}$$

$$= \frac{(5A+19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(5}{16a}$$

Mathematica [A] time = 0.62, size = 87, normalized size = 0.69

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((5A-13B)\cos(c+dx)+A-9B)+2(5A+19B)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]

[Out] (2*(5*A + 19*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (A - 9*B + (5*A - 13*B)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a*d*(a*(1 + Cos[c + d*x]))^(3/2))

fricas [B] time = 0.77, size = 223, normalized size = 1.77

$$\frac{\sqrt{2}\left((5A+19B)\cos(dx+c)^{3}+3(5A+19B)\cos(dx+c)^{2}+3(5A+19B)\cos(dx+c)+5A+19B\right)\sqrt{a}\log\left(\right.}{64\left(a^{3}d\cos(dx+c)^{3}+3a^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/64*(sqrt(2)*((5*A + 19*B)*cos(d*x + c)^3 + 3*(5*A + 19*B)*cos(d*x + c)^2 + 3*(5*A + 19*B)*cos(d*x + c) + 5*A + 19*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((5*A - 13*B)*cos(d*x + c) + A - 9*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

giac [A] time = 1.31, size = 134, normalized size = 1.06

$$-\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\left(\frac{2\sqrt{2}\left(Aa^{5}-Ba^{5}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{8}}-\frac{\sqrt{2}\left(3Aa^{5}-11Ba^{5}\right)}{a^{8}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{\sqrt{2}(5A+19B)\log\left(\left|-\sqrt{a}\tan\right.\right.}{}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] -1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/2*d*x + 1/2*c)^2/a^8 - sqrt(2)*(3*A*a^5 - 11*B*a^5)/a^8)*tan(1/2*d*x + 1/2*c) + sqrt(2)*(5*A + 19*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d

maple [B] time = 0.68, size = 292, normalized size = 2.32

$$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(5A\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\,\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+19B\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)

[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+19*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+5*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-13*B*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-2*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)

[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.119 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(3A+5B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] 1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/16*(3*A+5*B)*sin(d*x+c)/a/d
/(a+a*cos(d*x+c))^(3/2)+1/32*(3*A+5*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/
2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2750, 2650, 2649, 206}

$$\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(3A+5B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((3*A + 5*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x
]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d
*x])^(5/2)) + ((3*A + 5*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B)\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(3A+5B)\int \frac{1}{\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(3A+5B)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\right.}{16a^2 d}$$

$$= \frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A+5B)\sin}{16ad(a+a\cos(}$$

Mathematica [A] time = 0.52, size = 80, normalized size = 0.63

$$\frac{\sin(c+dx)((3A+5B)\cos(c+dx)+7A+B)+4(3A+5B)\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*(3*A + 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (7*A + B + (3
*A + 5*B)*Cos[c + d*x])*Sin[c + d*x])/(16*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [B] time = 0.54, size = 223, normalized size = 1.77

$$\frac{\sqrt{2}\left((3\,A+5\,B)\cos\left(dx+c\right)^3+3\,(3\,A+5\,B)\cos\left(dx+c\right)^2+3\,(3\,A+5\,B)\cos\left(dx+c\right)+3\,A+5\,B\right)\sqrt{a}\,\log\left(-\frac{a\,\mathrm{co}}{}\right.}{64\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*A + 5*B)*cos(d*x + c)^3 + 3*(3*A + 5*B)*cos(d*x + c)^2 +
3*(3*A + 5*B)*cos(d*x + c) + 3*A + 5*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*
sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) -
3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((3*A + 5*B)*cos(d*x + c) +
 7*A + B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*
a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 3.84, size = 134, normalized size = 1.06

$$\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\,\sqrt{2}\left(Aa^{5}-Ba^{5}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{8}}+\frac{\sqrt{2}\left(5\,Aa^{5}+3\,Ba^{5}\right)}{a^{8}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{\sqrt{2}\,(3\,A+5\,B)\log\left(\left|-\sqrt{a}\,\tan\left(\frac{1}{2}\right.\right.\right.}{}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/2
*d*x + 1/2*c)^2/a^8 + sqrt(2)*(5*A*a^5 + 3*B*a^5)/a^8)*tan(1/2*d*x + 1/2*c)
 - sqrt(2)*(3*A + 5*B)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

maple [B] time = 0.75, size = 292, normalized size = 2.32

$$\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(3A\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\,\left(\cos^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+5B\ln\left(\frac{4\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\,\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*ln(2*(2*a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*
d*x+1/2*c)^4*a+5*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(
1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+3*A*a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+5*B*a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+2*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^
```

```
2)^(1/2)*a^(1/2)-2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2
)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos\left(c + d\,x\right)}{{\left(a + a\,\cos\left(c + d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(5/2),x)
[Out] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.120 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=164

$$-\frac{(43A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-3B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] 2*A*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d-1/4*(A-B)*
sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(11*A-3*B)*sin(d*x+c)/a/d/(a+a*cos
(d*x+c))^(3/2)-1/32*(43*A-3*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*
cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2978, 2985, 2649, 206, 2773}

$$-\frac{(43A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-3B)\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d)
- ((43*A - 3*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c +
d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c
+ d*x])^(5/2)) - ((11*A - 3*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(
3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(4aA-\frac{3}{2}a(A-B)\cos(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(8a^2A-\frac{1}{4}a^2(1\right.}{}}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{A\int\sqrt{a+a\,c}}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(11A-3B)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{(2A)\,\mathrm{Subst}\Big(}{}$$

$$= \frac{2A\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - $$

Mathematica [A] time = 1.65, size = 126, normalized size = 0.77

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)((3B-11A)\cos(c+dx)-15A+7B)-2(43A-3B)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16ad(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(43*A - 3*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + 64*Sqrt[2]*
A*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-15*A + 7*B + (-1
1*A + 3*B)*Cos[c + d*x])*Tan[(c + d*x)/2])/(16*a*d*(a*(1 + Cos[c + d*x]))^(
3/2))
```

fricas [B] time = 0.81, size = 339, normalized size = 2.07

$$-\frac{\sqrt{2}\left((43\,A-3\,B)\cos(dx+c)^3+3\,(43\,A-3\,B)\cos(dx+c)^2+3\,(43\,A-3\,B)\cos(dx+c)+43\,A-3\,B\right)\sqrt{a}\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/64*(sqrt(2)*((43*A - 3*B)*cos(d*x + c)^3 + 3*(43*A - 3*B)*cos(d*x + c)^2
 + 3*(43*A - 3*B)*cos(d*x + c) + 43*A - 3*B)*sqrt(a)*log(-(a*cos(d*x + c)^2
 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x +
c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 32*(A*cos(d*x + c)^3 + 3
*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*s
in(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((11*A - 3*B)*cos
(d*x + c) + 15*A - 7*B)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d
*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 3.50, size = 250, normalized size = 1.52

$$-\frac{2\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\sqrt{2}\left(Aa^5-Ba^5\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^8}+\frac{\sqrt{2}\left(13Aa^5-5Ba^5\right)}{a^8}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{\sqrt{2}\left(43A\sqrt{a}-3B\sqrt{a}\right)\log\left(\cdots\right.}{\cdots}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] -1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(
1/2*d*x + 1/2*c)^2/a^8 + sqrt(2)*(13*A*a^5 - 5*B*a^5)/a^8)*tan(1/2*d*x + 1/
2*c) - sqrt(2)*(43*A*sqrt(a) - 3*B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/2*
c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^3 - 64*A*log(abs((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) +
3)))/a^(5/2) + 64*A*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(5/2))/d
```

maple [B] time = 1.58, size = 445, normalized size = 2.71

$$-\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(43A\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-3B\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A*ln(
2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2
)*cos(1/2*d*x+1/2*c)^4*a-3*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
```

```
+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-32*A*ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^4*a-32*A*ln(-4*(a*2^(1/2)
*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*
cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a+11*A*a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-3*B*a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+2*A*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^(1/2)-2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/
sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.121 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=207

$$-\frac{(5A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(115A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(35A-11B)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(15A}{16ad(a}$$

```
[Out] -(5*A-2*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d+1/3
2*(115*A-43*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2
))/a^(5/2)/d*2^(1/2)-1/4*(A-B)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(15
*A-7*B)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+1/16*(35*A-11*B)*tan(d*x+c)/a
^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2978, 2984, 2985, 2649, 206, 2773}

$$\frac{(35A-11B)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(5A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(115A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(15A-}{16ad(a\,c}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] -(((5*A - 2*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a
^(5/2)*d)) + ((115*A - 43*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Tan[c + d*x])/(4*d*
(a + a*Cos[c + d*x])^(5/2)) - ((15*A - 7*B)*Tan[c + d*x])/(16*a*d*(a + a*Co
s[c + d*x])^(3/2)) + ((35*A - 11*B)*Tan[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[
c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(a(5A-B)-\frac{5}{2}a(A-B)\cos(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{1}{2}a^2(35A-11\right.}{}}{}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(35A-11B)}{16a^2d\sqrt{a+a}}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(35A-11B)}{16a^2d\sqrt{a+a}}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(15A-7B)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{(35A-11B)}{16a^2d\sqrt{a+a}}$$

$$= -\frac{(5A-2B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{(115A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c-}{\sqrt{2}\sqrt{a+a\cos}}\right.}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [A] time = 3.47, size = 142, normalized size = 0.69

$$\frac{\tan(c+dx)(10(11A-3B)\cos(c+dx)+(35A-11B)\cos(2(c+dx))+67A-11B)+8(115A-43B)\cos^5\left(\frac{1}{2}(c+\right.}{32d(a(\cos(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (8*(115*A - 43*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 - 128*Sqrt[2
]*(5*A - 2*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + (67*A
- 11*B + 10*(11*A - 3*B)*Cos[c + d*x] + (35*A - 11*B)*Cos[2*(c + d*x)])*Tan
[c + d*x])/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [B] time = 1.48, size = 404, normalized size = 1.95

$$-\frac{\sqrt{2}\left((115\,A-43\,B)\cos\left(dx+c\right)^4+3\,(115\,A-43\,B)\cos\left(dx+c\right)^3+3\,(115\,A-43\,B)\cos\left(dx+c\right)^2+(115\,A-43\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/64*(sqrt(2)*((115*A - 43*B)*cos(d*x + c)^4 + 3*(115*A - 43*B)*cos(d*x +
c)^3 + 3*(115*A - 43*B)*cos(d*x + c)^2 + (115*A - 43*B)*cos(d*x + c))*sqrt(
a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(
d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) +
 16*((5*A - 2*B)*cos(d*x + c)^4 + 3*(5*A - 2*B)*cos(d*x + c)^3 + 3*(5*A - 2
*B)*cos(d*x + c)^2 + (5*A - 2*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^
3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((35*A - 11*
B)*cos(d*x + c)^2 + 5*(11*A - 3*B)*cos(d*x + c) + 16*A)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*
d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

giac [B] time = 3.95, size = 409, normalized size = 1.98

$$2\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\,\sqrt{2}\left(Aa^{5}-Ba^{5}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{8}}+\frac{\sqrt{2}\left(21\,Aa^{5}-13\,Ba^{5}\right)}{a^{8}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{\sqrt{2}\left(115\,A\sqrt{a}-43\,B\sqrt{a}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1
/2*d*x + 1/2*c)^2/a^8 + sqrt(2)*(21*A*a^5 - 13*B*a^5)/a^8)*tan(1/2*d*x + 1/
2*c) - sqrt(2)*(115*A*sqrt(a) - 43*B*sqrt(a))*log((sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^3 - 32*(5*A*sqrt(a) - 2*B*s
qrt(a))*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^3 + 32*(5*A*sqrt(a) - 2*B*sqrt(a))*log(a
bs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 +
a*(2*sqrt(2) - 3)))/a^3 + 128*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(a) - A*a^(3/2))/(((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*a^2))/d
```

maple [B] time = 1.71, size = 1122, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/16*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(230*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-86*B
*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^
(1/2)*cos(1/2*d*x+1/2*c)^6*a-160*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*
a))*cos(1/2*d*x+1/2*c)^6*a-160*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)
))*cos(1/2*d*x+1/2*c)^6*a+64*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
cos(1/2*d*x+1/2*c)^6*a+64*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*co
s(1/2*d*x+1/2*c)^6*a-115*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2
*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a+43*B*ln(2*(2*a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*
x+1/2*c)^4*a+70*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*
x+1/2*c)^4+80*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2
*c)^4*a+80*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c
)^4*a-22*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c
)^4-32*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^4*a
-32*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a-1
5*A*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+7*B
*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-2*A*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*B*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^(1/2))/a^(7/2)/cos(1/2*d*x+1/2*c)^3/(2*cos(1/2*d*x+1/2*c)+2^
(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2
*c)^2)^(1/2)/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^2\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2)),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+a*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.122 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=264

$$\frac{(39A-20B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(31A$$

```
[Out] 1/4*(39*A-20*B)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/
d-1/32*(219*A-115*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c)
)^(1/2))/a^(5/2)/d*2^(1/2)-1/4*(A-B)*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c
))^(5/2)-1/16*(19*A-11*B)*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)-
7/16*(9*A-5*B)*tan(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)+1/16*(31*A-15*B)*sec
(d*x+c)*tan(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.92, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2978, 2984, 2985, 2649, 206, 2773}

$$-\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(39A-20B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(31$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((39*A - 20*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4
*a^(5/2)*d) - ((219*A - 115*B)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (7*(9*A - 5*B)*Tan[c + d*x
])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]*Tan[c + d*x]
)/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((19*A - 11*B)*Sec[c + d*x]*Tan[c + d*
x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((31*A - 15*B)*Sec[c + d*x]*Tan[c
 + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(2a(3A-B)-\frac{7}{2}a(A-B)\cos(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-11B)\sec(c+dx)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-11B)\sec(c+dx)\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-}{16}$$

$$= -\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-}{16}$$

$$= -\frac{7(9A-5B)\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec(c+dx)\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(19A-}{16}$$

$$= \frac{(39A-20B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{5/2}d} - \frac{(219A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{2}\sqrt{a+a}}\right.}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [A] time = 6.20, size = 178, normalized size = 0.67

$$\frac{-8(219A-115B)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 32\sqrt{2}(39A-20B)\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sqrt{2}\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + a*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (-8*(219*A - 115*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + 32*Sqrt[
2]*(39*A - 20*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + (-1
58*A + 110*B + (-269*A + 169*B)*Cos[c + d*x] + (-190*A + 110*B)*Cos[2*(c +
d*x)] - 63*A*Cos[3*(c + d*x)] + 35*B*Cos[3*(c + d*x)])*Sec[c + d*x]^2*Tan[(
c + d*x)/2])/(64*a*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 0.66, size = 428, normalized size = 1.62

$$-\frac{\sqrt{2}\left((219\,A-115\,B)\cos\left(dx+c\right)^{5}+3\,(219\,A-115\,B)\cos\left(dx+c\right)^{4}+3\,(219\,A-115\,B)\cos\left(dx+c\right)^{3}+(219\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/64*(sqrt(2)*((219*A - 115*B)*cos(d*x + c)^5 + 3*(219*A - 115*B)*cos(d*x
+ c)^4 + 3*(219*A - 115*B)*cos(d*x + c)^3 + (219*A - 115*B)*cos(d*x + c)^2)
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1)) + 4*((39*A - 20*B)*cos(d*x + c)^5 + 3*(39*A - 20*B)*cos(d*x + c)^4 + 3
*(39*A - 20*B)*cos(d*x + c)^3 + (39*A - 20*B)*cos(d*x + c)^2)*sqrt(a)*log((
a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) +
 4*(7*(9*A - 5*B)*cos(d*x + c)^3 + 5*(19*A - 11*B)*cos(d*x + c)^2 + 4*(5*A
- 4*B)*cos(d*x + c) - 8*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*co
s(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(
d*x + c)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assum
es constant sign by intervals (correct if the argument is real):Check [abs(
cos((d*t_nostep+c)/2))]Discontinuities at zeroes of cos((d*t_nostep+c)/2) w
ere not checkedEvaluation time: 1.15Unable to divide, perhaps due to roundi
ng error%%%{%%{[%%%{%%{[66353586105696382734593O584064,0]:[1,0,-2]%%},[16]%
```

```
%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0]%%%} / %%%{%%%{%%{[9903520314283042199192
993792,0]:[1,0,-2]%%},[16]%%%},[0]%%%} Error: Bad Argument Value
```

maple [B] time = 1.99, size = 1610, normalized size = 6.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/8*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(320*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1
/2*c)+2*a))*cos(1/2*d*x+1/2*c)^8*a-624*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a))*cos(1/2*d*x+1/2*c)^8*a+320*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2
^(1/2)))*cos(1/2*d*x+1/2*c)^8*a-624*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^
(1/2)))*cos(1/2*d*x+1/2*c)^8*a+624*A*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(
1/2)))*cos(1/2*d*x+1/2*c)^6*a+624*A*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2
*a))*cos(1/2*d*x+1/2*c)^6*a-320*B*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2
)))*cos(1/2*d*x+1/2*c)^6*a-320*B*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*cos(1/2*d*x+1/2*c)^6*a-2*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
-876*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*
c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a+460*B*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^6*a-188*A*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-156*A*l
n(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*
a^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^4*a-156*A*ln(
-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(
1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a+80*B*ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1
/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^4*a+80*B*ln(-4*(a
*2^(1/2)*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-
2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a+19*A*a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2-11*B*a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)^2+100*B*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+2*A*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+219*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^
2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^4*a-115*B*ln(2
*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)
```

```
*cos(1/2*d*x+1/2*c)^4*a+252*A*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)*cos(1/2*d*x+1/2*c)^6-140*B*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)
*cos(1/2*d*x+1/2*c)^6+876*A*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*d*x+1/2*c)^8*a-460*B*ln(2*(2*a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*2^(1/2)*cos(1/2*
d*x+1/2*c)^8*a)/a^(7/2)/cos(1/2*d*x+1/2*c)^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))
^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c
)^2)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.123 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=159

$$\frac{10a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(A+B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a(9A+7B)\mathrm{si}}{}$$

```
[Out] 2/15*a*(9*A+7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/d+10/21*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*a*(9*A+7*B)*c
os(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a*(A+B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*
a*B*cos(d*x+c)^(7/2)*sin(d*x+c)/d+10/21*a*(A+B)*sin(d*x+c)*cos(d*x+c)^(1/2)
/d
```

Rubi [A] time = 0.20, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3023, 2748, 2635, 2639, 2641}

$$\frac{10a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a(A+B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a(9A+7B)\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a*(9*A + 7*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*a*(A + B)*Elliptic
F[(c + d*x)/2, 2])/(21*d) + (10*a*(A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(21*d) + (2*a*(9*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*a*(A
 + B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a*B*Cos[c + d*x]^(7/2)*Si
n[c + d*x])/(9*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^{\frac{5}{2}}(c+dx)\left(aA+(aA+aB)\cos(c+dx)+aB\,\mathrm{co}\right.$$

$$= \frac{2aB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2}{9}\int \cos^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}\right.$$

$$= \frac{2aB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + (a(A+B))\int \cos^{\frac{7}{2}}(c$$

$$= \frac{2a(9A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{2a(A+B)\,\mathrm{c}}{}$$

$$= \frac{2a(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10a(A+B)\sqrt{\cos(c+}}{21d}$$

$$= \frac{2a(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10a(A+B)F\left(\frac{1}{2}(c+\right.}{21d}$$

Mathematica [C] time = 6.35, size = 914, normalized size = 5.75

$$a\left(\sqrt{\cos(c+dx)}\,(\cos(c+dx)+1)\left(-\frac{(9A+7B)\cot(c)}{15d}+\frac{23(A+B)\cos(dx)\sin(c)}{84d}+\frac{(18A+19B)\cos(2dx)\sin(2c}{180d}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-1/15*((9*A
 + 7*B)*Cot[c])/d + (23*(A + B)*Cos[d*x]*Sin[c])/(84*d) + ((18*A + 19*B)*Cos
[2*d*x]*Sin[2*c])/(180*d) + ((A + B)*Cos[3*d*x]*Sin[3*c])/(28*d) + (B*Cos[4
*d*x]*Sin[4*c])/(72*d) + (23*(A + B)*Cos[c]*Sin[d*x])/(84*d) + ((18*A + 19*
B)*Cos[2*c]*Sin[2*d*x])/(180*d) + ((A + B)*Cos[3*c]*Sin[3*d*x])/(28*d) + (B
*Cos[4*c]*Sin[4*d*x])/(72*d)) - (5*A*(1 + Cos[c + d*x])*Csc[c]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*
Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[C
```

```
ot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (5*B*(1 + Cos[c + d*x])*Csc[c]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)
/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(
Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Ar
cTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*
Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Arc
Tan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTa
n[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan
[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sq
rt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (7*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 +
 (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(30*d))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba\cos(dx+c)^4+(A+B)a\cos(dx+c)^3+Aa\cos(dx+c)^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^4 + (A + B)*a*cos(d*x + c)^3 + A*a*cos(d*x + c)^
2)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

maple [B] time = 1.07, size = 411, normalized size = 2.58

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(-1120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720A+2960B)\left(\sin^8\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A+2960*B)*sin(1/2*d*x+1/2*c)^8
*cos(1/2*d*x+1/2*c)+(-1584*A-3152*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c
)+(1344*A+1792*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-366*A-408*B)*si
n(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*A*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.09, size = 177, normalized size = 1.11

$$-\frac{2\,A\,a\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\;{}_2F_1\left(\frac{1}{2},\frac{7}{4};\;\frac{11}{4};\;\cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}-\frac{2\,A\,a\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\;{}_2F_1\left(\frac{1}{2},\frac{9}{4};\;\frac{13}{4}\right.}{9\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] - (2*A*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
+ d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a*cos(c + d*x)^(9/2)*sin(c +
 d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1
/2)) - (2*B*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, c
os(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a*cos(c + d*x)^(11/2)*s
in(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*
x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.124 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=132

$$\frac{2a(7A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a(7A+5B)\sin$$

```
[Out] 6/5*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(7*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*(A+B)*cos(d*x+
c)^(3/2)*sin(d*x+c)/d+2/7*a*B*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/21*a*(7*A+5*B
)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.18, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a(7A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a(7A+5B)\sin$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*a*(A + B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(7*A + 5*B)*EllipticF[
(c + d*x)/2, 2])/(21*d) + (2*a*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(21*d) + (2*a*(A + B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*B*Cos[
c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^{\frac{3}{2}}(c+dx)\left(aA+(aA+aB)\cos(c+dx)+aB\right.$$

$$= \frac{2aB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \cos^{\frac{3}{2}}(c+dx)\left(\right.$$

$$= \frac{2aB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + (a(A+B))\int \cos^{\frac{5}{2}}$$

$$= \frac{2a(7A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2a(A+B}{}$$

$$= \frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(7A+5B)F\left(\frac{1}{2}(c+\right.}{21d}$$

Mathematica [C] time = 6.27, size = 872, normalized size = 6.61

$$a\left(\sqrt{\cos(c+dx)}\,(\cos(c+dx)+1)\left(-\frac{3(A+B)\cot(c)}{5d}+\frac{(28A+23B)\cos(dx)\sin(c)}{84d}+\frac{(A+B)\cos(2dx)\sin(2c)}{10d}+\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((-3*(A + B)*
Cot[c])/(5*d) + ((28*A + 23*B)*Cos[d*x]*Sin[c])/(84*d) + ((A + B)*Cos[2*d*x
]*Sin[2*c])/(10*d) + (B*Cos[3*d*x]*Sin[3*c])/(28*d) + ((28*A + 23*B)*Cos[c]
*Sin[d*x])/(84*d) + ((A + B)*Cos[2*c]*Sin[2*d*x])/(10*d) + (B*Cos[3*c]*Sin[
3*d*x])/(28*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*S
qrt[1 + Cot[c]^2]) - (5*B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
21*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2
]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqr
t[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])
/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]))/(10*d) - (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hyp
ergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan
[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^
2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/
(10*d))
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba\cos\left(dx+c\right)^{3}+(A+B)a\cos\left(dx+c\right)^{2}+Aa\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^3 + (A + B)*a*cos(d*x + c)^2 + A*a*cos(d*x + c))
*sqrt(cos(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [B] time = 0.98, size = 383, normalized size = 2.90

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-528B)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-528*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(308*A+448*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-11
2*A-122*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.61, size = 166, normalized size = 1.26

$$\frac{2\,A\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}-\frac{2\,A\,a\,\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right.\right.}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] (2*A*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) - (2*A*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos
(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a*cos(c + d*x)^(7/2)*sin(
c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)
^(1/2)) - (2*B*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4
, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.125 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aB\sin(c+dx)}{5}$$

```
[Out] 2/5*a*(5*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*B*cos(d*x+c)^(3
/2)*sin(d*x+c)/d+2/3*a*(A+B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aB\sin(c+dx)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*a*(5*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF[
(c + d*x)/2, 2])/(3*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d
) + (2*a*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \sqrt{\cos(c+dx)}\,\Big(aA+(aA+aB)\cos(c+dx)+aB$$

$$= \frac{2aB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\cos(c+dx)}\,\big($$

$$= \frac{2aB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + (a(A+B))\int \cos^{\frac{3}{2}}($$

$$= \frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)\sqrt{\cos(c+}}{3d}$$

$$= \frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)F\left(\frac{1}{2}(c+\right.}{3d}$$

Mathematica [C] time = 6.26, size = 830, normalized size = 8.22

$$a\left(\sqrt{\cos(c+dx)}\,(\cos(c+dx)+1)\left(-\frac{(5A+3B)\cot(c)}{5d}+\frac{(A+B)\cos(dx)\sin(c)}{3d}+\frac{B\cos(2dx)\sin(2c)}{10d}+\frac{(A+B)}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-1/5*((5*A +
 3*B)*Cot[c])/d + ((A + B)*Cos[d*x]*Sin[c])/(3*d) + (B*Cos[2*d*x]*Sin[2*c])
/(10*d) + ((A + B)*Cos[c]*Sin[d*x])/(3*d) + (B*Cos[2*c]*Sin[2*d*x])/(10*d))
 - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^
2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[
c]^2]) - (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[
c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2
*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^
2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (3*
B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2,
 -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]
*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^
2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba\cos\left(dx+c\right)^{2}+(A+B)a\cos\left(dx+c\right)+Aa\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)*sqrt(cos(d*x +
 c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

maple [B] time = 1.02, size = 355, normalized size = 3.51

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+44B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+44*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-10*A-16*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.52, size = 128, normalized size = 1.27

$$\frac{2\,A\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x)),x)
```

```
[Out] (2*A*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) + (2*B*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2))
)/(3*d) + (2*A*a*ellipticE(c/2 + (d*x)/2, 2))/d - (2*B*a*cos(c + d*x)^(7/2)
*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*
x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.126 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=70

$$\frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+2/3*a*(3*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*B*sin(d*x+c)*cos(d*
x+c)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3023, 2748, 2641, 2639}

$$\frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + B)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx &= \int \frac{aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2aB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{1}{2}a(3A+B)+\frac{3}{2}a(A+B)\,\cdots}{\sqrt{\cos(c+dx)}}\\ &= \frac{2aB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+(a(A+B))\int \sqrt{\cos(c+dx)}\,dx\,\cdots\\ &= \frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\cdots}{\cdots}\end{aligned}$$

Mathematica [C] time = 6.27, size = 784, normalized size = 11.20

$$a\left(-\frac{A\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c\cdots\right.\right.\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+\cdots}}}\cdots\right)}{2d}\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-(((A + B)*C
ot[c])/d) + (B*Cos[d*x]*Sin[c])/(3*d) + (B*Cos[c]*Sin[d*x])/(3*d)) - (A*(1
+ Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) - (B*(1
 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcT
an[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (A
*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2,
-1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c]
)/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*
Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2
]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Co
s[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (B*(1 + Cos[c
+ d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba\cos(dx+c)^2 + (A+B)a\cos(dx+c) + Aa}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)/sqrt(cos(d*x +
 c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c) + A)(a\cos(dx+c) + a)}{\sqrt{\cos(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

maple [B] time = 1.24, size = 321, normalized size = 4.59

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*B*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.53, size = 79, normalized size = 1.13

$$\frac{2\,B\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*B*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) + (2*B*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*a*(ellipticE(c/2 + (d*x)/
2, 2) + ellipticF(c/2 + (d*x)/2, 2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.127 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=66

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*a*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*A*sin(d*x+c)/d/cos(d*x+c
)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3021, 2748, 2641, 2639}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + B)*EllipticF[(c + d*
x)/2, 2])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}a(A+B)-\frac{1}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - (a(A-B))\int \sqrt{\cos(c+dx)}\,dx + (a(A+B))\int \\
&= -\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aA}{d\sqrt{c}}
\end{aligned}$$

Mathematica [C] time = 6.31, size = 783, normalized size = 11.86

$$a\left(\frac{A\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\,c}}\right.}{2d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-1/2*((-2*A
+ B + B*Cos[2*c])*Csc[c]*Sec[c])/d + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d) -
(A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) -
 (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
- ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2])
+ (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]
]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[
c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]
^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) - (B*(1 + Co
s[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[
1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba\cos\left(dx+c\right)^{2}+(A+B)a\cos\left(dx+c\right)+Aa}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)/cos(d*x + c)^(
3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

maple [B] time = 1.09, size = 240, normalized size = 3.64

$$-\frac{2a\left(A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2*a*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

mupad [B] time = 0.96, size = 90, normalized size = 1.36

$$\frac{2\,A\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}+\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*A*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a*(ellipticE(c/2 + (d*x)/2, 2)
 + ellipticF(c/2 + (d*x)/2, 2)))/d + (2*A*a*sin(c + d*x)*hypergeom([-1/4, 1
/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.128 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] -2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+2/3*a*(A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*A*sin(d*x+c)/d/cos
(d*x+c)^(3/2)+2*a*(A+B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(A
 + B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{3}{2}a(A+B)+\frac{1}{2}a(A+3B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{3}(a(A+3B)$$

$$= \frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(A+B)\sin(}{d\sqrt{\cos(c+}}$$

$$= -\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a}{3d}$$

Mathematica [C] time = 6.36, size = 813, normalized size = 8.56

$$a\left(\sqrt{\cos(c+dx)}\,(\cos(c+dx)+1)\left(\frac{A\sec(c)\sin(dx)\sec^2(c+dx)}{3d} + \frac{\sec(c)(A\sin(c)+3A\sin(dx)+3B\sin(dx))\,\mathrm{se}}{3d}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((A + B)*Csc
[c]*Sec[c])/d + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (Sec[c]*Sec[c +
d*x]*(A*Sin[c] + 3*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d)) - (A*(1 + Cos[c + d*x
])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (B*(1 + Cos[c +
d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]
^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTa
n[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sq
```

```
rt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) + (A*(1 + Cos[c +
 d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) + (B*(1 + Cos[c + d*x])*Csc[c
]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba\cos(dx+c)^2 + (A+B)a\cos(dx+c) + Aa}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)/cos(d*x + c)^(
5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c) + A)(a\cos(dx+c) + a)}{\cos(dx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

maple [B] time = 2.33, size = 426, normalized size = 4.48

$$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(\frac{B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}+\frac{\left(\frac{A}{2}+\frac{B}{2}\right)\left(\cdots\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(
1/2*A+1/2*B)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1)+1/2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.30, size = 150, normalized size = 1.58

$$\frac{2\,B\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,A\,a\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+\cdots\right.}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*B*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a*sin(c + d*x)*hypergeom([-1/4
, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
 + (2*A*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*co
s(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a*sin(c + d*x)*hypergeom([-
1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.129 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=132

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(3A+5B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a}{5d}$$

```
[Out] -2/5*a*(3*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*A*sin(d*x+c)/d
/cos(d*x+c)^(5/2)+2/3*a*(A+B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*a*(3*A+5*B)
*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(3A+5B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*a*(3*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF
[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(3*A + 5*B)*Sin[c
 + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{aA+(aA+aB)\cos(c+dx)+aB\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{5}{2}a(A+B)+\frac{1}{2}a(3A+5B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,d$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + \frac{1}{5}(a(3A+5$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(3A+5B)\sin(c}{5d\sqrt{\cos(c+dx}}$$

$$= -\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + $$

Mathematica [C] time = 6.43, size = 865, normalized size = 6.55

$$a\left(\sqrt{\cos(c+dx)}\,(\cos(c+dx)+1)\left(\frac{A\sec(c)\sin(dx)\sec^3(c+dx)}{5d}+\frac{\sec(c)(3A\sin(c)+5A\sin(dx)+5B\sin(dx))\,\mathrm{s}}{15d}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((3*A + 5*B)
*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*
Sec[c + d*x]^2*(3*A*Sin[c] + 5*A*Sin[d*x] + 5*B*Sin[d*x]))/(15*d) + (Sec[c]
*Sec[c + d*x]*(5*A*Sin[c] + 5*B*Sin[c] + 9*A*Sin[d*x] + 15*B*Sin[d*x]))/(15
*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot
[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 +
Cot[c]^2]) + (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + S
in[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d
) + (B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{
-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*
Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[
c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Ta
n[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[
c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba\cos(dx+c)^2+(A+B)a\cos(dx+c)+Aa}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)/cos(d*x + c)^(
7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

maple [B] time = 3.03, size = 661, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/10*A/(8*
sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(
1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)+1/2*B*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)
```

```
^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(1/2*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^
2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

mupad [B] time = 1.61, size = 177, normalized size = 1.34

$$\frac{2\,A\,a\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\,\frac{1}{4};\,\cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,A\,a\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos(c+d\,x)^2\right)}{5\,d\cos(c+d\,x)^{5/2}\,\sqrt{\sin(c+d\,x)^2}}+\frac{2\,B\,a\,\sin(\cdots}{d\,\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*A*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c
 + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*a*sin(c + d*x)*hypergeom([-5/4
, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/
2)) + (2*B*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*c
os(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a*sin(c + d*x)*hypergeom([
-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^
(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.130 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=194

$$\frac{4a^2(6A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(9A+8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(9A+11B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}+\frac{4a^2(9$$

```
[Out] 4/15*a^2*(9*A+8*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^2*(6*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/45*a^2*(9
*A+8*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/63*a^2*(9*A+11*B)*cos(d*x+c)^(5/2)*
sin(d*x+c)/d+2/9*B*cos(d*x+c)^(5/2)*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d+4/21*
a^2*(6*A+5*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.32, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^2(6A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(9A+8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(9A+11B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}+\frac{4a^2(9$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*a^2*(9*A + 8*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(6*A + 5*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(6*A + 5*B)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(21*d) + (4*a^2*(9*A + 8*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4
5*d) + (2*a^2*(9*A + 11*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*B*C
os[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2B\cos^{\frac{5}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{9d}$$

$$= \frac{2B\cos^{\frac{5}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{9d}$$

$$= \frac{2a^2(9A+11B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2B\cos}{}$$

$$= \frac{2a^2(9A+11B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2B\cos}{}$$

$$= \frac{4a^2(6A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{4a^2(9A}{}$$

$$= \frac{4a^2(9A+8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^2(6A+5B)F\left(\right.}{21d}$$

Mathematica [C] time = 6.30, size = 944, normalized size = 4.87

$$\sqrt{\cos(c+dx)}\,(\cos(c+dx)a+a)^2\left(-\frac{(9A+8B)\cot(c)}{15d} + \frac{(51A+46B)\cos(dx)\sin(c)}{168d} + \frac{(36A+37B)\cos(2dx)\sin(}{360d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/15*((9*A
 + 8*B)*Cot[c])/d + ((51*A + 46*B)*Cos[d*x]*Sin[c])/(168*d) + ((36*A + 37*B
)*Cos[2*d*x]*Sin[2*c])/(360*d) + ((A + 2*B)*Cos[3*d*x]*Sin[3*c])/(56*d) + (
B*Cos[4*d*x]*Sin[4*c])/(144*d) + ((51*A + 46*B)*Cos[c]*Sin[d*x])/(168*d) +
((36*A + 37*B)*Cos[2*c]*Sin[2*d*x])/(360*d) + ((A + 2*B)*Cos[3*c]*Sin[3*d*x
])/(56*d) + (B*Cos[4*c]*Sin[4*d*x])/(144*d)) - (2*A*(a + a*Cos[c + d*x])^2*
Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Se
c[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[
c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 +
 Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c +
 d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c
```

```
]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*A*(a +
 a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -
1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])
/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]
) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) - (4*B*(a + a*Co
s[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4},
 {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqr
t[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - (
(Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

integral $\left(\left(Ba^2\cos(dx+c)^4 + (A+2B)a^2\cos(dx+c)^3 + (2A+B)a^2\cos(dx+c)^2 + Aa^2\cos(dx+c)\right)\sqrt{\cos(dx}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^4 + (A + 2*B)*a^2*cos(d*x + c)^3 + (2*A + B)*a
^2*cos(d*x + c)^2 + A*a^2*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

maple [A] time = 1.10, size = 413, normalized size = 2.13

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-560B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(360A+1840B)\left(\sin^8\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(360*A+1840*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-1044*A-2368*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1134*A+1568*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-351*A-387*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+90*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-168*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

mupad [B] time = 1.07, size = 266, normalized size = 1.37

$$\frac{2\,A\,a^2\left(\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}-\frac{4\,A\,a^2\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\,\frac{11}{4};\,\cos(c+\right.}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(
3*d) - (4*A*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4,
 cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a^2*cos(c + d*x)^(9/2
)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d
*x)^2)^(1/2)) - (2*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/
4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*B*a^2*cos(c +
d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(
sin(c + d*x)^2)^(1/2)) - (2*B*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeo
m([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.131 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=161

$$\frac{4a^2(7A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(7A+9B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d}+\frac{4a^2(7A$$

```
[Out] 4/5*a^2*(4*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^2*(7*A+6*B)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/35*a^2*(7*
A+9*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*B*cos(d*x+c)^(3/2)*(a^2+a^2*cos(d*
x+c))*sin(d*x+c)/d+4/21*a^2*(7*A+6*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.29, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^2(7A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(7A+9B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d}+\frac{4a^2(7A$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*a^2*(4*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(7*A + 6*B)*El
lipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(7*A + 6*B)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(21*d) + (2*a^2*(7*A + 9*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35
*d) + (2*B*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2B\cos^{\frac{3}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d}$$

$$= \frac{2B\cos^{\frac{3}{2}}(c+dx)\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d}$$

$$= \frac{2a^2(7A+9B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2B\cos}{}$$

$$= \frac{2a^2(7A+9B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2B\cos}{}$$

$$= \frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(7A+6B)\sqrt{}}{}$$

$$= \frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(7A+6B)F}{21}$$

Mathematica [C] time = 6.27, size = 898, normalized size = 5.58

$$\sqrt{\cos(c+dx)}\,(\cos(c+dx)a+a)^2\left(-\frac{(4A+3B)\cot(c)}{5d} + \frac{(56A+51B)\cos(dx)\sin(c)}{168d} + \frac{(A+2B)\cos(2dx)\sin(2c)}{20d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/5*((4*A
+ 3*B)*Cot[c])/d + ((56*A + 51*B)*Cos[d*x]*Sin[c])/(168*d) + ((A + 2*B)*Cos
[2*d*x]*Sin[2*c])/(20*d) + (B*Cos[3*d*x]*Sin[3*c])/(56*d) + ((56*A + 51*B)*
Cos[c]*Sin[d*x])/(168*d) + ((A + 2*B)*Cos[2*c]*Sin[2*d*x])/(20*d) + (B*Cos[
3*c]*Sin[3*d*x])/(56*d)) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec
[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[
c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)
/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(
```

```
Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Ar
cTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (2*A*(a + a*Cos[c + d*x])^2*Csc[
c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x +
ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d) - (3*B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec
[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan
[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[T
an[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]
]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]]))/(10*d)
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^2\cos(dx+c)^3+(A+2B)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2\right)\sqrt{\cos(dx+c)}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

maple [A] time = 1.11, size = 385, normalized size = 2.39

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-84A-348B)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-348*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(224*A+378*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-9
1*A-117*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-84*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+30*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

mupad [B] time = 1.01, size = 231, normalized size = 1.43

$$\frac{2\,A\,a^2\,\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,B\,a^2\,\left(\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a^2\,\mathrm{E}\left(\frac{c}{2}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/
2, 2))/3))/d + (2*B*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 +
(d*x)/2, 2)))/(3*d) + (2*A*a^2*ellipticE(c/2 + (d*x)/2, 2))/d - (2*A*a^2*co
s(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/
(7*d*(sin(c + d*x)^2)^(1/2)) - (4*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hyp
ergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2
```

```
*B*a^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c +
d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.132 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=126

$$\frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2B\sin(}{}$$

```
[Out] 4/5*a^2*(5*A+4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/15*a^2*(5*A+7
*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+2/5*B*(a^2+a^2*cos(d*x+c))*sin(d*x+c)*cos
(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.27, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2B\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (4*a^2*(5*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(2*A + B)*Elli
pticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(15*d) + (2*B*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d
*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2B\sqrt{\cos(c+dx)}\,\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{(a+$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{\frac{1}{2}a^2(}{}$$

$$= \frac{2a^2(5A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2B\sqrt{\cos(c+dx)}\,\left(a^2\right.}{}$$

$$= \frac{2a^2(5A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2B\sqrt{\cos(c+dx)}\,\left(a^2\right.}{}$$

$$= \frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Mathematica [C] time = 6.30, size = 852, normalized size = 6.76

$$\sqrt{\cos(c+dx)}\,(\cos(c+dx)a+a)^2\left(-\frac{(5A+4B)\cot(c)}{5d} + \frac{(A+2B)\cos(dx)\sin(c)}{6d} + \frac{B\cos(2dx)\sin(2c)}{20d} + \frac{(A+2B)}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/5*((5*A
+ 4*B)*Cot[c])/d + ((A + 2*B)*Cos[d*x]*Sin[c])/(6*d) + (B*Cos[2*d*x]*Sin[2*
c])/(20*d) + ((A + 2*B)*Cos[c]*Sin[d*x])/(6*d) + (B*Cos[2*c]*Sin[2*d*x])/(2
0*d)) - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[
1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*
d*Sqrt[1 + Cot[c]^2]) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]
^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Si
```

```
n[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt
[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/
(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]))/(2*d) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((H
ypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x
+ ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos
[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + T
an[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c
]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])
)/(5*d)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{Ba^2\cos(dx+c)^3+(A+2B)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

maple [B] time = 0.99, size = 357, normalized size = 2.83

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-12B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(10A+32B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(10*A+32*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-5*A-13*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-12*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

mupad [B] time = 1.00, size = 153, normalized size = 1.21

$$\frac{2\,B\,a^2\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,A\,a^2\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+6\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\right|\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*B*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/
2, 2))/3))/d + (2*A*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + 6*ellipticE(c/2
+ (d*x)/2, 2) + 4*ellipticF(c/2 + (d*x)/2, 2)))/(3*d) + (2*B*a^2*ellipticE(
c/2 + (d*x)/2, 2))/d - (2*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([
1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.133 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{4a^2(3A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}+\frac{4}{}$$

```
[Out] 4*a^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+4/3*a^2*(3*A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*A*(a^2+a^2*cos(d*x+c)
)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-2/3*a^2*(3*A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)
/d
```

Rubi [A] time = 0.27, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^2(3A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}+\frac{4}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*B*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(3*A + 2*B)*EllipticF[(c + d
*x)/2, 2])/(3*d) - (2*a^2*(3*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
+ (2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+a\cos(c+dx))}{}$$

$$= \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}a^2(3A+B)+\left(-\right.}{}$$

$$= -\frac{2a^2(3A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2A\left(a^2+a^2\cos(c+\right.}{d\sqrt{\cos(c}}$$

$$= -\frac{2a^2(3A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2A\left(a^2+a^2\cos(c+\right.}{d\sqrt{\cos(c}}$$

$$= \frac{4a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^2(3A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2a^2(3}{}$$

Mathematica [C] time = 6.37, size = 623, normalized size = 5.28

$$-\frac{A\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\mathrm{ta}\right.}}{d\sqrt{\cot^2(}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/4*((-A +
 2*B + A*Cos[2*c] + 2*B*Cos[2*c])*Csc[c]*Sec[c])/d + (B*Cos[d*x]*Sin[c])/(6
*d) + (B*Cos[c]*Sin[d*x])/(6*d) + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/(2*d)) -
 (A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^
2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1
+ Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hype
```

```
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
2*d)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^2\cos(dx+c)^3+(A+2B)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

maple [B] time = 1.01, size = 388, normalized size = 3.29

$$-\frac{4a^2\left(2B\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -4/3*a^2*(2*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(3*A+B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

mupad [B] time = 1.14, size = 134, normalized size = 1.14

$$\frac{2\,B\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+6\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,A\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\right|}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*B*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + 6*ellipticE(c/2 + (d*x)/2, 2) +
 4*ellipticF(c/2 + (d*x)/2, 2)))/(3*d) + (2*A*a^2*ellipticE(c/2 + (d*x)/2,
2))/d + (4*A*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^2*sin(c + d*x)*hyp
ergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*
x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
[Out] Timed out
```

3.134 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(5A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}-\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+d\right.}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] -4*a^2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+4/3*a^2*(2*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*A*(a^2+a^2*cos(d*x
+c))*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/3*a^2*(5*A+3*B)*sin(d*x+c)/d/cos(d*x+c
)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(5A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}-\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)\left(a^2\cos(c+d\right.}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^2*A*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(2*A + 3*B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a^2*(5*A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]
]) + (2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+a\cos(c+dx)}{}$$

$$= \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}a^2(5A+3B)+(}{}$$

$$= \frac{2a^2(5A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+a}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(5A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+a}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2}{}$$

Mathematica [C] time = 6.45, size = 624, normalized size = 5.20

$$\frac{A\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(dx+\tan^{-1}(\tan(c))\right)\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}}}\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(-1/4*((-4*A
 - B + B*Cos[2*c])*Csc[c]*Sec[c])/d + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(6
*d) + (Sec[c]*Sec[c + d*x]*(A*Sin[c] + 6*A*Sin[d*x] + 3*B*Sin[d*x]))/(6*d))
 - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + C
ot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[
```

```
1 + Cot[c]^2]) + (A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((Hy
pergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
 ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[
d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Ta
n[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]
^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))
/(2*d)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^2\cos(dx+c)^3 + (A+2B)a^2\cos(dx+c)^2 + (2A+B)a^2\cos(dx+c) + Aa^2}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x
)
```

maple [B] time = 1.23, size = 513, normalized size = 4.28

$$-\frac{4\left(6\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(2A+B)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -4/3*(6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A+B)*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(7*A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2)))*sin(1/2*d*x+1/2*c)^2+2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+3*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*a^2/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*
x+1/2*c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x
)
```

mupad [B] time = 1.69, size = 196, normalized size = 1.63

$$\frac{2\,A\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,B\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,A\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*A*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a^2*ellipticE(c/2 + (d*x)/2,
 2))/d + (4*B*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (4*A*a^2*sin(c + d*x)*hy
pergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d
*x)^2)^(1/2)) + (2*A*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d
*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a^2*sin(c +
d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(si
n(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
[Out] Timed out
```

3.135 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=159

$$\frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(4A+5B)\sin(c+}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -4/5*a^2*(4*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/15*a^2*(7*A+
5*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*A*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d/
cos(d*x+c)^(5/2)+4/5*a^2*(4*A+5*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(4A+5B)\sin(c+}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-4*a^2*(4*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 2*B)*Ell
ipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(7*A + 5*B)*Sin[c + d*x])/(15*d*Cos[
c + d*x]^(3/2)) + (4*a^2*(4*A + 5*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*A*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\cos(c+dx)}{}$$

$$= \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}a^2(7A+5B)+(}{}$$

$$= \frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+a}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+a}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2(7A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a}{}$$

$$= -\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Mathematica [C] time = 6.53, size = 883, normalized size = 5.55

$$\sqrt{\cos(c+dx)}\,(\cos(c+dx)a+a)^2\left(\frac{A\sec(c)\sin(dx)\sec^3(c+dx)}{10d} + \frac{\sec(c)(3A\sin(c)+10A\sin(dx)+5B\sin(dx))\,\mathrm{s}}{30d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(((4*A + 5*B
)*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(10*d) + (Sec[c
]*Sec[c + d*x]^2*(3*A*Sin[c] + 10*A*Sin[d*x] + 5*B*Sin[d*x]))/(30*d) + (Sec
[c]*Sec[c + d*x]*(10*A*Sin[c] + 5*B*Sin[c] + 24*A*Sin[d*x] + 30*B*Sin[d*x])
)/(30*d)) - (A*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
```

```
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqr
t[1 + Cot[c]^2]) - (2*B*(a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]
^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(3*d*Sqrt[1 + Cot[c]^2]) + (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d
*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]
^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*
Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c]
)/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
 Tan[c]^2]]))/(5*d) + (B*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)/2]^4
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(2*d)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^2\cos(dx+c)^3 + (A+2B)a^2\cos(dx+c)^2 + (2A+B)a^2\cos(dx+c) + Aa^2}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c) + A)(a\cos(dx+c) + a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x
)
```

maple [B] time = 3.34, size = 741, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/20*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c
)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/4*A+1/2*B)*(-(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(1/2*A+1/4*B)*(-1/6*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos
(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x
)
```

mupad [B] time = 1.97, size = 229, normalized size = 1.44

$$\frac{6\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos\left(c+d\,x\right)^2\right)+20\,A\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\,\frac{1}{4};\,\cos\left(c+d\right.\right.}{15\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{1-\cos\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(7/2),x)
```

```
[Out] (6*A*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*A*a
^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) +
30*A*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*
x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (2*B*a^2*elli
pticF(c/2 + (d*x)/2, 2))/d + (4*B*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3
/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a
^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d
*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.136 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(6A+7B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(9A+7B)\sin(c}{35d\cos^{\frac{5}{2}}(c+dx}$$

```
[Out] -4/5*a^2*(3*A+4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^2*(6*A+7*B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/35*a^2*(9
*A+7*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+4/21*a^2*(6*A+7*B)*sin(d*x+c)/d/cos(d
*x+c)^(3/2)+2/7*A*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(7/2)+4/5*a^
2*(3*A+4*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(6A+7B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(9A+7B)\sin(c}{35d\cos^{\frac{5}{2}}(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-4*a^2*(3*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(6*A + 7*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(9*A + 7*B)*Sin[c + d*x])/(35*d*C
os[c + d*x]^(5/2)) + (4*a^2*(6*A + 7*B)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3
/2)) + (4*a^2*(3*A + 4*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*A*(a^
2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\cos(c+dx)}{}$$

$$= \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}a^2(9A+7B)+\left(}{}$$

$$= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+a}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2A\left(a^2+a^2\cos(c+dx)\right)\sin(c+a}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2(6A+7B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2(3}{5}$$

$$= -\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right.}{21d}$$

Mathematica [C] time = 6.63, size = 925, normalized size = 4.77

$$\sqrt{\cos(c+dx)}\,(\cos(c+dx)a+a)^2\left(\frac{A\sec(c)\sin(dx)\sec^4(c+dx)}{14d} + \frac{\sec(c)(5A\sin(c)+14A\sin(dx)+7B\sin(dx))\,\mathrm{s}}{70d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(((3*A + 4*B
)*Csc[c]*Sec[c])/(5*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(14*d) + (Sec[c
]*Sec[c + d*x]^3*(5*A*Sin[c] + 14*A*Sin[d*x] + 7*B*Sin[d*x]))/(70*d) + (Sec
[c]*Sec[c + d*x]^2*(42*A*Sin[c] + 21*B*Sin[c] + 60*A*Sin[d*x] + 70*B*Sin[d*
x]))/(210*d) + (Sec[c]*Sec[c + d*x]*(30*A*Sin[c] + 35*B*Sin[c] + 63*A*Sin[d
*x] + 84*B*Sin[d*x]))/(105*d)) - (2*A*(a + a*Cos[c + d*x])^2*Csc[c]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/
2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(S
```

```
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^2*Csc[c]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
 (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (3*A*(a + a*Cos[c + d*x])^
2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) + (2*B*(a + a*Cos[c + d*x])^2*Csc
[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x +
 ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + A
rcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan
[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^2\cos(dx+c)^3 + (A+2B)a^2\cos(dx+c)^2 + (2A+B)a^2\cos(dx+c) + Aa^2}{\cos(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)/cos(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(9/2), x
)
```

maple [B] time = 4.04, size = 851, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/5*(1/2
*A+1/4*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2
*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*
c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/4*B*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1
/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(1/4*A+1/2*B)*(-1/6*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d
*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))+1/4*A*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^4-5/42*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos
(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/cos(d*x + c)^(9/2), x
)
```

mupad [B] time = 2.30, size = 235, normalized size = 1.21

$$\frac{30\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\,-\frac{3}{4};\,\cos\left(c+d\,x\right)^2\right)+84\,A\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos\left(c-\right.\right.}{105\,d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{1-\cos\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/cos(c + d*x)^(9/2),x)
```

```
[Out] (30*A*a^2*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 84*A*
a^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2)
+ 70*A*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c +
d*x)^2))/(105*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2)) + (6*B*a^2*s
in(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*B*a^2*cos(c +
 d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 30*B*a^2*c
os(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(15
*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.137 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=237

$$\frac{4a^3(121A+105B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(17A+15B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{20a^3(22A+21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+d}{693d}$$

```
[Out] 4/15*a^3*(17*A+15*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/231*a^3*(121*A+105*B)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/45
*a^3*(17*A+15*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+20/693*a^3*(22*A+21*B)*cos(d
*x+c)^(5/2)*sin(d*x+c)/d+2/11*a*B*cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2*sin(d
*x+c)/d+2/99*(11*A+15*B)*cos(d*x+c)^(5/2)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d
+4/231*a^3*(121*A+105*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.48, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2976, 2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{4a^3(121A+105B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(17A+15B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{20a^3(22A+21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+d}{693d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*a^3*(17*A + 15*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(121*A + 10
5*B)*EllipticF[(c + d*x)/2, 2])/(231*d) + (4*a^3*(121*A + 105*B)*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(231*d) + (4*a^3*(17*A + 15*B)*Cos[c + d*x]^(3/2)*Si
n[c + d*x])/(45*d) + (20*a^3*(22*A + 21*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(693*d) + (2*a*B*Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(
11*d) + (2*(11*A + 15*B)*Cos[c + d*x]^(5/2)*(a^3 + a^3*Cos[c + d*x])*Sin[c
+ d*x])/(99*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{2aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{11d}$$

$$= \frac{2aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{11d}$$

$$= \frac{2aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{11d}$$

$$= \frac{20a^3(22A+21B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d} + \frac{2aB}{}$$

$$= \frac{20a^3(22A+21B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d} + \frac{2aB}{}$$

$$= \frac{4a^3(121A+105B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{231d} + \frac{4a^3}{}$$

$$= \frac{4a^3(17A+15B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(121A+10}{}$$

Mathematica [C] time = 6.32, size = 990, normalized size = 4.18

$$\sqrt{\cos(c+dx)}\,(\cos(c+dx)a+a)^3\left(-\frac{(17A+15B)\cot(c)}{30d} + \frac{(2134A+1953B)\cos(dx)\sin(c)}{7392d} + \frac{(73A+75B)\cos(2d}{720d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/30*((17*
A + 15*B)*Cot[c])/d + ((2134*A + 1953*B)*Cos[d*x]*Sin[c])/(7392*d) + ((73*A
 + 75*B)*Cos[2*d*x]*Sin[2*c])/(720*d) + (3*(44*A + 63*B)*Cos[3*d*x]*Sin[3*c
])/(4928*d) + ((A + 3*B)*Cos[4*d*x]*Sin[4*c])/(288*d) + (B*Cos[5*d*x]*Sin[5
*c])/(704*d) + ((2134*A + 1953*B)*Cos[c]*Sin[d*x])/(7392*d) + ((73*A + 75*B
)*Cos[2*c]*Sin[2*d*x])/(720*d) + (3*(44*A + 63*B)*Cos[3*c]*Sin[3*d*x])/(492
8*d) + ((A + 3*B)*Cos[4*c]*Sin[4*d*x])/(288*d) + (B*Cos[5*c]*Sin[5*d*x])/(7
04*d)) - (11*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
```

```
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sq
rt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x
- ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c
]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]
)/(22*d*Sqrt[1 + Cot[c]^2]) - (17*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 +
 (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(60*d) - (B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/
2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*
Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt
[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sq
rt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
)/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2]]))/(4*d)
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^3\cos(dx+c)^5 + (A+3B)a^3\cos(dx+c)^4 + 3(A+B)a^3\cos(dx+c)^3 + (3A+B)a^3\cos(dx+c)^2 + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^5 + (A + 3*B)*a^3*cos(d*x + c)^4 + 3*(A + B)*a
^3*cos(d*x + c)^3 + (3*A + B)*a^3*cos(d*x + c)^2 + A*a^3*cos(d*x + c))*sqrt
(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x
)
```

maple [A] time = 1.11, size = 441, normalized size = 1.86

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(10080B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-6160A-43680B)\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(10080*
B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-6160*A-43680*B)*sin(1/2*d*x+1/
2*c)^10*cos(1/2*d*x+1/2*c)+(24200*A+77280*B)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d
*x+1/2*c)+(-37532*A-72240*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(29722
*A+39270*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-8118*A-8820*B)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1815*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3927*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+1575*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3465*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x
)
```

mupad [B] time = 1.31, size = 360, normalized size = 1.52

$$\frac{A\,a^3\left(\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}-\frac{6\,A\,a^3\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\;\frac{11}{4};\;\cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}-\frac{2\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (A*a^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d - (6*A*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4],
 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a^3*cos(c + d*x
)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin
(c + d*x)^2)^(1/2)) - (2*A*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([
1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3
*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2
))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*
hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2)) -
 (6*B*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos
(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(13/2)*
sin(c + d*x)*hypergeom([1/2, 13/4], 17/4, cos(c + d*x)^2))/(13*d*(sin(c + d
*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.138 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=204

$$\frac{4a^3(13A+11B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(21A+17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(24A+23B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d}+$$

```
[Out] 4/15*a^3*(21*A+17*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^3*(13*A+11*B)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/105*a
^3*(24*A+23*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/9*a*B*cos(d*x+c)^(3/2)*(a+a*
cos(d*x+c))^2*sin(d*x+c)/d+2/63*(9*A+13*B)*cos(d*x+c)^(3/2)*(a^3+a^3*cos(d*
x+c))*sin(d*x+c)/d+4/21*a^3*(13*A+11*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.45, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2976, 2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{4a^3(13A+11B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(21A+17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(24A+23B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*a^3*(21*A + 17*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(13*A + 11*
B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(13*A + 11*B)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(21*d) + (4*a^3*(24*A + 23*B)*Cos[c + d*x]^(3/2)*Sin[c +
d*x])/(105*d) + (2*a*B*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2*Sin[c + d*
x])/(9*d) + (2*(9*A + 13*B)*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x])*Sin
[c + d*x])/(63*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{2aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{9d}$$

$$= \frac{2aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{9d}$$

$$= \frac{2aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\sin(c+dx)}{9d}$$

$$= \frac{4a^3(24A+23B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2aB}{}$$

$$= \frac{4a^3(24A+23B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2aB}{}$$

$$= \frac{4a^3(21A+17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(13A+11}{}$$

$$= \frac{4a^3(21A+17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(13A+11}{}$$

Mathematica [C] time = 6.30, size = 944, normalized size = 4.63

$$\sqrt{\cos(c+dx)}\,(\cos(c+dx)a+a)^3\left(-\frac{(21A+17B)\cot(c)}{30d} + \frac{(107A+97B)\cos(dx)\sin(c)}{336d} + \frac{(54A+73B)\cos(2dx)\sin}{720d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/30*((21*
A + 17*B)*Cot[c])/d + ((107*A + 97*B)*Cos[d*x]*Sin[c])/(336*d) + ((54*A + 7
3*B)*Cos[2*d*x]*Sin[2*c])/(720*d) + ((A + 3*B)*Cos[3*d*x]*Sin[3*c])/(112*d)
 + (B*Cos[4*d*x]*Sin[4*c])/(288*d) + ((107*A + 97*B)*Cos[c]*Sin[d*x])/(336*
d) + ((54*A + 73*B)*Cos[2*c]*Sin[2*d*x])/(720*d) + ((A + 3*B)*Cos[3*c]*Sin[
3*d*x])/(112*d) + (B*Cos[4*c]*Sin[4*d*x])/(288*d)) - (13*A*(a + a*Cos[c + d
*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcT
```

```
an[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*S
qrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (11*B*(a +
a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - Arc
Tan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) -
(7*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ
[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]
]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Ta
n[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 +
Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Co
s[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))
/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) - (17*B
*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Ta
n[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]
]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[
c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]
^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d)
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^3\cos(dx+c)^4 + (A+3B)a^3\cos(dx+c)^3 + 3(A+B)a^3\cos(dx+c)^2 + (3A+B)a^3\cos(dx+c) +\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)*sqrt(cos(d*x + c)),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c) + A)(a\cos(dx+c) + a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x
)
```

maple [A] time = 1.06, size = 413, normalized size = 2.02

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(-560B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(360A+2200B)\left(\sin^8\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(360*A+2200*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-1296*A-3412*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1806*A+2702*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-624*A-738*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+195*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-441*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+165*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x
)
```

mupad [B] time = 1.07, size = 323, normalized size = 1.58

$$\frac{2\left(A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+A\,a^3\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)\right)}{d}+\frac{B\,a^3\left(\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}(}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3,x)
```

```
[Out] (2*(A*a^3*ellipticE(c/2 + (d*x)/2, 2) + A*a^3*ellipticF(c/2 + (d*x)/2, 2) +
 A*a^3*cos(c + d*x)^(1/2)*sin(c + d*x)))/d + (B*a^3*((2*cos(c + d*x)^(1/2)*
sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d - (6*A*a^3*cos(c +
d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(
sin(c + d*x)^2)^(1/2)) - (2*A*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom
([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (6*B*a^3
*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2
))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*
hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2)) -
 (2*B*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos
(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.139 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=171

$$\frac{4a^3(21A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(42A+41B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{2\cdots}{\cdots}$$

```
[Out] 4/5*a^3*(9*A+7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^3*(21*A+13*B)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/105*a^3*
(42*A+41*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+2/7*a*B*(a+a*cos(d*x+c))^2*sin(d*
x+c)*cos(d*x+c)^(1/2)/d+2/35*(7*A+11*B)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)*cos
(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.43, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(21A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(42A+41B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d}+\frac{2\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (4*a^3*(9*A + 7*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(21*A + 13*B)*
EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(42*A + 41*B)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(105*d) + (2*a*B*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*S
in[c + d*x])/(7*d) + (2*(7*A + 11*B)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c +
d*x])*Sin[c + d*x])/(35*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2aB\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2}{7}\int \frac{(a+}{}$$

$$= \frac{2aB\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2(7A+11}{}$$

$$= \frac{2aB\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2(7A+11}{}$$

$$= \frac{4a^3(42A+41B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{105d} + \frac{2aB\sqrt{\cos(c+dx)}}{}$$

$$= \frac{4a^3(42A+41B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{105d} + \frac{2aB\sqrt{\cos(c+dx)}}{}$$

$$= \frac{4a^3(9A+7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(21A+13B)F\left(\frac{1}{2}(c+dx)\middle|\right.}{21d}$$

<u>Mathematica</u> [C] time = 6.36, size = 898, normalized size = 5.25

$$\sqrt{\cos(c+dx)}\,(\cos(c+dx)a+a)^3\left(-\frac{(9A+7B)\cot(c)}{10d} + \frac{(84A+107B)\cos(dx)\sin(c)}{336d} + \frac{(A+3B)\cos(2dx)\sin(2c)}{40d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/10*((9*A
 + 7*B)*Cot[c])/d + ((84*A + 107*B)*Cos[d*x]*Sin[c])/(336*d) + ((A + 3*B)*C
os[2*d*x]*Sin[2*c])/(40*d) + (B*Cos[3*d*x]*Sin[3*c])/(112*d) + ((84*A + 107
*B)*Cos[c]*Sin[d*x])/(336*d) + ((A + 3*B)*Cos[2*c]*Sin[2*d*x])/(40*d) + (B*
Cos[3*c]*Sin[3*d*x])/(112*d)) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^
6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (13*B*(a + a*Cos[c + d*x])^3*Csc[c]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
```

```
 (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (9*A*(a + a*Cos[c + d*x])
^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) - (7*B*(a + a*Cos[c + d*x])^3*Cs
c[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
+ ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]
]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[
Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^3\cos\left(dx+c\right)^4+(A+3B)a^3\cos\left(dx+c\right)^3+3(A+B)a^3\cos\left(dx+c\right)^2+(3A+B)a^3\cos\left(dx+c\right)+}{\sqrt{\cos\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/sqrt(cos(d*x + c)),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)^3}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x
)
```

maple [A] time = 1.02, size = 385, normalized size = 2.25

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-84A-432B)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-432*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(294*A+602*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
26*A-208*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+65*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x
)
```

mupad [B] time = 1.00, size = 255, normalized size = 1.49

$$\frac{2\left(B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+B\,a^3\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)\right)}{d}+\frac{6\,A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*(B*a^3*ellipticE(c/2 + (d*x)/2, 2) + B*a^3*ellipticF(c/2 + (d*x)/2, 2) +
 B*a^3*cos(c + d*x)^(1/2)*sin(c + d*x)))/d + (6*A*a^3*ellipticE(c/2 + (d*x)
/2, 2))/d + (4*A*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^3*cos(c + d*x)
^(1/2)*sin(c + d*x))/d - (2*A*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom
([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (6*B*a^3
*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2
))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*
hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.140 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=169

$$\frac{4a^3(5A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(5A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(5A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}-\frac{2(5A-$$

```
[Out] 4/5*a^3*(5*A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^3*(5*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*A*(a+a*co
s(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)-4/15*a^3*(5*A-6*B)*sin(d*x+c)*cos
(d*x+c)^(1/2)/d-2/5*(5*A-B)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^(1/2
)/d
```

Rubi [A] time = 0.43, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2975, 2976, 2968, 3023, 2748, 2641, 2639}

$$\frac{4a^3(5A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(5A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{4a^3(5A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}-\frac{2(5A-$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (4*a^3*(5*A + 9*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(5*A + 3*B)*El
lipticF[(c + d*x)/2, 2])/(3*d) - (4*a^3*(5*A - 6*B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(15*d) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]]) - (2*(5*A - B)*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x])*Sin[c
 + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+a\cos(c+dx))}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(5A-B)\sqrt{\cos(c+dx)}}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2(5A-B)\sqrt{\cos(c+dx)}}{}$$

$$= -\frac{4a^3(5A-6B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2aA(a+a\cos(c+}{d\sqrt{\cos(c}}$$

$$= -\frac{4a^3(5A-6B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2aA(a+a\cos(c+}{d\sqrt{\cos(c}}$$

$$= \frac{4a^3(5A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(5A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Mathematica [C] time = 6.46, size = 888, normalized size = 5.25

$$\sqrt{\cos(c+dx)}\,(\cos(c+dx)a+a)^3\left(-\frac{(15\cos(2c)A+5A+18B+18B\cos(2c))\csc(c)\sec(c)}{40d} + \frac{A\sec(c+dx)\sin(dx}{4d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/40*((5*A
 + 18*B + 15*A*Cos[2*c] + 18*B*Cos[2*c])*Csc[c]*Sec[c])/d + ((A + 3*B)*Cos[
d*x]*Sin[c])/(12*d) + (B*Cos[2*d*x]*Sin[2*c])/(40*d) + ((A + 3*B)*Cos[c]*Si
n[d*x])/(12*d) + (A*Sec[c]*Sec[c + d*x]*Sin[d*x])/(4*d) + (B*Cos[2*c]*Sin[2
*d*x])/(40*d)) - (5*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
```

```
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
6*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPF
Q[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[
d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + C
ot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c
]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 +
 (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(4*d) - (9*B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)
/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]
*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]))/(20*d)
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^3\cos(dx+c)^4 + (A+3B)a^3\cos(dx+c)^3 + 3(A+B)a^3\cos(dx+c)^2 + (3A+B)a^3\cos(dx+c) + A}{\cos(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(3/2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x
)
```

maple [B] time = 1.18, size = 519, normalized size = 3.07

$$-\frac{4a^3\left(-12B\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -4/15*a^3*(-12*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(5*A+21*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*A+9*B)*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+25*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)-15*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+15*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*B*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x
)
```

mupad [B] time = 1.04, size = 229, normalized size = 1.36

$$\frac{A\,a^3\,\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{6\,A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,B\,a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(3/2),x)
```

```
[Out] (A*a^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d + (6*A*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (6*A*a^3*ellipticF(c
/2 + (d*x)/2, 2))/d + (6*B*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (4*B*a^3*el
lipticF(c/2 + (d*x)/2, 2))/d + (2*B*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/d
+ (2*A*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos
(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) - (2*B*a^3*cos(c + d*x)^(7/2)*sin(c
 + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^
(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.141 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{20a^3(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{4a^3(4A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2(7A+3B)}{}$$

```
[Out] -4*a^3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/d+20/3*a^3*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*A*(a+a*cos(d*x+
c))^2*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/3*(7*A+3*B)*(a^3+a^3*cos(d*x+c))*sin(
d*x+c)/d/cos(d*x+c)^(1/2)-4/3*a^3*(4*A+B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.43, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2975, 2968, 3023, 2748, 2641, 2639}

$$\frac{20a^3(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{4a^3(4A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2(7A+3B)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^3*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*(A + B)*EllipticF[(c
 + d*x)/2, 2])/(3*d) - (4*a^3*(4*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
+ (2*(7*A + 3*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d
*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+a\cos(c+dx))}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(7A+3B)\left(a^3+a^3\cos\right.}{3d\sqrt{\cos(}}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(7A+3B)\left(a^3+a^3\cos\right.}{3d\sqrt{\cos(}}$$

$$= -\frac{4a^3(4A+B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aA(a+a\cos(c+}{3d\cos^{\frac{3}{2}}(c}$$

$$= -\frac{4a^3(4A+B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aA(a+a\cos(c+}{3d\cos^{\frac{3}{2}}(c}$$

$$= -\frac{4a^3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{20a^3(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} -$$

Mathematica [C] time = 6.54, size = 879, normalized size = 5.46

$$\sqrt{\cos(c+dx)}\,(\cos(c+dx)a+a)^3\left(\frac{A\sec(c)\sin(dx)\sec^2(c+dx)}{12d} + \frac{\sec(c)(A\sin(c)+9A\sin(dx)+3B\sin(dx))\sec}{12d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/8*((-5*A
 + B + A*Cos[2*c] + 3*B*Cos[2*c])*Csc[c]*Sec[c])/d + (B*Cos[d*x]*Sin[c])/(1
2*d) + (B*Cos[c]*Sin[d*x])/(12*d) + (A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(12*
d) + (Sec[c]*Sec[c + d*x]*(A*Sin[c] + 9*A*Sin[d*x] + 3*B*Sin[d*x]))/(12*d))
 - (5*A*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
```

```
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + C
ot[c]^2]) - (5*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*S
qrt[1 + Cot[c]^2]) + (A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*
((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + T
an[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
+ Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Co
s[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]))/(4*d) - (B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hyperg
eometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Arc
Tan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
+ ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]
^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 +
 Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*
d)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+A}{\cos(dx+c)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(5/2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x
)
```

maple [B] time = 1.27, size = 654, normalized size = 4.06

$$-\frac{4\left(-4B\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -4/3*(-4*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(9*A+5*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+2*B)*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
*sin(1/2*d*x+1/2*c)^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+5*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*a^3/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d
*x+1/2*c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^{3}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x
)
```

mupad [B] time = 1.63, size = 251, normalized size = 1.56

$$\frac{2\left(A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+3\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}+\frac{B\,a^3\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{6\,B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*(A*a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*A*a^3*ellipticF(c/2 + (d*x)/2, 2)
))/d + (B*a^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (
d*x)/2, 2))/3))/d + (6*B*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (6*B*a^3*elli
pticF(c/2 + (d*x)/2, 2))/d + (6*A*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3
/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*a
^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d
*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a^3*sin(c + d*x)*hypergeom([-1/4,
1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.142 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=171

$$\frac{4a^3(3A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(9A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(9A+5B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d\cos^{\frac{3}{2}}(c+dx)}+$$

```
[Out] -4/5*a^3*(9*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^3*(3*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*A*(a+a
*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/15*(9*A+5*B)*(a^3+a^3*cos(d*
x+c))*sin(d*x+c)/d/cos(d*x+c)^(3/2)+4/15*a^3*(21*A+20*B)*sin(d*x+c)/d/cos(d
*x+c)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2975, 2968, 3021, 2748, 2641, 2639}

$$\frac{4a^3(3A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(9A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(9A+5B)\sin(c+dx)\left(a^3\cos(c+dx)+a^3\right)}{15d\cos^{\frac{3}{2}}(c+dx)}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-4*a^3*(9*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(3*A + 5*B)*E
llipticF[(c + d*x)/2, 2])/(3*d) + (4*a^3*(21*A + 20*B)*Sin[c + d*x])/(15*d*
Sqrt[Cos[c + d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[
c + d*x]^(5/2)) + (2*(9*A + 5*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(15
*d*Cos[c + d*x]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\cos(c+dx))}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(9A+5B)\left(a^3+a^3\cos\right.}{15d\cos^{\frac{3}{2}}(}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(9A+5B)\left(a^3+a^3\cos\right.}{15d\cos^{\frac{3}{2}}(}$$

$$= \frac{4a^3(21A+20B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{2aA(a+a\cos(c+dx))^2\sin(c+}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{4a^3(21A+20B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}} + \frac{2aA(a+a\cos(c+dx))^2\sin(c+}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{4a^3(9A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(3A+5B)F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d}$$

Mathematica [C] time = 6.62, size = 890, normalized size = 5.20

$$\sqrt{\cos(c+dx)}\,(\cos(c+dx)a+a)^3\left(\frac{A\sec(c)\sin(dx)\sec^3(c+dx)}{20d} + \frac{\sec(c)(3A\sin(c)+15A\sin(dx)+5B\sin(dx))\,\mathrm{s}}{60d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-1/40*((-36
*A - 25*B + 5*B*Cos[2*c])*Csc[c]*Sec[c])/d + (A*Sec[c]*Sec[c + d*x]^3*Sin[d
*x])/(20*d) + (Sec[c]*Sec[c + d*x]^2*(3*A*Sin[c] + 15*A*Sin[d*x] + 5*B*Sin[
d*x]))/(60*d) + (Sec[c]*Sec[c + d*x]*(15*A*Sin[c] + 5*B*Sin[c] + 54*A*Sin[d
*x] + 45*B*Sin[d*x]))/(60*d)) - (A*(a + a*Cos[c + d*x])^3*Csc[c]*Hypergeome
tricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^
6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt
```

```
[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan
[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) - (5*B*(a + a*Cos[c + d*x])^3*Csc[c]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
(d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sq
rt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x
 - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) + (9*A*(a + a*Cos[c + d*x])^3
*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) + (B*(a + a*Cos[c + d*x])^3*Csc[c]
*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^3\cos(dx+c)^4+(A+3B)a^3\cos(dx+c)^3+3(A+B)a^3\cos(dx+c)^2+(3A+B)a^3\cos(dx+c)+A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(7/2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x
)
```

maple [B] time = 3.39, size = 916, normalized size = 5.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(60*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+108*A*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-216*A*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^6+100*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+60*B*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-180*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^6-60*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-108*A*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+246*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4-100*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-60*B*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+190*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^4+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-72*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-50*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x
)
```

mupad [B] time = 2.50, size = 287, normalized size = 1.68

$$\frac{2\left(B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+3\,B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{d}+\frac{2\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a^3\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right.\right.}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*(B*a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*B*a^3*ellipticF(c/2 + (d*x)/2, 2)
))/d + (2*A*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*A*a^3*sin(c + d*x)*hype
rgeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x
)^2)^(1/2)) + (2*A*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x
)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*a^3*sin(c + d*x)
*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin
(c + d*x)^2)^(1/2)) + (6*B*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos
(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a^3*sin(
c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/
2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.143 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=204

$$\frac{4a^3(13A+21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(7A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(11A+7B)\sin}{3}$$

```
[Out] -4/5*a^3*(7*A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^3*(13*A+21*B)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/105*a^3
*(41*A+42*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/7*a*A*(a+a*cos(d*x+c))^2*sin(d
*x+c)/d/cos(d*x+c)^(7/2)+2/35*(11*A+7*B)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d/
cos(d*x+c)^(5/2)+4/5*a^3*(7*A+9*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2975, 2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{4a^3(13A+21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(7A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(11A+7B)\sin}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-4*a^3*(7*A + 9*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*(13*A + 21*B)
*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(41*A + 42*B)*Sin[c + d*x])/(10
5*d*Cos[c + d*x]^(3/2)) + (4*a^3*(7*A + 9*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c
+ d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7
/2)) + (2*(11*A + 7*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(35*d*Cos[c +
 d*x]^(5/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\cos(c+dx))}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(11A+7B)\left(a^3+a^3\mathrm{co}\right.}{35d\cos^{\frac{5}{2}}}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(11A+7B)\left(a^3+a^3\mathrm{co}\right.}{35d\cos^{\frac{5}{2}}}$$

$$= \frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))^2\sin(c+}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))^2\sin(c+}{7d\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{4a^3(13A+21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{4a^3(41A+42B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{4a^3(7A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3(13A+21B)F\left(\frac{1}{2}(c+dx\right.}{21d}$$

Mathematica [C] time = 6.65, size = 925, normalized size = 4.53

$$\sqrt{\cos(c+dx)}\,(\cos(c+dx)a+a)^3\left(\frac{A\sec(c)\sin(dx)\sec^4(c+dx)}{28d} + \frac{\sec(c)(5A\sin(c)+21A\sin(dx)+7B\sin(dx))\,\mathrm{s}}{140d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(((7*A + 9*B
)*Csc[c]*Sec[c])/(10*d) + (A*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(28*d) + (Sec[
c]*Sec[c + d*x]^3*(5*A*Sin[c] + 21*A*Sin[d*x] + 7*B*Sin[d*x]))/(140*d) + (S
ec[c]*Sec[c + d*x]^2*(63*A*Sin[c] + 21*B*Sin[c] + 130*A*Sin[d*x] + 105*B*Si
```

```
n[d*x]))/(420*d) + (Sec[c]*Sec[c + d*x]*(130*A*Sin[c] + 105*B*Sin[c] + 294*
A*Sin[d*x] + 378*B*Sin[d*x]))/(420*d)) - (13*A*(a + a*Cos[c + d*x])^3*Csc[c
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]
*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[
d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (B*(a + a*Cos[c + d*x])
^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) + (7*A*(a + a*Cos[
c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[
1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d) + (9*B*(a + a*Cos[c + d
*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^3\cos(dx+c)^4 + (A+3B)a^3\cos(dx+c)^3 + 3(A+B)a^3\cos(dx+c)^2 + (3A+B)a^3\cos(dx+c) + \cdots}{\cos(dx+c)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(9/2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x
)
```

maple [B] time = 4.33, size = 929, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-1/5*(3/8*A+1/8*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/
2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/8*A+3/8*B)*(-(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(3/8*A+3/8*B)*
(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*A*(-1/56*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x
)
```

mupad [B] time = 2.67, size = 307, normalized size = 1.50

$$\frac{2\,B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{\frac{2\,A\,a^3\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\,-\frac{3}{4};\,\cos(c+d\,x)^2\right)}{7}+\frac{6\,A\,a^3\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos(c+d\,x)^2\right)}{5}+2\,A\,a^3\,\mathrm{c}\cdots}{d\,\mathrm{c}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(9/2),x)
```

```
[Out] (2*B*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + ((2*A*a^3*sin(c + d*x)*hypergeom(
[-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + (6*A*a^3*cos(c + d*x)*sin(c + d*x)*
hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/5 + 2*A*a^3*cos(c + d*x)^2*si
n(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 2*A*a^3*cos(c + d*
x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d
*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2)) + (6*B*a^3*sin(c + d*x)*hypergeom([-1
/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2
)) + (2*B*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*
cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a^3*sin(c + d*x)*hypergeo
m([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)
^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.144 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{4a^3(11A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(17A+21B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(11A+13B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^3(23A+24}{105d\cos}$$

```
[Out] -4/15*a^3*(17*A+21*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^3*(11*A+13*B)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/105*
a^3*(23*A+24*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+4/21*a^3*(11*A+13*B)*sin(d*x+
c)/d/cos(d*x+c)^(3/2)+2/9*a*A*(a+a*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(9
/2)+2/63*(13*A+9*B)*(a^3+a^3*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(7/2)+4/15
*a^3*(17*A+21*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2975, 2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{4a^3(11A+13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(17A+21B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^3(11A+13B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^3(23A+24}{105d\cos}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x]
```

```
[Out] (-4*a^3*(17*A + 21*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^3*(11*A + 13
*B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (4*a^3*(23*A + 24*B)*Sin[c + d*x])/
(105*d*Cos[c + d*x]^(5/2)) + (4*a^3*(11*A + 13*B)*Sin[c + d*x])/(21*d*Cos[c
 + d*x]^(3/2)) + (4*a^3*(17*A + 21*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]
]) + (2*a*A*(a + a*Cos[c + d*x])^2*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) +
 (2*(13*A + 9*B)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(63*d*Cos[c + d*x]^
(7/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+a\cos(c+dx))}{}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(13A+9B)\left(a^3+a^3\mathrm{c}\right.}{63d\cos^{\frac{7}{2}}}$$

$$= \frac{2aA(a+a\cos(c+dx))^2\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(13A+9B)\left(a^3+a^3\mathrm{c}\right.}{63d\cos^{\frac{7}{2}}}$$

$$= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))^2\sin(c+}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\cos(c+dx))^2\sin(c+}{9d\cos^{\frac{9}{2}}(c+dx)}$$

$$= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^3(11A+13B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} +$$

$$= -\frac{4a^3(17A+21B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{4a^3(11A+13B)F\left(\frac{1}{2}(c+\right.}{21d}$$

Mathematica [C] time = 6.71, size = 967, normalized size = 4.08

$$\sqrt{\cos(c+dx)}\,(\cos(c+dx)a+a)^3\left(\frac{A\sec(c)\sin(dx)\sec^5(c+dx)}{36d} + \frac{\sec(c)(7A\sin(c)+27A\sin(dx)+9B\sin(dx))\,\mathrm{s}}{252d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2)
,x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(((17*A + 21
*B)*Csc[c]*Sec[c])/(30*d) + (A*Sec[c]*Sec[c + d*x]^5*Sin[d*x])/(36*d) + (Se
c[c]*Sec[c + d*x]^4*(7*A*Sin[c] + 27*A*Sin[d*x] + 9*B*Sin[d*x]))/(252*d) +
(Sec[c]*Sec[c + d*x]*(55*A*Sin[c] + 65*B*Sin[c] + 119*A*Sin[d*x] + 147*B*Si
```

```
n[d*x]))/(210*d) + (Sec[c]*Sec[c + d*x]^3*(135*A*Sin[c] + 45*B*Sin[c] + 238
*A*Sin[d*x] + 189*B*Sin[d*x]))/(1260*d) + (Sec[c]*Sec[c + d*x]^2*(238*A*Sin
[c] + 189*B*Sin[c] + 330*A*Sin[d*x] + 390*B*Sin[d*x]))/(1260*d)) - (11*A*(a
 + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2])
 - (13*B*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 +
 Cot[c]^2]) + (17*A*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hy
pergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
 ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[
d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c
]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Ta
n[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]
^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))
/(60*d) + (7*B*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((Hyperge
ometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
 ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]
*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^
2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 +
Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*
d)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^3\cos(dx+c)^4 + (A+3B)a^3\cos(dx+c)^3 + 3(A+B)a^3\cos(dx+c)^2 + (3A+B)a^3\cos(dx+c) + A}{\cos(dx+c)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorith
m="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/cos(d*x + c)^(11/2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorith
m="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(11/2),
x)
```

maple [B] time = 4.92, size = 1178, normalized size = 4.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-1/5*(3/
8*A+3/8*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+1/8*A*(-1/144*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^
2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+1/8*B*(-(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(1/8*A+3/8*B)*(-1/6
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-
1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(3/8*A+1/8*B)*(-1/56*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/
2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)
```

```
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorith
m="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/cos(d*x + c)^(11/2),
x)
```

mupad [B] time = 3.03, size = 552, normalized size = 2.33

$$\frac{2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)\left(\frac{19\,A\,a^3\,\sin(c+d\,x)}{\cos(c+d\,x)^{3/2}\,\sqrt{1-\cos(c+d\,x)^2}}+\frac{9\,A\,a^3\,\sin(c+d\,x)}{\cos(c+d\,x)^{7/2}\,\sqrt{1-\cos(c+d\,x)^2}}+\frac{25\,B\,a^3\,\sin(c+d\,x)}{\cos(c+d\,x)^{3/2}\,\sqrt{1-\cos(c+d\,x)^2}}+\cdots\right.}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/cos(c + d*x)^(11/2),x)
```

```
[Out] (2*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2)*((19*A*a^3*sin(c + d*x))/(co
s(c + d*x)^(3/2)*(1 - cos(c + d*x)^2)^(1/2)) + (9*A*a^3*sin(c + d*x))/(cos(
c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2)) + (25*B*a^3*sin(c + d*x))/(cos(c
 + d*x)^(3/2)*(1 - cos(c + d*x)^2)^(1/2)) + (3*B*a^3*sin(c + d*x))/(cos(c +
 d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))))/(21*d) - (8*hypergeom([-1/4, 1/2]
, 7/4, cos(c + d*x)^2)*((34*A*a^3*sin(c + d*x))/(cos(c + d*x)^(1/2)*(1 - co
s(c + d*x)^2)^(1/2)) + (5*A*a^3*sin(c + d*x))/(cos(c + d*x)^(5/2)*(1 - cos(
c + d*x)^2)^(1/2)) + (27*B*a^3*sin(c + d*x))/(cos(c + d*x)^(1/2)*(1 - cos(c
 + d*x)^2)^(1/2))))/(135*d) + (8*((3*A*a^3*sin(c + d*x))/(cos(c + d*x)^(3/2
)*(1 - cos(c + d*x)^2)^(1/2)) + (B*a^3*sin(c + d*x))/(cos(c + d*x)^(3/2)*(1
 - cos(c + d*x)^2)^(1/2)))*hypergeom([-3/4, 1/2], 5/4, cos(c + d*x)^2))/(21
*d) + (2*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2)*((136*A*a^3*sin(c + d*
x))/(cos(c + d*x)^(1/2)*(1 - cos(c + d*x)^2)^(1/2)) + (39*A*a^3*sin(c + d*x
))/(cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (5*A*a^3*sin(c + d*x))
/(cos(c + d*x)^(9/2)*(1 - cos(c + d*x)^2)^(1/2)) + (153*B*a^3*sin(c + d*x))
/(cos(c + d*x)^(1/2)*(1 - cos(c + d*x)^2)^(1/2)) + (27*B*a^3*sin(c + d*x))/
(cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))))/(45*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
[Out] Timed out
```

3.145 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(5A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(5A-7B)\sin(c+dx)}{5ad}$$

```
[Out] -3/5*(5*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))/a/d+5/3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-1/5*(5*A-7*B)*cos(d*
x+c)^(3/2)*sin(d*x+c)/a/d+(A-B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+
c))+5/3*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.20, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2748, 2635, 2641, 2639}

$$\frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(5A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}+\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(5A-7B)\sin(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-3*(5*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(5*a*d) + (5*(A - B)*EllipticF[(
c + d*x)/2, 2])/(3*a*d) + (5*(A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*
d) - ((5*A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) + ((A - B)*Cos[c
 + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{1}{2}a(5\right.}{a^2}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(5A-7B)\int \cos^{\frac{5}{2}}(c+dx)\,dx}{2a} + \frac{(5}{}$$

$$= \frac{5(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{(5A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5ad}$$

$$= -\frac{3(5A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} + \frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{5(A-B)}{}$$

Mathematica [C] time = 6.62, size = 1182, normalized size = 7.58

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
(((21*I)/20)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
 (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((2*(5*A - 5*B + 10*A*Cos[c] - 16
*B*Cos[c])*Csc[c])/(5*d) + (4*(A - B)*Cos[d*x]*Sin[c])/(3*d) + (2*B*Cos[2*d
*x]*Sin[2*c])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Si
n[(d*x)/2]))/d + (4*(A - B)*Cos[c]*Sin[d*x])/(3*d) + (2*B*Cos[2*c]*Sin[2*d*
x])/(5*d)))/(a + a*Cos[c + d*x]) - (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d
*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Co
t[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]
]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (5*B*Cos[c/2 + (d*x)/2
]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]
^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*
Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d
*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(a*cos(d*
x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

maple [A] time = 1.16, size = 281, normalized size = 1.80

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(25A\,\mathrm{Ellipti}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+
1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*A*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*A*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2)))+48*B*sin(1/2*d*x+1/2*c)^8+(-40*A-56*B)*sin(1/2*d*x+1/2*c)
^6+(90*A-30*B)*sin(1/2*d*x+1/2*c)^4+(-35*A+23*B)*sin(1/2*d*x+1/2*c)^2)/a/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
[Out] Timed out
```

3.146 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=123

$$-\frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(3A-5B)\sin(c+d}{3ad}$$

```
[Out] 3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a/d-1/3*(3*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+(A-B)*cos(d*x+c)^(3/2)*
sin(d*x+c)/d/(a+a*cos(d*x+c))-1/3*(3*A-5*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.18, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2748, 2639, 2635, 2641}

$$-\frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}-\frac{(3A-5B)\sin(c+d}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((3*A - 5*B)*EllipticF[(c + d
*x)/2, 2])/(3*a*d) - ((3*A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
+ ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \sqrt{\cos(c+dx)}\,\left(\frac{3}{2}a(A-B)-\frac{1}{2}a(3}{a^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(3A-5B)\int \cos^{\frac{3}{2}}(c+dx)\,dx}{2a} + \frac{(3(}{}$$

$$= \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(3A-5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{(A-}{}$$

$$= \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{(3A-5B)\sqrt{c}}{}$$

Mathematica [C] time = 6.56, size = 1129, normalized size = 9.18

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (((3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
```

```
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - (
((3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (C
os[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(A - B)*(1 + 2*Cos[c])*Csc[c])/
d + (4*B*Cos[d*x]*Sin[c])/(3*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*
x)/2] - B*Sin[(d*x)/2]))/d + (4*B*Cos[c]*Sin[d*x])/(3*d)))/(a + a*Cos[c + d
*x]) + (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*S
qrt[1 + Cot[c]^2]) - (5*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Co
t[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a +
 a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c))/(a*cos(d*x
+ c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

maple [A] time = 1.07, size = 262, normalized size = 2.13

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(3A\,\mathrm{EllipticF}\left(c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
```

```
[Out] 1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/
2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))+8*B*sin(1/2*d*x+1/2*c)^6+(6*A-18*B)*sin(1/2*d*x+1/2*c)^4+(-3*A+
7*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{a+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c)),x)
[Out] Timed out
```

3.147 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] -(A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a/d+(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+(A-B)*sin(d*x+c)*cos(d*x+c)^(1
/2)/d/(a+a*cos(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2977, 2748, 2641, 2639}

$$\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(((A - 3*B)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((A - B)*EllipticF[(c + d*
x)/2, 2])/(a*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c
 + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{a+a\cos(c+dx)}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(A-B)-\frac{1}{2}a(A-3B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(A-3B)\int\sqrt{\cos(c+dx)}\,dx}{2a} + \cdots$$

$$= -\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-B)\sqrt{\cos\cdots}}{d(a+\cdots}$$

Mathematica [C] time = 6.45, size = 1098, normalized size = 12.92

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((-1/4*I)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + ((
(3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperge
ometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(
```

```
1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*S
qrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E
^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric
2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^
((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
+ E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)
*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Co
s[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(-A + B + 2*B*Cos[c])*Csc[c])/d
+ (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d))/(a
+ a*Cos[c + d*x]) - (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Co
s[c + d*x])*Sqrt[1 + Cot[c]^2]) + (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x
- ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c
]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]]
)/(d*(a + a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

maple [A] time = 1.03, size = 244, normalized size = 2.87

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \left(A\,\mathrm{EllipticF}\right.\right.}{a\cos\left(\frac{dx}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}\ \left(A+B\ \cos(c+d\,x)\right)}{a+a\ \cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.148 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=83

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] (A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a/d+(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)
/d/(a+a*cos(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2978, 2748, 2641, 2639}

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

```
[Out] ((A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((A + B)*EllipticF[(c + d*x)/2,
 2])/(a*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*
x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(A+B)+\frac{1}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{(A-B)\int \sqrt{\cos(c+dx)}\,dx}{2a} + \frac{(A$$

$$= \frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-B)\sqrt{\cos(c+}}{d(a+a\cos}$$

Mathematica [C] time = 6.49, size = 1094, normalized size = 13.18

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

```
[Out] ((I/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - ((I/4
)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeometric
2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
```

```
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)
*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*
Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[c/2 +
 (d*x)/2]^2*Sqrt[Cos[c + d*x]]*((-2*(A - B)*Csc[c])/d - (2*Sec[c/2]*Sec[c/2
 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d))/(a + a*Cos[c + d*x]) - (
A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[1 +
Cot[c]^2]) - (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqr
t[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
 ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d
*x])*Sqrt[1 + Cot[c]^2])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)^2+a\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^2 + a*cos(
d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

maple [A] time = 1.15, size = 243, normalized size = 2.93

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \left(A\,\mathrm{EllipticF}\left(\right.\right.\right.}{a\cos\left(\frac{dx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,\left(a+a\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.149 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=119

$$-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)}$$

```
[Out] -(3*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a/d-(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+(3*A-B)*sin(d*x+c)/a/d/cos(d*x
+c)^(1/2)-(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2978, 2748, 2636, 2639, 2641}

$$-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] -(((3*A - B)*EllipticE[(c + d*x)/2, 2])/(a*d)) - ((A - B)*EllipticF[(c + d*
x)/2, 2])/(a*d) + ((3*A - B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A -
 B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(3A-B)-\frac{1}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))} - \frac{(A-B)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{(3A-$$

$$= -\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(3A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A-B)\sin}{d\sqrt{\cos(c+dx)}\,(a-}$$

$$= -\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(3A-B)\sin(}{ad\sqrt{\cos(c+}}$$

Mathematica [C] time = 6.71, size = 1130, normalized size = 9.50

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) +
((I/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (Cos[
c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(((2*A + A*Cos[c] - B*Cos[c])*Csc[c/2]*
Sec[c/2]*Sec[c])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin
[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/(a + a*Cos[c + d*x])
 + (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])*Sqrt[
1 + Cot[c]^2]) - (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c
 + d*x])*Sqrt[1 + Cot[c]^2])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)^3+a\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^3 + a*cos(
d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

maple [A] time = 2.36, size = 319, normalized size = 2.68

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\sin\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A-B)*sin(1/2*d*x+1/2*c)^4+(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A-B)*sin(1/2*d*x+1/2*c)^2
)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
[Out] Timed out
```

3.150 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=153

$$\frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(5A-3B)\sin(c+dx}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a/d+1/3*(5*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+1/3*(5*A-3*B)*sin(d*x+c
)/a/d/cos(d*x+c)^(3/2)-(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))
-3*(A-B)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2978, 2748, 2636, 2641, 2639}

$$\frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{(5A-3B)\sin(c+dx}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((5*A - 3*B)*EllipticF[(c + d
*x)/2, 2])/(3*a*d) + ((5*A - 3*B)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
- (3*(A - B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - ((A - B)*Sin[c + d*x]
)/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(5A-3B)-\frac{3}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{(5A-3B)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{2a} - \frac{(3(A \cdots}{\cdots}$$

$$= \frac{(5A-3B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3(A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(A-B)\sin(c+\cdots}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos\cdots}$$

$$= \frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(5A-3B)\sin(\cdots}{3ad\cos^{\frac{3}{2}}(c\cdots}$$

Mathematica [C] time = 7.09, size = 1167, normalized size = 7.63

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (((3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) - (
((3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]) + (C
os[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(-(((A - B)*(2 + Cos[c])*Csc[c/2]*Se
c[c/2]*Sec[c])/d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[
(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c
 + d*x]*(A*Sin[c] - 3*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d)))/(a + a*Cos[c + d*
x]) - (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x]
)*Sqrt[1 + Cot[c]^2]) + (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a +
a*Cos[c + d*x])*Sqrt[1 + Cot[c]^2])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a\cos(dx+c)^4+a\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c)^4 + a*cos(
d*x + c)^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")

[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)

maple [B] time = 3.16, size = 493, normalized size = 3.22

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{(-2A+2B)\left(-\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((-2*A+2*B)*(-
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*A
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(A-B)*(cos(1/2*d*x+1/2*c)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,\left(a+a\,\cos\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.151 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=203

$$\frac{5(2A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7(5A-8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}+\frac{(2A-3B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{7(5A-8B)\sin(c}{15}$$

```
[Out] -7/5*(5*A-8*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))/a^2/d+5/3*(2*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-7/15*(5*A-8*
B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d+(2*A-3*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/
a^2/d/(1+cos(d*x+c))+1/3*(A-B)*cos(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c
))^2+5/3*(2*A-3*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.41, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2748, 2635, 2641, 2639}

$$\frac{5(2A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7(5A-8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}+\frac{(2A-3B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{7(5A-8B)\sin(c}{15}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-7*(5*A - 8*B)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) + (5*(2*A - 3*B)*Ellip
ticF[(c + d*x)/2, 2])/(3*a^2*d) + (5*(2*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(3*a^2*d) - (7*(5*A - 8*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*
d) + ((2*A - 3*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]
)) + ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7}{2}a(A-B)-\frac{1}{2}a(5A-11B)\cos(c\right.}{a+a\cos(c+dx)}}{3a^2}$$

$$= \frac{(2A-3B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{(2A-3B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{5(2A-3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d} - \frac{7(5A-8B)\cos^{\frac{3}{2}}(c+dx)\sin(}{15a^2d}$$

$$= -\frac{7(5A-8B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} + \frac{5(2A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{5(2A-}{}$$

Mathematica [C] time = 6.86, size = 1262, normalized size = 6.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] (((-7*I)/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2
+ (((28*I)/5)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
2 - (20*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}
, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])
^2*Sqrt[1 + Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcT
an[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(
a + a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[
c + d*x]]*((4*(15*A - 20*B + 20*A*Cos[c] - 36*B*Cos[c])*Csc[c])/(5*d) + (8*
(A - 2*B)*Cos[d*x]*Sin[c])/(3*d) + (4*B*Cos[2*d*x]*Sin[2*c])/(5*d) + (4*Sec
[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] - 4*B*Sin[(d*x)/2]))/d - (2*Sec[
c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (8*(A
- 2*B)*Cos[c]*Sin[d*x])/(3*d) + (4*B*Cos[2*c]*Sin[2*d*x])/(5*d) - (2*(A - B
)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{4}+A\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^4 + A*cos(d*x + c)^3)*sqrt(cos(d*x + c))/(a^2*cos(
d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x
)
```

maple [A] time = 1.05, size = 465, normalized size = 2.29

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;\left(96B\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+80A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-352B\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*B*cos(1/2
*d*x+1/2*c)^10+80*A*cos(1/2*d*x+1/2*c)^8-352*B*cos(1/2*d*x+1/2*c)^8+60*A*co
s(1/2*d*x+1/2*c)^6+100*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+210
*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+120*B*cos(1/2*d*x+1/2*c)^
6-150*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*B*cos(1/2*d*x+1/
```

```
2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-240*A*cos(1/2*d*x+1/2*c)^4+266*B*cos(1/2*d
*x+1/2*c)^4+105*A*cos(1/2*d*x+1/2*c)^2-135*B*cos(1/2*d*x+1/2*c)^2-5*A+5*B)/
a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{7/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.152 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=166

$$-\frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{5(A-2B)\sin(c$$

```
[Out] (4*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/a^2/d-5/3*(A-2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+1/3*(4*A-7*B)*cos(d
*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))+1/3*(A-B)*cos(d*x+c)^(5/2)*sin(
d*x+c)/d/(a+a*cos(d*x+c))^2-5/3*(A-2*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.39, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2748, 2639, 2635, 2641}

$$-\frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{5(A-2B)\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((4*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*(A - 2*B)*EllipticF[(c
 + d*x)/2, 2])/(3*a^2*d) - (5*(A - 2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3
*a^2*d) + ((4*A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c
 + d*x])) + ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d
*x])^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{3}{2}a(A-3B)\cos(c+dx\right.}{a+a\cos(c+dx)}}{3a^2}$$

$$= \frac{(4A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{(4A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5(A-2B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{(4A}{}$$

$$= \frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5(A-2B)\sqrt{}}{}$$

Mathematica [C] time = 6.76, size = 1218, normalized size = 7.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] ((2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (((
7*I)/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + (1
0*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqr
t[1 + Cot[c]^2]) - (20*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a +
a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c +
d*x]]*((-4*(2*A - 3*B + 2*A*Cos[c] - 4*B*Cos[c])*Csc[c])/d + (8*B*Cos[d*x]*
Sin[c])/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(2*A*Sin[(d*x)/2] - 3*B*Sin[
(d*x)/2]))/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*
x)/2]))/(3*d) + (8*B*Cos[c]*Sin[d*x])/(3*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(a^2*cos(
d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x
)
```

maple [B] time = 1.14, size = 435, normalized size = 2.62

$$\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-16B\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24A\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*B*cos(1/2*
d*x+1/2*c)^8+24*A*cos(1/2*d*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1
/2*d*x+1/2*c)^3+24*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*co
s(1/2*d*x+1/2*c)^6-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-42*B
*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1/2*c)^4+4
8*B*cos(1/2*d*x+1/2*c)^4+15*A*cos(1/2*d*x+1/2*c)^2-21*B*cos(1/2*d*x+1/2*c)^
2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{5/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.153 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=136

$$\frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(2A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+d$$

```
[Out] -(A-4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a^2/d+1/3*(2*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+1/3*(A-B)*cos(d*x+
c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+1/3*(2*A-5*B)*sin(d*x+c)*cos(d*x+c
)^(1/2)/a^2/d/(1+cos(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2977, 2748, 2641, 2639}

$$\frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(2A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)}{3d(a\cos(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -(((A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((2*A - 5*B)*EllipticF[(
c + d*x)/2, 2])/(3*a^2*d) + ((2*A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3*a^2*d*(1 + Cos[c + d*x])) + ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*
d*(a + a*Cos[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-B)-\frac{1}{2}a(A-7B)\cos(c+\right.}{a+a\cos(c+dx)}}{3a^2}$$

$$= \frac{(2A-5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+d}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{(2A-5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+d}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(2A-5B)}{3a^2}$$

Mathematica [C] time = 6.66, size = 1184, normalized size = 8.71

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] ((-1/2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
```

```
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 +
((2*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (4*
A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[
1 + Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*
Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*
x]]*((-4*(-A + 2*B + 2*B*Cos[c])*Csc[c])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]
*(A*Sin[(d*x)/2] - 2*B*Sin[(d*x)/2]))/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*
(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^2*
Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c))/(a^2*cos(d*
x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x
)
```

maple [B] time = 1.17, size = 421, normalized size = 3.10

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(12A\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*B*cos(1/2*d*x+1/2*c)^6-10*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-24*B*cos(1/2*d*x+1/2*c)^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4+38*B*cos(1/2*d*x+1/2*c)^4+9*A
*cos(1/2*d*x+1/2*c)^2-15*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)
^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\left(a+a\,\cos\left(c+d\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2,x)
[Out] Timed out
```

3.154 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=121

$$\frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a^2/d+1/3*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+B*sin(d*x+c)*cos(d*x+c)^(1
/2)/a^2/d/(1+cos(d*x+c))+1/3*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d
*x+c))^2
```

Rubi [A] time = 0.28, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2978, 2748, 2641, 2639}

$$\frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -((B*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((A + 2*B)*EllipticF[(c + d*x)/2
, 2])/(3*a^2*d) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d
*x])) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])
^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{1}{2}a(A+5B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{1}{2}$$

$$= \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{B\int$$

$$= -\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{B\sqrt{\cos(c+dx)}\,\sin}{a^2d(1+\cos(c+}$$

Mathematica [C] time = 6.52, size = 815, normalized size = 6.74

$$-\frac{iB\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(\frac{2e^{2idx}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt{e^{-idx}\left(2\left(1+e^{2idx}\right)\cos(c)+2i\left(-1+e^{2idx}\right)\sin(c)\right)}\,\sqrt{e^{2idx}\cos(2c)+ie^{2idx}\sin(2c)+1}}{3id\left(1+e^{2idx}\right)\cos(c)-3d\left(-1+e^{2idx}\right)\sin(c)}\right.}{2(\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] ((-1/2*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 -
(2*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sq
rt[1 + Cot[c]^2]) - (4*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a +
a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c +
d*x]]*((4*B*Csc[c])/d + (4*B*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d +
(2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) +
 (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^2 + 2*a^
2*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x
)
```

maple [B] time = 1.31, size = 350, normalized size = 2.89

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+12*B*cos(1/2*d*x+1/2*c)^6+4*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4-20*B*cos(1/2*d*x+1/2*c)^4-3*A*cos(
1/2*d*x+1/2*c)^2+9*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+dx)}\,\left(A+B\,\cos(c+dx)\right)}{\left(a+a\,\cos(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
[Out] Timed out
```

3.155 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=121

$$\frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a^2/d+1/3*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-A*sin(d*x+c)*cos(d*x+c)^(1/
2)/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*
x+c))^2
```

Rubi [A] time = 0.34, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2978, 2748, 2641, 2639}

$$\frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] (A*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((2*A + B)*EllipticF[(c + d*x)/2, 2
])/(3*a^2*d) - (A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]
)) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{\frac{1}{2}a(5A+B)-\frac{1}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{A\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+$$

$$= -\frac{A\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+$$

$$= \frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{A\sqrt{\cos(c+dx)}\,\sin}{a^2d(1+\cos(c+}$$

Mathematica [C] time = 6.55, size = 815, normalized size = 6.74

$$\frac{iA\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(\frac{2e^{2idx}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2idx}(\cos(c)+i\sin(c))^2\right)\sqrt{e^{-idx}\left(2\left(1+e^{2idx}\right)\cos(c)+2i\left(-1+e^{2idx}\right)\sin(c)\right)}\,\sqrt{e^{2idx}\cos(2c)+ie^{2idx}\sin(2c)+1}}{3id\left(1+e^{2idx}\right)\cos(c)-3d\left(-1+e^{2idx}\right)\sin(c)}\right.}{2(\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),
x]
```

```
[Out] ((I/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeom
etric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
```

```
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F
1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d
*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (4*
A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
- ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]
]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[
1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4,
 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*C
os[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x
]]*((-4*A*Csc[c])/d - (4*A*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (2
*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) - (
2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^3+2\,a^2\cos(dx+c)^2+a^2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^3 + 2*a^
2*cos(d*x + c)^2 + a^2*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

maple [B] time = 1.10, size = 350, normalized size = 2.89

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*d
*x+1/2*c)^6-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2*
d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^3-16*A*cos(1/2*d*x+1/2*c)^4-2*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1
/2*d*x+1/2*c)^2+3*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,\left(a+a\,\cos(c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.156 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=168

$$-\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{(5A-2B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(\cos(c+dx)+\ldots}$$

```
[Out] -(4*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a^2/d-1/3*(5*A-2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+(4*A-B)*sin(d*x+c)
/a^2/d/cos(d*x+c)^(1/2)-1/3*(5*A-2*B)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))/cos(d
*x+c)^(1/2)-1/3*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2978, 2748, 2636, 2639, 2641}

$$-\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{(5A-2B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(\cos(c+dx)+\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] -(((4*A - B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) - ((5*A - 2*B)*EllipticF[(
c + d*x)/2, 2])/(3*a^2*d) + ((4*A - B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*
x]]) - ((5*A - 2*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c +
d*x])) - ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]
)^2)
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(7A-B)-\frac{3}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(5A-2B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\cos(c+$$

$$= -\frac{(5A-2B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\cos(c+$$

$$= -\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(4A-B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(5A-2B}{3a^2d\sqrt{\cos(c+d}$$

$$= -\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{(4A-B)\sin}{a^2d\sqrt{\cos(}}$$

Mathematica [C] time = 6.81, size = 1217, normalized size = 7.24

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),
x]
```

```
[Out] ((-2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + ((
I/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2
*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[
-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*
I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^
((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x
))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 + (10*A
*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1
 + Cot[c]^2]) - (4*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Co
s[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]
]*((2*(2*A + 2*A*Cos[c] - B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/d + (2*Sec[c/
2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (4*Sec[c
/2]*Sec[c/2 + (d*x)/2]*(2*A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d + (8*A*Sec[c]
*Sec[c + d*x]*Sin[d*x])/d + (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)
))/(a + a*Cos[c + d*x])^2
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^4+2\,a^2\cos(dx+c)^3+a^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm ="fricas")

[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm ="giac")

[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)

maple [B] time = 1.35, size = 494, normalized size = 2.94

$$-\frac{2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(5A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x)

[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A-B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A-10*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(37*A-7*B)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\cos(c + d\,x)}{\cos(c + d\,x)^{3/2}\,(a + a\cos(c + d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.157 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=201

$$\frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A-4B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{5(2A-B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (7*A-4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/a^2/d+5/3*(2*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+5/3*(2*A-B)*sin(d*x
+c)/a^2/d/cos(d*x+c)^(3/2)-1/3*(7*A-4*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)/
(1+cos(d*x+c))-1/3*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2-(
7*A-4*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2978, 2748, 2636, 2641, 2639}

$$\frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A-4B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{5(2A-B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] ((7*A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (5*(2*A - B)*EllipticF[(c
 + d*x)/2, 2])/(3*a^2*d) + (5*(2*A - B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]
^(3/2)) - ((7*A - 4*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - ((7*A - 4
*B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])) - ((A - B
)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = -\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{3}{2}a(3A-B)-\frac{5}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{(7A-4B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+a}$$

$$= -\frac{(7A-4B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+a}$$

$$= \frac{5(2A-B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{(7A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(7A-4B)\sin(}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+}$$

$$= \frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{5(2A-B)\mathrm{si}}{3a^2d\cos^{\frac{3}{2}}}$$

Mathematica [C] time = 7.43, size = 1258, normalized size = 6.26

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),
x]
```

```
[Out] (((7*I)/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 -
 ((2*I)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
```

```
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (2
0*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqr
t[1 + Cot[c]^2]) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1
/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a +
a*Cos[c + d*x])^2*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c +
d*x]]*((-2*(4*A - 2*B + 3*A*Cos[c] - 2*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/
d - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] - 2*B*Sin[(d*x)/2]))/d
 - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d
) + (8*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (8*Sec[c]*Sec[c + d*x]*(A*
Sin[c] - 6*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d) - (2*(A - B)*Sec[c/2 + (d*x)/2
]^2*Tan[c/2])/(3*d)))/(a + a*Cos[c + d*x])^2
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^5+2\,a^2\cos(dx+c)^4+a^2\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^5 + 2*a^
2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)),
 x)
```

maple [B] time = 3.95, size = 750, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(A-
B)*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c
)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/(-1+sin(1/2*d*x+1/2*c)^2)+
(-8*A+4*B)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/
2*c)^2-1)+4*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(4*A-2*B)*(cos(1/2*
d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
[Out] Timed out
```

3.158 $\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=254

$$\frac{(11A-21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{7(17A-33B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{3(11A-21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{7(17A-3}{}$$

```
[Out] -7/10*(17*A-33*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/2*(11*A-21*B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-7/30*(1
7*A-33*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a^3/d+1/5*(A-B)*cos(d*x+c)^(9/2)*sin(
d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(7*A-12*B)*cos(d*x+c)^(7/2)*sin(d*x+c)/a/d
/(a+a*cos(d*x+c))^2+3/10*(11*A-21*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a^3+a^3
*cos(d*x+c))+1/2*(11*A-21*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.55, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2748, 2635, 2641, 2639}

$$\frac{(11A-21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{7(17A-33B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{3(11A-21B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{7(17A-3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-7*(17*A - 33*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((11*A - 21*B)*El
lipticF[(c + d*x)/2, 2])/(2*a^3*d) + ((11*A - 21*B)*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(2*a^3*d) - (7*(17*A - 33*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30
*a^3*d) + ((A - B)*Cos[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x
])^3) + ((7*A - 12*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c
 + d*x])^2) + (3*(11*A - 21*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3
+ a^3*Cos[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(\frac{9}{2}a(A-B)-\frac{5}{2}a(A-3B)\cos(c+dx\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(7A-12B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(7A-12B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(7A-12B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(11A-21B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a^3d} - \frac{7(17A-33B)\cos^{\frac{3}{2}}(c+dx)\sin}{30a^3d}$$

$$= -\frac{7(17A-33B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(11A-21B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{(11A}{}$$

Mathematica [C] time = 7.16, size = 1346, normalized size = 5.30

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] (((-119*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*H
ypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqr
t[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d
*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*
(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeo
metric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(
1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*S
qrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^
((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])
^3 + (((231*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*
x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
```

```
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I
)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hype
rgeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
+ E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d
*x])^3 - (22*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*
x])^3*Sqrt[1 + Cot[c]^2]) + (42*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[C
os[c + d*x]]*((4*(59*A - 99*B + 60*A*Cos[c] - 132*B*Cos[c])*Csc[c])/(5*d) +
 (16*(A - 3*B)*Cos[d*x]*Sin[c])/(3*d) + (8*B*Cos[2*d*x]*Sin[2*c])/(5*d) + (
4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(59*A*Sin[(d*x)/2] - 99*B*Sin[(d*x)/2]))/(5*d
) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(19*A*Sin[(d*x)/2] - 24*B*Sin[(d*x)/2]
))/(15*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/
2]))/(5*d) + (16*(A - 3*B)*Cos[c]*Sin[d*x])/(3*d) + (8*B*Cos[2*c]*Sin[2*d*x
])/(5*d) - (4*(19*A - 24*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A -
 B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{5}+A\cos\left(dx+c\right)^{4}\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^5 + A*cos(d*x + c)^4)*sqrt(cos(d*x + c))/(a^3*cos(
d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3, x
)
```

maple [A] time = 1.42, size = 493, normalized size = 1.94

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(192B\left(\cos^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+160A\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-864B\left(\cos^{10}\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(192*B*cos(1/
2*d*x+1/2*c)^12+160*A*cos(1/2*d*x+1/2*c)^10-864*B*cos(1/2*d*x+1/2*c)^10+468
*A*cos(1/2*d*x+1/2*c)^8+330*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^
5+714*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-228*B*cos(1/2*d*x+1/
2*c)^8-630*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-1386*B*cos(1/2*
d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*A*cos(1/2*d*x+1/2*c)^6+1590*B*c
os(1/2*d*x+1/2*c)^6+474*A*cos(1/2*d*x+1/2*c)^4-744*B*cos(1/2*d*x+1/2*c)^4-4
7*A*cos(1/2*d*x+1/2*c)^2+57*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x
+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{9/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(9/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
[Out] int((cos(c + d*x)^(9/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
[Out] Timed out
```

3.159 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=219

$$-\frac{(13A-33B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(7A-17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{7(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(13A-33}{}$$

```
[Out] 7/10*(7*A-17*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/6*(13*A-33*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*(A-B)
*cos(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/3*(A-2*B)*cos(d*x+c)^(5
/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+7/30*(7*A-17*B)*cos(d*x+c)^(3/2)*sin(
d*x+c)/d/(a^3+a^3*cos(d*x+c))-1/6*(13*A-33*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a
^3/d
```

Rubi [A] time = 0.52, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2748, 2639, 2635, 2641}

$$-\frac{(13A-33B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(7A-17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{7(7A-17B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(13A-33}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (7*(7*A - 17*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 33*B)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) - ((13*A - 33*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(6*a^3*d) + ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*C
os[c + d*x])^3) + ((A - 2*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*a*d*(a + a
*Cos[c + d*x])^2) + (7*(7*A - 17*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*d*
(a^3 + a^3*Cos[c + d*x]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7}{2}a(A-B)-\frac{1}{2}a(3A-13B)\cos(c+\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2}$$

$$= \frac{7(7A-17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-33B)\sqrt{\cos(c+dx)}\sin(c+dx)}{6a^3d}$$

$$= \frac{7(7A-17B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-33B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(13A-}{}$$

Mathematica [C] time = 6.99, size = 1306, normalized size = 5.96

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] (((49*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
 - (((119*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)
*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
```

```
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*
d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 +
E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x
])^3 + (26*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*
x])^3*Sqrt[1 + Cot[c]^2]) - (22*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeomet
ricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[C
os[c + d*x]]*((-4*(29*A - 59*B + 20*A*Cos[c] - 60*B*Cos[c])*Csc[c])/(5*d) +
 (16*B*Cos[d*x]*Sin[c])/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(29*A*Sin[(d
*x)/2] - 59*B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(14*A
*Sin[(d*x)/2] - 19*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]
^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) + (16*B*Cos[c]*Sin[d*x])/(3*d)
+ (4*(14*A - 19*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(A - B)*Sec[c
/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{4}+A\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^4 + A*cos(d*x + c)^3)*sqrt(cos(d*x + c))/(a^3*cos(
d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3, x
)

maple [A] time = 1.13, size = 465, normalized size = 2.12

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-160B\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+348A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130A\sqrt{\frac{1}{2}-\frac{\cos(dx+}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)

[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-160*B*cos(1/2*d*x+1/2*c)^10+348*A*cos(1/2*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-468*B*cos(1/2*d*x+1/2*c)^8-330*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-714*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+1/2*c)^6+1058*B*cos(1/2*d*x+1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-474*B*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2+47*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3, x
)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{7/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
[Out] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
[Out] Timed out
```

3.160 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=188

$$\frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-B)\sin(c+}{5d(a\cos(}$$

```
[Out] -1/10*(9*A-49*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(3*A-13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*(A-B)
*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(3*A-8*B)*cos(d*x+c)
^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+1/6*(3*A-13*B)*sin(d*x+c)*cos(d*x+
c)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.48, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2977, 2748, 2641, 2639}

$$\frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-B)\sin(c+}{5d(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -((9*A - 49*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A - 13*B)*Ellipt
icF[(c + d*x)/2, 2])/(6*a^3*d) + ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/
(5*d*(a + a*Cos[c + d*x])^3) + ((3*A - 8*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x]
)/(15*a*d*(a + a*Cos[c + d*x])^2) + ((3*A - 13*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{1}{2}a(A-11B)\cos(c\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\cos^{\frac{3}{2}}(c+dx)\sin(c+d}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\cos^{\frac{3}{2}}(c+dx)\sin(c+d}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(3A-8B)\cos^{\frac{3}{2}}(c+dx)\sin(c+d}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(A-B}{5}$$

Mathematica [C] time = 6.90, size = 1273, normalized size = 6.77

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] (((-9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
 + (((49*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*
Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sq
rt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*
d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d
*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperge
ometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E
^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]
)^3 - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^
3*Sqrt[1 + Cot[c]^2]) + (26*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTa
n[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Si
n[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*
(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos
[c + d*x]]*((-4*(-9*A + 29*B + 20*B*Cos[c])*Csc[c])/(5*d) + (4*Sec[c/2]*Sec
[c/2 + (d*x)/2]*(9*A*Sin[(d*x)/2] - 29*B*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]
*Sec[c/2 + (d*x)/2]^3*(9*A*Sin[(d*x)/2] - 14*B*Sin[(d*x)/2]))/(15*d) + (2*S
ec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) - (4*
(9*A - 14*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (
d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(cos(d*x + c))/(a^3*cos(
d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3, x
)
```

maple [B] time = 1.25, size = 451, normalized size = 2.40

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(108A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+30A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348*B*cos(1/2*d*x+1/2*c)^8-130*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-294*B*cos(1/2*d*x+1/2*c)^5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6+578*B*cos(1/2*d*x+1/2
*c)^6+114*A*cos(1/2*d*x+1/2*c)^4-264*B*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*
x+1/2*c)^2+37*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.161 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$\frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-B)\sin(c+dx)\,c}{5d(a\cos(c+dx}$$

```
[Out] -1/10*(A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*(A-B)*cos(d
*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+1/15*(A-6*B)*sin(d*x+c)*cos(d*x
+c)^(1/2)/a/d/(a+a*cos(d*x+c))^2+1/10*(A+9*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.47, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2978, 2748, 2641, 2639}

$$\frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A-B)\sin(c+dx)\,c}{5d(a\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -((A + 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*B)*EllipticF[(c
 + d*x)/2, 2])/(6*a^3*d) + ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(
a + a*Cos[c + d*x])^3) + ((A - 6*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*
d*(a + a*Cos[c + d*x])^2) + ((A + 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10
*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-B)+\frac{1}{2}a(A+9B)\cos(c\right.}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(A-B)\cos}{5d(a+}$$

Mathematica [C] time = 6.82, size = 1265, normalized size = 7.03

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]

[Out] ((-1/10*I)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 -
 (((9*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[
(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x
)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
```

```
+ E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqr
t[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((
2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
 - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[
Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^3
*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a +
 a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c +
 d*x]]*((4*(A + 9*B)*Csc[c])/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(4*A*
Sin[(d*x)/2] - 9*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5
*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*
(A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/(5*d) + (4*(4*A - 9*B)*Sec[c/2 + (d*x)
/2]^2*Tan[c/2])/(15*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/
(a + a*Cos[c + d*x])^3
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c))/(a^3*cos(d*
x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x
)
```

maple [B] time = 1.16, size = 451, normalized size = 2.51

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(12A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8+30*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*B*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6-198*B*cos(1/2*d*x+1/2*c)^6-
24*A*cos(1/2*d*x+1/2*c)^4+114*B*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c
)^2-27*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
[Out] Timed out
```

3.162 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)$$

```
[Out] 1/10*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*(A-B)*sin(d*x+c)
*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^3+1/15*(A+4*B)*sin(d*x+c)*cos(d*x+c)^(
1/2)/a/d/(a+a*cos(d*x+c))^2-1/10*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a
^3*cos(d*x+c))
```

Rubi [A] time = 0.46, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2977, 2978, 2748, 2641, 2639}

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(A+4B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((A - B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + B)*EllipticF[(c + d*
x)/2, 2])/(6*a^3*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a
*Cos[c + d*x])^3) + ((A + 4*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*(a
+ a*Cos[c + d*x])^2) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*d*(a^3
 + a^3*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{1}{2}a(3A+7B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A+4B)\sqrt{\cos(c+dx)}\,\sin(c+a}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A+4B)\sqrt{\cos(c+dx)}\,\sin(c+a}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A+4B)\sqrt{\cos(c+dx)}\,\sin(c+a}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(A-B)\sqrt{\cos(}}{5d(a+a}$$

Mathematica [C] time = 6.72, size = 1264, normalized size = 7.10

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] ((I/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeo
metric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^
((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2
F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*
d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 - ((
I/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeome
tric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((
2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1
[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2
```

```
*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E
^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*
x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 - (2*A
*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
 ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]
])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^3*Sqrt[1
 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Co
s[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]
]*((-4*(A - B)*Csc[c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/
2] - B*Sin[(d*x)/2]))/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)
/2] - B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x
)/2] + 4*B*Sin[(d*x)/2]))/(15*d) + (4*(A + 4*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/
2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c
 + d*x])^3
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^3 + 3*a^
3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x
)
```

maple [B] time = 1.28, size = 451, normalized size = 2.53

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(12A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8-10*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*B*cos(1/2*d*x+1/2*c)^5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6+2*B*cos(1/2*d*x+1/2*c)^6+6*A*
cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/2*c)^2-17*
B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}\ \left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.163 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=182

$$\frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A-B)\sin(c+dx)}{15ad(a\cos(c+}$$

```
[Out] 1/10*(9*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(3*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*(A-B)*sin(d*
x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^3-1/15*(6*A-B)*sin(d*x+c)*cos(d*x+
c)^(1/2)/a/d/(a+a*cos(d*x+c))^2-1/10*(9*A+B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/
(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.48, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2978, 2748, 2641, 2639}

$$\frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(6A-B)\sin(c+dx)}{15ad(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] ((9*A + B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A + B)*EllipticF[(c
+ d*x)/2, 2])/(6*a^3*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a
 + a*Cos[c + d*x])^3) - ((6*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d
*(a + a*Cos[c + d*x])^2) - ((9*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*
d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{\frac{1}{2}a(9A+B)-\frac{3}{2}a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(6A-B)\sqrt{\cos(c+dx)}\,\sin(c+d}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(6A-B)\sqrt{\cos(c+dx)}\,\sin(c+d}{15ad(a+a\cos(c+dx))^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(6A-B)\sqrt{\cos(c+dx)}\,\sin(c+d}{15ad(a+a\cos(c+dx))^2}$$

$$= \frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-B)\sqrt{\cos}}{5d(a+a}$$

Mathematica [C] time = 6.80, size = 1265, normalized size = 6.95

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),
x]
```

```
[Out] (((9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hype
rgeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(
2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)
]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1
+ E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeomet
ric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2
*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3
+ ((I/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 -
(2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d
*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot
[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^3*Sqrt
[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*
Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*
x]]*((-4*(9*A + B)*Csc[c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[
(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(6*A*S
in[(d*x)/2] - B*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(9*A
*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(5*d) - (4*(6*A - B)*Sec[c/2 + (d*x)/2]^2*
Tan[c/2])/(15*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a
*Cos[c + d*x])^3
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^4+3\,a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+a^3\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^4 + 3*a^
3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))),
 x)
```

maple [B] time = 1.23, size = 451, normalized size = 2.48

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(108A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-30A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/2
*d*x+1/2*c)^8-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos(
1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8-10*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*B*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-138*A*cos(1/2*d*x+1/2*c)^6-22*B*cos(1/2*d*x+1/2*c)^6+
24*A*cos(1/2*d*x+1/2*c)^4+6*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*x+1/2*c)^2
+7*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,\cos\left(c + d\,x\right)}{\sqrt{\cos\left(c + d\,x\right)}\,\left(a + a\,\cos\left(c + d\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.164 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(49A-9B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{(13A-3B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}$$

```
[Out] -1/10*(49*A-9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/6*(13*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/10*(49*
A-9*B)*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)-1/5*(A-B)*sin(d*x+c)/d/(a+a*cos(d*
x+c))^3/cos(d*x+c)^(1/2)-1/15*(8*A-3*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2/c
os(d*x+c)^(1/2)-1/6*(13*A-3*B)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))/cos(d*x+c)
^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2978, 2748, 2636, 2639, 2641}

$$-\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(49A-9B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{(13A-3B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] -((49*A - 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 3*B)*Ellipt
icF[(c + d*x)/2, 2])/(6*a^3*d) + ((49*A - 9*B)*Sin[c + d*x])/(10*a^3*d*Sqrt
[Cos[c + d*x]]) - ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos
[c + d*x])^3) - ((8*A - 3*B)*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a +
a*Cos[c + d*x])^2) - ((13*A - 3*B)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a
^3 + a^3*Cos[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(11A-B)-\frac{5}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3} - \frac{(8A-3B)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\cos(c$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3} - \frac{(8A-3B)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\cos(c$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3} - \frac{(8A-3B)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a+a\cos(c$$

$$= -\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(49A-9B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{(A-}{5d\sqrt{\cos(c+}$$

$$= -\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(49A-9}{10a^3d}$$

<u>**Mathematica**</u> **[C]** time = 7.11, size = 1305, normalized size = 5.90

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),
x]
```

```
[Out] (((-49*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
3 + (((9*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*
```

```
Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sq
rt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*
d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d
*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperge
ometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E
^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x]
)^3 + (26*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/
4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x
])^3*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*
(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos
[c + d*x]]*((2*(20*A + 29*A*Cos[c] - 9*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/
(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(29*A*Sin[(d*x)/2] - 9*B*Sin[(d*x)/2
]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(11*A*Sin[(d*x)/2] - 6*B*Sin[(
d*x)/2]))/(15*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin
[(d*x)/2]))/(5*d) + (16*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (4*(11*A - 6*B)
*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Ta
n[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^5+3\,a^3\cos(dx+c)^4+3\,a^3\cos(dx+c)^3+a^3\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^5 + 3*a^
3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)),
 x)
```

maple [B] time = 1.66, size = 685, normalized size = 3.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x
+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(49*A-9*B)*
sin(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(817*A-147*B)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(248*A-43*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(439*A-69*B)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*
d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos(c+dx)}{\cos(c+dx)^{3/2}\left(a+a\cos(c+dx)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
[Out] Timed out
```

3.165 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=254

$$\frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{7(17A-7B)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(33A-13B}{6a^3d\cos}$$

```
[Out] 7/10*(17*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(33*A-13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(33*A
-13*B)*sin(d*x+c)/a^3/d/cos(d*x+c)^(3/2)-1/5*(A-B)*sin(d*x+c)/d/cos(d*x+c)^
(3/2)/(a+a*cos(d*x+c))^3-1/3*(2*A-B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*c
os(d*x+c))^2-7/30*(17*A-7*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a^3+a^3*cos(d*x
+c))-7/10*(17*A-7*B)*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2978, 2748, 2636, 2641, 2639}

$$\frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{7(17A-7B)\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{(33A-13B}{6a^3d\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] (7*(17*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((33*A - 13*B)*Elli
pticF[(c + d*x)/2, 2])/(6*a^3*d) + ((33*A - 13*B)*Sin[c + d*x])/(6*a^3*d*Co
s[c + d*x]^(3/2)) - (7*(17*A - 7*B)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*
x]]) - ((A - B)*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^
3) - ((2*A - B)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]
)^2) - (7*(17*A - 7*B)*Sin[c + d*x])/(30*d*Cos[c + d*x]^(3/2)*(a^3 + a^3*Co
s[c + d*x]))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(13A-3B)-\frac{7}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{(2A-B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+$$

$$= \frac{(33A-13B)\sin(c+dx)}{6a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{7(17A-7B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{(A-B)}{5d\cos^{\frac{3}{2}}(c+dx)(}$$

$$= \frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(33A-}{6a^3c}$$

Mathematica [C] time = 7.81, size = 1346, normalized size = 5.30

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),
x]
```

```
[Out] (((119*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hy
pergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(
1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1
 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sq
rt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^(
```

```
(2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^
3 - (((49*I)/10)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)
*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*S
qrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I
*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*
d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 +
E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x
])^3 - (22*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Ar
cTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x]
)^3*Sqrt[1 + Cot[c]^2]) + (26*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - Arc
Tan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*
Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*
d*(a + a*Cos[c + d*x])^3*Sqrt[1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[C
os[c + d*x]]*((-2*(60*A - 20*B + 59*A*Cos[c] - 29*B*Cos[c])*Csc[c/2]*Sec[c/
2]*Sec[c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(59*A*Sin[(d*x)/2] - 29*B
*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(16*A*Sin[(d*x)/2]
 - 11*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*
x)/2] - B*Sin[(d*x)/2]))/(5*d) + (16*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d
) + (16*Sec[c]*Sec[c + d*x]*(A*Sin[c] - 9*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d)
 - (4*(16*A - 11*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(A - B)*Sec[
c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^6+3\,a^3\cos(dx+c)^5+3\,a^3\cos(dx+c)^4+a^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^6 + 3*a^
3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + a^3*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)),
 x)
```

maple [B] time = 1.68, size = 876, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/60*(4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(165*A*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-357*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-10*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(165*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*A*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
8*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(165*A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-357*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(165*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*A*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-168*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(17*A-7*B)*sin(1/2*d*x+1/2*c)^10+8*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(1167*A-482*B)*sin(1/2*d*x+1/2*c)
^8-10*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(1111*A-461*B)*s
in(1/2*d*x+1/2*c)^6+14*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(404*A-169*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(1029*A-439*B)*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(3/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)^5/a^3/sin(1/2*d*x+1/2*c)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.166 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=221

$$\frac{a(8A+7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{5a(8A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{5\sqrt{a}\,(8A+7B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}$$

```
[Out] 5/64*(8*A+7*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+
5/96*a*(8*A+7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/24*
a*(8*A+7*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a*B*co
s(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+5/64*a*(8*A+7*B)*sin(d*x
+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2981, 2770, 2774, 216}

$$\frac{a(8A+7B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{5a(8A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{5\sqrt{a}\,(8A+7B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (5*Sqrt[a]*(8*A + 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(64*d) + (5*a*(8*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqrt[
a + a*Cos[c + d*x]]) + (5*a*(8*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(9
6*d*Sqrt[a + a*Cos[c + d*x]]) + (a*(8*A + 7*B)*Cos[c + d*x]^(5/2)*Sin[c + d
*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(7/2)*Sin[c + d*x]
)/(4*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{1}{8}(8A+7B)\int \cos^{\frac{5}{2}}$$

$$= \frac{a(8A+7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{7}{2}}(c}{4d\sqrt{a+}}$$

$$= \frac{5a(8A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a(8A+7}{24}$$

$$= \frac{5a(8A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{5a(8A+}{9}$$

$$= \frac{5a(8A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{5a(8A+}{9}$$

$$= \frac{5\sqrt{a}\,(8A+7B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{5a(8A+7}{64}$$

Mathematica [A] time = 1.03, size = 135, normalized size = 0.61

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}(8A+7B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+d}\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(8*A + 7*B)*ArcSin
[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(152*A + 133*B + 2*(40*A
+ 53*B)*Cos[c + d*x] + 4*(8*A + 7*B)*Cos[2*(c + d*x)] + 12*B*Cos[3*(c + d*x
)])*Sin[(c + d*x)/2]))/(384*d)
```

fricas [A] time = 1.00, size = 151, normalized size = 0.68

$$\frac{\left(48\,B\cos\left(dx+c\right)^{3}+8\,(8\,A+7\,B)\cos\left(dx+c\right)^{2}+10\,(8\,A+7\,B)\cos\left(dx+c\right)+120\,A+105\,B\right)\sqrt{a\cos\left(dx+c\right)+}}{192\,(d\cos\left(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/192*((48*B*cos(d*x + c)^3 + 8*(8*A + 7*B)*cos(d*x + c)^2 + 10*(8*A + 7*B)
*cos(d*x + c) + 120*A + 105*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c) - 15*((8*A + 7*B)*cos(d*x + c) + 8*A + 7*B)*sqrt(a)*arctan(sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*
x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos\left(dx+c\right)+A)\sqrt{a\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2),
 x)
```

maple [B] time = 0.32, size = 428, normalized size = 1.94

$$\frac{(-1+\cos(dx+c))^4 \left(64A\sin(dx+c)\left(\cos^3(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+144A\sin(dx+c)\left(\cos^2(dx+c)\right)\left(\frac{\cos(dx}{1+\cos(dx}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/192/d*(-1+cos(d*x+c))^4*(64*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(
d*x+c)))^(3/2)+144*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)+48*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+200*A*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+56*B*sin(d*x+c)*cos(d
*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+120*A*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)+70*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)+105*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+120*A*c
os(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+1
05*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x
+c)))*cos(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^8/(cos(d*x+c)/(1
+cos(d*x+c)))^(7/2)
```

maxima [B] time = 2.98, size = 8220, normalized size = 37.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
```

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*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*si
n(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(
arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) -
cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2
*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(
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2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*A + (2*(cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)^(3/4)*((156*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - 2*cos(4*d*
x + 4*c) + 1)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + 39*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 39*sin(4*d*x + 4*c)^
3 + 156*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)
*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
 39*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c
) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) + sin(4*d*x + 4*c))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) + 156*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*si
n(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (32*
(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + s
in(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 +
 16*sin(4*d*x + 4*c)^2 - 55*cos(4*d*x + 4*c) + 39)*cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 55*sin(4*d*x + 4*c
))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 39*cos(4*d*x + 4*
c))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 156*(4*cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + sin(4*d*x
+ 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (39*cos(4*d*x + 4*c)^3 + 4*(39
*cos(4*d*x + 4*c)^3 + (39*cos(4*d*x + 4*c) - 8)*sin(4*d*x + 4*c)^2 - 86*cos
(4*d*x + 4*c)^2 + 55*cos(4*d*x + 4*c) - 8)*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c)))^2 + (39*cos(4*d*x + 4*c) - 8)*sin(4*d*x + 4*c)^2 + 4*(
39*cos(4*d*x + 4*c)^3 + (39*cos(4*d*x + 4*c) - 8)*sin(4*d*x + 4*c)^2 + 70*c
os(4*d*x + 4*c)^2 + 23*cos(4*d*x + 4*c) - 8)*sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))^2 - 8*cos(4*d*x + 4*c)^2 + (32*(cos(4*d*x + 4*c)^2 +
 sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2
*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^
2 - 55*cos(4*d*x + 4*c) + 39)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 55*sin(4*d*x + 4*c))*sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) - 39*cos(4*d*x + 4*c))*cos(3/4*arctan2(s
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in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(39*cos(4*d*x + 4*c)^3 + (39*cos(4*d*x + 4*c) - 8)*sin(4*d*x + 4*c)^2 - 47*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 39*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(39*cos(4*d*x + 4*c) - 8)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (39*cos(4*d*x + 4*c) - 8)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 6*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((4*(11*sin(4*d*x + 4*c)^3 + 11*(cos(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c) - 24*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 11*cos(4*d*x + 4*c)^2sin(4*d*x + 4*c) + 11*sin(4*d*x + 4*c)^3 + 4*(11*sin(4*d*x + 4*c)^3 + 11*(cos(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c) - 24*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(22*sin(4*d*x + 4*c)^3 + 22*(cos(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*x + 4*c) + 11*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - (48*cos(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)^2 - 37*cos(4*d*x + 4*c) - 11)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 11*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(8*(11*sin(4*d*x + 4*c)^2 - 24*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 11*(cos(4*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 22*sin(4*d*x + 4*c)^2 - 37*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (24*cos(4*d*x + 4*c)^2 + 24*sin(4*d*x + 4*c)^2 + 11*cos(4*d*x + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (11*cos(4*d*x + 4*c)^3 + 4*(11*cos(4*d*x + 4*c)^3 + (11*cos(4*d*x + 4*c) + 24)*sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c)^2 - 24*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 37*cos(4*d*x + 4*c) + 24)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (11*cos(4*d*x + 4*c) + 24)*sin(4*d*x + 4*c)^2 + 4*(11*cos(4*d*x + 4*c)^3 + (11*cos(4*d*x + 4*c) + 24)*sin(4*d*x + 4*c)^2 + 46*cos(4*d*x + 4*c)^2 - 24*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 59*cos(4*d*x + 4*c) + 24)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 24*cos(4*d*x + 4*c)^2 + 2*(22*cos

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(4*d*x + 4*c)^3 + 2*(11*cos(4*d*x + 4*c) + 24)*sin(4*d*x + 4*c)^2 + 26*cos(
4*d*x + 4*c)^2 - (48*cos(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)^2 - 37*cos(4*
d*x + 4*c) - 11)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 11*
sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 48*
cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (2
4*cos(4*d*x + 4*c)^2 + 24*sin(4*d*x + 4*c)^2 + 11*cos(4*d*x + 4*c))*cos(1/4
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(8*((11*cos(4*d*x + 4*c)
+ 24)*sin(4*d*x + 4*c) - 24*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 2*(11*cos(4*d*x + 4*c) + 24)*sin(4*d*x + 4*c) - 37*cos(1/4*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 11*(cos(4*d*x + 4*c) +
1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) - 11*sin(4*d*x + 4*c)*sin(1/4*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)))*sqrt(a) + 105*((4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2
- 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1
)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)
^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(
4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2
(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(
1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (4*(cos(4*d*x + 4
*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)
^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2
- cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c),
```

```
cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan
2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*si
n(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
)) - 1) - (4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c)
+ 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x
+ 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4
*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
+ 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))
 + 1) + (4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) +
1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x +
4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c
)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
```

```
+ 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) -
 1))*sqrt(a))*B/(4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x +
 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(
4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d
*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^{5/2}\,(A+B\,\cos(c+dx))\,\sqrt{a+a\,\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.167 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=176

$$\frac{a(6A+5B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\,(6A+5B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(6A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 1/8*(6*A+5*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+1
/12*a*(6*A+5*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a*
B*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/8*a*(6*A+5*B)*sin(
d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2981, 2770, 2774, 216}

$$\frac{a(6A+5B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\,(6A+5B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(6A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(8*d) + (a*(6*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*(6*A + 5*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt
[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a +
 a*Cos[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{6}(6A+5B)\int \cos^{\frac{3}{2}}$$

$$= \frac{a(6A+5B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{5}{2}}(c}{3d\sqrt{a+}}$$

$$= \frac{a(6A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a(6A+5B}{12d}$$

$$= \frac{a(6A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a(6A+5B}{12d}$$

$$= \frac{\sqrt{a}\,(6A+5B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a(6A+5B)}{8d\sqrt{}}$$

Mathematica [A] time = 0.55, size = 118, normalized size = 0.67

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,\left(3\sqrt{2}\,(6A+5B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+}\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(6*A + 5*B)*ArcSin[
Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(18*A + 19*B + 2*(6*A + 5*
B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

fricas [A] time = 1.00, size = 134, normalized size = 0.76

$$\frac{\left(8\,B\cos\left(dx+c\right)^{2}+2\left(6\,A+5\,B\right)\cos\left(dx+c\right)+18\,A+15\,B\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-3\,(\!(}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/24*((8*B*cos(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 18*A + 15*B)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((6*A + 5*B)*cos(d*
x + c) + 6*A + 5*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2),
 x)
```

maple [B] time = 0.39, size = 356, normalized size = 2.02

$$-\frac{\left(-1+\cos\left(dx+c\right)\right)^{3}\left(12A\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+30A\sin\left(dx+c\right)\cos\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^3*(12*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(3/2)+30*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
+8*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+18*A*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+10*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)+15*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)+18*A*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)/cos(d*x+c))+15*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/cos(d*x+c)))*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c
)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^6
```

maxima [B] time = 1.79, size = 2981, normalized size = 16.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*a
```

```
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - 1)))*A + (4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin
(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 5*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
- 4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sq
rt(a)*(arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2
(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(co
s(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
```

```
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*
sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*co
s(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*B)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^{3/2}\,(A+B\cos(c+dx))\,\sqrt{a+a\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.168 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{a}\,(4A+3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(4A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 1/4*(4*A+3*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+1
/2*a*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a*(4*A+3*B)
*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2981, 2770, 2774, 216}

$$\frac{\sqrt{a}\,(4A+3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(4A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(4*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/(4*d) + (a*(4*A + 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*
Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[
c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{4}(4A+3B)\int \sqrt{\mathrm{c}$$

$$= \frac{a(4A+3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{3}{2}}(}{2d\sqrt{a}}$$

$$= \frac{a(4A+3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{3}{2}}(}{2d\sqrt{a}}$$

$$= \frac{\sqrt{a}\,(4A+3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{a(4A+3B}{4d}$$

Mathematica [A] time = 0.30, size = 100, normalized size = 0.76

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\,(4A+3B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(4*A + 3*B)*ArcSin[Sq
rt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(4*A + 3*B + 2*B*Cos[c + d*x
])*Sin[(c + d*x)/2]))/(8*d)
```

fricas [A] time = 1.03, size = 117, normalized size = 0.89

$$\frac{(2\,B\cos\left(dx+c\right)+4\,A+3\,B)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-((4\,A+3\,B)\cos\left(dx+c\right)+4\,A+3}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/4*((2*B*cos(d*x + c) + 4*A + 3*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))*sin(d*x + c) - ((4*A + 3*B)*cos(d*x + c) + 4*A + 3*B)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(
d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.28, size = 284, normalized size = 2.17

$$\frac{(-1+\cos\left(dx+c\right))^{2}\left(4A\sin\left(dx+c\right)\cos\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+4A\sin\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+2B\sin\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))^2*(4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*B*sin(d*x+c)*c
os(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*B*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*A*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)
```

```
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))
^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4
```

maxima [B] time = 1.37, size = 1851, normalized size = 14.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A + (2*(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2
*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x +
 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d
*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2
*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - cos(2*d*
x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqr
t(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arcta
```

```
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(c+dx)}\ (A+B\ \cos(c+dx))\ \sqrt{a+a\ \cos(c+dx)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\cos(c+dx)+1)}\,(A+B\cos(c+dx))\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sqrt(cos(c + d*x))
, x)
```

3.169 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{a}\,(2A+B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*A+B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+a*B*sin
(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2981, 2774, 216}

$$\frac{\sqrt{a}\,(2A+B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(2*A + B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])
/d + (a*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{aB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{1}{2}(2A+B)\int \frac{\sqrt{a+a\cos(c+dx}}{\sqrt{\cos(c+dx)}}$$

$$= \frac{aB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{(2A+B)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\right.}{d}$$

$$= \frac{\sqrt{a}\,(2A+B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{aB\sqrt{\cos(c+dx)}\,\sin(c+dx}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.16, size = 83, normalized size = 1.06

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\,(2A+B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(2*A + B)*ArcSin[Sqrt
[2]*Sin[(c + d*x)/2]] + 2*B*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

fricas [A] time = 1.01, size = 97, normalized size = 1.24

$$\frac{\sqrt{a\cos(dx+c)+a}\,B\sqrt{\cos(dx+c)}\,\sin(dx+c) - ((2A+B)\cos(dx+c)+2A+B)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\text{co}}}{\sqrt{a}\,\sin(dx+c}\right.}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A + B)*co
s(d*x + c) + 2*A + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.25, size = 164, normalized size = 2.10

$$-\frac{(-1+\cos(dx+c))\left(B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+2A\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+B\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx}{1+\cos(d}}}{\cos(dx+c)}\right.\right.}{d\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))*(B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*A*ar
ctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+B*arctan(sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)^(1/2)*(a*(
1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2
```

maxima [B] time = 1.46, size = 939, normalized size = 12.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(4*A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
```

```
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}}{\sqrt{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),x)
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\cos(c+dx)+1)}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/sqrt(cos(c + d*x))
, x)
```

3.170 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}\,B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+2*a*A*sin(d
*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2980, 2774, 216}

$$\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}\,B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a]*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
```

```
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + B\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,d$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - \frac{(2B)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x\right.}{d}$$

$$= \frac{2\sqrt{a}\,B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx}}$$

Mathematica [A] time = 0.17, size = 86, normalized size = 1.13

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2A\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\,B\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*A*Sin[(c + d*x)/2]))/(d*Sqrt[Cos[c + d
*x]])
```

fricas [A] time = 1.07, size = 109, normalized size = 1.43

$$\frac{2\left(\sqrt{a\cos(dx+c)+a}\,A\sqrt{\cos(dx+c)}\,\sin(dx+c)-\left(B\cos(dx+c)^2+B\cos(dx+c)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}}{\sqrt{a}\,\sin(dx}\right.\right.}{d\cos(dx+c)^2+d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 2*(sqrt(a*cos(d*x + c) + a)*A*sqrt(cos(d*x + c))*sin(d*x + c) - (B*cos(d*x
+ c)^2 + B*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.25, size = 109, normalized size = 1.43

$$-\frac{2\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(-B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\sin\left(dx+c\right)+A\cos\left(dx+c\right)-A\right)}{d\sin\left(dx+c\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/d*(a*(1+cos(d*x+c)))^(1/2)*(-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+A*cos(d
*x+c)-A)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

maxima [B] time = 0.99, size = 245, normalized size = 3.22

$$\frac{B\sqrt{a}\arctan\left(\left(\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] (B*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos
```

```
(d*x + c)) + 2*A*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}}{\cos(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\cos(c+dx)+1)}\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/cos(c + d*x)**(3/2
), x)
```

3.171 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{2a(2A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/3*a*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+2/3*a*(2*A+3*B
)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2980, 2771}

$$\frac{2a(2A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
*a*(2*A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x
]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{3}(2A+3B)\int \frac{\sqrt{a+a\cos}}{\cos^{\frac{3}{2}}(c}$$
$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(2A+3B)\sin(c+}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.15, size = 57, normalized size = 0.67

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,((2A+3B)\cos(c+dx)+A)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(A + (2*A + 3*B)*Cos[c + d*x])*Tan[(c + d*x)/
2])/(3*d*Cos[c + d*x]^(3/2))
```

fricas [A] time = 0.78, size = 67, normalized size = 0.79

$$\frac{2\,((2\,A+3\,B)\cos(dx+c)+A)\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{3\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 2/3*((2*A + 3*B)*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.22, size = 62, normalized size = 0.73

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(2A\cos\left(dx+c\right)+3B\cos\left(dx+c\right)+A\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}}{3d\sin\left(dx+c\right)\cos\left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(2*A*cos(d*x+c)+3*B*cos(d*x+c)+A)*(a*(1+cos(d*x+c)))
^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

maxima [B] time = 0.96, size = 289, normalized size = 3.40

$$\frac{2\left(\frac{3B\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}+\frac{A\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*B*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c) + 1) + 1
)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)) + A*(3*sqrt(2)*sqrt(a
)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)*(sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) +
1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

mupad [B] time = 1.56, size = 112, normalized size = 1.32

$$\frac{2\sqrt{a}\,\left(\cos\left(c+d\,x\right)+1\right)\,\left(2\,A\,\sin\left(c+d\,x\right)+3\,B\,\sin\left(c+d\,x\right)+2\,A\,\sin\left(2\,c+2\,d\,x\right)+2\,A\,\sin\left(3\,c+3\,d\,x\right)+3\,B}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(3\,\cos\left(c+d\,x\right)+2\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(3\,c+3\,d\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*(a*(cos(c + d*x) + 1))^(1/2)*(2*A*sin(c + d*x) + 3*B*sin(c + d*x) + 2*A*
sin(2*c + 2*d*x) + 2*A*sin(3*c + 3*d*x) + 3*B*sin(3*c + 3*d*x)))/(3*d*cos(c
 + d*x)^(1/2)*(3*cos(c + d*x) + 2*cos(2*c + 2*d*x) + cos(3*c + 3*d*x) + 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\left(A+B\cos\left(c+dx\right)\right)}{\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/cos(c + d*x)**(5/2
), x)
```

3.172 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{2a(4A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a(4A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/5*a*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+2/15*a*(4*A+5*
B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+4/15*a*(4*A+5*B)*si
n(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2980, 2772, 2771}

$$\frac{2a(4A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a(4A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*
x]]) + (4*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{5}(4A+5B)\int \frac{\sqrt{a+a\cos}}{\cos^{\frac{5}{2}}(c}$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(4A+5B)\sin(c+}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,c}}$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(4A+5B)\sin(c+}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,c}}$$

Mathematica [A] time = 0.25, size = 78, normalized size = 0.60

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,((4A+5B)\cos(c+dx)+(4A+5B)\cos(2(c+dx))+7A+5B)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(7*A + 5*B + (4*A + 5*B)*Cos[c + d*x] + (4*A
+ 5*B)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

fricas [A] time = 1.02, size = 86, normalized size = 0.66

$$\frac{2\left(2\,(4\,A+5\,B)\cos(dx+c)^2+(4\,A+5\,B)\cos(dx+c)+3\,A\right)\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{15\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(2*(4*A + 5*B)*cos(d*x + c)^2 + (4*A + 5*B)*cos(d*x + c) + 3*A)*sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*co
s(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.24, size = 86, normalized size = 0.66

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(8A\left(\cos^{2}\left(dx+c\right)\right)+10B\left(\cos^{2}\left(dx+c\right)\right)+4A\cos\left(dx+c\right)+5B\cos\left(dx+c\right)+3A\right)\sqrt{a\,(}}{15d\sin\left(dx+c\right)\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(8*A*cos(d*x+c)^2+10*B*cos(d*x+c)^2+4*A*cos(d*x+c)+
5*B*cos(d*x+c)+3*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

maxima [B] time = 0.73, size = 428, normalized size = 3.29

$$\frac{2\left(\frac{5B\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}+\frac{A\left(\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\right.}\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(5*B*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sq
rt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x
 + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
```

```
(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 + 1)) + A*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*
sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*
(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

mupad [B] time = 3.25, size = 194, normalized size = 1.49

$$\frac{4\sqrt{a}\,\left(\cos\left(c+d\,x\right)+1\right)\,\left(14\,A\,\sin\left(c+d\,x\right)+10\,B\,\sin\left(c+d\,x\right)+8\,A\,\sin\left(2\,c+2\,d\,x\right)+18\,A\,\sin\left(3\,c+3\,d\,x\right)+4\cdots\right.}{15\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2),x)
```

```
[Out] (4*(a*(cos(c + d*x) + 1))^(1/2)*(14*A*sin(c + d*x) + 10*B*sin(c + d*x) + 8*
A*sin(2*c + 2*d*x) + 18*A*sin(3*c + 3*d*x) + 4*A*sin(4*c + 4*d*x) + 4*A*sin
(5*c + 5*d*x) + 10*B*sin(2*c + 2*d*x) + 15*B*sin(3*c + 3*d*x) + 5*B*sin(4*c
 + 4*d*x) + 5*B*sin(5*c + 5*d*x)))/(15*d*cos(c + d*x)^(1/2)*(10*cos(c + d*x
) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos(4*c + 4*d*x) + cos(5*c
+ 5*d*x) + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.173 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{8a(6A+7B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a(6A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx}}$$

```
[Out] 2/7*a*A*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+2/35*a*(6*A+7*
B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+8/105*a*(6*A+7*B)*s
in(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+16/105*a*(6*A+7*B)*sin(
d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2980, 2772, 2771}

$$\frac{8a(6A+7B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a(6A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*a*A*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
*a*(6*A + 7*B)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*
x]]) + (8*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*
Cos[c + d*x]]) + (16*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}(6A+7B)\int \frac{\sqrt{a+a\cos}}{\cos^{\frac{7}{2}}(c}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(6A+7B)\sin(c+}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\,c}}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(6A+7B)\sin(c+}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\,c}}$$

$$= \frac{2aA\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a(6A+7B)\sin(c+}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\,c}}$$

Mathematica [A] time = 0.40, size = 102, normalized size = 0.58

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,(9(6A+7B)\cos(c+dx)+2(6A+7B)\cos(2(c+dx))+12A\cos(3(c+dx)}{105d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2
),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(27*A + 14*B + 9*(6*A + 7*B)*Cos[c + d*x] + 2
*(6*A + 7*B)*Cos[2*(c + d*x)] + 12*A*Cos[3*(c + d*x)] + 14*B*Cos[3*(c + d*x
)])*Tan[(c + d*x)/2])/(105*d*Cos[c + d*x]^(7/2))
```

fricas [A] time = 0.96, size = 104, normalized size = 0.59

$$\frac{2\left(8\,(6\,A+7\,B)\cos\left(dx+c\right)^{3}+4\,(6\,A+7\,B)\cos\left(dx+c\right)^{2}+3\,(6\,A+7\,B)\cos\left(dx+c\right)+15\,A\right)\sqrt{a\cos\left(dx+c\right)+}}{105\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(8*(6*A + 7*B)*cos(d*x + c)^3 + 4*(6*A + 7*B)*cos(d*x + c)^2 + 3*(6*A
 + 7*B)*cos(d*x + c) + 15*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*si
n(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.24, size = 108, normalized size = 0.62

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(48A\left(\cos^{3}\left(dx+c\right)\right)+56B\left(\cos^{3}\left(dx+c\right)\right)+24A\left(\cos^{2}\left(dx+c\right)\right)+28B\left(\cos^{2}\left(dx+c\right)\right)\right.}{105d\sin\left(dx+c\right)\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(48*A*cos(d*x+c)^3+56*B*cos(d*x+c)^3+24*A*cos(d*x+
c)^2+28*B*cos(d*x+c)^2+18*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*(a*(1+cos(d*x+
c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

maxima [B] time = 1.04, size = 522, normalized size = 2.98

$$\frac{2\left(\frac{7\,B\left(\frac{15\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{17\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{7\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{3}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}+\frac{3\,A\left(\frac{35\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{70\,\sqrt{2}\,\sqrt{a}\,\sin(dx}{(\cos(dx+c)+1}\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx}{\cos(dx+}\right.}\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 2/105*(7*B*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 3*A*(35*sqrt(2)*sqrt(a)*sin(d*x
+ c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 58*sqrt(2)
*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x +
c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(co
s(d*x + c) + 1)^8 + 1)))/d
```

mupad [B] time = 6.23, size = 479, normalized size = 2.74

$$\frac{\sqrt{a+a\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}\right)}\,\left(\frac{(96\,A+112\,B)\,1\mathrm{i}}{105\,d}-\frac{B\,\mathrm{e}^{c\,3\mathrm{i}+\cdots}}{3\cdots}\right.}{\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+3\,\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}+3\,\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\sqrt{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\cdots}}{2}\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] ((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(((96*A + 11
2*B)*1i)/(105*d) - (B*exp(c*3i + d*x*3i)*8i)/(3*d) + (B*exp(c*4i + d*x*4i)*
8i)/(3*d) - (exp(c*7i + d*x*7i)*(96*A + 112*B)*1i)/(105*d) + (exp(c*2i + d*
x*2i)*(336*A + 392*B)*1i)/(105*d) - (exp(c*5i + d*x*5i)*(336*A + 392*B)*1i)
/(105*d)))/((exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*1
i + d*x*1i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c
*2i + d*x*2i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp
(c*3i + d*x*3i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*e
xp(c*4i + d*x*4i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3
*exp(c*5i + d*x*5i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) +
 exp(c*6i + d*x*6i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) +
 exp(c*7i + d*x*7i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.174 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=227

$$\frac{a^{3/2}(88A+75B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)\cos^{\frac{3}{2}}(\ldots}{96d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 1/64*a^(3/2)*(88*A+75*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/
d+1/96*a^2*(88*A+75*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
+1/24*a^2*(8*A+9*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/
64*a^2*(88*A+75*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/4
*a*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.50, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2981, 2770, 2774, 216}

$$\frac{a^2(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(88A+75B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(\ldots}{\sqrt{a\cos(c\ldots}}\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(64*d) + (a^2*(88*A + 75*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqr
t[a + a*Cos[c + d*x]]) + (a^2*(88*A + 75*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x]
)/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(8*A + 9*B)*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(5/2)*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{4d} +$$

$$= \frac{a^2(8A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos^{\frac{5}{2}}(c}{}$$

$$= \frac{a^2(88A+75B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(8A}{}$$

$$= \frac{a^2(88A+75B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(88A}{}$$

$$= \frac{a^2(88A+75B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(88A}{}$$

$$= \frac{a^{3/2}(88A+75B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^2(88A+}{}$$

Mathematica [A] time = 1.12, size = 136, normalized size = 0.60

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\,(88A+75B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c}\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(88*A + 75*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(296*A + 285*B + 2*(88
*A + 93*B)*Cos[c + d*x] + 4*(8*A + 15*B)*Cos[2*(c + d*x)] + 12*B*Cos[3*(c +
 d*x)])*Sin[(c + d*x)/2]))/(384*d)
```

fricas [A] time = 1.17, size = 162, normalized size = 0.71

$$\frac{\left(48\,Ba\cos\left(dx+c\right)^{3}+8\,(8\,A+15\,B)a\cos\left(dx+c\right)^{2}+2\,(88\,A+75\,B)a\cos\left(dx+c\right)+3\,(88\,A+75\,B)a\right)\sqrt{a\cos\left(d\right.}}{192\,(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/192*((48*B*a*cos(d*x + c)^3 + 8*(8*A + 15*B)*a*cos(d*x + c)^2 + 2*(88*A +
 75*B)*a*cos(d*x + c) + 3*(88*A + 75*B)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))*sin(d*x + c) - 3*((88*A + 75*B)*a*cos(d*x + c) + (88*A + 75*B)*
a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(
d*x + c))))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2
), x)
```

maple [B] time = 0.25, size = 429, normalized size = 1.89

$$-\frac{a(-1+\cos(dx+c))^3\left(64A\sin(dx+c)\left(\cos^3(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+240A\sin(dx+c)\left(\cos^2(dx+c)\right)\left(\frac{\cos}{1+\cos}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -1/192/d*a*(-1+cos(d*x+c))^3*(64*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)+240*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(3/2)+48*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+440*A
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+120*B*sin(d*x+c)*c
os(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+264*A*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)+150*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)+225*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+26
4*A*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+
c))+225*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x
+c)))^(5/2)/sin(d*x+c)^6
```

maxima [B] time = 3.18, size = 8904, normalized size = 39.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*
d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqr
t(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/
4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(si
n(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1)
 - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
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)) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*arc
tan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqrt(
a))*A + 3*(2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((9*a*cos(4*d*x + 4*c)^2*sin(4*
d*x + 4*c) + 9*a*sin(4*d*x + 4*c)^3 + 36*(a*sin(4*d*x + 4*c)^3 + (a*cos(4*d
*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(4*d*x + 4*c))*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 36*(a*sin(4*d*x + 4*c)^3 + (a*cos(4
*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(4*d*x + 4*c))*sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 9*(2*a*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c) - 2*(a*cos
(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*co
s(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 36*(a*sin(4*d*x + 4*c)
^3 + (a*cos(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (8*a*cos(4*d*x + 4*c)^2 + 32
*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*a*sin(4*d*x + 4*c
)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c
) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 9*a*cos(4*d
*x + 4*c) + 2*(16*a*cos(4*d*x + 4*c)^2 + 16*a*sin(4*d*x + 4*c)^2 - 25*a*cos
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(4*d*x + 4*c) + 9*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 25*a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 36*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (9*a*cos(4*d*x + 4*c)^3 - 8*a*cos(4*d*x + 4*c)^2 + 4*(9*a*cos(4*d*x + 4*c)^3 - 26*a*cos(4*d*x + 4*c)^2 + (9*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 25*a*cos(4*d*x + 4*c) - 8*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (9*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 4*(9*a*cos(4*d*x + 4*c)^3 + 10*a*cos(4*d*x + 4*c)^2 + (9*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 - 7*a*cos(4*d*x + 4*c) - 8*a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (8*a*cos(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*a*sin(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 9*a*cos(4*d*x + 4*c) + 2*(16*a*cos(4*d*x + 4*c)^2 + 16*a*sin(4*d*x + 4*c)^2 - 25*a*cos(4*d*x + 4*c) + 9*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 25*a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(9*a*cos(4*d*x + 4*c)^3 - 17*a*cos(4*d*x + 4*c)^2 + (9*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 8*a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 9*(2*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c) - 2*(a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(9*a*cos(4*d*x + 4*c) - 8*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (9*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((7*a*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 7*a*sin(4*d*x + 4*c)^3 - 48*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 4*(7*a*sin(4*d*x + 4*c)^3 + 7*(a*cos(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(4*d*x + 4*c) - 68*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 7*a*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 4*(7*a*sin(4*d*x + 4*c)^3 + 48*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (7*a*cos(4*

d*x + 4*c)^2 + 14*a*cos(4*d*x + 4*c) + 19*a)*sin(4*d*x + 4*c) - 68*(a*cos(4
*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/4*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + 2*(14*a*sin(4*d*x + 4*c)^3 + 7*a*cos(1/4*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 14*(a*cos(4*d*x + 4
*c)^2 - a*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) - (136*a*cos(4*d*x + 4*c)^2 +
136*a*sin(4*d*x + 4*c)^2 - 129*a*cos(4*d*x + 4*c) - 7*a)*sin(1/4*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) - 2*(6*a*cos(4*d*x + 4*c)^2 + 24*(a*cos(4*d*x + 4*c)^2 + a*si
n(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))^2 + 20*a*sin(4*d*x + 4*c)^2 - 129*a*sin(4*d*x + 4*c)
*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*(3*a*cos(4*d*x +
4*c)^2 + 10*a*sin(4*d*x + 4*c)^2 - 68*a*sin(4*d*x + 4*c)*sin(1/4*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) - 3*a*cos(4*d*x + 4*c))*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 7*(a*cos(4*d*x + 4*c) + a)*cos(1/4*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) - (68*a*cos(4*d*x + 4*c)^2 + 68*a*sin(4*d*x + 4*c)^2
 + 7*a*cos(4*d*x + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (7*a*cos(4*d*x
+ 4*c)^3 - 48*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x
+ 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 56*a*c
os(4*d*x + 4*c)^2 + 4*(7*a*cos(4*d*x + 4*c)^3 + 30*a*cos(4*d*x + 4*c)^2 + (
7*a*cos(4*d*x + 4*c) + 44*a)*sin(4*d*x + 4*c)^2 - 93*a*cos(4*d*x + 4*c) - 4
4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*
cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 56*a)*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 7*(a*cos(4*d*x + 4*c) + 8*a)*sin
(4*d*x + 4*c)^2 + 4*(7*a*cos(4*d*x + 4*c)^3 + 70*a*cos(4*d*x + 4*c)^2 + 7*(
a*cos(4*d*x + 4*c) + 8*a)*sin(4*d*x + 4*c)^2 + 119*a*cos(4*d*x + 4*c) - 12*
(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 44*(a*cos(4*d*x + 4*c)
^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 56*a)*sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 - 7*a*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 2*(14*a*cos(4*d*x + 4*c)^3 + 92*a*cos(4*d*x + 4*c)^2
 + 2*(7*a*cos(4*d*x + 4*c) + 53*a)*sin(4*d*x + 4*c)^2 - 7*a*sin(4*d*x + 4*c
)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 112*a*cos(4*d*x +
4*c) - (88*a*cos(4*d*x + 4*c)^2 + 88*a*sin(4*d*x + 4*c)^2 - 81*a*cos(4*d*x
+ 4*c) - 7*a)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (44*a*cos(4*d*x + 4*c)^2 +
44*a*sin(4*d*x + 4*c)^2 + 7*a*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) + 2*(96*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))^2*sin(4*d*x + 4*c) + 81*a*cos(1/4*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(44*a*cos(1/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - (7*a*cos(4*d*x + 4*c) + 53*a)*sin

```
(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 14*(a
*cos(4*d*x + 4*c) + 8*a)*sin(4*d*x + 4*c) + 7*(a*cos(4*d*x + 4*c) + a)*sin(
1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 1)))*sqrt(a) + 75*((a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*
sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2
 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4
*c)^2 - a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d
*x + 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)
*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) -
 (a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2
*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^
2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x +
4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*
x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*ar
```

```
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*
arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a*cos(4*d*x + 4*c)^2
 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) +
a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x + 4
*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*
c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*cos(4
*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^
(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
 cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1)
 + (a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 -
 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c
)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x
+ 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*
d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 1)) - 1))*sqrt(a))*B/(4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4
*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*
c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x
+ 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 -
4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)
```

```
+ sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))))/
d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^{3/2} \, (A+B\cos(c+dx)) \, (a+a\cos(c+dx))^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.175 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=180

$$\frac{a^{3/2}(14A+11B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(6A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(14A+11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 1/8*a^(3/2)*(14*A+11*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d
+1/12*a^2*(6*A+7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/
8*a^2*(14*A+11*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/3*
a*B*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.41, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2981, 2770, 2774, 216}

$$\frac{a^2(6A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(14A+11B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(14A+11B)\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(8*d) + (a^2*(14*A + 11*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[
a + a*Cos[c + d*x]]) + (a^2*(6*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(1
2*d*Sqrt[a + a*Cos[c + d*x]]) + (a*B*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c +
d*x]]*Sin[c + d*x])/(3*d)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d}$$

$$= \frac{a^2(6A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{aB\cos$$

$$= \frac{a^2(14A+11B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(6$$

$$= \frac{a^2(14A+11B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(6$$

$$= \frac{a^{3/2}(14A+11B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a^2(14A$$

Mathematica [A] time = 0.69, size = 119, normalized size = 0.66

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\,(14A+11B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(14*A + 11*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(42*A + 37*B + 2*(6*A
+ 11*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

fricas [A] time = 1.35, size = 144, normalized size = 0.80

$$\frac{\left(8\,Ba\cos\left(dx+c\right)^2+2\left(6\,A+11\,B\right)a\cos\left(dx+c\right)+3\left(14\,A+11\,B\right)a\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+\right.}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/24*((8*B*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*cos(d*x + c) + 3*(14*A + 11*
B)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((14*A +
 11*B)*a*cos(d*x + c) + (14*A + 11*B)*a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
 + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)
), x)
```

maple [B] time = 0.35, size = 357, normalized size = 1.98

$$\frac{a(-1+\cos(dx+c))^2\left(12A\sin(dx+c)\left(\cos^2(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+54A\sin(dx+c)\cos(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/24/d*a*(-1+cos(d*x+c))^2*(12*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(3/2)+54*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)+8*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+42*A*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+22*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+33*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)+42*A*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/cos(d*x+c))+33*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+
c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4
```

maxima [B] time = 2.09, size = 3023, normalized size = 16.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (4*(a*cos(3/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(
3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(3/4)*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c),
```

cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) -
 (3*a*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1))) + 1) - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/
4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))) - 1) - a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/
2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(
1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2
((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(

```
sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)) - 1))*sqrt(a))*B)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(c+dx)}\ (A+B\ \cos(c+dx))\ (a+a\ \cos(c+dx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.176 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=133

$$\frac{a^{3/2}(12A+7B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}{2d}$$

```
[Out] 1/4*a^(3/2)*(12*A+7*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+
1/4*a^2*(4*A+5*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/2*
a*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.33, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2976, 2981, 2774, 216}

$$\frac{a^{3/2}(12A+7B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(4*d) + (a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a +
 a*Cos[c + d*x]]) + (a*B*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c
+ d*x])/(2*d)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{aB\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{2d}+\frac{1}{2}\int \frac{\sqrt{a+\cdots}}{\cdots}$$

$$= \frac{a^2(4A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{aB\sqrt{\cos(c+dx)}\,\sqrt{a\cdots}}{\cdots}$$

$$= \frac{a^2(4A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{aB\sqrt{\cos(c+dx)}\,\sqrt{a\cdots}}{\cdots}$$

$$= \frac{a^{3/2}(12A+7B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}+\frac{a^2(4A+5B)\sqrt{\cos(c\cdots}}{4d\sqrt{a+a\cos\cdots}}$$

Mathematica [A] time = 0.38, size = 101, normalized size = 0.76

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\,(12A+7B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+\cdots}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(12*A + 7*B)*ArcSin
[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(4*A + 7*B + 2*B*Cos[c +
d*x])*Sin[(c + d*x)/2]))/(8*d)
```

fricas [A] time = 1.24, size = 125, normalized size = 0.94

$$\frac{(2\,Ba\cos\left(dx+c\right)+(4\,A+7\,B)a)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-((12\,A+7\,B)a\cos\left(dx+c\right)+(\ldots}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*((2*B*a*cos(d*x + c) + (4*A + 7*B)*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))*sin(d*x + c) - ((12*A + 7*B)*a*cos(d*x + c) + (12*A + 7*B)*a)*sq
rt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
 c))))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.31, size = 283, normalized size = 2.13

$$-\frac{a\left(-1+\cos\left(dx+c\right)\right)\left(4A\sin\left(dx+c\right)\cos\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+4A\sin\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+2B\sin\left(dx+\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*a*(-1+cos(d*x+c))*(4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*B*sin(d*x+c)*
```

```
cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*B*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)+12*A*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+7*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/cos(d*x+
c)^(1/2)/sin(d*x+c)^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

maxima [B] time = 1.68, size = 1884, normalized size = 14.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A +
(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c)
+ a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^2 + s
```

```
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2
((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - 1))*sqrt(a))*B)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+d\,x))\,(a+a\cos(c+d\,x))^{3/2}}{\sqrt{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\cos(c+dx)+1))^{\frac{3}{2}}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)*(A + B*cos(c + d*x))/sqrt(cos(c + d*
x)), x)
```

3.177 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{a^{3/2}(2A+3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] a^(3/2)*(2*A+3*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d-a^2*(
2*A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+2*a*A*sin(d*x+c
)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2981, 2774, 216}

$$\frac{a^{3/2}(2A+3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^(3/2)*(2*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d - (a^2*(2*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\sqrt{a+a\cos(c+dx}}{}$$

$$= -\frac{a^2(2A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+}}{d\sqrt{\cos(c}}$$

$$= -\frac{a^2(2A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+}}{d\sqrt{\cos(c}}$$

$$= \frac{a^{3/2}(2A+3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^2(2A-B)\sqrt{\cos(c+d}}{d\sqrt{a+a\cos(c}}$$

Mathematica [A] time = 0.33, size = 107, normalized size = 0.85

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\,(2A+3B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+d\right.\right.}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(2*A + 3*B)*ArcSin[
Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*A + B*Cos[c + d*x])*Sin
[(c + d*x)/2]))/(2*d*Sqrt[Cos[c + d*x]])
```

fricas [A] time = 1.15, size = 135, normalized size = 1.07

$$\frac{(Ba\cos(dx+c)+2\,Aa)\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)-\left((2\,A+3\,B)a\cos(dx+c)^2+(2\,A+3\,B\right.}{d\cos(dx+c)^2+d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] ((B*a*cos(d*x + c) + 2*A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin
(d*x + c) - ((2*A + 3*B)*a*cos(d*x + c)^2 + (2*A + 3*B)*a*cos(d*x + c))*sqr
t(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
c))))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.28, size = 300, normalized size = 2.38

$$\frac{\sqrt{a\,(1+\cos(dx+c))}\,\left(2A\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+3B\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c)))^(1/2)*(2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*B*cos
```

```
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c))+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+B*cos(d*x+c)*sin(
d*x+c)+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*A*sin(d*x+c))*a/(1+cos(d*x+c))/cos(d*x+
c)^(1/2)
```

maxima [B] time = 1.60, size = 1801, normalized size = 14.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*B + 2*(
(a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*
```

```
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*s
qrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+d\,x))\,(a+a\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\cos(c+dx)+1))^{\frac{3}{2}}\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)*(A + B*cos(c + d*x))/cos(c + d*x)**(
3/2), x)
```

3.178 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{2a^{3/2}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2*a^(3/2)*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2/3*a^2*(4*
A+3*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/3*a*A*sin(d*x
+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A] time = 0.32, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2980, 2774, 216}

$$\frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a^(3/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^2*(4*A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c +
d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3
/2))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\sqrt{a+a\cos(c+d}}{}$$

$$= \frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx}}{3d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx}}{3d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{2a^{3/2}B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^2(4A+3B)\sin(c+dx}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c}}$$

Mathematica [A] time = 0.39, size = 106, normalized size = 0.85

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,\left(2\sin\left(\frac{1}{2}(c+dx)\right)((5A+3B)\cos(c+dx)+A)+3\sqrt{2}\,B\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}\right.\right.\right.}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*B*ArcSin[Sqrt[2]*
Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(A + (5*A + 3*B)*Cos[c + d*x])*Sin
[(c + d*x)/2]))/(3*d*Cos[c + d*x]^(3/2))
```

fricas [A] time = 0.88, size = 133, normalized size = 1.06

$$\frac{2\left(((5\,A+3\,B)a\cos\left(dx+c\right)+Aa)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-3\left(Ba\cos\left(dx+c\right)^{3}+Ba\cos\left(\right.\right.\right.}{3\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 2/3*(((5*A + 3*B)*a*cos(d*x + c) + A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c) - 3*(B*a*cos(d*x + c)^3 + B*a*cos(d*x + c)^2)*sqrt(a)
*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.29, size = 211, normalized size = 1.69

$$-\frac{2a\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-3B\cos\left(dx+c\right)\sin\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)-3B\sin\left(dx+\right.\right.}{3d\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*a*(a*(1+cos(d*x+c)))^(1/2)*(-3*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c))-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+5*A*cos(d*x+c)^2+3*B*cos(d*
x+c)^2-4*A*cos(d*x+c)-3*B*cos(d*x+c)-A)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

maxima [B] time = 0.96, size = 1124, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/6*(3*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)))*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4) + 8*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*
sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin
```

```
(d*x + c)^5/(cos(d*x + c) + 1)^5)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\ \cos(c + d\,x))\ (a + a\ \cos(c + d\,x))^{3/2}}{\cos(c + d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\cos(c + dx) + 1))^{\frac{3}{2}}\,(A + B\cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)*(A + B*cos(c + d*x))/cos(c + d*x)**(
5/2), x)
```

3.179 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{2a^2(6A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(18A+25B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 2/15*a^2*(6*A+5*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+2/1
5*a^2*(18*A+25*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/5*
a*A*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)
```

Rubi [A] time = 0.34, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2975, 2980, 2771}

$$\frac{2a^2(6A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(18A+25B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a^2*(6*A + 5*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a^2*(18*A + 25*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*
Cos[c + d*x]^(5/2))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
```

```
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{\sqrt{a+a\cos(c+dx)}}{}$$

$$= \frac{2a^2(6A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a\cos(c+dx)}}{5d\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2a^2(6A+5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(18A+25B)\,\mathrm{si}}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+}}$$

Mathematica [A] time = 0.33, size = 80, normalized size = 0.60

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,(2(9A+5B)\cos(c+dx)+(18A+25B)\cos(2(c+dx))+24A+25B)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(24*A + 25*B + 2*(9*A + 5*B)*Cos[c + d*x] + (
18*A + 25*B)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

fricas [A] time = 1.35, size = 88, normalized size = 0.66

$$\frac{2\left((18\,A+25\,B)a\cos\left(dx+c\right)^2+(9\,A+5\,B)a\cos\left(dx+c\right)+3\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*((18*A + 25*B)*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) + 3*A*a)*
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4
+ d*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.23, size = 87, normalized size = 0.65

$$-\frac{2a\left(-1+\cos\left(dx+c\right)\right)\left(18A\left(\cos^{2}\left(dx+c\right)\right)+25B\left(\cos^{2}\left(dx+c\right)\right)+9A\cos\left(dx+c\right)+5B\cos\left(dx+c\right)+3A\right)\sqrt{}}{15d\sin\left(dx+c\right)\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*a*(-1+cos(d*x+c))*(18*A*cos(d*x+c)^2+25*B*cos(d*x+c)^2+9*A*cos(d*x+
c)+5*B*cos(d*x+c)+3*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

maxima [B] time = 0.87, size = 344, normalized size = 2.57

$$\frac{4\left(\frac{5\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)B}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}+\frac{3\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)A\left(\cdots\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\cdots\right.}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 4/15*(5*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3
/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/
```

```
(cos(d*x + c) + 1)^5)*B/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(
d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)) + 3*(5*sqrt(2)*a^(3/2)*sin(d*x + c)
/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/
2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos
(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

mupad [B] time = 3.09, size = 195, normalized size = 1.46

$$\frac{2\,a\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(48\,A\,\sin\left(c+d\,x\right)+50\,B\,\sin\left(c+d\,x\right)+36\,A\,\sin\left(2\,c+2\,d\,x\right)+66\,A\,\sin\left(3\,c+3\,d\,x\right)\right.}{15\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*a*(a*(cos(c + d*x) + 1))^(1/2)*(48*A*sin(c + d*x) + 50*B*sin(c + d*x) +
36*A*sin(2*c + 2*d*x) + 66*A*sin(3*c + 3*d*x) + 18*A*sin(4*c + 4*d*x) + 18*
A*sin(5*c + 5*d*x) + 20*B*sin(2*c + 2*d*x) + 75*B*sin(3*c + 3*d*x) + 10*B*s
in(4*c + 4*d*x) + 25*B*sin(5*c + 5*d*x)))/(15*d*cos(c + d*x)^(1/2)*(10*cos(
c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos(4*c + 4*d*x) + c
os(5*c + 5*d*x) + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.180 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=181

$$\frac{2a^2(52A+63B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/35*a^2*(8*A+7*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+2/1
05*a^2*(52*A+63*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+4/1
05*a^2*(52*A+63*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/7
*a*A*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

Rubi [A] time = 0.43, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2980, 2772, 2771}

$$\frac{2a^2(52A+63B)\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*(8*A + 7*B)*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c
+ d*x]]) + (2*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqr
t[a + a*Cos[c + d*x]]) + (4*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sin[
c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{2}{7}\int \frac{\sqrt{a+a\cos(c+dx)}}{} \\ &= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2aA\sqrt{a+a\cos(c+dx)}}{7d\cos^{\frac{7}{2}}(c+} \\ &= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(52A+63B)\,\mathrm{s}}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+}} \\ &= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a^2(52A+63B)\,\mathrm{s}}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+}}\end{aligned}$$

Mathematica [A] time = 0.54, size = 102, normalized size = 0.56

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,(3(78A+77B)\cos(c+dx)+(52A+63B)\cos(2(c+dx))+52A\cos(3(c+}{105d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(82*A + 63*B + 3*(78*A + 77*B)*Cos[c + d*x] +
 (52*A + 63*B)*Cos[2*(c + d*x)] + 52*A*Cos[3*(c + d*x)] + 63*B*Cos[3*(c + d
*x)])*Tan[(c + d*x)/2])/(105*d*Cos[c + d*x]^(7/2))
```

fricas [A] time = 0.69, size = 107, normalized size = 0.59

$$\frac{2\left(2(52A+63B)a\cos(dx+c)^{3}+(52A+63B)a\cos(dx+c)^{2}+3(13A+7B)a\cos(dx+c)+15Aa\right)\sqrt{a\cos(}}{105\left(d\cos(dx+c)^{5}+d\cos(dx+c)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(2*(52*A + 63*B)*a*cos(d*x + c)^3 + (52*A + 63*B)*a*cos(d*x + c)^2 +
3*(13*A + 7*B)*a*cos(d*x + c) + 15*A*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.24, size = 109, normalized size = 0.60

$$-\frac{2a(-1+\cos(dx+c))\left(104A\left(\cos^{3}(dx+c)\right)+126B\left(\cos^{3}(dx+c)\right)+52A\left(\cos^{2}(dx+c)\right)+63B\left(\cos^{2}(dx+c\right.\right.}{105d\sin(dx+c)\cos(dx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(104*A*cos(d*x+c)^3+126*B*cos(d*x+c)^3+52*A*cos(
d*x+c)^2+63*B*cos(d*x+c)^2+39*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*(a*(1+cos(
d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

maxima [B] time = 0.96, size = 481, normalized size = 2.66

$$\frac{4\left(\frac{21\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)B\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}+\frac{\left(\frac{105\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{245\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\cdots\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\cdots\right.}\cdots\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 4/105*(21*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
+ (105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a^(3/2
)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 1)))/d
```

mupad [B] time = 6.72, size = 236, normalized size = 1.30

$$\frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\left(-\frac{8\,a\,e^{\frac{c\,7\mathrm{i}}{2}+\frac{d\,x\,7\mathrm{i}}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A+3\,B\right)}{3\,d}+\frac{16\,a\,e^{\frac{c\,7\mathrm{i}}{2}+\frac{d\,x\,7\mathrm{i}}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\left(13\,A+12\,B\right)}{15\,d}+\cdots\right.}{6\,\sqrt{\cos\left(c+d\,x\right)}\,e^{\frac{c\,7\mathrm{i}}{2}+\frac{d\,x\,7\mathrm{i}}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,\sqrt{\cos\left(c+d\,x\right)}\,e^{\frac{c\,7\mathrm{i}}{2}+\frac{d\,x\,7\mathrm{i}}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+2\,\sqrt{\cos\left(c+d\,x\right)}\,e^{\frac{c\,7\mathrm{i}}{2}+\frac{d\,x\,7\mathrm{i}}{2}}\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] ((a + a*cos(c + d*x))^(1/2)*((16*a*exp((c*7i)/2 + (d*x*7i)/2)*sin((3*c)/2 +
 (3*d*x)/2)*(13*A + 12*B))/(15*d) - (8*a*exp((c*7i)/2 + (d*x*7i)/2)*sin(c/2
 + (d*x)/2)*(2*A + 3*B))/(3*d) + (8*a*exp((c*7i)/2 + (d*x*7i)/2)*sin((7*c)/
2 + (7*d*x)/2)*(52*A + 63*B))/(105*d)))/(6*cos(c + d*x)^(1/2)*exp((c*7i)/2
+ (d*x*7i)/2)*cos(c/2 + (d*x)/2) + 6*cos(c + d*x)^(1/2)*exp((c*7i)/2 + (d*x
*7i)/2)*cos((3*c)/2 + (3*d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*7i)/2 + (d*x
*7i)/2)*cos((5*c)/2 + (5*d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*7i)/2 + (d*x
*7i)/2)*cos((7*c)/2 + (7*d*x)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.181 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{8a^2(34A+39B)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(34A+39B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}$$

```
[Out] 2/63*a^2*(10*A+9*B)*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+2/
105*a^2*(34*A+39*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+8/
315*a^2*(34*A+39*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+16
/315*a^2*(34*A+39*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2
/9*a*A*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)
```

Rubi [A] time = 0.51, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2980, 2772, 2771}

$$\frac{8a^2(34A+39B)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(34A+39B)\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x
]
```

```
[Out] (2*a^2*(10*A + 9*B)*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a^2*(34*A + 39*B)*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)*Sq
rt[a + a*Cos[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sin[c + d*x])/(315*d*Cos[c +
 d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a^2*(34*A + 39*B)*Sin[c + d*x])
/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Co
s[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\sqrt{a+a\cos(c+dx)}}{}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a+a\cos(c+dx)}}{9d\cos^{\frac{9}{2}}(c+}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(34A+39B)\,\mathrm{s}}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+}}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(34A+39B)\,\mathrm{s}}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+}}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(34A+39B)\,\mathrm{s}}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+}}$$

Mathematica [A] time = 0.69, size = 124, normalized size = 0.54

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,((374A+324B)\cos(c+dx)+11(34A+39B)\cos(2(c+dx))+68A\cos(3(}{315d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
1/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(376*A + 351*B + (374*A + 324*B)*Cos[c + d*x]
 + 11*(34*A + 39*B)*Cos[2*(c + d*x)] + 68*A*Cos[3*(c + d*x)] + 78*B*Cos[3*(
c + d*x)] + 68*A*Cos[4*(c + d*x)] + 78*B*Cos[4*(c + d*x)])*Tan[(c + d*x)/2]
)/(315*d*Cos[c + d*x]^(9/2))
```

fricas [A] time = 0.85, size = 126, normalized size = 0.55

$$\frac{2\left(8\,(34\,A+39\,B)a\cos\left(dx+c\right)^4+4\,(34\,A+39\,B)a\cos\left(dx+c\right)^3+3\,(34\,A+39\,B)a\cos\left(dx+c\right)^2+5\,(17\,A+9\,}{315\left(d\cos\left(dx+c\right)^6+d\cos\left(dx+c\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(8*(34*A + 39*B)*a*cos(d*x + c)^4 + 4*(34*A + 39*B)*a*cos(d*x + c)^3
+ 3*(34*A + 39*B)*a*cos(d*x + c)^2 + 5*(17*A + 9*B)*a*cos(d*x + c) + 35*A*a
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^
6 + d*cos(d*x + c)^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.27, size = 131, normalized size = 0.57

$$-\frac{2a\left(-1+\cos\left(dx+c\right)\right)\left(272A\left(\cos^{4}\left(dx+c\right)\right)+312B\left(\cos^{4}\left(dx+c\right)\right)+136A\left(\cos^{3}\left(dx+c\right)\right)+156B\left(\cos^{3}\left(dx\right.\right.\right.}{315d\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(272*A*cos(d*x+c)^4+312*B*cos(d*x+c)^4+136*A*cos
(d*x+c)^3+156*B*cos(d*x+c)^3+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+85*A*cos
(d*x+c)+45*B*cos(d*x+c)+35*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c
)^(9/2)
```

maxima [B] time = 1.18, size = 573, normalized size = 2.51

$$\frac{4\left(\frac{3\left(\frac{105\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{245\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{273\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{171\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{38\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)B\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{3}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}+\frac{315\sqrt{2}a^{\frac{3}{2}}\sin(\ldots}{\cos(dx+c)+\ldots}\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 4/315*(3*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)
*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x +
```

```
c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*B*(sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) +
 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + 1)) + (315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x +
c) + 1) - 840*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sq
rt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1242*sqrt(2)*a^(3/2)*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7 + 517*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9 - 94*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11
)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c
) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1
)))/d
```

mupad [B] time = 7.02, size = 289, normalized size = 1.27

$$\frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}\,\left(-\frac{16\,B\,a\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{3\,d}+\frac{16\,a\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)\,(34\,A\cdots}{35\,d}\cdots\right.}{12\,\sqrt{\cos\left(c+d\,x\right)}\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\,\sqrt{\cos\left(c+d\,x\right)}\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+8\,\sqrt{\cos\left(c+d\,x\right)}\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/cos(c + d*x)^(11/2),x
)
```

```
[Out] ((a + a*cos(c + d*x))^(1/2)*((16*a*exp((c*9i)/2 + (d*x*9i)/2)*sin((5*c)/2 +
 (5*d*x)/2)*(34*A + 39*B))/(35*d) - (16*B*a*exp((c*9i)/2 + (d*x*9i)/2)*sin(
(3*c)/2 + (3*d*x)/2))/(3*d) + (32*a*exp((c*9i)/2 + (d*x*9i)/2)*sin((9*c)/2
+ (9*d*x)/2)*(34*A + 39*B))/(315*d) + (96*a*exp((c*9i)/2 + (d*x*9i)/2)*sin(
c/2 + (d*x)/2)*(A + B))/(5*d)))/(12*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*
9i)/2)*cos(c/2 + (d*x)/2) + 8*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)
*cos((3*c)/2 + (3*d*x)/2) + 8*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)
*cos((5*c)/2 + (5*d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)
*cos((7*c)/2 + (7*d*x)/2) + 2*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)
*cos((9*c)/2 + (9*d*x)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

3.182 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=274

$$\frac{a^{5/2}(326A+283B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^3(170A+157B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{240d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(326A+283B)\sin(c-}{192d\sqrt{a\cos(c+}$$

```
[Out] 1/128*a^(5/2)*(326*A+283*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2
))/d+1/5*a*B*cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+1/192*a^3
*(326*A+283*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/240*a
^3*(170*A+157*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/128
*a^3*(326*A+283*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/4
0*a^2*(10*A+13*B)*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [**A**] time = 0.71, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2981, 2770, 2774, 216}

$$\frac{a^3(170A+157B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{240d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(326A+283B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(10A+13B)\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(128*d) + (a^3*(326*A + 283*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(128
*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(326*A + 283*B)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(170*A + 157*B)*Cos[c + d
*x]^(5/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(10*A + 13*
B)*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d) + (a*B*
Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
```

```
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

$$= \frac{a^2(10A+13B)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\ \mathrm{s}}{40d}$$

$$= \frac{a^3(170A+157B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(}{}$$

$$= \frac{a^3(326A+283B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(}{}$$

$$= \frac{a^3(326A+283B)\sqrt{\cos(c+dx)}\ \sin(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^3}{}$$

$$= \frac{a^3(326A+283B)\sqrt{\cos(c+dx)}\ \sin(c+dx)}{128d\sqrt{a+a\cos(c+dx)}} + \frac{a^3}{}$$

$$= \frac{a^{5/2}(326A+283B)\sin^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{128d} + \frac{a^3(32}{}$$

Mathematica [A] time = 1.98, size = 159, normalized size = 0.58

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}\,(326A+283B)\sin^{-1}\left(\sqrt{2}\ \sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*(326*A + 283*B
)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(5810*A + 5521*B
+ (3620*A + 3874*B)*Cos[c + d*x] + 4*(230*A + 331*B)*Cos[2*(c + d*x)] + 120
*A*Cos[3*(c + d*x)] + 348*B*Cos[3*(c + d*x)] + 48*B*Cos[4*(c + d*x)])*Sin[(
c + d*x)/2]))/(3840*d)
```

fricas [A] time = 1.20, size = 194, normalized size = 0.71

$$\frac{\left(384\,Ba^2\cos(dx+c)^4+48\,(10\,A+29\,B)a^2\cos(dx+c)^3+8\,(230\,A+283\,B)a^2\cos(dx+c)^2+10\,(326\,A+28\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/1920*((384*B*a^2*cos(d*x + c)^4 + 48*(10*A + 29*B)*a^2*cos(d*x + c)^3 + 8
*(230*A + 283*B)*a^2*cos(d*x + c)^2 + 10*(326*A + 283*B)*a^2*cos(d*x + c) +
 15*(326*A + 283*B)*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*
x + c) - 15*((326*A + 283*B)*a^2*cos(d*x + c) + (326*A + 283*B)*a^2)*sqrt(a
)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))
))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2
), x)
```

maple [B] time = 0.26, size = 503, normalized size = 1.84

$$-\frac{a^2(-1+\cos(dx+c))^3\left(480A\sin(dx+c)\left(\cos^4(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+2320A\sin(dx+c)\left(\cos^3(dx+c)\right)\left(\frac{c}{1}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -1/1920/d*a^2*(-1+cos(d*x+c))^3*(480*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)+2320*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)+384*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
+5100*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1392*B*si
n(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+8150*A*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2264*B*sin(d*x+c)*cos(d*x+c)^3*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4890*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)+2830*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
+4245*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4890*A*cos(
```

```
d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+4245
*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c
)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3/2)/sin(d*x+c)^6/(cos(d*x+c)/(1+c
os(d*x+c)))^(5/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{3/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.183 $\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=227

$$\frac{a^{5/2}(200A+163B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(104A+95B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^3(200A+163B)\sin(c+d}{64d\sqrt{a\cos(c+dx}$$

```
[Out] 1/64*a^(5/2)*(200*A+163*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2)
)/d+1/4*a*B*cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d+1/96*a^3*(
104*A+95*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/64*a^3*(
200*A+163*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/24*a^2*
(8*A+11*B)*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.71, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2981, 2770, 2774, 216}

$$\frac{a^3(104A+95B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{24d}+\frac{a^{5/2}(200A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]])/(64*d) + (a^3*(200*A + 163*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d
*Sqrt[a + a*Cos[c + d*x]]) + (a^3*(104*A + 95*B)*Cos[c + d*x]^(3/2)*Sin[c +
 d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*(8*A + 11*B)*Cos[c + d*x]^(3/
2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(24*d) + (a*B*Cos[c + d*x]^(3/2)*
(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{aB\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{4d}$$

$$= \frac{a^2(8A+11B)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,\sin}{24d}$$

$$= \frac{a^3(104A+95B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(8}{}$$

$$= \frac{a^3(200A+163B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(}{}$$

$$= \frac{a^3(200A+163B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{a^3(}{}$$

$$= \frac{a^{5/2}(200A+163B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{a^3(20}{}$$

Mathematica [A] time = 1.25, size = 137, normalized size = 0.60

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\,(200A+163B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos}\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(200*A + 163*B)
*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(632*A + 581*B + (
272*A + 362*B)*Cos[c + d*x] + 4*(8*A + 23*B)*Cos[2*(c + d*x)] + 12*B*Cos[3*
(c + d*x)])*Sin[(c + d*x)/2]))/(384*d)
```

fricas [A] time = 1.28, size = 174, normalized size = 0.77

$$\frac{\left(48\,Ba^2\cos\left(dx+c\right)^3+8\,(8\,A+23\,B)a^2\cos\left(dx+c\right)^2+2\,(136\,A+163\,B)a^2\cos\left(dx+c\right)+3\,(200\,A+163\,B)a^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/192*((48*B*a^2*cos(d*x + c)^3 + 8*(8*A + 23*B)*a^2*cos(d*x + c)^2 + 2*(13
6*A + 163*B)*a^2*cos(d*x + c) + 3*(200*A + 163*B)*a^2)*sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((200*A + 163*B)*a^2*cos(d*x + c)
+ (200*A + 163*B)*a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)
), x)
```

maple [B] time = 0.20, size = 431, normalized size = 1.90

$$\frac{a^2(-1+\cos(dx+c))^2\left(64A\sin(dx+c)\left(\cos^3(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+336A\sin(dx+c)\left(\cos^2(dx+c)\right)\left(\frac{\cos}{1+\cos}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/192/d*a^2*(-1+cos(d*x+c))^2*(64*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(3/2)+336*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)+48*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+872*
A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+184*B*sin(d*x+c)*
cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+600*A*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)+326*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)+489*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+6
00*A*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x
+c))+489*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)^4/(cos(d*x
+c)/(1+cos(d*x+c)))^(3/2)
```

maximà [B] time = 2.90, size = 9415, normalized size = 41.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(
3*d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(
1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c),
```

```
cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
))) - 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2
*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) - 1))*sqrt(a))*A + (10*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(
4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^3 + 12*(a^2*sin(4*
d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(
4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 12*(
a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) +
 a^2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + 3*(2*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d
*x + 4*c) + a^2*sin(4*d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)
^2 - a^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c)
^2 - 3*a^2*cos(4*d*x + 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 +
2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
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 4*c)))^2 + 2*(16*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2 - 19*a
^2*cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) - 2*(64*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin
(4*d*x + 4*c) + 19*a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) -
12*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x +
4*c)^2 + a^2*sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (3*a^2*
cos(4*d*x + 4*c)^3 - 8*a^2*cos(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3
 - 14*a^2*cos(4*d*x + 4*c)^2 + 19*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x +
 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 - 8*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c)))^2 + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^
2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 2*a^2*cos(4*d*x + 4*c)^2 - 13*a^2*cos(4*d
*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 - 8*a^2)*si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (8*a^2*cos(4*d*x + 4
*c)^2 + 8*a^2*sin(4*d*x + 4*c)^2 - 3*a^2*cos(4*d*x + 4*c) + 32*(a^2*cos(4*d
*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(a^2*cos(4*d*x + 4*c)
^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(16*a^2*cos(4*d*x + 4*c)^2 + 16
*a^2*sin(4*d*x + 4*c)^2 - 19*a^2*cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a^2*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 19*a^2*sin(4*d*x + 4*c))*sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 11*a^2*cos(4*d*
x + 4*c)^2 + 8*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(
4*d*x + 4*c)^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 3*(2
*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)
+ a^2*sin(4*d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) - 4*(4*(3*a^2*cos(4*d*x + 4*c) - 8*a^2)*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) -
 8*a^2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 6*(co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))) + 1)^(1/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3
*a^2*sin(4*d*x + 4*c)^3 + 3*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^3 + 4*(3*a^2*sin(4*d*x + 4*c)^3 + 3*(a^2*cos(4*d*x + 4*c)^
2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4
*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arct
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an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 4*(3*a^2*sin(4*d*x + 4*c)^3 + 160*a^2*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x
 + 4*c)^2 + 6*a^2*cos(4*d*x + 4*c) + 43*a^2)*sin(4*d*x + 4*c) - 160*(a^2*co
s(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*s
in(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(6*a^2*sin(4*d*x + 4*c)^3 + 3*a^2*cos(
1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 6*(a^2*
cos(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) - (320*a^2*cos(
4*d*x + 4*c)^2 + 320*a^2*sin(4*d*x + 4*c)^2 - 317*a^2*cos(4*d*x + 4*c) - 3*
a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(20*a^2*cos(4*d*x + 4*c)^2 + 26*a^
2*sin(4*d*x + 4*c)^2 - 317*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) + 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*
c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))^2 + 8*(10*a^2*cos(4*d*x + 4*c)^2 + 13*a^2*sin(4*d*x + 4*c)^2
 - 160*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) - 10*a^2*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))) + 3*(a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 - (160*a^2*cos(4*d*x + 4*c)^2 + 160*a^2*sin(4*d*x + 4*c)^2 + 3*a^2*cos(4*d
*x + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*ar
ctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (3*a^2*cos(4*d*x + 4*c)^3 + 1
20*a^2*cos(4*d*x + 4*c)^2 - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4
*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^3 - 3*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 4*(3*a^2*cos(4*d*x + 4*c)^3 + 74*a^2*cos(4*d*x + 4*c
)^2 - 197*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) + 80*a^2)*sin(4*d*
x + 4*c)^2 + 120*a^2 - 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2
- 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(a^2*
cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3
 + 126*a^2*cos(4*d*x + 4*c)^2 + 243*a^2*cos(4*d*x + 4*c) + 3*(a^2*cos(4*d*x
 + 4*c) + 40*a^2)*sin(4*d*x + 4*c)^2 + 120*a^2 - 40*(a^2*cos(4*d*x + 4*c)^2
 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) - 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(
4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + 2*(6*a^2*cos(4*d*x + 4*c)^3 + 214*a^2*cos(4*d*x + 4*c)^2 - 3*a^2*si
n(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 240*a
^2*cos(4*d*x + 4*c) + 2*(3*a^2*cos(4*d*x + 4*c) + 110*a^2)*sin(4*d*x + 4*c)
^2 - (160*a^2*cos(4*d*x + 4*c)^2 + 160*a^2*sin(4*d*x + 4*c)^2 - 157*a^2*cos
(4*d*x + 4*c) - 3*a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (80*a^2*cos(4*d*x
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+ 4*c)^2 + 80*a^2*sin(4*d*x + 4*c)^2 + 3*a^2*cos(4*d*x + 4*c))*cos(1/4*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2*(320*a^2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2*sin(4*d*x + 4*c) + 157*a^2*cos(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(80*a^2*cos(1/4
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - (3*a^2*cos
(4*d*x + 4*c) + 110*a^2)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) - 6*(a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c) +
 3*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*ar
ctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 489*((a^2*cos(4*d*x
+ 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d
*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^
2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*co
s(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4
*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)
+ a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/
4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
 cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (a^2*co
s(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2
*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x
 + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2
- a^2*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) - 4*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x
 + 4*c) + a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2
```

*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + (a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))

```
, cos(4*d*x + 4*c))) + 1)) - 1))*sqrt(a))*B/(4*(cos(4*d*x + 4*c)^2 + sin(4*
d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d
*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos
(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x +
4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*
c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x
+ 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.184 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=180

$$\frac{a^{5/2}(38A+25B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^3(54A+49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(2A+3B)\sin(c+dx)\sqrt{\mathrm{co}}}{4}$$

```
[Out] 1/8*a^(5/2)*(38*A+25*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d
+1/3*a*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+1/24*a^3*(54*
A+49*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a^2*(2*A+3
*B)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.55, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2976, 2981, 2774, 216}

$$\frac{a^{5/2}(38A+25B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^3(54A+49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2(2A+3B)\sin(c+dx)\sqrt{\mathrm{co}}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(8*d) + (a^3*(54*A + 49*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt
[a + a*Cos[c + d*x]]) + (a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(4*d) + (a*B*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{aB\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\int \frac{(a+$$

$$= \frac{a^2(2A+3B)\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{aB$$

$$= \frac{a^3(54A+49B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(2A+3B)\sqrt{\cos(}$$

$$= \frac{a^3(54A+49B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2(2A+3B)\sqrt{\cos(}$$

$$= \frac{a^{5/2}(38A+25B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{a^3(54A+49B)\sqrt{\cos(}}{24d\sqrt{a+a\,c}}$$

Mathematica [A] time = 0.78, size = 121, normalized size = 0.67

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(38A+25B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(}\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(38*A + 25*B)*A
rcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(66*A + 79*B + 2*(6*
A + 17*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

fricas [A] time = 1.13, size = 154, normalized size = 0.86

$$\frac{\left(8\,Ba^2\cos\left(dx+c\right)^2+2\left(6\,A+17\,B\right)a^2\cos\left(dx+c\right)+3\left(22\,A+25\,B\right)a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(\right.}{24\left(d\cos\left(dx+c\right)\right.-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/24*((8*B*a^2*cos(d*x + c)^2 + 2*(6*A + 17*B)*a^2*cos(d*x + c) + 3*(22*A +
 25*B)*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((
38*A + 25*B)*a^2*cos(d*x + c) + (38*A + 25*B)*a^2)*sqrt(a)*arctan(sqrt(a*co
s(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)
 + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.34, size = 357, normalized size = 1.98

$$-\frac{a^2\left(-1+\cos\left(dx+c\right)\right)\left(12A\sin\left(dx+c\right)\left(\cos^2\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+78A\sin\left(dx+c\right)\cos\left(dx+c\right)\left(\frac{\cos(dx+c}{1+\cos(dx+}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/24/d*a^2*(-1+cos(d*x+c))*(12*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)+78*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/
2)+8*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+66*A*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+34*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)+75*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)+114*A*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)/cos(d*x+c))+75*B*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^
(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

maxima [B] time = 2.09, size = 3071, normalized size = 17.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*
d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
```

n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (4*
(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c)
 - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*
a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqr
t(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(
1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(
sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) +
1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +

```
3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(
cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a
^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1
))*sqrt(a))*B)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\cos(c + dx))\,(a + a\cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
[Out] Timed out
```

3.185 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{a^{5/2}(20A+19B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(4A-9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(4A-B)\sin(c+dx)\sqrt{\cos(c+}}{2d}$$

```
[Out] 1/4*a^(5/2)*(20*A+19*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d
+2*a*A*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-1/4*a^3*(4*A-9*
B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)-1/2*a^2*(4*A-B)*sin
(d*x+c)*cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.55, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2975, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(20A+19B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(4A-9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{a^2(4A-B)\sin(c+dx)\sqrt{\cos(c+}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]])/(4*d) - (a^3*(4*A - 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a
+ a*Cos[c + d*x]]) - (a^2*(4*A - B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d
*x]]*Sin[c + d*x])/(2*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+a\cos(c+dx)}{}$$

$$= -\frac{a^2(4A-B)\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{2d} + \frac{2a}{}$$

$$= -\frac{a^3(4A-9B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(4A-B)\sqrt{\cos(c+}}{}$$

$$= -\frac{a^3(4A-9B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} - \frac{a^2(4A-B)\sqrt{\cos(c+}}{}$$

$$= \frac{a^{5/2}(20A+19B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} - \frac{a^3(4A-9B)\sqrt{\cos(c+}}{4d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.69, size = 126, normalized size = 0.71

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\,(20A+19B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2\sin\left(\frac{1}{2}(c+\right.\right.}{8d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*(20*A + 19*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(8*A + B + (4*A + 11*B
)*Cos[c + d*x] + B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d*Sqrt[Cos[c + d
*x]])
```

fricas [A] time = 0.88, size = 164, normalized size = 0.92

$$\frac{\left(2\,Ba^2\cos\left(dx+c\right)^2+(4\,A+11\,B)a^2\cos\left(dx+c\right)+8\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\left((20\right.}{4\left(d\cos\left(dx+c\right)^2+d\cos\left(d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*((2*B*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 8*A*a^2)*sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - ((20*A + 19*B)*a^2*
cos(d*x + c)^2 + (20*A + 19*B)*a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^2
 + d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.31, size = 336, normalized size = 1.89

$$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(20A\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\;\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+2B\sin\left(dx+c\right)\left(\cos^2\left(dx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(20*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*B*
sin(d*x+c)*cos(d*x+c)^2+19*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+4*A*cos(d*x+
c)*sin(d*x+c)+20*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+11*B*cos(d*x+c)*sin(d*x+c)+19*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/cos(d*x+c))+8*A*sin(d*x+c))*a^2/(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

maxima [B] time = 1.89, size = 2080, normalized size = 11.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*((2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*d*
x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2*a
rctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*B + 4*(2*
(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c)
- (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
```

```
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 8*(a
^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) -
(a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)))*sqrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\cos(c + dx))\ (a + a\cos(c + dx))^{5/2}}{\cos(c + dx)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),x)
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
[Out] Timed out
```

3.186 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=173

$$\frac{a^{5/2}(2A+5B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3(14A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(2A+B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] a^(5/2)*(2*A+5*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2/3*a
*A*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)-1/3*a^3*(14*A+3*B)*
sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)+2*a^2*(2*A+B)*sin(d*x+
c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2981, 2774, 216}

$$\frac{a^{5/2}(2A+5B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3(14A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(2A+B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
])/d - (a^3*(14*A + 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^2*(2*A + B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*
Sqrt[Cos[c + d*x]]) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*
Cos[c + d*x]^(3/2))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+a\cos(c+d$$

$$= \frac{2a^2(2A+B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aA(a+a\cos}{3d\,c}$$

$$= -\frac{a^3(14A+3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(2A+B)\sqrt{a+}}{d\sqrt{}}$$

$$= -\frac{a^3(14A+3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(2A+B)\sqrt{a+}}{d\sqrt{}}$$

$$= \frac{a^{5/2}(2A+5B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^3(14A+3B)\sqrt{\cos(c+}}{3d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.71, size = 130, normalized size = 0.75

$$\frac{a^2 \sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\,(2A+5B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+\sin\left(\frac{1}{2}(c+dx\right.\right.}{6d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*(2*A + 5*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (4*A + 3*B + 4*(8*A + 3*
B)*Cos[c + d*x] + 3*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*d*Cos[c + d*x
]^(3/2))
```

fricas [A] time = 1.01, size = 169, normalized size = 0.98

$$\frac{\left(3\,Ba^2\cos\left(dx+c\right)^2+2\,(8\,A+3\,B)a^2\cos\left(dx+c\right)+2\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-3\left((2\right.}{3\left(d\cos\left(dx+c\right)^3+d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((3*B*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 2*A*a^2)*sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*((2*A + 5*B)*a^2
*cos(d*x + c)^3 + (2*A + 5*B)*a^2*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^
3 + d*cos(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.28, size = 484, normalized size = 2.80

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(\sin^{2}\left(dx+c\right)\right)\left(6A\left(\cos^{2}\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+15B\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/3/d*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2*(6*A*cos(d*x+c)^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c))+15*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+12*A*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)/cos(d*x+c))+30*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+6*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))+15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*B*sin(d*x+c)*cos(d*x+c)^2+16*A*cos(d*x
+c)*sin(d*x+c)+6*B*cos(d*x+c)*sin(d*x+c)+2*A*sin(d*x+c))*a^2/(-1+cos(d*x+c)
)/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)
```

maxima [B] time = 1.68, size = 2370, normalized size = 13.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(3*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
```

n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 8*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4) + 2*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((co

```
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2
*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - 1))*sqrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{5/2}}{\cos(c+dx)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2),x)
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
[Out] Timed out
```

3.187 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2a^{5/2}B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2*a^(5/2)*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+2/5*a*A*(a+
a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/15*a^3*(32*A+35*B)*sin(
d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/15*a^2*(8*A+5*B)*sin(d*x
+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A] time = 0.51, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2980, 2774, 216}

$$\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{2a^{5/2}B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a^(5/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (
2*a^3*(32*A + 35*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(15*d
*Cos[c + d*x]^(3/2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d
*Cos[c + d*x]^(5/2))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\cos(c+d$$

$$= \frac{2a^2(8A+5B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+a\cos$$

$$= \frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+5B)\sqrt{a+a}}{15d\cos}$$

$$= \frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(8A+5B)\sqrt{a+a}}{15d\cos}$$

$$= \frac{2a^{5/2}B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^3(32A+35B)\sin(c+a}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c}$$

Mathematica [A] time = 0.78, size = 130, normalized size = 0.76

$$\frac{a^2 \sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)(2(14A+5B)\cos(c+dx)+(43A+40B)\cos(2(c+dx)\right.}{30d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(30*Sqrt[2]*B*ArcSin[Sqrt[
2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(49*A + 40*B + 2*(14*A + 5*B)*C
os[c + d*x] + (43*A + 40*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(30*d*Cos[
c + d*x]^(5/2))
```

fricas [A] time = 0.99, size = 161, normalized size = 0.94

$$\frac{2\left(\left((43\,A+40\,B)a^2\cos\left(dx+c\right)^2+(14\,A+5\,B)a^2\cos\left(dx+c\right)+3\,Aa^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx\right.\right.}{15\left(d\cos\left(dx+c\right)^4+d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(((43*A + 40*B)*a^2*cos(d*x + c)^2 + (14*A + 5*B)*a^2*cos(d*x + c) + 3
*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 15*(B*a^
2*cos(d*x + c)^4 + B*a^2*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
 + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^4 + d*cos
(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.39, size = 306, normalized size = 1.78

$$-\frac{2a^{2}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(-15B\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)-30B\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*a^2*(a*(1+cos(d*x+c)))^(1/2)*(-15*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)/cos(d*x+c))-30*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-15*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))+43*A*cos(d*x+c)^3+40*B*cos(d*x+c)^3-29*A*cos(
d*x+c)^2-35*B*cos(d*x+c)^2-11*A*cos(d*x+c)-5*B*cos(d*x+c)-3*A)/sin(d*x+c)/c
os(d*x+c)^(5/2)
```

maxima [B] time = 1.18, size = 1548, normalized size = 9.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/30*(5*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2
*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3
*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)
 + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
```

```
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x +
 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*co
s(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqr
t(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
+ 16*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^(5/
2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{5/2}}{\cos(c+dx)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
[Out] Timed out
```

3.188 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=181

$$\frac{2a^3(10A+11B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(230A+301B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(10A+7B)\sin(c+dx)\sqrt{a\cos}}{35d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 2/7*a*A*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/15*a^3*(10*A
+11*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+2/105*a^3*(230*
A+301*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/35*a^2*(10*
A+7*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)
```

Rubi [A] time = 0.55, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2975, 2980, 2771}

$$\frac{2a^3(10A+11B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(10A+7B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3(230A+301B)\sin}{105d\sqrt{\cos(c+dx)}\sqrt{a\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*a^3*(10*A + 11*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[
c + d*x]]) + (2*a^3*(230*A + 301*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(10*A + 7*B)*Sqrt[a + a*Cos[c + d*x]]*S
in[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*
Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
```

```
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\cos(c+d}{}$$

$$= \frac{2a^2(10A+7B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\,c}{7}$$

$$= \frac{2a^3(10A+11B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(10A+7B)\sqrt{a+a}}{35d\cos}$$

$$= \frac{2a^3(10A+11B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(230A+301B}{105d\sqrt{\cos(c+dx)}\,\sqrt{a}}$$

Mathematica [A] time = 0.63, size = 104, normalized size = 0.57

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,((930A+987B)\cos(c+dx)+2(115A+98B)\cos(2(c+dx))+230A\cos}{210d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(290*A + 196*B + (930*A + 987*B)*Cos[c + d*
x] + 2*(115*A + 98*B)*Cos[2*(c + d*x)] + 230*A*Cos[3*(c + d*x)] + 301*B*Cos
[3*(c + d*x)])*Tan[(c + d*x)/2])/(210*d*Cos[c + d*x]^(7/2))
```

fricas [A] time = 0.99, size = 114, normalized size = 0.63

$$\frac{2\left((230\,A+301\,B)a^2\cos\left(dx+c\right)^3+(115\,A+98\,B)a^2\cos\left(dx+c\right)^2+3\,(20\,A+7\,B)a^2\cos\left(dx+c\right)+15\,Aa^2\right)\sqrt{a}}{105\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*((230*A + 301*B)*a^2*cos(d*x + c)^3 + (115*A + 98*B)*a^2*cos(d*x + c)
^2 + 3*(20*A + 7*B)*a^2*cos(d*x + c) + 15*A*a^2)*sqrt(a*cos(d*x + c) + a)*s
qrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.23, size = 111, normalized size = 0.61

$$-\frac{2a^2\left(-1+\cos\left(dx+c\right)\right)\left(230A\left(\cos^3\left(dx+c\right)\right)+301B\left(\cos^3\left(dx+c\right)\right)+115A\left(\cos^2\left(dx+c\right)\right)+98B\left(\cos^2\left(dx+\right.\right.\right.}{105d\sin\left(dx+c\right)\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*a^2*(-1+cos(d*x+c))*(230*A*cos(d*x+c)^3+301*B*cos(d*x+c)^3+115*A*c
os(d*x+c)^2+98*B*cos(d*x+c)^2+60*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*(a*(1+c
os(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

maxima [B] time = 1.31, size = 396, normalized size = 2.19

$$\frac{8\left(\frac{7\left(\frac{15\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{28\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)B}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}+\frac{5\left(\frac{21\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{56\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}}\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 8/105*(7*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7)*B/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d
*x + c) + 1) + 1)^(7/2)) + 5*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c)
 + 1) - 56*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)
*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1
)))/d
```

mupad [B] time = 6.86, size = 551, normalized size = 3.04

$$\frac{\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\left(\frac{a^2\,(230\,A+301\,B)\,2i}{105\,d}-\frac{a^2\,e^{c\,3i+d\,x\,3i}\,(10\,A+17\,B)\,2i}{3\,d}+\frac{a^2\,e^{c\,4i+d\,x}}{}\right.}{\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+e^{c\,1i+d\,x\,1i}\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+3\,e^{c\,2i+d\,x\,2i}\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+3\,e^{c\,3i+d\,x\,3i}\sqrt{\frac{e^{-c\,1i-}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] ((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(230*A
 + 301*B)*2i)/(105*d) - (a^2*exp(c*3i + d*x*3i)*(10*A + 17*B)*2i)/(3*d) + (
a^2*exp(c*4i + d*x*4i)*(10*A + 17*B)*2i)/(3*d) + (a^2*exp(c*2i + d*x*2i)*(1
00*A + 113*B)*2i)/(15*d) - (a^2*exp(c*5i + d*x*5i)*(100*A + 113*B)*2i)/(15*
d) - (a^2*exp(c*7i + d*x*7i)*(230*A + 301*B)*2i)/(105*d) - (B*a^2*exp(c*1i
+ d*x*1i)*2i)/d + (B*a^2*exp(c*6i + d*x*6i)*2i)/d))/((exp(- c*1i - d*x*1i)/
2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*1i + d*x*1i)*(exp(- c*1i - d*x*1i)/
2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c*2i + d*x*2i)*(exp(- c*1i - d*x*1i
)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c*3i + d*x*3i)*(exp(- c*1i - d*x*
1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c*4i + d*x*4i)*(exp(- c*1i - d*
x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 3*exp(c*5i + d*x*5i)*(exp(- c*1i -
d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*6i + d*x*6i)*(exp(- c*1i -
d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*7i + d*x*7i)*(exp(- c*1i -
d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.189 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{2a^3(292A+345B)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a^3(292A+345B)\sin(c+d}{315d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+d}}$$

```
[Out] 2/9*a*A*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)+2/315*a^3*(124
*A+135*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+2/315*a^3*(2
92*A+345*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+4/315*a^3*
(292*A+345*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/21*a^2
*(4*A+3*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

Rubi [A] time = 0.70, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2980, 2772, 2771}

$$\frac{2a^3(292A+345B)\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{a\cos}}{21d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x
]
```

```
[Out] (2*a^3*(124*A + 135*B)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3
/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(4*A + 3*B)*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)) + (2*a*A*(a + a*Co
s[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a + a\cos(c+dx))^{5/2}(A + B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a + a\cos(c+dx))^{3/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a + a\cos(c + d}{}$$

$$= \frac{2a^2(4A+3B)\sqrt{a + a\cos(c+dx)}\,\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aA(a + a\,\mathrm{cc}}{9d}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a + a\cos(c+dx)}} + \frac{2a^2(4A+3B)\sqrt{a + a}}{21d\,\mathrm{cc}}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a + a\cos(c+dx)}} + \frac{2a^3(292A+345}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\cos^{\frac{5}{2}}(c+dx)\sqrt{a + a\cos(c+dx)}} + \frac{2a^3(292A+345}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{}}$$

Mathematica [A] time = 0.86, size = 126, normalized size = 0.55

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,((1396A+1215B)\cos(c+dx)+2(803A+870B)\cos(2(c+dx))+292A}{630d\cos^{\frac{9}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
1/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(1454*A + 1395*B + (1396*A + 1215*B)*Cos[c
+ d*x] + 2*(803*A + 870*B)*Cos[2*(c + d*x)] + 292*A*Cos[3*(c + d*x)] + 345*
B*Cos[3*(c + d*x)] + 292*A*Cos[4*(c + d*x)] + 345*B*Cos[4*(c + d*x)])*Tan[(
c + d*x)/2])/(630*d*Cos[c + d*x]^(9/2))
```

fricas [A] time = 1.03, size = 135, normalized size = 0.59

$$\frac{2\left(2\,(292\,A+345\,B)a^2\cos(dx+c)^4+(292\,A+345\,B)a^2\cos(dx+c)^3+3\,(73\,A+60\,B)a^2\cos(dx+c)^2+5\,(26}{315\left(d\cos(dx+c)^6+d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(2*(292*A + 345*B)*a^2*cos(d*x + c)^4 + (292*A + 345*B)*a^2*cos(d*x +
 c)^3 + 3*(73*A + 60*B)*a^2*cos(d*x + c)^2 + 5*(26*A + 9*B)*a^2*cos(d*x + c
) + 35*A*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*c
os(d*x + c)^6 + d*cos(d*x + c)^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.28, size = 133, normalized size = 0.58

$$-\frac{2a^{2}\left(-1+\cos\left(dx+c\right)\right)\left(584A\left(\cos^{4}\left(dx+c\right)\right)+690B\left(\cos^{4}\left(dx+c\right)\right)+292A\left(\cos^{3}\left(dx+c\right)\right)+345B\left(\cos^{3}\left(dx\right.\right.\right.}{315d\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(584*A*cos(d*x+c)^4+690*B*cos(d*x+c)^4+292*A*c
os(d*x+c)^3+345*B*cos(d*x+c)^3+219*A*cos(d*x+c)^2+180*B*cos(d*x+c)^2+130*A*
cos(d*x+c)+45*B*cos(d*x+c)+35*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*
x+c)^(9/2)
```

maxima [B] time = 0.96, size = 533, normalized size = 2.34

$$\frac{8\left(\frac{15\left(\frac{21\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{56\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{63\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{36\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)B\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}+\frac{\left(\frac{315\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}\right.}{}\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 8/315*(15*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*
a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c)
```

```
 + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*B*(sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + (315*sqrt(2)*a
^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a^(5/2)*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 572*sqrt
(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*sqrt(2)*a^(5/2)*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

mupad [B] time = 8.24, size = 647, normalized size = 2.84

$$\frac{\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,\left(\frac{a^2\,(292\,A+345\,B)\,4i}{315\,d}-\frac{a^2}{}\right.}{\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+e^{c\,1i+d\,x\,1i}\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+4\,e^{c\,2i+d\,x\,2i}\sqrt{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}+4\,e^{c\,3i+d\,x\,3i}\sqrt{\frac{e^{-c\,1i-}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2),x
)
```

```
[Out] ((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(292*A
 + 345*B)*4i)/(315*d) - (a^2*exp(c*3i + d*x*3i)*(2*A + 5*B)*4i)/(3*d) + (a^
2*exp(c*6i + d*x*6i)*(2*A + 5*B)*4i)/(3*d) + (a^2*exp(c*4i + d*x*4i)*(24*A
+ 25*B)*4i)/(5*d) - (a^2*exp(c*5i + d*x*5i)*(24*A + 25*B)*4i)/(5*d) + (a^2*
exp(c*2i + d*x*2i)*(146*A + 155*B)*4i)/(35*d) - (a^2*exp(c*7i + d*x*7i)*(14
6*A + 155*B)*4i)/(35*d) - (a^2*exp(c*9i + d*x*9i)*(292*A + 345*B)*4i)/(315*
d)))/((exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*1i + d*
x*1i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 4*exp(c*2i +
d*x*2i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 4*exp(c*3i
+ d*x*3i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 6*exp(c*4
i + d*x*4i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 6*exp(c
*5i + d*x*5i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 4*exp
(c*6i + d*x*6i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + 4*e
xp(c*7i + d*x*7i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + e
xp(c*8i + d*x*8i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2) + e
xp(c*9i + d*x*9i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
[Out] Timed out
```

3.190 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=275

$$\frac{8a^3(710A+803B)\sin(c+dx)}{3465d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(710A+803B)\sin(c+dx)}{1155d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(194A+209B)\sin(c+}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+}$$

```
[Out] 2/11*a*A*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(11/2)+2/693*a^3*(1
94*A+209*B)*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+2/1155*a^3
*(710*A+803*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+8/3465*
a^3*(710*A+803*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+16/3
465*a^3*(710*A+803*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+
2/99*a^2*(14*A+11*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)
```

Rubi [A] time = 0.71, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2975, 2980, 2772, 2771}

$$\frac{8a^3(710A+803B)\sin(c+dx)}{3465d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(710A+803B)\sin(c+dx)}{1155d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(194A+209B)\sin(c+}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2),x
]
```

```
[Out] (2*a^3*(194*A + 209*B)*Sin[c + d*x])/(693*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(710*A + 803*B)*Sin[c + d*x])/(1155*d*Cos[c + d*x]^(
5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sin[c + d*x])/(3465
*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a^3*(710*A + 803*B)*S
in[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/
2)) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(1
1/2))
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2aA(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+a\cos(c+\ldots}{\ldots}$$

$$= \frac{2a^2(14A+11B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2aA(a+a\ldots}{1\ldots}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(14A+11B)\sqrt{a\ldots}}{99d\,c\ldots}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(710A+803\ldots}{1155d\cos^{\frac{5}{2}}(c+dx)\ldots}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(710A+803\ldots}{1155d\cos^{\frac{5}{2}}(c+dx)\ldots}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^3(710A+803\ldots}{1155d\cos^{\frac{5}{2}}(c+dx)\ldots}$$

Mathematica [A] time = 0.99, size = 147, normalized size = 0.53

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\,((25070A+24827B)\cos(c+dx)+(9230A+9284B)\cos(2(c+dx))+9\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
3/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(9070*A + 7678*B + (25070*A + 24827*B)*Cos[
c + d*x] + (9230*A + 9284*B)*Cos[2*(c + d*x)] + 9230*A*Cos[3*(c + d*x)] + 1
0439*B*Cos[3*(c + d*x)] + 1420*A*Cos[4*(c + d*x)] + 1606*B*Cos[4*(c + d*x)]
 + 1420*A*Cos[5*(c + d*x)] + 1606*B*Cos[5*(c + d*x)])*Tan[(c + d*x)/2])/(69
30*d*Cos[c + d*x]^(11/2))
```

fricas [A] time = 0.95, size = 156, normalized size = 0.57

$$\frac{2\left(8\,(710\,A+803\,B)a^2\cos(dx+c)^5+4\,(710\,A+803\,B)a^2\cos(dx+c)^4+3\,(710\,A+803\,B)a^2\cos(dx+c)^3+5\ldots}{3465\left(d\cos(\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(8*(710*A + 803*B)*a^2*cos(d*x + c)^5 + 4*(710*A + 803*B)*a^2*cos(d*
x + c)^4 + 3*(710*A + 803*B)*a^2*cos(d*x + c)^3 + 5*(355*A + 286*B)*a^2*cos
(d*x + c)^2 + 35*(32*A + 11*B)*a^2*cos(d*x + c) + 315*A*a^2)*sqrt(a*cos(d*x
 + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^7 + d*cos(d*x +
c)^6)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.31, size = 155, normalized size = 0.56

$$-\frac{2a^2\left(-1+\cos\left(dx+c\right)\right)\left(5680A\left(\cos^5\left(dx+c\right)\right)+6424B\left(\cos^5\left(dx+c\right)\right)+2840A\left(\cos^4\left(dx+c\right)\right)+3212B\left(\cos^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x)
```

```
[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(5680*A*cos(d*x+c)^5+6424*B*cos(d*x+c)^5+2840
*A*cos(d*x+c)^4+3212*B*cos(d*x+c)^4+2130*A*cos(d*x+c)^3+2409*B*cos(d*x+c)^3
+1775*A*cos(d*x+c)^2+1430*B*cos(d*x+c)^2+1120*A*cos(d*x+c)+385*B*cos(d*x+c)
+315*A)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(11/2)
```

maxima [B] time = 0.99, size = 626, normalized size = 2.28

$$8\left(\frac{11\left(\frac{315\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{945\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1449\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1287\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{572\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{104\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)B\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] 8/3465*(11*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(
2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
104*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 1)) + 5*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1
) - 2310*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4620*sqrt(2)
*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5478*sqrt(2)*a^(5/2)*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 + 3575*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x
 + c) + 1)^9 - 1300*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 +
 200*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2
)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

mupad [B] time = 7.32, size = 773, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/cos(c + d*x)^(13/2),x
)
```

```
[Out] ((a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(710*A
 + 803*B)*16i)/(3465*d) - (a^2*exp(c*5i + d*x*5i)*(30*A + 41*B)*8i)/(15*d)
+ (a^2*exp(c*6i + d*x*6i)*(30*A + 41*B)*8i)/(15*d) + (a^2*exp(c*4i + d*x*4i
)*(160*A + 157*B)*8i)/(35*d) - (a^2*exp(c*7i + d*x*7i)*(160*A + 157*B)*8i)/
(35*d) + (a^2*exp(c*2i + d*x*2i)*(710*A + 803*B)*8i)/(315*d) - (a^2*exp(c*9
i + d*x*9i)*(710*A + 803*B)*8i)/(315*d) - (a^2*exp(c*11i + d*x*11i)*(710*A
+ 803*B)*16i)/(3465*d) - (B*a^2*exp(c*3i + d*x*3i)*8i)/(3*d) + (B*a^2*exp(c
*8i + d*x*8i)*8i)/(3*d)))/((exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^
(1/2) + exp(c*1i + d*x*1i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2)^
(1/2) + 5*exp(c*2i + d*x*2i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2
)^(1/2) + 5*exp(c*3i + d*x*3i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)
/2)^(1/2) + 10*exp(c*4i + d*x*4i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*
1i)/2)^(1/2) + 10*exp(c*5i + d*x*5i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d
*x*1i)/2)^(1/2) + 10*exp(c*6i + d*x*6i)*(exp(- c*1i - d*x*1i)/2 + exp(c*1i
+ d*x*1i)/2)^(1/2) + 10*exp(c*7i + d*x*7i)*(exp(- c*1i - d*x*1i)/2 + exp(c*
```

```
1i + d*x*1i)/2)^(1/2) + 5*exp(c*8i + d*x*8i)*(exp(- c*1i - d*x*1i)/2 + exp(
c*1i + d*x*1i)/2)^(1/2) + 5*exp(c*9i + d*x*9i)*(exp(- c*1i - d*x*1i)/2 + ex
p(c*1i + d*x*1i)/2)^(1/2) + exp(c*10i + d*x*10i)*(exp(- c*1i - d*x*1i)/2 +
exp(c*1i + d*x*1i)/2)^(1/2) + exp(c*11i + d*x*11i)*(exp(- c*1i - d*x*1i)/2
+ exp(c*1i + d*x*1i)/2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

3.191 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=190

$$-\frac{(4A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}\,d}+\frac{(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -1/4*(4*A-7*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)+
(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c
))^(1/2))*2^(1/2)/d/a^(1/2)+1/2*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*
x+c))^(1/2)+1/4*(4*A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2
)
```

Rubi [A] time = 0.59, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2983, 2982, 2782, 205, 2774, 216}

$$-\frac{(4A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}\,d}+\frac{(4A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((4*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*S
qrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Co
s[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + ((4*A - B)*Sqrt[Cos[c
 + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3aB}{2}+\frac{1}{2}a(4A-B)\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}dx}{2a}$$

$$= \frac{(4A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \int$$

$$= \frac{(4A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - (4$$

$$(4$$

$$= \frac{(4A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{(4A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}\,d} + \frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{}}\right.}{\sqrt{a}\,d}$$

Mathematica [C] time = 1.97, size = 348, normalized size = 1.83

$$\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,(4A+2B\cos(c+dx)-B)}{d} + \frac{\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(-8i\sqrt{2}(A-B)\log\left(1+e^{i(c+dx)}\right)+i(4\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (Cos[(c + d*x)/2]*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x))]*(-4*A*d*x + 7*B*d*x + I*(4*A - 7*B)*ArcSinh[E^(I*(c +
d*x))] - (8*I)*Sqrt[2]*(A - B)*Log[1 + E^(I*(c + d*x))] - (4*I)*A*Log[1 + S
qrt[1 + E^((2*I)*(c + d*x))]] + (7*I)*B*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x)
)]] + (8*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(
c + d*x))]] - (8*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^
((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]) + (4*Sqrt[Cos[c + d
*x]]*(4*A - B + 2*B*Cos[c + d*x])*Sin[(c + d*x)/2])/d))/(8*Sqrt[a*(1 + Cos[
c + d*x])])
```

fricas [A] time = 8.76, size = 184, normalized size = 0.97

$$\frac{(2\,B\cos\left(dx+c\right)+4\,A-B)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+((4\,A-7\,B)\cos\left(dx+c\right)+4\,A-7\,B}{4\,(ad\cos\left(dx+c\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*((2*B*cos(d*x + c) + 4*A - B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c
))*sin(d*x + c) + ((4*A - 7*B)*cos(d*x + c) + 4*A - 7*B)*sqrt(a)*arctan(sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 4*sqrt(2
)*((A - B)*a*cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) +
 a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) +
 a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos\left(dx+c\right)+A)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a),
 x)
```

maple [B] time = 0.32, size = 346, normalized size = 1.82

$$\frac{\left(\cos^{\frac{3}{2}}\left(dx+c\right)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{3}\left(-4A\sin\left(dx+c\right)\cos\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}-4A\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(-4*A*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-4*A*sin(d*x+c)*(cos(d*
```

```
x+c)/(1+cos(d*x+c)))^(3/2)-2*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)+4*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*2^(1/2)-4*B
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*2^(1/2)+B*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*A*cos(d*x+c)*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-7*B*cos(d*x+c)*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))/sin(d*x+c)^6/(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.192 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{(2A-B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*A-B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-(A-B)*a
rctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2
))*2^(1/2)/d/a^(1/2)+B*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2983, 2982, 2782, 205, 2774, 216}

$$\frac{(2A-B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((2*A - B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a
]*d) - (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (B*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\frac{aB}{2}+\frac{1}{2}a(2A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{(2A-B)\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{2a} + (-A+B)$$

$$= \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{(2a(A-B))\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{}{\sqrt{\text{co}}}\right.}{d}$$

$$= \frac{(2A-B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a}}\right.}{\sqrt{a}\,d}$$

Mathematica [C] time = 1.27, size = 222, normalized size = 1.57

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{4B\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}}{d} - \frac{i\sqrt{2}\,e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left((2A-B)\sinh^{-1}\left(e^{i(c+dx)}\right)+2\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{1-e^{i(c+d}}{\sqrt{2}\,\sqrt{1+e^{2i(}}}\right.\right.}{d\sqrt{1+e^{2i(c+dx)}}}\right.}{2\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (Cos[(c + d*x)/2]*(((-I)*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c
+ d*x)))/E^(I*(c + d*x))]*((2*A - B)*ArcSinh[E^(I*(c + d*x))] + 2*Sqrt[2]*(
A - B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]
)] + (-2*A + B)*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*
I)*(c + d*x))]) + (4*B*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2])/d))/(2*Sqrt[a*(
1 + Cos[c + d*x])])
```

fricas [A] time = 4.16, size = 168, normalized size = 1.19

$$\frac{\sqrt{a\cos(dx+c)+a}\,B\sqrt{\cos(dx+c)}\,\sin(dx+c) - ((2A-B)\cos(dx+c)+2A-B)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos}}{\sqrt{a}\,\sin(dx+c)}\right.}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A - B)*co
s(d*x + c) + 2*A - B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) + sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a
)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(a*cos(d*x + c) + a),
 x)
```

maple [A] time = 0.26, size = 216, normalized size = 1.53

$$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\,(-1+\cos(dx+c))^2\left(A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}+B\sin(dx+c)\sqrt{\frac{\cos(d}{1+\cos(}}\right.}{d\sin(dx+c)^4\left(\frac{\cos}{1+\mathrm{co}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(A*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)-B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)+2*A*arctan(sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-B*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))/sin(d*x+c)^4/(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(1/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/sqrt(a*(cos(c + d*x) + 1))
, x)
```

3.193 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)+(A-B)*arcta
n(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2
^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2982, 2782, 205, 2774, 216}

$$\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) +
 (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx = (A-B)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx + \frac{B\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{a}$$

$$= -\frac{(2a(A-B))\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{(2B)\,\text{S}}{}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.15, size = 82, normalized size = 0.82

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)+\sqrt{2}\,B\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]
),x]
```

```
[Out] (2*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + (A - B)*ArcTan[Sin[(c + d*
x)/2]/Sqrt[Cos[c + d*x]]])*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 3.88, size = 96, normalized size = 0.96

$$-\frac{\sqrt{2}\,(A-B)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)+2\,B\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -(sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*B*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

maple [A] time = 0.23, size = 149, normalized size = 1.49

$$\frac{\sqrt{a\,(1+\cos(dx+c))}\,\left(\sqrt{\cos}\,(dx+c)\right)(-1+\cos(dx+c))\left(A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}\,-B\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\right.}{d\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\;a\sin(dx+c)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*(A*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*2^(1/2)-B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/
2)-2*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))/(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)/a/sin(d*x+c)^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*sqrt(cos(c + d*x)
)), x)
```

3.194 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=99

$$\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))*2^(1/2)/d/a^(1/2)+2*A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x
+c))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2984, 12, 2782, 205}

$$\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + \frac{2\int -\frac{a(A-B)}{2\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + (-A+B)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+a$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}} + \frac{(2a(A-B))\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x}{d}$$

$$= -\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2A\sin(c+}{d\sqrt{\cos(c+dx)}\,\sqrt{a+}$$

Mathematica [C] time = 1.64, size = 203, normalized size = 2.05

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(10B\cos(c+dx)-(A-B)\left(\frac{1}{2}\sin(c+dx)\tan(c+dx)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\sec(c+dx)\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]
),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*(10*B*Cos[c + d*x] - (A - B)*((-5*(1 +
 4*Cos[c + d*x] + Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^4*(1 - Cos[c + d*x] +
```

```
ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*S
ec[c + d*x]]))/4 + (Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c +
d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/2)))/(5*d*Cos[c + d*x]^(3/2)*Sqrt[a*
(1 + Cos[c + d*x])])
```

fricas [A] time = 1.18, size = 143, normalized size = 1.44

$$\frac{2\sqrt{a\cos(dx+c)+a}\,A\sqrt{\cos(dx+c)}\,\sin(dx+c)-\frac{\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+}}{2\left(\cos(dx+c)^2+\cos(dx+\right.}\right.}{\sqrt{a}}}{ad\cos(dx+c)^2+ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] (2*sqrt(a*cos(d*x + c) + a)*A*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(2)*((A
 - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*arctan(1/2*sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*
x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

maple [B] time = 0.26, size = 230, normalized size = 2.32

$$\frac{\sqrt{a(1+\cos(dx+c))}\left(A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)-B\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}\sqrt{\frac{\mathrm{co}}{1+\mathrm{c}}}\right.}{da(1+\mathrm{cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c)))^(1/2)*(A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-B*arcsin((-1+cos(d*x+c))/sin(d
```

```
*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+A*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-B*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*A*sin
(d*x+c))/a/(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(3/
2)), x)
```

3.195 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=142

$$-\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c}$$

```
[Out] (A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c
))^(1/2))*2^(1/2)/d/a^(1/2)+2/3*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*
x+c))^(1/2)-2/3*(A-3*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2
)
```

Rubi [A] time = 0.34, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2984, 12, 2782, 205}

$$-\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 3*B)*Sin[c + d*x])/(3*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-3B)+aA\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+d}$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+d}$$

$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+d}$$

$$= \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2A\sin(c+d}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a}$$

Mathematica [C] time = 6.81, size = 627, normalized size = 4.42

$$\frac{2(A-B)\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-12\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-12\left(3\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]
),x]
```

```
[Out] (4*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(3*d*Sqrt[a*(1 + Cos[c + d*x])]
*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (8*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d
*x)/2])/(3*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) +
 (2*(A - B)*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*(-12*Cos[(c + d*x)/2]^4
*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin
[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/
2, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]
^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[-(Sin[c/2
 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3
*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7*Sin[c/2 +
(d*x)/2]^2)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))] - 3*
ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*(1 - 2*
Sin[c/2 + (d*x)/2]^2))))/(63*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 +
(d*x)/2]^2)^(7/2))
```

fricas [A] time = 0.75, size = 163, normalized size = 1.15

$$-\frac{2\left((A-3B)\cos\left(dx+c\right)-A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\frac{3\sqrt{2}\left((A-B)a\cos(dx+c)^3+(A-B)a\cos(dx\right.}{}}{3\left(ad\cos\left(dx+c\right)^3+ad\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*(2*((A - 3*B)*cos(d*x + c) - A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))*sin(d*x + c) - 3*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*
x + c)^2)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*si
n(d*x + c)/((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x
 + c)^3 + a*d*cos(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\,\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

maple [B] time = 0.34, size = 383, normalized size = 2.70

$$\frac{\left(\sin^2(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\,\left(3A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^2(dx+c)\right)\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}-3B\arcsin\left(\frac{-1\cdots}{\mathrm{s}\cdots}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3/d*sin(d*x+c)^2*(a*(1+cos(d*x+c)))^(1/2)*(3*A*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-3*B*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)+6*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*2^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(3/2)-6*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+3*A*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-3*B*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*A*cos(d*x+c)*sin(d*
x+c)-6*B*cos(d*x+c)*sin(d*x+c)-2*A*sin(d*x+c))/a/(-1+cos(d*x+c))/(1+cos(d*x
+c))^2/cos(d*x+c)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,\sqrt{a+a\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.196 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=187

$$-\frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a}}\right.}{\sqrt{a}\,d}$$

```
[Out] -(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))*2^(1/2)/d/a^(1/2)+2/5*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d
*x+c))^(1/2)-2/15*(A-5*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1
/2)+2/15*(13*A-5*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2984, 12, 2782, 205}

$$-\frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a}}\right.}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*Cos[c
+ d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 5*B)*Sin[c + d*x])/(15*d*C
os[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*(13*A - 5*B)*Sin[c + d*x])
/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-5B)+2aA\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{5a}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+}$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2A\sin(c}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a}}$$

Mathematica [C] time = 7.82, size = 1728, normalized size = 9.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]
),x]
```

```
[Out] (4*B*Cos[c/2 + (d*x)/2]*Sin[c/2 + (d*x)/2])/(5*d*Sqrt[a*(1 + Cos[c + d*x])]
*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) + (16*B*Cos[c/2 + (d*x)/2]*(Sin[c/2 +
(d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (2*Sin[c/2 + (d*x)/2])/Sqrt[1
 - 2*Sin[c/2 + (d*x)/2]^2]))/(15*d*Sqrt[a*(1 + Cos[c + d*x])]) - (2*(A - B)
*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*(4725*Sin[c/2 + (d*x)/2]^2 - 48825
*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x
)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2,
 Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10
- 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin
[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 51
8760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*S
in[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)
/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2
+ (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[
c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt
[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*
Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^
16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d
*x)/2]^4*HypergeometricPFQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2
```

```
]^2)))/(675*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)
*(-1 + 2*Sin[c/2 + (d*x)/2]^2))
```

fricas [A] time = 1.17, size = 180, normalized size = 0.96

$$\frac{2\left((13\,A-5\,B)\cos\left(dx+c\right)^2-(A-5\,B)\cos\left(dx+c\right)+3\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\frac{15\cdots}{}}{15\left(ad\cos\left(dx+c\right)^4+ad\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(2*((13*A - 5*B)*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) + 3*A)*sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*((A - B)*a*
cos(d*x + c)^4 + (A - B)*a*cos(d*x + c)^3)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x + c)
)*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{a\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(7/2)
), x)
```

maple [B] time = 0.36, size = 519, normalized size = 2.78

$$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(\sin^4\left(dx+c\right)\right)\left(15A\left(\cos^3\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{2}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-15B\left(\cos^3\left(\cdots\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^4*(15*A*cos(d*x+c)^3*(cos(d*x+c)
/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-15*B*cos(
```

```
d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arcsin((-1+cos(d*x+c))/s
in(d*x+c))+45*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arcs
in((-1+cos(d*x+c))/sin(d*x+c))-45*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
)^(5/2)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+45*A*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-45*B*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))+15*A*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arcsin((-1+cos(d
*x+c))/sin(d*x+c))-15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)*arcsin((-
1+cos(d*x+c))/sin(d*x+c))+26*A*cos(d*x+c)^2*sin(d*x+c)-10*B*sin(d*x+c)*cos(
d*x+c)^2-2*A*cos(d*x+c)*sin(d*x+c)+10*B*cos(d*x+c)*sin(d*x+c)+6*A*sin(d*x+c
))/a/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3/cos(d*x+c)^(5/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a*cos(c + d*x))^(1/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.197 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=197

$$\frac{(2A-3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(5A-9B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (2*A-3*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d+1/2*(
A-B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-1/4*(5*A-9*B)*arc
tan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))
/a^(3/2)/d*2^(1/2)-1/2*(A-3*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x
+c))^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(2A-3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(5A-9B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((2*A - 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3
/2)*d) - ((5*A - 9*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) - ((A - 3*B)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-B)-a(A-3B)\cos(c+d\right.}{\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A-3B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{2ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A-3B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{2ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A-3B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{2ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(2A-3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-9B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a}}\right.}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] time = 2.20, size = 362, normalized size = 1.84

$$\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{2\sqrt{\cos(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)(-A+2B\cos(c+dx)+3B)}{d} + \frac{\sqrt{2}e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(i\sqrt{2}(5A-9B)\log\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(4*A*d*x - 6*B*d*x - (2*I)*(2*A - 3*B)*ArcSinh[E^(I*
(c + d*x))] + I*Sqrt[2]*(5*A - 9*B)*Log[1 + E^(I*(c + d*x))] + (4*I)*A*Log[
1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (6*I)*B*Log[1 + Sqrt[1 + E^((2*I)*(c +
 d*x))]] - (5*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2
*I)*(c + d*x))]] + (9*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1
 + E^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]) + (2*Sqrt[Cos[
c + d*x]]*(-A + 3*B + 2*B*Cos[c + d*x])*Sec[(c + d*x)/2]*Tan[(c + d*x)/2])/
d))/(2*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 15.06, size = 237, normalized size = 1.20

$$\sqrt{2}\left((5\,A-9\,B)\cos\left(dx+c\right)^{2}+2\,(5\,A-9\,B)\cos\left(dx+c\right)+5\,A-9\,B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5*A
 - 9*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(sqrt(a)*sin(d*x + c))) + 2*(2*B*cos(d*x + c) - A + 3*B)*sqrt(a*cos(d*x + c
) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 4*((2*A - 3*B)*cos(d*x + c)^2 + 2*
(2*A - 3*B)*cos(d*x + c) + 2*A - 3*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) +
a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^
2*d*cos(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos\left(dx+c\right)+A)\cos\left(dx+c\right)^{\frac{3}{2}}}{(a\cos\left(dx+c\right)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

maple [B] time = 0.30, size = 379, normalized size = 1.92

$$-\frac{\left(\cos^{\frac{3}{2}}\left(dx+c\right)\right)\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{3}\left(2A\left(\cos^{2}\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+5A\arcsin\left(\frac{-1+\mathrm{co}}{\sin(}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(2*A*cos
(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+5*A*arcsin((-1+cos(d*x+c))/sin(
```

```
d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-9*B*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*sin(d*x+c)*cos(d*x+c)*2^(1/2)-4*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)+8*A*sin(d*x+c)*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)/cos(d*x+c))-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-12*B*sin(d*x+c
)*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)-2*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+6*B*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^7/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/
a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{3/2}\,(A+B\,\cos(c+d\,x))}{(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.198 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{(A-5B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d+1/4*(A-5*B)
*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1
/2))/a^(3/2)/d*2^(1/2)+1/2*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x
+c))^(3/2)
```

Rubi [A] time = 0.40, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2977, 2982, 2782, 205, 2774, 216}

$$\frac{(A-5B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
 ((A - 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(A-B)+2aB\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(A-5B)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(A-5B)\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\right.}{2d}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{(A-5B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right.}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] time = 1.91, size = 226, normalized size = 1.56

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{(A-B)\sqrt{\cos(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{d} - \frac{ie^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(-\sqrt{2}\,(A-5B)\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+\right.}{\sqrt{2}\,d\sqrt{1+e^{2i(c+dx)}}}\right.}{(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(((-I)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)
))/E^(I*(c + d*x))]*(4*B*ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*(A - 5*B)*ArcTa
nh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 4*B*Arc
Tanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x
))]) + ((A - B)*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Tan[(c + d*x)/2])/d))/(
a*(1 + Cos[c + d*x]))^(3/2)
```

fricas [A] time = 10.31, size = 203, normalized size = 1.40

$$-\frac{\sqrt{2}\left((A-5\,B)\cos\left(dx+c\right)^2+2\,(A-5\,B)\cos\left(dx+c\right)+A-5\,B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-2\,\sqrt{a}}{4\left(a^2d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*
B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(A - B)*sqrt(cos(d*x + c))*
sin(d*x + c) + 8*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3/2
), x)
```

maple [B] time = 0.28, size = 298, normalized size = 2.06

$$-\frac{\left(\sqrt{\cos}\ (dx+c)\right)\sqrt{a\,(1+\cos(dx+c))}\ (-1+\cos(dx+c))^{2}\left(2A\left(\cos^{2}(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+A\arcsin\left(\frac{-1+\cdots}{\sin\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(2*A*cos
(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+A*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
sin(d*x+c)*cos(d*x+c)*2^(1/2)-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-8*B*sin
(d*x+c)*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(
d*x+c))-2*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*B*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^5/(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(3/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(3
/2), x)
```

3.199 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{(3A+B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 1/4*(3*A+B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos
(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(
a+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.22, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2978, 12, 2782, 205}

$$\frac{(3A+B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((3*A + B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{a(3A+B)}{2\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(3A+B)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c-}}{4a}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{(3A+B)\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x\right.}{2d}$$

$$= \frac{(3A+B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{(A-B)\sqrt{\cos(c+dx)}\,s}{2d(a+a\cos(c+d}$$

Mathematica [C]   time = 1.15, size = 212, normalized size = 1.98

$$\frac{\frac{1}{2}i(A-B)e^{-\frac{1}{2}i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,\sqrt{\cos(c+dx)}\,\cos\left(\frac{1}{2}(c+dx)\right)+i(3A+B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e\right.}}{d\sqrt{1+e^{2i(c+dx)}}\,(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] (I*(3*A + B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d
*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])
]*Cos[(c + d*x)/2]^3 + ((I/2)*(A - B)*(-1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]])/E^((I/2)*(c + d*x)))/(
d*Sqrt[1 + E^((2*I)*(c + d*x))]*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 0.92, size = 164, normalized size = 1.53

$$\frac{\sqrt{2}\left((3A+B)\cos(dx+c)^2+2(3A+B)\cos(dx+c)+3A+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A + B
)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x
+ c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x
+ c) + a)*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 +
2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

maple [B] time = 0.26, size = 246, normalized size = 2.30

$$\frac{\sqrt{a(1+\cos(dx+c))}\ (-1+\cos(dx+c))\left(-2A\left(\cos^2(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+3A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sin(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(-2*A*cos(d*x+c)^2*(cos(d*x+
c)/(1+cos(d*x+c)))^(3/2)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*
cos(d*x+c)*2^(1/2)+B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+
c)*2^(1/2)+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*B*cos(d*x+c)^2*(cos(d*x+
```

```
c)/(1+cos(d*x+c)))^(1/2)-2*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/a^2/cos(d*x+c)^(1/2)/sin(d*x+c)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+a\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(cos(c + d
*x))), x)
```

3.200 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{(7A-3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+}{2d\sqrt{\cos(c+dx)}(a\cos(c}$$

```
[Out] -1/4*(7*A-3*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*
cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c)
)^(3/2)/cos(d*x+c)^(1/2)+1/2*(5*A-B)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*c
os(d*x+c))^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 2984, 12, 2782, 205}

$$-\frac{(7A-3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+}{2d\sqrt{\cos(c+dx)}(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] -((7*A - 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*
d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((5*A - B)*Sin[c + d*x])
/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
```

```
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(5A-B)-a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sin(c+}{2ad\sqrt{\cos(c+dx)}\,\sqrt{a+a\,c}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sin(c+}{2ad\sqrt{\cos(c+dx)}\,\sqrt{a+a\,c}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sin(c+}{2ad\sqrt{\cos(c+dx)}\,\sqrt{a+a\,c}}$$

$$= -\frac{(7A-3B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{(A-B)\,\mathrm{si}}{2d\sqrt{\cos(c+dx)}\,(a}$$

Mathematica [C] time = 3.87, size = 423, normalized size = 2.71

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{(A+3B)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(5(4\cos(c+dx)+\cos(2(c+dx))+1)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{2-2\sec(c+dx)}\,\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)(}\right.\right.\right.}{2\cos^{\frac{3}{2}}(c+dx)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(30*(A - B)*ArcTan[(1 - 2*Sin[(c + d*x)/2])/Sqrt[Cos[c
+ d*x]]] - 30*(A - B)*ArcTan[(1 + 2*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]]] -
 (20*(A - B)*Sqrt[Cos[c + d*x]])/(-1 + Sin[(c + d*x)/2]) - (20*(A - B)*Sqrt
[Cos[c + d*x]])/(1 + Sin[(c + d*x)/2]) + (5*(A - B)*(-1 + 2*Sin[(c + d*x)/2
]))/(Sqrt[Cos[c + d*x]]*(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2) - (5*(A -
B)*(1 + 2*Sin[(c + d*x)/2]))/(Sqrt[Cos[c + d*x]]*(-1 + Sin[(c + d*x)/2])) +
 ((A + 3*B)*Csc[(c + d*x)/2]^3*(5*(1 + 4*Cos[c + d*x] + Cos[2*(c + d*x)])*(
1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c
+ d*x]*Sqrt[2 - 2*Sec[c + d*x]]) - 2*Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c
 + d*x]*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^4*Sin[c + d*x]*Tan[c + d*x]))
/(2*Cos[c + d*x]^(3/2))))/(10*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 1.06, size = 201, normalized size = 1.29

$$-\frac{\sqrt{2}\left((7A-3B)\cos(dx+c)^3+2(7A-3B)\cos(dx+c)^2+(7A-3B)\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+}}{2\left(a\cos(d\right.}\right.}{4\left(a^2d\cos(dx+c)^3+2a^2d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^3 + 2*(7*A - 3*B)*cos(d*x + c)^2 +
(7*A - 3*B)*cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) +
a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x +
c))) - 2*((5*A - B)*cos(d*x + c) + 4*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2
*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

maple [B] time = 0.28, size = 299, normalized size = 1.92

$$-\frac{\sqrt{a(1+\cos(dx+c))}\left(-7A\cos(dx+c)\sin(dx+c)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3B\cos(dx+c)\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*(-7*A*cos(d*x+c)*sin(d*x+c)*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*B*cos(d*x+c
)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)-7*A*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*(c
```

```
os(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*B*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d
*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+10*A*cos(d*x+c)^2-2*B*cos(
d*x+c)^2-2*A*cos(d*x+c)+2*B*cos(d*x+c)-8*A)/a^2/sin(d*x+c)/(1+cos(d*x+c))/c
os(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^{3/2}\,(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a*(cos(c + d*x) + 1))**(3/2)*cos(c + d*x)**
(3/2)), x)
```

3.201 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=203

$$\frac{(11A-7B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(7A-3B)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+a}$$

```
[Out] -1/2*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+1/4*(11*A-7
*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2))/a^(3/2)/d*2^(1/2)+1/6*(7*A-3*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+
a*cos(d*x+c))^(1/2)-1/6*(19*A-15*B)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*co
s(d*x+c))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 2984, 12, 2782, 205}

$$\frac{(11A-7B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(7A-3B)\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((11*A - 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*
d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((7*A - 3*B)*Sin[c + d*x
])/(6*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - ((19*A - 15*B)*Sin
[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(7A-3B)-2a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sin(c+dx}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sin(c+dx}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sin(c+dx}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sin(c+dx}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos}}$$

$$= \frac{(11A-7B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{(A-B)\sin(}{2d\cos^{\frac{3}{2}}(c+dx)(a+a}$$

Mathematica [C] time = 6.81, size = 1054, normalized size = 5.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] -1/6*((A - B)*Cos[c/2 + (d*x)/2]^3*(1 - 2*Sin[c/2 + (d*x)/2]))/(d*(a*(1 + C
os[c + d*x]))^(3/2)*(1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(
3/2)) + ((A - B)*Cos[c/2 + (d*x)/2]^3*(1 + 2*Sin[c/2 + (d*x)/2]))/(6*d*(a*(
1 + Cos[c + d*x]))^(3/2)*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]
^2)^(3/2)) - ((A - B)*Cos[c/2 + (d*x)/2]^3*(5*ArcTan[(1 - 2*Sin[c/2 + (d*x)
/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (1 + Sin[c/2 + (d*x)/2])/((1 - Sin
[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2
+ (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
+ ((A - B)*Cos[c/2 + (d*x)/2]^3*(5*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1
 - 2*Sin[c/2 + (d*x)/2]^2]] + (1 - Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x
```

```
)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^
2])/(1 + Sin[c/2 + (d*x)/2])))/(d*(a*(1 + Cos[c + d*x]))^(3/2)) + ((A + 3*B
)*Cot[c/2 + (d*x)/2]^3*Csc[c/2 + (d*x)/2]^2*(-12*Cos[(c + d*x)/2]^4*Hyperge
ometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (
d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2F1[2, 7/2, 9/2, -(Sin
[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8*(4 -
7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*Sqrt[-(Sin[c/2 + (d*x)
/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*(15 - 2
0*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 - 7*Sin[c/2 + (d*x)/2]
^2)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))] - 3*ArcTanh[
Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]]*(1 - 2*Sin[c/2
+ (d*x)/2]^2))))/(63*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1 - 2*Sin[c/2 + (d*x)/
2]^2)^(7/2))
```

fricas [A] time = 0.98, size = 221, normalized size = 1.09

$$\frac{3\sqrt{2}\left((11A-7B)\cos(dx+c)^4+2(11A-7B)\cos(dx+c)^3+(11A-7B)\cos(dx+c)^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{a\cos\cdots}}{2(\cdots}\right.}{12\left(a^2d\cos(dx+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^4 + 2*(11*A - 7*B)*cos(d*x + c)^
3 + (11*A - 7*B)*cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x
+ c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos
(d*x + c))) - 2*((19*A - 15*B)*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) - 4
*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*
x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

maple [B] time = 0.34, size = 443, normalized size = 2.18

$$\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\ \sin\left(dx+c\right)\left(33A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)\sqrt{2}\ \left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}-21B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/12/d*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*(33*A*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-
21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)+66*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(
d*x+c)*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-42*B*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)+33*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(3/2)-21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin
(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-38*A*cos(d*x+c)^3+30*B*co
s(d*x+c)^3+14*A*cos(d*x+c)^2-6*B*cos(d*x+c)^2+32*A*cos(d*x+c)-24*B*cos(d*x+
c)-8*A)/a^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(3/2)), x
)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.202 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=246

$$\frac{(2A-5B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(43A-115B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(11A-35B)\sin(c+dx)\sqrt{\cos}}{16a^2d\sqrt{a\cos(c+dx)+}}$$

```
[Out] (2*A-5*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d+1/4*(
A-B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/16*(7*A-15*B)*c
os(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)-1/32*(43*A-115*B)*arc
tan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))
/a^(5/2)/d*2^(1/2)-1/16*(11*A-35*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*
cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(2A-5B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(43A-115B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(}}\right)}{16\sqrt{2}\,a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((2*A - 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5
/2)*d) - ((43*A - 115*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
+ d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c
 + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((7*A - 15*B
)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) - ((
11*A - 35*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c +
d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
```

```
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-a(A-5B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(7A-15B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(7A-15B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(7A-15B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(7A-15B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(2A-5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-115B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a}}\right.}{16\sqrt{2}a^{5/2}d}$$

Mathematica [C] time = 3.53, size = 376, normalized size = 1.53

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((55B-15A)\cos(c+dx)-11A+8B\cos(2(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d
*x)))/E^(I*(c + d*x))]*(32*A*d*x - 80*B*d*x - (16*I)*(2*A - 5*B)*ArcSinh[E^
(I*(c + d*x))] + I*Sqrt[2]*(43*A - 115*B)*Log[1 + E^(I*(c + d*x))] + (32*I)
```

```
*A*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (80*I)*B*Log[1 + Sqrt[1 + E^((2
*I)*(c + d*x))]] - (43*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[2]*Sqrt[
1 + E^((2*I)*(c + d*x))]] + (115*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x)) + Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d*x))] + Sqrt[
Cos[c + d*x]]*(-11*A + 43*B + (-15*A + 55*B)*Cos[c + d*x] + 8*B*Cos[2*(c +
d*x)])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/2]))/(8*d*(a*(1 + Cos[c + d*x]))^(5
/2))
```

fricas [A] time = 28.56, size = 302, normalized size = 1.23

$$\frac{\sqrt{2}\left((43\,A-115\,B)\cos\left(dx+c\right)^{3}+3\,(43\,A-115\,B)\cos\left(dx+c\right)^{2}+3\,(43\,A-115\,B)\cos\left(dx+c\right)+43\,A-115\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^3 + 3*(43*A - 115*B)*cos(d*x + c
)^2 + 3*(43*A - 115*B)*cos(d*x + c) + 43*A - 115*B)*sqrt(a)*arctan(sqrt(2)*
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(16
*B*cos(d*x + c)^2 - 5*(3*A - 11*B)*cos(d*x + c) - 11*A + 35*B)*sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 32*((2*A - 5*B)*cos(d*x + c)
^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + 3*(2*A - 5*B)*cos(d*x + c) + 2*A - 5*B)
*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*
x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
+ c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos\left(dx+c\right)+A)\cos\left(dx+c\right)^{\frac{5}{2}}}{(a\cos\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

maple [B] time = 0.34, size = 647, normalized size = 2.63

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{5}\left(\cos^{\frac{5}{2}}\left(dx+c\right)\right)\left(30A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^{3}\left(dx+c\right)\right)+43A\arcsin\left(\frac{-1}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*cos(d*x+c)^(5/2)*(30*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^3+43*A*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+22*A*cos(d*x+c)^2*(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)-115*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*
x+c)^2*sin(d*x+c)-32*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+43*A*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-30*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)+64*A*arctan(sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-115*B*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-78*B*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)-160*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-22*A*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)+64*A*sin(d*x+c)*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))+40*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)-160*B*sin(d*x+c)*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)/cos(d*x+c))+70*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/
sin(d*x+c)^11/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.203 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=194

$$\frac{(3A-43B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{(3A$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d+1/4*(A-B)*c
os(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+1/32*(3*A-43*B)*arctan(
1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(
5/2)/d*2^(1/2)+1/16*(3*A-11*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x
+c))^(3/2)
```

Rubi [A] time = 0.58, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2977, 2982, 2782, 205, 2774, 216}

$$\frac{(3A-43B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{(3A$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
 ((3*A - 43*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((3*A - 11*B)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-B)+4aB\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}dx}{4a^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A-11B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A-11B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(3A-11B)\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{(3A-43B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}$$

Mathematica [C] time = 2.17, size = 246, normalized size = 1.27

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((7A-15B)\cos(c+dx)+3A-11B)-\frac{i\sqrt{2}e^{\frac{1}{2}i(c+dx)}}{}\right.}{8d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-I)*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(
c + d*x)))/E^(I*(c + d*x))]*(32*B*ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*(3*A -
 43*B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]
)] - 32*B*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d
*x))] + Sqrt[Cos[c + d*x]]*(3*A - 11*B + (7*A - 15*B)*Cos[c + d*x])*Sec[(c
+ d*x)/2]^3*Tan[(c + d*x)/2]))/(8*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 24.44, size = 267, normalized size = 1.38

$$-\frac{\sqrt{2}\left((3A-43B)\cos(dx+c)^3+3(3A-43B)\cos(dx+c)^2+3(3A-43B)\cos(dx+c)+3A-43B\right)\sqrt{a}\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^2
 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((7*A - 15
*B)*cos(d*x + c) + 3*A - 11*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c) + 64*(B*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c)
 + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*s
in(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos
(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

maple [B] time = 0.31, size = 515, normalized size = 2.65

$$-\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\,(-1+\cos(dx+c))^4\left(14A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^3(dx+c)\right)+3A\arcsin\left(\frac{-1}{\ldots}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*(14*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^3+3*A*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+6*A*cos(d*x+c)^2*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)-43*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c
)^2*sin(d*x+c)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)
*2^(1/2)-14*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)-43*B*arcsin((-1+
```

```
cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-64*B*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-30*
B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-6*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)-64*B*sin(d*x+c)*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c))+8*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)+22*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^9/(cos(d*
x+c)/(1+cos(d*x+c)))^(5/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\cos^{\frac{3}{2}}(c+dx)}{(a(\cos(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)**(3/2)/(a*(cos(c + d*x) + 1))**(
5/2), x)
```

3.204 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=154

$$\frac{(5A+3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] 1/32*(5*A+3*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*
cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+1/4*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/
d/(a+a*cos(d*x+c))^(5/2)+1/16*(A+7*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*
cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.38, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2977, 2978, 12, 2782, 205}

$$\frac{(5A+3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((5*A + 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) + ((A + 7*B)*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
```

```
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(A-B)+a(A+3B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B)\sqrt{\cos(c+dx)}\,\sin(c+a}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B)\sqrt{\cos(c+dx)}\,\sin(c+a}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{(A+7B)\sqrt{\cos(c+dx)}\,\sin(c+a}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(5A+3B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{(A-B)\sqrt{\cos(c+dx)}}{4d(a+a\cos(c+}$$

Mathematica [C] time = 1.51, size = 198, normalized size = 1.29

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\sqrt{\cos(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((A+7B)\cos(c+dx)+5A+3B)+\frac{i(5A+3B)e}{}\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((I*(5*A + 3*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*
(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1
+ E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + (Sqrt[Cos[c + d*x
]]*(5*A + 3*B + (A + 7*B)*Cos[c + d*x])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/2]
)/2))/(4*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 0.89, size = 215, normalized size = 1.40

$$\frac{\sqrt{2}\left((5A+3B)\cos(dx+c)^3+3(5A+3B)\cos(dx+c)^2+3(5A+3B)\cos(dx+c)+5A+3B\right)\sqrt{a}\,\arctan\left(\right.}{32\left(a^3d\cos(dx+c)^3+3a^3d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 +
3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*c
os(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
+ a*cos(d*x + c))) + 2*((A + 7*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*
cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5/2
), x)
```

maple [B] time = 0.32, size = 413, normalized size = 2.68

$$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{a\,(1+\cos(dx+c))}\,\,(-1+\cos(dx+c))^{3}\left(2A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^{3}(dx+c)\right)+10A\left(\cos^{2}(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(2*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^3+10*A*cos(d*x+c)^2*(cos(d*x+c)/(
1+cos(d*x+c)))^(3/2)+5*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x
+c)^2*sin(d*x+c)+3*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^
2*sin(d*x+c)-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)+5*A*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+14*B*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin
(d*x+c)*cos(d*x+c)*2^(1/2)-10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-8*B*cos(d
*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-6*B*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2))/sin(d*x+c)^7/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)}{{\left(a+a\,\cos(c+d\,x)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(5/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{(a(\cos(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(5
/2), x)
```

3.205 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$\frac{(19A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+}}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] 1/32*(19*A+5*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)
/d/(a+a*cos(d*x+c))^(5/2)-1/16*(9*A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a
*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.44, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2978, 12, 2782, 205}

$$\frac{(19A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] ((19*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((9*A - B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
```

```
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(7A+B)-a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(9A-B)\sqrt{\cos(c+dx)}\,\sin(c+}{16ad(a+a\cos(c+dx))^{3/}}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(9A-B)\sqrt{\cos(c+dx)}\,\sin(c+}{16ad(a+a\cos(c+dx))^{3/}}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(9A-B)\sqrt{\cos(c+dx)}\,\sin(c+}{16ad(a+a\cos(c+dx))^{3/}}$$

$$= \frac{(19A+5B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{(A-B)\sqrt{\cos(c+}}{4d(a+a\cos(}$$

Mathematica [C] time = 1.47, size = 200, normalized size = 1.28

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-\frac{1}{2}\sqrt{\cos(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((9A-B)\cos(c+dx)+13A-5B)+\frac{i(19A+}{}\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((I*(19*A + 5*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)
*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1
 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - (Sqrt[Cos[c + d*
x]]*(13*A - 5*B + (9*A - B)*Cos[c + d*x])*Sec[(c + d*x)/2]^3*Tan[(c + d*x)/
2])/2))/(4*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 1.13, size = 217, normalized size = 1.39

$$\frac{\sqrt{2}\left((19\,A+5\,B)\cos\left(dx+c\right)^{3}+3\,(19\,A+5\,B)\cos\left(dx+c\right)^{2}+3\,(19\,A+5\,B)\cos\left(dx+c\right)+19\,A+5\,B\right)\sqrt{a}\,\operatorname{arcta}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2
+ 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt
(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c
)^2 + a*cos(d*x + c))) - 2*((9*A - B)*cos(d*x + c) + 13*A - 5*B)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

maple [B] time = 0.27, size = 413, normalized size = 2.65

$$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\ \left(-1+\cos\left(dx+c\right)\right)^{2}\left(18A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^{3}\left(dx+c\right)\right)+26A\left(\cos^{2}\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(18*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(3/2)*cos(d*x+c)^3+26*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)-19*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c
)-5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-18
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)-19*A*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-2*B*cos(d*x+c)^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)-5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*
x+c)*2^(1/2)-26*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-8*B*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)+10*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/a^3/cos(d*x+c)^(1/2)/sin(d*x+c)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.206 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{(75A-19B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(49A-9B)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}-\frac{(13A-5B)\,\mathrm{si}}{16ad\sqrt{\cos(c+dx)}\,(a\,\mathrm{c}}$$

```
[Out] -1/32*(75*A-19*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a
+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x
+c))^(5/2)/cos(d*x+c)^(1/2)-1/16*(13*A-5*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))
^(3/2)/cos(d*x+c)^(1/2)+1/16*(49*A-9*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(
a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 2984, 12, 2782, 205}

$$\frac{(49A-9B)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}-\frac{(75A-19B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(13A-5B)\,\mathrm{sin}}{16ad\sqrt{\cos(c+dx)}\,(a\,\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] -((75*A - 19*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/
(4*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) - ((13*A - 5*B)*Sin[c +
 d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)) + ((49*A - 9*
B)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(9A-B)-2a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,d}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+}{16ad\sqrt{\cos(c+dx)}\,(a+a\cos}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+}{16ad\sqrt{\cos(c+dx)}\,(a+a\cos}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+}{16ad\sqrt{\cos(c+dx)}\,(a+a\cos}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sin(c+}{16ad\sqrt{\cos(c+dx)}\,(a+a\cos}$$

$$= -\frac{(75A-19B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{(A-B)\,\mathrm{s}}{4d\sqrt{\cos(c+dx)}\,(a}$$

Mathematica [C] time = 2.77, size = 217, normalized size = 1.07

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)(2(85A-13B)\cos(c+dx)+(49A-9B)\cos(2(c+dx))+113A-9B)}{4\sqrt{\cos(c+dx)}} - \frac{i(75A-19B)e^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}(}}{\sqrt{1}}\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-I)*(75*A - 19*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((
2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + ((113*A - 9*
B + 2*(85*A - 13*B)*Cos[c + d*x] + (49*A - 9*B)*Cos[2*(c + d*x)])*Sec[(c +
d*x)/2]^3*Tan[(c + d*x)/2])/(4*Sqrt[Cos[c + d*x]])))/(4*d*(a*(1 + Cos[c + d
*x]))^(5/2))
```

fricas [A] time = 0.84, size = 248, normalized size = 1.22

$$-\frac{\sqrt{2}\left((75\,A-19\,B)\cos\left(dx+c\right)^{4}+3\,(75\,A-19\,B)\cos\left(dx+c\right)^{3}+3\,(75\,A-19\,B)\cos\left(dx+c\right)^{2}+(75\,A-19\,B)\right.}{32\left(a^{3}d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^4 + 3*(75*A - 19*B)*cos(d*x + c)
^3 + 3*(75*A - 19*B)*cos(d*x + c)^2 + (75*A - 19*B)*cos(d*x + c))*sqrt(a)*a
rctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d
*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((49*A - 9*B)*cos(d*x + c)
^2 + (85*A - 13*B)*cos(d*x + c) + 32*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2)), x)
```

maple [B] time = 0.30, size = 443, normalized size = 2.18

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)\left(75A\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+}{1+\cos(dx}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(75*A*sin(d*x+c)*cos(d*x+c
)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)-19*B*sin(d*x+c)*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/
```

```
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+150*A*cos(d*x+c)*sin(d*x+c)*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-38*B*cos
(d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)+75*A*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-19*B*sin(d*x+c)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-98*A*cos(d*x+c)^3+
18*B*cos(d*x+c)^3-72*A*cos(d*x+c)^2+8*B*cos(d*x+c)^2+106*A*cos(d*x+c)-26*B*
cos(d*x+c)+64*A)/a^3/sin(d*x+c)^3/(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.207 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=250

$$\frac{(163A-75B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(95A-39B)\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(299A-147B)}{48a^2d\sqrt{\cos(c+dx)}}$$

```
[Out] -1/4*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2)-1/16*(17*A-
9*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+1/32*(163*A-75*
B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^
(1/2))/a^(5/2)/d*2^(1/2)+1/48*(95*A-39*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)
/(a+a*cos(d*x+c))^(1/2)-1/48*(299*A-147*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2
)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 2984, 12, 2782, 205}

$$\frac{(95A-39B)\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(299A-147B)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{(163A-75B)\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{\cos(}}\right)}{16\sqrt{2}\,a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] ((163*A - 75*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/
(4*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) - ((17*A - 9*B)*Sin[c +
 d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + ((95*A - 39
*B)*Sin[c + d*x])/(48*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) -
((299*A - 147*B)*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(11A-3B)-3a(A-B)\cos(c+d}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c-}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c-}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c-}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c-}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sin(c-}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{co}}$$

$$= \frac{(163A-75B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)}{4d\cos^{\frac{3}{2}}(c+dx)(}$$

Mathematica [C] time = 3.58, size = 239, normalized size = 0.96

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)((1537A-825B)\cos(c+dx)+2(503A-255B)\cos(2(c+dx))+299A\cos(3(c+dx))+878A-147B\,\mathrm{co}}{8\cos^{\frac{3}{2}}(c+dx)}\right.}{12d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((3*I)*(163*A - 75*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^
((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*
```

```
Sqrt[1 + E^((2*I)*(c + d*x))]])])/Sqrt[1 + E^((2*I)*(c + d*x))] - ((878*A -
510*B + (1537*A - 825*B)*Cos[c + d*x] + 2*(503*A - 255*B)*Cos[2*(c + d*x)]
+ 299*A*Cos[3*(c + d*x)] - 147*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^3*Tan[(
c + d*x)/2])/(8*Cos[c + d*x]^(3/2))))/(12*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 1.01, size = 270, normalized size = 1.08

$$3\sqrt{2}\left((163\,A-75\,B)\cos\left(dx+c\right)^{5}+3\,(163\,A-75\,B)\cos\left(dx+c\right)^{4}+3\,(163\,A-75\,B)\cos\left(dx+c\right)^{3}+(163\,A-75\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^5 + 3*(163*A - 75*B)*cos(d*x +
 c)^4 + 3*(163*A - 75*B)*cos(d*x + c)^3 + (163*A - 75*B)*cos(d*x + c)^2)*sq
rt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((299*A - 147*B)*co
s(d*x + c)^3 + (503*A - 255*B)*cos(d*x + c)^2 + 32*(5*A - 3*B)*cos(d*x + c)
 - 32*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*c
os(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos
(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/
2)), x)
```

maple [B] time = 0.24, size = 571, normalized size = 2.28

$$\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-489A\left(\cos^{3}\left(dx+c\right)\right)\sin\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{2}+225B\left(\cos^{3}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/96/d*(a*(1+cos(d*x+c)))^(1/2)*(-489*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)+225*B*cos
(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*2^(1/2)-1467*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)
*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+675*B*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)-1467*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c
)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+675*B*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-4
89*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)+225*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+598*A*cos(d*x+c)^4-294*B*cos(d*x+c)^4+
408*A*cos(d*x+c)^3-216*B*cos(d*x+c)^3-686*A*cos(d*x+c)^2+318*B*cos(d*x+c)^2
-384*A*cos(d*x+c)+192*B*cos(d*x+c)+64*A)/a^3/sin(d*x+c)/(1+cos(d*x+c))^2/co
s(d*x+c)^(3/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(5/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.208 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=293

$$\frac{(2A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{(177A-637B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{7(7A-27B)\sin(c+dx)\sqrt{\mathrm{co}}}{64a^3d\sqrt{a\cos(c+dx)}}$$

```
[Out] (2*A-7*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d+1/6*(
A-B)*cos(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)+1/16*(3*A-7*B)*co
s(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)+1/192*(79*A-259*B)*cos
(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)-1/128*(177*A-637*B)*a
rctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2
))/a^(7/2)/d*2^(1/2)-7/64*(7*A-27*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/d/(a+a
*cos(d*x+c))^(1/2)
```

Rubi [A] time = 1.04, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(79A-259B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(2A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{7(7A-27B)\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((2*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7
/2)*d) - ((177*A - 637*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c
 + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Cos[
c + d*x]^(7/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((3*A - 7*B
)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) + ((
79*A - 259*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*
x])^(3/2)) - (7*(7*A - 27*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*a^3*d*Sqr
t[a + a*Cos[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
```

```
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7}{2}a(A-B)-a(A-7B)\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}}{6a^2}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(3A-7B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(2A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} - \frac{(177A-637B)\tan^{-1}\left(\frac{\sqrt{a}\sin(}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{}}\right.}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] time = 5.82, size = 396, normalized size = 1.35

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{4}\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)((3172B-724A)\cos(c+dx)+(1099B-247A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*((3*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c +
 d*x)))/E^(I*(c + d*x))]*(128*A*d*x - 448*B*d*x - (64*I)*(2*A - 7*B)*ArcSin
h[E^(I*(c + d*x))] + I*Sqrt[2]*(177*A - 637*B)*Log[1 + E^(I*(c + d*x))] + (
128*I)*A*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - (448*I)*B*Log[1 + Sqrt[1
+ E^((2*I)*(c + d*x))]] - (177*I)*Sqrt[2]*A*Log[1 - E^(I*(c + d*x)) + Sqrt[
2]*Sqrt[1 + E^((2*I)*(c + d*x))]] + (637*I)*Sqrt[2]*B*Log[1 - E^(I*(c + d*x
)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(c + d*x))]
 + (Sqrt[Cos[c + d*x]]*(-541*A + 2233*B + (-724*A + 3172*B)*Cos[c + d*x] +
(-247*A + 1099*B)*Cos[2*(c + d*x)] + 96*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2
]^5*Tan[(c + d*x)/2])/4))/(48*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

fricas [A] time = 43.47, size = 368, normalized size = 1.26

$$3\sqrt{2}\left((177\,A - 637\,B)\cos\left(dx + c\right)^{4} + 4\,(177\,A - 637\,B)\cos\left(dx + c\right)^{3} + 6\,(177\,A - 637\,B)\cos\left(dx + c\right)^{2} + 4\,(177$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((177*A - 637*B)*cos(d*x + c)^4 + 4*(177*A - 637*B)*cos(d*
x + c)^3 + 6*(177*A - 637*B)*cos(d*x + c)^2 + 4*(177*A - 637*B)*cos(d*x + c
) + 177*A - 637*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(192*B*cos(d*x + c)^3 - (247*A - 109
9*B)*cos(d*x + c)^2 - 2*(181*A - 721*B)*cos(d*x + c) - 147*A + 567*B)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 384*((2*A - 7*B)*cos(
d*x + c)^4 + 4*(2*A - 7*B)*cos(d*x + c)^3 + 6*(2*A - 7*B)*cos(d*x + c)^2 +
4*(2*A - 7*B)*cos(d*x + c) + 2*A - 7*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos\left(dx + c\right) + A)\cos\left(dx + c\right)^{\frac{7}{2}}}{(a\cos\left(dx + c\right) + a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

maple [B] time = 0.38, size = 887, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] -1/384/d*cos(d*x+c)^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^7*(494*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4+531*A*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+724*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*cos(d*x+c)^3-1911*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*c
os(d*x+c)^3*sin(d*x+c)-384*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^5
+1062*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+
768*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d
*x+c)^3*sin(d*x+c)-200*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-382
2*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-1814
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4-2688*B*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^3*sin(d*x+c)+531*A
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+1536*A*ar
ctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2*
sin(d*x+c)-724*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)-1911*B*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-686*B*cos(d*x+c)
^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5376*B*arctan(sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+768*A*sin(d*x+c)*c
os(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-2
94*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1750*B*cos(d*x+c)^2*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)-2688*B*sin(d*x+c)*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+1134*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2))/sin(d*x+c)^15/(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{7/2}\,\left(A+B\,\cos\left(c+d\,x\right)\right)}{\left(a+a\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),x)
[Out] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

3.209 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=241

$$\frac{(5A-177B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}+\frac{(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d+1/6*(A-B)*c
os(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)+1/48*(5*A-17*B)*cos(d*x
+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)+1/128*(5*A-177*B)*arctan(1/
2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/
2)/d*2^(1/2)+1/64*(5*A-49*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x
+c))^(3/2)
```

Rubi [A] time = 0.77, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2977, 2982, 2782, 205, 2774, 216}

$$\frac{(5A-49B)\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(5A-177B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7/2)*d) +
 ((5*A - 177*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Cos[c + d*x]^(
5/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((5*A - 17*B)*Cos[c +
 d*x]^(3/2)*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) + ((5*A - 49*
B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}a(A-B)+6aB\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(5A-17B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(5A-17B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(5A-17B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(5A-17B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} + \frac{(5A-177B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+d}}\right.}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] time = 3.24, size = 266, normalized size = 1.10

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{4}\sqrt{\cos(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(4(25A-181B)\cos(c+dx)+(67A-247B)\mathrm{co}\right.}{48}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((-3*I)*Sqrt[2]*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)
*(c + d*x)))/E^(I*(c + d*x))]*(128*B*ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*(5*
A - 177*B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*
x))])] - 128*B*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/Sqrt[1 + E^((2*I)*(
c + d*x))] + (Sqrt[Cos[c + d*x]]*(97*A - 541*B + 4*(25*A - 181*B)*Cos[c + d
*x] + (67*A - 247*B)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2])
/4))/(48*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

fricas [A] time = 24.59, size = 327, normalized size = 1.36

$$-\frac{3\sqrt{2}\left((5A-177B)\cos(dx+c)^4+4(5A-177B)\cos(dx+c)^3+6(5A-177B)\cos(dx+c)^2+4(5A-177\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((5*A - 177*B)*cos(d*x + c)^4 + 4*(5*A - 177*B)*cos(d*x +
 c)^3 + 6*(5*A - 177*B)*cos(d*x + c)^2 + 4*(5*A - 177*B)*cos(d*x + c) + 5*A
 - 177*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)/(sqrt(a)*sin(d*x + c))) - 2*((67*A - 247*B)*cos(d*x + c)^2 + 2*(25*A - 18
1*B)*cos(d*x + c) + 15*A - 147*B)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c
))*sin(d*x + c) + 768*(B*cos(d*x + c)^4 + 4*B*cos(d*x + c)^3 + 6*B*cos(d*x
+ c)^2 + 4*B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos
(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

maple [B] time = 0.37, size = 703, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^6*cos(d*x+c)^(5/2)*(134*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4+15*A*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+100*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*cos(d*x+c)^3-531*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos
```

```
(d*x+c)^3*sin(d*x+c)+30*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*
x+c)^2*sin(d*x+c)-104*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-768*
B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c
)^3*sin(d*x+c)-1062*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)
^2*sin(d*x+c)-494*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+15*A*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-100*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)-1536*B*arctan(sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-531*B*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-230*B*cos(d*x+c)^3*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)-30*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-768*
B*sin(d*x+c)*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
/cos(d*x+c))+430*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+294*B*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^13/(cos(d*x+c)/(1+cos
(d*x+c)))^(7/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{5/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+a\,\cos(c+d\,x)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

3.210 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=201

$$\frac{(7A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c}{6d(a\cos(c+dx)+a)^7}$$

```
[Out] 1/6*(A-B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)+1/128*(7*A+5
*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))
^(1/2))/a^(7/2)/d*2^(1/2)+1/48*(A-13*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+
a*cos(d*x+c))^(5/2)+1/192*(17*A+67*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+
a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.59, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2977, 2978, 12, 2782, 205}

$$\frac{(17A+67B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(7A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c}{6d(a\cos(c+dx)+a)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((7*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((A - 13*B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) + ((17*A + 67*B)*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a(A-B)+a(A+5B)\cos(c+dx\right.}{(a+a\cos(c+dx))^{5/2}}}{6a^2}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A-13B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A-13B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A-13B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A-13B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(7A+5B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} + \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{6d(a+a\cos(c+dx}$$

Mathematica [C] time = 2.36, size = 217, normalized size = 1.08

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{8}\sqrt{\cos(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(20(7A+5B)\cos(c+dx)+(17A+67B)\cos(2\right.}{24d(a(\cos(c+dx)+1))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((3*I)*(7*A + 5*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2
*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + (Sqrt[Cos[c +
 d*x]]*(59*A + 97*B + 20*(7*A + 5*B)*Cos[c + d*x] + (17*A + 67*B)*Cos[2*(c
+ d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2])/8))/(24*d*(a*(1 + Cos[c + d*x
]))^(7/2))
```

fricas [A] time = 0.60, size = 266, normalized size = 1.32

$$\frac{3\sqrt{2}\left((7A+5B)\cos(dx+c)^4+4(7A+5B)\cos(dx+c)^3+6(7A+5B)\cos(dx+c)^2+4(7A+5B)\cos(dx\right.}{384\left(a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((7*A + 5*B)*cos(d*x + c)^4 + 4*(7*A + 5*B)*cos(d*x + c)^3
 + 6*(7*A + 5*B)*cos(d*x + c)^2 + 4*(7*A + 5*B)*cos(d*x + c) + 7*A + 5*B)*s
qrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c
))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*((17*A + 67*B)*cos
(d*x + c)^2 + 10*(7*A + 5*B)*cos(d*x + c) + 21*A + 15*B)*sqrt(a*cos(d*x + c
) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos
(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

maple [B] time = 0.34, size = 549, normalized size = 2.73

$$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\,(-1+\cos(dx+c))^{5}\left(34A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^{4}(dx+c)\right)+140A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] 1/384/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*(34*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4+140*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*cos(d*x+c)^3+21*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos
(d*x+c)^3*sin(d*x+c)+15*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*
x+c)^3*sin(d*x+c)+8*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+42*A*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+134*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+30*B*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-140*A*(cos(d*x+c)/(1+cos(d*x+c)))^
```

```
(3/2)*cos(d*x+c)+21*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x
+c)*2^(1/2)-34*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*B*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)-42*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)-70*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-
30*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^11/(cos(d*x+c
)/(1+cos(d*x+c)))^(5/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)}{{\left(a+a\,\cos(c+d\,x)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.211 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=201

$$\frac{(13A+7B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(A+3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+$$

```
[Out] 1/128*(13*A+7*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+
a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)+1/6*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2
)/d/(a+a*cos(d*x+c))^(7/2)+1/16*(A+3*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+
a*cos(d*x+c))^(5/2)-1/192*(5*A-17*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a
*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.58, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2977, 2978, 12, 2782, 205}

$$-\frac{(5A-17B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(13A+7B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{(A+3B)\sin(c+dx)\sqrt{c}}{16ad(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((13*A + 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + ((A + 3*B)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) - ((5*A - 17*B)*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{1}{2}a(A-B)+2a(A+2B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A+3B)\sqrt{\cos(c+dx)}\,\sin(c+a}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A+3B)\sqrt{\cos(c+dx)}\,\sin(c+a}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A+3B)\sqrt{\cos(c+dx)}\,\sin(c+a}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{(A+3B)\sqrt{\cos(c+dx)}\,\sin(c+a}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(13A+7B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} + \frac{(A-B)\sqrt{\cos(c+dx}}{6d(a+a\cos(c}$$

Mathematica [C] time = 2.14, size = 215, normalized size = 1.07

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{8}\sqrt{\cos(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(4(A+35B)\cos(c+dx)+(17B-5A)\cos(2(\right.}{24d(a(\cos(c+dx)+1))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((3*I)*(13*A + 7*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((
2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + (Sqrt[Cos[c
+ d*x]]*(73*A + 59*B + 4*(A + 35*B)*Cos[c + d*x] + (-5*A + 17*B)*Cos[2*(c +
 d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2])/8))/(24*d*(a*(1 + Cos[c + d*x]
))^(7/2))
```

fricas [A] time = 1.75, size = 264, normalized size = 1.31

$$\frac{3\sqrt{2}\left((13A+7B)\cos(dx+c)^4+4(13A+7B)\cos(dx+c)^3+6(13A+7B)\cos(dx+c)^2+4(13A+7B)\cos\right.}{384\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((13*A + 7*B)*cos(d*x + c)^4 + 4*(13*A + 7*B)*cos(d*x + c)
^3 + 6*(13*A + 7*B)*cos(d*x + c)^2 + 4*(13*A + 7*B)*cos(d*x + c) + 13*A + 7
*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*
x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((5*A - 17*B)
*cos(d*x + c)^2 - 2*(A + 35*B)*cos(d*x + c) - 39*A - 21*B)*sqrt(a*cos(d*x +
 c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*c
os(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(7/2
), x)
```

maple [B] time = 0.34, size = 549, normalized size = 2.73

$$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{a\,(1+\cos(dx+c))}\,\left(-1+\cos(dx+c)\right)^4\left(10A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^4(dx+c)\right)-39A\arcsin\left(\frac{-1+}{\mathrm{si}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] 1/384/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*(10*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^4-39*A*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)*cos(d*x+c)^3-21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+
c)^3*sin(d*x+c)-78*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^
2*sin(d*x+c)-88*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-34*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4-42*B*arcsin((-1+cos(d*x+c))/sin(d
*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-39*A*arcsin((-1+cos(d*x+c))/sin(d*x+
```

```
c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos
(d*x+c)-106*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-21*B*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+78*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)+98*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+42*
B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^9/(cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(7/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)}{{\left(a+a\,\cos(c+d\,x)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + a*cos(c + d*x))^(7/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.212 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=203

$$\frac{(63A+13B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{(5A-B)\sin(c+dx)\sqrt{\mathrm{co}}}{16ad(a\cos(c+dx)\,\cdot}$$

```
[Out] 1/128*(63*A+13*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a
+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)-1/6*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/
2)/d/(a+a*cos(d*x+c))^(7/2)-1/16*(5*A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a
+a*cos(d*x+c))^(5/2)-1/192*(103*A+5*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a
+a*cos(d*x+c))^(3/2)
```

Rubi [A] time = 0.59, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2978, 12, 2782, 205}

$$-\frac{(103A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{(63A+13B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{(5A-B)\sin(c+dx)\sqrt{}}{16ad(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] ((63*A + 13*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - ((A - B)*Sqrt[Cos[c + d*
x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) - ((5*A - B)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) - ((103*A + 5*B)*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{1}{2}a(11A+B)-2a(A-B)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}\,d\ldots}{6a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(5A-B)\sqrt{\cos(c+dx)}\,\sin(c \ldots}{16ad(a+a\cos(c+dx))^{5/\ldots}}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(5A-B)\sqrt{\cos(c+dx)}\,\sin(c \ldots}{16ad(a+a\cos(c+dx))^{5/\ldots}}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(5A-B)\sqrt{\cos(c+dx)}\,\sin(c \ldots}{16ad(a+a\cos(c+dx))^{5/\ldots}}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(5A-B)\sqrt{\cos(c+dx)}\,\sin(c \ldots}{16ad(a+a\cos(c+dx))^{5/\ldots}}$$

$$= \frac{(63A+13B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} - \frac{(A-B)\sqrt{\cos(c+\ldots}}{6d(a+a\cos\ldots}$$

Mathematica [C] time = 2.16, size = 216, normalized size = 1.06

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(-\frac{1}{8}\sqrt{\cos(c+dx)}\,\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)((532A-4B)\cos(c+dx)+(103A+5B)\cos$$
$$24d(a(\cos(c+dx)+1))^{7/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((3*I)*(63*A + 13*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^(
(2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*S
qrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - (Sqrt[Cos[c
 + d*x]]*(493*A - 73*B + (532*A - 4*B)*Cos[c + d*x] + (103*A + 5*B)*Cos[2*(
c + d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2])/8))/(24*d*(a*(1 + Cos[c + d
*x]))^(7/2))
```

fricas [A] time = 1.05, size = 266, normalized size = 1.31

$$\frac{3\sqrt{2}\left((63\,A+13\,B)\cos\left(dx+c\right)^4+4\,(63\,A+13\,B)\cos\left(dx+c\right)^3+6\,(63\,A+13\,B)\cos\left(dx+c\right)^2+4\,(63\,A+13\,B}{38}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((63*A + 13*B)*cos(d*x + c)^4 + 4*(63*A + 13*B)*cos(d*x +
c)^3 + 6*(63*A + 13*B)*cos(d*x + c)^2 + 4*(63*A + 13*B)*cos(d*x + c) + 63*A
 + 13*B)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(c
os(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*((103*A
+ 5*B)*cos(d*x + c)^2 + 2*(133*A - B)*cos(d*x + c) + 195*A - 39*B)*sqrt(a*c
os(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4
*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4
*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sqrt(cos(d*x + c
))), x)
```

maple [B] time = 0.33, size = 549, normalized size = 2.70

$$\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{3}\left(-206A\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\left(\cos^{4}\left(dx+c\right)\right)+189A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] 1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(-206*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*cos(d*x+c)^4+189*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(
1/2)*cos(d*x+c)^3*sin(d*x+c)-532*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*
x+c)^3+39*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^3*sin(d*x
+c)+378*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c
)-184*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+78*B*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-10*B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)^4+189*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d
*x+c)*cos(d*x+c)*2^(1/2)+532*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)
+39*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+14*B
*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+390*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)+74*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-78*B*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
a^4/cos(d*x+c)^(1/2)/sin(d*x+c)^7
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(7/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.213 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{3(121A-21B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{(691A-103B)\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(199A-}{192a^2d\sqrt{\cos(c+}}$$

```
[Out] -3/128*(121*A-21*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/
(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)-1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d
*x+c))^(7/2)/cos(d*x+c)^(1/2)-1/48*(19*A-7*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c
))^(5/2)/cos(d*x+c)^(1/2)-1/192*(199*A-43*B)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+
c))^(3/2)/cos(d*x+c)^(1/2)+1/192*(691*A-103*B)*sin(d*x+c)/a^3/d/cos(d*x+c)^
(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 2984, 12, 2782, 205}

$$\frac{(691A-103B)\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{(199A-43B)\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{3(121A-21B)\tan^{-1}\left(\frac{}{\sqrt{2}}\right.}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] (-3*(121*A - 21*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - ((A - B)*Sin[c + d*x
])/(6*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)) - ((19*A - 7*B)*Sin[
c + d*x])/(48*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)) - ((199*A
- 43*B)*Sin[c + d*x])/(192*a^2*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3
/2)) + ((691*A - 103*B)*Sin[c + d*x])/(192*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A + B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{1}{2}a(13A-B)-3a(A-B)\cos(c+dx}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}}{6a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sin(c}{48ad\sqrt{\cos(c+dx)}\,(a+a\,c}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sin(c}{48ad\sqrt{\cos(c+dx)}\,(a+a\,c}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sin(c}{48ad\sqrt{\cos(c+dx)}\,(a+a\,c}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sin(c}{48ad\sqrt{\cos(c+dx)}\,(a+a\,c}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sin(c}{48ad\sqrt{\cos(c+dx)}\,(a+a\,c}$$

$$= -\frac{3(121A-21B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} - \frac{(A-}{6d\sqrt{\cos(c+dx}}$$

Mathematica [C] time = 2.92, size = 240, normalized size = 0.96

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(9(941A-121B)\cos(c+dx)+4(937A-133B)\cos(2(c+dx))+691A\cos(3(c+dx))+5284A-103B\cos}{16\sqrt{\cos(c+dx)}}\right.}{24d(a(\cos(c+dx)+1))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((-9*I)*(121*A - 21*B)*E^((I/2)*(c + d*x))*Sqrt[(1 + E
^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]
*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] + ((5284*A
- 532*B + 9*(941*A - 121*B)*Cos[c + d*x] + 4*(937*A - 133*B)*Cos[2*(c + d*x
```

```
)] + 691*A*Cos[3*(c + d*x)] - 103*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^5*Ta
n[(c + d*x)/2])/(16*Sqrt[Cos[c + d*x]])))/(24*d*(a*(1 + Cos[c + d*x]))^(7/2
))
```

fricas [A] time = 0.82, size = 298, normalized size = 1.19

$$-\frac{9\sqrt{2}\left((121\,A-21\,B)\cos\left(dx+c\right)^{5}+4\,(121\,A-21\,B)\cos\left(dx+c\right)^{4}+6\,(121\,A-21\,B)\cos\left(dx+c\right)^{3}+4\,(121\,A-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(9*sqrt(2)*((121*A - 21*B)*cos(d*x + c)^5 + 4*(121*A - 21*B)*cos(d*x
 + c)^4 + 6*(121*A - 21*B)*cos(d*x + c)^3 + 4*(121*A - 21*B)*cos(d*x + c)^2
 + (121*A - 21*B)*cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x +
 c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(
d*x + c))) - 2*((691*A - 103*B)*cos(d*x + c)^3 + 2*(937*A - 133*B)*cos(d*x
+ c)^2 + 39*(41*A - 5*B)*cos(d*x + c) + 384*A)*sqrt(a*cos(d*x + c) + a)*sqr
t(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(d*x + c)^
4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(3/
2)), x)
```

maple [B] time = 0.38, size = 581, normalized size = 2.32

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{2}\left(-1089A\left(\cos^{3}\left(dx+c\right)\right)\sin\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin\left(\frac{-1+\cos(dx+}{\sin(dx+c)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(-1089*A*cos(d*x+c)^3*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+
c))*2^(1/2)+189*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)-3267*A*sin(d*x+c)*cos(d*x+c)^2*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+567*B*sin(d*x+c)*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3267*A*cos(d*x+c)*sin(d*x+c)*arcsin((-1+c
os(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+567*B*cos(
d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)+1382*A*cos(d*x+c)^4-1089*A*sin(d*x+c)*arcsin((-1+cos(d*
x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-206*B*cos(d*x+c
)^4+189*B*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)+2366*A*cos(d*x+c)^3-326*B*cos(d*x+c)^3-550*A*cos(d*x
+c)^2+142*B*cos(d*x+c)^2-2430*A*cos(d*x+c)+390*B*cos(d*x+c)-768*A)/a^4/sin(
d*x+c)^5/(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(7/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.214 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=297

$$\frac{(1015A-363B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{(579A-199B)\sin(c+dx)}{192a^3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(1887A-69}{192a^3d\sqrt{\cos(c+dx}}$$

```
[Out] -1/6*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2)-1/48*(23*A-
11*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2)-1/64*(109*A-41
*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+1/128*(1015*A-
363*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))/a^(7/2)/d*2^(1/2)+1/192*(579*A-199*B)*sin(d*x+c)/a^3/d/cos(d*x+c
)^(3/2)/(a+a*cos(d*x+c))^(1/2)-1/192*(1887*A-691*B)*sin(d*x+c)/a^3/d/cos(d*
x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 1.03, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 2984, 12, 2782, 205}

$$\frac{(579A-199B)\sin(c+dx)}{192a^3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{(109A-41B)\sin(c+dx)}{64a^2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-\frac{(1887A-691B)\sin(c}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] ((1015*A - 363*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - ((A - B)*Sin[c + d*x]
)/(6*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)) - ((23*A - 11*B)*Sin[
c + d*x])/(48*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) - ((109*A
- 41*B)*Sin[c + d*x])/(64*a^2*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/
2)) + ((579*A - 199*B)*Sin[c + d*x])/(192*a^3*d*Cos[c + d*x]^(3/2)*Sqrt[a +
 a*Cos[c + d*x]]) - ((1887*A - 691*B)*Sin[c + d*x])/(192*a^3*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{3}{2}a(5A-B)-4a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,d}{6a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sin(c+}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sin(c+}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sin(c+}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sin(c+}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sin(c+}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sin(c+}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos}$$

$$= \frac{(1015A-363B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}\,a^{7/2}d} - \frac{(A-B)}{6d\cos^{\frac{3}{2}}(c+dx)(}$$

Mathematica [C] time = 5.40, size = 262, normalized size = 0.88

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(4(9415A-3579B)\cos(c+dx)+8(3069A-1145B)\cos(2(c+dx))+10164A\cos(3(c+dx))+1887A\cos}{32\cos^{\frac{3}{2}}(c+dx)}\right.}{24d(a(\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((3*I)*(1015*A - 363*B)*E^((I/2)*(c + d*x))*Sqrt[(1 +
E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[1 + E^((2*I)*(c + d*x))] - ((21641*
A - 8469*B + 4*(9415*A - 3579*B)*Cos[c + d*x] + 8*(3069*A - 1145*B)*Cos[2*(
c + d*x)] + 10164*A*Cos[3*(c + d*x)] - 3748*B*Cos[3*(c + d*x)] + 1887*A*Cos
[4*(c + d*x)] - 691*B*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^5*Tan[(c + d*x)/2]
)/(32*Cos[c + d*x]^(3/2))))/(24*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

fricas [A] time = 0.83, size = 319, normalized size = 1.07

$$3\sqrt{2}\left(\left(1015\,A-363\,B\right)\cos\left(dx+c\right)^{6}+4\left(1015\,A-363\,B\right)\cos\left(dx+c\right)^{5}+6\left(1015\,A-363\,B\right)\cos\left(dx+c\right)^{4}+4\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((1015*A - 363*B)*cos(d*x + c)^6 + 4*(1015*A - 363*B)*cos(
d*x + c)^5 + 6*(1015*A - 363*B)*cos(d*x + c)^4 + 4*(1015*A - 363*B)*cos(d*x
 + c)^3 + (1015*A - 363*B)*cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(
a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)
^2 + a*cos(d*x + c))) - 2*((1887*A - 691*B)*cos(d*x + c)^4 + 2*(2541*A - 93
7*B)*cos(d*x + c)^3 + 39*(109*A - 41*B)*cos(d*x + c)^2 + 128*(7*A - 3*B)*co
s(d*x + c) - 128*A)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c
))/(a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4
+ 4*a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(5/
2)), x)
```

maple [B] time = 0.24, size = 715, normalized size = 2.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(-3045*A*cos(d*x+c)^4*2^(
1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))+1089*B*cos(d*x+c)^4*2^(1/2)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-12180*A*cos(d*x+c)^3*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*2^(1/2
)+4356*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*2^(1/2)-18270*A*arcsin((-1+cos(d*x+c))/sin(d*x+c
))*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+6534*B
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)-12180*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d
*x+c)*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+4356*B*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(3/2)-3045*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*2^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1089*B*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+3774*A*cos(d*x+c)^5-
1382*B*cos(d*x+c)^5+6390*A*cos(d*x+c)^4-2366*B*cos(d*x+c)^4-1662*A*cos(d*x+
c)^3+550*B*cos(d*x+c)^3-6710*A*cos(d*x+c)^2+2430*B*cos(d*x+c)^2-2048*A*cos(
d*x+c)+768*B*cos(d*x+c)+256*A)/a^4/sin(d*x+c)^3/(1+cos(d*x+c))^2/cos(d*x+c)
^(3/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(7/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

3.215 $\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=105

$$-\frac{(aB+Ab)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{(4aA+3bB)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4aA+3bB)+\frac{bB\sin(}{}$$

```
[Out] 1/8*(4*A*a+3*B*b)*x+(A*b+B*a)*sin(d*x+c)/d+1/8*(4*A*a+3*B*b)*cos(d*x+c)*sin
(d*x+c)/d+1/4*b*B*cos(d*x+c)^3*sin(d*x+c)/d-1/3*(A*b+B*a)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3023, 2748, 2635, 8, 2633}

$$-\frac{(aB+Ab)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{(4aA+3bB)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4aA+3bB)+\frac{bB\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((4*a*A + 3*b*B)*x)/8 + ((A*b + a*B)*Sin[c + d*x])/d + ((4*a*A + 3*b*B)*Cos
[c + d*x]*Sin[c + d*x])/(8*d) + (b*B*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (
(A*b + a*B)*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^2(c+dx)\left(aA+(Ab+aB)\cos(c+dx)+bB\,c\right.$$
$$= \frac{bB\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}\int \cos^2(c+dx)(4a$$
$$= \frac{bB\cos^3(c+dx)\sin(c+dx)}{4d} + (Ab+aB)\int \cos^3(c$$
$$= \frac{(4aA+3bB)\cos(c+dx)\sin(c+dx)}{8d} + \frac{bB\cos^3(c+}{}$$
$$= \frac{1}{8}(4aA+3bB)x + \frac{(Ab+aB)\sin(c+dx)}{d} + \frac{(4aA+}{}$$

Mathematica [A] time = 0.22, size = 91, normalized size = 0.87

$$\frac{-32(aB+Ab)\sin^3(c+dx)+96(aB+Ab)\sin(c+dx)+24(aA+bB)\sin(2(c+dx))+48aAc+48aAdx+3bB}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (48*a*A*c + 36*b*B*c + 48*a*A*d*x + 36*b*B*d*x + 96*(A*b + a*B)*Sin[c + d*x
] - 32*(A*b + a*B)*Sin[c + d*x]^3 + 24*(a*A + b*B)*Sin[2*(c + d*x)] + 3*b*B
*Sin[4*(c + d*x)])/(96*d)
```

fricas [A] time = 1.00, size = 81, normalized size = 0.77

$$\frac{3\,(4\,Aa+3\,Bb)dx+\left(6\,Bb\cos\left(dx+c\right)^{3}+8\,(Ba+Ab)\cos\left(dx+c\right)^{2}+16\,Ba+16\,Ab+3\,(4\,Aa+3\,Bb)\cos\left(dx+c\right)\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(4*A*a + 3*B*b)*d*x + (6*B*b*cos(d*x + c)^3 + 8*(B*a + A*b)*cos(d*x
 + c)^2 + 16*B*a + 16*A*b + 3*(4*A*a + 3*B*b)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.41, size = 89, normalized size = 0.85

$$\frac{1}{8}\,(4\,Aa+3\,Bb)x+\frac{Bb\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{(Ba+Ab)\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{(Aa+Bb)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{3\,(Ba+Ab)\sin\left(dx\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/8*(4*A*a + 3*B*b)*x + 1/32*B*b*sin(4*d*x + 4*c)/d + 1/12*(B*a + A*b)*sin(
3*d*x + 3*c)/d + 1/4*(A*a + B*b)*sin(2*d*x + 2*c)/d + 3/4*(B*a + A*b)*sin(d
*x + c)/d
```

maple [A] time = 0.05, size = 107, normalized size = 1.02

$$\frac{Bb\left(\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ab\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}+\frac{aB\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}+aA\left(\frac{\cos(dx+c)\sin(d}{2}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(B*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A
*b*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a*B*(2+cos(d*x+c)^2)*sin(d*x+c)+a*A*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 1.37, size = 101, normalized size = 0.96

$$\frac{24\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa-32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ba-32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)A}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a - 32*(sin(d*x + c)^3 - 3*sin(
d*x + c))*B*a - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b + 3*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b)/d
```

mupad [B] time = 0.47, size = 117, normalized size = 1.11

$$\frac{A\,a\,x}{2}+\frac{3\,B\,b\,x}{8}+\frac{3\,A\,b\,\sin\left(c+d\,x\right)}{4\,d}+\frac{3\,B\,a\,\sin\left(c+d\,x\right)}{4\,d}+\frac{A\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,b\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{B\,a\,\sin\left(3\,c\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

```
[Out] (A*a*x)/2 + (3*B*b*x)/8 + (3*A*b*sin(c + d*x))/(4*d) + (3*B*a*sin(c + d*x))
/(4*d) + (A*a*sin(2*c + 2*d*x))/(4*d) + (A*b*sin(3*c + 3*d*x))/(12*d) + (B*
a*sin(3*c + 3*d*x))/(12*d) + (B*b*sin(2*c + 2*d*x))/(4*d) + (B*b*sin(4*c +
4*d*x))/(32*d)
```

sympy [A] time = 1.04, size = 252, normalized size = 2.40

$$\begin{cases} \frac{Aax\sin^2(c+dx)}{2} + \frac{Aax\cos^2(c+dx)}{2} + \frac{Aa\sin(c+dx)\cos(c+dx)}{2d} + \frac{2Ab\sin^3(c+dx)}{3d} + \frac{Ab\sin(c+dx)\cos^2(c+dx)}{d} + \frac{2Ba\sin^3(c+dx)}{3d} + \frac{Ba\sin}{} \\ x(A+B\cos(c))(a+b\cos(c))\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
d*x)*cos(c + d*x)/(2*d) + 2*A*b*sin(c + d*x)**3/(3*d) + A*b*sin(c + d*x)*co
s(c + d*x)**2/d + 2*B*a*sin(c + d*x)**3/(3*d) + B*a*sin(c + d*x)*cos(c + d*
x)**2/d + 3*B*b*x*sin(c + d*x)**4/8 + 3*B*b*x*sin(c + d*x)**2*cos(c + d*x)*
*2/4 + 3*B*b*x*cos(c + d*x)**4/8 + 3*B*b*sin(c + d*x)**3*cos(c + d*x)/(8*d)
 + 5*B*b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c))*(
a + b*cos(c))*cos(c)**2, True))
```

3.216 $\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=84

$$\frac{(3aA+2bB)\sin(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(aB+Ab)+\frac{bB\sin(c+dx)\cos^2(c+dx)}{3d}$$

```
[Out] 1/2*(A*b+B*a)*x+1/3*(3*A*a+2*B*b)*sin(d*x+c)/d+1/2*(A*b+B*a)*cos(d*x+c)*sin
(d*x+c)/d+1/3*b*B*cos(d*x+c)^2*sin(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2968, 3023, 2734}

$$\frac{(3aA+2bB)\sin(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(aB+Ab)+\frac{bB\sin(c+dx)\cos^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((A*b + a*B)*x)/2 + ((3*a*A + 2*b*B)*Sin[c + d*x])/(3*d) + ((A*b + a*B)*Cos
[c + d*x]*Sin[c + d*x])/(2*d) + (b*B*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos(c+dx)\left(aA+(Ab+aB)\cos(c+dx)+bB\cos\right.$$
$$= \frac{bB\cos^2(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\int \cos(c+dx)(3aA$$
$$= \frac{1}{2}(Ab+aB)x + \frac{(3aA+2bB)\sin(c+dx)}{3d} + \frac{(Ab+aB}{}$$

Mathematica [A] time = 0.16, size = 75, normalized size = 0.89

$$\frac{3(4aA+3bB)\sin(c+dx)+3(aB+Ab)\sin(2(c+dx))+6aBc+6aBdx+6Abc+6Abdx+bB\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*A*b*c + 6*a*B*c + 6*A*b*d*x + 6*a*B*d*x + 3*(4*a*A + 3*b*B)*Sin[c + d*x]
 + 3*(A*b + a*B)*Sin[2*(c + d*x)] + b*B*Sin[3*(c + d*x)])/(12*d)
```

fricas [A] time = 1.03, size = 60, normalized size = 0.71

$$\frac{3(Ba+Ab)dx+\left(2\,Bb\cos\left(dx+c\right)^2+6\,Aa+4\,Bb+3\,(Ba+Ab)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/6*(3*(B*a + A*b)*d*x + (2*B*b*cos(d*x + c)^2 + 6*A*a + 4*B*b + 3*(B*a + A
*b)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.31, size = 68, normalized size = 0.81

$$\frac{1}{2}(Ba+Ab)x + \frac{Bb\sin\left(3\,dx+3\,c\right)}{12\,d} + \frac{(Ba+Ab)\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{(4\,Aa+3\,Bb)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(B*a + A*b)*x + 1/12*B*b*sin(3*d*x + 3*c)/d + 1/4*(B*a + A*b)*sin(2*d*x
 + 2*c)/d + 1/4*(4*A*a + 3*B*b)*sin(d*x + c)/d
```

maple [A] time = 0.05, size = 85, normalized size = 1.01

$$\frac{\frac{Bb\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3} + Ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + aB\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + aA\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/3*B*b*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+a*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*A*sin(d*x+c))
```

maxima [A] time = 0.31, size = 79, normalized size = 0.94

$$\frac{3\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Ba + 3\,(2\,dx + 2\,c + \sin(2\,dx + 2\,c))Ab - 4\left(\sin(dx+c)^3 - 3\,\sin(dx+c)\right)Bb + 12}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a + 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*A*b - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*b + 12*A*a*sin(d*x + c))
/d
```

mupad [B] time = 0.40, size = 84, normalized size = 1.00

$$\frac{A\,b\,x}{2}+\frac{B\,a\,x}{2}+\frac{A\,a\,\sin(c+d\,x)}{d}+\frac{3\,B\,b\,\sin(c+d\,x)}{4\,d}+\frac{A\,b\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{B\,a\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{B\,b\,\sin(3\,c+3\,d}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

```
[Out] (A*b*x)/2 + (B*a*x)/2 + (A*a*sin(c + d*x))/d + (3*B*b*sin(c + d*x))/(4*d) +
 (A*b*sin(2*c + 2*d*x))/(4*d) + (B*a*sin(2*c + 2*d*x))/(4*d) + (B*b*sin(3*c
 + 3*d*x))/(12*d)
```

sympy [A] time = 0.51, size = 168, normalized size = 2.00

$$\begin{cases} \frac{Aa\sin(c+dx)}{d} + \frac{Abx\sin^2(c+dx)}{2} + \frac{Abx\cos^2(c+dx)}{2} + \frac{Ab\sin(c+dx)\cos(c+dx)}{2d} + \frac{Bax\sin^2(c+dx)}{2} + \frac{Bax\cos^2(c+dx)}{2} + \frac{Ba\sin(c+dx)\cos(c+}{2d} \\ x\,(A + B\cos(c))\,(a + b\cos(c))\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*sin(c + d*x)/d + A*b*x*sin(c + d*x)**2/2 + A*b*x*cos(c + d*x
)**2/2 + A*b*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*x*sin(c + d*x)**2/2 + B*
a*x*cos(c + d*x)**2/2 + B*a*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*b*sin(c +
 d*x)**3/(3*d) + B*b*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*c
os(c))*(a + b*cos(c))*cos(c), True))
```

3.217 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{1}{2}x(2aA+bB)+\frac{bB\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] 1/2*(2*A*a+B*b)*x+(A*b+B*a)*sin(d*x+c)/d+1/2*b*B*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2734}

$$\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{1}{2}x(2aA+bB)+\frac{bB\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((2*a*A + b*B)*x)/2 + ((A*b + a*B)*Sin[c + d*x])/d + (b*B*Cos[c + d*x]*Sin[
c + d*x])/(2*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\,dx=\frac{1}{2}(2aA+bB)x+\frac{(Ab+aB)\sin(c+dx)}{d}+\frac{bB\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.09, size = 51, normalized size = 0.98

$$\frac{4(aB+Ab)\sin(c+dx)+4aAdx+bB\sin(2(c+dx))+2bBc+2bBdx}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

[Out] (2*b*B*c + 4*a*A*d*x + 2*b*B*d*x + 4*(A*b + a*B)*Sin[c + d*x] + b*B*Sin[2*(
c + d*x)])/(4*d)

fricas [A] time = 0.79, size = 42, normalized size = 0.81

$$\frac{(2\,Aa + Bb)dx + (Bb\cos{(dx + c)} + 2\,Ba + 2\,Ab)\sin{(dx + c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] 1/2*((2*A*a + B*b)*d*x + (B*b*cos(d*x + c) + 2*B*a + 2*A*b)*sin(d*x + c))/d

giac [A] time = 0.30, size = 45, normalized size = 0.87

$$\frac{1}{2}\,(2\,Aa + Bb)x + \frac{Bb\sin{(2\,dx + 2\,c)}}{4\,d} + \frac{(Ba + Ab)\sin{(dx + c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] 1/2*(2*A*a + B*b)*x + 1/4*B*b*sin(2*d*x + 2*c)/d + (B*a + A*b)*sin(d*x + c)
/d

maple [A] time = 0.05, size = 57, normalized size = 1.10

$$\frac{Bb\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + Ab\sin{(dx + c)} + aB\sin{(dx + c)} + aA\,(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)

[Out] 1/d*(B*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b*sin(d*x+c)+a*B*sin(d
*x+c)+a*A*(d*x+c))

maxima [A] time = 0.60, size = 55, normalized size = 1.06

$$\frac{4\,(dx + c)Aa + (2\,dx + 2\,c + \sin{(2\,dx + 2\,c)})Bb + 4\,Ba\sin{(dx + c)} + 4\,Ab\sin{(dx + c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/4*(4*(d*x + c)*A*a + (2*d*x + 2*c + sin(2*d*x + 2*c))*B*b + 4*B*a*sin(d*x
 + c) + 4*A*b*sin(d*x + c))/d

mupad [B] time = 0.36, size = 50, normalized size = 0.96

$$A\,a\,x+\frac{B\,b\,x}{2}+\frac{A\,b\,\sin\left(c+d\,x\right)}{d}+\frac{B\,a\,\sin\left(c+d\,x\right)}{d}+\frac{B\,b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

```
[Out] A*a*x + (B*b*x)/2 + (A*b*sin(c + d*x))/d + (B*a*sin(c + d*x))/d + (B*b*sin(
2*c + 2*d*x))/(4*d)
```

sympy [A] time = 0.25, size = 94, normalized size = 1.81

$$\begin{cases} Aax + \frac{Ab\sin\left(c+dx\right)}{d} + \frac{Ba\sin\left(c+dx\right)}{d} + \frac{Bbx\sin^2\left(c+dx\right)}{2} + \frac{Bbx\cos^2\left(c+dx\right)}{2} + \frac{Bb\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d} & \text{for}\ d \neq 0 \\ x\left(A+B\cos\left(c\right)\right)\left(a+b\cos\left(c\right)\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x + A*b*sin(c + d*x)/d + B*a*sin(c + d*x)/d + B*b*x*sin(c +
d*x)**2/2 + B*b*x*cos(c + d*x)**2/2 + B*b*sin(c + d*x)*cos(c + d*x)/(2*d),
Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c)), True))
```

3.218 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=35

$$x(aB+Ab)+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\sin(c+dx)}{d}$$

```
[Out] (A*b+B*a)*x+a*A*arctanh(sin(d*x+c))/d+b*B*sin(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2968, 3023, 2735, 3770}

$$x(aB+Ab)+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (A*b + a*B)*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*Sin[c + d*x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec(c+dx)\,dx = \int \left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\sec$$
$$= \frac{bB\sin(c+dx)}{d} + \int (aA+(Ab+aB)\cos(c+dx))\sec($$
$$= (Ab+aB)x + \frac{bB\sin(c+dx)}{d} + (aA)\int \sec(c+dx)\,dx$$
$$= (Ab+aB)x + \frac{aA\tanh^{-1}(\sin(c+dx))}{d} + \frac{bB\sin(c+dx}{d}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.31

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d} + aBx + Abx + \frac{bB\sin(c)\cos(dx)}{d} + \frac{bB\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] A*b*x + a*B*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*Cos[d*x]*Sin[c])/d + (
b*B*Cos[c]*Sin[d*x])/d
```

fricas [A] time = 1.23, size = 54, normalized size = 1.54

$$\frac{2\,(Ba+Ab)dx + Aa\log\left(\sin\left(dx+c\right)+1\right) - Aa\log\left(-\sin\left(dx+c\right)+1\right) + 2\,Bb\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(B*a + A*b)*d*x + A*a*log(sin(d*x + c) + 1) - A*a*log(-sin(d*x + c)
+ 1) + 2*B*b*sin(d*x + c))/d
```

giac [B] time = 0.67, size = 79, normalized size = 2.26

$$\frac{Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - Aa\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + (Ba+Ab)(dx+c) + \frac{2\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

```
[Out] (A*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - A*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (B*a + A*b)*(d*x + c) + 2*B*b*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

maple [A] time = 0.09, size = 56, normalized size = 1.60

$$Abx + aBx + \frac{aA\ln\left(\sec\left(dx + c\right) + \tan\left(dx + c\right)\right)}{d} + \frac{Abc}{d} + \frac{bB\sin\left(dx + c\right)}{d} + \frac{Bac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] A*b*x+a*B*x+1/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*b*c+b*B*sin(d*x+c)/d+1/
d*B*a*c
```

maxima [A] time = 0.30, size = 47, normalized size = 1.34

$$\frac{(dx + c)Ba + (dx + c)Ab + Aa\log\left(\sec\left(dx + c\right) + \tan\left(dx + c\right)\right) + Bb\sin\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima
")
```

```
[Out] ((d*x + c)*B*a + (d*x + c)*A*b + A*a*log(sec(d*x + c) + tan(d*x + c)) + B*b
*sin(d*x + c))/d
```

mupad [B] time = 0.48, size = 100, normalized size = 2.86

$$\frac{B\,b\,\sin\left(c+d\,x\right)}{d}+\frac{2\,A\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x),x)
```

```
[Out] (B*b*sin(c + d*x))/d + (2*A*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d + (2*A*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))(a + b\cos(c + dx))\sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c),x)
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x), x)
```

3.219 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=35

$$\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+bBx$$

```
[Out] b*B*x+(A*b+B*a)*arctanh(sin(d*x+c))/d+a*A*tan(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3021, 2735, 3770}

$$\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\tan(c+dx)}{d}+bBx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] b*B*x + ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (a*A*Tan[c + d*x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \int \left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\mathrm{se}$$
$$= \frac{aA\tan(c+dx)}{d} + \int (Ab+aB+bB\cos(c+dx))\sec($$
$$= bBx + \frac{aA\tan(c+dx)}{d} - (-Ab-aB)\int \sec(c+dx)\,dx$$
$$= bBx + \frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{d} + \frac{aA\tan(c+}{d}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.23

$$\frac{aA\tan(c+dx)}{d} + \frac{aB\tanh^{-1}(\sin(c+dx))}{d} + \frac{Ab\tanh^{-1}(\sin(c+dx))}{d} + bBx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] b*B*x + (A*b*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*
A*Tan[c + d*x])/d
```

fricas [B] time = 0.62, size = 85, normalized size = 2.43

$$\frac{2\,Bbdx\cos(dx+c) + (Ba+Ab)\cos(dx+c)\log(\sin(dx+c)+1) - (Ba+Ab)\cos(dx+c)\log(-\sin(dx+c)+}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fric
as")
```

```
[Out] 1/2*(2*B*b*d*x*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)*log(sin(d*x + c) + 1
) - (B*a + A*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*A*a*sin(d*x + c))/(
d*cos(d*x + c))
```

giac [B] time = 0.43, size = 84, normalized size = 2.40

$$\frac{(dx+c)Bb+(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac
")
```

```
[Out] ((d*x + c)*B*b + (B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (B*a + A*
b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

maple [A] time = 0.10, size = 65, normalized size = 1.86

$$bBx+\frac{aA\tan(dx+c)}{d}+\frac{Ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{aB\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{Bbc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] b*B*x+a*A*tan(d*x+c)/d+1/d*A*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*B*ln(sec(d*x
+c)+tan(d*x+c))+1/d*B*b*c
```

maxima [B] time = 0.31, size = 73, normalized size = 2.09

$$\frac{2\,(dx+c)Bb+Ba\Big(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\Big)+Ab\Big(\log(\sin(dx+c)+1)-\log(\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(d*x + c)*B*b + B*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ A*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*A*a*tan(d*x + c))
/d
```

mupad [B] time = 0.48, size = 114, normalized size = 3.26

$$\frac{2\,B\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a\,\sin(c+d\,x)}{d\,\cos(c+d\,x)}-\frac{A\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)2\mathrm{i}}{d}-\frac{B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)2\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] (2*B*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (B*a*atan((sin(c/2
+ (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/d - (A*b*atan((sin(c/2 + (d*x)/2)*1i
)/cos(c/2 + (d*x)/2))*2i)/d + (A*a*sin(c + d*x))/(d*cos(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))(a + b\cos(c + dx))\sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

3.220 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=61

$$\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(aA+2bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 1/2*(A*a+2*B*b)*arctanh(sin(d*x+c))/d+(A*b+B*a)*tan(d*x+c)/d+1/2*a*A*sec(d*
x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3021, 2748, 3767, 8, 3770}

$$\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(aA+2bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] ((a*A + 2*b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((A*b + a*B)*Tan[c + d*x])/d
+ (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \int \left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\text{se}$$

$$= \frac{aA\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int (2(Ab+aB)+(aA$$

$$= \frac{aA\sec(c+dx)\tan(c+dx)}{2d} + (Ab+aB)\int \sec^2(c+a$$

$$= \frac{(aA+2bB)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{aA\sec(c+dx)\tan}{2d}$$

$$= \frac{(aA+2bB)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{(Ab+aB)\tan(c+}{d}$$

Mathematica [A] time = 0.02, size = 75, normalized size = 1.23

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA\tan(c+dx)\sec(c+dx)}{2d}+\frac{aB\tan(c+dx)}{d}+\frac{Ab\tan(c+dx)}{d}+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/(2*d) + (b*B*ArcTanh[Sin[c + d*x]])/d + (A*b*Ta
n[c + d*x])/d + (a*B*Tan[c + d*x])/d + (a*A*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

fricas [A] time = 0.53, size = 96, normalized size = 1.57

$$\frac{(Aa+2\,Bb)\cos(dx+c)^2\log(\sin(dx+c)+1)-(Aa+2\,Bb)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,(Aa+2\,(}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fric
as")
```

```
[Out] 1/4*((A*a + 2*B*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a + 2*B*b)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a + 2*(B*a + A*b)*cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.46, size = 151, normalized size = 2.48

$$\frac{(Aa+2\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa+2\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ba\tan\left(\frac{1}{2}\,d\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac
")
```

```
[Out] 1/2*((A*a + 2*B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + 2*B*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1
/2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c)
 + 2*B*a*tan(1/2*d*x + 1/2*c) + 2*A*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.11, size = 86, normalized size = 1.41

$$\frac{aA\sec(dx+c)\tan(dx+c)}{2d}+\frac{aA\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{aB\tan(dx+c)}{d}+\frac{Ab\tan(dx+c)}{d}+\frac{Bb\ln(\sec}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a*A*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*A*ln(sec(d*x+c)+tan(d*x+c))+1/d*a*B
*tan(d*x+c)+1/d*A*b*tan(d*x+c)+1/d*B*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.47, size = 95, normalized size = 1.56

$$-\frac{Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-2\,Bb\left(\log(\sin(dx+c)+1)-\log(\sin(dx+c\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxi
ma")
```

```
[Out] -1/4*(A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 2*B*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 4*B*a*tan(d*x + c) - 4*A*b*tan(d*x + c))/d
```

mupad [B] time = 1.27, size = 104, normalized size = 1.70

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A\,a+2\,A\,b+2\,B\,a\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(2\,A\,b-A\,a+2\,B\,a\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A\,a+2\,B\,b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(A*a + 2*A*b + 2*B*a) - tan(c/2 + (d*x)/2)^3*(2*A*b - A
*a + 2*B*a))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (ata
nh(tan(c/2 + (d*x)/2))*(A*a + 2*B*b))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))\,(a + b\cos(c + dx))\sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x)**3, x)
```

3.221 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=93

$$\frac{(2aA+3bB)\tan(c+dx)}{3d}+\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aB+Ab)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

```
[Out] 1/2*(A*b+B*a)*arctanh(sin(d*x+c))/d+1/3*(2*A*a+3*B*b)*tan(d*x+c)/d+1/2*(A*b
+B*a)*sec(d*x+c)*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2968, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(2aA+3bB)\tan(c+dx)}{3d}+\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aB+Ab)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aA\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a*A + 3*b*B)*Tan[c + d*x])/
(3*d) + ((A*b + a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*Sec[c + d*x]^2
*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= \int \left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\mathrm{se}\\ &= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (3(Ab+aB)+(2a\\ &= \frac{aA\sec^2(c+dx)\tan(c+dx)}{3d}+(Ab+aB)\int \sec^3(c+\\ &= \frac{(Ab+aB)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aA\sec^2(c+dx)}{3d}\\ &= \frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(2aA+3bB)\tan(c}{3d}\end{aligned}$$

Mathematica [A] time = 0.28, size = 67, normalized size = 0.72

$$\frac{3(aB+Ab)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3(aB+Ab)\sec(c+dx)+2aA\tan^2(c+dx)+6aA+6bB\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (3*(A*b + a*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*a*A + 6*b*B + 3*(A*b
 + a*B)*Sec[c + d*x] + 2*a*A*Tan[c + d*x]^2))/(6*d)
```

fricas [A] time = 0.99, size = 115, normalized size = 1.24

$$\frac{3\,(Ba+Ab)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(Ba+Ab)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(2\,Aa\right.}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(B*a + A*b)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a + A*b)*co
s(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(2*A*a + 3*B*b)*cos(d*x + c)^2 +
 2*A*a + 3*(B*a + A*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [B] time = 0.41, size = 210, normalized size = 2.26

$$3\,(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac
")
```

```
[Out] 1/6*(3*(B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a + A*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 - 3*B*a*tan
(1/2*d*x + 1/2*c)^5 - 3*A*b*tan(1/2*d*x + 1/2*c)^5 + 6*B*b*tan(1/2*d*x + 1/
2*c)^5 - 4*A*a*tan(1/2*d*x + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3 + 6*A
*a*tan(1/2*d*x + 1/2*c) + 3*B*a*tan(1/2*d*x + 1/2*c) + 3*A*b*tan(1/2*d*x +
1/2*c) + 6*B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.12, size = 128, normalized size = 1.38

$$\frac{2aA\tan(dx+c)}{3d}+\frac{aA\left(\sec^2(dx+c)\right)\tan(dx+c)}{3d}+\frac{aB\sec(dx+c)\tan(dx+c)}{2d}+\frac{aB\ln(\sec(dx+c)+\tan(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 2/3*a*A*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*a*B*sec(d*x+c)
*tan(d*x+c)+1/2/d*a*B*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*A*b*sec(d*x+c)*tan(d*
x+c)+1/2/d*A*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*b*tan(d*x+c)
```

maxima [A] time = 0.47, size = 127, normalized size = 1.37

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa-3Ba\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-3Ab\left(\frac{}{s}\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a - 3*B*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*A*b*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*B*b*tan(d*x + c))/d
```

mupad [B] time = 2.57, size = 145, normalized size = 1.56

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(Ab+Ba)}{d}-\frac{(2Aa-Ab-Ba+2Bb)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\frac{4Aa}{3}-4Bb\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(A*b + B*a))/d - (tan(c/2 + (d*x)/2)*(2*A*a + A*
b + B*a + 2*B*b) - tan(c/2 + (d*x)/2)^3*((4*A*a)/3 + 4*B*b) + tan(c/2 + (d*
x)/2)^5*(2*A*a - A*b - B*a + 2*B*b))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2
 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(A+B\cos(c+dx))(a+b\cos(c+dx))\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sec(c + d*x)**4, x)
```

3.222 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=114

$$\frac{(aB+Ab)\tan^3(c+dx)}{3d}+\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(3aA+4bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3aA+4bB)\tan(c+dx)\sec}{8d}$$

[Out] 1/8*(3*A*a+4*B*b)*arctanh(sin(d*x+c))/d+(A*b+B*a)*tan(d*x+c)/d+1/8*(3*A*a+4*B*b)*sec(d*x+c)*tan(d*x+c)/d+1/4*a*A*sec(d*x+c)^3*tan(d*x+c)/d+1/3*(A*b+B*a)*tan(d*x+c)^3/d

Rubi [A] time = 0.18, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3021, 2748, 3767, 3768, 3770}

$$\frac{(aB+Ab)\tan^3(c+dx)}{3d}+\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(3aA+4bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3aA+4bB)\tan(c+dx)\sec}{8d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]

[Out] ((3*a*A + 4*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((A*b + a*B)*Tan[c + d*x])/d + ((3*a*A + 4*b*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((A*b + a*B)*Tan[c + d*x]^3)/(3*d)

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
```

```
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \int \left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)$$
$$= \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (4(Ab+aB)+($$
$$= \frac{aA\sec^3(c+dx)\tan(c+dx)}{4d}+(Ab+aB)\int \sec^4(c$$
$$= \frac{(3aA+4bB)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aA\sec^3(c+}{}$$
$$= \frac{(3aA+4bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(Ab+aB)\tan(}{d}$$

Mathematica [A] time = 0.62, size = 85, normalized size = 0.75

$$\frac{3(3aA+4bB)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\left(8(aB+Ab)(\cos(2(c+dx))+2)\sec(c+dx)+6aA\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*a*A + 4*b*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(9*a*A + 12*b*B + 8
*(A*b + a*B)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*a*A*Sec[c + d*x]^2)*Ta
n[c + d*x])/(24*d)
```

fricas [A] time = 0.65, size = 136, normalized size = 1.19

$$\frac{3\,(3\,Aa+4\,Bb)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,(3\,Aa+4\,Bb)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\,\Big(16}{48\,d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(3*A*a + 4*B*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A*a + 4
*B*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(B*a + A*b)*cos(d*x + c
)^3 + 3*(3*A*a + 4*B*b)*cos(d*x + c)^2 + 6*A*a + 8*(B*a + A*b)*cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 0.51, size = 304, normalized size = 2.67

$$3\,(3\,Aa+4\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(3\,Aa+4\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\Big(15\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac
")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a + 4*B
*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a*tan(1/2*d*x + 1/2*c)^7 -
 24*B*a*tan(1/2*d*x + 1/2*c)^7 - 24*A*b*tan(1/2*d*x + 1/2*c)^7 + 12*B*b*tan
(1/2*d*x + 1/2*c)^7 + 9*A*a*tan(1/2*d*x + 1/2*c)^5 + 40*B*a*tan(1/2*d*x + 1
/2*c)^5 + 40*A*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*b*tan(1/2*d*x + 1/2*c)^5 + 9
*A*a*tan(1/2*d*x + 1/2*c)^3 - 40*B*a*tan(1/2*d*x + 1/2*c)^3 - 40*A*b*tan(1/
2*d*x + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3 + 15*A*a*tan(1/2*d*x + 1/2
*c) + 24*B*a*tan(1/2*d*x + 1/2*c) + 24*A*b*tan(1/2*d*x + 1/2*c) + 12*B*b*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.14, size = 171, normalized size = 1.50

$$\frac{aA\left(\sec^3(dx+c)\right)\tan(dx+c)}{4d}+\frac{3aA\sec(dx+c)\tan(dx+c)}{8d}+\frac{3aA\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{2aB\tan(d}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/4*a*A*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*A*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*A
*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a*B*tan(d*x+c)+1/3/d*a*B*tan(d*x+c)*sec(d*
x+c)^2+2/3/d*A*b*tan(d*x+c)+1/3/d*A*b*tan(d*x+c)*sec(d*x+c)^2+1/2/d*B*b*tan
(d*x+c)*sec(d*x+c)+1/2/d*B*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 1.30, size = 163, normalized size = 1.43

$$16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba+16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ab-3Aa\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*A*b - 3*A*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*B*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

mupad [B] time = 3.86, size = 194, normalized size = 1.70

$$\frac{\left(\frac{5Aa}{4}-2Ab-2Ba+Bb\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{3Aa}{4}+\frac{10Ab}{3}+\frac{10Ba}{3}-Bb\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{3Aa}{4}-\frac{10Ab}{3}-\frac{10Ba}{3}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((5*A*a)/4 + 2*A*b + 2*B*a + B*b) + tan(c/2 + (d*x)/2)^
7*((5*A*a)/4 - 2*A*b - 2*B*a + B*b) - tan(c/2 + (d*x)/2)^3*((10*A*b)/3 - (3
*A*a)/4 + (10*B*a)/3 + B*b) + tan(c/2 + (d*x)/2)^5*((3*A*a)/4 + (10*A*b)/3
+ (10*B*a)/3 - B*b))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 -
```

```
4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (atanh(tan(c/2 + (d*x
)/2))*((3*A*a)/4 + B*b))/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.223 $\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=189

$$\frac{\left(4a^2A+6abB+3Ab^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2A+6abB+3Ab^2\right)-\frac{\left(5a(aB+2Ab)+4b^2B\right)\sin^3(c+d}{15d}$$

```
[Out] 1/8*(4*A*a^2+3*A*b^2+6*B*a*b)*x+1/5*(4*b^2*B+5*a*(2*A*b+B*a))*sin(d*x+c)/d+
1/8*(4*A*a^2+3*A*b^2+6*B*a*b)*cos(d*x+c)*sin(d*x+c)/d+1/20*b*(5*A*b+6*B*a)*
cos(d*x+c)^3*sin(d*x+c)/d+1/5*b*B*cos(d*x+c)^3*(a+b*cos(d*x+c))*sin(d*x+c)/
d-1/15*(4*b^2*B+5*a*(2*A*b+B*a))*sin(d*x+c)^3/d
```

Rubi [A] time = 0.31, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2990, 3023, 2748, 2635, 8, 2633}

$$\frac{\left(4a^2A+6abB+3Ab^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2A+6abB+3Ab^2\right)-\frac{\left(5a(aB+2Ab)+4b^2B\right)\sin^3(c+d}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*x)/8 + ((4*b^2*B + 5*a*(2*A*b + a*B))*Sin[c
+ d*x])/(5*d) + ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + (b*(5*A*b + 6*a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(20*d) + (b*B*Cos[c
+ d*x]^3*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d) - ((4*b^2*B + 5*a*(2*A*b
+ a*B))*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{bB\cos^3(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{5d} + \frac{1}{5}$$
$$= \frac{b(5Ab+6aB)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{bB\cos^3(}{}$$
$$= \frac{b(5Ab+6aB)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{bB\cos^3(}{}$$
$$= \frac{\left(4a^2A+3Ab^2+6abB\right)\cos(c+dx)\sin(c+dx)}{8d} +$$
$$= \frac{1}{8}\left(4a^2A+3Ab^2+6abB\right)x + \frac{\left(4b^2B+5a(2Ab+a\right.}{5d}$$

Mathematica [A] time = 0.48, size = 146, normalized size = 0.77

$$\frac{60(c+dx)\left(4a^2A+6abB+3Ab^2\right)+60\left(6a^2B+12aAb+5b^2B\right)\sin(c+dx)+120\left(a^2A+2abB+Ab^2\right)\sin(2(c}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (60*(4*a^2*A + 3*A*b^2 + 6*a*b*B)*(c + d*x) + 60*(12*a*A*b + 6*a^2*B + 5*b^
2*B)*Sin[c + d*x] + 120*(a^2*A + A*b^2 + 2*a*b*B)*Sin[2*(c + d*x)] + 10*(8*
a*A*b + 4*a^2*B + 5*b^2*B)*Sin[3*(c + d*x)] + 15*b*(A*b + 2*a*B)*Sin[4*(c +
 d*x)] + 6*b^2*B*Sin[5*(c + d*x)])/(480*d)
```

fricas [A] time = 0.65, size = 142, normalized size = 0.75

$$\frac{15\left(4Aa^2+6Bab+3Ab^2\right)dx+\left(24Bb^2\cos(dx+c)^4+30\left(2Bab+Ab^2\right)\cos(dx+c)^3+80Ba^2+160Aab+6\right.}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*d*x + (24*B*b^2*cos(d*x + c)^4 + 30
*(2*B*a*b + A*b^2)*cos(d*x + c)^3 + 80*B*a^2 + 160*A*a*b + 64*B*b^2 + 8*(5*
B*a^2 + 10*A*a*b + 4*B*b^2)*cos(d*x + c)^2 + 15*(4*A*a^2 + 6*B*a*b + 3*A*b^
2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.40, size = 156, normalized size = 0.83

$$\frac{Bb^2\sin(5dx+5c)}{80d}+\frac{1}{8}\left(4Aa^2+6Bab+3Ab^2\right)x+\frac{\left(2Bab+Ab^2\right)\sin(4dx+4c)}{32d}+\frac{\left(4Ba^2+8Aab+5Bb^2\right)\sin}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/80*B*b^2*sin(5*d*x + 5*c)/d + 1/8*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*x + 1/32*
(2*B*a*b + A*b^2)*sin(4*d*x + 4*c)/d + 1/48*(4*B*a^2 + 8*A*a*b + 5*B*b^2)*s
in(3*d*x + 3*c)/d + 1/4*(A*a^2 + 2*B*a*b + A*b^2)*sin(2*d*x + 2*c)/d + 1/8*
(6*B*a^2 + 12*A*a*b + 5*B*b^2)*sin(d*x + c)/d
```

maple [A] time = 0.05, size = 184, normalized size = 0.97

$$\frac{a^2 A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{B a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+\frac{2Aab\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2Bab\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right.}{4}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*B*a^2*(2+cos(d*x+c
)^2)*sin(d*x+c)+2/3*A*a*b*(2+cos(d*x+c)^2)*sin(d*x+c)+2*B*a*b*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*b^2*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5*b^2*B*(8/3+cos(d*x+c)^4+4/3*cos
(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.83, size = 176, normalized size = 0.93

$$\frac{120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2-160\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^2-320\left(\sin(dx+c)^3-3\,\sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 - 160*(sin(d*x + c)^3 - 3
*sin(d*x + c))*B*a^2 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b + 30*(12
*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a*b + 15*(12*d*x + 1
2*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*b^2 + 32*(3*sin(d*x + c)^5 -
 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*b^2)/d
```

mupad [B] time = 3.93, size = 307, normalized size = 1.62

$$\frac{x\left(A\,a^2+\frac{3\,B\,a\,b}{2}+\frac{3\,A\,b^2}{4}\right)}{2}+\frac{\left(2\,B\,a^2-\frac{5\,A\,b^2}{4}-A\,a^2+2\,B\,b^2+4\,A\,a\,b-\frac{5\,B\,a\,b}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{16\,B\,a^2}{3}-\frac{A\,b^2}{2}-2\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (x*(A*a^2 + (3*A*b^2)/4 + (3*B*a*b)/2))/2 + (tan(c/2 + (d*x)/2)^5*((20*B*a^
2)/3 + (116*B*b^2)/15 + (40*A*a*b)/3) - tan(c/2 + (d*x)/2)^9*(A*a^2 + (5*A*
b^2)/4 - 2*B*a^2 - 2*B*b^2 - 4*A*a*b + (5*B*a*b)/2) + tan(c/2 + (d*x)/2)^3*
(2*A*a^2 + (A*b^2)/2 + (16*B*a^2)/3 + (8*B*b^2)/3 + (32*A*a*b)/3 + B*a*b) -
 tan(c/2 + (d*x)/2)^7*(2*A*a^2 + (A*b^2)/2 - (16*B*a^2)/3 - (8*B*b^2)/3 - (
32*A*a*b)/3 + B*a*b) + tan(c/2 + (d*x)/2)*(A*a^2 + (5*A*b^2)/4 + 2*B*a^2 +
2*B*b^2 + 4*A*a*b + (5*B*a*b)/2))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 +
 (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 +
(d*x)/2)^10 + 1))
```

sympy [A] time = 2.47, size = 459, normalized size = 2.43

$$\begin{cases} \frac{Aa^2x\sin^2(c+dx)}{2} + \frac{Aa^2x\cos^2(c+dx)}{2} + \frac{Aa^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{4Aab\sin^3(c+dx)}{3d} + \frac{2Aab\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3Ab^2x\sin^4(c+dx)}{8} \\ x(A+B\cos(c))(a+b\cos(c))^2\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**2/2 + A*a**2
*sin(c + d*x)*cos(c + d*x)/(2*d) + 4*A*a*b*sin(c + d*x)**3/(3*d) + 2*A*a*b*
sin(c + d*x)*cos(c + d*x)**2/d + 3*A*b**2*x*sin(c + d*x)**4/8 + 3*A*b**2*x*
sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*A*b**2*x*cos(c + d*x)**4/8 + 3*A*b**2
*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*A*b**2*sin(c + d*x)*cos(c + d*x)**3
/(8*d) + 2*B*a**2*sin(c + d*x)**3/(3*d) + B*a**2*sin(c + d*x)*cos(c + d*x)*
*2/d + 3*B*a*b*x*sin(c + d*x)**4/4 + 3*B*a*b*x*sin(c + d*x)**2*cos(c + d*x)
**2/2 + 3*B*a*b*x*cos(c + d*x)**4/4 + 3*B*a*b*sin(c + d*x)**3*cos(c + d*x)/
(4*d) + 5*B*a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 8*B*b**2*sin(c + d*x)*
*5/(15*d) + 4*B*b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + B*b**2*sin(c +
 d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))**2*cos
(c)**2, True))
```

3.224 $\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=170

$$\frac{\left(-2a^2B+8aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(4a^2B+8aAb+3b^2B\right)+\frac{\left(a^3(-B)+4a^2Ab+8ab^2B+4Ab^3\right)}{6bd}$$

```
[Out] 1/8*(8*A*a*b+4*B*a^2+3*B*b^2)*x+1/6*(4*A*a^2*b+4*A*b^3-B*a^3+8*B*a*b^2)*sin
(d*x+c)/b/d+1/24*(8*A*a*b-2*B*a^2+9*B*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/12*(4*
A*b-B*a)*(a+b*cos(d*x+c))^2*sin(d*x+c)/b/d+1/4*B*(a+b*cos(d*x+c))^3*sin(d*x
+c)/b/d
```

Rubi [A] time = 0.23, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3023, 2753, 2734}

$$\frac{\left(4a^2Ab+a^3(-B)+8ab^2B+4Ab^3\right)\sin(c+dx)}{6bd}+\frac{\left(-2a^2B+8aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(4a^2B+8a\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((8*a*A*b + 4*a^2*B + 3*b^2*B)*x)/8 + ((4*a^2*A*b + 4*A*b^3 - a^3*B + 8*a*b
^2*B)*Sin[c + d*x])/(6*b*d) + ((8*a*A*b - 2*a^2*B + 9*b^2*B)*Cos[c + d*x]*S
in[c + d*x])/(24*d) + ((4*A*b - a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
12*b*d) + (B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*b*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^2\left(A\cos(c+dx)+B\cos^2(c+\right.$$

$$= \frac{B(a+b\cos(c+dx))^3\sin(c+dx)}{4bd} + \frac{\int (a+b\cos(c+}{}$$

$$= \frac{(4Ab-aB)(a+b\cos(c+dx))^2\sin(c+dx)}{12bd} + \frac{B(a+}{}$$

$$= \frac{1}{8}\left(8aAb+4a^2B+3b^2B\right)x + \frac{\left(4a^2Ab+4Ab^3-a^3B\right.}{6b}$$

Mathematica [A] time = 0.46, size = 118, normalized size = 0.69

$$\frac{12(c+dx)\left(4a^2B+8aAb+3b^2B\right)+24\left(4a^2A+6abB+3Ab^2\right)\sin(c+dx)+24\left(a^2B+2aAb+b^2B\right)\sin(2(c+}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (12*(8*a*A*b + 4*a^2*B + 3*b^2*B)*(c + d*x) + 24*(4*a^2*A + 3*A*b^2 + 6*a*b
*B)*Sin[c + d*x] + 24*(2*a*A*b + a^2*B + b^2*B)*Sin[2*(c + d*x)] + 8*b*(A*b
 + 2*a*B)*Sin[3*(c + d*x)] + 3*b^2*B*Sin[4*(c + d*x)])/(96*d)
```

fricas [A] time = 0.75, size = 114, normalized size = 0.67

$$\frac{3\left(4\,Ba^2+8\,Aab+3\,Bb^2\right)dx+\left(6\,Bb^2\cos(dx+c)^3+24\,Aa^2+32\,Bab+16\,Ab^2+8\left(2\,Bab+Ab^2\right)\cos(dx+c)^2\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*d*x + (6*B*b^2*cos(d*x + c)^3 + 24*A*
a^2 + 32*B*a*b + 16*A*b^2 + 8*(2*B*a*b + A*b^2)*cos(d*x + c)^2 + 3*(4*B*a^2
 + 8*A*a*b + 3*B*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.38, size = 124, normalized size = 0.73

$$\frac{Bb^2\sin(4\,dx+4\,c)}{32\,d}+\frac{1}{8}\left(4\,Ba^2+8\,Aab+3\,Bb^2\right)x+\frac{\left(2\,Bab+Ab^2\right)\sin(3\,dx+3\,c)}{12\,d}+\frac{\left(Ba^2+2\,Aab+Bb^2\right)\sin(2\,dx}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/32*B*b^2*sin(4*d*x + 4*c)/d + 1/8*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*x + 1/12*
(2*B*a*b + A*b^2)*sin(3*d*x + 3*c)/d + 1/4*(B*a^2 + 2*A*a*b + B*b^2)*sin(2*
d*x + 2*c)/d + 1/4*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*sin(d*x + c)/d
```

maple [A] time = 0.05, size = 152, normalized size = 0.89

$$\frac{a^2A\sin(dx+c)+B\,a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2Aab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{2Bab\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(a^2*A*sin(d*x+c)+B*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*A*a
*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2/3*B*a*b*(2+cos(d*x+c)^2)*sin
(d*x+c)+1/3*A*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+b^2*B*(1/4*(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.53, size = 142, normalized size = 0.84

$$\frac{24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2+48\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aab-64\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 + 48*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a*b - 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a*b - 32*(sin(d*x
 + c)^3 - 3*sin(d*x + c))*A*b^2 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*s
in(2*d*x + 2*c))*B*b^2 + 96*A*a^2*sin(d*x + c))/d
```

mupad [B] time = 0.51, size = 169, normalized size = 0.99

$$\frac{B\,a^2\,x}{2}+\frac{3\,B\,b^2\,x}{8}+\frac{A\,a^2\,\sin\left(c+d\,x\right)}{d}+\frac{3\,A\,b^2\,\sin\left(c+d\,x\right)}{4\,d}+A\,a\,b\,x+\frac{B\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,b^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (B*a^2*x)/2 + (3*B*b^2*x)/8 + (A*a^2*sin(c + d*x))/d + (3*A*b^2*sin(c + d*x
))/(4*d) + A*a*b*x + (B*a^2*sin(2*c + 2*d*x))/(4*d) + (A*b^2*sin(3*c + 3*d*
x))/(12*d) + (B*b^2*sin(2*c + 2*d*x))/(4*d) + (B*b^2*sin(4*c + 4*d*x))/(32*
d) + (3*B*a*b*sin(c + d*x))/(2*d) + (A*a*b*sin(2*c + 2*d*x))/(2*d) + (B*a*b
*sin(3*c + 3*d*x))/(6*d)
```

sympy [A] time = 1.19, size = 338, normalized size = 1.99

$$\begin{cases} \frac{Aa^2\sin\left(c+dx\right)}{d}+Aabx\sin^2\left(c+dx\right)+Aabx\cos^2\left(c+dx\right)+\frac{Aab\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{2Ab^2\sin^3\left(c+dx\right)}{3d}+\frac{Ab^2\sin\left(c+dx\right)\cos^2}{d} \\ x\left(A+B\cos\left(c\right)\right)\left(a+b\cos\left(c\right)\right)^2\cos\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*sin(c + d*x)/d + A*a*b*x*sin(c + d*x)**2 + A*a*b*x*cos(c
+ d*x)**2 + A*a*b*sin(c + d*x)*cos(c + d*x)/d + 2*A*b**2*sin(c + d*x)**3/(3
*d) + A*b**2*sin(c + d*x)*cos(c + d*x)**2/d + B*a**2*x*sin(c + d*x)**2/2 +
B*a**2*x*cos(c + d*x)**2/2 + B*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 4*B*a
*b*sin(c + d*x)**3/(3*d) + 2*B*a*b*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*b**
2*x*sin(c + d*x)**4/8 + 3*B*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B*
b**2*x*cos(c + d*x)**4/8 + 3*B*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*
B*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c))*(a
+ b*cos(c))**2*cos(c), True))
```

3.225 $\int (a + b\cos(c + dx))^2 (A + B\cos(c + dx))\, dx$

Optimal. Leaf size=107

$$\frac{2\left(a^2B + 3aAb + b^2B\right)\sin(c + dx)}{3d} + \frac{1}{2}x\left(2a^2A + 2abB + Ab^2\right) + \frac{b(2aB + 3Ab)\sin(c + dx)\cos(c + dx)}{6d} + \frac{B\sin(c + \cdots}{}$$

```
[Out] 1/2*(2*A*a^2+A*b^2+2*B*a*b)*x+2/3*(3*A*a*b+B*a^2+B*b^2)*sin(d*x+c)/d+1/6*b*
(3*A*b+2*B*a)*cos(d*x+c)*sin(d*x+c)/d+1/3*B*(a+b*cos(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$\frac{2\left(a^2B + 3aAb + b^2B\right)\sin(c + dx)}{3d} + \frac{1}{2}x\left(2a^2A + 2abB + Ab^2\right) + \frac{b(2aB + 3Ab)\sin(c + dx)\cos(c + dx)}{6d} + \frac{B\sin(c + \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((2*a^2*A + A*b^2 + 2*a*b*B)*x)/2 + (2*(3*a*A*b + a^2*B + b^2*B)*Sin[c + d*
x])/(3*d) + (b*(3*A*b + 2*a*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (B*(a + b
*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{B(a+b\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{1}{3}\int (a+b\cos(c+dx))(3aA$$
$$= \frac{1}{2}\left(2a^2A+Ab^2+2abB\right)x + \frac{2\left(3aAb+a^2B+b^2B\right)\sin(c+dx)}{3d}$$

Mathematica [A] time = 0.23, size = 90, normalized size = 0.84

$$\frac{6(c+dx)\left(2a^2A+2abB+Ab^2\right)+3\left(4a^2B+8aAb+3b^2B\right)\sin(c+dx)+3b(2aB+Ab)\sin(2(c+dx))+b^2B\sin}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*(2*a^2*A + A*b^2 + 2*a*b*B)*(c + d*x) + 3*(8*a*A*b + 4*a^2*B + 3*b^2*B)*
Sin[c + d*x] + 3*b*(A*b + 2*a*B)*Sin[2*(c + d*x)] + b^2*B*Sin[3*(c + d*x)])
/(12*d)
```

fricas [A] time = 0.66, size = 85, normalized size = 0.79

$$\frac{3\left(2\,Aa^2+2\,Bab+Ab^2\right)dx+\left(2\,Bb^2\cos\left(dx+c\right)^2+6\,Ba^2+12\,Aab+4\,Bb^2+3\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)\right)\sin}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*A*a^2 + 2*B*a*b + A*b^2)*d*x + (2*B*b^2*cos(d*x + c)^2 + 6*B*a^2
+ 12*A*a*b + 4*B*b^2 + 3*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.47, size = 93, normalized size = 0.87

$$\frac{Bb^2\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{1}{2}\left(2\,Aa^2+2\,Bab+Ab^2\right)x+\frac{\left(2\,Bab+Ab^2\right)\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{\left(4\,Ba^2+8\,Aab+3\,Bb^2\right)\sin\left(d\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*B*b^2*sin(3*d*x + 3*c)/d + 1/2*(2*A*a^2 + 2*B*a*b + A*b^2)*x + 1/4*(2*
B*a*b + A*b^2)*sin(2*d*x + 2*c)/d + 1/4*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*sin(d
*x + c)/d
```

maple [A] time = 0.05, size = 114, normalized size = 1.07

$$\frac{\frac{b^{2}B\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}+A\,b^{2}\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2Bab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2Aab\sin\left(dx+c\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/3*b^2*B*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b^2*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+2*B*a*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*A*a*b*s
in(d*x+c)+B*a^2*sin(d*x+c)+a^2*A*(d*x+c))
```

maxima [A] time = 0.31, size = 108, normalized size = 1.01

$$\frac{12\,(dx+c)Aa^{2}+6\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Bab+3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ab^{2}-4\left(\sin\left(dx+c\right)^{3}-3\,\mathrm{s}\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*A*a^2 + 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b + 3*(2*
d*x + 2*c + sin(2*d*x + 2*c))*A*b^2 - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*B
*b^2 + 12*B*a^2*sin(d*x + c) + 24*A*a*b*sin(d*x + c))/d
```

mupad [B] time = 0.45, size = 115, normalized size = 1.07

$$A\,a^{2}\,x+\frac{A\,b^{2}\,x}{2}+\frac{B\,a^{2}\,\sin\left(c+d\,x\right)}{d}+\frac{3\,B\,b^{2}\,\sin\left(c+d\,x\right)}{4\,d}+B\,a\,b\,x+\frac{A\,b^{2}\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,b^{2}\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{2\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

```
[Out] A*a^2*x + (A*b^2*x)/2 + (B*a^2*sin(c + d*x))/d + (3*B*b^2*sin(c + d*x))/(4*
d) + B*a*b*x + (A*b^2*sin(2*c + 2*d*x))/(4*d) + (B*b^2*sin(3*c + 3*d*x))/(1
2*d) + (2*A*a*b*sin(c + d*x))/d + (B*a*b*sin(2*c + 2*d*x))/(2*d)
```

sympy [A] time = 0.59, size = 199, normalized size = 1.86

$$\begin{cases} Aa^{2}x+\frac{2Aab\sin\left(c+dx\right)}{d}+\frac{Ab^{2}x\sin^{2}\left(c+dx\right)}{2}+\frac{Ab^{2}x\cos^{2}\left(c+dx\right)}{2}+\frac{Ab^{2}\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{Ba^{2}\sin\left(c+dx\right)}{d}+Babx\sin^{2}\left(c+dx\right) \\ x\left(A+B\cos\left(c\right)\right)\left(a+b\cos\left(c\right)\right)^{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x + 2*A*a*b*sin(c + d*x)/d + A*b**2*x*sin(c + d*x)**2/2 +
 A*b**2*x*cos(c + d*x)**2/2 + A*b**2*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*
*2*sin(c + d*x)/d + B*a*b*x*sin(c + d*x)**2 + B*a*b*x*cos(c + d*x)**2 + B*a
*b*sin(c + d*x)*cos(c + d*x)/d + 2*B*b**2*sin(c + d*x)**3/(3*d) + B*b**2*si
n(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))**
2, True))
```

3.226 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=86

$$\frac{1}{2}x\left(2a^2B+4aAb+b^2B\right)+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(3aB+2Ab)\sin(c+dx)}{2d}+\frac{bB\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

```
[Out] 1/2*(4*A*a*b+2*B*a^2+B*b^2)*x+a^2*A*arctanh(sin(d*x+c))/d+1/2*b*(2*A*b+3*B*
a)*sin(d*x+c)/d+1/2*b*B*(a+b*cos(d*x+c))*sin(d*x+c)/d
```

Rubi [A] time = 0.18, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2990, 3023, 2735, 3770}

$$\frac{1}{2}x\left(2a^2B+4aAb+b^2B\right)+\frac{a^2A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b(3aB+2Ab)\sin(c+dx)}{2d}+\frac{bB\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] ((4*a*A*b + 2*a^2*B + b^2*B)*x)/2 + (a^2*A*ArcTanh[Sin[c + d*x]])/d + (b*(2
*A*b + 3*a*B)*Sin[c + d*x])/(2*d) + (b*B*(a + b*Cos[c + d*x])*Sin[c + d*x])
/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{bB(a+b\cos(c+dx))\sin(c+dx)}{2d}+\frac{1}{2}\int \left(2a^2A+\left(\right.\right.$$
$$=\frac{b(2Ab+3aB)\sin(c+dx)}{2d}+\frac{bB(a+b\cos(c+dx))\,\mathrm{s}}{2d}$$
$$=\frac{1}{2}\left(4aAb+2a^2B+b^2B\right)x+\frac{b(2Ab+3aB)\sin(c+d}{2d}$$
$$=\frac{1}{2}\left(4aAb+2a^2B+b^2B\right)x+\frac{a^2A\tanh^{-1}(\sin(c+dx)}{d}$$

Mathematica [A] time = 0.23, size = 120, normalized size = 1.40

$$\frac{2(c+dx)\left(2a^2B+4aAb+b^2B\right)-4a^2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4a^2A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\mathrm{c}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*(4*a*A*b + 2*a^2*B + b^2*B)*(c + d*x) - 4*a^2*A*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 4*a^2*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*b*(
A*b + 2*a*B)*Sin[c + d*x] + b^2*B*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.73, size = 87, normalized size = 1.01

$$\frac{Aa^2\log(\sin(dx+c)+1)-Aa^2\log(-\sin(dx+c)+1)+\left(2Ba^2+4Aab+Bb^2\right)dx+\left(Bb^2\cos(dx+c)+4Bab\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/2*(A*a^2*log(sin(d*x + c) + 1) - A*a^2*log(-sin(d*x + c) + 1) + (2*B*a^2
+ 4*A*a*b + B*b^2)*d*x + (B*b^2*cos(d*x + c) + 4*B*a*b + 2*A*b^2)*sin(d*x +
 c))/d
```

giac [B] time = 0.45, size = 178, normalized size = 2.07

$$\frac{2\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Aa^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\left(2\,Ba^2+4\,Aab+Bb^2\right)(dx+c)+\frac{2\left(4\,Bab\,\mathrm{t}\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/2*(2*A*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*A*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (2*B*a^2 + 4*A*a*b + B*b^2)*(d*x + c) + 2*(4*B*a*b*tan(
1/2*d*x + 1/2*c)^3 + 2*A*b^2*tan(1/2*d*x + 1/2*c)^3 - B*b^2*tan(1/2*d*x + 1
/2*c)^3 + 4*B*a*b*tan(1/2*d*x + 1/2*c) + 2*A*b^2*tan(1/2*d*x + 1/2*c) + B*b
^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.10, size = 120, normalized size = 1.40

$$\frac{a^2A\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+a^2Bx+\frac{B\,a^2c}{d}+2Aabx+\frac{2Aabc}{d}+\frac{2Bab\sin\left(dx+c\right)}{d}+\frac{A\,b^2\sin\left(dx+c\right)}{d}+\frac{b^2B\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+a^2*B*x+1/d*B*a^2*c+2*A*a*b*x+2/d*A*a*b
*c+2/d*B*a*b*sin(d*x+c)+1/d*A*b^2*sin(d*x+c)+1/2/d*b^2*B*cos(d*x+c)*sin(d*x
+c)+1/2*b^2*B*x+1/2/d*b^2*B*c
```

maxima [A] time = 1.21, size = 92, normalized size = 1.07

$$\frac{4\,(dx+c)Ba^2+8\,(dx+c)Aab+(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Bb^2+4\,Aa^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+8\,Ba}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(4*(d*x + c)*B*a^2 + 8*(d*x + c)*A*a*b + (2*d*x + 2*c + sin(2*d*x + 2*c
))*B*b^2 + 4*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 8*B*a*b*sin(d*x + c)
+ 4*A*b^2*sin(d*x + c))/d
```

mupad [B] time = 0.69, size = 169, normalized size = 1.97

$$\frac{A\,b^2\,\sin\left(c+d\,x\right)}{d}+\frac{2\,A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{B\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{B\,b^2\,\sin\left(2\,c+2\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x),x)
```

```
[Out] (A*b^2*sin(c + d*x))/d + (2*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/d + (2*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (B*b^2*a
tan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (B*b^2*sin(2*c + 2*d*x))/(4
*d) + (2*B*a*b*sin(c + d*x))/d + (4*A*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\,(a+b\cos(c+dx))^2\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sec(c + d*x), x)
```

3.227 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=60

$$\frac{a^2A\tan(c+dx)}{d}+\frac{a(aB+2Ab)\tanh^{-1}(\sin(c+dx))}{d}+bx(2aB+Ab)+\frac{b^2B\sin(c+dx)}{d}$$

```
[Out] b*(A*b+2*B*a)*x+a*(2*A*b+B*a)*arctanh(sin(d*x+c))/d+b^2*B*sin(d*x+c)/d+a^2*
A*tan(d*x+c)/d
```

Rubi [A] time = 0.17, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2988, 3023, 2735, 3770}

$$\frac{a^2A\tan(c+dx)}{d}+\frac{a(aB+2Ab)\tanh^{-1}(\sin(c+dx))}{d}+bx(2aB+Ab)+\frac{b^2B\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] b*(A*b + 2*a*B)*x + (a*(2*A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (b^2*B*Sin[
c + d*x])/d + (a^2*A*Tan[c + d*x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{a^2A\tan(c+dx)}{d} - \int \left(-a(2Ab+aB)-b(Ab+2aB\right.$$

$$= \frac{b^2B\sin(c+dx)}{d} + \frac{a^2A\tan(c+dx)}{d} - \int (-a(2Ab+$$

$$= b(Ab+2aB)x + \frac{b^2B\sin(c+dx)}{d} + \frac{a^2A\tan(c+dx}{d}$$

$$= b(Ab+2aB)x + \frac{a(2Ab+aB)\tanh^{-1}(\sin(c+dx))}{d}$$

Mathematica [A] time = 0.50, size = 109, normalized size = 1.82

$$\frac{a^2A\tan(c+dx) + b(c+dx)(2aB+Ab) - a(aB+2Ab)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + a(aB+2Ab)\,\mathrm{lo}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (b*(A*b + 2*a*B)*(c + d*x) - a*(2*A*b + a*B)*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + a*(2*A*b + a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^2
*B*Sin[c + d*x] + a^2*A*Tan[c + d*x])/d
```

fricas [A] time = 0.61, size = 117, normalized size = 1.95

$$\frac{2\left(2\,Bab+Ab^2\right)dx\cos(dx+c) + \left(Ba^2+2\,Aab\right)\cos(dx+c)\log(\sin(dx+c)+1) - \left(Ba^2+2\,Aab\right)\cos(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*(2*(2*B*a*b + A*b^2)*d*x*cos(d*x + c) + (B*a^2 + 2*A*a*b)*cos(d*x + c)*
log(sin(d*x + c) + 1) - (B*a^2 + 2*A*a*b)*cos(d*x + c)*log(-sin(d*x + c) +
1) + 2*(B*b^2*cos(d*x + c) + A*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 0.73, size = 152, normalized size = 2.53

$$\frac{\left(2\,Bab + Ab^2\right)(dx + c) + \left(Ba^2 + 2\,Aab\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(Ba^2 + 2\,Aab\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] ((2*B*a*b + A*b^2)*(d*x + c) + (B*a^2 + 2*A*a*b)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (B*a^2 + 2*A*a*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a^2*
tan(1/2*d*x + 1/2*c)^3 - B*b^2*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*x +
 1/2*c) + B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

maple [A] time = 0.12, size = 104, normalized size = 1.73

$$A\,b^2x+2Babx+\frac{a^2A\tan(dx+c)}{d}+\frac{2Aab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{A\,b^2c}{d}+\frac{b^2B\sin(dx+c)}{d}+\frac{B\,a^2\ln(\sec(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] A*b^2*x+2*B*a*b*x+a^2*A*tan(d*x+c)/d+2/d*A*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/
d*A*b^2*c+b^2*B*sin(d*x+c)/d+1/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*B*a*b*
c
```

maxima [A] time = 0.55, size = 103, normalized size = 1.72

$$\frac{4\,(dx+c)Bab + 2\,(dx+c)Ab^2 + Ba^2\left(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\right) + 2\,Aab\left(\log(\sin(dx+c)+1\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*(4*(d*x + c)*B*a*b + 2*(d*x + c)*A*b^2 + B*a^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 2*A*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*B*b^2*sin(d*x + c) + 2*A*a^2*tan(d*x + c))/d
```

mupad [B] time = 0.88, size = 169, normalized size = 2.82

$$\frac{A\,a^2\tan\left(c+d\,x\right)}{d}+\frac{2\,A\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{B\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d\,\cos\left(c+d\,x\right)}+\frac{4\,B\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] (A*a^2*tan(c + d*x))/d + (2*A*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d - (B*a^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/d + (B*
b^2*sin(2*c + 2*d*x))/(2*d*cos(c + d*x)) - (A*a*b*atan((sin(c/2 + (d*x)/2)*
1i)/cos(c/2 + (d*x)/2))*4i)/d + (4*B*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))(a + b\cos(c + dx))^2 \sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sec(c + d*x)**2, x)
```

3.228 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=80

$$\frac{\left(a^2A+4abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2A\tan(c+dx)\sec(c+dx)}{2d}+\frac{a(aB+2Ab)\tan(c+dx)}{d}+b^2Bx$$

```
[Out] b^2*B*x+1/2*(A*a^2+2*A*b^2+4*B*a*b)*arctanh(sin(d*x+c))/d+a*(2*A*b+B*a)*tan
(d*x+c)/d+1/2*a^2*A*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2988, 3021, 2735, 3770}

$$\frac{\left(a^2A+4abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2A\tan(c+dx)\sec(c+dx)}{2d}+\frac{a(aB+2Ab)\tan(c+dx)}{d}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] b^2*B*x + ((a^2*A + 2*A*b^2 + 4*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2
*A*b + a*B)*Tan[c + d*x])/d + (a^2*A*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^3(c+dx)\,dx &= \frac{a^2A\sec(c+dx)\tan(c+dx)}{2d}-\frac{1}{2}\int \Big(-2a(2Ab+aB\\ &= \frac{a(2Ab+aB)\tan(c+dx)}{d}+\frac{a^2A\sec(c+dx)\tan(c-\\ &= b^2Bx+\frac{a(2Ab+aB)\tan(c+dx)}{d}+\frac{a^2A\sec(c+dx}{2d}\\ &= b^2Bx+\frac{\left(a^2A+2Ab^2+4abB\right)\tanh^{-1}(\sin(c+dx))}{2d}\end{aligned}$$

Mathematica [A] time = 0.27, size = 67, normalized size = 0.84

$$\frac{\left(a^2A+4abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))+a\tan(c+dx)(aA\sec(c+dx)+2aB+4Ab)+2b^2Bdx}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^2*B*d*x + (a^2*A + 2*A*b^2 + 4*a*b*B)*ArcTanh[Sin[c + d*x]] + a*(4*A*b
 + 2*a*B + a*A*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

fricas [A] time = 0.61, size = 136, normalized size = 1.70

$$\frac{4\,Bb^2dx\cos\left(dx+c\right)^2+\left(Aa^2+4\,Bab+2\,Ab^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(Aa^2+4\,Bab+2\,Ab^2\right)\cos}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*B*b^2*d*x*cos(d*x + c)^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*cos(d*x + c)^
2*log(sin(d*x + c) + 1) - (A*a^2 + 4*B*a*b + 2*A*b^2)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) + 2*(A*a^2 + 2*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.52, size = 190, normalized size = 2.38

$$\frac{2\,(dx+c)Bb^2 + \left(Aa^2 + 4\,Bab + 2\,Ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(Aa^2 + 4\,Bab + 2\,Ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right.\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*(d*x + c)*B*b^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x +
1/2*c) + 1)) - (A*a^2 + 4*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) + 2*(A*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*A
*a*b*tan(1/2*d*x + 1/2*c)^3 + A*a^2*tan(1/2*d*x + 1/2*c) + 2*B*a^2*tan(1/2*
d*x + 1/2*c) + 4*A*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2
)/d
```

maple [A] time = 0.12, size = 133, normalized size = 1.66

$$\frac{a^2A\sec(dx+c)\tan(dx+c)}{2d} + \frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{B\,a^2\tan(dx+c)}{d} + \frac{2Aab\tan(dx+c)}{d} + \frac{2Bab\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^2*A*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+1/d
*B*a^2*tan(d*x+c)+2/d*A*a*b*tan(d*x+c)+2/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+
1/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+b^2*B*x+1/d*b^2*B*c
```

maxima [A] time = 0.41, size = 140, normalized size = 1.75

$$\frac{4\,(dx+c)Bb^2 - Aa^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 4\,Bab\left(\log(\sin(dx+c)+1) - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(4*(d*x + c)*B*b^2 - A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*B*a*b*(log(sin(d*x + c) + 1)
- log(sin(d*x + c) - 1)) + 2*A*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c
) - 1)) + 4*B*a^2*tan(d*x + c) + 8*A*a*b*tan(d*x + c))/d
```

mupad [B] time = 0.98, size = 176, normalized size = 2.20

$$\frac{2\left(\frac{A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+A\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+B\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+2\,B\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\right)}{d}+\frac{\frac{B\,a^2\,\sin(2\,c}{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^3,x)
```

```
[Out] (2*((A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + A*b^2*atanh(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + B*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)) + 2*B*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/d +
((B*a^2*sin(2*c + 2*d*x))/2 + (A*a^2*sin(c + d*x))/2 + A*a*b*sin(2*c + 2*d*
x))/(d*(cos(2*c + 2*d*x)/2 + 1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))\,(a + b\cos(c + dx))^2\sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sec(c + d*x)**3, x)
```

3.229 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=116

$$\frac{\left(2a^2A+6abB+3Ab^2\right)\tan(c+dx)}{3d}+\frac{\left(a^2B+2aAb+2b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}+$$

```
[Out] 1/2*(2*A*a*b+B*a^2+2*B*b^2)*arctanh(sin(d*x+c))/d+1/3*(2*A*a^2+3*A*b^2+6*B*
a*b)*tan(d*x+c)/d+1/2*a*(2*A*b+B*a)*sec(d*x+c)*tan(d*x+c)/d+1/3*a^2*A*sec(d
*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.27, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2988, 3021, 2748, 3767, 8, 3770}

$$\frac{\left(2a^2A+6abB+3Ab^2\right)\tan(c+dx)}{3d}+\frac{\left(a^2B+2aAb+2b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2A\tan(c+dx)\sec^2(c+dx)}{3d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((2*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*a^2*A + 3*A
*b^2 + 6*a*b*B)*Tan[c + d*x])/(3*d) + (a*(2*A*b + a*B)*Sec[c + d*x]*Tan[c +
 d*x])/(2*d) + (a^2*A*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
```

```
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{a^2A\sec^2(c+dx)\tan(c+dx)}{3d} - \frac{1}{3}\int \Big(-3a(2Ab+a$$

$$= \frac{a(2Ab+aB)\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^2A\sec^2(c$$

$$= \frac{a(2Ab+aB)\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^2A\sec^2(c$$

$$= \frac{\left(2aAb+a^2B+2b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a(2A$$

$$= \frac{\left(2aAb+a^2B+2b^2B\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\Big(2a^2A$$

Mathematica [A] time = 0.47, size = 92, normalized size = 0.79

$$\frac{3\left(a^2B+2aAb+2b^2B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(2\left(a^2A\tan^2(c+dx)+3a^2A+6abB+3Ab^2\right)+3a(aB\ldots\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (3*(2*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*a*(2
*A*b + a*B)*Sec[c + d*x] + 2*(3*a^2*A + 3*A*b^2 + 6*a*b*B + a^2*A*Tan[c + d
*x]^2)))/(6*d)
```

fricas [A] time = 1.06, size = 150, normalized size = 1.29

$$\frac{3\left(Ba^2+2\,Aab+2\,Bb^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(Ba^2+2\,Aab+2\,Bb^2\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx\ldots\right.\right.}{12\,d\cos\left(dx\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) -
3*(B*a^2 + 2*A*a*b + 2*B*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*
A*a^2 + 2*(2*A*a^2 + 6*B*a*b + 3*A*b^2)*cos(d*x + c)^2 + 3*(B*a^2 + 2*A*a*b
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [B] time = 0.82, size = 294, normalized size = 2.53

$$\frac{3\left(Ba^2+2\,Aab+2\,Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(Ba^2+2\,Aab+2\,Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\ldots\right.}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(
B*a^2 + 2*A*a*b + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^2*
tan(1/2*d*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b*tan(1/2*d
*x + 1/2*c)^5 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2
*c)^5 - 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 -
12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*ta
```

```
n(1/2*d*x + 1/2*c) + 6*A*a*b*tan(1/2*d*x + 1/2*c) + 12*B*a*b*tan(1/2*d*x +
1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.13, size = 174, normalized size = 1.50

$$\frac{2a^2A\tan(dx+c)}{3d}+\frac{a^2A\left(\sec^2(dx+c)\right)\tan(dx+c)}{3d}+\frac{B\,a^2\sec(dx+c)\tan(dx+c)}{2d}+\frac{B\,a^2\ln(\sec(dx+c)+\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 2/3*a^2*A*tan(d*x+c)/d+1/3*a^2*A*sec(d*x+c)^2*tan(d*x+c)/d+1/2/d*B*a^2*sec(
d*x+c)*tan(d*x+c)+1/2/d*B*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a*b*sec(d*x+c
)*tan(d*x+c)+1/d*A*a*b*ln(sec(d*x+c)+tan(d*x+c))+2/d*B*a*b*tan(d*x+c)+1/d*A
*b^2*tan(d*x+c)+1/d*b^2*B*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.44, size = 172, normalized size = 1.48

$$4\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^2-3\,Ba^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-6\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 - 3*B*a^2*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 6*A*
a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 6*B*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
24*B*a*b*tan(d*x + c) + 12*A*b^2*tan(d*x + c))/d
```

mupad [B] time = 3.66, size = 227, normalized size = 1.96

$$\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{B\,a^2}{2}+A\,a\,b+B\,b^2\right)}{2\,B\,a^2+4\,A\,a\,b+4\,B\,b^2}\right)\left(B\,a^2+2\,A\,a\,b+2\,B\,b^2\right)}{d}-\left(2\,A\,a^2+2\,A\,b^2-B\,a^2-2\,A\,a\,b+4\,B\,a\,b\right)\,\tan\left(\frac{c}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^4,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((B*a^2)/2 + B*b^2 + A*a*b))/(2*B*a^2 + 4*B*b^
2 + 4*A*a*b))*(B*a^2 + 2*B*b^2 + 2*A*a*b))/d - (tan(c/2 + (d*x)/2)*(2*A*a^2
 + 2*A*b^2 + B*a^2 + 2*A*a*b + 4*B*a*b) - tan(c/2 + (d*x)/2)^3*((4*A*a^2)/3
 + 4*A*b^2 + 8*B*a*b) + tan(c/2 + (d*x)/2)^5*(2*A*a^2 + 2*A*b^2 - B*a^2 - 2
*A*a*b + 4*B*a*b))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + ta
n(c/2 + (d*x)/2)^6 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.230 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=156

$$\frac{(2a^2B+4aAb+3b^2B)\tan(c+dx)}{3d}+\frac{(3a^2A+8abB+4Ab^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3a^2A+8abB+4Ab^2)\tan(\ldots}{8d}$$

```
[Out] 1/8*(3*A*a^2+4*A*b^2+8*B*a*b)*arctanh(sin(d*x+c))/d+1/3*(4*A*a*b+2*B*a^2+3*
B*b^2)*tan(d*x+c)/d+1/8*(3*A*a^2+4*A*b^2+8*B*a*b)*sec(d*x+c)*tan(d*x+c)/d+1
/3*a*(2*A*b+B*a)*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a^2*A*sec(d*x+c)^3*tan(d*x+c
)/d
```

Rubi [A] time = 0.29, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2988, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(2a^2B+4aAb+3b^2B)\tan(c+dx)}{3d}+\frac{(3a^2A+8abB+4Ab^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3a^2A+8abB+4Ab^2)\tan(\ldots}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a*A*b + 2
*a^2*B + 3*b^2*B)*Tan[c + d*x])/(3*d) + ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (a*(2*A*b + a*B)*Sec[c + d*x]^2*Tan[c + d*x]
)/(3*d) + (a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
```

```
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{a^2A\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{1}{4}\int \Big(-4a(2Ab+a$$
$$= \frac{a(2Ab+aB)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{a^2A\sec^3(}{}$$
$$= \frac{a(2Ab+aB)\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{a^2A\sec^3(}{}$$
$$= \frac{\left(3a^2A+4Ab^2+8abB\right)\sec(c+dx)\tan(c+dx)}{8d} + $$
$$= \frac{\left(3a^2A+4Ab^2+8abB\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(4a}{}$$

Mathematica [A] time = 0.72, size = 120, normalized size = 0.77

$$\frac{3\left(3a^2A+8abB+4Ab^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3\left(3a^2A+8abB+4Ab^2\right)\sec(c+dx)+24\left(a^2B+\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*(
2*a*A*b + a^2*B + b^2*B) + 3*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*Sec[c + d*x] + 6
*a^2*A*Sec[c + d*x]^3 + 8*a*(2*A*b + a*B)*Tan[c + d*x]^2))/(24*d)
```

fricas [A] time = 0.92, size = 180, normalized size = 1.15

$$\frac{3\left(3\,Aa^2+8\,Bab+4\,Ab^2\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(3\,Aa^2+8\,Bab+4\,Ab^2\right)\cos(dx+c)^4\log(-\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
- 3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2
*(8*(2*B*a^2 + 4*A*a*b + 3*B*b^2)*cos(d*x + c)^3 + 6*A*a^2 + 3*(3*A*a^2 + 8
*B*a*b + 4*A*b^2)*cos(d*x + c)^2 + 8*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 0.55, size = 478, normalized size = 3.06

$$3\left(3\,Aa^2+8\,Bab+4\,Ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(3\,Aa^2+8\,Bab+4\,Ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*
A*a^2*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b*t
an(1/2*d*x + 1/2*c)^7 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^2*tan(1/2*
d*x + 1/2*c)^7 - 24*B*b^2*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^2*tan(1/2*d*x + 1/
2*c)^5 + 40*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 80*A*a*b*tan(1/2*d*x + 1/2*c)^5
- 24*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*B*
b^2*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^2*tan(
1/2*d*x + 1/2*c)^3 - 80*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x
 + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*B*b^2*tan(1/2*d*x + 1/2*
c)^3 + 15*A*a^2*tan(1/2*d*x + 1/2*c) + 24*B*a^2*tan(1/2*d*x + 1/2*c) + 48*A
*a*b*tan(1/2*d*x + 1/2*c) + 24*B*a*b*tan(1/2*d*x + 1/2*c) + 12*A*b^2*tan(1/
2*d*x + 1/2*c) + 24*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^4)/d
```

maple [A] time = 0.13, size = 241, normalized size = 1.54

$$\frac{a^2A\left(\sec^3(dx+c)\right)\tan(dx+c)}{4d}+\frac{3a^2A\sec(dx+c)\tan(dx+c)}{8d}+\frac{3a^2A\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{2B\,a^2\tan}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/4*a^2*A*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a^2*A*sec(d*x+c)*tan(d*x+c)/d+3/8/d
*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*B*a^2*tan(d*x+c)+1/3/d*B*a^2*tan(d*x
+c)*sec(d*x+c)^2+4/3/d*A*a*b*tan(d*x+c)+2/3/d*A*a*b*tan(d*x+c)*sec(d*x+c)^2
+1/d*B*a*b*tan(d*x+c)*sec(d*x+c)+1/d*B*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*
A*b^2*tan(d*x+c)*sec(d*x+c)+1/2/d*A*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^2*B
*tan(d*x+c)
```

maxima [A] time = 0.58, size = 228, normalized size = 1.46

$$16\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ba^2+32\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aab-3\,Aa^2\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 + 32*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a*b - 3*A*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 24*B*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 12*A*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*B*b^2*tan(d*x +
 c))/d
```

mupad [B] time = 3.87, size = 314, normalized size = 2.01

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,A\,a^2}{8}+B\,a\,b+\frac{A\,b^2}{2}\right)}{\frac{3\,A\,a^2}{2}+4\,B\,a\,b+2\,A\,b^2}\right)\left(\frac{3\,A\,a^2}{4}+2\,B\,a\,b+A\,b^2\right)}{d}+\frac{\left(\frac{5\,A\,a^2}{4}+A\,b^2-2\,B\,a^2-2\,B\,b^2-4\,A\,a\,b+2\,B\,a\,b\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((3*A*a^2)/8 + (A*b^2)/2 + B*a*b))/((3*A*a^2)/
2 + 2*A*b^2 + 4*B*a*b))*((3*A*a^2)/4 + A*b^2 + 2*B*a*b))/d + (tan(c/2 + (d*
x)/2)^7*((5*A*a^2)/4 + A*b^2 - 2*B*a^2 - 2*B*b^2 - 4*A*a*b + 2*B*a*b) - tan
(c/2 + (d*x)/2)^3*(A*b^2 - (3*A*a^2)/4 + (10*B*a^2)/3 + 6*B*b^2 + (20*A*a*b
)/3 + 2*B*a*b) + tan(c/2 + (d*x)/2)^5*((3*A*a^2)/4 - A*b^2 + (10*B*a^2)/3 +
 6*B*b^2 + (20*A*a*b)/3 - 2*B*a*b) + tan(c/2 + (d*x)/2)*((5*A*a^2)/4 + A*b^
2 + 2*B*a^2 + 2*B*b^2 + 4*A*a*b + 2*B*a*b))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*
tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.231 $\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=269

$$\frac{b\left(14a^2B+18aAb+5b^2B\right)\sin(c+dx)\cos^3(c+dx)}{24d}-\frac{\left(5a^3B+15a^2Ab+12ab^2B+4Ab^3\right)\sin^3(c+dx)}{15d}+\frac{\left(5a^3B+1\right.}{}$$

```
[Out] 1/16*(8*A*a^3+18*A*a*b^2+18*B*a^2*b+5*B*b^3)*x+1/5*(15*A*a^2*b+4*A*b^3+5*B*
a^3+12*B*a*b^2)*sin(d*x+c)/d+1/16*(8*A*a^3+18*A*a*b^2+18*B*a^2*b+5*B*b^3)*c
os(d*x+c)*sin(d*x+c)/d+1/24*b*(18*A*a*b+14*B*a^2+5*B*b^2)*cos(d*x+c)^3*sin(
d*x+c)/d+1/15*b^2*(3*A*b+4*B*a)*cos(d*x+c)^4*sin(d*x+c)/d+1/6*b*B*cos(d*x+c
)^3*(a+b*cos(d*x+c))^2*sin(d*x+c)/d-1/15*(15*A*a^2*b+4*A*b^3+5*B*a^3+12*B*a
*b^2)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.51, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2990, 3033, 3023, 2748, 2635, 8, 2633}

$$-\frac{\left(15a^2Ab+5a^3B+12ab^2B+4Ab^3\right)\sin^3(c+dx)}{15d}+\frac{\left(15a^2Ab+5a^3B+12ab^2B+4Ab^3\right)\sin(c+dx)}{5d}+\frac{b\left(14a^2B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((8*a^3*A + 18*a*A*b^2 + 18*a^2*b*B + 5*b^3*B)*x)/16 + ((15*a^2*A*b + 4*A*b
^3 + 5*a^3*B + 12*a*b^2*B)*Sin[c + d*x])/(5*d) + ((8*a^3*A + 18*a*A*b^2 + 1
8*a^2*b*B + 5*b^3*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (b*(18*a*A*b + 14*
a^2*B + 5*b^2*B)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*(3*A*b + 4*a*B)
*Cos[c + d*x]^4*Sin[c + d*x])/(15*d) + (b*B*Cos[c + d*x]^3*(a + b*Cos[c + d
*x])^2*Sin[c + d*x])/(6*d) - ((15*a^2*A*b + 4*A*b^3 + 5*a^3*B + 12*a*b^2*B)
*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{bB\cos^3(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{6d}+\frac{1}{6}$$
$$=\frac{b^2(3Ab+4aB)\cos^4(c+dx)\sin(c+dx)}{15d}+\frac{bB\cos^3(c$$
$$=\frac{b\left(18aAb+14a^2B+5b^2B\right)\cos^3(c+dx)\sin(c+dx)}{24d}$$
$$=\frac{b\left(18aAb+14a^2B+5b^2B\right)\cos^3(c+dx)\sin(c+dx)}{24d}$$
$$=\frac{\left(8a^3A+18aAb^2+18a^2bB+5b^3B\right)\cos(c+dx)\sin(}{16d}$$
$$=\frac{1}{16}\left(8a^3A+18aAb^2+18a^2bB+5b^3B\right)x+\frac{\left(15a^2Ab\right.}{}$$

Mathematica [A] time = 0.69, size = 289, normalized size = 1.07

$$\frac{480a^3Ac+480a^3Adx+80a^3B\sin(3(c+dx))+240a^2Ab\sin(3(c+dx))+90a^2bB\sin(4(c+dx))+1080a^2bBc+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (480*a^3*A*c + 1080*a*A*b^2*c + 1080*a^2*b*B*c + 300*b^3*B*c + 480*a^3*A*d*
x + 1080*a*A*b^2*d*x + 1080*a^2*b*B*d*x + 300*b^3*B*d*x + 120*(18*a^2*A*b +
 5*A*b^3 + 6*a^3*B + 15*a*b^2*B)*Sin[c + d*x] + 15*(16*a^3*A + 48*a*A*b^2 +
 48*a^2*b*B + 15*b^3*B)*Sin[2*(c + d*x)] + 240*a^2*A*b*Sin[3*(c + d*x)] + 1
00*A*b^3*Sin[3*(c + d*x)] + 80*a^3*B*Sin[3*(c + d*x)] + 300*a*b^2*B*Sin[3*(
c + d*x)] + 90*a*A*b^2*Sin[4*(c + d*x)] + 90*a^2*b*B*Sin[4*(c + d*x)] + 45*
b^3*B*Sin[4*(c + d*x)] + 12*A*b^3*Sin[5*(c + d*x)] + 36*a*b^2*B*Sin[5*(c +
d*x)] + 5*b^3*B*Sin[6*(c + d*x)])/(960*d)
```

fricas [A] time = 0.96, size = 211, normalized size = 0.78

$$\frac{15\left(8\,Aa^3+18\,Ba^2b+18\,Aab^2+5\,Bb^3\right)dx+\left(40\,Bb^3\cos\left(dx+c\right)^5+48\left(3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)^4+160\,Ba^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 5*B*b^3)*d*x + (40*B*b^3*cos
(d*x + c)^5 + 48*(3*B*a*b^2 + A*b^3)*cos(d*x + c)^4 + 160*B*a^3 + 480*A*a^2
*b + 384*B*a*b^2 + 128*A*b^3 + 10*(18*B*a^2*b + 18*A*a*b^2 + 5*B*b^3)*cos(d
*x + c)^3 + 16*(5*B*a^3 + 15*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x + c)^2
 + 15*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 5*B*b^3)*cos(d*x + c))*sin(d*x +
 c))/d
```

giac [A] time = 0.47, size = 230, normalized size = 0.86

$$\frac{Bb^3\sin(6\,dx+6\,c)}{192\,d}+\frac{1}{16}\left(8\,Aa^3+18\,Ba^2b+18\,Aab^2+5\,Bb^3\right)x+\frac{\left(3\,Bab^2+Ab^3\right)\sin(5\,dx+5\,c)}{80\,d}+\frac{3\left(2\,Ba^2b+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*B*b^3*sin(6*d*x + 6*c)/d + 1/16*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 +
5*B*b^3)*x + 1/80*(3*B*a*b^2 + A*b^3)*sin(5*d*x + 5*c)/d + 3/64*(2*B*a^2*b
+ 2*A*a*b^2 + B*b^3)*sin(4*d*x + 4*c)/d + 1/48*(4*B*a^3 + 12*A*a^2*b + 15*B
*a*b^2 + 5*A*b^3)*sin(3*d*x + 3*c)/d + 1/64*(16*A*a^3 + 48*B*a^2*b + 48*A*a
*b^2 + 15*B*b^3)*sin(2*d*x + 2*c)/d + 1/8*(6*B*a^3 + 18*A*a^2*b + 15*B*a*b^
2 + 5*A*b^3)*sin(d*x + c)/d
```

maple [A] time = 0.10, size = 270, normalized size = 1.00

$$\frac{A\,a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^3B\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+A\,a^2b\left(2+\cos^2(dx+c)\right)\sin(dx+c)+3a^2bB\left(\frac{(\cos\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^3*B*(2+cos(d*x+c
)^2)*sin(d*x+c)+A*a^2*b*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^2*b*B*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*A*a*b^2*(1/4*(cos(d*x+c)^
3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3/5*B*a*b^2*(8/3+cos(d*x+c)^4+4
/3*cos(d*x+c)^2)*sin(d*x+c)+1/5*A*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*s
in(d*x+c)+b^3*B*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*
x+c)+5/16*d*x+5/16*c))
```

maxima [A] time = 0.57, size = 266, normalized size = 0.99

$$\frac{240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^3-320\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^3-960\left(\sin(dx+c)^3-3\,\sin(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 - 320*(sin(d*x + c)^3 - 3
*sin(d*x + c))*B*a^3 - 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2*b + 90*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2*b + 90*(12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b^2 + 192*(3*sin(d*x +
 c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a*b^2 + 64*(3*sin(d*x + c)^5
 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*b^3 - 5*(4*sin(2*d*x + 2*c)^3 - 6
0*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*b^3)/d
```

mupad [B] time = 1.11, size = 352, normalized size = 1.31

$$\frac{A\,a^3\,x}{2}+\frac{5\,B\,b^3\,x}{16}+\frac{9\,A\,a\,b^2\,x}{8}+\frac{9\,B\,a^2\,b\,x}{8}+\frac{5\,A\,b^3\,\sin\left(c+d\,x\right)}{8\,d}+\frac{3\,B\,a^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{A\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{5\,A\,b^3\,}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

```
[Out] (A*a^3*x)/2 + (5*B*b^3*x)/16 + (9*A*a*b^2*x)/8 + (9*B*a^2*b*x)/8 + (5*A*b^3
*sin(c + d*x))/(8*d) + (3*B*a^3*sin(c + d*x))/(4*d) + (A*a^3*sin(2*c + 2*d*
x))/(4*d) + (5*A*b^3*sin(3*c + 3*d*x))/(48*d) + (B*a^3*sin(3*c + 3*d*x))/(1
2*d) + (A*b^3*sin(5*c + 5*d*x))/(80*d) + (15*B*b^3*sin(2*c + 2*d*x))/(64*d)
 + (3*B*b^3*sin(4*c + 4*d*x))/(64*d) + (B*b^3*sin(6*c + 6*d*x))/(192*d) + (
3*A*a*b^2*sin(2*c + 2*d*x))/(4*d) + (A*a^2*b*sin(3*c + 3*d*x))/(4*d) + (3*A
*a*b^2*sin(4*c + 4*d*x))/(32*d) + (3*B*a^2*b*sin(2*c + 2*d*x))/(4*d) + (5*B
*a*b^2*sin(3*c + 3*d*x))/(16*d) + (3*B*a^2*b*sin(4*c + 4*d*x))/(32*d) + (3*
B*a*b^2*sin(5*c + 5*d*x))/(80*d) + (9*A*a^2*b*sin(c + d*x))/(4*d) + (15*B*a
*b^2*sin(c + d*x))/(8*d)
```

sympy [A] time = 4.62, size = 721, normalized size = 2.68

$$\begin{cases} \frac{Aa^3x\sin^2(c+dx)}{2}+\frac{Aa^3x\cos^2(c+dx)}{2}+\frac{Aa^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{2Aa^2b\sin^3(c+dx)}{d}+\frac{3Aa^2b\sin(c+dx)\cos^2(c+dx)}{d}+\frac{9Aab^2x\sin^4(c+dx)}{8} \\ x\left(A+B\cos(c)\right)\left(a+b\cos(c)\right)^3\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*x*sin(c + d*x)**2/2 + A*a**3*x*cos(c + d*x)**2/2 + A*a**3
*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*a**2*b*sin(c + d*x)**3/d + 3*A*a**2*
b*sin(c + d*x)*cos(c + d*x)**2/d + 9*A*a*b**2*x*sin(c + d*x)**4/8 + 9*A*a*b
```

```
**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 9*A*a*b**2*x*cos(c + d*x)**4/8 +
9*A*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 15*A*a*b**2*sin(c + d*x)*co
s(c + d*x)**3/(8*d) + 8*A*b**3*sin(c + d*x)**5/(15*d) + 4*A*b**3*sin(c + d*
x)**3*cos(c + d*x)**2/(3*d) + A*b**3*sin(c + d*x)*cos(c + d*x)**4/d + 2*B*a
**3*sin(c + d*x)**3/(3*d) + B*a**3*sin(c + d*x)*cos(c + d*x)**2/d + 9*B*a**
2*b*x*sin(c + d*x)**4/8 + 9*B*a**2*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
9*B*a**2*b*x*cos(c + d*x)**4/8 + 9*B*a**2*b*sin(c + d*x)**3*cos(c + d*x)/(8
*d) + 15*B*a**2*b*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*B*a*b**2*sin(c + d
*x)**5/(5*d) + 4*B*a*b**2*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*B*a*b**2*si
n(c + d*x)*cos(c + d*x)**4/d + 5*B*b**3*x*sin(c + d*x)**6/16 + 15*B*b**3*x*
sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*B*b**3*x*sin(c + d*x)**2*cos(c + d*
x)**4/16 + 5*B*b**3*x*cos(c + d*x)**6/16 + 5*B*b**3*sin(c + d*x)**5*cos(c +
 d*x)/(16*d) + 5*B*b**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*B*b**3*s
in(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos
(c))**3*cos(c)**2, True))
```

3.232 $\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=243

$$\frac{\left(-3a^2B+15aAb+16b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^2}{60bd}+\frac{\left(-6a^3B+30a^2Ab+71ab^2B+45Ab^3\right)\sin(c+dx)\cos}{120d}$$

```
[Out] 1/8*(12*A*a^2*b+3*A*b^3+4*B*a^3+9*B*a*b^2)*x+1/30*(15*A*a^3*b+60*A*a*b^3-3*
B*a^4+52*B*a^2*b^2+16*B*b^4)*sin(d*x+c)/b/d+1/120*(30*A*a^2*b+45*A*b^3-6*B*
a^3+71*B*a*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/60*(15*A*a*b-3*B*a^2+16*B*b^2)*(a
+b*cos(d*x+c))^2*sin(d*x+c)/b/d+1/20*(5*A*b-B*a)*(a+b*cos(d*x+c))^3*sin(d*x
+c)/b/d+1/5*B*(a+b*cos(d*x+c))^4*sin(d*x+c)/b/d
```

Rubi [A] time = 0.33, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3023, 2753, 2734}

$$\frac{\left(15a^3Ab+52a^2b^2B-3a^4B+60aAb^3+16b^4B\right)\sin(c+dx)}{30bd}+\frac{\left(-3a^2B+15aAb+16b^2B\right)\sin(c+dx)(a+b\cos(c+}{60bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*x)/8 + ((15*a^3*A*b + 60*a*A*
b^3 - 3*a^4*B + 52*a^2*b^2*B + 16*b^4*B)*Sin[c + d*x])/(30*b*d) + ((30*a^2*
A*b + 45*A*b^3 - 6*a^3*B + 71*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(120*d) +
 ((15*a*A*b - 3*a^2*B + 16*b^2*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*
b*d) + ((5*A*b - a*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(20*b*d) + (B*(a
 + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*b*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
```

```
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^3\left(A\cos(c+dx)+B\cos^2(c+\right.$$

$$= \frac{B(a+b\cos(c+dx))^4\sin(c+dx)}{5bd} + \frac{\int (a+b\cos(c+}{}$$

$$= \frac{(5Ab-aB)(a+b\cos(c+dx))^3\sin(c+dx)}{20bd} + \frac{B(a+}{}$$

$$= \frac{\left(15aAb-3a^2B+16b^2B\right)(a+b\cos(c+dx))^2\sin(c}{60bd}$$

$$= \frac{1}{8}\left(12a^2Ab+3Ab^3+4a^3B+9ab^2B\right)x + \frac{\left(15a^3Ab+\right.}{}$$

Mathematica [A] time = 0.72, size = 176, normalized size = 0.72

$$\frac{10b\left(12a^2B+12aAb+5b^2B\right)\sin(3(c+dx))+60(c+dx)\left(4a^3B+12a^2Ab+9ab^2B+3Ab^3\right)+60\left(8a^3A+18a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (60*(12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*(c + d*x) + 60*(8*a^3*A +
18*a*A*b^2 + 18*a^2*b*B + 5*b^3*B)*Sin[c + d*x] + 120*(3*a^2*A*b + A*b^3 +
a^3*B + 3*a*b^2*B)*Sin[2*(c + d*x)] + 10*b*(12*a*A*b + 12*a^2*B + 5*b^2*B)*
Sin[3*(c + d*x)] + 15*b^2*(A*b + 3*a*B)*Sin[4*(c + d*x)] + 6*b^3*B*Sin[5*(c
 + d*x)])/(480*d)
```

fricas [A] time = 0.70, size = 174, normalized size = 0.72

$$\frac{15\left(4\,Ba^3+12\,Aa^2b+9\,Bab^2+3\,Ab^3\right)dx+\left(24\,Bb^3\cos\left(dx+c\right)^4+120\,Aa^3+240\,Ba^2b+240\,Aab^2+64\,Bb^3+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/120*(15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*d*x + (24*B*b^3*cos(
d*x + c)^4 + 120*A*a^3 + 240*B*a^2*b + 240*A*a*b^2 + 64*B*b^3 + 30*(3*B*a*b
^2 + A*b^3)*cos(d*x + c)^3 + 8*(15*B*a^2*b + 15*A*a*b^2 + 4*B*b^3)*cos(d*x
+ c)^2 + 15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*cos(d*x + c))*sin(
d*x + c))/d
```

giac [A] time = 0.51, size = 188, normalized size = 0.77

$$\frac{Bb^3\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{1}{8}\left(4\,Ba^3+12\,Aa^2b+9\,Bab^2+3\,Ab^3\right)x+\frac{\left(3\,Bab^2+Ab^3\right)\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{\left(12\,Ba^2b+12\,Aa\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/80*B*b^3*sin(5*d*x + 5*c)/d + 1/8*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A
*b^3)*x + 1/32*(3*B*a*b^2 + A*b^3)*sin(4*d*x + 4*c)/d + 1/48*(12*B*a^2*b +
12*A*a*b^2 + 5*B*b^3)*sin(3*d*x + 3*c)/d + 1/4*(B*a^3 + 3*A*a^2*b + 3*B*a*b
^2 + A*b^3)*sin(2*d*x + 2*c)/d + 1/8*(8*A*a^3 + 18*B*a^2*b + 18*A*a*b^2 + 5
*B*b^3)*sin(d*x + c)/d
```

maple [A] time = 0.05, size = 227, normalized size = 0.93

$$\frac{A\,a^3\sin\left(dx+c\right)+a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3A\,a^2b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2bB\left(2+\cos^2\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(A*a^3*sin(d*x+c)+a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*A*a
^2*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*b*B*(2+cos(d*x+c)^2)*sin
(d*x+c)+A*b^2*a*(2+cos(d*x+c)^2)*sin(d*x+c)+3*B*a*b^2*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*
x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5*b^3*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c))
```

maxima [A] time = 0.32, size = 217, normalized size = 0.89

$$\frac{120\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba^3+360\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa^2b-480\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3 + 360*(2*d*x + 2*c + sin(
2*d*x + 2*c))*A*a^2*b - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2*b - 480
*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b^2 + 45*(12*d*x + 12*c + sin(4*d*x
+ 4*c) + 8*sin(2*d*x + 2*c))*B*a*b^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c)
 + 8*sin(2*d*x + 2*c))*A*b^3 + 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 1
5*sin(d*x + c))*B*b^3 + 480*A*a^3*sin(d*x + c))/d
```

mupad [B] time = 0.78, size = 277, normalized size = 1.14

$$\frac{3\,A\,b^3\,x}{8}+\frac{B\,a^3\,x}{2}+\frac{3\,A\,a^2\,b\,x}{2}+\frac{9\,B\,a\,b^2\,x}{8}+\frac{A\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{5\,B\,b^3\,\sin\left(c+d\,x\right)}{8\,d}+\frac{A\,b^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a^3\,\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

```
[Out] (3*A*b^3*x)/8 + (B*a^3*x)/2 + (3*A*a^2*b*x)/2 + (9*B*a*b^2*x)/8 + (A*a^3*si
n(c + d*x))/d + (5*B*b^3*sin(c + d*x))/(8*d) + (A*b^3*sin(2*c + 2*d*x))/(4*
d) + (B*a^3*sin(2*c + 2*d*x))/(4*d) + (A*b^3*sin(4*c + 4*d*x))/(32*d) + (5*
B*b^3*sin(3*c + 3*d*x))/(48*d) + (B*b^3*sin(5*c + 5*d*x))/(80*d) + (3*A*a^2
*b*sin(2*c + 2*d*x))/(4*d) + (A*a*b^2*sin(3*c + 3*d*x))/(4*d) + (3*B*a*b^2*
sin(2*c + 2*d*x))/(4*d) + (B*a^2*b*sin(3*c + 3*d*x))/(4*d) + (3*B*a*b^2*sin
(4*c + 4*d*x))/(32*d) + (9*A*a*b^2*sin(c + d*x))/(4*d) + (9*B*a^2*b*sin(c +
 d*x))/(4*d)
```

sympy [A] time = 2.76, size = 551, normalized size = 2.27

$$\begin{cases} \frac{Aa^3\sin(c+dx)}{d} + \frac{3Aa^2bx\sin^2(c+dx)}{2} + \frac{3Aa^2bx\cos^2(c+dx)}{2} + \frac{3Aa^2b\sin(c+dx)\cos(c+dx)}{2d} + \frac{2Aab^2\sin^3(c+dx)}{d} + \frac{3Aab^2\sin(c+dx)\cos^2(c+dx)}{d} \\ x(A+B\cos(c))(a+b\cos(c))^3\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*sin(c + d*x)/d + 3*A*a**2*b*x*sin(c + d*x)**2/2 + 3*A*a**
2*b*x*cos(c + d*x)**2/2 + 3*A*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*A*
a*b**2*sin(c + d*x)**3/d + 3*A*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*
b**3*x*sin(c + d*x)**4/8 + 3*A*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3
*A*b**3*x*cos(c + d*x)**4/8 + 3*A*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) +
 5*A*b**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**3*x*sin(c + d*x)**2/2 +
 B*a**3*x*cos(c + d*x)**2/2 + B*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*
a**2*b*sin(c + d*x)**3/d + 3*B*a**2*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*B*
a*b**2*x*sin(c + d*x)**4/8 + 9*B*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4
 + 9*B*a*b**2*x*cos(c + d*x)**4/8 + 9*B*a*b**2*sin(c + d*x)**3*cos(c + d*x)
/(8*d) + 15*B*a*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*B*b**3*sin(c +
d*x)**5/(15*d) + 4*B*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + B*b**3*si
n(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))**
3*cos(c), True))
```

3.233 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=171

$$\frac{b\left(6a^2B+20aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{\left(3a^3B+16a^2Ab+12ab^2B+4Ab^3\right)\sin(c+dx)}{6d}+\frac{1}{8}x\left(8a^3A\right.$$

```
[Out] 1/8*(8*A*a^3+12*A*a*b^2+12*B*a^2*b+3*B*b^3)*x+1/6*(16*A*a^2*b+4*A*b^3+3*B*a
^3+12*B*a*b^2)*sin(d*x+c)/d+1/24*b*(20*A*a*b+6*B*a^2+9*B*b^2)*cos(d*x+c)*si
n(d*x+c)/d+1/12*(4*A*b+3*B*a)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/4*B*(a+b*co
s(d*x+c))^3*sin(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$\frac{\left(16a^2Ab+3a^3B+12ab^2B+4Ab^3\right)\sin(c+dx)}{6d}+\frac{b\left(6a^2B+20aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(8a^3A\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*x)/8 + ((16*a^2*A*b + 4*A*b^
3 + 3*a^3*B + 12*a*b^2*B)*Sin[c + d*x])/(6*d) + (b*(20*a*A*b + 6*a^2*B + 9*
b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + ((4*A*b + 3*a*B)*(a + b*Cos[c +
d*x])^2*Sin[c + d*x])/(12*d) + (B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d
)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{B(a+b\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{4}\int (a+b\cos(c+dx))^2(4aA$$
$$= \frac{(4Ab+3aB)(a+b\cos(c+dx))^2\sin(c+dx)}{12d} + \frac{B(a+b\cos(c+d}{4d}$$
$$= \frac{1}{8}\left(8a^3A+12aAb^2+12a^2bB+3b^3B\right)x + \frac{\left(16a^2Ab+4Ab^3+3a^3}{6}$$

Mathematica [A] time = 0.42, size = 140, normalized size = 0.82

$$\frac{24b\left(3a^2B+3aAb+b^2B\right)\sin(2(c+dx))+12(c+dx)\left(8a^3A+12a^2bB+12aAb^2+3b^3B\right)+24\left(4a^3B+12a^2Ab+\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (12*(8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*(c + d*x) + 24*(12*a^2*A*
b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*Sin[c + d*x] + 24*b*(3*a*A*b + 3*a^2*B +
 b^2*B)*Sin[2*(c + d*x)] + 8*b^2*(A*b + 3*a*B)*Sin[3*(c + d*x)] + 3*b^3*B*S
in[4*(c + d*x)])/(96*d)
```

fricas [A] time = 1.16, size = 136, normalized size = 0.80

$$\frac{3\left(8\,Aa^3+12\,Ba^2b+12\,Aab^2+3\,Bb^3\right)dx+\left(6\,Bb^3\cos\left(dx+c\right)^3+24\,Ba^3+72\,Aa^2b+48\,Bab^2+16\,Ab^3+8\left(3\,B\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*d*x + (6*B*b^3*cos(d*
x + c)^3 + 24*B*a^3 + 72*A*a^2*b + 48*B*a*b^2 + 16*A*b^3 + 8*(3*B*a*b^2 + A
*b^3)*cos(d*x + c)^2 + 9*(4*B*a^2*b + 4*A*a*b^2 + B*b^3)*cos(d*x + c))*sin(
d*x + c))/d
```

giac [A] time = 0.59, size = 148, normalized size = 0.87

$$\frac{Bb^3\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{1}{8}\left(8\,Aa^3+12\,Ba^2b+12\,Aab^2+3\,Bb^3\right)x+\frac{\left(3\,Bab^2+Ab^3\right)\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{\left(3\,Ba^2b+3\,Aab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*B*b^3*sin(4*d*x + 4*c)/d + 1/8*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*
B*b^3)*x + 1/12*(3*B*a*b^2 + A*b^3)*sin(3*d*x + 3*c)/d + 1/4*(3*B*a^2*b + 3
*A*a*b^2 + B*b^3)*sin(2*d*x + 2*c)/d + 1/4*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^
2 + 3*A*b^3)*sin(d*x + c)/d
```

maple [A] time = 0.06, size = 180, normalized size = 1.05

$$b^3B\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{A\,b^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+B a\,b^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(b^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*A*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+3*A*
a*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*B*(1/2*cos(d*x+c)*s
in(d*x+c)+1/2*d*x+1/2*c)+3*A*a^2*b*sin(d*x+c)+a^3*B*sin(d*x+c)+A*a^3*(d*x+c
))
```

maxima [A] time = 0.65, size = 171, normalized size = 1.00

$$96\,(dx+c)Aa^3+72\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2b+72\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aab^2-96\left(\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(96*(d*x + c)*A*a^3 + 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2*b + 72
*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b^2 - 96*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*B*a*b^2 - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b^3 + 3*(12*d*x + 12
*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b^3 + 96*B*a^3*sin(d*x + c) +
 288*A*a^2*b*sin(d*x + c))/d
```

mupad [B] time = 0.57, size = 202, normalized size = 1.18

$$A\,a^3\,x+\frac{3\,B\,b^3\,x}{8}+\frac{3\,A\,a\,b^2\,x}{2}+\frac{3\,B\,a^2\,b\,x}{2}+\frac{3\,A\,b^3\,\sin(c+d\,x)}{4\,d}+\frac{B\,a^3\,\sin(c+d\,x)}{d}+\frac{A\,b^3\,\sin(3\,c+3\,d\,x)}{12\,d}+\frac{B\,b^3\,\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

```
[Out] A*a^3*x + (3*B*b^3*x)/8 + (3*A*a*b^2*x)/2 + (3*B*a^2*b*x)/2 + (3*A*b^3*sin(
c + d*x))/(4*d) + (B*a^3*sin(c + d*x))/d + (A*b^3*sin(3*c + 3*d*x))/(12*d)
+ (B*b^3*sin(2*c + 2*d*x))/(4*d) + (B*b^3*sin(4*c + 4*d*x))/(32*d) + (3*A*a
*b^2*sin(2*c + 2*d*x))/(4*d) + (3*B*a^2*b*sin(2*c + 2*d*x))/(4*d) + (B*a*b^
2*sin(3*c + 3*d*x))/(4*d) + (3*A*a^2*b*sin(c + d*x))/d + (9*B*a*b^2*sin(c +
 d*x))/(4*d)
```

sympy [A] time = 1.32, size = 386, normalized size = 2.26

$$\begin{cases} Aa^3x + \frac{3Aa^2b\sin(c+dx)}{d} + \frac{3Aab^2x\sin^2(c+dx)}{2} + \frac{3Aab^2x\cos^2(c+dx)}{2} + \frac{3Aab^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{2Ab^3\sin^3(c+dx)}{3d} + \frac{Ab^3\sin(c+dx)}{d} \\ x(A+B\cos(c))(a+b\cos(c))^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*x + 3*A*a**2*b*sin(c + d*x)/d + 3*A*a*b**2*x*sin(c + d*x)
**2/2 + 3*A*a*b**2*x*cos(c + d*x)**2/2 + 3*A*a*b**2*sin(c + d*x)*cos(c + d*
x)/(2*d) + 2*A*b**3*sin(c + d*x)**3/(3*d) + A*b**3*sin(c + d*x)*cos(c + d*x
)**2/d + B*a**3*sin(c + d*x)/d + 3*B*a**2*b*x*sin(c + d*x)**2/2 + 3*B*a**2*
b*x*cos(c + d*x)**2/2 + 3*B*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*B*a*
b**2*sin(c + d*x)**3/d + 3*B*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*b*
*3*x*sin(c + d*x)**4/8 + 3*B*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*B
*b**3*x*cos(c + d*x)**4/8 + 3*B*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5
*B*b**3*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + B*cos(c))*(a
 + b*cos(c))**3, True))
```

3.234 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=137

$$\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\left(8a^2B+9aAb+2b^2B\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(2a^3B+6a^2Ab+3ab^2B+Ab^3\right)+\frac{b^2(5aB+$$

```
[Out] 1/2*(6*A*a^2*b+A*b^3+2*B*a^3+3*B*a*b^2)*x+a^3*A*arctanh(sin(d*x+c))/d+1/3*b
*(9*A*a*b+8*B*a^2+2*B*b^2)*sin(d*x+c)/d+1/6*b^2*(3*A*b+5*B*a)*cos(d*x+c)*si
n(d*x+c)/d+1/3*b*B*(a+b*cos(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A] time = 0.32, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2990, 3033, 3023, 2735, 3770}

$$\frac{b\left(8a^2B+9aAb+2b^2B\right)\sin(c+dx)}{3d}+\frac{1}{2}x\left(6a^2Ab+2a^3B+3ab^2B+Ab^3\right)+\frac{a^3A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2(5aB+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] ((6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*x)/2 + (a^3*A*ArcTanh[Sin[c + d*
x]])/d + (b*(9*a*A*b + 8*a^2*B + 2*b^2*B)*Sin[c + d*x])/(3*d) + (b^2*(3*A*b
 + 5*a*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (b*B*(a + b*Cos[c + d*x])^2*Si
n[c + d*x])/(3*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
```

```
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{bB(a+b\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (a+b\cos($$

$$= \frac{b^2(3Ab+5aB)\cos(c+dx)\sin(c+dx)}{6d}+\frac{bB(a+b\cos$$

$$= \frac{b\left(9aAb+8a^2B+2b^2B\right)\sin(c+dx)}{3d}+\frac{b^2(3Ab+5aB$$

$$= \frac{1}{2}\left(6a^2Ab+Ab^3+2a^3B+3ab^2B\right)x+\frac{b\left(9aAb+8a^2\right.$$

$$= \frac{1}{2}\left(6a^2Ab+Ab^3+2a^3B+3ab^2B\right)x+\frac{a^3A\tanh^{-1}(\sin}{d}$$

Mathematica [A] time = 0.40, size = 159, normalized size = 1.16

$$\frac{-12a^3A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^3A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+9b\left(4a^2B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (6*(6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*(c + d*x) - 12*a^3*A*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^3*A*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 9*b*(4*a*A*b + 4*a^2*B + b^2*B)*Sin[c + d*x] + 3*b^2*(A*b + 3*a*
B)*Sin[2*(c + d*x)] + b^3*B*Sin[3*(c + d*x)])/(12*d)
```

fricas [A] time = 0.54, size = 131, normalized size = 0.96

$$\frac{3\,Aa^3\log\left(\sin\left(dx+c\right)+1\right)-3\,Aa^3\log\left(-\sin\left(dx+c\right)+1\right)+3\left(2\,Ba^3+6\,Aa^2b+3\,Bab^2+Ab^3\right)dx+\left(2\,Bb^3\,\mathrm{co}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*A*a^3*log(sin(d*x + c) + 1) - 3*A*a^3*log(-sin(d*x + c) + 1) + 3*(2*
B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*d*x + (2*B*b^3*cos(d*x + c)^2 + 18*B
*a^2*b + 18*A*a*b^2 + 4*B*b^3 + 3*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x
 + c))/d
```

giac [B] time = 0.65, size = 314, normalized size = 2.29

$$\frac{6\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(2\,Ba^3+6\,Aa^2b+3\,Bab^2+Ab^3\right)(dx\,\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/6*(6*A*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*A*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*(d*x + c) +
 2*(18*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 -
 9*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^
```

```
3*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*A*a*b^2*t
an(1/2*d*x + 1/2*c)^3 + 4*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 18*B*a^2*b*tan(1/2
*d*x + 1/2*c) + 18*A*a*b^2*tan(1/2*d*x + 1/2*c) + 9*B*a*b^2*tan(1/2*d*x + 1
/2*c) + 3*A*b^3*tan(1/2*d*x + 1/2*c) + 6*B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.11, size = 207, normalized size = 1.51

$$\frac{A\,a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+a^3Bx+\frac{a^3Bc}{d}+3A\,a^2bx+\frac{3A\,a^2bc}{d}+\frac{3a^2bB\sin(dx+c)}{d}+\frac{3A\,b^2a\sin(dx+c)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*B*x+1/d*a^3*B*c+3*A*a^2*b*x+3/d*A*a
^2*b*c+3/d*a^2*b*B*sin(d*x+c)+3/d*A*b^2*a*sin(d*x+c)+3/2/d*B*a*b^2*cos(d*x+
c)*sin(d*x+c)+3/2*B*a*b^2*x+3/2/d*B*a*b^2*c+1/2/d*A*b^3*cos(d*x+c)*sin(d*x+
c)+1/2*A*b^3*x+1/2/d*A*b^3*c+1/3/d*B*sin(d*x+c)*cos(d*x+c)^2*b^3+2/3/d*b^3*
B*sin(d*x+c)
```

maxima [A] time = 0.32, size = 145, normalized size = 1.06

$$\frac{12\,(dx+c)Ba^3+36\,(dx+c)Aa^2b+9\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Bab^2+3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ab^3-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(12*(d*x + c)*B*a^3 + 36*(d*x + c)*A*a^2*b + 9*(2*d*x + 2*c + sin(2*d*
x + 2*c))*B*a*b^2 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^3 - 4*(sin(d*x +
 c)^3 - 3*sin(d*x + c))*B*b^3 + 12*A*a^3*log(sec(d*x + c) + tan(d*x + c)) +
 36*B*a^2*b*sin(d*x + c) + 36*A*a*b^2*sin(d*x + c))/d
```

mupad [B] time = 1.91, size = 1924, normalized size = 14.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(A*b^3 + 2*B*b^3 + 6*A*a*b^2 + 3*B*a*b^2 + 6*B*a^2*b) +
 tan(c/2 + (d*x)/2)^3*((4*B*b^3)/3 + 12*A*a*b^2 + 12*B*a^2*b) + tan(c/2 + (
d*x)/2)^5*(2*B*b^3 - A*b^3 + 6*A*a*b^2 - 3*B*a*b^2 + 6*B*a^2*b))/(d*(3*tan(
```

```
c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (a
tan(((((A*b^3*1i)/2 + B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*(32*A*a^3 + 1
6*A*b^3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a*b^2) + tan(c/2 + (d*x)/2)*(32*A^2*
a^6 + 8*A^2*b^6 + 32*B^2*a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2 + 72*B^2*a^
2*b^4 + 96*B^2*a^4*b^2 + 48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B*a^3*b^3))*(
(A*b^3*1i)/2 + B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*1i - (((A*b^3*1i)/2
+ B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*(32*A*a^3 + 16*A*b^3 + 32*B*a^3 +
 96*A*a^2*b + 48*B*a*b^2) - tan(c/2 + (d*x)/2)*(32*A^2*a^6 + 8*A^2*b^6 + 32
*B^2*a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2 + 72*B^2*a^2*b^4 + 96*B^2*a^4*b
^2 + 48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B*a^3*b^3))*((A*b^3*1i)/2 + B*a^3
*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*1i)/((((A*b^3*1i)/2 + B*a^3*1i + A*a^2*b
*3i + (B*a*b^2*3i)/2)*(32*A*a^3 + 16*A*b^3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a
*b^2) + tan(c/2 + (d*x)/2)*(32*A^2*a^6 + 8*A^2*b^6 + 32*B^2*a^6 + 96*A^2*a^
2*b^4 + 288*A^2*a^4*b^2 + 72*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + 48*A*B*a*b^5 +
192*A*B*a^5*b + 320*A*B*a^3*b^3))*((A*b^3*1i)/2 + B*a^3*1i + A*a^2*b*3i + (
B*a*b^2*3i)/2) + (((A*b^3*1i)/2 + B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2)*(
32*A*a^3 + 16*A*b^3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a*b^2) - tan(c/2 + (d*x)
/2)*(32*A^2*a^6 + 8*A^2*b^6 + 32*B^2*a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2
 + 72*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + 48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B
*a^3*b^3))*((A*b^3*1i)/2 + B*a^3*1i + A*a^2*b*3i + (B*a*b^2*3i)/2) + 64*A*B
^2*a^9 - 64*A^2*B*a^9 - 192*A^3*a^8*b + 16*A^3*a^3*b^6 + 192*A^3*a^5*b^4 -
32*A^3*a^6*b^3 + 576*A^3*a^7*b^2 + 384*A^2*B*a^8*b + 144*A*B^2*a^5*b^4 + 19
2*A*B^2*a^7*b^2 + 96*A^2*B*a^4*b^5 + 640*A^2*B*a^6*b^3 - 96*A^2*B*a^7*b^2))
*(A*b^3 + 2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2))/d - (A*a^3*atan((A*a^3*(tan(c/2
 + (d*x)/2)*(32*A^2*a^6 + 8*A^2*b^6 + 32*B^2*a^6 + 96*A^2*a^2*b^4 + 288*A^2
*a^4*b^2 + 72*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + 48*A*B*a*b^5 + 192*A*B*a^5*b +
 320*A*B*a^3*b^3) + A*a^3*(32*A*a^3 + 16*A*b^3 + 32*B*a^3 + 96*A*a^2*b + 48
*B*a*b^2))*1i + A*a^3*(tan(c/2 + (d*x)/2)*(32*A^2*a^6 + 8*A^2*b^6 + 32*B^2*
a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2 + 72*B^2*a^2*b^4 + 96*B^2*a^4*b^2 +
48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B*a^3*b^3) - A*a^3*(32*A*a^3 + 16*A*b^
3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a*b^2))*1i)/(64*A*B^2*a^9 - 64*A^2*B*a^9 -
 192*A^3*a^8*b + A*a^3*(tan(c/2 + (d*x)/2)*(32*A^2*a^6 + 8*A^2*b^6 + 32*B^2
*a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2 + 72*B^2*a^2*b^4 + 96*B^2*a^4*b^2 +
 48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B*a^3*b^3) + A*a^3*(32*A*a^3 + 16*A*b
^3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a*b^2)) - A*a^3*(tan(c/2 + (d*x)/2)*(32*A
^2*a^6 + 8*A^2*b^6 + 32*B^2*a^6 + 96*A^2*a^2*b^4 + 288*A^2*a^4*b^2 + 72*B^2
*a^2*b^4 + 96*B^2*a^4*b^2 + 48*A*B*a*b^5 + 192*A*B*a^5*b + 320*A*B*a^3*b^3)
 - A*a^3*(32*A*a^3 + 16*A*b^3 + 32*B*a^3 + 96*A*a^2*b + 48*B*a*b^2)) + 16*A
^3*a^3*b^6 + 192*A^3*a^5*b^4 - 32*A^3*a^6*b^3 + 576*A^3*a^7*b^2 + 384*A^2*B
*a^8*b + 144*A*B^2*a^5*b^4 + 192*A*B^2*a^7*b^2 + 96*A^2*B*a^4*b^5 + 640*A^2
*B*a^6*b^3 - 96*A^2*B*a^7*b^2))*2i)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))(a + b\cos(c + dx))^3 \sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3*sec(c + d*x), x)
```

3.235 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=131

$$-\frac{b(2a^2A-3abB-Ab^2)\sin(c+dx)}{d}+\frac{1}{2}bx(6a^2B+6aAb+b^2B)+\frac{a^2(aB+3Ab)\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2(2aA-$$

```
[Out] 1/2*b*(6*A*a*b+6*B*a^2+B*b^2)*x+a^2*(3*A*b+B*a)*arctanh(sin(d*x+c))/d-b*(2*
A*a^2-A*b^2-3*B*a*b)*sin(d*x+c)/d-1/2*b^2*(2*A*a-B*b)*cos(d*x+c)*sin(d*x+c)
/d+a*A*(a+b*cos(d*x+c))^2*tan(d*x+c)/d
```

Rubi [A] time = 0.33, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2989, 3033, 3023, 2735, 3770}

$$-\frac{b(2a^2A-3abB-Ab^2)\sin(c+dx)}{d}+\frac{1}{2}bx(6a^2B+6aAb+b^2B)+\frac{a^2(aB+3Ab)\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2(2aA-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (b*(6*a*A*b + 6*a^2*B + b^2*B)*x)/2 + (a^2*(3*A*b + a*B)*ArcTanh[Sin[c + d*
x]])/d - (b*(2*a^2*A - A*b^2 - 3*a*b*B)*Sin[c + d*x])/d - (b^2*(2*a*A - b*B
)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a*A*(a + b*Cos[c + d*x])^2*Tan[c + d*
x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^2\tan(c+dx)}{d} + \int (a+b\cos(c$$

$$= -\frac{b^2(2aA-bB)\cos(c+dx)\sin(c+dx)}{2d} + \frac{aA(a+b\,c}{}$$

$$= -\frac{b\left(2a^2A-Ab^2-3abB\right)\sin(c+dx)}{d} - \frac{b^2(2aA-bB}{}$$

$$= \frac{1}{2}b\left(6aAb+6a^2B+b^2B\right)x - \frac{b\left(2a^2A-Ab^2-3abB\right.}{d}$$

$$= \frac{1}{2}b\left(6aAb+6a^2B+b^2B\right)x + \frac{a^2(3Ab+aB)\tanh^{-1}(\mathrm{s}}{d}$$

Mathematica [A] time = 0.68, size = 217, normalized size = 1.66

$$\frac{\frac{4a^3A\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{4a^3A\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}+2b(c+dx)\left(6a^2B+6aAb+b^2B\right)-4a^2(aB+3Ab)\log\left(\cos\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (2*b*(6*a*A*b + 6*a^2*B + b^2*B)*(c + d*x) - 4*a^2*(3*A*b + a*B)*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] + 4*a^2*(3*A*b + a*B)*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + (4*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]) + (4*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]) + 4*b^2*(A*b + 3*a*B)*Sin[c + d*x] + b^3*B*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.97, size = 152, normalized size = 1.16

$$\frac{\left(6\,Ba^2b+6\,Aab^2+Bb^3\right)dx\cos\left(dx+c\right)+\left(Ba^3+3\,Aa^2b\right)\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(Ba^3+3\,Aa^2b\right)\cos}{2\,d\cos\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*((6*B*a^2*b + 6*A*a*b^2 + B*b^3)*d*x*cos(d*x + c) + (B*a^3 + 3*A*a^2*b)
*cos(d*x + c)*log(sin(d*x + c) + 1) - (B*a^3 + 3*A*a^2*b)*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (B*b^3*cos(d*x + c)^2 + 2*A*a^3 + 2*(3*B*a*b^2 + A*b^3
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 1.29, size = 234, normalized size = 1.79

$$-\frac{\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\left(6\,Ba^2b+6\,Aab^2+Bb^3\right)(dx+c)-2\left(Ba^3+3\,Aa^2b\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+2\left(Ba^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (6*B*a^2*
b + 6*A*a*b^2 + B*b^3)*(d*x + c) - 2*(B*a^3 + 3*A*a^2*b)*log(abs(tan(1/2*d*
```

```
x + 1/2*c) + 1)) + 2*(B*a^3 + 3*A*a^2*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 - 2*(6*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2*c)^3 - B
*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*B*a*b^2*tan(1/2*d*x + 1/2*c) + 2*A*b^3*tan(
1/2*d*x + 1/2*c) + B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^2)/d
```

maple [A] time = 0.13, size = 168, normalized size = 1.28

$$\frac{A\,a^3\tan(dx+c)}{d}+\frac{a^3B\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{3A\,a^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}+3a^2bBx+\frac{3B\,a^2bc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] 1/d*A*a^3*tan(d*x+c)+1/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3/d*A*a^2*b*ln(sec
(d*x+c)+tan(d*x+c))+3*a^2*b*B*x+3/d*B*a^2*b*c+3*A*b^2*a*x+3/d*A*a*b^2*c+3/d
*B*b^2*a*sin(d*x+c)+1/d*A*b^3*sin(d*x+c)+1/2/d*b^3*B*cos(d*x+c)*sin(d*x+c)+
1/2*b^3*B*x+1/2/d*b^3*B*c
```

maxima [A] time = 0.32, size = 144, normalized size = 1.10

$$\frac{12\,(dx+c)Ba^2b+12\,(dx+c)Aab^2+(2\,dx+2\,c+\sin(2\,dx+2\,c))Bb^3+2\,Ba^3\left(\log(\sin(dx+c)+1)-\log(\sin(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/4*(12*(d*x + c)*B*a^2*b + 12*(d*x + c)*A*a*b^2 + (2*d*x + 2*c + sin(2*d*x
 + 2*c))*B*b^3 + 2*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) +
6*A*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*B*a*b^2*sin(
d*x + c) + 4*A*b^3*sin(d*x + c) + 4*A*a^3*tan(d*x + c))/d
```

mupad [B] time = 1.35, size = 236, normalized size = 1.80

$$\frac{B\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+6\,A\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+6\,B\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)2\mathrm{i}-A\,a^2\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] (B*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - B*a^3*atan((sin(c/2 +
(d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i + 6*A*a*b^2*atan(sin(c/2 + (d*x)/2)/cos
```

```
(c/2 + (d*x)/2)) - A*a^2*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))
*6i + 6*B*a^2*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + ((A*b^3*si
n(2*c + 2*d*x))/2 + (B*b^3*sin(3*c + 3*d*x))/8 + A*a^3*sin(c + d*x) + (B*b^
3*sin(c + d*x))/8 + (3*B*a*b^2*sin(2*c + 2*d*x))/2)/(d*cos(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))(a + b\cos(c + dx))^3 \sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3*sec(c + d*x)**2, x)
```

3.236 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=124

$$\frac{a\left(a^2A+6abB+6Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(aB+2Ab)\tan(c+dx)}{d}-\frac{b^2(aA-2bB)\sin(c+dx)}{2d}+b^2x(3aB+A$$

```
[Out] b^2*(A*b+3*B*a)*x+1/2*a*(A*a^2+6*A*b^2+6*B*a*b)*arctanh(sin(d*x+c))/d-1/2*b
^2*(A*a-2*B*b)*sin(d*x+c)/d+a^2*(2*A*b+B*a)*tan(d*x+c)/d+1/2*a*A*(a+b*cos(d
*x+c))^2*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.34, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2989, 3031, 3023, 2735, 3770}

$$\frac{a\left(a^2A+6abB+6Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(aB+2Ab)\tan(c+dx)}{d}-\frac{b^2(aA-2bB)\sin(c+dx)}{2d}+b^2x(3aB+A$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] b^2*(A*b + 3*a*B)*x + (a*(a^2*A + 6*A*b^2 + 6*a*b*B)*ArcTanh[Sin[c + d*x]])
/(2*d) - (b^2*(a*A - 2*b*B)*Sin[c + d*x])/(2*d) + (a^2*(2*A*b + a*B)*Tan[c
+ d*x])/d + (a*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}$$

$$= \frac{a^2(2Ab+aB)\tan(c+dx)}{d} + \frac{aA(a+b\cos(c+dx))}{}$$

$$= -\frac{b^2(aA-2bB)\sin(c+dx)}{2d} + \frac{a^2(2Ab+aB)\tan(c}{d}$$

$$= b^2(Ab+3aB)x - \frac{b^2(aA-2bB)\sin(c+dx)}{2d} + \frac{a^2(2}{}$$

$$= b^2(Ab+3aB)x + \frac{a\left(a^2A+6Ab^2+6abB\right)\tanh^{-1}(\mathrm{s}}{2d}$$

Mathematica [B] time = 2.10, size = 277, normalized size = 2.23

$$\frac{\frac{a^3A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{a^3A}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}-2a\left(a^2A+6abB+6Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (4*b^2*(A*b + 3*a*B)*(c + d*x) - 2*a*(a^2*A + 6*A*b^2 + 6*a*b*B)*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] + 2*a*(a^2*A + 6*A*b^2 + 6*a*b*B)*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])^2 + (4*a^2*(3*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]) - (a^3*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a^2*(3*A*b
 + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 4*b^3*B*S
in[c + d*x])/(4*d)
```

fricas [A] time = 0.90, size = 167, normalized size = 1.35

$$\frac{4\left(3\,Bab^2+Ab^3\right)dx\cos\left(dx+c\right)^2+\left(Aa^3+6\,Ba^2b+6\,Aab^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(Aa^3+6\,Ba^2b\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(4*(3*B*a*b^2 + A*b^3)*d*x*cos(d*x + c)^2 + (A*a^3 + 6*B*a^2*b + 6*A*a*
b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a^3 + 6*B*a^2*b + 6*A*a*b^2)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*B*b^3*cos(d*x + c)^2 + A*a^3
+ 2*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.57, size = 239, normalized size = 1.93

$$\frac{\frac{4\,Bb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+2\left(3\,Bab^2+Ab^3\right)(dx+c)+\left(Aa^3+6\,Ba^2b+6\,Aab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(Aa^3+6\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*B*b^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(3*B*a*b
^2 + A*b^3)*(d*x + c) + (A*a^3 + 6*B*a^2*b + 6*A*a*b^2)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - (A*a^3 + 6*B*a^2*b + 6*A*a*b^2)*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^3*tan(1/2*d*x + 1/2*c)^
3 - 6*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + A*a^3*tan(1/2*d*x + 1/2*c) + 2*B*a^3
*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^2)/d
```

maple [A] time = 0.13, size = 172, normalized size = 1.39

$$\frac{A a^3 \sec(dx+c)\tan(dx+c)}{2d}+\frac{A a^3 \ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{a^3 B\tan(dx+c)}{d}+\frac{3A a^2 b\tan(dx+c)}{d}+\frac{3a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2/d*A*a^3*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d
*a^3*B*tan(d*x+c)+3/d*A*a^2*b*tan(d*x+c)+3/d*a^2*b*B*ln(sec(d*x+c)+tan(d*x+
c))+3/d*A*b^2*a*ln(sec(d*x+c)+tan(d*x+c))+3*B*a*b^2*x+3/d*B*a*b^2*c+A*b^3*x
+1/d*A*b^3*c+1/d*b^3*B*sin(d*x+c)
```

maxima [A] time = 1.54, size = 169, normalized size = 1.36

$$\frac{12\,(dx+c)Bab^2+4\,(dx+c)Ab^3-Aa^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6\,Ba^2b\left(\mathrm{lo}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(12*(d*x + c)*B*a*b^2 + 4*(d*x + c)*A*b^3 - A*a^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*B*a^2*
b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*A*a*b^2*(log(sin(d*x
+ c) + 1) - log(sin(d*x + c) - 1)) + 4*B*b^3*sin(d*x + c) + 4*B*a^3*tan(d*x
 + c) + 12*A*a^2*b*tan(d*x + c))/d
```

mupad [B] time = 1.56, size = 249, normalized size = 2.01

$$\frac{\frac{B a^3\sin(2c+2dx)}{2}+\frac{B b^3\sin(3c+3dx)}{4}+\frac{A a^3\sin(c+dx)}{2}+\frac{B b^3\sin(c+dx)}{4}+\frac{3A a^2 b\sin(2c+2dx)}{2}}{d\left(\frac{\cos(2c+2dx)}{2}+\frac{1}{2}\right)}-\frac{2\left(\frac{A a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)1i}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)1i}{2}-A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^3,x)
```

```
[Out] ((B*a^3*sin(2*c + 2*d*x))/2 + (B*b^3*sin(3*c + 3*d*x))/4 + (A*a^3*sin(c + d
*x))/2 + (B*b^3*sin(c + d*x))/4 + (3*A*a^2*b*sin(2*c + 2*d*x))/2)/(d*(cos(2
*c + 2*d*x)/2 + 1/2)) - (2*((A*a^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (
d*x)/2))*1i)/2 - A*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + A*a*b^
2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i - 3*B*a*b^2*atan(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + B*a^2*b*atan((sin(c/2 + (d*x)/2)*1i)/c
os(c/2 + (d*x)/2))*3i))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.237 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=145

$$\frac{a\left(2a^2A+9abB+8Ab^2\right)\tan(c+dx)}{3d}+\frac{a^2(3aB+5Ab)\tan(c+dx)\sec(c+dx)}{6d}+\frac{\left(a^3B+3a^2Ab+6ab^2B+2Ab^3\right)}{2d}$$

```
[Out] b^3*B*x+1/2*(3*A*a^2*b+2*A*b^3+B*a^3+6*B*a*b^2)*arctanh(sin(d*x+c))/d+1/3*a
*(2*A*a^2+8*A*b^2+9*B*a*b)*tan(d*x+c)/d+1/6*a^2*(5*A*b+3*B*a)*sec(d*x+c)*ta
n(d*x+c)/d+1/3*a*A*(a+b*cos(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.35, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2989, 3031, 3021, 2735, 3770}

$$\frac{a\left(2a^2A+9abB+8Ab^2\right)\tan(c+dx)}{3d}+\frac{\left(3a^2Ab+a^3B+6ab^2B+2Ab^3\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(3aB+5Ab)\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] b^3*B*x + ((3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*ArcTanh[Sin[c + d*x]])
/(2*d) + (a*(2*a^2*A + 8*A*b^2 + 9*a*b*B)*Tan[c + d*x])/(3*d) + (a^2*(5*A*b
 + 3*a*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (a*A*(a + b*Cos[c + d*x])^2*Se
c[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d} +$$

$$= \frac{a^2(5Ab+3aB)\sec(c+dx)\tan(c+dx)}{6d} + \frac{aA(a+b}{}$$

$$= \frac{a\left(2a^2A+8Ab^2+9abB\right)\tan(c+dx)}{3d} + \frac{a^2(5Ab+3}{}$$

$$= b^3Bx + \frac{a\left(2a^2A+8Ab^2+9abB\right)\tan(c+dx)}{3d} + \frac{a^2(}{}$$

$$= b^3Bx + \frac{\left(3a^2Ab+2Ab^3+a^3B+6ab^2B\right)\tanh^{-1}(\sin}{2d}$$

Mathematica [A] time = 0.59, size = 108, normalized size = 0.74

$$\frac{2a^3A\tan^3(c+dx)+3a\tan(c+dx)\left(2a^2A+a(aB+3Ab)\sec(c+dx)+6abB+6Ab^2\right)+3\left(a^3B+3a^2Ab+6ab\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (6*b^3*B*d*x + 3*(3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*ArcTanh[Sin[c +
d*x]] + 3*a*(2*a^2*A + 6*A*b^2 + 6*a*b*B + a*(3*A*b + a*B)*Sec[c + d*x])*Ta
n[c + d*x] + 2*a^3*A*Tan[c + d*x]^3)/(6*d)
```

fricas [A] time = 0.52, size = 189, normalized size = 1.30

$$\frac{12\,Bb^3dx\cos(dx+c)^3+3\left(Ba^3+3\,Aa^2b+6\,Bab^2+2\,Ab^3\right)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\left(Ba^3+3\,Aa\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(12*B*b^3*d*x*cos(d*x + c)^3 + 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*
b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^
2 + 2*A*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*A*a^3 + 2*(2*A*a^
3 + 9*B*a^2*b + 9*A*a*b^2)*cos(d*x + c)^2 + 3*(B*a^3 + 3*A*a^2*b)*cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

giac [B] time = 0.44, size = 336, normalized size = 2.32

$$6\,(dx+c)Bb^3+3\left(Ba^3+3\,Aa^2b+6\,Bab^2+2\,Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(Ba^3+3\,Aa^2b+6\,Bab^2+2\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(d*x + c)*B*b^3 + 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*log(ab
s(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*
B*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*B*a^2*
b*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^3*tan(
1/2*d*x + 1/2*c)^3 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*tan(1/2
*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c) + 3*B*a^3*tan(1/2*d*x + 1/2*
c) + 9*A*a^2*b*tan(1/2*d*x + 1/2*c) + 18*B*a^2*b*tan(1/2*d*x + 1/2*c) + 18*
A*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

maple [A] time = 0.14, size = 223, normalized size = 1.54

$$\frac{2A\,a^3\tan(dx+c)}{3d}+\frac{A\,a^3\tan(dx+c)\left(\sec^2(dx+c)\right)}{3d}+\frac{a^3B\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^3B\ln(\sec(dx+c)+\tan(d}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 2/3/d*A*a^3*tan(d*x+c)+1/3/d*A*a^3*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^3*B*sec(
d*x+c)*tan(d*x+c)+1/2/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*A*a^2*b*sec(d
*x+c)*tan(d*x+c)+3/2/d*A*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^2*b*B*tan(d*
x+c)+3/d*A*b^2*a*tan(d*x+c)+3/d*B*b^2*a*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*b^3
*ln(sec(d*x+c)+tan(d*x+c))+b^3*B*x+1/d*b^3*B*c
```

maxima [A] time = 0.65, size = 216, normalized size = 1.49

$$4\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^3+12\,(dx+c)Bb^3-3\,Ba^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 12*(d*x + c)*B*b^3 - 3*B*
a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 9*A*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 18*B*a*b^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 6*A*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 36*B*a^2*b*tan(d*x + c) + 36*A*a*b^2*tan(d*x + c))/d
```

mupad [B] time = 1.95, size = 526, normalized size = 3.63

$$\frac{A a^3 \sin(3c+3dx)}{6} + \frac{B a^3 \sin(2c+2dx)}{4} + \frac{A a^3 \sin(c+dx)}{2} + \frac{3 A a b^2 \sin(c+dx)}{4} + \frac{3 B a^2 b \sin(c+dx)}{4} - \frac{A b^3 \cos(c+dx)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right) 1i}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right) 3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] ((A*a^3*sin(3*c + 3*d*x))/6 + (B*a^3*sin(2*c + 2*d*x))/4 + (A*a^3*sin(c + d
*x))/2 + (3*A*a*b^2*sin(c + d*x))/4 + (3*B*a^2*b*sin(c + d*x))/4 - (A*b^3*c
os(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/2 - (B*a^3
*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/4 + (3*B
*b^3*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (3*A*a^2
*b*sin(2*c + 2*d*x))/4 + (3*A*a*b^2*sin(3*c + 3*d*x))/4 + (3*B*a^2*b*sin(3*
c + 3*d*x))/4 - (A*b^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos
(3*c + 3*d*x)*1i)/2 - (B*a^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2
))*cos(3*c + 3*d*x)*1i)/4 + (B*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2))*cos(3*c + 3*d*x))/2 - (A*a^2*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (
d*x)/2))*cos(3*c + 3*d*x)*3i)/4 - (B*a*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos
(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*3i)/2 - (A*a^2*b*cos(c + d*x)*atan((sin(c
/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*9i)/4 - (B*a*b^2*cos(c + d*x)*atan((s
in(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*9i)/2)/(d*((3*cos(c + d*x))/4 + c
os(3*c + 3*d*x)/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.238 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=188

$$\frac{a\left(3a^2A+12abB+10Ab^2\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{a^2(2aB+3Ab)\tan(c+dx)\sec^2(c+dx)}{6d}+\frac{\left(2a^3B+6a^2Ab+\right.}{}$$

[Out] 1/8*(3*A*a^3+12*A*a*b^2+12*B*a^2*b+8*B*b^3)*arctanh(sin(d*x+c))/d+1/3*(6*A*a^2*b+3*A*b^3+2*B*a^3+9*B*a*b^2)*tan(d*x+c)/d+1/8*a*(3*A*a^2+10*A*b^2+12*B*a*b)*sec(d*x+c)*tan(d*x+c)/d+1/6*a^2*(3*A*b+2*B*a)*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a*A*(a+b*cos(d*x+c))^2*sec(d*x+c)^3*tan(d*x+c)/d

Rubi [A] time = 0.46, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2989, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{\left(6a^2Ab+2a^3B+9ab^2B+3Ab^3\right)\tan(c+dx)}{3d}+\frac{\left(3a^3A+12a^2bB+12aAb^2+8b^3B\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(3a^2A\right.}{}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]

[Out] ((3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((6*a^2*A*b + 3*A*b^3 + 2*a^3*B + 9*a*b^2*B)*Tan[c + d*x])/(3*d) + (a*(3*a^2*A + 10*A*b^2 + 12*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(3*A*b + 2*a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a*A*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
```

```
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}$$
$$= \frac{a^2(3Ab+2aB)\sec^2(c+dx)\tan(c+dx)}{6d} + \frac{aA(a+b}{}$$
$$= \frac{a\left(3a^2A+10Ab^2+12abB\right)\sec(c+dx)\tan(c+dx)}{8d}$$
$$= \frac{a\left(3a^2A+10Ab^2+12abB\right)\sec(c+dx)\tan(c+dx)}{8d}$$
$$= \frac{\left(3a^3A+12aAb^2+12a^2bB+8b^3B\right)\tanh^{-1}(\sin(c+}{8d}$$
$$= \frac{\left(3a^3A+12aAb^2+12a^2bB+8b^3B\right)\tanh^{-1}(\sin(c+}{8d}$$

Mathematica [A] time = 0.84, size = 140, normalized size = 0.74

$$\frac{3\left(3a^3A+12a^2bB+12aAb^2+8b^3B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(6a^3A\sec^3(c+dx)+9a\left(a^2A+4abB+\right.\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(24*(3*a^2*A*b + A*b^3 + a^3*B + 3*a*b^2*B) + 9*a*(a^2*A + 4*A*b
^2 + 4*a*b*B)*Sec[c + d*x] + 6*a^3*A*Sec[c + d*x]^3 + 8*a^2*(3*A*b + a*B)*T
an[c + d*x]^2))/(24*d)
```

fricas [A] time = 0.90, size = 211, normalized size = 1.12

$$\frac{3\left(3\,Aa^3+12\,Ba^2b+12\,Aab^2+8\,Bb^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,Aa^3+12\,Ba^2b+12\,Aab^2+8\,Bb\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*cos(d*x + c)^4*log(si
n(d*x + c) + 1) - 3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1) + 2*(6*A*a^3 + 8*(2*B*a^3 + 6*A*a^2*b + 9*B*a*
b^2 + 3*A*b^3)*cos(d*x + c)^3 + 9*(A*a^3 + 4*B*a^2*b + 4*A*a*b^2)*cos(d*x +
```

```
 c)^2 + 8*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
)
```

giac [B] time = 0.69, size = 586, normalized size = 3.12

$$3\left(3\,Aa^{3}+12\,Ba^{2}b+12\,Aab^{2}+8\,Bb^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(3\,Aa^{3}+12\,Ba^{2}b+12\,Aab^{2}+8\,Bb^{3}\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) - 3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^3*tan
(1/2*d*x + 1/2*c)^7 - 72*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 36*B*a^2*b*tan(1/
2*d*x + 1/2*c)^7 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*B*a*b^2*tan(1/2*d
*x + 1/2*c)^7 - 24*A*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^3*tan(1/2*d*x + 1/2
*c)^5 + 40*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 120*A*a^2*b*tan(1/2*d*x + 1/2*c)^
5 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 +
 216*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A
*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^2*b
*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 216*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 72*A*b^3*tan(1/
2*d*x + 1/2*c)^3 + 15*A*a^3*tan(1/2*d*x + 1/2*c) + 24*B*a^3*tan(1/2*d*x + 1
/2*c) + 72*A*a^2*b*tan(1/2*d*x + 1/2*c) + 36*B*a^2*b*tan(1/2*d*x + 1/2*c) +
 36*A*a*b^2*tan(1/2*d*x + 1/2*c) + 72*B*a*b^2*tan(1/2*d*x + 1/2*c) + 24*A*b
^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.14, size = 290, normalized size = 1.54

$$\frac{A\,a^{3}\tan\left(dx+c\right)\left(\sec^{3}\left(dx+c\right)\right)}{4d}+\frac{3A\,a^{3}\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d}+\frac{3A\,a^{3}\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d}+\frac{2a^{3}B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/4/d*A*a^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*a^3*sec(d*x+c)*tan(d*x+c)+3/8/d
*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a^3*B*tan(d*x+c)+1/3/d*a^3*B*tan(d*x
+c)*sec(d*x+c)^2+2/d*A*a^2*b*tan(d*x+c)+1/d*A*a^2*b*tan(d*x+c)*sec(d*x+c)^2
+3/2/d*a^2*b*B*tan(d*x+c)*sec(d*x+c)+3/2/d*a^2*b*B*ln(sec(d*x+c)+tan(d*x+c)
)+3/2/d*A*a*b^2*tan(d*x+c)*sec(d*x+c)+3/2/d*A*b^2*a*ln(sec(d*x+c)+tan(d*x+c
```

```
))+3/d*B*a*b^2*tan(d*x+c)+1/d*A*b^3*tan(d*x+c)+1/d*b^3*B*ln(sec(d*x+c)+tan(
d*x+c))
```

maxima [A] time = 0.35, size = 273, normalized size = 1.45

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{3}+48\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{2}b-3\,Aa^{3}\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 + 48*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a^2*b - 3*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(
d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*
x + c) - 1)) - 36*B*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) - 36*A*a*b^2*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*B*b^3*(lo
g(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 144*B*a*b^2*tan(d*x + c) + 4
8*A*b^3*tan(d*x + c))/d
```

mupad [B] time = 3.95, size = 395, normalized size = 2.10

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,A\,a^3}{8}+\frac{3\,B\,a^2\,b}{2}+\frac{3\,A\,a\,b^2}{2}+B\,b^3\right)}{\frac{3\,A\,a^3}{2}+6\,B\,a^2\,b+6\,A\,a\,b^2+4\,B\,b^3}\right)\left(\frac{3\,A\,a^3}{4}+3\,B\,a^2\,b+3\,A\,a\,b^2+2\,B\,b^3\right)}{d}-\frac{\left(2\,A\,b^3-\frac{5\,A\,a^3}{4}+2\,B\,a^3-3\,A\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((3*A*a^3)/8 + B*b^3 + (3*A*a*b^2)/2 + (3*B*a^
2*b)/2))/((3*A*a^3)/2 + 4*B*b^3 + 6*A*a*b^2 + 6*B*a^2*b))*((3*A*a^3)/4 + 2*
B*b^3 + 3*A*a*b^2 + 3*B*a^2*b))/d - (tan(c/2 + (d*x)/2)^7*(2*A*b^3 - (5*A*a
^3)/4 + 2*B*a^3 - 3*A*a*b^2 + 6*A*a^2*b + 6*B*a*b^2 - 3*B*a^2*b) + tan(c/2
+ (d*x)/2)^3*(6*A*b^3 - (3*A*a^3)/4 + (10*B*a^3)/3 + 3*A*a*b^2 + 10*A*a^2*b
 + 18*B*a*b^2 + 3*B*a^2*b) - tan(c/2 + (d*x)/2)^5*((3*A*a^3)/4 + 6*A*b^3 +
(10*B*a^3)/3 - 3*A*a*b^2 + 10*A*a^2*b + 18*B*a*b^2 - 3*B*a^2*b) - tan(c/2 +
 (d*x)/2)*((5*A*a^3)/4 + 2*A*b^3 + 2*B*a^3 + 3*A*a*b^2 + 6*A*a^2*b + 6*B*a*
b^2 + 3*B*a^2*b))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*t
an(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
[Out] Timed out
```

3.239 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=236

$$\frac{a\left(4a^2A+15abB+12Ab^2\right)\tan(c+dx)\sec^2(c+dx)}{15d}+\frac{a^2(5aB+7Ab)\tan(c+dx)\sec^3(c+dx)}{20d}+\frac{\left(8a^3A+30a^2bB\right.}{}$$

```
[Out] 1/8*(9*A*a^2*b+4*A*b^3+3*B*a^3+12*B*a*b^2)*arctanh(sin(d*x+c))/d+1/15*(8*A*
a^3+30*A*a*b^2+30*B*a^2*b+15*B*b^3)*tan(d*x+c)/d+1/8*(9*A*a^2*b+4*A*b^3+3*B
*a^3+12*B*a*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/15*a*(4*A*a^2+12*A*b^2+15*B*a*b)
*sec(d*x+c)^2*tan(d*x+c)/d+1/20*a^2*(7*A*b+5*B*a)*sec(d*x+c)^3*tan(d*x+c)/d
+1/5*a*A*(a+b*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d
```

Rubi [A] time = 0.49, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2989, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{\left(8a^3A+30a^2bB+30aAb^2+15b^3B\right)\tan(c+dx)}{15d}+\frac{\left(9a^2Ab+3a^3B+12ab^2B+4Ab^3\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(4a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] ((9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*ArcTanh[Sin[c + d*x]])/(8*d)
+ ((8*a^3*A + 30*a*A*b^2 + 30*a^2*b*B + 15*b^3*B)*Tan[c + d*x])/(15*d) + ((
9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d
) + (a*(4*a^2*A + 12*A*b^2 + 15*a*b*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d)
+ (a^2*(7*A*b + 5*a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*A*(a + b*Co
s[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}$$
$$=\frac{a^2(7Ab+5aB)\sec^3(c+dx)\tan(c+dx)}{20d}+\frac{aA(a+b}{}$$
$$=\frac{a\left(4a^2A+12Ab^2+15abB\right)\sec^2(c+dx)\tan(c+dx)}{15d}$$
$$=\frac{a\left(4a^2A+12Ab^2+15abB\right)\sec^2(c+dx)\tan(c+dx)}{15d}$$
$$=\frac{\left(9a^2Ab+4Ab^3+3a^3B+12ab^2B\right)\sec(c+dx)\tan(c}{8d}$$
$$=\frac{\left(9a^2Ab+4Ab^3+3a^3B+12ab^2B\right)\tanh^{-1}(\sin(c+d}{8d}$$

Mathematica [A] time = 3.26, size = 181, normalized size = 0.77

$$\frac{15\left(3a^3B+9a^2Ab+12ab^2B+4Ab^3\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(30a^2(aB+3Ab)\sec^3(c+dx)+8\left(3a^3A\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (15*(9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(15*(9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Sec[c + d*x] +
30*a^2*(3*A*b + a*B)*Sec[c + d*x]^3 + 8*(15*(a^3*A + 3*a*A*b^2 + 3*a^2*b*B
+ b^3*B) + 5*a*(2*a^2*A + 3*A*b^2 + 3*a*b*B)*Tan[c + d*x]^2 + 3*a^3*A*Tan[c
 + d*x]^4)))/(120*d)
```

fricas [A] time = 0.86, size = 249, normalized size = 1.06

$$\frac{15\left(3\,Ba^3+9\,Aa^2b+12\,Bab^2+4\,Ab^3\right)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\left(3\,Ba^3+9\,Aa^2b+12\,Bab^2+4\,Ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x + c)^5*log(s
in(d*x + c) + 1) - 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x
+ c)^5*log(-sin(d*x + c) + 1) + 2*(8*(8*A*a^3 + 30*B*a^2*b + 30*A*a*b^2 + 1
5*B*b^3)*cos(d*x + c)^4 + 24*A*a^3 + 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 +
 4*A*b^3)*cos(d*x + c)^3 + 8*(4*A*a^3 + 15*B*a^2*b + 15*A*a*b^2)*cos(d*x +
c)^2 + 30*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5
)
```

giac [B] time = 0.89, size = 722, normalized size = 3.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*B*a^3*t
an(1/2*d*x + 1/2*c)^9 - 225*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*B*a^2*b*ta
n(1/2*d*x + 1/2*c)^9 + 360*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 180*B*a*b^2*tan
(1/2*d*x + 1/2*c)^9 - 60*A*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*B*b^3*tan(1/2*d
*x + 1/2*c)^9 - 160*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 30*B*a^3*tan(1/2*d*x + 1
/2*c)^7 + 90*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 960*B*a^2*b*tan(1/2*d*x + 1/2
*c)^7 - 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 360*B*a*b^2*tan(1/2*d*x + 1/2*
c)^7 + 120*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 480*B*b^3*tan(1/2*d*x + 1/2*c)^7
+ 464*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 +
1200*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 720*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 16
0*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 90*A*a^2
*b*tan(1/2*d*x + 1/2*c)^3 - 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*A*a*b^
2*tan(1/2*d*x + 1/2*c)^3 - 360*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*A*b^3*t
an(1/2*d*x + 1/2*c)^3 - 480*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^3*tan(1/
2*d*x + 1/2*c) + 75*B*a^3*tan(1/2*d*x + 1/2*c) + 225*A*a^2*b*tan(1/2*d*x +
1/2*c) + 360*B*a^2*b*tan(1/2*d*x + 1/2*c) + 360*A*a*b^2*tan(1/2*d*x + 1/2*c
) + 180*B*a*b^2*tan(1/2*d*x + 1/2*c) + 60*A*b^3*tan(1/2*d*x + 1/2*c) + 120*
B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

maple [A] time = 0.15, size = 382, normalized size = 1.62

$$\frac{8A\,a^3\tan\left(dx+c\right)}{15d}+\frac{A\,a^3\tan\left(dx+c\right)\left(\sec^4\left(dx+c\right)\right)}{5d}+\frac{4A\,a^3\tan\left(dx+c\right)\left(\sec^2\left(dx+c\right)\right)}{15d}+\frac{a^3B\tan\left(dx+c\right)\left(\sec^3\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 8/15/d*A*a^3*tan(d*x+c)+1/5/d*A*a^3*tan(d*x+c)*sec(d*x+c)^4+4/15/d*A*a^3*ta
n(d*x+c)*sec(d*x+c)^2+1/4/d*a^3*B*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^3*B*sec(d
*x+c)*tan(d*x+c)+3/8/d*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3/4/d*A*a^2*b*tan(d*
x+c)*sec(d*x+c)^3+9/8/d*A*a^2*b*sec(d*x+c)*tan(d*x+c)+9/8/d*A*a^2*b*ln(sec(
d*x+c)+tan(d*x+c))+2/d*a^2*b*B*tan(d*x+c)+1/d*a^2*b*B*tan(d*x+c)*sec(d*x+c)
^2+2/d*A*b^2*a*tan(d*x+c)+1/d*A*a*b^2*tan(d*x+c)*sec(d*x+c)^2+3/2/d*B*a*b^2
*tan(d*x+c)*sec(d*x+c)+3/2/d*B*b^2*a*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*A*b^3*
tan(d*x+c)*sec(d*x+c)+1/2/d*A*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^3*B*tan(d
*x+c)
```

maxima [A] time = 0.51, size = 341, normalized size = 1.44

$$\frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^3+240\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2b+240\left(\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
240*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2*b + 240*(tan(d*x + c)^3 + 3*tan
(d*x + c))*A*a*b^2 - 15*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d
*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
 + c) - 1)) - 45*A*a^2*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 180*B*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c
) + 1) + log(sin(d*x + c) - 1)) - 60*A*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*B*b^3*tan(d*x +
 c))/d
```

mupad [B] time = 3.89, size = 470, normalized size = 1.99

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3Ba^3}{8}+\frac{9Aa^2b}{8}+\frac{3Bab^2}{2}+\frac{Ab^3}{2}\right)}{\frac{3Ba^3}{2}+\frac{9Aa^2b}{2}+6Bab^2+2Ab^3}\right)\left(\frac{3Ba^3}{4}+\frac{9Aa^2b}{4}+3Bab^2+Ab^3\right)}{d}-\frac{\left(2Aa^3-Ab^3-\frac{5Ba^3}{4}+2Bb^3+6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^6,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((A*b^3)/2 + (3*B*a^3)/8 + (9*A*a^2*b)/8 + (3*
B*a*b^2)/2))/(2*A*b^3 + (3*B*a^3)/2 + (9*A*a^2*b)/2 + 6*B*a*b^2))*(A*b^3 +
(3*B*a^3)/4 + (9*A*a^2*b)/4 + 3*B*a*b^2))/d - (tan(c/2 + (d*x)/2)*(2*A*a^3
+ A*b^3 + (5*B*a^3)/4 + 2*B*b^3 + 6*A*a*b^2 + (15*A*a^2*b)/4 + 3*B*a*b^2 +
6*B*a^2*b) + tan(c/2 + (d*x)/2)^5*((116*A*a^3)/15 + 12*B*b^3 + 20*A*a*b^2 +
 20*B*a^2*b) + tan(c/2 + (d*x)/2)^9*(2*A*a^3 - A*b^3 - (5*B*a^3)/4 + 2*B*b^
3 + 6*A*a*b^2 - (15*A*a^2*b)/4 - 3*B*a*b^2 + 6*B*a^2*b) - tan(c/2 + (d*x)/2
)^3*((8*A*a^3)/3 + 2*A*b^3 + (B*a^3)/2 + 8*B*b^3 + 16*A*a*b^2 + (3*A*a^2*b)
/2 + 6*B*a*b^2 + 16*B*a^2*b) - tan(c/2 + (d*x)/2)^7*((8*A*a^3)/3 - 2*A*b^3
- (B*a^3)/2 + 8*B*b^3 + 16*A*a*b^2 - (3*A*a^2*b)/2 - 6*B*a*b^2 + 16*B*a^2*b
))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x
)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.240 $\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=366

$$\frac{b^2\left(31a^2B+49aAb+18b^2B\right)\sin(c+dx)\cos^4(c+dx)}{105d}+\frac{b\left(104a^3B+224a^2Ab+140ab^2B+35Ab^3\right)\sin(c+dx)\cos}{168d}$$

```
[Out] 1/16*(8*A*a^4+36*A*a^2*b^2+5*A*b^4+24*B*a^3*b+20*B*a*b^3)*x+1/35*(140*A*a^3
*b+112*A*a*b^3+35*B*a^4+168*B*a^2*b^2+24*B*b^4)*sin(d*x+c)/d+1/16*(8*A*a^4+
36*A*a^2*b^2+5*A*b^4+24*B*a^3*b+20*B*a*b^3)*cos(d*x+c)*sin(d*x+c)/d+1/168*b
*(224*A*a^2*b+35*A*b^3+104*B*a^3+140*B*a*b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/1
05*b^2*(49*A*a*b+31*B*a^2+18*B*b^2)*cos(d*x+c)^4*sin(d*x+c)/d+1/42*b*(7*A*b
+10*B*a)*cos(d*x+c)^3*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/7*b*B*cos(d*x+c)^3*
(a+b*cos(d*x+c))^3*sin(d*x+c)/d-1/105*(140*A*a^3*b+112*A*a*b^3+35*B*a^4+168
*B*a^2*b^2+24*B*b^4)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.84, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2990, 3049, 3033, 3023, 2748, 2635, 8, 2633}

$$-\frac{\left(140a^3Ab+168a^2b^2B+35a^4B+112aAb^3+24b^4B\right)\sin^3(c+dx)}{105d}+\frac{\left(140a^3Ab+168a^2b^2B+35a^4B+112aAb^3\right.}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((8*a^4*A + 36*a^2*A*b^2 + 5*A*b^4 + 24*a^3*b*B + 20*a*b^3*B)*x)/16 + ((140
*a^3*A*b + 112*a*A*b^3 + 35*a^4*B + 168*a^2*b^2*B + 24*b^4*B)*Sin[c + d*x])
/(35*d) + ((8*a^4*A + 36*a^2*A*b^2 + 5*A*b^4 + 24*a^3*b*B + 20*a*b^3*B)*Cos
[c + d*x]*Sin[c + d*x])/(16*d) + (b*(224*a^2*A*b + 35*A*b^3 + 104*a^3*B + 1
40*a*b^2*B)*Cos[c + d*x]^3*Sin[c + d*x])/(168*d) + (b^2*(49*a*A*b + 31*a^2*
B + 18*b^2*B)*Cos[c + d*x]^4*Sin[c + d*x])/(105*d) + (b*(7*A*b + 10*a*B)*Co
s[c + d*x]^3*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(42*d) + (b*B*Cos[c + d*x
]^3*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(7*d) - ((140*a^3*A*b + 112*a*A*b^
3 + 35*a^4*B + 168*a^2*b^2*B + 24*b^4*B)*Sin[c + d*x]^3)/(105*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
```

```
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{bB\cos^3(c+dx)(a+b\cos(c+dx))^3\sin(c+dx)}{7d} + \frac{1}{7}$$

$$= \frac{b(7Ab+10aB)\cos^3(c+dx)(a+b\cos(c+dx))^2\sin(}{42d}$$

$$= \frac{b^2\left(49aAb+31a^2B+18b^2B\right)\cos^4(c+dx)\sin(c+d}{105d}$$

$$= \frac{b\left(224a^2Ab+35Ab^3+104a^3B+140ab^2B\right)\cos^3(c+}{168d}$$

$$= \frac{b\left(224a^2Ab+35Ab^3+104a^3B+140ab^2B\right)\cos^3(c+}{168d}$$

$$= \frac{\left(8a^4A+36a^2Ab^2+5Ab^4+24a^3bB+20ab^3B\right)\cos(}{16d}$$

$$= \frac{1}{16}\left(8a^4A+36a^2Ab^2+5Ab^4+24a^3bB+20ab^3B\right)x$$

Mathematica [A] time = 0.88, size = 408, normalized size = 1.11

$$\frac{3360a^4Ac+3360a^4Adx+560a^4B\sin(3(c+dx))+2240a^3Ab\sin(3(c+dx))+840a^3bB\sin(4(c+dx))+10080a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3360*a^4*A*c + 15120*a^2*A*b^2*c + 2100*A*b^4*c + 10080*a^3*b*B*c + 8400*a
*b^3*B*c + 3360*a^4*A*d*x + 15120*a^2*A*b^2*d*x + 2100*A*b^4*d*x + 10080*a^
3*b*B*d*x + 8400*a*b^3*B*d*x + 105*(192*a^3*A*b + 160*a*A*b^3 + 48*a^4*B +
240*a^2*b^2*B + 35*b^4*B)*Sin[c + d*x] + 105*(16*a^4*A + 96*a^2*A*b^2 + 15*
A*b^4 + 64*a^3*b*B + 60*a*b^3*B)*Sin[2*(c + d*x)] + 2240*a^3*A*b*Sin[3*(c +
 d*x)] + 2800*a*A*b^3*Sin[3*(c + d*x)] + 560*a^4*B*Sin[3*(c + d*x)] + 4200*
a^2*b^2*B*Sin[3*(c + d*x)] + 735*b^4*B*Sin[3*(c + d*x)] + 1260*a^2*A*b^2*Si
n[4*(c + d*x)] + 315*A*b^4*Sin[4*(c + d*x)] + 840*a^3*b*B*Sin[4*(c + d*x)]
+ 1260*a*b^3*B*Sin[4*(c + d*x)] + 336*a*A*b^3*Sin[5*(c + d*x)] + 504*a^2*b^
2*B*Sin[5*(c + d*x)] + 147*b^4*B*Sin[5*(c + d*x)] + 35*A*b^4*Sin[6*(c + d*x
)] + 140*a*b^3*B*Sin[6*(c + d*x)] + 15*b^4*B*Sin[7*(c + d*x)])/(6720*d)
```

fricas [A] time = 1.60, size = 289, normalized size = 0.79

$$\frac{105\left(8\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+20\,Bab^3+5\,Ab^4\right)dx+\left(240\,Bb^4\cos\left(dx+c\right)^6+280\left(4\,Bab^3+Ab^4\right)\cos\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1680*(105*(8*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*d*
x + (240*B*b^4*cos(d*x + c)^6 + 280*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^5 + 11
20*B*a^4 + 4480*A*a^3*b + 5376*B*a^2*b^2 + 3584*A*a*b^3 + 768*B*b^4 + 96*(2
1*B*a^2*b^2 + 14*A*a*b^3 + 3*B*b^4)*cos(d*x + c)^4 + 70*(24*B*a^3*b + 36*A*
a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*cos(d*x + c)^3 + 16*(35*B*a^4 + 140*A*a^3*b
 + 168*B*a^2*b^2 + 112*A*a*b^3 + 24*B*b^4)*cos(d*x + c)^2 + 105*(8*A*a^4 +
24*B*a^3*b + 36*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*cos(d*x + c))*sin(d*x + c
))/d
```

giac [A] time = 0.52, size = 313, normalized size = 0.86

$$\frac{Bb^4\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{1}{16}\left(8\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+20\,Bab^3+5\,Ab^4\right)x+\frac{\left(4\,Bab^3+Ab^4\right)\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/448*B*b^4*sin(7*d*x + 7*c)/d + 1/16*(8*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2
+ 20*B*a*b^3 + 5*A*b^4)*x + 1/192*(4*B*a*b^3 + A*b^4)*sin(6*d*x + 6*c)/d +
1/320*(24*B*a^2*b^2 + 16*A*a*b^3 + 7*B*b^4)*sin(5*d*x + 5*c)/d + 1/64*(8*B*
a^3*b + 12*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*sin(4*d*x + 4*c)/d + 1/192*(16
```

```
*B*a^4 + 64*A*a^3*b + 120*B*a^2*b^2 + 80*A*a*b^3 + 21*B*b^4)*sin(3*d*x + 3*
c)/d + 1/64*(16*A*a^4 + 64*B*a^3*b + 96*A*a^2*b^2 + 60*B*a*b^3 + 15*A*b^4)*
sin(2*d*x + 2*c)/d + 1/64*(48*B*a^4 + 192*A*a^3*b + 240*B*a^2*b^2 + 160*A*a
*b^3 + 35*B*b^4)*sin(d*x + c)/d
```

maple [A] time = 0.06, size = 368, normalized size = 1.01

$$A\,a^{4}\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{a^{4}B\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}+\frac{4A\,a^{3}b\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}+4B\,a^{3}b\left(\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx}{2}\right.}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*a^4*B*(2+cos(d*x+c
)^2)*sin(d*x+c)+4/3*A*a^3*b*(2+cos(d*x+c)^2)*sin(d*x+c)+4*B*a^3*b*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6*A*a^2*b^2*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6/5*B*a^2*b^2*(8/3+cos(d*
x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4/5*A*a*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d
*x+c)^2)*sin(d*x+c)+4*B*a*b^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(
d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+A*b^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)
^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+1/7*B*b^4*(16/5+cos(d*x+c)^
6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

maxima [A] time = 0.39, size = 366, normalized size = 1.00

$$1680\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^{4}-2240\left(\sin(dx+c)^{3}-3\,\sin(dx+c)\right)Ba^{4}-8960\left(\sin(dx+c)^{3}-3\,\sin(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/6720*(1680*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 - 2240*(sin(d*x + c)^3
- 3*sin(d*x + c))*B*a^4 - 8960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3*b +
840*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3*b + 1260*
(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2*b^2 + 2688*(3
*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^2*b^2 + 1792*(3*
sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a*b^3 - 140*(4*sin(
2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*
B*a*b^3 - 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 4
8*sin(2*d*x + 2*c))*A*b^4 - 192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*
sin(d*x + c)^3 - 35*sin(d*x + c))*B*b^4)/d
```

mupad [B] time = 2.64, size = 436, normalized size = 1.19

$$\frac{420\,A\,a^4\,\sin\left(2\,c+2\,d\,x\right)+\frac{1575\,A\,b^4\,\sin(2\,c+2\,d\,x)}{4}+140\,B\,a^4\,\sin\left(3\,c+3\,d\,x\right)+\frac{315\,A\,b^4\,\sin(4\,c+4\,d\,x)}{4}+\frac{35\,A\,b^4\,\sin(6\,c+6\,d}{4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4,x)
```

```
[Out] (420*A*a^4*sin(2*c + 2*d*x) + (1575*A*b^4*sin(2*c + 2*d*x))/4 + 140*B*a^4*s
in(3*c + 3*d*x) + (315*A*b^4*sin(4*c + 4*d*x))/4 + (35*A*b^4*sin(6*c + 6*d*
x))/4 + (735*B*b^4*sin(3*c + 3*d*x))/4 + (147*B*b^4*sin(5*c + 5*d*x))/4 + (
15*B*b^4*sin(7*c + 7*d*x))/4 + 1260*B*a^4*sin(c + d*x) + (3675*B*b^4*sin(c
+ d*x))/4 + 4200*A*a*b^3*sin(c + d*x) + 5040*A*a^3*b*sin(c + d*x) + 840*A*a
^4*d*x + 525*A*b^4*d*x + 700*A*a*b^3*sin(3*c + 3*d*x) + 560*A*a^3*b*sin(3*c
 + 3*d*x) + 84*A*a*b^3*sin(5*c + 5*d*x) + 1575*B*a*b^3*sin(2*c + 2*d*x) + 1
680*B*a^3*b*sin(2*c + 2*d*x) + 315*B*a*b^3*sin(4*c + 4*d*x) + 210*B*a^3*b*s
in(4*c + 4*d*x) + 35*B*a*b^3*sin(6*c + 6*d*x) + 6300*B*a^2*b^2*sin(c + d*x)
 + 2520*A*a^2*b^2*sin(2*c + 2*d*x) + 315*A*a^2*b^2*sin(4*c + 4*d*x) + 1050*
B*a^2*b^2*sin(3*c + 3*d*x) + 126*B*a^2*b^2*sin(5*c + 5*d*x) + 2100*B*a*b^3*
d*x + 2520*B*a^3*b*d*x + 3780*A*a^2*b^2*d*x)/(1680*d)
```

sympy [A] time = 8.20, size = 1017, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**4*x*sin(c + d*x)**2/2 + A*a**4*x*cos(c + d*x)**2/2 + A*a**4
*sin(c + d*x)*cos(c + d*x)/(2*d) + 8*A*a**3*b*sin(c + d*x)**3/(3*d) + 4*A*a
**3*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*A*a**2*b**2*x*sin(c + d*x)**4/4 +
9*A*a**2*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 9*A*a**2*b**2*x*cos(c +
 d*x)**4/4 + 9*A*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 15*A*a**2*b
**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 32*A*a*b**3*sin(c + d*x)**5/(15*d)
 + 16*A*a*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 4*A*a*b**3*sin(c + d
*x)*cos(c + d*x)**4/d + 5*A*b**4*x*sin(c + d*x)**6/16 + 15*A*b**4*x*sin(c +
 d*x)**4*cos(c + d*x)**2/16 + 15*A*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/1
6 + 5*A*b**4*x*cos(c + d*x)**6/16 + 5*A*b**4*sin(c + d*x)**5*cos(c + d*x)/(
16*d) + 5*A*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*A*b**4*sin(c +
d*x)*cos(c + d*x)**5/(16*d) + 2*B*a**4*sin(c + d*x)**3/(3*d) + B*a**4*sin(c
 + d*x)*cos(c + d*x)**2/d + 3*B*a**3*b*x*sin(c + d*x)**4/2 + 3*B*a**3*b*x*s
in(c + d*x)**2*cos(c + d*x)**2 + 3*B*a**3*b*x*cos(c + d*x)**4/2 + 3*B*a**3*
b*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 5*B*a**3*b*sin(c + d*x)*cos(c + d*x)
```

```
**3/(2*d) + 16*B*a**2*b**2*sin(c + d*x)**5/(5*d) + 8*B*a**2*b**2*sin(c + d*
x)**3*cos(c + d*x)**2/d + 6*B*a**2*b**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*
B*a*b**3*x*sin(c + d*x)**6/4 + 15*B*a*b**3*x*sin(c + d*x)**4*cos(c + d*x)**
2/4 + 15*B*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/4 + 5*B*a*b**3*x*cos(c
+ d*x)**6/4 + 5*B*a*b**3*sin(c + d*x)**5*cos(c + d*x)/(4*d) + 10*B*a*b**3*s
in(c + d*x)**3*cos(c + d*x)**3/(3*d) + 11*B*a*b**3*sin(c + d*x)*cos(c + d*x
)**5/(4*d) + 16*B*b**4*sin(c + d*x)**7/(35*d) + 8*B*b**4*sin(c + d*x)**5*co
s(c + d*x)**2/(5*d) + 2*B*b**4*sin(c + d*x)**3*cos(c + d*x)**4/d + B*b**4*s
in(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))*
*4*cos(c)**2, True))
```

3.241 $\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=325

$$\frac{\left(-4a^2B+24aAb+25b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^3}{120bd}+\frac{\left(-4a^3B+24a^2Ab+53ab^2B+32Ab^3\right)\sin(c+dx)(}{120bd}$$

```
[Out] 1/16*(32*A*a^3*b+24*A*a*b^3+8*B*a^4+36*B*a^2*b^2+5*B*b^4)*x+1/60*(24*A*a^4*
b+224*A*a^2*b^3+32*A*b^5-4*B*a^5+121*B*a^3*b^2+128*B*a*b^4)*sin(d*x+c)/b/d+
1/240*(48*A*a^3*b+232*A*a*b^3-8*B*a^4+178*B*a^2*b^2+75*B*b^4)*cos(d*x+c)*si
n(d*x+c)/d+1/120*(24*A*a^2*b+32*A*b^3-4*B*a^3+53*B*a*b^2)*(a+b*cos(d*x+c))^
2*sin(d*x+c)/b/d+1/120*(24*A*a*b-4*B*a^2+25*B*b^2)*(a+b*cos(d*x+c))^3*sin(d
*x+c)/b/d+1/30*(6*A*b-B*a)*(a+b*cos(d*x+c))^4*sin(d*x+c)/b/d+1/6*B*(a+b*cos
(d*x+c))^5*sin(d*x+c)/b/d
```

Rubi [A] time = 0.51, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2968, 3023, 2753, 2734}

$$\frac{\left(224a^2Ab^3+24a^4Ab+121a^3b^2B-4a^5B+128ab^4B+32Ab^5\right)\sin(c+dx)}{60bd}+\frac{\left(-4a^2B+24aAb+25b^2B\right)\sin(c+}{120bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B)*x)/16 + ((24*
a^4*A*b + 224*a^2*A*b^3 + 32*A*b^5 - 4*a^5*B + 121*a^3*b^2*B + 128*a*b^4*B)
*Sin[c + d*x])/(60*b*d) + ((48*a^3*A*b + 232*a*A*b^3 - 8*a^4*B + 178*a^2*b^
2*B + 75*b^4*B)*Cos[c + d*x]*Sin[c + d*x])/(240*d) + ((24*a^2*A*b + 32*A*b^
3 - 4*a^3*B + 53*a*b^2*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(120*b*d) +
((24*a*A*b - 4*a^2*B + 25*b^2*B)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(120*
b*d) + ((6*A*b - a*B)*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(30*b*d) + (B*(a
 + b*Cos[c + d*x])^5*Sin[c + d*x])/(6*b*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^4\left(A\cos(c+dx)+B\cos^2(c+dx\right.$$

$$= \frac{B(a+b\cos(c+dx))^5\sin(c+dx)}{6bd} + \frac{\int (a+b\cos(c+}{}$$

$$= \frac{(6Ab-aB)(a+b\cos(c+dx))^4\sin(c+dx)}{30bd} + \frac{B(a+b}{}$$

$$= \frac{\left(24aAb-4a^2B+25b^2B\right)(a+b\cos(c+dx))^3\sin(c+}{120bd}$$

$$= \frac{\left(24a^2Ab+32Ab^3-4a^3B+53ab^2B\right)(a+b\cos(c+d}{120bd}$$

$$= \frac{1}{16}\left(32a^3Ab+24aAb^3+8a^4B+36a^2b^2B+5b^4B\right)x$$

Mathematica [A] time = 1.16, size = 333, normalized size = 1.02

$$\frac{480a^4Bc + 480a^4Bdx + 1920a^3Abc + 1920a^3Abdx + 320a^3bB\sin(3(c+dx)) + 480a^2Ab^2\sin(3(c+dx)) + 180a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (1920*a^3*A*b*c + 1440*a*A*b^3*c + 480*a^4*B*c + 2160*a^2*b^2*B*c + 300*b^4
*B*c + 1920*a^3*A*b*d*x + 1440*a*A*b^3*d*x + 480*a^4*B*d*x + 2160*a^2*b^2*B
*d*x + 300*b^4*B*d*x + 120*(8*a^4*A + 36*a^2*A*b^2 + 5*A*b^4 + 24*a^3*b*B +
 20*a*b^3*B)*Sin[c + d*x] + 15*(64*a^3*A*b + 64*a*A*b^3 + 16*a^4*B + 96*a^2
*b^2*B + 15*b^4*B)*Sin[2*(c + d*x)] + 480*a^2*A*b^2*Sin[3*(c + d*x)] + 100*
A*b^4*Sin[3*(c + d*x)] + 320*a^3*b*B*Sin[3*(c + d*x)] + 400*a*b^3*B*Sin[3*(
c + d*x)] + 120*a*A*b^3*Sin[4*(c + d*x)] + 180*a^2*b^2*B*Sin[4*(c + d*x)] +
 45*b^4*B*Sin[4*(c + d*x)] + 12*A*b^4*Sin[5*(c + d*x)] + 48*a*b^3*B*Sin[5*(
c + d*x)] + 5*b^4*B*Sin[6*(c + d*x)])/(960*d)
```

fricas [A] time = 0.85, size = 243, normalized size = 0.75

$$\frac{15\left(8\,Ba^4 + 32\,Aa^3b + 36\,Ba^2b^2 + 24\,Aab^3 + 5\,Bb^4\right)dx + \left(40\,Bb^4\cos\left(dx+c\right)^5 + 240\,Aa^4 + 640\,Ba^3b + 960\,Aa^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/240*(15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*d*x
+ (40*B*b^4*cos(d*x + c)^5 + 240*A*a^4 + 640*B*a^3*b + 960*A*a^2*b^2 + 512*
B*a*b^3 + 128*A*b^4 + 48*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^4 + 10*(36*B*a^2*
b^2 + 24*A*a*b^3 + 5*B*b^4)*cos(d*x + c)^3 + 32*(10*B*a^3*b + 15*A*a^2*b^2
+ 8*B*a*b^3 + 2*A*b^4)*cos(d*x + c)^2 + 15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2
*b^2 + 24*A*a*b^3 + 5*B*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.47, size = 263, normalized size = 0.81

$$\frac{Bb^4\sin\left(6\,dx+6\,c\right)}{192\,d} + \frac{1}{16}\left(8\,Ba^4 + 32\,Aa^3b + 36\,Ba^2b^2 + 24\,Aab^3 + 5\,Bb^4\right)x + \frac{\left(4\,Bab^3 + Ab^4\right)\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/192*B*b^4*sin(6*d*x + 6*c)/d + 1/16*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2
+ 24*A*a*b^3 + 5*B*b^4)*x + 1/80*(4*B*a*b^3 + A*b^4)*sin(5*d*x + 5*c)/d + 1
/64*(12*B*a^2*b^2 + 8*A*a*b^3 + 3*B*b^4)*sin(4*d*x + 4*c)/d + 1/48*(16*B*a^
3*b + 24*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*sin(3*d*x + 3*c)/d + 1/64*(16*B*
a^4 + 64*A*a^3*b + 96*B*a^2*b^2 + 64*A*a*b^3 + 15*B*b^4)*sin(2*d*x + 2*c)/d
 + 1/8*(8*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 20*B*a*b^3 + 5*A*b^4)*sin(d*x
 + c)/d
```

maple [A] time = 0.06, size = 316, normalized size = 0.97

$$A\,a^{4}\sin\left(dx+c\right)+a^{4}B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4A\,a^{3}b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{4B\,a^{3}b\left(2+\cos^{2}(dx+c)\right)\sin(dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(A*a^4*sin(d*x+c)+a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*A*a
^3*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4/3*B*a^3*b*(2+cos(d*x+c)^2)
*sin(d*x+c)+2*A*a^2*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+6*B*a^2*b^2*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*A*a*b^3*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/5*B*a*b^3*(8/3+cos(d*x+c)^4
+4/3*cos(d*x+c)^2)*sin(d*x+c)+1/5*A*b^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)
*sin(d*x+c)+B*b^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(
d*x+c)+5/16*d*x+5/16*c))
```

maxima [A] time = 0.48, size = 307, normalized size = 0.94

$$\frac{240\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ba^{4}+960\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa^{3}b-1280\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/960*(240*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 + 960*(2*d*x + 2*c + sin(
2*d*x + 2*c))*A*a^3*b - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3*b - 19
20*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2*b^2 + 180*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2*b^2 + 120*(12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b^3 + 256*(3*sin(d*x + c)^5 - 10*sin(d*x
 + c)^3 + 15*sin(d*x + c))*B*a*b^3 + 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)
^3 + 15*sin(d*x + c))*A*b^4 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*s
in(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*B*b^4 + 960*A*a^4*sin(d*x + c))/d
```

mupad [B] time = 1.37, size = 403, normalized size = 1.24

$$\frac{B\,a^4\,x}{2}+\frac{5\,B\,b^4\,x}{16}+\frac{3\,A\,a\,b^3\,x}{2}+2\,A\,a^3\,b\,x+\frac{A\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{5\,A\,b^4\,\sin\left(c+d\,x\right)}{8\,d}+\frac{9\,B\,a^2\,b^2\,x}{4}+\frac{B\,a^4\,\sin\left(2\,c+2\,d\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4,x)
```

```
[Out] (B*a^4*x)/2 + (5*B*b^4*x)/16 + (3*A*a*b^3*x)/2 + 2*A*a^3*b*x + (A*a^4*sin(c
 + d*x))/d + (5*A*b^4*sin(c + d*x))/(8*d) + (9*B*a^2*b^2*x)/4 + (B*a^4*sin(
2*c + 2*d*x))/(4*d) + (5*A*b^4*sin(3*c + 3*d*x))/(48*d) + (A*b^4*sin(5*c +
5*d*x))/(80*d) + (15*B*b^4*sin(2*c + 2*d*x))/(64*d) + (3*B*b^4*sin(4*c + 4*
d*x))/(64*d) + (B*b^4*sin(6*c + 6*d*x))/(192*d) + (A*a*b^3*sin(2*c + 2*d*x)
)/d + (A*a^3*b*sin(2*c + 2*d*x))/d + (A*a*b^3*sin(4*c + 4*d*x))/(8*d) + (9*
A*a^2*b^2*sin(c + d*x))/(2*d) + (5*B*a*b^3*sin(3*c + 3*d*x))/(12*d) + (B*a^
3*b*sin(3*c + 3*d*x))/(3*d) + (B*a*b^3*sin(5*c + 5*d*x))/(20*d) + (A*a^2*b^
2*sin(3*c + 3*d*x))/(2*d) + (3*B*a^2*b^2*sin(2*c + 2*d*x))/(2*d) + (3*B*a^2
*b^2*sin(4*c + 4*d*x))/(16*d) + (5*B*a*b^3*sin(c + d*x))/(2*d) + (3*B*a^3*b
*sin(c + d*x))/d
```

sympy [A] time = 4.99, size = 811, normalized size = 2.50

$$\begin{cases} \frac{Aa^4\sin\left(c+dx\right)}{d}+2Aa^3bx\sin^2\left(c+dx\right)+2Aa^3bx\cos^2\left(c+dx\right)+\frac{2Aa^3b\sin\left(c+dx\right)\cos\left(c+dx\right)}{d}+\frac{4Aa^2b^2\sin^3\left(c+dx\right)}{d}+\frac{6Aa^2b^2}{} \\ x\left(A+B\cos\left(c\right)\right)\left(a+b\cos\left(c\right)\right)^4\cos\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**4*sin(c + d*x)/d + 2*A*a**3*b*x*sin(c + d*x)**2 + 2*A*a**3*
b*x*cos(c + d*x)**2 + 2*A*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*A*a**2*b**
2*sin(c + d*x)**3/d + 6*A*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*A*a*
b**3*x*sin(c + d*x)**4/2 + 3*A*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3
*A*a*b**3*x*cos(c + d*x)**4/2 + 3*A*a*b**3*sin(c + d*x)**3*cos(c + d*x)/(2*
d) + 5*A*a*b**3*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 8*A*b**4*sin(c + d*x)*
*5/(15*d) + 4*A*b**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + A*b**4*sin(c +
 d*x)*cos(c + d*x)**4/d + B*a**4*x*sin(c + d*x)**2/2 + B*a**4*x*cos(c + d*x
)**2/2 + B*a**4*sin(c + d*x)*cos(c + d*x)/(2*d) + 8*B*a**3*b*sin(c + d*x)**
3/(3*d) + 4*B*a**3*b*sin(c + d*x)*cos(c + d*x)**2/d + 9*B*a**2*b**2*x*sin(c
 + d*x)**4/4 + 9*B*a**2*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 9*B*a**2
*b**2*x*cos(c + d*x)**4/4 + 9*B*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d
) + 15*B*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 32*B*a*b**3*sin(c +
 d*x)**5/(15*d) + 16*B*a*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 4*B*a
```

```
*b**3*sin(c + d*x)*cos(c + d*x)**4/d + 5*B*b**4*x*sin(c + d*x)**6/16 + 15*B
*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*B*b**4*x*sin(c + d*x)**2*co
s(c + d*x)**4/16 + 5*B*b**4*x*cos(c + d*x)**6/16 + 5*B*b**4*sin(c + d*x)**5
*cos(c + d*x)/(16*d) + 5*B*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*
B*b**4*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(A + B*cos(c))*(a
 + b*cos(c))**4*cos(c), True))
```

3.242 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=241

$$\frac{(12a^2B+35aAb+16b^2B)\sin(c+dx)(a+b\cos(c+dx))^2}{60d}+\frac{b(24a^3B+130a^2Ab+116ab^2B+45Ab^3)\sin(c+d}{120d}$$

```
[Out] 1/8*(8*A*a^4+24*A*a^2*b^2+3*A*b^4+16*B*a^3*b+12*B*a*b^3)*x+1/30*(95*A*a^3*b
+80*A*a*b^3+12*B*a^4+112*B*a^2*b^2+16*B*b^4)*sin(d*x+c)/d+1/120*b*(130*A*a^
2*b+45*A*b^3+24*B*a^3+116*B*a*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/60*(35*A*a*b+1
2*B*a^2+16*B*b^2)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/20*(5*A*b+4*B*a)*(a+b*c
os(d*x+c))^3*sin(d*x+c)/d+1/5*B*(a+b*cos(d*x+c))^4*sin(d*x+c)/d
```

Rubi [A] time = 0.34, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2753, 2734}

$$\frac{(95a^3Ab+112a^2b^2B+12a^4B+80aAb^3+16b^4B)\sin(c+dx)}{30d}+\frac{(12a^2B+35aAb+16b^2B)\sin(c+dx)(a+b\cos}{60d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((8*a^4*A + 24*a^2*A*b^2 + 3*A*b^4 + 16*a^3*b*B + 12*a*b^3*B)*x)/8 + ((95*a
^3*A*b + 80*a*A*b^3 + 12*a^4*B + 112*a^2*b^2*B + 16*b^4*B)*Sin[c + d*x])/(3
0*d) + (b*(130*a^2*A*b + 45*A*b^3 + 24*a^3*B + 116*a*b^2*B)*Cos[c + d*x]*Si
n[c + d*x])/(120*d) + ((35*a*A*b + 12*a^2*B + 16*b^2*B)*(a + b*Cos[c + d*x]
)^2*Sin[c + d*x])/(60*d) + ((5*A*b + 4*a*B)*(a + b*Cos[c + d*x])^3*Sin[c +
d*x])/(20*d) + (B*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
&& IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\,dx = \frac{B(a+b\cos(c+dx))^4\sin(c+dx)}{5d} + \frac{1}{5}\int (a+b\cos(c+dx))^3(5aA$$
$$= \frac{(5Ab+4aB)(a+b\cos(c+dx))^3\sin(c+dx)}{20d} + \frac{B(a+b\cos(c+d}{5d}$$
$$= \frac{\left(35aAb+12a^2B+16b^2B\right)(a+b\cos(c+dx))^2\sin(c+dx)}{60d} + \frac{(5A}{}$$
$$= \frac{1}{8}\left(8a^4A+24a^2Ab^2+3Ab^4+16a^3bB+12ab^3B\right)x + \frac{\left(95a^3Ab+\right.}{}$$

Mathematica [A] time = 0.66, size = 263, normalized size = 1.09

$$\frac{480a^4Ac+480a^4Adx+960a^3bBc+960a^3bBdx+1440a^2Ab^2c+1440a^2Ab^2dx+240a^2b^2B\sin(3(c+dx))+120}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x]),x]
```

```
[Out] (480*a^4*A*c + 1440*a^2*A*b^2*c + 180*A*b^4*c + 960*a^3*b*B*c + 720*a*b^3*B
*c + 480*a^4*A*d*x + 1440*a^2*A*b^2*d*x + 180*A*b^4*d*x + 960*a^3*b*B*d*x +
 720*a*b^3*B*d*x + 60*(32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5
*b^4*B)*Sin[c + d*x] + 120*b*(6*a^2*A*b + A*b^3 + 4*a^3*B + 4*a*b^2*B)*Sin[
2*(c + d*x)] + 160*a*A*b^3*Sin[3*(c + d*x)] + 240*a^2*b^2*B*Sin[3*(c + d*x)
] + 50*b^4*B*Sin[3*(c + d*x)] + 15*A*b^4*Sin[4*(c + d*x)] + 60*a*b^3*B*Sin[
4*(c + d*x)] + 6*b^4*B*Sin[5*(c + d*x)])/(480*d)
```

fricas [A] time = 1.14, size = 197, normalized size = 0.82

$$\frac{15\left(8\,Aa^4+16\,Ba^3b+24\,Aa^2b^2+12\,Bab^3+3\,Ab^4\right)dx+\left(24\,Bb^4\cos\left(dx+c\right)^4+120\,Ba^4+480\,Aa^3b+480\,Ba^2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*d*x
+ (24*B*b^4*cos(d*x + c)^4 + 120*B*a^4 + 480*A*a^3*b + 480*B*a^2*b^2 + 320*
A*a*b^3 + 64*B*b^4 + 30*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^3 + 16*(15*B*a^2*b
```

```
^2 + 10*A*a*b^3 + 2*B*b^4)*cos(d*x + c)^2 + 15*(16*B*a^3*b + 24*A*a^2*b^2 +
 12*B*a*b^3 + 3*A*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.51, size = 212, normalized size = 0.88

$$\frac{Bb^4\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{8}\left(8\,Aa^4+16\,Ba^3b+24\,Aa^2b^2+12\,Bab^3+3\,Ab^4\right)x+\frac{\left(4\,Bab^3+Ab^4\right)\sin(4\,dx+4\,c)}{32\,d}+\frac{(24\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/80*B*b^4*sin(5*d*x + 5*c)/d + 1/8*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 +
12*B*a*b^3 + 3*A*b^4)*x + 1/32*(4*B*a*b^3 + A*b^4)*sin(4*d*x + 4*c)/d + 1/4
8*(24*B*a^2*b^2 + 16*A*a*b^3 + 5*B*b^4)*sin(3*d*x + 3*c)/d + 1/4*(4*B*a^3*b
 + 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*sin(2*d*x + 2*c)/d + 1/8*(8*B*a^4 + 32*
A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*sin(d*x + c)/d
```

maple [A] time = 0.05, size = 258, normalized size = 1.07

$$\frac{\frac{B\,b^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+A\,b^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+4Ba\,b^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\cdots\right.}{4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*B*b^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*b^4*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*B*a*b^3*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*A*a*b^3*(2+cos(d*x+c)
^2)*sin(d*x+c)+2*B*a^2*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+6*A*a^2*b^2*(1/2*cos
(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*B*a^3*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*
d*x+1/2*c)+4*A*a^3*b*sin(d*x+c)+a^4*B*sin(d*x+c)+A*a^4*(d*x+c))
```

maxima [A] time = 0.58, size = 246, normalized size = 1.02

$$\frac{480\,(dx+c)Aa^4+480\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^3b+720\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2b^2-960\left(\sin\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(480*(d*x + c)*A*a^4 + 480*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3*b +
 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2*b^2 - 960*(sin(d*x + c)^3 - 3*s
```

```
in(d*x + c))*B*a^2*b^2 - 640*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a*b^3 + 60
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a*b^3 + 15*(12*d
*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*b^4 + 32*(3*sin(d*x +
c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*b^4 + 480*B*a^4*sin(d*x + c)
+ 1920*A*a^3*b*sin(d*x + c))/d
```

mupad [B] time = 0.88, size = 307, normalized size = 1.27

$$A\,a^4\,x+\frac{3\,A\,b^4\,x}{8}+\frac{3\,B\,a\,b^3\,x}{2}+2\,B\,a^3\,b\,x+\frac{B\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{5\,B\,b^4\,\sin\left(c+d\,x\right)}{8\,d}+3\,A\,a^2\,b^2\,x+\frac{A\,b^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4,x)
```

```
[Out] A*a^4*x + (3*A*b^4*x)/8 + (3*B*a*b^3*x)/2 + 2*B*a^3*b*x + (B*a^4*sin(c + d*
x))/d + (5*B*b^4*sin(c + d*x))/(8*d) + 3*A*a^2*b^2*x + (A*b^4*sin(2*c + 2*d
*x))/(4*d) + (A*b^4*sin(4*c + 4*d*x))/(32*d) + (5*B*b^4*sin(3*c + 3*d*x))/(
48*d) + (B*b^4*sin(5*c + 5*d*x))/(80*d) + (A*a*b^3*sin(3*c + 3*d*x))/(3*d)
+ (B*a*b^3*sin(2*c + 2*d*x))/d + (B*a^3*b*sin(2*c + 2*d*x))/d + (B*a*b^3*si
n(4*c + 4*d*x))/(8*d) + (9*B*a^2*b^2*sin(c + d*x))/(2*d) + (3*A*a^2*b^2*sin
(2*c + 2*d*x))/(2*d) + (B*a^2*b^2*sin(3*c + 3*d*x))/(2*d) + (3*A*a*b^3*sin(
c + d*x))/d + (4*A*a^3*b*sin(c + d*x))/d
```

sympy [A] time = 2.83, size = 580, normalized size = 2.41

$$\begin{cases} Aa^4x + \frac{4Aa^3b\sin(c+dx)}{d} + 3Aa^2b^2x\sin^2(c+dx) + 3Aa^2b^2x\cos^2(c+dx) + \frac{3Aa^2b^2\sin(c+dx)\cos(c+dx)}{d} + \frac{8Aab^3\sin^3(c+dx)}{3d} \\ x\left(A+B\cos(c)\right)\left(a+b\cos(c)\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*a**4*x + 4*A*a**3*b*sin(c + d*x)/d + 3*A*a**2*b**2*x*sin(c + d
*x)**2 + 3*A*a**2*b**2*x*cos(c + d*x)**2 + 3*A*a**2*b**2*sin(c + d*x)*cos(c
 + d*x)/d + 8*A*a*b**3*sin(c + d*x)**3/(3*d) + 4*A*a*b**3*sin(c + d*x)*cos(
c + d*x)**2/d + 3*A*b**4*x*sin(c + d*x)**4/8 + 3*A*b**4*x*sin(c + d*x)**2*c
os(c + d*x)**2/4 + 3*A*b**4*x*cos(c + d*x)**4/8 + 3*A*b**4*sin(c + d*x)**3*
cos(c + d*x)/(8*d) + 5*A*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) + B*a**4*s
in(c + d*x)/d + 2*B*a**3*b*x*sin(c + d*x)**2 + 2*B*a**3*b*x*cos(c + d*x)**2
 + 2*B*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*B*a**2*b**2*sin(c + d*x)**3/d
 + 6*B*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*B*a*b**3*x*sin(c + d*x)
**4/2 + 3*B*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3*B*a*b**3*x*cos(c +
 d*x)**4/2 + 3*B*a*b**3*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 5*B*a*b**3*sin
```

```
(c + d*x)*cos(c + d*x)**3/(2*d) + 8*B*b**4*sin(c + d*x)**5/(15*d) + 4*B*b**
4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + B*b**4*sin(c + d*x)*cos(c + d*x)*
*4/d, Ne(d, 0)), (x*(A + B*cos(c))*(a + b*cos(c))**4, True))
```

3.243 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=200

$$\frac{a^4A\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\left(26a^2B+32aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{b\left(19a^3B+34a^2Ab+16ab^2B+4\right.}{6d}$$

```
[Out] 1/8*(32*A*a^3*b+16*A*a*b^3+8*B*a^4+24*B*a^2*b^2+3*B*b^4)*x+a^4*A*arctanh(si
n(d*x+c))/d+1/6*b*(34*A*a^2*b+4*A*b^3+19*B*a^3+16*B*a*b^2)*sin(d*x+c)/d+1/2
4*b^2*(32*A*a*b+26*B*a^2+9*B*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/12*b*(4*A*b+7*B
*a)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+1/4*b*B*(a+b*cos(d*x+c))^3*sin(d*x+c)/d
```

Rubi [A] time = 0.55, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2990, 3049, 3033, 3023, 2735, 3770}

$$\frac{b\left(34a^2Ab+19a^3B+16ab^2B+4Ab^3\right)\sin(c+dx)}{6d}+\frac{b^2\left(26a^2B+32aAb+9b^2B\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(32a\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] ((32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4*B)*x)/8 + (a^4*A
*ArcTanh[Sin[c + d*x]])/d + (b*(34*a^2*A*b + 4*A*b^3 + 19*a^3*B + 16*a*b^2*
B)*Sin[c + d*x])/(6*d) + (b^2*(32*a*A*b + 26*a^2*B + 9*b^2*B)*Cos[c + d*x]*
Sin[c + d*x])/(24*d) + (b*(4*A*b + 7*a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*
x])/(12*d) + (b*B*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
```

```
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{bB(a+b\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}\int (a+b\cos($$
$$= \frac{b(4Ab+7aB)(a+b\cos(c+dx))^2\sin(c+dx)}{12d}+\frac{bB(a$$
$$= \frac{b^2\left(32aAb+26a^2B+9b^2B\right)\cos(c+dx)\sin(c+dx)}{24d}$$
$$= \frac{b\left(34a^2Ab+4Ab^3+19a^3B+16ab^2B\right)\sin(c+dx)}{6d}+$$
$$= \frac{1}{8}\left(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B\right)x+$$
$$= \frac{1}{8}\left(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B\right)x+$$

Mathematica [A] time = 0.61, size = 210, normalized size = 1.05

$$\frac{-96a^4A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+96a^4A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+24b^2\left(6a^2B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (12*(32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4*B)*(c + d*x)
- 96*a^4*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 96*a^4*A*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 24*b*(24*a^2*A*b + 3*A*b^3 + 16*a^3*B + 12*a
*b^2*B)*Sin[c + d*x] + 24*b^2*(4*a*A*b + 6*a^2*B + b^2*B)*Sin[2*(c + d*x)]
+ 8*b^3*(A*b + 4*a*B)*Sin[3*(c + d*x)] + 3*b^4*B*Sin[4*(c + d*x)])/(96*d)
```

fricas [A] time = 1.52, size = 183, normalized size = 0.92

$$\frac{12\,Aa^4\log\left(\sin\left(dx+c\right)+1\right)-12\,Aa^4\log\left(-\sin\left(dx+c\right)+1\right)+3\left(8\,Ba^4+32\,Aa^3b+24\,Ba^2b^2+16\,Aab^3+3\,Bb\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fric
as")
```

```
[Out] 1/24*(12*A*a^4*log(sin(d*x + c) + 1) - 12*A*a^4*log(-sin(d*x + c) + 1) + 3*
(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*d*x + (6*B*b^4
```

```
*cos(d*x + c)^3 + 96*B*a^3*b + 144*A*a^2*b^2 + 64*B*a*b^3 + 16*A*b^4 + 8*(4
*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 3*(24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*
cos(d*x + c))*sin(d*x + c))/d
```

giac [B] time = 0.64, size = 603, normalized size = 3.02

$$\frac{24\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,Aa^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(8\,Ba^4+32\,Aa^3b+24\,Ba^2b^2+16\,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac
")
```

```
[Out] 1/24*(24*A*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*A*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3
+ 3*B*b^4)*(d*x + c) + 2*(96*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 144*A*a^2*b^2
*tan(1/2*d*x + 1/2*c)^7 - 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b^3*
tan(1/2*d*x + 1/2*c)^7 + 96*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*A*b^4*tan(1
/2*d*x + 1/2*c)^7 - 15*B*b^4*tan(1/2*d*x + 1/2*c)^7 + 288*B*a^3*b*tan(1/2*d
*x + 1/2*c)^5 + 432*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 72*B*a^2*b^2*tan(1/2
*d*x + 1/2*c)^5 - 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 160*B*a*b^3*tan(1/2*d
*x + 1/2*c)^5 + 40*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 9*B*b^4*tan(1/2*d*x + 1/2
*c)^5 + 288*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 432*A*a^2*b^2*tan(1/2*d*x + 1/
2*c)^3 + 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 48*A*a*b^3*tan(1/2*d*x + 1/2
*c)^3 + 160*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 40*A*b^4*tan(1/2*d*x + 1/2*c)^
3 - 9*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 96*B*a^3*b*tan(1/2*d*x + 1/2*c) + 144*
A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 48*A*a
*b^3*tan(1/2*d*x + 1/2*c) + 96*B*a*b^3*tan(1/2*d*x + 1/2*c) + 24*A*b^4*tan(
1/2*d*x + 1/2*c) + 15*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 +
 1)^4)/d
```

maple [A] time = 0.12, size = 319, normalized size = 1.60

$$\frac{A\,a^4\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+a^4Bx+\frac{a^4Bc}{d}+4A\,a^3bx+\frac{4A\,a^3bc}{d}+\frac{4B\,a^3b\sin\left(dx+c\right)}{d}+\frac{6A\,a^2b^2\sin\left(dx+c\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 1/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*B*x+1/d*a^4*B*c+4*A*a^3*b*x+4/d*A*a
^3*b*c+4/d*B*a^3*b*sin(d*x+c)+6/d*A*a^2*b^2*sin(d*x+c)+3/d*B*a^2*b^2*cos(d*
x+c)*sin(d*x+c)+3*B*a^2*b^2*x+3/d*B*a^2*b^2*c+2/d*A*a*b^3*cos(d*x+c)*sin(d*
```

```
x+c)+2*A*a*b^3*x+2/d*A*a*b^3*c+4/3/d*B*sin(d*x+c)*cos(d*x+c)^2*a*b^3+8/3/d*
B*a*b^3*sin(d*x+c)+1/3/d*A*sin(d*x+c)*cos(d*x+c)^2*b^4+2/3/d*A*b^4*sin(d*x+
c)+1/4/d*B*b^4*sin(d*x+c)*cos(d*x+c)^3+3/8/d*B*b^4*cos(d*x+c)*sin(d*x+c)+3/
8*b^4*B*x+3/8/d*B*b^4*c
```

maxima [A] time = 0.34, size = 208, normalized size = 1.04

$$\frac{96\,(dx+c)Ba^4+384\,(dx+c)Aa^3b+144\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2b^2+96\,(2\,dx+2\,c+\sin(2\,dx+2\,c))A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(96*(d*x + c)*B*a^4 + 384*(d*x + c)*A*a^3*b + 144*(2*d*x + 2*c + sin(2
*d*x + 2*c))*B*a^2*b^2 + 96*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b^3 - 128*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a*b^3 - 32*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*A*b^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*b
^4 + 96*A*a^4*log(sec(d*x + c) + tan(d*x + c)) + 384*B*a^3*b*sin(d*x + c) +
 576*A*a^2*b^2*sin(d*x + c))/d
```

mupad [B] time = 1.42, size = 369, normalized size = 1.84

$$\frac{3\,A\,b^4\,\sin\left(c+d\,x\right)}{4\,d}+\frac{2\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,B\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}+\frac{A\,b^4\,\sin\left(3\,c+\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x),x)
```

```
[Out] (3*A*b^4*sin(c + d*x))/(4*d) + (2*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/d + (2*B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3
*B*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*d) + (A*b^4*sin(3*c
+ 3*d*x))/(12*d) + (B*b^4*sin(2*c + 2*d*x))/(4*d) + (B*b^4*sin(4*c + 4*d*x)
)/(32*d) + (4*A*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*A
*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a*b^3*sin(2*c +
2*d*x))/d + (6*A*a^2*b^2*sin(c + d*x))/d + (B*a*b^3*sin(3*c + 3*d*x))/(3*d)
 + (6*B*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*B*a^2*b
^2*sin(2*c + 2*d*x))/(2*d) + (3*B*a*b^3*sin(c + d*x))/d + (4*B*a^3*b*sin(c
+ d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\,(a+b\cos(c+dx))^4\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**4*sec(c + d*x), x)
```

3.244 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=195

$$\frac{a^3(aB+4Ab)\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\left(6a^2A-8abB-3Ab^2\right)\sin(c+dx)\cos(c+dx)}{6d}-\frac{b\left(6a^3A-17a^2bB-12aA\right.}{3d}$$

```
[Out] 1/2*b*(12*A*a^2*b+A*b^3+8*B*a^3+4*B*a*b^2)*x+a^3*(4*A*b+B*a)*arctanh(sin(d*
x+c))/d-1/3*b*(6*A*a^3-12*A*a*b^2-17*B*a^2*b-2*B*b^3)*sin(d*x+c)/d-1/6*b^2*
(6*A*a^2-3*A*b^2-8*B*a*b)*cos(d*x+c)*sin(d*x+c)/d-1/3*b*(3*A*a-B*b)*(a+b*co
s(d*x+c))^2*sin(d*x+c)/d+a*A*(a+b*cos(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A] time = 0.57, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3049, 3033, 3023, 2735, 3770}

$$-\frac{b\left(6a^3A-17a^2bB-12aAb^2-2b^3B\right)\sin(c+dx)}{3d}-\frac{b^2\left(6a^2A-8abB-3Ab^2\right)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}bx\left(12a^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (b*(12*a^2*A*b + A*b^3 + 8*a^3*B + 4*a*b^2*B)*x)/2 + (a^3*(4*A*b + a*B)*Arc
Tanh[Sin[c + d*x]])/d - (b*(6*a^3*A - 12*a*A*b^2 - 17*a^2*b*B - 2*b^3*B)*Si
n[c + d*x])/(3*d) - (b^2*(6*a^2*A - 3*A*b^2 - 8*a*b*B)*Cos[c + d*x]*Sin[c +
 d*x])/(6*d) - (b*(3*a*A - b*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
+ (a*A*(a + b*Cos[c + d*x])^3*Tan[c + d*x])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
```

```
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\tan(c+dx)}{d} + \int (a+b\cos(c$$

$$= -\frac{b(3aA-bB)(a+b\cos(c+dx))^2\sin(c+dx)}{3d} + \frac{aA($$

$$= -\frac{b^2\left(6a^2A-3Ab^2-8abB\right)\cos(c+dx)\sin(c+dx)}{6d}$$

$$= -\frac{b\left(6a^3A-12aAb^2-17a^2bB-2b^3B\right)\sin(c+dx)}{3d} -$$

$$= \frac{1}{2}b\left(12a^2Ab+Ab^3+8a^3B+4ab^2B\right)x - \frac{b\left(6a^3A-1\right.}{}$$

$$= \frac{1}{2}b\left(12a^2Ab+Ab^3+8a^3B+4ab^2B\right)x + \frac{a^3(4Ab+a}{}$$

Mathematica [A] time = 1.07, size = 257, normalized size = 1.32

$$\frac{\frac{12a^4A\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{12a^4A\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)} - 12a^3(aB+4Ab)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 12a^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (6*b*(12*a^2*A*b + A*b^3 + 8*a^3*B + 4*a*b^2*B)*(c + d*x) - 12*a^3*(4*A*b +
 a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^3*(4*A*b + a*B)*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*a^4*A*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]) + (12*a^4*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]) + 3*b^2*(16*a*A*b + 24*a^2*B + 3*b^2*B)*Sin[c + d*x]
+ 3*b^3*(A*b + 4*a*B)*Sin[2*(c + d*x)] + b^4*B*Sin[3*(c + d*x)])/(12*d)
```

fricas [A] time = 1.59, size = 196, normalized size = 1.01

$$\frac{3\left(8\,Ba^3b+12\,Aa^2b^2+4\,Bab^3+Ab^4\right)dx\cos(dx+c) + 3\left(Ba^4+4\,Aa^3b\right)\cos(dx+c)\log(\sin(dx+c)+1) - 3\left(B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(8*B*a^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*d*x*cos(d*x + c) + 3*
(B*a^4 + 4*A*a^3*b)*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*(B*a^4 + 4*A*a^3
*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + (2*B*b^4*cos(d*x + c)^3 + 6*A*a^4
 + 3*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 4*(9*B*a^2*b^2 + 6*A*a*b^3 + B*b^
4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 1.23, size = 371, normalized size = 1.90

$$-\frac{\frac{12\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - 3\left(8\,Ba^3b + 12\,Aa^2b^2 + 4\,Bab^3 + Ab^4\right)(dx+c) - 6\left(Ba^4 + 4\,Aa^3b\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(8*B*a
^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c) - 6*(B*a^4 + 4*A*a^3*b)*
log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6*(B*a^4 + 4*A*a^3*b)*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(36*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 24*A*a*b^3*ta
n(1/2*d*x + 1/2*c)^5 - 12*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^4*tan(1/2*
d*x + 1/2*c)^5 + 6*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 72*B*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*B*b^4*tan(1/2*d*x + 1/2*
c)^3 + 36*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 24*A*a*b^3*tan(1/2*d*x + 1/2*c)
+ 12*B*a*b^3*tan(1/2*d*x + 1/2*c) + 3*A*b^4*tan(1/2*d*x + 1/2*c) + 6*B*b^4*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

maple [A] time = 0.13, size = 255, normalized size = 1.31

$$\frac{A\,a^4\tan(dx+c)}{d}+\frac{a^4B\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{4A\,a^3b\ln(\sec(dx+c)+\tan(dx+c))}{d}+4B\,a^3bx+\frac{4B\,a^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] 1/d*A*a^4*tan(d*x+c)+1/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+4/d*A*a^3*b*ln(sec
(d*x+c)+tan(d*x+c))+4*B*a^3*b*x+4/d*B*a^3*b*c+6*A*a^2*b^2*x+6/d*A*a^2*b^2*c
+6/d*B*a^2*b^2*sin(d*x+c)+4/d*A*a*b^3*sin(d*x+c)+2/d*B*a*b^3*cos(d*x+c)*sin
(d*x+c)+2*B*a*b^3*x+2/d*B*a*b^3*c+1/2/d*A*b^4*cos(d*x+c)*sin(d*x+c)+1/2*A*b
^4*x+1/2/d*A*b^4*c+1/3/d*B*sin(d*x+c)*cos(d*x+c)^2*b^4+2/3/d*B*b^4*sin(d*x+
c)
```

maxima [A] time = 0.32, size = 197, normalized size = 1.01

$$\frac{48\,(dx+c)Ba^3b + 72\,(dx+c)Aa^2b^2 + 12\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Bab^3 + 3\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Ab^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="ma
xima")
```

```
[Out] 1/12*(48*(d*x + c)*B*a^3*b + 72*(d*x + c)*A*a^2*b^2 + 12*(2*d*x + 2*c + sin
(2*d*x + 2*c))*B*a*b^3 + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^4 - 4*(sin(
d*x + c)^3 - 3*sin(d*x + c))*B*b^4 + 6*B*a^4*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 24*A*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1)) + 72*B*a^2*b^2*sin(d*x + c) + 48*A*a*b^3*sin(d*x + c) + 12*A*a^4*tan(d*
x + c))/d
```

mupad [B] time = 2.27, size = 2522, normalized size = 12.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(2*A*a^4 + A*b^4 + 2*B*b^4 + 12*B*a^2*b^2 + 8*A*a*b^3 +
 4*B*a*b^3) + tan(c/2 + (d*x)/2)^7*(2*A*a^4 + A*b^4 - 2*B*b^4 - 12*B*a^2*b^
2 - 8*A*a*b^3 + 4*B*a*b^3) + tan(c/2 + (d*x)/2)^3*(6*A*a^4 - A*b^4 - (2*B*b
^4)/3 + 12*B*a^2*b^2 + 8*A*a*b^3 - 4*B*a*b^3) - tan(c/2 + (d*x)/2)^5*(A*b^4
 - 6*A*a^4 - (2*B*b^4)/3 + 12*B*a^2*b^2 + 8*A*a*b^3 + 4*B*a*b^3))/(d*(2*tan
(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 1)) - (
atan(((B*a^4 + 4*A*a^3*b)*((B*a^4 + 4*A*a^3*b)*(16*A*b^4 + 32*B*a^4 + 192*A
*a^2*b^2 + 128*A*a^3*b + 64*B*a*b^3 + 128*B*a^3*b) + tan(c/2 + (d*x)/2)*(8*
A^2*b^8 + 32*B^2*a^8 + 192*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*b^2
 + 128*B^2*a^2*b^6 + 512*B^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B*a*b^7 + 256
*A*B*a^7*b + 896*A*B*a^3*b^5 + 1536*A*B*a^5*b^3))*1i - (B*a^4 + 4*A*a^3*b)*
((B*a^4 + 4*A*a^3*b)*(16*A*b^4 + 32*B*a^4 + 192*A*a^2*b^2 + 128*A*a^3*b + 6
4*B*a*b^3 + 128*B*a^3*b) - tan(c/2 + (d*x)/2)*(8*A^2*b^8 + 32*B^2*a^8 + 192
*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*b^2 + 128*B^2*a^2*b^6 + 512*B
^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B*a*b^7 + 256*A*B*a^7*b + 896*A*B*a^3*b
^5 + 1536*A*B*a^5*b^3))*1i)/((B*a^4 + 4*A*a^3*b)*((B*a^4 + 4*A*a^3*b)*(16*A
*b^4 + 32*B*a^4 + 192*A*a^2*b^2 + 128*A*a^3*b + 64*B*a*b^3 + 128*B*a^3*b) +
 tan(c/2 + (d*x)/2)*(8*A^2*b^8 + 32*B^2*a^8 + 192*A^2*a^2*b^6 + 1152*A^2*a^
4*b^4 + 512*A^2*a^6*b^2 + 128*B^2*a^2*b^6 + 512*B^2*a^4*b^4 + 512*B^2*a^6*b
^2 + 64*A*B*a*b^7 + 256*A*B*a^7*b + 896*A*B*a^3*b^5 + 1536*A*B*a^5*b^3)) +
(B*a^4 + 4*A*a^3*b)*((B*a^4 + 4*A*a^3*b)*(16*A*b^4 + 32*B*a^4 + 192*A*a^2*b
```

```
^2 + 128*A*a^3*b + 64*B*a*b^3 + 128*B*a^3*b) - tan(c/2 + (d*x)/2)*(8*A^2*b^
8 + 32*B^2*a^8 + 192*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*b^2 + 128
*B^2*a^2*b^6 + 512*B^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B*a*b^7 + 256*A*B*a
^7*b + 896*A*B*a^3*b^5 + 1536*A*B*a^5*b^3)) - 256*B^3*a^11*b + 64*A^3*a^3*b
^9 + 1536*A^3*a^5*b^7 - 512*A^3*a^6*b^6 + 9216*A^3*a^7*b^5 - 6144*A^3*a^8*b
^4 + 256*B^3*a^6*b^6 + 1024*B^3*a^8*b^4 - 128*B^3*a^9*b^3 + 1024*B^3*a^10*b
^2 + 1152*A*B^2*a^5*b^7 + 5888*A*B^2*a^7*b^5 - 1056*A*B^2*a^8*b^4 + 7168*A*
B^2*a^9*b^3 - 2432*A*B^2*a^10*b^2 + 528*A^2*B*a^4*b^8 + 7552*A^2*B*a^6*b^6
- 2304*A^2*B*a^7*b^5 + 14592*A^2*B*a^8*b^4 - 7168*A^2*B*a^9*b^3))*(B*a^4*2i
 + A*a^3*b*8i))/d - (b*atan(((b*(tan(c/2 + (d*x)/2)*(8*A^2*b^8 + 32*B^2*a^8
 + 192*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*b^2 + 128*B^2*a^2*b^6 +
 512*B^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B*a*b^7 + 256*A*B*a^7*b + 896*A*B
*a^3*b^5 + 1536*A*B*a^5*b^3) - (b*(A*b^3 + 8*B*a^3 + 12*A*a^2*b + 4*B*a*b^2
)*(16*A*b^4 + 32*B*a^4 + 192*A*a^2*b^2 + 128*A*a^3*b + 64*B*a*b^3 + 128*B*a
^3*b)*1i)/2)*(A*b^3 + 8*B*a^3 + 12*A*a^2*b + 4*B*a*b^2))/2 + (b*(tan(c/2 +
(d*x)/2)*(8*A^2*b^8 + 32*B^2*a^8 + 192*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512
*A^2*a^6*b^2 + 128*B^2*a^2*b^6 + 512*B^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B
*a*b^7 + 256*A*B*a^7*b + 896*A*B*a^3*b^5 + 1536*A*B*a^5*b^3) + (b*(A*b^3 +
8*B*a^3 + 12*A*a^2*b + 4*B*a*b^2)*(16*A*b^4 + 32*B*a^4 + 192*A*a^2*b^2 + 12
8*A*a^3*b + 64*B*a*b^3 + 128*B*a^3*b)*1i)/2)*(A*b^3 + 8*B*a^3 + 12*A*a^2*b
+ 4*B*a*b^2))/2)/(64*A^3*a^3*b^9 - 256*B^3*a^11*b + 1536*A^3*a^5*b^7 - 512*
A^3*a^6*b^6 + 9216*A^3*a^7*b^5 - 6144*A^3*a^8*b^4 + 256*B^3*a^6*b^6 + 1024*
B^3*a^8*b^4 - 128*B^3*a^9*b^3 + 1024*B^3*a^10*b^2 - (b*(tan(c/2 + (d*x)/2)*
(8*A^2*b^8 + 32*B^2*a^8 + 192*A^2*a^2*b^6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*
b^2 + 128*B^2*a^2*b^6 + 512*B^2*a^4*b^4 + 512*B^2*a^6*b^2 + 64*A*B*a*b^7 +
256*A*B*a^7*b + 896*A*B*a^3*b^5 + 1536*A*B*a^5*b^3) - (b*(A*b^3 + 8*B*a^3 +
 12*A*a^2*b + 4*B*a*b^2)*(16*A*b^4 + 32*B*a^4 + 192*A*a^2*b^2 + 128*A*a^3*b
 + 64*B*a*b^3 + 128*B*a^3*b)*1i)/2)*(A*b^3 + 8*B*a^3 + 12*A*a^2*b + 4*B*a*b
^2)*1i)/2 + (b*(tan(c/2 + (d*x)/2)*(8*A^2*b^8 + 32*B^2*a^8 + 192*A^2*a^2*b^
6 + 1152*A^2*a^4*b^4 + 512*A^2*a^6*b^2 + 128*B^2*a^2*b^6 + 512*B^2*a^4*b^4
+ 512*B^2*a^6*b^2 + 64*A*B*a*b^7 + 256*A*B*a^7*b + 896*A*B*a^3*b^5 + 1536*A
*B*a^5*b^3) + (b*(A*b^3 + 8*B*a^3 + 12*A*a^2*b + 4*B*a*b^2)*(16*A*b^4 + 32*
B*a^4 + 192*A*a^2*b^2 + 128*A*a^3*b + 64*B*a*b^3 + 128*B*a^3*b)*1i)/2)*(A*b
^3 + 8*B*a^3 + 12*A*a^2*b + 4*B*a*b^2)*1i)/2 + 1152*A*B^2*a^5*b^7 + 5888*A*
B^2*a^7*b^5 - 1056*A*B^2*a^8*b^4 + 7168*A*B^2*a^9*b^3 - 2432*A*B^2*a^10*b^2
 + 528*A^2*B*a^4*b^8 + 7552*A^2*B*a^6*b^6 - 2304*A^2*B*a^7*b^5 + 14592*A^2*
B*a^8*b^4 - 7168*A^2*B*a^9*b^3))*(A*b^3 + 8*B*a^3 + 12*A*a^2*b + 4*B*a*b^2)
)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.245 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=209

$$\frac{a^2\left(a^2A+8abB+12Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\left(2a^2B+6aAb-b^2B\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}b^2x\left(12a^2B\right.$$

```
[Out] 1/2*b^2*(8*A*a*b+12*B*a^2+B*b^2)*x+1/2*a^2*(A*a^2+12*A*b^2+8*B*a*b)*arctanh
(sin(d*x+c))/d-1/2*b*(13*A*a^2*b-2*A*b^3+4*B*a^3-8*B*a*b^2)*sin(d*x+c)/d-1/
2*b^2*(6*A*a*b+2*B*a^2-B*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/2*a*(5*A*b+2*B*a)*(
a+b*cos(d*x+c))^2*tan(d*x+c)/d+1/2*a*A*(a+b*cos(d*x+c))^3*sec(d*x+c)*tan(d*
x+c)/d
```

Rubi [A] time = 0.62, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3047, 3033, 3023, 2735, 3770}

$$-\frac{b\left(13a^2Ab+4a^3B-8ab^2B-2Ab^3\right)\sin(c+dx)}{2d}+\frac{a^2\left(a^2A+8abB+12Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\left(2a^2B+6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (b^2*(8*a*A*b + 12*a^2*B + b^2*B)*x)/2 + (a^2*(a^2*A + 12*A*b^2 + 8*a*b*B)*
ArcTanh[Sin[c + d*x]])/(2*d) - (b*(13*a^2*A*b - 2*A*b^3 + 4*a^3*B - 8*a*b^2
*B)*Sin[c + d*x])/(2*d) - (b^2*(6*a*A*b + 2*a^2*B - b^2*B)*Cos[c + d*x]*Sin
[c + d*x])/(2*d) + (a*(5*A*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/
(2*d) + (a*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
```

```
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}$$
$$= \frac{a(5Ab+2aB)(a+b\cos(c+dx))^2\tan(c+dx)}{2d} + \frac{aA}{}$$
$$= -\frac{b^2\left(6aAb+2a^2B-b^2B\right)\cos(c+dx)\sin(c+dx)}{2d}$$
$$= -\frac{b\left(13a^2Ab-2Ab^3+4a^3B-8ab^2B\right)\sin(c+dx)}{2d}$$
$$= \frac{1}{2}b^2\left(8aAb+12a^2B+b^2B\right)x - \frac{b\left(13a^2Ab-2Ab^3\right.}{}$$
$$= \frac{1}{2}b^2\left(8aAb+12a^2B+b^2B\right)x + \frac{a^2\left(a^2A+12Ab^2+\right.}{}$$

Mathematica [A] time = 1.85, size = 310, normalized size = 1.48

$$\frac{\frac{a^4A}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{a^4A}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{4a^3(aB+4Ab)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{4a^3(aB+4Ab)\sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)} + 2b^2(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^2*(8*a*A*b + 12*a^2*B + b^2*B)*(c + d*x) - 2*a^2*(a^2*A + 12*A*b^2 + 8
*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*(a^2*A + 12*A*b^2
+ 8*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^4*A)/(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^2 + (4*a^3*(4*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]) - (a^4*A)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2 + (4*a^3*(4*A*b + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]) + 4*b^3*(A*b + 4*a*B)*Sin[c + d*x] + b^4*B*Sin[2*(c + d*x)])/(4*d
)
```

fricas [A] time = 0.70, size = 202, normalized size = 0.97

$$\frac{2\left(12\,Ba^2b^2+8\,Aab^3+Bb^4\right)dx\cos\left(dx+c\right)^2+\left(Aa^4+8\,Ba^3b+12\,Aa^2b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fr
icas")
```

```
[Out] 1/4*(2*(12*B*a^2*b^2 + 8*A*a*b^3 + B*b^4)*d*x*cos(d*x + c)^2 + (A*a^4 + 8*B
*a^3*b + 12*A*a^2*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a^4 + 8*B*
a^3*b + 12*A*a^2*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(B*b^4*cos(
d*x + c)^3 + A*a^4 + 2*(4*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 2*(B*a^4 + 4*A*
a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.69, size = 526, normalized size = 2.52

$$\frac{\left(12\,Ba^2b^2 + 8\,Aab^3 + Bb^4\right)(dx + c) + \left(Aa^4 + 8\,Ba^3b + 12\,Aa^2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(Aa^4 + 8\,Ba^3b + 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*((12*B*a^2*b^2 + 8*A*a*b^3 + B*b^4)*(d*x + c) + (A*a^4 + 8*B*a^3*b + 12
*A*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a^4 + 8*B*a^3*b + 12*A*
a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^4*tan(1/2*d*x + 1/2*c)
^7 - 2*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 8*
B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 2*A*b^4*tan(1/2*d*x + 1/2*c)^7 - B*b^4*tan
(1/2*d*x + 1/2*c)^7 + 3*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 2*B*a^4*tan(1/2*d*x
+ 1/2*c)^5 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 8*B*a*b^3*tan(1/2*d*x + 1/2
*c)^5 - 2*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 3
*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 8*A*a^3*b*
tan(1/2*d*x + 1/2*c)^3 - 8*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*A*b^4*tan(1/2
*d*x + 1/2*c)^3 - 3*B*b^4*tan(1/2*d*x + 1/2*c)^3 + A*a^4*tan(1/2*d*x + 1/2*
c) + 2*B*a^4*tan(1/2*d*x + 1/2*c) + 8*A*a^3*b*tan(1/2*d*x + 1/2*c) + 8*B*a*
b^3*tan(1/2*d*x + 1/2*c) + 2*A*b^4*tan(1/2*d*x + 1/2*c) + B*b^4*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

maple [A] time = 0.15, size = 236, normalized size = 1.13

$$\frac{A\,a^4\sec(dx + c)\tan(dx + c)}{2d} + \frac{A\,a^4\ln(\sec(dx + c) + \tan(dx + c))}{2d} + \frac{a^4B\tan(dx + c)}{d} + \frac{4A\,a^3b\tan(dx + c)}{d} + \frac{4B\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2/d*A*a^4*sec(d*x+c)*tan(d*x+c)+1/2/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+1/d
*a^4*B*tan(d*x+c)+4/d*A*a^3*b*tan(d*x+c)+4/d*B*a^3*b*ln(sec(d*x+c)+tan(d*x+
```

```
c))+6/d*A*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+6*B*a^2*b^2*x+6/d*B*a^2*b^2*c+4
*A*a*b^3*x+4/d*A*a*b^3*c+4/d*B*a*b^3*sin(d*x+c)+1/d*A*b^4*sin(d*x+c)+1/2/d*
B*b^4*cos(d*x+c)*sin(d*x+c)+1/2*b^4*B*x+1/2/d*B*b^4*c
```

maxima [A] time = 1.21, size = 209, normalized size = 1.00

$$24\,(dx+c)Ba^2b^2+16\,(dx+c)Aab^3+(2\,dx+2\,c+\sin(2\,dx+2\,c))Bb^4-Aa^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="ma
xima")
```

```
[Out] 1/4*(24*(d*x + c)*B*a^2*b^2 + 16*(d*x + c)*A*a*b^3 + (2*d*x + 2*c + sin(2*d
*x + 2*c))*B*b^4 - A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 8*B*a^3*b*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 12*A*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x +
c) - 1)) + 16*B*a*b^3*sin(d*x + c) + 4*A*b^4*sin(d*x + c) + 4*B*a^4*tan(d*x
 + c) + 16*A*a^3*b*tan(d*x + c))/d
```

mupad [B] time = 2.31, size = 330, normalized size = 1.58

$$\frac{2\left(\frac{A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{B\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+4\,A\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+4\,B\,a^3\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+6\,A\,a^2\,b^2\,\mathrm{a}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^3,x)
```

```
[Out] (2*((A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (B*b^4*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + 4*A*a*b^3*atan(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)) + 4*B*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)) + 6*A*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 6*B*a^2*b^2
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/d + ((B*a^4*sin(2*c + 2*d*x)
)/2 + (A*b^4*sin(3*c + 3*d*x))/4 + (B*b^4*sin(2*c + 2*d*x))/8 + (B*b^4*sin(
4*c + 4*d*x))/16 + (A*a^4*sin(c + d*x))/2 + (A*b^4*sin(c + d*x))/4 + B*a*b^
3*sin(c + d*x) + 2*A*a^3*b*sin(2*c + 2*d*x) + B*a*b^3*sin(3*c + 3*d*x))/(d*
(cos(2*c + 2*d*x)/2 + 1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.246 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=198

$$-\frac{b^2\left(3a^2B+8aAb-6b^2B\right)\sin(c+dx)}{6d}+\frac{a^2\left(2a^2A+9abB+9Ab^2\right)\tan(c+dx)}{3d}+\frac{a\left(a^3B+4a^2Ab+12ab^2B+8A\right.}{2d}$$

```
[Out] b^3*(A*b+4*B*a)*x+1/2*a*(4*A*a^2*b+8*A*b^3+B*a^3+12*B*a*b^2)*arctanh(sin(d*
x+c))/d-1/6*b^2*(8*A*a*b+3*B*a^2-6*B*b^2)*sin(d*x+c)/d+1/3*a^2*(2*A*a^2+9*A
*b^2+9*B*a*b)*tan(d*x+c)/d+1/2*a*(2*A*b+B*a)*(a+b*cos(d*x+c))^2*sec(d*x+c)*
tan(d*x+c)/d+1/3*a*A*(a+b*cos(d*x+c))^3*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] time = 0.58, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3047, 3031, 3023, 2735, 3770}

$$-\frac{b^2\left(3a^2B+8aAb-6b^2B\right)\sin(c+dx)}{6d}+\frac{a^2\left(2a^2A+9abB+9Ab^2\right)\tan(c+dx)}{3d}+\frac{a\left(4a^2Ab+a^3B+12ab^2B+8A\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] b^3*(A*b + 4*a*B)*x + (a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b^2*B)*ArcTanh
[Sin[c + d*x]])/(2*d) - (b^2*(8*a*A*b + 3*a^2*B - 6*b^2*B)*Sin[c + d*x])/(6
*d) + (a^2*(2*a^2*A + 9*A*b^2 + 9*a*b*B)*Tan[c + d*x])/(3*d) + (a*(2*A*b +
a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*A*(a + b*
Cos[c + d*x])^3*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
```

```
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)}{3d} +$$

$$= \frac{a(2Ab+aB)(a+b\cos(c+dx))^2\sec(c+dx)\tan(c$$

$$= \frac{a^2\left(2a^2A+9Ab^2+9abB\right)\tan(c+dx)}{3d} + \frac{a(2Ab+$$

$$= -\frac{b^2\left(8aAb+3a^2B-6b^2B\right)\sin(c+dx)}{6d} + \frac{a^2\left(2a^2A$$

$$= b^3(Ab+4aB)x - \frac{b^2\left(8aAb+3a^2B-6b^2B\right)\sin(c+$$

$$= b^3(Ab+4aB)x + \frac{a\left(4a^2Ab+8Ab^3+a^3B+12ab^2$$

Mathematica [B] time = 5.95, size = 415, normalized size = 2.10

$$\frac{\frac{2a^4A\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2a^4A\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{a^3(a(A+3B)+12Ab)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{a^3(a(A+3B)+12Ab)}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{8a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (12*b^3*(A*b + 4*a*B)*(c + d*x) - 6*a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b
^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*a*(4*a^2*A*b + 8*A*b^3 +
 a^3*B + 12*a*b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*(12*A*
b + a*(A + 3*B)))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*a^4*A*Sin[(c
 + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (8*a^2*(a^2*A + 9*A*b
^2 + 6*a*b*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (2*
a^4*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - (a^3*(12*
A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (8*a^2*(a^2*A
 + 9*A*b^2 + 6*a*b*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]) + 12*b^4*B*Sin[c + d*x])/(12*d)
```

fricas [A] time = 1.62, size = 219, normalized size = 1.11

$$\frac{12\left(4Bab^3+Ab^4\right)dx\cos(dx+c)^3 + 3\left(Ba^4+4Aa^3b+12Ba^2b^2+8Aab^3\right)\cos(dx+c)^3\log(\sin(dx+c)+1) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fr
icas")
```

```
[Out] 1/12*(12*(4*B*a*b^3 + A*b^4)*d*x*cos(d*x + c)^3 + 3*(B*a^4 + 4*A*a^3*b + 12
*B*a^2*b^2 + 8*A*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^4 + 4
*A*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1)
+ 2*(6*B*b^4*cos(d*x + c)^3 + 2*A*a^4 + 4*(A*a^4 + 6*B*a^3*b + 9*A*a^2*b^2)
*cos(d*x + c)^2 + 3*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^3)
```

giac [B] time = 0.68, size = 387, normalized size = 1.95

$$\frac{\frac{12\,Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+6\left(4\,Bab^3+Ab^4\right)(dx+c)+3\left(Ba^4+4\,Aa^3b+12\,Ba^2b^2+8\,Aab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="gi
ac")
```

```
[Out] 1/6*(12*B*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(4*B*a*
b^3 + A*b^4)*(d*x + c) + 3*(B*a^4 + 4*A*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*l
og(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a^4 + 4*A*a^3*b + 12*B*a^2*b^2 + 8
*A*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a^4*tan(1/2*d*x + 1/2
*c)^5 - 3*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^3*b*tan(1/2*d*x + 1/2*c)^5
+ 24*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 36*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 -
 4*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*A*
a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^4*tan(1/2*d*x + 1/2*c) + 3*B*a^4*tan
(1/2*d*x + 1/2*c) + 12*A*a^3*b*tan(1/2*d*x + 1/2*c) + 24*B*a^3*b*tan(1/2*d*
x + 1/2*c) + 36*A*a^2*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^3)/d
```

maple [A] time = 0.16, size = 262, normalized size = 1.32

$$\frac{2A\,a^4\tan(dx+c)}{3d}+\frac{A\,a^4\tan(dx+c)\left(\sec^2(dx+c)\right)}{3d}+\frac{a^4B\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^4B\ln(\sec(dx+c)+\tan(d}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 2/3/d*A*a^4*tan(d*x+c)+1/3/d*A*a^4*tan(d*x+c)*sec(d*x+c)^2+1/2/d*a^4*B*sec(
d*x+c)*tan(d*x+c)+1/2/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+2/d*A*a^3*b*sec(d*x
```

```
+c)*tan(d*x+c)+2/d*A*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+4/d*B*a^3*b*tan(d*x+c)
+6/d*A*a^2*b^2*tan(d*x+c)+6/d*B*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+4/d*A*a*b
^3*ln(sec(d*x+c)+tan(d*x+c))+4*B*a*b^3*x+4/d*B*a*b^3*c+A*b^4*x+1/d*A*b^4*c+
1/d*B*b^4*sin(d*x+c)
```

maxima [A] time = 0.91, size = 245, normalized size = 1.24

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{4}+48\,(dx+c)Bab^{3}+12\,(dx+c)Ab^{4}-3\,Ba^{4}\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="ma
xima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 48*(d*x + c)*B*a*b^3 + 12
*(d*x + c)*A*b^4 - 3*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*A*a^3*b*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 36*B*a^2*b^
2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 24*A*a*b^3*(log(sin(d*x
 + c) + 1) - log(sin(d*x + c) - 1)) + 12*B*b^4*sin(d*x + c) + 48*B*a^3*b*ta
n(d*x + c) + 72*A*a^2*b^2*tan(d*x + c))/d
```

mupad [B] time = 2.83, size = 636, normalized size = 3.21

$$\frac{\frac{A\,a^4\,\sin(3\,c+3\,d\,x)}{6}+\frac{B\,a^4\,\sin(2\,c+2\,d\,x)}{4}+\frac{B\,b^4\,\sin(2\,c+2\,d\,x)}{4}+\frac{B\,b^4\,\sin(4\,c+4\,d\,x)}{8}+\frac{A\,a^4\,\sin(c+d\,x)}{2}+B\,a^3\,b\,\sin\left(c+d\,x\right)+\frac{3\,A\,b^4\,\mathrm{c}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^4,x)
```

```
[Out] ((A*a^4*sin(3*c + 3*d*x))/6 + (B*a^4*sin(2*c + 2*d*x))/4 + (B*b^4*sin(2*c +
 2*d*x))/4 + (B*b^4*sin(4*c + 4*d*x))/8 + (A*a^4*sin(c + d*x))/2 + B*a^3*b*
sin(c + d*x) + (3*A*b^4*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/2 - (B*a^4*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)
/2))*3i)/4 + A*a^3*b*sin(2*c + 2*d*x) + (3*A*a^2*b^2*sin(c + d*x))/2 + B*a^
3*b*sin(3*c + 3*d*x) + (A*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*c
os(3*c + 3*d*x))/2 - (B*a^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2)
)*cos(3*c + 3*d*x)*1i)/4 + (3*A*a^2*b^2*sin(3*c + 3*d*x))/2 - A*a*b^3*atan(
(sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*2i - A*a^3*b*a
tan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*1i + 2*B*a
```

```
*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x) - B*a^2*b
^2*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*9i - B*a^2
*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*3i -
 A*a*b^3*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*6i -
 A*a^3*b*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i +
 6*B*a*b^3*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*((3
*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.247 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=216

$$\frac{a^2\left(9a^2A+32abB+26Ab^2\right)\tan(c+dx)\sec(c+dx)}{24d}+\frac{a\left(4a^3B+16a^2Ab+34ab^2B+19Ab^3\right)\tan(c+dx)}{6d}+\frac{(3a^4}{}$$

```
[Out] b^4*B*x+1/8*(3*A*a^4+24*A*a^2*b^2+8*A*b^4+16*B*a^3*b+32*B*a*b^3)*arctanh(si
n(d*x+c))/d+1/6*a*(16*A*a^2*b+19*A*b^3+4*B*a^3+34*B*a*b^2)*tan(d*x+c)/d+1/2
4*a^2*(9*A*a^2+26*A*b^2+32*B*a*b)*sec(d*x+c)*tan(d*x+c)/d+1/12*a*(7*A*b+4*B
*a)*(a+b*cos(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a*A*(a+b*cos(d*x+c))^3
*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] time = 0.60, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3047, 3031, 3021, 2735, 3770}

$$\frac{a\left(16a^2Ab+4a^3B+34ab^2B+19Ab^3\right)\tan(c+dx)}{6d}+\frac{\left(24a^2Ab^2+3a^4A+16a^3bB+32ab^3B+8Ab^4\right)\tanh^{-1}(\sin}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] b^4*B*x + ((3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B)*Arc
Tanh[Sin[c + d*x]])/(8*d) + (a*(16*a^2*A*b + 19*A*b^3 + 4*a^3*B + 34*a*b^2*
B)*Tan[c + d*x])/(6*d) + (a^2*(9*a^2*A + 26*A*b^2 + 32*a*b*B)*Sec[c + d*x]*
Tan[c + d*x])/(24*d) + (a*(7*A*b + 4*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*
x]^2*Tan[c + d*x])/(12*d) + (a*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3*Tan[
c + d*x])/(4*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
```

```
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^5(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\sec^3(c+dx)\tan(c+dx)}{4d} +$$
$$= \frac{a(7Ab+4aB)(a+b\cos(c+dx))^2\sec^2(c+dx)\tan}{12d}$$
$$= \frac{a^2\left(9a^2A+26Ab^2+32abB\right)\sec(c+dx)\tan(c+d}{24d}$$
$$= \frac{a\left(16a^2Ab+19Ab^3+4a^3B+34ab^2B\right)\tan(c+dx)}{6d}$$
$$= b^4Bx + \frac{a\left(16a^2Ab+19Ab^3+4a^3B+34ab^2B\right)\tan(}{6d}$$
$$= b^4Bx + \frac{\left(3a^4A+24a^2Ab^2+8Ab^4+16a^3bB+32a\right.}{8d}$$

Mathematica [A] time = 1.08, size = 160, normalized size = 0.74

$$\frac{8a^3(aB+4Ab)\tan^3(c+dx)+3a\tan(c+dx)\left(2a^3A\sec^3(c+dx)+a\left(3a^2A+16abB+24Ab^2\right)\sec(c+dx)+8\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (24*b^4*B*d*x + 3*(3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3
*B)*ArcTanh[Sin[c + d*x]] + 3*a*(8*(4*a^2*A*b + 4*A*b^3 + a^3*B + 6*a*b^2*B
) + a*(3*a^2*A + 24*A*b^2 + 16*a*b*B)*Sec[c + d*x] + 2*a^3*A*Sec[c + d*x]^3
)*Tan[c + d*x] + 8*a^3*(4*A*b + a*B)*Tan[c + d*x]^3)/(24*d)
```

fricas [A] time = 0.84, size = 250, normalized size = 1.16

$$\frac{48\,Bb^4dx\cos(dx+c)^4+3\left(3\,Aa^4+16\,Ba^3b+24\,Aa^2b^2+32\,Bab^3+8\,Ab^4\right)\cos(dx+c)^4\log(\sin(dx+c)+1)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fr
icas")
```

```
[Out] 1/48*(48*B*b^4*d*x*cos(d*x + c)^4 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2
+ 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*A*a^4 +
 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^4*log(-sin(
d*x + c) + 1) + 2*(6*A*a^4 + 16*(B*a^4 + 4*A*a^3*b + 9*B*a^2*b^2 + 6*A*a*b^
3)*cos(d*x + c)^3 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2)*cos(d*x + c)^2
+ 8*(B*a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

giac [B] time = 0.69, size = 635, normalized size = 2.94

$$\frac{24\,(dx+c)Bb^4 + 3\left(3\,Aa^4 + 16\,Ba^3b + 24\,Aa^2b^2 + 32\,Bab^3 + 8\,Ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\left(3\,Aa^4 + 16\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="gi
ac")
```

```
[Out] 1/24*(24*(d*x + c)*B*b^4 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a*
b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a^4 + 16*B*a^3*b
 + 24*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
+ 2*(15*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 96
*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*A*
a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 96*
A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 40*B*a^4*
tan(1/2*d*x + 1/2*c)^5 + 160*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 48*B*a^3*b*ta
n(1/2*d*x + 1/2*c)^5 - 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 432*B*a^2*b^2*
tan(1/2*d*x + 1/2*c)^5 + 288*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^4*tan(1
/2*d*x + 1/2*c)^3 - 40*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*A*a^3*b*tan(1/2*d
*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*b^2*tan(1/2*d*
x + 1/2*c)^3 - 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*A*a*b^3*tan(1/2*d
*x + 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x + 1/2*c) + 24*B*a^4*tan(1/2*d*x + 1/2*
c) + 96*A*a^3*b*tan(1/2*d*x + 1/2*c) + 48*B*a^3*b*tan(1/2*d*x + 1/2*c) + 72
*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 144*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 96*A
*a*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

maple [A] time = 0.16, size = 338, normalized size = 1.56

$$\frac{A\,a^4\tan\left(dx+c\right)\left(\sec^3\left(dx+c\right)\right)}{4d} + \frac{3A\,a^4\sec\left(dx+c\right)\tan\left(dx+c\right)}{8d} + \frac{3A\,a^4\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8d} + \frac{2a^4B\,\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/4/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+3/8/d*A*a^4*sec(d*x+c)*tan(d*x+c)+3/8/d
*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a^4*B*tan(d*x+c)+1/3/d*a^4*B*tan(d*x
+c)*sec(d*x+c)^2+8/3/d*A*a^3*b*tan(d*x+c)+4/3/d*A*a^3*b*tan(d*x+c)*sec(d*x+
c)^2+2/d*B*a^3*b*tan(d*x+c)*sec(d*x+c)+2/d*B*a^3*b*ln(sec(d*x+c)+tan(d*x+c)
)+3/d*A*a^2*b^2*tan(d*x+c)*sec(d*x+c)+3/d*A*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c
))+6/d*B*a^2*b^2*tan(d*x+c)+4/d*A*a*b^3*tan(d*x+c)+4/d*B*a*b^3*ln(sec(d*x+c
)+tan(d*x+c))+1/d*A*b^4*ln(sec(d*x+c)+tan(d*x+c))+b^4*B*x+1/d*B*b^4*c
```

maxima [A] time = 0.58, size = 317, normalized size = 1.47

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{4}+64\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{3}b+48\left(dx+c\right)Bb^{4}-3\,Aa^{4}\left(\frac{2\left(3\right.}{\sin\left(\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^4 + 64*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*a^3*b + 48*(d*x + c)*B*b^4 - 3*A*a^4*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 48*B*a^3*b*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 72*A*a^2*b^2*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 96*B*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 24*
A*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 288*B*a^2*b^2*tan(d
*x + c) + 192*A*a*b^3*tan(d*x + c))/d
```

mupad [B] time = 2.98, size = 1969, normalized size = 9.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^5,x)
```

```
[Out] ((27*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 + 9*A*b^4*atanh(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (9*A*a^4*sin(3*c + 3*d*x))/8 + 4*B
*a^4*sin(2*c + 2*d*x) + B*a^4*sin(4*c + 4*d*x) + 9*B*b^4*atan((9*A^2*a^8*si
n(c/2 + (d*x)/2) + 64*A^2*b^8*sin(c/2 + (d*x)/2) + 64*B^2*b^8*sin(c/2 + (d*
x)/2) + 384*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624*A^2*a^4*b^4*sin(c/2 + (d*x
)/2) + 144*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 1024*B^2*a^2*b^6*sin(c/2 + (d*x
)/2) + 1024*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 256*B^2*a^6*b^2*sin(c/2 + (d*x
)/2) + 1792*A*B*a^3*b^5*sin(c/2 + (d*x)/2) + 960*A*B*a^5*b^3*sin(c/2 + (d*x
```

```
)/2) + 512*A*B*a*b^7*sin(c/2 + (d*x)/2) + 96*A*B*a^7*b*sin(c/2 + (d*x)/2))/
(cos(c/2 + (d*x)/2)*(9*A^2*a^8 + 64*A^2*b^8 + 64*B^2*b^8 + 384*A^2*a^2*b^6
+ 624*A^2*a^4*b^4 + 144*A^2*a^6*b^2 + 1024*B^2*a^2*b^6 + 1024*B^2*a^4*b^4 +
 256*B^2*a^6*b^2 + 512*A*B*a*b^7 + 96*A*B*a^7*b + 1792*A*B*a^3*b^5 + 960*A*
B*a^5*b^3))) + (33*A*a^4*sin(c + d*x))/8 + 12*B*b^4*cos(2*c + 2*d*x)*atan((
9*A^2*a^8*sin(c/2 + (d*x)/2) + 64*A^2*b^8*sin(c/2 + (d*x)/2) + 64*B^2*b^8*s
in(c/2 + (d*x)/2) + 384*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624*A^2*a^4*b^4*si
n(c/2 + (d*x)/2) + 144*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 1024*B^2*a^2*b^6*si
n(c/2 + (d*x)/2) + 1024*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 256*B^2*a^6*b^2*si
n(c/2 + (d*x)/2) + 1792*A*B*a^3*b^5*sin(c/2 + (d*x)/2) + 960*A*B*a^5*b^3*si
n(c/2 + (d*x)/2) + 512*A*B*a*b^7*sin(c/2 + (d*x)/2) + 96*A*B*a^7*b*sin(c/2
+ (d*x)/2))/(cos(c/2 + (d*x)/2)*(9*A^2*a^8 + 64*A^2*b^8 + 64*B^2*b^8 + 384*
A^2*a^2*b^6 + 624*A^2*a^4*b^4 + 144*A^2*a^6*b^2 + 1024*B^2*a^2*b^6 + 1024*B
^2*a^4*b^4 + 256*B^2*a^6*b^2 + 512*A*B*a*b^7 + 96*A*B*a^7*b + 1792*A*B*a^3*
b^5 + 960*A*B*a^5*b^3))) + 3*B*b^4*cos(4*c + 4*d*x)*atan((9*A^2*a^8*sin(c/2
 + (d*x)/2) + 64*A^2*b^8*sin(c/2 + (d*x)/2) + 64*B^2*b^8*sin(c/2 + (d*x)/2)
 + 384*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624*A^2*a^4*b^4*sin(c/2 + (d*x)/2)
+ 144*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 1024*B^2*a^2*b^6*sin(c/2 + (d*x)/2)
+ 1024*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 256*B^2*a^6*b^2*sin(c/2 + (d*x)/2)
+ 1792*A*B*a^3*b^5*sin(c/2 + (d*x)/2) + 960*A*B*a^5*b^3*sin(c/2 + (d*x)/2)
+ 512*A*B*a*b^7*sin(c/2 + (d*x)/2) + 96*A*B*a^7*b*sin(c/2 + (d*x)/2))/(cos(
c/2 + (d*x)/2)*(9*A^2*a^8 + 64*A^2*b^8 + 64*B^2*b^8 + 384*A^2*a^2*b^6 + 624
*A^2*a^4*b^4 + 144*A^2*a^6*b^2 + 1024*B^2*a^2*b^6 + 1024*B^2*a^4*b^4 + 256*
B^2*a^6*b^2 + 512*A*B*a*b^7 + 96*A*B*a^7*b + 1792*A*B*a^3*b^5 + 960*A*B*a^5
*b^3))) + 6*B*a^3*b*sin(c + d*x) + 36*B*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2)) + 18*B*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) +
 12*A*a*b^3*sin(2*c + 2*d*x) + 16*A*a^3*b*sin(2*c + 2*d*x) + 6*A*a*b^3*sin(
4*c + 4*d*x) + 4*A*a^3*b*sin(4*c + 4*d*x) + 9*A*a^2*b^2*sin(c + d*x) + 6*B*
a^3*b*sin(3*c + 3*d*x) + (9*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2))*cos(2*c + 2*d*x))/2 + (9*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2))*cos(4*c + 4*d*x))/8 + 27*A*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)) + 12*A*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c +
 2*d*x) + 3*A*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(4*c + 4*
d*x) + 9*A*a^2*b^2*sin(3*c + 3*d*x) + 18*B*a^2*b^2*sin(2*c + 2*d*x) + 9*B*a
^2*b^2*sin(4*c + 4*d*x) + 48*B*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*cos(2*c + 2*d*x) + 24*B*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*cos(2*c + 2*d*x) + 12*B*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*cos(4*c + 4*d*x) + 6*B*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2))*cos(4*c + 4*d*x) + 36*A*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2))*cos(2*c + 2*d*x) + 9*A*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*cos(4*c + 4*d*x))/(12*d*(cos(2*c + 2*d*x)/2 + cos(4*c + 4*d*x)/8 +
 3/8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.248 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=267

$$\frac{a^2\left(8a^2A+25abB+18Ab^2\right)\tan(c+dx)\sec^2(c+dx)}{30d}+\frac{a\left(15a^3B+60a^2Ab+110ab^2B+56Ab^3\right)\tan(c+dx)\sec(c+dx)}{40d}$$

```
[Out] 1/8*(12*A*a^3*b+16*A*a*b^3+3*B*a^4+24*B*a^2*b^2+8*B*b^4)*arctanh(sin(d*x+c)
)/d+1/15*(8*A*a^4+60*A*a^2*b^2+15*A*b^4+40*B*a^3*b+60*B*a*b^3)*tan(d*x+c)/d
+1/40*a*(60*A*a^2*b+56*A*b^3+15*B*a^3+110*B*a*b^2)*sec(d*x+c)*tan(d*x+c)/d+
1/30*a^2*(8*A*a^2+18*A*b^2+25*B*a*b)*sec(d*x+c)^2*tan(d*x+c)/d+1/20*a*(8*A*
b+5*B*a)*(a+b*cos(d*x+c))^2*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a*A*(a+b*cos(d*x+
c))^3*sec(d*x+c)^4*tan(d*x+c)/d
```

Rubi [A] time = 0.72, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2989, 3047, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{\left(60a^2Ab^2+8a^4A+40a^3bB+60ab^3B+15Ab^4\right)\tan(c+dx)}{15d}+\frac{\left(12a^3Ab+24a^2b^2B+3a^4B+16aAb^3+8b^4B\right)\tan}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] ((12*a^3*A*b + 16*a*A*b^3 + 3*a^4*B + 24*a^2*b^2*B + 8*b^4*B)*ArcTanh[Sin[c
 + d*x]])/(8*d) + ((8*a^4*A + 60*a^2*A*b^2 + 15*A*b^4 + 40*a^3*b*B + 60*a*b
^3*B)*Tan[c + d*x])/(15*d) + (a*(60*a^2*A*b + 56*A*b^3 + 15*a^3*B + 110*a*b
^2*B)*Sec[c + d*x]*Tan[c + d*x])/(40*d) + (a^2*(8*a^2*A + 18*A*b^2 + 25*a*b
*B)*Sec[c + d*x]^2*Tan[c + d*x])/(30*d) + (a*(8*A*b + 5*a*B)*(a + b*Cos[c +
 d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a*A*(a + b*Cos[c + d*x])^3*
Sec[c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
```

```
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^6(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}$$

$$= \frac{a(8Ab+5aB)(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c}{20d}$$

$$= \frac{a^2\left(8a^2A+18Ab^2+25abB\right)\sec^2(c+dx)\tan(c+dx}{30d}$$

$$= \frac{a\left(60a^2Ab+56Ab^3+15a^3B+110ab^2B\right)\sec(c+dx)}{40d}$$

$$= \frac{a\left(60a^2Ab+56Ab^3+15a^3B+110ab^2B\right)\sec(c+dx)}{40d}$$

$$= \frac{\left(12a^3Ab+16aAb^3+3a^4B+24a^2b^2B+8b^4B\right)\tanh}{8d}$$

$$= \frac{\left(12a^3Ab+16aAb^3+3a^4B+24a^2b^2B+8b^4B\right)\tanh}{8d}$$

Mathematica [A] time = 4.25, size = 198, normalized size = 0.74

$$\frac{15\left(3a^4B+12a^3Ab+24a^2b^2B+16aAb^3+8b^4B\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(24a^4A\tan^4(c+dx)+30a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (15*(12*a^3*A*b + 16*a*A*b^3 + 3*a^4*B + 24*a^2*b^2*B + 8*b^4*B)*ArcTanh[Si
n[c + d*x]] + Tan[c + d*x]*(120*(a^4*A + 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B +
4*a*b^3*B) + 15*a*(12*a^2*A*b + 16*A*b^3 + 3*a^3*B + 24*a*b^2*B)*Sec[c + d*
x] + 30*a^3*(4*A*b + a*B)*Sec[c + d*x]^3 + 80*a^2*(a^2*A + 3*A*b^2 + 2*a*b*
B)*Tan[c + d*x]^2 + 24*a^4*A*Tan[c + d*x]^4))/(120*d)
```

fricas [A] time = 0.66, size = 281, normalized size = 1.05

$$\frac{15\left(3\,Ba^4+12\,Aa^3b+24\,Ba^2b^2+16\,Aab^3+8\,Bb^4\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(3\,Ba^4+12\,Aa^3b+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*cos(
d*x + c)^5*log(sin(d*x + c) + 1) - 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2
+ 16*A*a*b^3 + 8*B*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(24*A*a^4
 + 8*(8*A*a^4 + 40*B*a^3*b + 60*A*a^2*b^2 + 60*B*a*b^3 + 15*A*b^4)*cos(d*x
+ c)^4 + 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3)*cos(d*x + c)
^3 + 16*(2*A*a^4 + 10*B*a^3*b + 15*A*a^2*b^2)*cos(d*x + c)^2 + 30*(B*a^4 +
4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

giac [B] time = 0.54, size = 850, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*log(
abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 +
16*A*a*b^3 + 8*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^4*tan
(1/2*d*x + 1/2*c)^9 - 75*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 300*A*a^3*b*tan(1/2
*d*x + 1/2*c)^9 + 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 720*A*a^2*b^2*tan(1/
2*d*x + 1/2*c)^9 - 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 240*A*a*b^3*tan(1
/2*d*x + 1/2*c)^9 + 480*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*A*b^4*tan(1/2*
d*x + 1/2*c)^9 - 160*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 30*B*a^4*tan(1/2*d*x +
1/2*c)^7 + 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 1280*B*a^3*b*tan(1/2*d*x +
1/2*c)^7 - 1920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*B*a^2*b^2*tan(1/2*d*
x + 1/2*c)^7 + 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 1920*B*a*b^3*tan(1/2*d*
x + 1/2*c)^7 - 480*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 464*A*a^4*tan(1/2*d*x + 1
/2*c)^5 + 1600*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 2400*A*a^2*b^2*tan(1/2*d*x
```

```
+ 1/2*c)^5 + 2880*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 720*A*b^4*tan(1/2*d*x +
1/2*c)^5 - 160*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^4*tan(1/2*d*x + 1/2*c)
^3 - 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1280*B*a^3*b*tan(1/2*d*x + 1/2*c)
^3 - 1920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 720*B*a^2*b^2*tan(1/2*d*x + 1/
2*c)^3 - 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 1920*B*a*b^3*tan(1/2*d*x + 1/
2*c)^3 - 480*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^4*tan(1/2*d*x + 1/2*c)
+ 75*B*a^4*tan(1/2*d*x + 1/2*c) + 300*A*a^3*b*tan(1/2*d*x + 1/2*c) + 480*B*
a^3*b*tan(1/2*d*x + 1/2*c) + 720*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 360*B*a^2
*b^2*tan(1/2*d*x + 1/2*c) + 240*A*a*b^3*tan(1/2*d*x + 1/2*c) + 480*B*a*b^3*
tan(1/2*d*x + 1/2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c
)^2 - 1)^5)/d
```

maple [A] time = 0.15, size = 431, normalized size = 1.61

$$\frac{8A\,a^4\tan(dx+c)}{15d}+\frac{A\,a^4\tan(dx+c)\left(\sec^4(dx+c)\right)}{5d}+\frac{4A\,a^4\tan(dx+c)\left(\sec^2(dx+c)\right)}{15d}+\frac{a^4B\tan(dx+c)\left(\sec^3(\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 8/15/d*A*a^4*tan(d*x+c)+1/5/d*A*a^4*tan(d*x+c)*sec(d*x+c)^4+4/15/d*A*a^4*ta
n(d*x+c)*sec(d*x+c)^2+1/4/d*a^4*B*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^4*B*sec(d
*x+c)*tan(d*x+c)+3/8/d*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+1/d*A*a^3*b*tan(d*x+
c)*sec(d*x+c)^3+3/2/d*A*a^3*b*sec(d*x+c)*tan(d*x+c)+3/2/d*A*a^3*b*ln(sec(d*
x+c)+tan(d*x+c))+8/3/d*B*a^3*b*tan(d*x+c)+4/3/d*B*a^3*b*tan(d*x+c)*sec(d*x+
c)^2+4/d*A*a^2*b^2*tan(d*x+c)+2/d*A*a^2*b^2*tan(d*x+c)*sec(d*x+c)^2+3/d*B*a
^2*b^2*tan(d*x+c)*sec(d*x+c)+3/d*B*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*A*
a*b^3*tan(d*x+c)*sec(d*x+c)+2/d*A*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+4/d*B*a*b
^3*tan(d*x+c)+1/d*A*b^4*tan(d*x+c)+1/d*B*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.59, size = 386, normalized size = 1.45

$$\frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^4+320\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^3b+480\left(\text{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
320*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3*b + 480*(tan(d*x + c)^3 + 3*tan
(d*x + c))*A*a^2*b^2 - 15*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d
```

```
*x + c) - 1)) - 60*A*a^3*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x
+ c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x +
c) - 1)) - 360*B*a^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) - 240*A*a*b^3*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 120*B*b^4*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 960*B*a*b^3*tan(d*x + c) +
240*A*b^4*tan(d*x + c))/d
```

mupad [B] time = 3.88, size = 555, normalized size = 2.08

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,B\,a^4}{8}+\frac{3\,A\,a^3\,b}{2}+3\,B\,a^2\,b^2+2\,A\,a\,b^3+B\,b^4\right)}{\frac{3\,B\,a^4}{2}+6\,A\,a^3\,b+12\,B\,a^2\,b^2+8\,A\,a\,b^3+4\,B\,b^4}\right)\left(\frac{3\,B\,a^4}{4}+3\,A\,a^3\,b+6\,B\,a^2\,b^2+4\,A\,a\,b^3+2\,B\,b^4\right)}{d}-\frac{\left(2\,A\,a^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^6,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((3*B*a^4)/8 + B*b^4 + 3*B*a^2*b^2 + 2*A*a*b^3
 + (3*A*a^3*b)/2))/((3*B*a^4)/2 + 4*B*b^4 + 12*B*a^2*b^2 + 8*A*a*b^3 + 6*A*
a^3*b))*((3*B*a^4)/4 + 2*B*b^4 + 6*B*a^2*b^2 + 4*A*a*b^3 + 3*A*a^3*b))/d -
(tan(c/2 + (d*x)/2)*(2*A*a^4 + 2*A*b^4 + (5*B*a^4)/4 + 12*A*a^2*b^2 + 6*B*a
^2*b^2 + 4*A*a*b^3 + 5*A*a^3*b + 8*B*a*b^3 + 8*B*a^3*b) + tan(c/2 + (d*x)/2
)^5*((116*A*a^4)/15 + 12*A*b^4 + 40*A*a^2*b^2 + 48*B*a*b^3 + (80*B*a^3*b)/3
) + tan(c/2 + (d*x)/2)^9*(2*A*a^4 + 2*A*b^4 - (5*B*a^4)/4 + 12*A*a^2*b^2 -
6*B*a^2*b^2 - 4*A*a*b^3 - 5*A*a^3*b + 8*B*a*b^3 + 8*B*a^3*b) - tan(c/2 + (d
*x)/2)^3*((8*A*a^4)/3 + 8*A*b^4 + (B*a^4)/2 + 32*A*a^2*b^2 + 12*B*a^2*b^2 +
 8*A*a*b^3 + 2*A*a^3*b + 32*B*a*b^3 + (64*B*a^3*b)/3) - tan(c/2 + (d*x)/2)^
7*((8*A*a^4)/3 + 8*A*b^4 - (B*a^4)/2 + 32*A*a^2*b^2 - 12*B*a^2*b^2 - 8*A*a*
b^3 - 2*A*a^3*b + 32*B*a*b^3 + (64*B*a^3*b)/3))/(d*(5*tan(c/2 + (d*x)/2)^2
- 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^
8 + tan(c/2 + (d*x)/2)^10 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.249 $\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx$

Optimal. Leaf size=324

$$\frac{a^2\left(25a^2A+72abB+48Ab^2\right)\tan(c+dx)\sec^3(c+dx)}{120d}+\frac{a\left(4a^3B+16a^2Ab+27ab^2B+13Ab^3\right)\tan(c+dx)\sec^2(c+dx)}{15d}$$

```
[Out] 1/16*(5*A*a^4+36*A*a^2*b^2+8*A*b^4+24*B*a^3*b+32*B*a*b^3)*arctanh(sin(d*x+c
))/d+1/15*(32*A*a^3*b+40*A*a*b^3+8*B*a^4+60*B*a^2*b^2+15*B*b^4)*tan(d*x+c)/
d+1/16*(5*A*a^4+36*A*a^2*b^2+8*A*b^4+24*B*a^3*b+32*B*a*b^3)*sec(d*x+c)*tan(
d*x+c)/d+1/15*a*(16*A*a^2*b+13*A*b^3+4*B*a^3+27*B*a*b^2)*sec(d*x+c)^2*tan(d
*x+c)/d+1/120*a^2*(25*A*a^2+48*A*b^2+72*B*a*b)*sec(d*x+c)^3*tan(d*x+c)/d+1/
10*a*(3*A*b+2*B*a)*(a+b*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d+1/6*a*A*(a+
b*cos(d*x+c))^3*sec(d*x+c)^5*tan(d*x+c)/d
```

Rubi [A] time = 0.80, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2989, 3047, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{\left(32a^3Ab+60a^2b^2B+8a^4B+40aAb^3+15b^4B\right)\tan(c+dx)}{15d}+\frac{\left(36a^2Ab^2+5a^4A+24a^3bB+32ab^3B+8Ab^4\right)\tan(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

```
[Out] ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*ArcTanh[Sin[c
 + d*x]])/(16*d) + ((32*a^3*A*b + 40*a*A*b^3 + 8*a^4*B + 60*a^2*b^2*B + 15*
b^4*B)*Tan[c + d*x])/(15*d) + ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b
*B + 32*a*b^3*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a*(16*a^2*A*b + 13*A*
b^3 + 4*a^3*B + 27*a*b^2*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d) + (a^2*(25*
a^2*A + 48*A*b^2 + 72*a*b*B)*Sec[c + d*x]^3*Tan[c + d*x])/(120*d) + (a*(3*A
*b + 2*a*B)*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(10*d) + (a
*A*(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
```

```
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4(A+B\cos(c+dx))\sec^7(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^3\sec^5(c+dx)\tan(c+dx)}{6d} +$$

$$= \frac{a(3Ab+2aB)(a+b\cos(c+dx))^2\sec^4(c+dx)\tan}{10d}$$

$$= \frac{a^2\left(25a^2A+48Ab^2+72abB\right)\sec^3(c+dx)\tan(c+}{120d}$$

$$= \frac{a\left(16a^2Ab+13Ab^3+4a^3B+27ab^2B\right)\sec^2(c+dx)}{15d}$$

$$= \frac{a\left(16a^2Ab+13Ab^3+4a^3B+27ab^2B\right)\sec^2(c+dx)}{15d}$$

$$= \frac{\left(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B\right)\sec}{16d}$$

$$= \frac{\left(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B\right)\tan}{16d}$$

Mathematica [A] time = 2.77, size = 244, normalized size = 0.75

$$\frac{15\left(5a^4A+24a^3bB+36a^2Ab^2+32ab^3B+8Ab^4\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(40a^4A\sec^5(c+dx)+48a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*(A + B*Cos[c + d*x])*Sec[c + d*x]^7,x]
```

```
[Out] (15*(5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*ArcTanh[Si
n[c + d*x]] + Tan[c + d*x]*(240*(4*a^3*A*b + 4*a*A*b^3 + a^4*B + 6*a^2*b^2*
B + b^4*B) + 15*(5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B
)*Sec[c + d*x] + 10*a^2*(5*a^2*A + 36*A*b^2 + 24*a*b*B)*Sec[c + d*x]^3 + 40
*a^4*A*Sec[c + d*x]^5 + 160*a*(4*a^2*A*b + 2*A*b^3 + a^3*B + 3*a*b^2*B)*Tan
[c + d*x]^2 + 48*a^3*(4*A*b + a*B)*Tan[c + d*x]^4))/(240*d)
```

fricas [A] time = 1.10, size = 327, normalized size = 1.01

$$\frac{15\left(5\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+32\,Bab^3+8\,Ab^4\right)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\left(5\,Aa^4+24\,Ba^3b+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(
d*x + c)^6*log(sin(d*x + c) + 1) - 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2
+ 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(8*B*
a^4 + 32*A*a^3*b + 60*B*a^2*b^2 + 40*A*a*b^3 + 15*B*b^4)*cos(d*x + c)^5 + 4
0*A*a^4 + 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*c
os(d*x + c)^4 + 32*(2*B*a^4 + 8*A*a^3*b + 15*B*a^2*b^2 + 10*A*a*b^3)*cos(d*
x + c)^3 + 10*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2)*cos(d*x + c)^2 + 48*(B*
a^4 + 4*A*a^3*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6)
```

giac [B] time = 0.93, size = 1186, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*log(
abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 +
32*B*a*b^3 + 8*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(165*A*a^4*tan
(1/2*d*x + 1/2*c)^11 - 240*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*A*a^3*b*tan(
1/2*d*x + 1/2*c)^11 + 600*B*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*A*a^2*b^2*t
an(1/2*d*x + 1/2*c)^11 - 1440*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*A*a*b
^3*tan(1/2*d*x + 1/2*c)^11 + 480*B*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*A*b^
4*tan(1/2*d*x + 1/2*c)^11 - 240*B*b^4*tan(1/2*d*x + 1/2*c)^11 + 25*A*a^4*ta
n(1/2*d*x + 1/2*c)^9 + 560*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 2240*A*a^3*b*tan(
1/2*d*x + 1/2*c)^9 - 840*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 1260*A*a^2*b^2*ta
n(1/2*d*x + 1/2*c)^9 + 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 3520*A*a*b^3
*tan(1/2*d*x + 1/2*c)^9 - 1440*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 360*A*b^4*t
an(1/2*d*x + 1/2*c)^9 + 1200*B*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*A*a^4*tan(1
/2*d*x + 1/2*c)^7 - 1248*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 4992*A*a^3*b*tan(1/
2*d*x + 1/2*c)^7 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*A*a^2*b^2*tan(1
/2*d*x + 1/2*c)^7 - 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*A*a*b^3*ta
n(1/2*d*x + 1/2*c)^7 + 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*A*b^4*tan(1
/2*d*x + 1/2*c)^7 - 2400*B*b^4*tan(1/2*d*x + 1/2*c)^7 + 450*A*a^4*tan(1/2*d
*x + 1/2*c)^5 + 1248*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 4992*A*a^3*b*tan(1/2*d*
x + 1/2*c)^5 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 360*A*a^2*b^2*tan(1/2*d
*x + 1/2*c)^5 + 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 5760*A*a*b^3*tan(1/
2*d*x + 1/2*c)^5 + 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 240*A*b^4*tan(1/2*d
*x + 1/2*c)^5 + 2400*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 25*A*a^4*tan(1/2*d*x +
1/2*c)^3 - 560*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*A*a^3*b*tan(1/2*d*x + 1/
2*c)^3 - 840*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1260*A*a^2*b^2*tan(1/2*d*x +
1/2*c)^3 - 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*A*a*b^3*tan(1/2*d*x
 + 1/2*c)^3 - 1440*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 360*A*b^4*tan(1/2*d*x +
 1/2*c)^3 - 1200*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 165*A*a^4*tan(1/2*d*x + 1/2
```

```
*c) + 240*B*a^4*tan(1/2*d*x + 1/2*c) + 960*A*a^3*b*tan(1/2*d*x + 1/2*c) + 6
00*B*a^3*b*tan(1/2*d*x + 1/2*c) + 900*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 1440
*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 960*A*a*b^3*tan(1/2*d*x + 1/2*c) + 480*B*
a*b^3*tan(1/2*d*x + 1/2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*c) + 240*B*b^4*tan
(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

maple [A] time = 0.16, size = 550, normalized size = 1.70

$$\frac{A\,b^4\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{4A\,a^3b\tan(dx+c)\left(\sec^4(dx+c)\right)}{5d}+\frac{2Ba\,b^3\tan(dx+c)\sec(dx+c)}{d}+\frac{2B\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x)
```

```
[Out] 1/2/d*A*b^4*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*A*a^2*b^2*tan(d*x+c)*sec(d*x+c)
^3+2/d*B*a*b^3*tan(d*x+c)*sec(d*x+c)+1/d*B*b^4*tan(d*x+c)+8/15/d*a^4*B*tan(
d*x+c)+1/2/d*A*b^4*tan(d*x+c)*sec(d*x+c)+1/6/d*A*a^4*tan(d*x+c)*sec(d*x+c)^
5+1/5/d*a^4*B*tan(d*x+c)*sec(d*x+c)^4+4/5/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^4
+2/d*B*a^2*b^2*tan(d*x+c)*sec(d*x+c)^2+4/3/d*A*a*b^3*tan(d*x+c)*sec(d*x+c)^
2+5/16/d*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+1/d*B*a^3*b*tan(d*x+c)*sec(d*x+c)^
3+32/15/d*A*a^3*b*tan(d*x+c)+3/2/d*B*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+9/4/d*
A*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+5/24/d*A*a^4*tan(d*x+c)*sec(d*x+c)^3+4/
15/d*a^4*B*tan(d*x+c)*sec(d*x+c)^2+16/15/d*A*a^3*b*tan(d*x+c)*sec(d*x+c)^2+
3/2/d*B*a^3*b*tan(d*x+c)*sec(d*x+c)+9/4/d*A*a^2*b^2*tan(d*x+c)*sec(d*x+c)+5
/16/d*A*a^4*sec(d*x+c)*tan(d*x+c)+4/d*B*a^2*b^2*tan(d*x+c)+8/3/d*A*a*b^3*ta
n(d*x+c)+2/d*B*a*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.44, size = 474, normalized size = 1.46

$$\frac{32\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ba^4+128\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*(A+B*cos(d*x+c))*sec(d*x+c)^7,x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 +
128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3*b + 960*
(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2*b^2 + 640*(tan(d*x + c)^3 + 3*tan(d
*x + c))*A*a*b^3 - 5*A*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*s
in(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 1
5*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 120*B*a^3*b*(2*(3*sin
(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*l
```

```
og(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 180*A*a^2*b^2*(2*(3*sin(d
*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 480*B*a*b^3*(2*sin(d*x + c)
/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12
0*A*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) + 480*B*b^4*tan(d*x + c))/d
```

mupad [B] time = 3.75, size = 706, normalized size = 2.18

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{5\,A\,a^4}{16}+\frac{3\,B\,a^3\,b}{2}+\frac{9\,A\,a^2\,b^2}{4}+2\,B\,a\,b^3+\frac{A\,b^4}{2}\right)}{\frac{5\,A\,a^4}{4}+6\,B\,a^3\,b+9\,A\,a^2\,b^2+8\,B\,a\,b^3+2\,A\,b^4}\right)\left(\frac{5\,A\,a^4}{8}+3\,B\,a^3\,b+\frac{9\,A\,a^2\,b^2}{2}+4\,B\,a\,b^3+A\,b^4\right)}{d}+\frac{\left(\frac{11\,A\,a^4}{8}+A\,b^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^4)/cos(c + d*x)^7,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((5*A*a^4)/16 + (A*b^4)/2 + (9*A*a^2*b^2)/4 +
2*B*a*b^3 + (3*B*a^3*b)/2))/((5*A*a^4)/4 + 2*A*b^4 + 9*A*a^2*b^2 + 8*B*a*b^
3 + 6*B*a^3*b))*((5*A*a^4)/8 + A*b^4 + (9*A*a^2*b^2)/2 + 4*B*a*b^3 + 3*B*a^
3*b))/d + (tan(c/2 + (d*x)/2)*((11*A*a^4)/8 + A*b^4 + 2*B*a^4 + 2*B*b^4 + (
15*A*a^2*b^2)/2 + 12*B*a^2*b^2 + 8*A*a*b^3 + 8*A*a^3*b + 4*B*a*b^3 + 5*B*a^
3*b) + tan(c/2 + (d*x)/2)^11*((11*A*a^4)/8 + A*b^4 - 2*B*a^4 - 2*B*b^4 + (1
5*A*a^2*b^2)/2 - 12*B*a^2*b^2 - 8*A*a*b^3 - 8*A*a^3*b + 4*B*a*b^3 + 5*B*a^3
*b) - tan(c/2 + (d*x)/2)^3*(3*A*b^4 - (5*A*a^4)/24 + (14*B*a^4)/3 + 10*B*b^
4 + (21*A*a^2*b^2)/2 + 44*B*a^2*b^2 + (88*A*a*b^3)/3 + (56*A*a^3*b)/3 + 12*
B*a*b^3 + 7*B*a^3*b) + tan(c/2 + (d*x)/2)^9*((5*A*a^4)/24 - 3*A*b^4 + (14*B
*a^4)/3 + 10*B*b^4 - (21*A*a^2*b^2)/2 + 44*B*a^2*b^2 + (88*A*a*b^3)/3 + (56
*A*a^3*b)/3 - 12*B*a*b^3 - 7*B*a^3*b) + tan(c/2 + (d*x)/2)^5*((15*A*a^4)/4
+ 2*A*b^4 + (52*B*a^4)/5 + 20*B*b^4 + 3*A*a^2*b^2 + 72*B*a^2*b^2 + 48*A*a*b
^3 + (208*A*a^3*b)/5 + 8*B*a*b^3 + 2*B*a^3*b) + tan(c/2 + (d*x)/2)^7*((15*A
*a^4)/4 + 2*A*b^4 - (52*B*a^4)/5 - 20*B*b^4 + 3*A*a^2*b^2 - 72*B*a^2*b^2 -
48*A*a*b^3 - (208*A*a^3*b)/5 + 8*B*a*b^3 + 2*B*a^3*b))/(d*(15*tan(c/2 + (d*
x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (
d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*(A+B*cos(d*x+c))*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

3.250 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=178

$$-\frac{2a^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(2a^2+b^2\right)(Ab-aB)}{2b^4}-\frac{\left(-3a^2B+3aAb-2b^2B\right)\sin(c+dx)}{3b^3d}+\frac{(Ab-a}{}$$

```
[Out] 1/2*(2*a^2+b^2)*(A*b-B*a)*x/b^4-1/3*(3*A*a*b-3*B*a^2-2*B*b^2)*sin(d*x+c)/b^
3/d+1/2*(A*b-B*a)*cos(d*x+c)*sin(d*x+c)/b^2/d+1/3*B*cos(d*x+c)^2*sin(d*x+c)
/b/d-2*a^3*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^4
/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2990, 3049, 3023, 2735, 2659, 205}

$$-\frac{\left(-3a^2B+3aAb-2b^2B\right)\sin(c+dx)}{3b^3d}-\frac{2a^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(2a^2+b^2\right)(Ab-aB)}{2b^4}+\frac{(Ab-a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*a^2 + b^2)*(A*b - a*B)*x)/(2*b^4) - (2*a^3*(A*b - a*B)*ArcTan[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) - ((3*a
*A*b - 3*a^2*B - 2*b^2*B)*Sin[c + d*x])/(3*b^3*d) + ((A*b - a*B)*Cos[c + d*
x]*Sin[c + d*x])/(2*b^2*d) + (B*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}dx = \frac{B\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{\cos(c+dx)\left(2aB+2bB\cos(c+dx)+3(Ab-aB)\cos^2(c+\right.}{a+b\cos(c+dx)}}{3b}$$

$$= \frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{B\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{3}{-}}{}$$

$$= -\frac{\left(3aAb-3a^2B-2b^2B\right)\sin(c+dx)}{3b^3d} + \frac{(Ab-aB)\cos(c+dx)\sin(c+}{2b^2d}$$

$$= \frac{\left(2a^2+b^2\right)(Ab-aB)x}{2b^4} - \frac{\left(3aAb-3a^2B-2b^2B\right)\sin(c+dx)}{3b^3d} + \frac{(Ab-}{}$$

$$= \frac{\left(2a^2+b^2\right)(Ab-aB)x}{2b^4} - \frac{\left(3aAb-3a^2B-2b^2B\right)\sin(c+dx)}{3b^3d} + \frac{(Ab-}{}$$

$$= \frac{\left(2a^2+b^2\right)(Ab-aB)x}{2b^4} - \frac{2a^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^4\sqrt{a+b}\,d} - \frac{(3a}{}$$

Mathematica [A] time = 0.47, size = 152, normalized size = 0.85

$$\frac{6\left(2a^2+b^2\right)(c+dx)(Ab-aB)+3b\left(4a^2B-4aAb+3b^2B\right)\sin(c+dx)-\frac{24a^3(aB-Ab)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+3b^2(}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (6*(2*a^2 + b^2)*(A*b - a*B)*(c + d*x) - (24*a^3*(-(A*b) + a*B)*ArcTanh[((a
 - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 3*b*(-4*a*A*b
 + 4*a^2*B + 3*b^2*B)*Sin[c + d*x] + 3*b^2*(A*b - a*B)*Sin[2*(c + d*x)] + b
^3*B*Sin[3*(c + d*x)])/(12*b^4*d)
```

fricas [A] time = 1.11, size = 541, normalized size = 3.04

$$\left[-\frac{3\left(2Ba^5-2Aa^4b-Ba^3b^2+Aa^2b^3-Bab^4+Ab^5\right)dx-3\left(Ba^4-Aa^3b\right)\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos}{b^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [-1/6*(3*(2*B*a^5 - 2*A*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*d*
x - 3*(B*a^4 - A*a^3*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
 - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*B*a^4
*b - 6*A*a^3*b^2 - 2*B*a^2*b^3 + 6*A*a*b^4 - 4*B*b^5 + 2*(B*a^2*b^3 - B*b^5
)*cos(d*x + c)^2 - 3*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c)
)*sin(d*x + c))/((a^2*b^4 - b^6)*d), -1/6*(3*(2*B*a^5 - 2*A*a^4*b - B*a^3*b
^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*d*x - 6*(B*a^4 - A*a^3*b)*sqrt(a^2 - b^2)
*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*B*a^4*b
- 6*A*a^3*b^2 - 2*B*a^2*b^3 + 6*A*a*b^4 - 4*B*b^5 + 2*(B*a^2*b^3 - B*b^5)*c
os(d*x + c)^2 - 3*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c))*s
in(d*x + c))/((a^2*b^4 - b^6)*d)]
```

giac [B] time = 0.49, size = 360, normalized size = 2.02

$$-\frac{\frac{3\left(2\,Ba^{3}-2\,Aa^{2}b+Bab^{2}-Ab^{3}\right)(dx+c)}{b^{4}}+\frac{12\left(Ba^{4}-Aa^{3}b\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,b^{4}}-\frac{2\left(6\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(3*(2*B*a^3 - 2*A*a^2*b + B*a*b^2 - A*b^3)*(d*x + c)/b^4 + 12*(B*a^4 -
 A*a^3*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 -
b^2)*b^4) - 2*(6*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b*tan(1/2*d*x + 1/2*c
)^5 + 3*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 3*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*B
*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*A*a*b*ta
n(1/2*d*x + 1/2*c)^3 + 4*B*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*tan(1/2*d*x
 + 1/2*c) - 6*A*a*b*tan(1/2*d*x + 1/2*c) - 3*B*a*b*tan(1/2*d*x + 1/2*c) + 3
*A*b^2*tan(1/2*d*x + 1/2*c) + 6*B*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
 1/2*c)^2 + 1)^3*b^3))/d
```

maple [B] time = 0.09, size = 641, normalized size = 3.60

$$-\frac{2a^{3}\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)A}{d\,b^{3}\sqrt{(a-b)\,(a+b)}}+\frac{2a^{4}\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)B}{d\,b^{4}\sqrt{(a-b)\,(a+b)}}-\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)Aa}{d\,b^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{db\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d*a^3/b^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))*A+2/d*a^4/b^4/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b
)/((a-b)*(a+b))^(1/2))*B-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2
*c)^5*A*a-1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*A+2/d/b^3/(
1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*a^2*B+1/d/b^2/(1+tan(1/2*d*x
+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*B*a+2/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(
1/2*d*x+1/2*c)^5*B-4/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*
A*a+4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*a^2*B+4/3/d/b/(
1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*B-2/d/b^2/(1+tan(1/2*d*x+1/2
*c)^2)^3*tan(1/2*d*x+1/2*c)*A*a+2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*
d*x+1/2*c)*a^2*B+2/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*B+1/d/
b/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*A-1/d/b^2/(1+tan(1/2*d*x+1/
2*c)^2)^3*tan(1/2*d*x+1/2*c)*B*a+2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*A*a^2+1
/d/b*arctan(tan(1/2*d*x+1/2*c))*A-2/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^3*B-
1/d/b^2*arctan(tan(1/2*d*x+1/2*c))*B*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 5.09, size = 4568, normalized size = 25.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)

[Out] ((tan(c/2 + (d*x)/2)*(A*b^2 + 2*B*a^2 + 2*B*b^2 - 2*A*a*b - B*a*b))/b^3 + (tan(c/2 + (d*x)/2)^5*(2*B*a^2 - A*b^2 + 2*B*b^2 - 2*A*a*b + B*a*b))/b^3 + (4*tan(c/2 + (d*x)/2)^3*(3*B*a^2 + B*b^2 - 3*A*a*b))/(3*b^3))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (atan((((2*a^2 + b^2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 + ((2*a^2 + b^2)*(A*b - B*a)*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 - (tan(c/2 + (d*x)/2)*(2*a^2 + b^2)*(A*b - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*4i)/b^10)*1i)/(2*b^4)))/(2*b^4) + ((2*a^2 + b^2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 - ((2*a^2 + b^2)*(A*b - B*a)*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 + (tan(c/2 + (d*x)/2)*(2*a^2 + b^2)*(A*b - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*4i)/b^10)*1i)/(2*b^4)))/(2*b^4))/((16*(4*B^3*a^11 - 6*B^3*a^10*b + A^3*a^3*b^8 - 2*A^3*a^4*b^7 + 5*A^3*a^5*b^6 - 6*A^3*a^6*b^5 + 6*A^3*a^7*b^4 - 4*A^3*a^8*b^3 - B^3*a^6*b^5 + 2*B^3*a^7*b^4 - 5*B^3*a^8*b^3 + 6*B^3*a^9*b^2 - 12*A*B^2*a^10*b + 3*A*B^2*a^5*b^6 - 6*A*B^2*a^6*b^5 + 15*A*B^2*a^7*b^4 - 18*A*B^2*a^8*b^3 + 18*A*B^2*a^9*b^2 - 3*A^2*B*a^4*b^7 + 6*A^2*B*a^5*b^6 - 15*A^2*B*a^6*b^5 + 18*A^2*B*a^7*b^4 - 18*A^2*B*a^8*b^3 + 12*A^2*B*a^9*b^2))/b^9 - ((2*a^2 + b^2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 + ((2*a^2 + b^2)*(A*b - B*a)*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 - (tan(c/2 + (d*x)/2)*(2*a^2 + b^2)*(A*b - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*4i)/b^10)*1i)/(2*b^4))*1i)/(2*b^4) + ((2*a^2 + b^2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B *a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 - ((2*a^2 + b^2)*(A*b - B*a)*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 + (tan(c/2 + (d*x)/2)*(2*a^2 + b^2)*(A*b - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*4i)/b^10)*1i)/(2*b^4))*1i)/(2*b^4)))*(2*a^2 + b^2)*(A*b - B*a))/(b^4*d) + (a^3*atan(((a^3*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 + (a^3*(-(a + b)*(a - b))^(1/2)*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 - (8*a^3*tan(c/2 + (d*x)/2)*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8))/(b^6*(b^6 - a^2*b^4)))*(A*b - B*a))/(b^6 - a^2*b^4))*1i)/(b^6 - a^2*b^4) + (a^3*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 - (a^3*(-(a + b)*(a - b))^(1/2)*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 + (8*a^3*tan(c/2 + (d*x)/2)*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8))/(b^6*(b^6 - a^2*b^4)))*(A*b - B*a))/(b^6 - a^2*b^4))*1i)/(b^6 - a^2*b^4))/((16*(4*B^3*a^11 - 6*B^3*a^10*b + A^3*a^3*b^8 - 2*A^3*a^4*b^7 + 5*A^3*a^5*b^6 - 6*A^3*a^6*b^5 + 6*A^3*a^7*b^4 - 4*A^3*a^8*b^3 - B^3*a^6*b^5 + 2*B^3*a^7*b^4 - 5*B^3*a^8*b^3 + 6*B^3*a^9*b^2 - 12*A*B^2*a^10*b + 3*A*B^2*a^5*b^6 - 6*A*B^2*a^6*b^5 + 15*A*B^2*a^7*b^4 - 18*A*B^2*a^8*b^3 + 18*A*B^2*a^9*b^2 - 3*A^2*B*a^4*b^7 + 6*A^2*B*a^5*b^6 - 15*A^2*B*a^6*b^5 + 18*A^2*B*a^7*b^4 - 18*A^2*B*a^8*b^3 + 12*A^2*B*a^9*b^2))/b^9 - (a^3*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 + (a^3*(-(a + b)*(a - b))^(1/2)*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 - (8*a^3*tan(c/2 + (d*x)/2)*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8))/(b^6*(b^6 - a^2*b^4)))*(A*b - B*a))/(b^6 - a^2*b^4)))/(b^6 - a^2*b^4) + (a^3*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9

```
 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4
*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^
2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^
2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^
4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 - (a^3*(-(a
+ b)*(a - b))^(1/2)*((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b
^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 -
 2*B*a*b^12))/b^9 + (8*a^3*tan(c/2 + (d*x)/2)*(-(a + b)*(a - b))^(1/2)*(A*b
 - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8))/(b^6*(b^6 - a^2*b^4)))*(A*b -
B*a))/(b^6 - a^2*b^4)))/(b^6 - a^2*b^4)))*(-(a + b)*(a - b))^(1/2)*(A*b - B
*a)*2i)/(d*(b^6 - a^2*b^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.251 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{2a^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\,\sqrt{a+b}}-\frac{x\left(-2a^2B+2aAb-b^2B\right)}{2b^3}+\frac{(Ab-aB)\sin(c+dx)}{b^2d}+\frac{B\sin(c+dx)\cos(c+$$

```
[Out] -1/2*(2*A*a*b-2*B*a^2-B*b^2)*x/b^3+(A*b-B*a)*sin(d*x+c)/b^2/d+1/2*B*cos(d*x
+c)*sin(d*x+c)/b/d+2*a^2*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a
+b)^(1/2))/b^3/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2990, 3023, 2735, 2659, 205}

$$\frac{2a^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\,\sqrt{a+b}}-\frac{x\left(-2a^2B+2aAb-b^2B\right)}{2b^3}+\frac{(Ab-aB)\sin(c+dx)}{b^2d}+\frac{B\sin(c+dx)\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] -((2*a*A*b - 2*a^2*B - b^2*B)*x)/(2*b^3) + (2*a^2*(A*b - a*B)*ArcTan[(Sqrt[
a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) + ((
A*b - a*B)*Sin[c + d*x])/(b^2*d) + (B*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{B\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{aB+bB\cos(c+dx)+2(Ab-aB)\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2b}$$

$$= \frac{(Ab-aB)\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \frac{abB-\left(2aAb-2a^2B-b\right.}{a+b\cos(c+}}{2b^2}$$

$$= -\frac{\left(2aAb-2a^2B-b^2B\right)x}{2b^3} + \frac{(Ab-aB)\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(}{2bd}$$

$$= -\frac{\left(2aAb-2a^2B-b^2B\right)x}{2b^3} + \frac{(Ab-aB)\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(}{2bd}$$

$$= -\frac{\left(2aAb-2a^2B-b^2B\right)x}{2b^3} + \frac{2a^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d} + \frac{(A}{}$$

Mathematica [A] time = 0.31, size = 121, normalized size = 0.90

$$\frac{2(c+dx)\left(2a^2B-2aAb+b^2B\right)+\frac{8a^2(aB-Ab)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+4b(Ab-aB)\sin(c+dx)+b^2B\sin(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(-2*a*A*b + 2*a^2*B + b^2*B)*(c + d*x) + (8*a^2*(-(A*b) + a*B)*ArcTanh[(
(a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 4*b*(A*b -
a*B)*Sin[c + d*x] + b^2*B*Sin[2*(c + d*x)])/(4*b^3*d)
```

fricas [A] time = 1.15, size = 426, normalized size = 3.18

$$\left[\frac{\left(2\,Ba^4-2\,Aa^3b-Ba^2b^2+2\,Aab^3-Bb^4\right)dx+\left(Ba^3-Aa^2b\right)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a}}{b^2\cos(dx+c)^2+2\,a}\right.}{2\left(a^2b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/2*((2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b^4)*d*x + (B*a^3 -
A*a^2*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
 c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (2*B*a^3*b - 2*A*a^2*b^
2 - 2*B*a*b^3 + 2*A*b^4 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))*sin(d*x + c))/(
(a^2*b^3 - b^5)*d), 1/2*((2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b
^4)*d*x - 2*(B*a^3 - A*a^2*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/
(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*
A*b^4 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d)
]
```

giac [A] time = 0.49, size = 227, normalized size = 1.69

$$\frac{\frac{\left(2\,Ba^2-2\,Aab+Bb^2\right)(dx+c)}{b^3}+\frac{4\left(Ba^3-Aa^2b\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^3}-\frac{2\left(2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ab\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((2*B*a^2 - 2*A*a*b + B*b^2)*(d*x + c)/b^3 + 4*(B*a^3 - A*a^2*b)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*
c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^3) - 2*(2
*B*a*tan(1/2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 + B*b*tan(1/2*d*
x + 1/2*c)^3 + 2*B*a*tan(1/2*d*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c) - B*
b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

maple [B] time = 0.08, size = 367, normalized size = 2.74

$$\frac{2a^2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)A}{d\,b^2\sqrt{(a-b)\,(a+b)}}-\frac{2a^3\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)B}{d\,b^3\sqrt{(a-b)\,(a+b)}}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)Ba}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d*a^2/b^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))*A-2/d*a^3/b^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)
/((a-b)*(a+b))^(1/2))*B+2/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)
^3*A-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B*a-1/d/b/(1+t
an(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B+2/d/b/(1+tan(1/2*d*x+1/2*c)^2
)^2*tan(1/2*d*x+1/2*c)*A-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2
*c)*B*a+1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B-2/d/b^2*arcta
n(tan(1/2*d*x+1/2*c))*A*a+2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2*B+1/d/b*ar
ctan(tan(1/2*d*x+1/2*c))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 4.00, size = 3761, normalized size = 28.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(2*A*b - 2*B*a + B*b))/b^2 - (tan(c/2 + (d*x)/2)^3*(2*
B*a - 2*A*b + B*b))/b^2)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4
+ 1)) - (atan((((((((8*(2*B*b^10 + 8*A*a^2*b^8 - 4*A*a^3*b^7 + 2*B*a^2*b^8 -
 6*B*a^3*b^7 + 4*B*a^4*b^6 - 4*A*a*b^9 - 2*B*a*b^9))/b^6 - (4*tan(c/2 + (d*
x)/2)*(B*a^2*2i + B*b^2*1i - A*a*b*2i)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/
b^7)*(B*a^2*2i + B*b^2*1i - A*a*b*2i))/(2*b^3) - (8*tan(c/2 + (d*x)/2)*(8*B
^2*a^7 - B^2*b^7 + 3*B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A^2*a^3*
b^4 - 16*A^2*a^4*b^3 + 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b^4 - 16*
B^2*a^4*b^3 + 16*B^2*a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B*a^2*b^5
+ 20*A*B*a^3*b^4 - 28*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4)*(B*a^2*2i + B*b^2
*1i - A*a*b*2i)*1i)/(2*b^3) - (((((8*(2*B*b^10 + 8*A*a^2*b^8 - 4*A*a^3*b^7
+ 2*B*a^2*b^8 - 6*B*a^3*b^7 + 4*B*a^4*b^6 - 4*A*a*b^9 - 2*B*a*b^9))/b^6 + (
4*tan(c/2 + (d*x)/2)*(B*a^2*2i + B*b^2*1i - A*a*b*2i)*(8*a*b^8 - 16*a^2*b^7
 + 8*a^3*b^6))/b^7)*(B*a^2*2i + B*b^2*1i - A*a*b*2i))/(2*b^3) + (8*tan(c/2
+ (d*x)/2)*(8*B^2*a^7 - B^2*b^7 + 3*B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^
5 + 12*A^2*a^3*b^4 - 16*A^2*a^4*b^3 + 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^
2*a^3*b^4 - 16*B^2*a^4*b^3 + 16*B^2*a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b -
12*A*B*a^2*b^5 + 20*A*B*a^3*b^4 - 28*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4)*(B
*a^2*2i + B*b^2*1i - A*a*b*2i)*1i)/(2*b^3))/((16*(4*B^3*a^8 - 6*B^3*a^7*b +
 4*A^3*a^4*b^4 - 4*A^3*a^5*b^3 - B^3*a^3*b^5 + 2*B^3*a^4*b^4 - 5*B^3*a^5*b^
3 + 6*B^3*a^6*b^2 - 12*A*B^2*a^7*b + A*B^2*a^2*b^6 - 2*A*B^2*a^3*b^5 + 9*A*
B^2*a^4*b^4 - 12*A*B^2*a^5*b^3 + 16*A*B^2*a^6*b^2 - 4*A^2*B*a^3*b^5 + 6*A^2
*B*a^4*b^4 - 14*A^2*B*a^5*b^3 + 12*A^2*B*a^6*b^2))/b^6 + (((((8*(2*B*b^10 +
 8*A*a^2*b^8 - 4*A*a^3*b^7 + 2*B*a^2*b^8 - 6*B*a^3*b^7 + 4*B*a^4*b^6 - 4*A*
a*b^9 - 2*B*a*b^9))/b^6 - (4*tan(c/2 + (d*x)/2)*(B*a^2*2i + B*b^2*1i - A*a*
b*2i)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/b^7)*(B*a^2*2i + B*b^2*1i - A*a*b
*2i))/(2*b^3) - (8*tan(c/2 + (d*x)/2)*(8*B^2*a^7 - B^2*b^7 + 3*B^2*a*b^6 -
16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A^2*a^3*b^4 - 16*A^2*a^4*b^3 + 8*A^2*a^5*
b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b^4 - 16*B^2*a^4*b^3 + 16*B^2*a^5*b^2 + 4*
A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B*a^2*b^5 + 20*A*B*a^3*b^4 - 28*A*B*a^4*b^3
 + 32*A*B*a^5*b^2))/b^4)*(B*a^2*2i + B*b^2*1i - A*a*b*2i))/(2*b^3) + (((((8
*(2*B*b^10 + 8*A*a^2*b^8 - 4*A*a^3*b^7 + 2*B*a^2*b^8 - 6*B*a^3*b^7 + 4*B*a^
4*b^6 - 4*A*a*b^9 - 2*B*a*b^9))/b^6 + (4*tan(c/2 + (d*x)/2)*(B*a^2*2i + B*b
^2*1i - A*a*b*2i)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/b^7)*(B*a^2*2i + B*b^
2*1i - A*a*b*2i))/(2*b^3) + (8*tan(c/2 + (d*x)/2)*(8*B^2*a^7 - B^2*b^7 + 3*
B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A^2*a^3*b^4 - 16*A^2*a^4*b^3
+ 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b^4 - 16*B^2*a^4*b^3 + 16*B^2*
a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B*a^2*b^5 + 20*A*B*a^3*b^4 - 28
*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4)*(B*a^2*2i + B*b^2*1i - A*a*b*2i))/(2*b
^3)))*(B*a^2*2i + B*b^2*1i - A*a*b*2i)*1i)/(b^3*d) + (a^2*atan(((a^2*(-(a +
 b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(8*B^2*a^7 - B^2*b^7
```

```
+ 3*B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A^2*a^3*b^4 - 16*A^2*a^4*
b^3 + 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b^4 - 16*B^2*a^4*b^3 + 16*
B^2*a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B*a^2*b^5 + 20*A*B*a^3*b^4
- 28*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4 + (a^2*(-(a + b)*(a - b))^(1/2)*(A*
b - B*a)*((8*(2*B*b^10 + 8*A*a^2*b^8 - 4*A*a^3*b^7 + 2*B*a^2*b^8 - 6*B*a^3*
b^7 + 4*B*a^4*b^6 - 4*A*a*b^9 - 2*B*a*b^9))/b^6 + (8*a^2*tan(c/2 + (d*x)/2)
*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/(
b^4*(b^5 - a^2*b^3))))/(b^5 - a^2*b^3))*1i)/(b^5 - a^2*b^3) + (a^2*(-(a + b
)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(8*B^2*a^7 - B^2*b^7 +
3*B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A^2*a^3*b^4 - 16*A^2*a^4*b^
3 + 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b^4 - 16*B^2*a^4*b^3 + 16*B^
2*a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B*a^2*b^5 + 20*A*B*a^3*b^4 -
28*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4 - (a^2*(-(a + b)*(a - b))^(1/2)*(A*b
- B*a)*((8*(2*B*b^10 + 8*A*a^2*b^8 - 4*A*a^3*b^7 + 2*B*a^2*b^8 - 6*B*a^3*b^
7 + 4*B*a^4*b^6 - 4*A*a*b^9 - 2*B*a*b^9))/b^6 - (8*a^2*tan(c/2 + (d*x)/2)*(
-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/(b^
4*(b^5 - a^2*b^3))))/(b^5 - a^2*b^3))*1i)/(b^5 - a^2*b^3))/((16*(4*B^3*a^8
- 6*B^3*a^7*b + 4*A^3*a^4*b^4 - 4*A^3*a^5*b^3 - B^3*a^3*b^5 + 2*B^3*a^4*b^4
 - 5*B^3*a^5*b^3 + 6*B^3*a^6*b^2 - 12*A*B^2*a^7*b + A*B^2*a^2*b^6 - 2*A*B^2
*a^3*b^5 + 9*A*B^2*a^4*b^4 - 12*A*B^2*a^5*b^3 + 16*A*B^2*a^6*b^2 - 4*A^2*B*
a^3*b^5 + 6*A^2*B*a^4*b^4 - 14*A^2*B*a^5*b^3 + 12*A^2*B*a^6*b^2))/b^6 + (a^
2*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(8*B^2*a^7 -
B^2*b^7 + 3*B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A^2*a^3*b^4 - 16*
A^2*a^4*b^3 + 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b^4 - 16*B^2*a^4*b
^3 + 16*B^2*a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B*a^2*b^5 + 20*A*B*
a^3*b^4 - 28*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4 + (a^2*(-(a + b)*(a - b))^(
1/2)*(A*b - B*a)*((8*(2*B*b^10 + 8*A*a^2*b^8 - 4*A*a^3*b^7 + 2*B*a^2*b^8 -
6*B*a^3*b^7 + 4*B*a^4*b^6 - 4*A*a*b^9 - 2*B*a*b^9))/b^6 + (8*a^2*tan(c/2 +
(d*x)/2)*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3
*b^6))/(b^4*(b^5 - a^2*b^3))))/(b^5 - a^2*b^3)))/(b^5 - a^2*b^3) - (a^2*(-(
a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(8*B^2*a^7 - B^2*b
^7 + 3*B^2*a*b^6 - 16*B^2*a^6*b - 4*A^2*a^2*b^5 + 12*A^2*a^3*b^4 - 16*A^2*a
^4*b^3 + 8*A^2*a^5*b^2 - 7*B^2*a^2*b^5 + 13*B^2*a^3*b^4 - 16*B^2*a^4*b^3 +
16*B^2*a^5*b^2 + 4*A*B*a*b^6 - 16*A*B*a^6*b - 12*A*B*a^2*b^5 + 20*A*B*a^3*b
^4 - 28*A*B*a^4*b^3 + 32*A*B*a^5*b^2))/b^4 - (a^2*(-(a + b)*(a - b))^(1/2)*
(A*b - B*a)*((8*(2*B*b^10 + 8*A*a^2*b^8 - 4*A*a^3*b^7 + 2*B*a^2*b^8 - 6*B*a
^3*b^7 + 4*B*a^4*b^6 - 4*A*a*b^9 - 2*B*a*b^9))/b^6 - (8*a^2*tan(c/2 + (d*x)
/2)*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6)
)/(b^4*(b^5 - a^2*b^3))))/(b^5 - a^2*b^3)))/(b^5 - a^2*b^3)))*(-(a + b)*(a
- b))^(1/2)*(A*b - B*a)*2i)/(d*(b^5 - a^2*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.252 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=89

$$-\frac{2a(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\,\sqrt{a+b}}+\frac{x(Ab-aB)}{b^2}+\frac{B\sin(c+dx)}{bd}$$

```
[Out] (A*b-B*a)*x/b^2+B*sin(d*x+c)/b/d-2*a*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d
*x+1/2*c)/(a+b)^(1/2))/b^2/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3023, 12, 2735, 2659, 205}

$$-\frac{2a(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\,\sqrt{a+b}}+\frac{x(Ab-aB)}{b^2}+\frac{B\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 - (2*a*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (B*Sin[c + d*x])/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx &= \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx\\
&= \frac{B\sin(c+dx)}{bd}+\frac{\int \frac{(Ab-aB)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}\\
&= \frac{B\sin(c+dx)}{bd}+\frac{(Ab-aB)\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}\\
&= \frac{(Ab-aB)x}{b^2}+\frac{B\sin(c+dx)}{bd}-\frac{(a(Ab-aB))\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2}\\
&= \frac{(Ab-aB)x}{b^2}+\frac{B\sin(c+dx)}{bd}-\frac{(2a(Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x
\end{aligned}
$$

$$
= \frac{(Ab-aB)x}{b^2}-\frac{2a(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d}+\frac{B\sin(c+dx)}{bd}
$$

Mathematica [A] time = 0.21, size = 85, normalized size = 0.96

$$-\frac{\frac{2a(aB-Ab)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+(c+dx)(Ab-aB)+bB\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*(c + d*x) - (2*a*(-(A*b) + a*B)*ArcTanh[((a - b)*Tan[(c + d*x)
/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + b*B*Sin[c + d*x])/(b^2*d)
```

fricas [A] time = 0.94, size = 322, normalized size = 3.62

$$\left[-\frac{2\left(Ba^3-Aa^2b-Bab^2+Ab^3\right)dx-\left(Ba^2-Aab\right)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right.}{2\left(a^2b^2-b^4\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] [-1/2*(2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a^2 - A*a*b)*sqrt(-a^
2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a
^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*
b^2 - b^4)*d), -((B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a^2 - A*a*b)*
sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))
) - (B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d)]
```

giac [A] time = 0.45, size = 142, normalized size = 1.60

$$-\frac{\frac{(Ba-Ab)(dx+c)}{b^2}-\frac{2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}+\frac{2\left(Ba^2-Aab\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -((B*a - A*b)*(d*x + c)/b^2 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 + 1)*b) + 2*(B*a^2 - A*a*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
 + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^
2 - b^2)))/(sqrt(a^2 - b^2)*b^2))/d
```

maple [B] time = 0.08, size = 172, normalized size = 1.93

$$-\frac{2a\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)A}{db\sqrt{(a-b)(a+b)}}+\frac{2a^2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)B}{d\,b^2\sqrt{(a-b)(a+b)}}+\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d*a/b/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^
(1/2))*A+2/d*a^2/b^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((
a-b)*(a+b))^(1/2))*B+2/d/b*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2/
d/b*arctan(tan(1/2*d*x+1/2*c))*A-2/d/b^2*arctan(tan(1/2*d*x+1/2*c))*B*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.12, size = 541, normalized size = 6.08

$$\frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,\left(a^2-b^2\right)}-\frac{2\,A\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,\left(a^2-b^2\right)}-\frac{B\,b\,\sin\left(c+d\,x\right)}{d\,\left(a^2-b^2\right)}+\frac{2\,A\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d\,\left(a^2-b^2\right)}-\frac{2\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

[Out] (2*B*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (2*A*
b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (B*b*sin(c
 + d*x))/(d*(a^2 - b^2)) + (2*A*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2)))/(b*d*(a^2 - b^2)) - (2*B*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/(b^2*d*(a^2 - b^2)) + (A*a*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*
x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b*d*(b^
2 - a^2)^(1/2)) - (A*a*log((b*sin(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2) + c
os(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b*d*(b^2 - a^2)^
(1/2)) - (B*a^2*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b^2*d*(b^2 - a^2)^(1/2)
) + (B*a^2*log((b*sin(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2) + cos(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b^2*d*(b^2 - a^2)^(1/2)) + (
B*a^2*sin(c + d*x))/(b*d*(a^2 - b^2))

sympy [A] time = 118.45, size = 3225, normalized size = 36.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)

[Out] Piecewise((zoo*x*(A + B*cos(c)), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (A*d*x*ta
n(c/2 + d*x/2)**3/(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + A*d*x*
tan(c/2 + d*x/2)/(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + A*tan(c
/2 + d*x/2)**2/(b*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + A/(b*d*ta
n(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + B*d*x*tan(c/2 + d*x/2)**3/(b*d*
tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + B*d*x*tan(c/2 + d*x/2)/(b*d*t
an(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + 3*B*tan(c/2 + d*x/2)**2/(b*d*t
an(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + B/(b*d*tan(c/2 + d*x/2)**3 + b
*d*tan(c/2 + d*x/2)), Eq(a, -b)), ((A*sin(c + d*x)/d + B*x*sin(c + d*x)**2/
2 + B*x*cos(c + d*x)**2/2 + B*sin(c + d*x)*cos(c + d*x)/(2*d))/a, Eq(b, 0))
, (x*(A + B*cos(c))*cos(c)/(a + b*cos(c)), Eq(d, 0)), (A*d*x*tan(c/2 + d*x/
2)**2/(b*d*tan(c/2 + d*x/2)**2 + b*d) + A*d*x/(b*d*tan(c/2 + d*x/2)**2 + b*
d) - A*tan(c/2 + d*x/2)**3/(b*d*tan(c/2 + d*x/2)**2 + b*d) - A*tan(c/2 + d*
x/2)/(b*d*tan(c/2 + d*x/2)**2 + b*d) - B*d*x*tan(c/2 + d*x/2)**2/(b*d*tan(c
/2 + d*x/2)**2 + b*d) - B*d*x/(b*d*tan(c/2 + d*x/2)**2 + b*d) + B*tan(c/2 +
 d*x/2)**3/(b*d*tan(c/2 + d*x/2)**2 + b*d) + 3*B*tan(c/2 + d*x/2)/(b*d*tan(
c/2 + d*x/2)**2 + b*d), Eq(a, b)), (A*a*b*d*x*sqrt(-a/(a - b) - b/(a - b))*
tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)
2 + a*b2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a
 - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) + A*a*b*d
*x*sqrt(-a/(a - b) - b/(a - b))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(
c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a
 - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b)
)) - A*a*b*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))*tan(c/2 +

d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b*
*2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan
(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - A*a*b*log(-sqrt(-
a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b**2*d*sqrt(-a/(a - b) - b/(a
 - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d
*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b)
- b/(a - b))) + A*a*b*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))*
tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)
2 + a*b2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a
 - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) + A*a*b*l
og(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b**2*d*sqrt(-a/(a -
b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b))
 - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a
/(a - b) - b/(a - b))) - A*b**2*d*x*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 +
d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b*
*2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan
(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - A*b**2*d*x*sqrt(-
a/(a - b) - b/(a - b))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x
/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b
/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - B*a*
*2*d*x*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(-a/(
a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a -
 b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqr
t(-a/(a - b) - b/(a - b))) - B*a**2*d*x*sqrt(-a/(a - b) - b/(a - b))/(a*b**
2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a
- b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2
 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) + B*a**2*log(-sqrt(-a/(a - b) - b/(
a - b)) + tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b) -
 b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b
3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)2 - b**3*d*sqrt(-a/(a
- b) - b/(a - b))) + B*a**2*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d
*x/2))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*
d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/
2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - B*a**2*log(sqrt(-a/(
a - b) - b/(a - b)) + tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(
-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/
(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d
*sqrt(-a/(a - b) - b/(a - b))) - B*a**2*log(sqrt(-a/(a - b) - b/(a - b)) +
tan(c/2 + d*x/2))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**
2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a -
 b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) + B*a*b*d*x
*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(-a/(a - b)
 - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) -
 b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(
a - b) - b/(a - b))) + B*a*b*d*x*sqrt(-a/(a - b) - b/(a - b))/(a*b**2*d*sqr

```
t(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) -
b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3
*d*sqrt(-a/(a - b) - b/(a - b))) + 2*B*a*b*sqrt(-a/(a - b) - b/(a - b))*tan
(c/2 + d*x/2)/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 +
a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))
*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - 2*B*b**2*sqrt
(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)/(a*b**2*d*sqrt(-a/(a - b) - b/(a
- b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*
sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) -
 b/(a - b))), True))
```

3.253 $\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\,\sqrt{a+b}}+\frac{Bx}{b}$$

```
[Out] B*x/b+2*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b/d/(a
-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2735, 2659, 205}

$$\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\,\sqrt{a+b}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*x)/b + (2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{Bx}{b} - \frac{(-Ab+aB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b}$$
$$= \frac{Bx}{b} + \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$
$$= \frac{Bx}{b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}$$

Mathematica [A] time = 0.12, size = 68, normalized size = 1.01

$$\frac{\frac{2(aB-Ab)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + B(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*(c + d*x) + (2*(-(A*b) + a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a
^2 + b^2]])/Sqrt[-a^2 + b^2])/(b*d)
```

fricas [A] time = 0.72, size = 242, normalized size = 3.61

$$\left[\frac{2\left(Ba^2-Bb^2\right)dx + (Ba-Ab)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\left(a^2b-b^3\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(B*a^2 - B*b^2)*d*x + (B*a - A*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d
*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
 + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
+ a^2)))/((a^2*b - b^3)*d), ((B*a^2 - B*b^2)*d*x - (B*a - A*b)*sqrt(a^2 - b
^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))))/((a^2*b -
 b^3)*d)]
```

giac [B] time = 0.66, size = 296, normalized size = 4.42

$$-\frac{\frac{\left(\sqrt{a^2-b^2}\,B(2\,a-b)|a-b|-\sqrt{a^2-b^2}\,Ab|a-b|-\sqrt{a^2-b^2}\,A|a-b||b|+\sqrt{a^2-b^2}\,B|a-b||b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{\frac{2\,a+\sqrt{-4\,(a+b)(a-b)+4\,a^2}}{a-b}}}\right)\right)}{\left(a^2-2\,ab+b^2\right)b^2+\left(a^3-2\,a^2b+ab^2\right)|b|}+\frac{(2\,Ba-Ab-Bb\cdots}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -((sqrt(a^2 - b^2)*B*(2*a - b)*abs(a - b) - sqrt(a^2 - b^2)*A*b*abs(a - b)
- sqrt(a^2 - b^2)*A*abs(a - b)*abs(b) + sqrt(a^2 - b^2)*B*abs(a - b)*abs(b)
)*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*
c)/sqrt((2*a + sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/((a^2 - 2*a*b +
 b^2)*b^2 + (a^3 - 2*a^2*b + a*b^2)*abs(b)) + (2*B*a - A*b - B*b + A*abs(b)
 - B*abs(b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2
*d*x + 1/2*c)/sqrt((2*a - sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/(b^2
 - a*abs(b)))/d
```

maple [A] time = 0.06, size = 113, normalized size = 1.69

$$\frac{2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)A}{d\sqrt{(a-b)\,(a+b)}}-\frac{2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)aB}{db\sqrt{(a-b)\,(a+b)}}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)
)*A-2/d/b/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))
^(1/2))*a*B+2/d/b*arctan(tan(1/2*d*x+1/2*c))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.72, size = 344, normalized size = 5.13

$$a\left(B\ln\left(\frac{b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sqrt{-(a+b)\,(a-b)}-B\ln\left(\frac{a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x)),x)
```

```
[Out] (a*(B*log((b*sin(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2))*(-(a + b)*(a - b))^(1/2) - B*log((a
*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)
^(1/2))/cos(c/2 + (d*x)/2))*(b^2 - a^2)^(1/2)) - A*b*log((b*sin(c/2 + (d*x)
/2) - a*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2
+ (d*x)/2))*(-(a + b)*(a - b))^(1/2) + A*b*log((a*sin(c/2 + (d*x)/2) - b*si
n(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2)
)*(b^2 - a^2)^(1/2))/(b*d*(a^2 - b^2)) + (2*B*atan(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2)))/(b*d)
```

sympy [A] time = 24.72, size = 524, normalized size = 7.82

$$\begin{cases} \frac{\tilde{\infty}x(A+B\cos(c))}{\cos(c)} \\ \frac{A}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{Bx}{b}+\frac{B}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} \\ \frac{Ax+\frac{B\sin(c+dx)}{d}}{a} \\ \frac{x(A+B\cos(c))}{a+b\cos(c)} \\ \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd}+\frac{Bx}{b}-\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} \\ \frac{Ab\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{Ab\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}+\frac{Badx\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{Ba\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\right.}{abd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-b^2d\sqrt{-}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cos(c))/cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (A
/(b*d*tan(c/2 + d*x/2)) + B*x/b + B/(b*d*tan(c/2 + d*x/2)), Eq(a, -b)), ((A
*x + B*sin(c + d*x)/d)/a, Eq(b, 0)), (x*(A + B*cos(c))/(a + b*cos(c)), Eq(d
, 0)), (A*tan(c/2 + d*x/2)/(b*d) + B*x/b - B*tan(c/2 + d*x/2)/(b*d), Eq(a,
b)), (A*b*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqrt
(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) - A*b*log(s
qrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqrt(-a/(a - b) - b/
(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) + B*a*d*x*sqrt(-a/(a - b) -
 b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) -
b/(a - b))) - B*a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*
b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) + B
*a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqrt(-a/(a -
 b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) - B*b*d*x*sqrt(-a/(
a - b) - b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a
 - b) - b/(a - b))), True))
```

3.254 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] A*arctanh(sin(d*x+c))/a/d-2*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)
/(a+b)^(1/2))/a/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3001, 3770, 2659, 205}

$$\frac{A\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt
[a - b]*Sqrt[a + b]*d) + (A*ArcTanh[Sin[c + d*x]])/(a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\int \sec(c+dx)\,dx}{a} + \frac{(-Ab+aB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{ad} - \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\right.\right.}{ad}$$

$$= -\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}\,d} + \frac{A\tanh^{-1}(\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.16, size = 112, normalized size = 1.47

$$\frac{\frac{2(Ab-aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + A\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+d\right.\right.\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*(A*b - a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[
-a^2 + b^2] + A*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]))/(a*d)
```

fricas [A] time = 1.91, size = 304, normalized size = 4.00

$$\left[\frac{(Ba-Ab)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right) + \left(Aa^2-Ab^2\right)\text{lo}}{2\left(a^3-ab^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] [1/2*((B*a - A*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*
cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (A*a^2 - A*b^2
)*log(sin(d*x + c) + 1) - (A*a^2 - A*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a
*b^2)*d), 1/2*(2*(B*a - A*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(
sqrt(a^2 - b^2)*sin(d*x + c))) + (A*a^2 - A*b^2)*log(sin(d*x + c) + 1) - (A
*a^2 - A*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d)]
```

giac [A] time = 0.47, size = 127, normalized size = 1.67

$$\frac{\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)(Ba-Ab)}{\sqrt{a^2-b^2}\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] (A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - A*log(abs(tan(1/2*d*x + 1/2*c) -
1))/a + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(B*a - A*b)/(s
qrt(a^2 - b^2)*a))/d
```

maple [A] time = 0.12, size = 135, normalized size = 1.78

$$-\frac{2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)Ab}{da\sqrt{(a-b)(a+b)}}+\frac{2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)B}{d\sqrt{(a-b)(a+b)}}-\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}+\frac{A\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d/a/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1
/2))*A*b+2/d/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))*B-1/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)+1/a/d*A*ln(tan(1/2*d*x+1/2*c)
+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.60, size = 342, normalized size = 4.50

$$\frac{2\,A\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}+\frac{b\,\left(A\,\ln\left(\frac{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{-\left(a+b\right)\,\left(a-b\right)}-A\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))),x)
```

```
[Out] (2*A*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) + (b*(A*log((a*cos
(c/2 + (d*x)/2) + b*cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/
2))/cos(c/2 + (d*x)/2))*(-(a + b)*(a - b))^(1/2) - A*log((a*sin(c/2 + (d*x)
/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2
+ (d*x)/2))*(b^2 - a^2)^(1/2)) - B*a*log((a*cos(c/2 + (d*x)/2) + b*cos(c/2
+ (d*x)/2) + sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2))*(-(a
 + b)*(a - b))^(1/2) + B*a*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2)
 + cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/cos(c/2 + (d*x)/2))*(b^2 - a^2)^(1
/2))/(a*d*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos{(c + dx)})\sec{(c + dx)}}{a + b\cos{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

3.255 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=99

$$\frac{2b(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c+dx)}{ad}$$

```
[Out] -(A*b-B*a)*arctanh(sin(d*x+c))/a^2/d+2*b*(A*b-B*a)*arctan((a-b)^(1/2)*tan(1
/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/d/(a-b)^(1/2)/(a+b)^(1/2)+A*tan(d*x+c)/a/d
```

Rubi [A] time = 0.18, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 12, 2747, 3770, 2659, 205}

$$\frac{2b(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*b*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*S
qrt[a - b]*Sqrt[a + b]*d) - ((A*b - a*B)*ArcTanh[Sin[c + d*x]])/(a^2*d) + (
A*Tan[c + d*x])/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\tan(c+dx)}{ad}+\frac{\int \frac{(-Ab+aB)\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$

$$= \frac{A\tan(c+dx)}{ad}+\frac{(-Ab+aB)\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$

$$= \frac{A\tan(c+dx)}{ad}-\frac{(Ab-aB)\int \sec(c+dx)\,dx}{a^2}+\frac{(b(Ab-aB))\int \frac{1}{a+b\cos(}$$

$$= -\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{A\tan(c+dx)}{ad}+\frac{(2b(Ab-aB))\,\mathrm{Su}}{}$$

$$= \frac{2b(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}\,d}-\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx)}{a^2d}$$

Mathematica [A] time = 0.56, size = 129, normalized size = 1.30

$$\frac{-\frac{2b(Ab-aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+(Ab-aB)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-2*b*(A*b - a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sq
rt[-a^2 + b^2] + (A*b - a*B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + a*A*Tan[c + d*x])/(a^2*d)
```

fricas [B] time = 0.95, size = 460, normalized size = 4.65

$$\left[\frac{\left(Bab-Ab^2\right)\sqrt{-a^2+b^2}\cos(dx+c)\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2+2\sqrt{-a^2+b^2}\left(a\cos(dx+c)+b\right)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)+\left(B\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/2*((B*a*b - A*b^2)*sqrt(-a^2 + b^2)*cos(d*x + c)*log((2*a*b*cos(d*x + c)
 + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*s
in(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))
 + (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) -
 (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) +
2*(A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*x + c)), -1/2*(2
*(B*a*b - A*b^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b
^2)*sin(d*x + c)))*cos(d*x + c) - (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d
*x + c)*log(sin(d*x + c) + 1) + (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x
 + c)*log(-sin(d*x + c) + 1) - 2*(A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^
2*b^2)*d*cos(d*x + c))]
```

giac [A] time = 0.97, size = 175, normalized size = 1.77

$$\frac{\frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a}+\frac{2\left(Bab-Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\right.}{\sqrt{a^2-b^2}\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] ((B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - (B*a - A*b)*log(abs(t
an(1/2*d*x + 1/2*c) - 1))/a^2 - 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/
2*c)^2 - 1)*a) + 2*(B*a*b - A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2
*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(
a^2 - b^2)))/(sqrt(a^2 - b^2)*a^2))/d
```

maple [B] time = 0.15, size = 228, normalized size = 2.30

$$\frac{2b^2 \arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) A}{d\,a^2\sqrt{(a-b)\,(a+b)}}-\frac{2b \arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) B}{da\sqrt{(a-b)\,(a+b)}}-\frac{A}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) Ab}{d\,a^2}-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d*b^2/a^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))*A-2/d*b/a/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a
-b)*(a+b))^(1/2))*B-1/a/d*A/(tan(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tan(1/2*d*x+1
/2*c)-1)*A*b-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B-1/a/d*A/(tan(1/2*d*x+1/2*c)+1
)-1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*A*b+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.99, size = 675, normalized size = 6.82

$$\frac{A\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)2\mathrm{i}}{d\,\left(a^2-b^2\right)}-\frac{B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)2\mathrm{i}}{d\,\left(a^2-b^2\right)}+\frac{A\,a\tan\left(c+d\,x\right)}{d\,\left(a^2-b^2\right)}-\frac{A\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)1\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)2\mathrm{i}}{a^2\,d\,\left(a^2-b^2\right)}+\frac{B\,b^2\,\mathrm{atan}}{a\,d\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))),x)
```

```
[Out] (A*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/(d*(a^2 - b^2)) -
 (B*a*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/(d*(a^2 - b^2))
+ (A*a*tan(c + d*x))/(d*(a^2 - b^2)) - (A*b^3*atan((sin(c/2 + (d*x)/2)*1i)/
cos(c/2 + (d*x)/2))*2i)/(a^2*d*(a^2 - b^2)) + (B*b^2*atan((sin(c/2 + (d*x)/
2)*1i)/cos(c/2 + (d*x)/2))*2i)/(a*d*(a^2 - b^2)) - (A*b^2*tan(c + d*x))/(a*
d*(a^2 - b^2)) - (B*b*atan(((a^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 2*b
^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 2*b^5*sin(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2) + 3*a^2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^3*b^2*sin(c/
2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^4*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)
)*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)^2))*(-(a + b)*(a - b))^(1/2)*2i)/(a
*d*(a^2 - b^2)) + (A*b^2*atan(((a^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) +
2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 2*b^5*sin(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2) + 3*a^2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^3*b^2*sin
(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^4*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1
/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)^2))*(-(a + b)*(a - b))^(1/2)*2i)
/(a^2*d*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x)), x)
```

3.256 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=143

$$-\frac{2b^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}-\frac{(Ab-aB)\tan(c+dx)}{a^2d}+\frac{\left(a^2A-2abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{A}{}$$

```
[Out] 1/2*(A*a^2+2*A*b^2-2*B*a*b)*arctanh(sin(d*x+c))/a^3/d-2*b^2*(A*b-B*a)*arcta
n((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/d/(a-b)^(1/2)/(a+b)^(1/2)
-(A*b-B*a)*tan(d*x+c)/a^2/d+1/2*A*sec(d*x+c)*tan(d*x+c)/a/d
```

Rubi [A] time = 0.49, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(a^2A-2abB+2Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{(Ab-aB)\tan(c+dx)}{a^2d}+\frac{A}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*b^2*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
3*Sqrt[a - b]*Sqrt[a + b]*d) + ((a^2*A + 2*A*b^2 - 2*a*b*B)*ArcTanh[Sin[c +
 d*x]])/(2*a^3*d) - ((A*b - a*B)*Tan[c + d*x])/(a^2*d) + (A*Sec[c + d*x]*Ta
n[c + d*x])/(2*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(-2(Ab-aB)+aA\cos(c+dx)+Ab\cos^2(c+dx)\right)\sec^2(c}{a+b\cos(c+dx)}}{2a}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{a^2d} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad} + \int \frac{\left(a^2A+2Ab^2-2a\right.}{}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{a^2d} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\left(b^2(Ab-aB)\right)}{}$$

$$= \frac{\left(a^2A+2Ab^2-2abB\right)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{(Ab-aB)\tan(c+dx)}{a^2d} +$$

$$= -\frac{2b^2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}\,d} + \frac{\left(a^2A+2Ab^2-2abB\right)\tanh}{2a^3d}$$

Mathematica [B] time = 1.78, size = 300, normalized size = 2.10

$$\frac{\frac{8b^2(Ab-aB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - 2\left(a^2A-2abB+2Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2\left(a^2A-2ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((8*b^2*(A*b - a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/S
qrt[-a^2 + b^2] - 2*(a^2*A + 2*A*b^2 - 2*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]] + 2*(a^2*A + 2*A*b^2 - 2*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + (a^2*A)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*a*(-(A*b
) + a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (a^2*A)/
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a*(-(A*b) + a*B)*Sin[(c + d*x)
/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(4*a^3*d)
```

fricas [B] time = 8.21, size = 589, normalized size = 4.12

$$\left[\frac{2\left(Bab^2-Ab^3\right)\sqrt{-a^2+b^2}\,\cos(dx+c)^2\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2-2\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/4*(2*(B*a*b^2 - A*b^3)*sqrt(-a^2 + b^2)*cos(d*x + c)^2*log((2*a*b*cos(d*
x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
+ b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
 a^2)) + (A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*cos(d*x + c)
^2*log(sin(d*x + c) + 1) - (A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A
*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a^4 - A*a^2*b^2 + 2*(B*a
^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3
*b^2)*d*cos(d*x + c)^2), 1/4*(4*(B*a*b^2 - A*b^3)*sqrt(a^2 - b^2)*arctan(-(
a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 + (A*a^4
 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - (A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*cos(d*x +
 c)^2*log(-sin(d*x + c) + 1) + 2*(A*a^4 - A*a^2*b^2 + 2*(B*a^4 - A*a^3*b -
B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x
 + c)^2)]
```

giac [B] time = 0.77, size = 269, normalized size = 1.88

$$\frac{\frac{\left(Aa^2-2\,Bab+2\,Ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{\left(Aa^2-2\,Bab+2\,Ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{4\left(Bab^2-Ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\cdots\right.\right.}{\sqrt{a^2-b^2}\,a^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((A*a^2 - 2*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - (
A*a^2 - 2*B*a*b + 2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 4*(B*a*
b^2 - A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2
 - b^2)*a^3) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3
 + 2*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + 2*B*a*tan(1/2*
d*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*
a^2))/d
```

maple [B] time = 0.16, size = 410, normalized size = 2.87

$$-\frac{2b^{3}\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)A}{d\,a^{3}\sqrt{(a-b)\,(a+b)}}+\frac{2b^{2}\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)B}{d\,a^{2}\sqrt{(a-b)\,(a+b)}}+\frac{A}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{A}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d*b^3/a^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))*A+2/d*b^2/a^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b
)/((a-b)*(a+b))^(1/2))*B+1/2/a/d*A/(tan(1/2*d*x+1/2*c)-1)^2+1/2/a/d*A/(tan(
1/2*d*x+1/2*c)-1)+1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*A*b-1/a/d/(tan(1/2*d*x+1/2
*c)-1)*B-1/2/a/d*A*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1
)*A*b^2+1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*B*b-1/2/a/d*A/(tan(1/2*d*x+1/2*c)+
1)^2+1/2/a/d*A/(tan(1/2*d*x+1/2*c)+1)+1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*A*b-1/
a/d/(tan(1/2*d*x+1/2*c)+1)*B+1/2/a/d*A*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^3*ln(
tan(1/2*d*x+1/2*c)+1)*A*b^2-1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 4.21, size = 4051, normalized size = 28.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))),x)
```

```
[Out] (B*a*sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*b*
sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*a*sin(c
 + d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*a*atan((sin(c/2
```

+ (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2
 + 1/2)) + (B*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(d*(a^
2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*b^2*atan((sin(c/2 + (d*x)/2)*1i)/
cos(c/2 + (d*x)/2))*1i)/(2*a*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A
*b^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(a^3*d*(a^2 - b^2
)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2
 + (d*x)/2))*1i)/(a^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*b^3*si
n(2*c + 2*d*x))/(2*a^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b^2*s
in(2*c + 2*d*x))/(2*a*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*a*atan
((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(2*d*(a^2
 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B*b*atan((sin(c/2 + (d*x)/2)*1i)/cos
(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 +
1/2)) - (A*b^2*sin(c + d*x))/(2*a*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2))
 + (A*b^3*atan(((A^2*a^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*A^2*b^7*s
in(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*A^2*b^9*sin(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2) - A^2*a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*A^2*a^2*b^7*
sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*A^2*a^4*b^5*sin(c/2 + (d*x)/2)*(b^
2 - a^2)^(1/2) - 3*A^2*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*A^2
*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 2*A^2*a^7*b^2*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2) + 8*B^2*a^2*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2
) - 8*B^2*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 12*B^2*a^4*b^5*sin
(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4*B^2*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 -
 a^2)^(1/2) - 4*B^2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 4*B^2*a^
7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 16*A*B*a*b^6*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(3/2) + 16*A*B*a*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4
*A*B*a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 20*A*B*a^3*b^6*sin(c/2 +
(d*x)/2)*(b^2 - a^2)^(1/2) + 4*A*B*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(
1/2) + 4*A*B*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))*1i)/(cos(c/2 + (
d*x)/2)*(a*b^2 - a^3)*(A^2*a^7 - 3*A^2*a^3*b^4 + 2*A^2*a^5*b^2 - 4*B^2*a^3*
b^4 + 4*B^2*a^5*b^2 - 4*A*B*a^6*b + 4*A*B*a^2*b^5)))*(-(a + b)*(a - b))^(1/
2)*1i)/(a^3*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*b^2*atan((sin(c/
2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(2*a*d*(a^2 - b^2
)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*b^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2
 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(a^3*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 +
1/2)) - (B*b^2*atan(((A^2*a^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*A^2*
b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*A^2*b^9*sin(c/2 + (d*x)/2)*(b^
2 - a^2)^(1/2) - A^2*a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*A^2*a^2
*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*A^2*a^4*b^5*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2) - 3*A^2*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) -
2*A^2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 2*A^2*a^7*b^2*sin(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2) + 8*B^2*a^2*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)
^(3/2) - 8*B^2*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 12*B^2*a^4*b^
5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4*B^2*a^5*b^4*sin(c/2 + (d*x)/2)*(
b^2 - a^2)^(1/2) - 4*B^2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 4*B
^2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 16*A*B*a*b^6*sin(c/2 + (d

x)/2)(b^2 - a^2)^(3/2) + 16*A*B*a*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
) - 4*A*B*a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 20*A*B*a^3*b^6*sin(c
/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 4*A*B*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2) + 4*A*B*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))*1i)/(cos(c/
2 + (d*x)/2)*(a*b^2 - a^3)*(A^2*a^7 - 3*A^2*a^3*b^4 + 2*A^2*a^5*b^2 - 4*B^2
*a^3*b^4 + 4*B^2*a^5*b^2 - 4*A*B*a^6*b + 4*A*B*a^2*b^5)))*(-(a + b)*(a - b)
)^(1/2)*1i)/(a^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b^3*atan((s
in(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(a^2*d*(a^2
- b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*b^3*atan(((A^2*a^9*sin(c/2 + (d*x)/
2)*(b^2 - a^2)^(1/2) + 8*A^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*A
^2*b^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - A^2*a^8*b*sin(c/2 + (d*x)/2)*
(b^2 - a^2)^(1/2) + 8*A^2*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*
A^2*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 3*A^2*a^5*b^4*sin(c/2 +
(d*x)/2)*(b^2 - a^2)^(1/2) - 2*A^2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(
1/2) + 2*A^2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*B^2*a^2*b^5*s
in(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*B^2*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2
 - a^2)^(1/2) + 12*B^2*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4*B^2
*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4*B^2*a^6*b^3*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2) + 4*B^2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
) - 16*A*B*a*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 16*A*B*a*b^8*sin(c/
2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4*A*B*a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)
^(1/2) - 20*A*B*a^3*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 4*A*B*a^4*b^
5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 4*A*B*a^7*b^2*sin(c/2 + (d*x)/2)*(
b^2 - a^2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(A^2*a^7 - 3*A^2*a^
3*b^4 + 2*A^2*a^5*b^2 - 4*B^2*a^3*b^4 + 4*B^2*a^5*b^2 - 4*A*B*a^6*b + 4*A*B
*a^2*b^5)))*cos(2*c + 2*d*x)*(-(a + b)*(a - b))^(1/2)*1i)/(a^3*d*(a^2 - b^2
)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b^2*atan(((A^2*a^9*sin(c/2 + (d*x)/2)*(b
^2 - a^2)^(1/2) + 8*A^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*A^2*b^
9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - A^2*a^8*b*sin(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2) + 8*A^2*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*A^2*a
^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 3*A^2*a^5*b^4*sin(c/2 + (d*x)
/2)*(b^2 - a^2)^(1/2) - 2*A^2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)
+ 2*A^2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*B^2*a^2*b^5*sin(c/
2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*B^2*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^
2)^(1/2) + 12*B^2*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4*B^2*a^5*
b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 4*B^2*a^6*b^3*sin(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2) + 4*B^2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 1
6*A*B*a*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 16*A*B*a*b^8*sin(c/2 + (
d*x)/2)*(b^2 - a^2)^(1/2) - 4*A*B*a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
) - 20*A*B*a^3*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 4*A*B*a^4*b^5*sin
(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 4*A*B*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 -
 a^2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(A^2*a^7 - 3*A^2*a^3*b^4
 + 2*A^2*a^5*b^2 - 4*B^2*a^3*b^4 + 4*B^2*a^5*b^2 - 4*A*B*a^6*b + 4*A*B*a^2*
b^5)))*cos(2*c + 2*d*x)*(-(a + b)*(a - b))^(1/2)*1i)/(a^2*d*(a^2 - b^2)*(co
s(2*c + 2*d*x)/2 + 1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{a + b\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x)), x)
```

3.257 $\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{2b^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{(Ab-aB)\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{\left(a^2+2b^2\right)(Ab-aB)\tanh^{-1}(\sin(c+$$

```
[Out] -1/2*(a^2+2*b^2)*(A*b-B*a)*arctanh(sin(d*x+c))/a^4/d+2*b^3*(A*b-B*a)*arctan
((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/d/(a-b)^(1/2)/(a+b)^(1/2)+
1/3*(2*A*a^2+3*A*b^2-3*B*a*b)*tan(d*x+c)/a^3/d-1/2*(A*b-B*a)*sec(d*x+c)*tan
(d*x+c)/a^2/d+1/3*A*sec(d*x+c)^2*tan(d*x+c)/a/d
```

Rubi [A] time = 0.77, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$\frac{2b^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(2a^2A-3abB+3Ab^2\right)\tan(c+dx)}{3a^3d}-\frac{\left(a^2+2b^2\right)(Ab-aB)\tanh^{-1}(\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*b^3*(A*b - a*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4
*Sqrt[a - b]*Sqrt[a + b]*d) - ((a^2 + 2*b^2)*(A*b - a*B)*ArcTanh[Sin[c + d*
x]])/(2*a^4*d) + ((2*a^2*A + 3*A*b^2 - 3*a*b*B)*Tan[c + d*x])/(3*a^3*d) - (
(A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + (A*Sec[c + d*x]^2*Tan[c
+ d*x])/(3*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{A\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{\left(-3(Ab-aB)+2aA\cos(c+dx)+2Ab\cos^2(c+dx)\right)\mathrm{se}}{a+b\cos(c+dx)}}{3a}$$

$$= -\frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{A\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int}{}$$

$$= \frac{\left(2a^2A+3Ab^2-3abB\right)\tan(c+dx)}{3a^3d} - \frac{(Ab-aB)\sec(c+dx)\tan(c+a}{2a^2d}$$

$$= \frac{\left(2a^2A+3Ab^2-3abB\right)\tan(c+dx)}{3a^3d} - \frac{(Ab-aB)\sec(c+dx)\tan(c+a}{2a^2d}$$

$$= -\frac{\left(a^2+2b^2\right)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2a^4d} + \frac{\left(2a^2A+3Ab^2-3abB\right)}{3a^3d}$$

$$= \frac{2b^3(Ab-aB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}\,d} - \frac{\left(a^2+2b^2\right)(Ab-aB)\tanh^{-1}}{2a^4d}$$

Mathematica [B] time = 2.27, size = 422, normalized size = 2.26

$$\frac{\frac{2a^3A\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2a^3A\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{4a\left(2a^2A-3abB+3Ab^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{4a\left(2a^2A-3abB+3Ab^2\right)\sin\left(\frac{1}{2}(c+d\right.}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((24*b^3*(-(A*b) + a*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])/Sqrt[-a^2 + b^2] - 6*(a^2 + 2*b^2)*(-(A*b) + a*B)*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] + 6*(a^2 + 2*b^2)*(-(A*b) + a*B)*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + (a^2*(-3*A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^2 + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])^3 + (4*a*(2*a^2*A + 3*A*b^2 - 3*a*b*B)*Sin[(c + d*x)/2])/(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]) + (2*a^3*A*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^3 - (a^2*(-3*A*b + a*(A + 3*B)))/(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 + (4*a*(2*a^2*A + 3*A*b^2 - 3*a*b*B)*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]))/(12*a^4*d)
```

fricas [A] time = 2.72, size = 729, normalized size = 3.90

$$\left[\frac{6\left(Bab^3 - Ab^4\right)\sqrt{-a^2+b^2}\cos\left(dx+c\right)^3\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/12*(6*(B*a*b^3 - A*b^4)*sqrt(-a^2 + b^2)*cos(d*x + c)^3*log((2*a*b*cos(d
*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
 + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
+ a^2)) + 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)
*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*
a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2
*A*a^5 - 2*A*a^3*b^2 + 2*(2*A*a^5 - 3*B*a^4*b + A*a^3*b^2 + 3*B*a^2*b^3 - 3
*A*a*b^4)*cos(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3*b^2 + A*a^2*b^3)*cos(
d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x + c)^3), -1/12*(12*(B*a*
b^3 - A*b^4)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*
sin(d*x + c)))*cos(d*x + c)^3 - 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3
- 2*B*a*b^4 + 2*A*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*(B*a^5 - A*
a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) - 2*(2*A*a^5 - 2*A*a^3*b^2 + 2*(2*A*a^5 - 3*B*a^4*b + A*a^3
*b^2 + 3*B*a^2*b^3 - 3*A*a*b^4)*cos(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3
*b^2 + A*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x +
c)^3)]
```

giac [B] time = 0.62, size = 412, normalized size = 2.20

$$\frac{\frac{3\left(Ba^3-Aa^2b+2\,Bab^2-2\,Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{3\left(Ba^3-Aa^2b+2\,Bab^2-2\,Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{12\left(Bab^3-Ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(B*a^3 - A*a^2*b + 2*B*a*b^2 - 2*A*b^3)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/a^4 - 3*(B*a^3 - A*a^2*b + 2*B*a*b^2 - 2*A*b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^4 + 12*(B*a*b^3 - A*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
```

```
*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)
^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*
a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^2*tan(1
/2*d*x + 1/2*c)^3 + 12*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c) -
3*A*a*b*tan(1/2*d*x + 1/2*c) - 6*B*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1
/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^3))/d
```

maple [B] time = 0.18, size = 688, normalized size = 3.68

$$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) A\,b^{3}}{d\,a^{4}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) B\,b^{2}}{d\,a^{3}}-\frac{A\,b^{2}}{d\,a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{Bb}{d\,a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{}{2d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*A*b^3-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B*b
^2-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*A*b^2+1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B*b+
1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2*A*b-1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*A*b
^3+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B*b^2-1/3/a/d*A/(tan(1/2*d*x+1/2*c)-1)^
3+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2*B-1/3/a/d*A/(tan(1/2*d*x+1/2*c)+1)^3-1/2
/a/d/(tan(1/2*d*x+1/2*c)+1)^2*B+1/2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)*A*b-1/2/
a/d*ln(tan(1/2*d*x+1/2*c)-1)*B-1/a/d*A/(tan(1/2*d*x+1/2*c)+1)+1/2/a/d*ln(ta
n(1/2*d*x+1/2*c)+1)*B-1/a/d*A/(tan(1/2*d*x+1/2*c)-1)-1/2/d/a^2/(tan(1/2*d*x
+1/2*c)-1)^2*A*b-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*A*b^2+1/d/a^2/(tan(1/2*d*x+
1/2*c)+1)*B*b+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)*B+1/2/a/d*A/(tan(1/2*d*x+1/2*c
)+1)^2+1/2/a/d/(tan(1/2*d*x+1/2*c)+1)*B-1/2/a/d*A/(tan(1/2*d*x+1/2*c)-1)^2-
1/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*A*b-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)*A*b
-1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)*A*b+2/d*b^4/a^4/((a-b)*(a+b))^(1/2)*arcta
n(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-2/d*b^3/a^3/((a-b)*(a+b))
^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 4.90, size = 4696, normalized size = 25.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^4*(a + b*cos(c + d*x))),x)
```

```
[Out] (atan(((((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*
a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 +
3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b
^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B
*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4
+ 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 + (((8*(2*B*a^13 - 4*A*a^8*b^5 + 6*A
*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3 + 2*B*a
^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 - (4*tan(c/2 + (d*x)/2)*(8*a^10*b +
 8*a^8*b^3 - 16*a^9*b^2)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/a^10)*(2*
A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/(2*a^4))*(2*A*b^3 - B*a^3 + A*a^2*b -
 2*B*a*b^2)*1i)/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16
*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5
 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3
*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 1
6*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^
5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 - (((8*(2*B*a^13
- 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6
*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 + (4*tan(c/2 + (
d*x)/2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*
B*a*b^2))/a^10)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/(2*a^4))*(2*A*b^3
- B*a^3 + A*a^2*b - 2*B*a*b^2)*1i)/(2*a^4))/((16*(4*A^3*b^11 - 6*A^3*a*b^10
 + 6*A^3*a^2*b^9 - 5*A^3*a^3*b^8 + 2*A^3*a^4*b^7 - A^3*a^5*b^6 - 4*B^3*a^3*
b^8 + 6*B^3*a^4*b^7 - 6*B^3*a^5*b^6 + 5*B^3*a^6*b^5 - 2*B^3*a^7*b^4 + B^3*a
^8*b^3 - 12*A^2*B*a*b^10 + 12*A*B^2*a^2*b^9 - 18*A*B^2*a^3*b^8 + 18*A*B^2*a
^4*b^7 - 15*A*B^2*a^5*b^6 + 6*A*B^2*a^6*b^5 - 3*A*B^2*a^7*b^4 + 18*A^2*B*a^
2*b^9 - 18*A^2*B*a^3*b^8 + 15*A^2*B*a^4*b^7 - 6*A^2*B*a^5*b^6 + 3*A^2*B*a^6
*b^5))/a^9 + (((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 +
3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*
b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2
*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 +
 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^
5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 + (((8*(2*B*a^13 - 4*A*a^8*b^5
 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3 +
 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 - (4*tan(c/2 + (d*x)/2)*(8*a^
10*b + 8*a^8*b^3 - 16*a^9*b^2)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/a^1
```

0)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/(2*a^4))*(2*A*b^3 - B*a^3 + A*a
^2*b - 2*B*a*b^2))/(2*a^4) - (((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 -
16*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*
b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*
a^3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2
- 16*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4
*b^5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 - (((8*(2*B*a^
13 - 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4
- 6*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 + (4*tan(c/2
+ (d*x)/2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2)*(2*A*b^3 - B*a^3 + A*a^2*b -
2*B*a*b^2))/a^10)*(2*A*b^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/(2*a^4))*(2*A*b
^3 - B*a^3 + A*a^2*b - 2*B*a*b^2))/(2*a^4)))*(2*A*b^3 - B*a^3 + A*a^2*b - 2
*B*a*b^2)*1i)/(a^4*d) - ((tan(c/2 + (d*x)/2)*(2*A*a^2 + 2*A*b^2 + B*a^2 - A
*a*b - 2*B*a*b))/a^3 + (tan(c/2 + (d*x)/2)^5*(2*A*a^2 + 2*A*b^2 - B*a^2 + A
*a*b - 2*B*a*b))/a^3 - (4*tan(c/2 + (d*x)/2)^3*(A*a^2 + 3*A*b^2 - 3*B*a*b))
/(3*a^3))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (
d*x)/2)^6 - 1)) + (b^3*atan(((b^3*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*
tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*a^8*b + 16*A
^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^
3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*
a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B*a^8*b + 32*
A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4 + 14*A*B*a^6
*b^3 - 6*A*B*a^7*b^2))/a^6 + (b^3*(-(a + b)*(a - b))^(1/2)*((8*(2*B*a^13 -
4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6*B
*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 - (8*b^3*tan(c/2 +
(d*x)/2)*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a^10*b + 8*a^8*b^3 - 16*a
^9*b^2))/(a^6*(a^6 - a^4*b^2)))*(A*b - B*a))/(a^6 - a^4*b^2))*1i)/(a^6 - a^
4*b^2) + (b^3*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(
8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*
a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*
B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6
*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B
*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2
))/a^6 - (b^3*(-(a + b)*(a - b))^(1/2)*((8*(2*B*a^13 - 4*A*a^8*b^5 + 6*A*a^
9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3 + 2*B*a^11
*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 + (8*b^3*tan(c/2 + (d*x)/2)*(-(a + b)*
(a - b))^(1/2)*(A*b - B*a)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6 -
a^4*b^2)))*(A*b - B*a))/(a^6 - a^4*b^2))*1i)/(a^6 - a^4*b^2))/((16*(4*A^3*
b^11 - 6*A^3*a*b^10 + 6*A^3*a^2*b^9 - 5*A^3*a^3*b^8 + 2*A^3*a^4*b^7 - A^3*a
^5*b^6 - 4*B^3*a^3*b^8 + 6*B^3*a^4*b^7 - 6*B^3*a^5*b^6 + 5*B^3*a^6*b^5 - 2*
B^3*a^7*b^4 + B^3*a^8*b^3 - 12*A^2*B*a*b^10 + 12*A*B^2*a^2*b^9 - 18*A*B^2*a
^3*b^8 + 18*A*B^2*a^4*b^7 - 15*A*B^2*a^5*b^6 + 6*A*B^2*a^6*b^5 - 3*A*B^2*a^
7*b^4 + 18*A^2*B*a^2*b^9 - 18*A^2*B*a^3*b^8 + 15*A^2*B*a^4*b^7 - 6*A^2*B*a^
5*b^6 + 3*A^2*B*a^6*b^5))/a^9 + (b^3*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(
(8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*a^8*b + 1

```
6*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6
*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b^5 - 16*B
^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B*a^8*b +
32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4 + 14*A*B*
a^6*b^3 - 6*A*B*a^7*b^2))/a^6 + (b^3*(-(a + b)*(a - b))^(1/2)*((8*(2*B*a^13
 - 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 -
6*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 - (8*b^3*tan(c/
2 + (d*x)/2)*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a^10*b + 8*a^8*b^3 - 1
6*a^9*b^2))/(a^6*(a^6 - a^4*b^2)))*(A*b - B*a))/(a^6 - a^4*b^2)))/(a^6 - a^
4*b^2) - (b^3*(-(a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(
8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*
a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*
B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6
*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B
*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2
))/a^6 - (b^3*(-(a + b)*(a - b))^(1/2)*((8*(2*B*a^13 - 4*A*a^8*b^5 + 6*A*a^
9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3 + 2*B*a^11
*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 + (8*b^3*tan(c/2 + (d*x)/2)*(-(a + b)*
(a - b))^(1/2)*(A*b - B*a)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6 -
 a^4*b^2)))*(A*b - B*a))/(a^6 - a^4*b^2)))/(a^6 - a^4*b^2)))*(-(a + b)*(a -
 b))^(1/2)*(A*b - B*a)*2i)/(d*(a^6 - a^4*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^4(c + dx)}{a + b\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**4/(a + b*cos(c + d*x)), x)
```

3.258 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=263

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\cos(c+dx)}{2b^2d\left(a^2-b^2\right)}-\frac{x\left(-6a^2B+4aAb-b^2B\right)}{2b^4}$$

```
[Out] -1/2*(4*A*a*b-6*B*a^2-B*b^2)*x/b^4+2*a^2*(2*A*a^2*b-3*A*b^3-3*B*a^3+4*B*a*b
^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^4/(a+b
)^(3/2)/d+(2*A*a^2*b-A*b^3-3*B*a^3+2*B*a*b^2)*sin(d*x+c)/b^3/(a^2-b^2)/d-1/
2*(2*A*a*b-3*B*a^2+B*b^2)*cos(d*x+c)*sin(d*x+c)/b^2/(a^2-b^2)/d+a*(A*b-B*a)
*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.66, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3049, 3023, 2735, 2659, 205}

$$\frac{\left(2a^2Ab-3a^3B+2ab^2B-Ab^3\right)\sin(c+dx)}{b^3d\left(a^2-b^2\right)}+\frac{2a^2\left(2a^2Ab-3a^3B+4ab^2B-3Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((4*a*A*b - 6*a^2*B - b^2*B)*x)/(2*b^4) + (2*a^2*(2*a^2*A*b - 3*A*b^3 - 3*
a^3*B + 4*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(3/2)*b^4*(a + b)^(3/2)*d) + ((2*a^2*A*b - A*b^3 - 3*a^3*B + 2*a*b^2*B
)*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) - ((2*a*A*b - 3*a^2*B + b^2*B)*Cos[c +
d*x]*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^2*Si
n[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(-2a(Ab-aB)+b(Ab-aB)\cos\ldots\right.}{a+b\cos\ldots}}{b\left(a\ldots\right.}$$

$$= -\frac{\left(2aAb-3a^2B+b^2B\right)\cos(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\cos^2(c+\ldots}{b\left(a^2-b^2\right)d(a+\ldots}$$

$$= \frac{\left(2a^2Ab-Ab^3-3a^3B+2ab^2B\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{\left(2aAb-3a^2B+b^2B\right)\mathrm{c}\ldots}{2b^2\left(a^2\ldots\right.}$$

$$= -\frac{\left(4aAb-6a^2B-b^2B\right)x}{2b^4} + \frac{\left(2a^2Ab-Ab^3-3a^3B+2ab^2B\right)\sin(c+d\ldots}{b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(4aAb-6a^2B-b^2B\right)x}{2b^4} + \frac{\left(2a^2Ab-Ab^3-3a^3B+2ab^2B\right)\sin(c+d\ldots}{b^3\left(a^2-b^2\right)d}$$

$$= -\frac{\left(4aAb-6a^2B-b^2B\right)x}{2b^4} + \frac{2a^2\left(2a^2Ab-3Ab^3-3a^3B+4ab^2B\right)\tan^{-\ldots}}{(a-b)^{3/2}b^4(a+b)^{3/2}d}$$

Mathematica [A] time = 1.07, size = 184, normalized size = 0.70

$$\frac{\frac{4a^3b(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + 2(c+dx)\left(6a^2B-4aAb+b^2B\right) - \frac{8a^2\left(3a^3B-2a^2Ab-4ab^2B+3Ab^3\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + 4b(Ab\ldots}{4b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*(-4*a*A*b + 6*a^2*B + b^2*B)*(c + d*x) - (8*a^2*(-2*a^2*A*b + 3*A*b^3 +
3*a^3*B - 4*a*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/
(-a^2 + b^2)^(3/2) + 4*b*(A*b - 2*a*B)*Sin[c + d*x] + (4*a^3*b*(A*b - a*B)*
Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])) + b^2*B*Sin[2*(c + d*x
)])/(4*b^4*d)
```

fricas [A] time = 1.57, size = 965, normalized size = 3.67

$$\left[\frac{\left(6\,Ba^6b-4\,Aa^5b^2-11\,Ba^4b^3+8\,Aa^3b^4+4\,Ba^2b^5-4\,Aab^6+Bb^7\right)dx\cos\left(dx+c\right)+\left(6\,Ba^7-4\,Aa^6b-11\,Ba^5b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*((6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 -
 4*A*a*b^6 + B*b^7)*d*x*cos(d*x + c) + (6*B*a^7 - 4*A*a^6*b - 11*B*a^5*b^2
+ 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*d*x - (3*B*a^6 - 2*A*a
^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3 + (3*B*a^5*b - 2*A*a^4*b^2 - 4*B*a^3*b^3 +
 3*A*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a
^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
+ c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*B
*a^6*b - 4*A*a^5*b^2 - 10*B*a^4*b^3 + 6*A*a^3*b^4 + 4*B*a^2*b^5 - 2*A*a*b^6
 - (B*a^4*b^3 - 2*B*a^2*b^5 + B*b^7)*cos(d*x + c)^2 + (3*B*a^5*b^2 - 2*A*a^
4*b^3 - 6*B*a^3*b^4 + 4*A*a^2*b^5 + 3*B*a*b^6 - 2*A*b^7)*cos(d*x + c))*sin(
d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^
6 + a*b^8)*d), 1/2*((6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 +
 4*B*a^2*b^5 - 4*A*a*b^6 + B*b^7)*d*x*cos(d*x + c) + (6*B*a^7 - 4*A*a^6*b -
 11*B*a^5*b^2 + 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*d*x - 2*
(3*B*a^6 - 2*A*a^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3 + (3*B*a^5*b - 2*A*a^4*b^2
 - 4*B*a^3*b^3 + 3*A*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(
d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*B*a^6*b - 4*A*a^5*b^2 -
10*B*a^4*b^3 + 6*A*a^3*b^4 + 4*B*a^2*b^5 - 2*A*a*b^6 - (B*a^4*b^3 - 2*B*a^2
*b^5 + B*b^7)*cos(d*x + c)^2 + (3*B*a^5*b^2 - 2*A*a^4*b^3 - 6*B*a^3*b^4 + 4
*A*a^2*b^5 + 3*B*a*b^6 - 2*A*b^7)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2
*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*d)]
```

giac [A] time = 0.83, size = 338, normalized size = 1.29

$$\frac{\frac{4\left(3\,Ba^5-2\,Aa^4b-4\,Ba^3b^2+3\,Aa^2b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{a^2-b^2}}-\frac{4\left(Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Aa^3b\tan\left(\frac{1}{2}\,\right.\right.}{\left(a^2b^3-b^5\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*(3*B*a^5 - 2*A*a^4*b - 4*B*a^3*b^2 + 3*A*a^2*b^3)*(pi*floor(1/2*(d*x
 + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) - 4*(B
*a^4*tan(1/2*d*x + 1/2*c) - A*a^3*b*tan(1/2*d*x + 1/2*c))/((a^2*b^3 - b^5)*
(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + (6*B*a^2 -
 4*A*a*b + B*b^2)*(d*x + c)/b^4 - 2*(4*B*a*tan(1/2*d*x + 1/2*c)^3 - 2*A*b*t
an(1/2*d*x + 1/2*c)^3 + B*b*tan(1/2*d*x + 1/2*c)^3 + 4*B*a*tan(1/2*d*x + 1/
2*c) - 2*A*b*tan(1/2*d*x + 1/2*c) - B*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
 + 1/2*c)^2 + 1)^2*b^3))/d
```

maple [B] time = 0.09, size = 643, normalized size = 2.44

$$\frac{2a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A}{d\,b^{2}\left(a^{2}-b^{2}\right)\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}-\frac{2a^{4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{d\,b^{3}\left(a^{2}-b^{2}\right)\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)*A-2/d*a^4/b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x
+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B+4/d*a^4/b^3/(a-b)/(a+b)/((a-b)*(a+b
))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-6/d*a^2/b/(
a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)
)^(1/2))*A-6/d*a^5/b^4/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1
/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+8/d*a^3/b^2/(a-b)/(a+b)/((a-b)*(a+b))^(1
/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+2/d/b^2/(1+tan(1
/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A-4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^
2*tan(1/2*d*x+1/2*c)^3*B*a-1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1
/2*c)^3*B+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*A-4/d/b^3/(
1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B*a+1/d/b^2/(1+tan(1/2*d*x+1/2
*c)^2)^2*tan(1/2*d*x+1/2*c)*B-4/d/b^3*arctan(tan(1/2*d*x+1/2*c))*A*a+6/d/b^
4*arctan(tan(1/2*d*x+1/2*c))*a^2*B+1/d/b^2*arctan(tan(1/2*d*x+1/2*c))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.21, size = 6744, normalized size = 25.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (a^2*atan(((a^2*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*B^2
*a^10 + B^2*b^10 - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3
*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 +
32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*
a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9
- 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*
a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8*b^2))/(a*b^8 + b^9
 - a^2*b^7 - a^3*b^6) + (a^2*((8*(2*B*b^15 + 12*A*a^2*b^13 + 12*A*a^3*b^12
- 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6*b^9 + 6*B*a^2*b^13 - 16*B*a^3*b^12
 - 14*B*a^4*b^11 + 28*B*a^5*b^10 + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^14)
)/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (8*a^2*tan(c/2 + (d*x)/2)*(-(a + b
)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2)*(8*a*b^13
- 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8
+ b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(-(a
+ b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2))/(b^10
- 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*
b^2)*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + (a^2*(-(a + b)^3*(a - b
)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 + B^2*b^10 - 2*B^2*a*b^9 - 7
2*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5
*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 -
20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2
*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 -
40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*
B*a^7*b^3 + 96*A*B*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a^2*((8*(
2*B*b^15 + 12*A*a^2*b^13 + 12*A*a^3*b^12 - 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8
*A*a^6*b^9 + 6*B*a^2*b^13 - 16*B*a^3*b^12 - 14*B*a^4*b^11 + 28*B*a^5*b^10 +
 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^14))/(a*b^11 + b^12 - a^2*b^10 - a^3*
b^9) + (8*a^2*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*
B*a^3 - 2*A*a^2*b - 4*B*a*b^2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^
4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 -
 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 +
 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)
)*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2)*1i)/(b^10 - 3*a^2*b^8 + 3*a^4
*b^6 - a^6*b^4))/((16*(108*B^3*a^11 - 54*B^3*a^10*b - 48*A^3*a^4*b^7 - 24*A
^3*a^5*b^6 + 80*A^3*a^6*b^5 + 16*A^3*a^7*b^4 - 32*A^3*a^8*b^3 + 4*B^3*a^3*b
```

^8 - 4*B^3*a^4*b^7 + 41*B^3*a^5*b^6 - 9*B^3*a^6*b^5 + 63*B^3*a^7*b^4 + 81*B
^3*a^8*b^3 - 216*B^3*a^9*b^2 - 216*A*B^2*a^10*b - 3*A*B^2*a^2*b^9 + 3*A*B^2
*a^3*b^8 - 63*A*B^2*a^4*b^7 + 15*A*B^2*a^5*b^6 - 186*A*B^2*a^6*b^5 - 162*A*
B^2*a^7*b^4 + 468*A*B^2*a^8*b^3 + 108*A*B^2*a^9*b^2 + 24*A^2*B*a^3*b^8 - 6*
A^2*B*a^4*b^7 + 168*A^2*B*a^5*b^6 + 108*A^2*B*a^6*b^5 - 336*A^2*B*a^7*b^4 -
 72*A^2*B*a^8*b^3 + 144*A^2*B*a^9*b^2))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9
) - (a^2*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 +
 B^2*b^10 - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 +
20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*
a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5
 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*
B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5
 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8*b^2))/(a*b^8 + b^9 - a^2*
b^7 - a^3*b^6) + (a^2*((8*(2*B*b^15 + 12*A*a^2*b^13 + 12*A*a^3*b^12 - 20*A*
a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6*b^9 + 6*B*a^2*b^13 - 16*B*a^3*b^12 - 14*B
*a^4*b^11 + 28*B*a^5*b^10 + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^14))/(a*b^
11 + b^12 - a^2*b^10 - a^3*b^9) - (8*a^2*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a
 - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2)*(8*a*b^13 - 8*a^2
*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 -
 a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(-(a + b)^3*
(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2))/(b^10 - 3*a^2
*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2))/(
b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + (a^2*(-(a + b)^3*(a - b)^3)^(1/2)
*((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 + B^2*b^10 - 2*B^2*a*b^9 - 72*B^2*a^9*
b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*
A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3
*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 -
 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3
*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3
+ 96*A*B*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a^2*((8*(2*B*b^15 +
 12*A*a^2*b^13 + 12*A*a^3*b^12 - 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6*b^9
 + 6*B*a^2*b^13 - 16*B*a^3*b^12 - 14*B*a^4*b^11 + 28*B*a^5*b^10 + 6*B*a^6*b
^9 - 12*B*a^7*b^8 - 8*A*a*b^14))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (8*
a^2*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 2*
A*a^2*b - 4*B*a*b^2)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8
*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8
 + 3*a^4*b^6 - a^6*b^4)))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 -
 2*A*a^2*b - 4*B*a*b^2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*A*b^3
 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^
4)))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 2*A*a^2*b - 4*B*a*b^
2)*2i)/(d*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)) - (atan(-((((8*tan(c/2
+ (d*x)/2)*(72*B^2*a^10 + B^2*b^10 - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^
2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 -
 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^2
*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8*

```
b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*B
*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8*
b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (((8*(2*B*b^15 + 12*A*a^2*b^13 +
12*A*a^3*b^12 - 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6*b^9 + 6*B*a^2*b^13 -
 16*B*a^3*b^12 - 14*B*a^4*b^11 + 28*B*a^5*b^10 + 6*B*a^6*b^9 - 12*B*a^7*b^8
 - 8*A*a*b^14))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (4*tan(c/2 + (d*x)/2
)*(B*a^2*6i + B*b^2*1i - A*a*b*4i)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 1
6*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)
))*(B*a^2*6i + B*b^2*1i - A*a*b*4i))/(2*b^4))*(B*a^2*6i + B*b^2*1i - A*a*b*
4i)*1i)/(2*b^4) + (((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 + B^2*b^10 - 2*B^2*a
*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64
*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^
2*b^8 - 20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 +
 120*B^2*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^
2*b^8 - 40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4
 + 176*A*B*a^7*b^3 + 96*A*B*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - ((
(8*(2*B*b^15 + 12*A*a^2*b^13 + 12*A*a^3*b^12 - 20*A*a^4*b^11 - 4*A*a^5*b^10
 + 8*A*a^6*b^9 + 6*B*a^2*b^13 - 16*B*a^3*b^12 - 14*B*a^4*b^11 + 28*B*a^5*b^
10 + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^14))/(a*b^11 + b^12 - a^2*b^10 -
a^3*b^9) + (4*tan(c/2 + (d*x)/2)*(B*a^2*6i + B*b^2*1i - A*a*b*4i)*(8*a*b^13
 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a
*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(B*a^2*6i + B*b^2*1i - A*a*b*4i))/(2*b^4)
)*(B*a^2*6i + B*b^2*1i - A*a*b*4i)*1i)/(2*b^4))/((16*(108*B^3*a^11 - 54*B^3
*a^10*b - 48*A^3*a^4*b^7 - 24*A^3*a^5*b^6 + 80*A^3*a^6*b^5 + 16*A^3*a^7*b^4
 - 32*A^3*a^8*b^3 + 4*B^3*a^3*b^8 - 4*B^3*a^4*b^7 + 41*B^3*a^5*b^6 - 9*B^3*
a^6*b^5 + 63*B^3*a^7*b^4 + 81*B^3*a^8*b^3 - 216*B^3*a^9*b^2 - 216*A*B^2*a^1
0*b - 3*A*B^2*a^2*b^9 + 3*A*B^2*a^3*b^8 - 63*A*B^2*a^4*b^7 + 15*A*B^2*a^5*b
^6 - 186*A*B^2*a^6*b^5 - 162*A*B^2*a^7*b^4 + 468*A*B^2*a^8*b^3 + 108*A*B^2*
a^9*b^2 + 24*A^2*B*a^3*b^8 - 6*A^2*B*a^4*b^7 + 168*A^2*B*a^5*b^6 + 108*A^2*
B*a^6*b^5 - 336*A^2*B*a^7*b^4 - 72*A^2*B*a^8*b^3 + 144*A^2*B*a^9*b^2))/(a*b
^11 + b^12 - a^2*b^10 - a^3*b^9) - (((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 + B
^2*b^10 - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20
*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^
8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5 +
 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*B*
a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5 -
 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8*b^2))/(a*b^8 + b^9 - a^2*b^
7 - a^3*b^6) + (((8*(2*B*b^15 + 12*A*a^2*b^13 + 12*A*a^3*b^12 - 20*A*a^4*b^
11 - 4*A*a^5*b^10 + 8*A*a^6*b^9 + 6*B*a^2*b^13 - 16*B*a^3*b^12 - 14*B*a^4*b
^11 + 28*B*a^5*b^10 + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^14))/(a*b^11 + b
^12 - a^2*b^10 - a^3*b^9) - (4*tan(c/2 + (d*x)/2)*(B*a^2*6i + B*b^2*1i - A*
a*b*4i)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*
a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(B*a^2*6i + B*b^2*1i - A
*a*b*4i))/(2*b^4))*(B*a^2*6i + B*b^2*1i - A*a*b*4i))/(2*b^4) + (((8*tan(c/2
 + (d*x)/2)*(72*B^2*a^10 + B^2*b^10 - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a
```

```
^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4
- 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^
2*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8
*b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*
B*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8
*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (((8*(2*B*b^15 + 12*A*a^2*b^13 +
 12*A*a^3*b^12 - 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6*b^9 + 6*B*a^2*b^13
- 16*B*a^3*b^12 - 14*B*a^4*b^11 + 28*B*a^5*b^10 + 6*B*a^6*b^9 - 12*B*a^7*b^
8 - 8*A*a*b^14))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (4*tan(c/2 + (d*x)/
2)*(B*a^2*6i + B*b^2*1i - A*a*b*4i)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 +
16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6
)))*(B*a^2*6i + B*b^2*1i - A*a*b*4i))/(2*b^4))*(B*a^2*6i + B*b^2*1i - A*a*b
*4i))/(2*b^4)))*(B*a^2*6i + B*b^2*1i - A*a*b*4i)*1i)/(b^4*d) - ((tan(c/2 +
(d*x)/2)^5*(6*B*a^4 - 2*A*b^4 + B*b^4 + 2*A*a^2*b^2 - 5*B*a^2*b^2 + 2*A*a*b
^3 - 4*A*a^3*b + 3*B*a*b^3 - 3*B*a^3*b))/((a*b^3 - b^4)*(a + b)) + (tan(c/2
 + (d*x)/2)*(2*A*b^4 + 6*B*a^4 + B*b^4 - 2*A*a^2*b^2 - 5*B*a^2*b^2 + 2*A*a*
b^3 - 4*A*a^3*b - 3*B*a*b^3 + 3*B*a^3*b))/((a*b^3 - b^4)*(a + b)) - (2*tan(
c/2 + (d*x)/2)^3*(B*b^4 - 6*B*a^4 + 3*B*a^2*b^2 - 2*A*a*b^3 + 4*A*a^3*b))/(
b*(a*b^2 - b^3)*(a + b)))/(d*(a + b + tan(c/2 + (d*x)/2)^2*(3*a + b) + tan(
c/2 + (d*x)/2)^6*(a - b) + tan(c/2 + (d*x)/2)^4*(3*a - b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.259 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=155

$$-\frac{a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{2a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{x(Ab-2aB)}{b^3}+$$

```
[Out] (A*b-2*B*a)*x/b^3-2*a*(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^2)*arctan((a-b)^(1/2
)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^3/(a+b)^(3/2)/d+B*sin(d*x+c
)/b^2/d-a^2*(A*b-B*a)*sin(d*x+c)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.44, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2988, 3023, 2735, 2659, 205}

$$-\frac{2a\left(a^2Ab-2a^3B+3ab^2B-2Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{x(Ab-2aB)}{b^3}+B-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*x)/b^3 - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a*b^2*B)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^
(3/2)*d) + (B*Sin[c + d*x])/(b^2*d) - (a^2*(A*b - a*B)*Sin[c + d*x])/(b^2*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = -\frac{a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{ab(Ab-aB)+\left(a^2-b^2\right)(Ab-aB)\cos(c+dx)+b}{a+b\cos(c+dx)}}{b^2\left(a^2-b^2\right)}$$

$$= \frac{B\sin(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{ab^2(Ab-aB)+b\left(a^2-b^2\right)}{a+b\cos(}}{b^3\left(a^2\right.}$$

$$= \frac{(Ab-2aB)x}{b^3} + \frac{B\sin(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{(a(}{}$$

$$= \frac{(Ab-2aB)x}{b^3} + \frac{B\sin(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{(2a}{}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{2a\left(a^2Ab-2Ab^3-2a^3B+3ab^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+d\right.}{\sqrt{a+b}}\right.}{(a-b)^{3/2}b^3(a+b)^{3/2}d}$$

Mathematica [A] time = 0.85, size = 147, normalized size = 0.95

$$\frac{\frac{a^2b(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + \frac{2a\left(2a^3B-a^2Ab-3ab^2B+2Ab^3\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + (c+dx)(Ab-2aB) + bB\sin(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*(c + d*x) + (2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*B
)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2)
+ b*B*Sin[c + d*x] + (a^2*b*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(
a + b*Cos[c + d*x])))/(b^3*d)
```

fricas [B] time = 1.25, size = 788, normalized size = 5.08

$$\left[-\frac{2\left(2\,Ba^5b - Aa^4b^2 - 4\,Ba^3b^3 + 2\,Aa^2b^4 + 2\,Bab^5 - Ab^6\right)dx\cos\left(dx+c\right) + 2\left(2\,Ba^6 - Aa^5b - 4\,Ba^4b^2 + 2\,Aa^3b^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(2*(2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A
*b^6)*d*x*cos(d*x + c) + 2*(2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 +
 2*B*a^2*b^4 - A*a*b^5)*d*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^
3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(-a
^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-
a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3
+ A*a^2*b^4 + B*a*b^5 + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6)*cos(d*x + c))*sin
(d*x + c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b
^5 + a*b^7)*d), -((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*
a*b^5 - A*b^6)*d*x*cos(d*x + c) + (2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^
3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*d*x - (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A
*a^2*b^3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*
sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))
) - (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b^5 + (B*a^4*b^2
 - 2*B*a^2*b^4 + B*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^4 - 2*a^2*b^6 +
 b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d)]
```

giac [B] time = 3.22, size = 1116, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] ((4*B*a^6*b^2 - 2*A*a^5*b^3 - 2*B*a^5*b^3 + A*a^4*b^4 - 9*B*a^4*b^4 + 5*A*a
^3*b^5 + 4*B*a^3*b^5 - 2*A*a^2*b^6 + 5*B*a^2*b^6 - 3*A*a*b^7 - 2*B*a*b^7 +
A*b^8 + 2*B*a^3*abs(-a^2*b^3 + b^5) - A*a^2*b*abs(-a^2*b^3 + b^5) - B*a^2*b
*abs(-a^2*b^3 + b^5) + A*a*b^2*abs(-a^2*b^3 + b^5) - 2*B*a*b^2*abs(-a^2*b^3
 + b^5) + A*b^3*abs(-a^2*b^3 + b^5))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + ar
ctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*a^3*b^2 - 2*a*b^4 + sqrt(-4*(
a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*(a^3*b^2 - a^2*b^3 - a*b^4 + b^5) + 4*(a^3
*b^2 - a*b^4)^2))/(a^3*b^2 - a^2*b^3 - a*b^4 + b^5))))/(a^3*b^2*abs(-a^2*b^
3 + b^5) - a*b^4*abs(-a^2*b^3 + b^5) + (a^2*b^3 - b^5)^2) + ((a^2*b - a*b^2
 - b^3)*sqrt(a^2 - b^2)*A*abs(-a^2*b^3 + b^5)*abs(-a + b) - (2*a^3 - a^2*b
- 2*a*b^2)*sqrt(a^2 - b^2)*B*abs(-a^2*b^3 + b^5)*abs(-a + b) - (2*a^5*b^3 -
 a^4*b^4 - 5*a^3*b^5 + 2*a^2*b^6 + 3*a*b^7 - b^8)*sqrt(a^2 - b^2)*A*abs(-a
+ b) + (4*a^6*b^2 - 2*a^5*b^3 - 9*a^4*b^4 + 4*a^3*b^5 + 5*a^2*b^6 - 2*a*b^7
)*sqrt(a^2 - b^2)*B*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan
(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*a^3*b^2 - 2*a*b^4 - sqrt(-4*(a^3*
b^2 + a^2*b^3 - a*b^4 - b^5)*(a^3*b^2 - a^2*b^3 - a*b^4 + b^5) + 4*(a^3*b^2
```

```
 - a*b^4)^2))/(a^3*b^2 - a^2*b^3 - a*b^4 + b^5))))/((a^2*b^3 - b^5)^2*(a^2
- 2*a*b + b^2) - (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*abs(-a^2*b^3 + b
^5)) + 2*(2*B*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
 B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*b^3*ta
n(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d*x + 1/2*c) - A*a^2*b*tan(1/2*d*x +
 1/2*c) + B*a^2*b*tan(1/2*d*x + 1/2*c) - B*a*b^2*tan(1/2*d*x + 1/2*c) - B*b
^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c
)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2*b^2 - b^4)))/d
```

maple [B] time = 0.10, size = 445, normalized size = 2.87

$$-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A}{db\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{d\,b^2\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d*a^2/b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)*A+2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B-2/d*a^3/b^2/(a-b)/(a+b)/((a-b)*(a+b)
)^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+4/d*a/(a-b)/
(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/
2))*A+4/d*a^4/b^3/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)
*(a-b)/((a-b)*(a+b))^(1/2))*B-6/d*a^2/b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arc
tan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+2/d/b^2*B*tan(1/2*d*x+1
/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2/d/b^2*A*arctan(tan(1/2*d*x+1/2*c))-4/d/b^3
*B*arctan(tan(1/2*d*x+1/2*c))*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 5.17, size = 3276, normalized size = 21.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1i)*(A*b - 2*B*a)*1i)/(b^3*d) - ((2*tan(c/2 + (d*
x)/2)^3*(A*a^2*b - B*b^3 - 2*B*a^3 + B*a*b^2 + B*a^2*b))/(b^2*(a + b)*(a -
b)) + (2*tan(c/2 + (d*x)/2)*(B*b^3 - 2*B*a^3 + A*a^2*b + B*a*b^2 - B*a^2*b)
)/(b^2*(a + b)*(a - b)))/(d*(a + b + tan(c/2 + (d*x)/2)^4*(a - b) + 2*a*tan
(c/2 + (d*x)/2)^2)) - (log(tan(c/2 + (d*x)/2) - 1i)*(A*b*1i - B*a*2i))/(b^3
*d) - (a*atan(((a*((32*tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^
7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5
*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B
^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8
*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b
^7 - a^2*b^5 - a^3*b^4) + (a*((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^
5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6
+ 2*A*a*b^11 + 2*B*a*b^11))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*a*tan(c
/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3
*B*a*b^2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^
6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a
^6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a
*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(-(a + b)^3*(a - b)^3)^(1/2
)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^
5 - a^6*b^3) + (a*((32*tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^
7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5
*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B
^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8
*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b
^7 - a^2*b^5 - a^3*b^4) - (a*((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^
5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6
+ 2*A*a*b^11 + 2*B*a*b^11))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a*tan(c
/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3
*B*a*b^2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^
6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a
^6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a
*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(-(a + b)^3*(a - b)^3)^(1/2
)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^
5 - a^6*b^3))/((64*(8*B^3*a^8 - 2*A^3*a*b^7 - 4*B^3*a^7*b - 2*A^3*a^2*b^6 +
 3*A^3*a^3*b^5 + A^3*a^4*b^4 - A^3*a^5*b^3 + 12*B^3*a^4*b^4 + 6*B^3*a^5*b^3
 - 20*B^3*a^6*b^2 - 12*A*B^2*a^7*b - 20*A*B^2*a^3*b^5 - 13*A*B^2*a^4*b^4 +
32*A*B^2*a^5*b^3 + 8*A*B^2*a^6*b^2 + 11*A^2*B*a^2*b^6 + 9*A^2*B*a^3*b^5 - 1
7*A^2*B*a^4*b^4 - 5*A^2*B*a^5*b^3 + 6*A^2*B*a^6*b^2))/(a*b^8 + b^9 - a^2*b^
7 - a^3*b^6) - (a*((32*tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^
7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5
*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B
```

```
^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8
*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b
^7 - a^2*b^5 - a^3*b^4) + (a*((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^
5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6
+ 2*A*a*b^11 + 2*B*a*b^11))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*a*tan(c
/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3
*B*a*b^2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^
6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a
^6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a
*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(-(a + b)^3*(a - b)^3)^(1/2
)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 -
 a^6*b^3) + (a*((32*tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 -
 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^
3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*
a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*
B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b^7
- a^2*b^5 - a^3*b^4) - (a*((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^5*b
^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2
*A*a*b^11 + 2*B*a*b^11))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a*tan(c/2
+ (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*
a*b^2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b
^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*
b^3)))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^
2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(-(a + b)^3*(a - b)^3)^(1/2)*(
2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^
6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*
b^2)*2i)/(d*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.260 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=122

$$-\frac{2\left(a^3B-2ab^2B+Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{Bx}{b^2}$$

```
[Out] B*x/b^2-2*(A*b^3+B*a^3-2*B*a*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+
b)^(1/2))/(a-b)^(3/2)/b^2/(a+b)^(3/2)/d+a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/
d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.24, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2968, 3021, 2735, 2659, 205}

$$-\frac{2\left(a^3B-2ab^2B+Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{Bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*x)/b^2 - (2*(A*b^3 + a^3*B - 2*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) + (a*(A*b - a*B)*Si
n[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx \\
&= \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{b(Ab-aB)-\left(a^2-b^2\right)B\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)} \\
&= \frac{Bx}{b^2} + \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(Ab^3+a\left(a^2-2b^2\right)B\right)\int \frac{1}{a+b\cos(\ldots}}{b^2\left(a^2-b^2\right)} \\
&= \frac{Bx}{b^2} + \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(2\left(Ab^3+a\left(a^2-2b^2\right)B\right)\right)\mathrm{Subs}\ldots}{b^2\left(\ldots\right.} \\
&= \frac{Bx}{b^2} - \frac{2\left(Ab^3+a^3B-2ab^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d} + \frac{a(Ab-aB)\ldots}{b\left(a^2-b^2\right)d(a\ldots}
\end{aligned}
$$

Mathematica [A] time = 0.55, size = 119, normalized size = 0.98

$$\frac{-\frac{2\left(aB\left(a^2-2b^2\right)+Ab^3\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+\frac{ab(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}+B(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*(c + d*x) - (2*(A*b^3 + a*(a^2 - 2*b^2)*B)*ArcTanh[((a - b)*Tan[(c + d*x
)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + (a*b*(A*b - a*B)*Sin[c + d*x]
)/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/(b^2*d)
```

fricas [B] time = 0.73, size = 552, normalized size = 4.52

$$\left[\frac{2\left(Ba^4b-2\,Ba^2b^3+Bb^5\right)dx\cos\left(dx+c\right)+2\left(Ba^5-2\,Ba^3b^2+Bab^4\right)dx-\left(Ba^4-2\,Ba^2b^2+Aab^3+\left(Ba^3b-2\,B\right.\right.}{2\left(\left(a^4b^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*(2*(B*a^4*b - 2*B*a^2*b^3 + B*b^5)*d*x*cos(d*x + c) + 2*(B*a^5 - 2*B*a
^3*b^2 + B*a*b^4)*d*x - (B*a^4 - 2*B*a^2*b^2 + A*a*b^3 + (B*a^3*b - 2*B*a*b
^3 + A*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2
 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x +
c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(B*a
^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/((a^4*b^3 - 2*a^2*b^5
 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d), ((B*a^4*b - 2*B*
a^2*b^3 + B*b^5)*d*x*cos(d*x + c) + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*d*x - (
B*a^4 - 2*B*a^2*b^2 + A*a*b^3 + (B*a^3*b - 2*B*a*b^3 + A*b^4)*cos(d*x + c))
*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)
)) - (B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/((a^4*b^3 -
2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d)]
```

giac [A] time = 0.44, size = 199, normalized size = 1.63

$$\frac{\frac{2\left(Ba^3-2\,Bab^2+Ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{a^2-b^2}}+\frac{(dx+c)B}{b^2}-\frac{2\left(Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Aab\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(a^2b-b^3\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] (2*(B*a^3 - 2*B*a*b^2 + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2
- b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) + (d*x + c)*B/b^2 - 2*(B*a^2*tan
(1/2*d*x + 1/2*c) - A*a*b*tan(1/2*d*x + 1/2*c))/((a^2*b - b^3)*(a*tan(1/2*d
*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)))/d
```

maple [B] time = 0.08, size = 320, normalized size = 2.62

$$\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A}{d\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{db\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)*A-2/d/b*a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B-2/d*b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arcta
n(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-2/d*a^3/b^2/(a-b)/(a+b)/(
(a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+4
/d/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(
a+b))^(1/2))*B*a+2/d/b^2*arctan(tan(1/2*d*x+1/2*c))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.79, size = 3775, normalized size = 30.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)

[Out] (2*B*atan(((B*((B*((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (B*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))*1i)/b^2 + (32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))/b^2 - (B*((B*((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (B*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))*1i)/b^2 - (32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))/b^2)/((64*(B^3*a^5 - A*B^2*b^5 + A^2*B*b^5 + 2*B^3*a*b^4 - B^3*a^4*b + 2*B^3*a^2*b^3 - 3*B^3*a^3*b^2 - 3*A*B^2*a*b^4 + A*B^2*a^2*b^3 + A*B^2*a^3*b^2))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (B*((B*((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (B*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))*1i)/b^2 + (32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))*1i)/b^2 + (B*((B*((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (B*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)))*1i)/b^2 - (32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))*1i)/b^2)))/(b^2*d) + (atan((((((32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + (((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2)*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (((32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*

```
b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3
*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (((32*(A*a^2*b^7 - B*b^9 - A*b^9
 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/
(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a -
 b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 +
 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^
8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3
+ B*a^3 - 2*B*a*b^2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(-(a + b)^3*
(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2)*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b
^4 - a^6*b^2))/((64*(B^3*a^5 - A*B^2*b^5 + A^2*B*b^5 + 2*B^3*a*b^4 - B^3*a^
4*b + 2*B^3*a^2*b^3 - 3*B^3*a^3*b^2 - 3*A*B^2*a*b^4 + A*B^2*a^2*b^3 + A*B^2
*a^3*b^2))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (((32*tan(c/2 + (d*x)/2)*(A^
2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4
*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 -
 a^2*b^3 - a^3*b^2) + (((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*
b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^
4 - a^3*b^3) - (32*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 +
 B*a^3 - 2*B*a*b^2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^
5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4
*b^4 - a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2))
/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b
^3 + B*a^3 - 2*B*a*b^2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) - (((32*ta
n(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b
 + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*
b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (((32*(A*a^2*b^7 - B*b^9 - A*b^9
- A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(
a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a -
b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 +
4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8
 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 +
 B*a^3 - 2*B*a*b^2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(-(a + b)^3*(
a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 -
 a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2)*2i)/(d
*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)) - (2*tan(c/2 + (d*x)/2)*(B*a^2 -
A*a*b))/(d*(a + b)*(a*b - b^2)*(a + b + tan(c/2 + (d*x)/2)^2*(a - b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.261 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{2(aA-bB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] 2*(A*a-B*b)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/
(a+b)^(3/2)/d-(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2754, 12, 2659, 205}

$$\frac{2(aA-bB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(3/2)*(a + b)^(3/2)*d) - ((A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*
Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\int \frac{-aA+bB}{a+b\cos(c+dx)}\,dx}{-a^2+b^2}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{(aA-bB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2-b^2}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{(2(aA-bB))\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+\right.\right.}{\left(a^2-b^2\right)d}$$

$$= \frac{2(aA-bB)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}$$

Mathematica [A] time = 0.35, size = 97, normalized size = 0.97

$$\frac{\frac{2(aA-bB)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+\frac{(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*(a*A - b*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2
 + b^2)^(3/2) + ((-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c
 + d*x])))/d
```

fricas [A] time = 0.78, size = 379, normalized size = 3.79

$$\left[-\frac{\left(Aa^2 - Bab + \left(Aab - Bb^2\right)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right.}{2\left(\left(a^4b - 2\,a^2b^3 + b^5\right)d\cos\left(dx+c\right) + \left(a^5 - 2\,a^3b^2 + ab\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((A*a^2 - B*a*b + (A*a*b - B*b^2)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(
(2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*
cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*c
os(d*x + c) + a^2)) - 2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*sin(d*x + c))/(
(a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), ((
A*a^2 - B*a*b + (A*a*b - B*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*co
s(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (B*a^3 - A*a^2*b - B*a*b^
2 + A*b^3)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 -
 2*a^3*b^2 + a*b^4)*d)]
```

giac [A] time = 0.41, size = 159, normalized size = 1.59

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)(Aa-Bb)}{\left(a^2-b^2\right)^{\frac{3}{2}}} - \frac{Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)\left(a^2-b^2\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(A*a - B*b)/(a^2 -
 b^2)^(3/2) - (B*a*tan(1/2*d*x + 1/2*c) - A*b*tan(1/2*d*x + 1/2*c))/((a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2 - b^2)))/d
```

maple [B] time = 0.07, size = 234, normalized size = 2.34

$$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)Ab}{d\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)aB}{d\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)*A*b+2/d/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)*a*B+2/d*a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(t
an(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-2/d/(a-b)/(a+b)/((a-b)*(a+b)
)^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*b
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 0.73, size = 113, normalized size = 1.13

$$\frac{2\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)}{2\,\sqrt{a+b}\,\sqrt{a-b}}\right)\,\left(A\,a-B\,b\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A\,b-B\,a\right)}{d\,\left(a+b\right)\,\left(a-b\right)\,\left(\left(a-b\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*atan((tan(c/2 + (d*x)/2)*(2*a - 2*b))/(2*(a + b)^(1/2)*(a - b)^(1/2)))*(
A*a - B*b))/(d*(a + b)^(3/2)*(a - b)^(3/2)) - (2*tan(c/2 + (d*x)/2)*(A*b -
B*a))/(d*(a + b)*(a - b)*(a + b + tan(c/2 + (d*x)/2)^2*(a - b)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.262 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{2\left(a^3(-B)+2a^2Ab-Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] -2*(2*A*a^2*b-A*b^3-B*a^3)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2
))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+A*arctanh(sin(d*x+c))/a^2/d+b*(A*b-B*a)*si
n(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.28, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3000, 3001, 3770, 2659, 205}

$$-\frac{2\left(2a^2Ab+a^3(-B)-Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{A\tanh^{-1}(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*(2*a^2*A*b - A*b^3 - a^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (A*ArcTanh[Sin[c + d*x]])/(a
^2*d) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
```

```
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(A\left(a^2-b^2\right)-a(Ab-aB)\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{A\int\sec(c+dx)\,dx}{a^2} - \frac{\left(2a^2Ab-Ab^3-\right.}{a^2}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(2\left(2a^2Ab-\right.\right.}{}$$

$$= -\frac{2\left(2a^2Ab-Ab^3-a^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{A\tanh^{-1}(\sin(c+d}{a^2d}$$

Mathematica [A] time = 0.63, size = 191, normalized size = 1.44

$$\frac{\cos(c+dx)(A\sec(c+dx)+B)\left(\frac{2\left(a^3B-2a^2Ab+Ab^3\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+\frac{ab(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}-A\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.}{a^2d(A+B\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*((2*(-2*a^2*A*b + A*b^3 + a^3*B)*ArcTanh
[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - A*Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + (a*b*(A*b - a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x]
))))/(a^2*d*(A + B*Cos[c + d*x]))
```

fricas [B] time = 7.69, size = 684, normalized size = 5.14

$$\left[\frac{\left(Ba^4-2\,Aa^3b+Aab^3+\left(Ba^3b-2\,Aa^2b^2+Ab^4\right)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\cdots}{b^2\cos(dx+c)^2+\cdots}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((B*a^4 - 2*A*a^3*b + A*a*b^3 + (B*a^3*b - 2*A*a^2*b^2 + A*b^4)*cos(d*
x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (A*a^5 - 2*A*a^3*b^2 + A
*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1
) - (A*a^5 - 2*A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*
x + c))*log(-sin(d*x + c) + 1) - 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b
^4)*sin(d*x + c))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*
a^5*b^2 + a^3*b^4)*d), 1/2*(2*(B*a^4 - 2*A*a^3*b + A*a*b^3 + (B*a^3*b - 2*A
*a^2*b^2 + A*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b
)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (A*a^5 - 2*A*a^3*b^2 + A*a*b^4 + (A*a^4
*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) - (A*a^5 - 2*
A*a^3*b^2 + A*a*b^4 + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*cos(d*x + c))*log(-si
n(d*x + c) + 1) - 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c
))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b
^4)*d)]
```

giac [A] time = 1.49, size = 223, normalized size = 1.68

$$\frac{\frac{2\left(Ba^3-2\,Aa^2b+Ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-a^2b^2\right)\sqrt{a^2-b^2}}+\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] (2*(B*a^3 - 2*A*a^2*b + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 -
b^2)))/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) + A*log(abs(tan(1/2*d*x + 1/2*c) +
 1))/a^2 - A*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 - 2*(B*a*b*tan(1/2*d*x
+ 1/2*c) - A*b^2*tan(1/2*d*x + 1/2*c))/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*
c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)))/d
```

maple [B] time = 0.14, size = 342, normalized size = 2.57

$$\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A}{da\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{d\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/a*b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)*A-2/d*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B-4/d*b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arcta
n(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+2/d/a^2/(a-b)/(a+b)/((a-b
)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^3+2
/d/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(
a+b))^(1/2))*B*a-1/d/a^2*A*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^2*A*ln(tan(1/2*d*
x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.81, size = 3763, normalized size = 28.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] - (A*atan(((A*((A*((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2
 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a
^4*b^2) - (32*A*tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6
*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2))))/a^
2 - (32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2
*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b +
2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))*1i)/a^2 - (A*((A*((32*(A
*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2
+ 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*A*tan(c/2 +
 (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*
b^2))/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2))))/a^2 + (32*tan(c/2 + (d*x)/2
)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^
4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b +
a^5 - a^2*b^3 - a^3*b^2))*1i)/a^2)/((A*((A*((32*(A*a^4*b^5 - B*a^9 - A*a^9
- 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b))/(
a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (32*A*tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^
4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^5 -
 a^2*b^3 - a^3*b^2))))/a^2 - (32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 +
B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2
*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))
)/a^2 - (64*(A^3*b^5 + A*B^2*a^5 - A^2*B*a^5 - A^3*a*b^4 + 2*A^3*a^4*b - 3*
A^3*a^2*b^3 + 2*A^3*a^3*b^2 - 3*A^2*B*a^4*b + A^2*B*a^2*b^3 + A^2*B*a^3*b^2
))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (A*((A*((32*(A*a^4*b^5 - B*a^9 - A*a
^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b)
)/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*A*tan(c/2 + (d*x)/2)*(2*a^9*b - 2
*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/(a^2*(a^4*b + a^
5 - a^2*b^3 - a^3*b^2))))/a^2 + (32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6
 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*
A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^
2)))/a^2))*2i)/(a^2*d) - (atan((((-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2
```

+ (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A ^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/ (a^4*b + a^5 - a^2*b^3 - a^3*b^2) + (((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3*A* a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^ (1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6* b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2 *b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(A*b^3 + B*a^3 - 2* A*a^2*b)*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) + ((-(a + b)^3*(a - b) ^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a* b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a ^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) - (((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^ 8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (32*tan(c/2 + (d*x)/2)* (-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b)*(2*a^9*b - 2*a^4*b ^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^ 3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(-(a + b)^3*(a - b)^ 3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^ 2))*(A*b^3 + B*a^3 - 2*A*a^2*b)*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))/((64*(A^3*b^5 + A*B^2*a^5 - A^2*B*a^5 - A^3*a*b^4 + 2*A^3*a^4*b - 3*A^3*a ^2*b^3 + 2*A^3*a^3*b^2 - 3*A^2*B*a^4*b + A^2*B*a^2*b^3 + A^2*B*a^3*b^2))/(a ^5*b + a^6 - a^3*b^3 - a^4*b^2) - ((-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/ 2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5 *A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) + (((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3* A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b))/(a^5* b + a^6 - a^3*b^3 - a^4*b^2) + (32*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^ 6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a ^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a ^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(A*b^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) + ((-(a + b)^3*(a - b)^ 3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b ^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^ 5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) - (((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8 *b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (32*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b)*(2*a^9*b - 2*a^4*b^ 6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(A*b^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b)*2i)/(d*(a^8 - a^2*b ^6 + 3*a^4*b^4 - 3*a^6*b^2)) - (2*tan(c/2 + (d*x)/2)*(A*b^2 - B*a*b))/(d*(a

```
+ b)*(a*b - a^2)*(a + b + tan(c/2 + (d*x)/2)^2*(a - b)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**2, x)
```

3.263 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$-\frac{(2Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{\left(a^2A+abB-2Ab^2\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{2b\left(-2a^3B\right.}{}$$

```
[Out] 2*b*(3*A*a^2*b-2*A*b^3-2*B*a^3+B*a*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*
c)/(a+b)^(1/2))/a^3/(a-b)^(3/2)/(a+b)^(3/2)/d-(2*A*b-B*a)*arctanh(sin(d*x+c
))/a^3/d+(A*a^2-2*A*b^2+B*a*b)*tan(d*x+c)/a^2/(a^2-b^2)/d+b*(A*b-B*a)*tan(d
*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.67, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$\frac{2b\left(3a^2Ab-2a^3B+ab^2B-2Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(a^2A+abB-2Ab^2\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{b(Ab-aB)\,\mathrm{ta}}{ad\left(a^2-b^2\right)(a+}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*b*(3*a^2*A*b - 2*A*b^3 - 2*a^3*B + a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c +
 d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2*A*b - a*B
)*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((a^2*A - 2*A*b^2 + a*b*B)*Tan[c + d*x])
/(a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b
*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2A-2Ab^2+abB-a(Ab-aB)\cos(c+dx)+b(Ab\right.}{a+b\cos(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2A-2Ab^2+abB\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + $$

$$= \frac{\left(a^2A-2Ab^2+abB\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - ($$

$$= -\frac{(2Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{\left(a^2A-2Ab^2+abB\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + $$

$$= \frac{2b\left(3a^2Ab-2Ab^3-2a^3B+ab^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} - \frac{(2Ab-a}{}$$

Mathematica [A] time = 1.95, size = 240, normalized size = 1.27

$$\frac{-\frac{2b\left(2a^3B-3a^2Ab-ab^2B+2Ab^3\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + \frac{ab^2(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + aA\tan(c+dx) - aB\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((-2*b*(-3*a^2*A*b + 2*A*b^3 + 2*a^3*B - a*b^2*B)*ArcTanh[((a - b)*Tan[(c +
 d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + 2*A*b*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] - a*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*A*
b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a*B*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + (a*b^2*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a +
b*Cos[c + d*x])) + a*A*Tan[c + d*x])/(a^3*d)
```

fricas [B] time = 21.47, size = 1088, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(((2*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + 2*A*b^5)*cos(d*x + c)^2 + (2
*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b
^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 +
b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + a^2)) - ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a
^2*b^4 + B*a*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b
^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(sin(d*x + c) +
1) + ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^
6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*
b^4 - 2*A*a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(A*a^6 - 2*A*a^4*
b^2 + A*a^2*b^4 + (A*a^5*b + B*a^4*b^2 - 3*A*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^
5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c
)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c)), -1/2*(2*((2*B*a^3*b^2 -
3*A*a^2*b^3 - B*a*b^4 + 2*A*b^5)*cos(d*x + c)^2 + (2*B*a^4*b - 3*A*a^3*b^2
- B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x +
 c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^
3*b^3 + 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*
b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(si
n(d*x + c) + 1) + ((B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^4 + B*a
*b^5 - 2*A*b^6)*cos(d*x + c)^2 + (B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3
*b^3 + B*a^2*b^4 - 2*A*a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(A*a
^6 - 2*A*a^4*b^2 + A*a^2*b^4 + (A*a^5*b + B*a^4*b^2 - 3*A*a^3*b^3 - B*a^2*b
^4 + 2*A*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*
d*cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c))]
```

giac [B] time = 0.80, size = 404, normalized size = 2.14

$$\frac{2\left(2\,Ba^{3}b-3\,Aa^{2}b^{2}-Bab^{3}+2\,Ab^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{5}-a^{3}b^{2}\right)\sqrt{a^{2}-b^{2}}}-\frac{2\left(Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-Aa^{2}b\tan\left(\frac{1}{2}\,dx+\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] (2*(2*B*a^3*b - 3*A*a^2*b^2 - B*a*b^3 + 2*A*b^4)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) - 2*(A*a^3*tan
(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - A*a*b^2*tan(1/2*d*x
+ 1/2*c)^3 - B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2*c)^
```

```
3 + A*a^3*tan(1/2*d*x + 1/2*c) + A*a^2*b*tan(1/2*d*x + 1/2*c) - A*a*b^2*tan
(1/2*d*x + 1/2*c) + B*a*b^2*tan(1/2*d*x + 1/2*c) - 2*A*b^3*tan(1/2*d*x + 1/
2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d
*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)) + (B*a - 2*A*b)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1))/a^3 - (B*a - 2*A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3)
/d
```

maple [B] time = 0.18, size = 502, normalized size = 2.66

$$-\frac{2b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A}{d\,a^2\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}+\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{da\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d*b^3/a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b+a+b)*A+2/d*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B+6/d*b^2/a/(a-b)/(a+b)/((a-b)*(a+b))^
(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-4/d*b^4/a^3/(a
-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))
^(1/2))*A-4/d/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-
b)/((a-b)*(a+b))^(1/2))*B*b+2/d*b^3/a^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arc
tan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-1/d/a^2*A/(tan(1/2*d*x+
1/2*c)-1)+2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*A*b-1/d/a^2*ln(tan(1/2*d*x+1/2*c
)-1)*B-1/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)-2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*A*
b+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 8.52, size = 5464, normalized size = 28.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^2),x)
```

```
[Out] (atan(((((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*
a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4
*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^
3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*
b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 - a^4*b
^3 - a^5*b^2) + (((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4 - 3*A
*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*a^11*b
 + 2*B*a^11*b))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*tan(c/2 + (d*x)/2)*
(2*A*b - B*a)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2
*a^10*b^2))/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2)))*(2*A*b - B*a))/a^3)*(2
*A*b - B*a)*1i)/a^3 + (((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2
*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*
A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4
 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^
6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*
b + a^7 - a^4*b^3 - a^5*b^2) - (((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*
A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b
^2 + 2*A*a^11*b + 2*B*a^11*b))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*tan(
c/2 + (d*x)/2)*(2*A*b - B*a)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4
- 4*a^9*b^3 - 2*a^10*b^2))/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5*b^2)))*(2*A*b
- B*a))/a^3)*(2*A*b - B*a)*1i)/a^3)/((64*(8*A^3*b^8 - 4*A^3*a*b^7 - 2*B^3*a
^7*b - 20*A^3*a^2*b^6 + 6*A^3*a^3*b^5 + 12*A^3*a^4*b^4 - B^3*a^3*b^5 + B^3*
a^4*b^4 + 3*B^3*a^5*b^3 - 2*B^3*a^6*b^2 - 12*A^2*B*a*b^7 + 6*A*B^2*a^2*b^6
- 5*A*B^2*a^3*b^5 - 17*A*B^2*a^4*b^4 + 9*A*B^2*a^5*b^3 + 11*A*B^2*a^6*b^2 +
 8*A^2*B*a^2*b^6 + 32*A^2*B*a^3*b^5 - 13*A^2*B*a^4*b^4 - 20*A^2*B*a^5*b^3))
/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 +
B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A
^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5
- 5*B^2*a^4*b^4 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b
 + 8*A*B*a^2*b^6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B*
a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (((32*(A*a^7*b^5 - 2*A*a^6*b^
6 - B*a^12 + 5*A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a^9
*b^3 + B*a^10*b^2 + 2*A*a^11*b + 2*B*a^11*b))/(a^8*b + a^9 - a^6*b^3 - a^7*
b^2) + (32*tan(c/2 + (d*x)/2)*(2*A*b - B*a)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b
^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/(a^3*(a^6*b + a^7 - a^4*b^3 - a^5
*b^2)))*(2*A*b - B*a))/a^3)*(2*A*b - B*a))/a^3 - (((32*tan(c/2 + (d*x)/2)*(
8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a
^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*
B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 -
 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*
b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (((32*(A*a^7*b^5
```

- 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b
^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*a^11*b + 2*B*a^11*b))/(a^8*b + a^9 - a^
6*b^3 - a^7*b^2) - (32*tan(c/2 + (d*x)/2)*(2*A*b - B*a)*(2*a^11*b - 2*a^6*b
^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/(a^3*(a^6*b + a^7 - a
^4*b^3 - a^5*b^2)))*(2*A*b - B*a))/a^3)*(2*A*b - B*a))/a^3))*(2*A*b - B*a)*
2i)/(a^3*d) - ((2*tan(c/2 + (d*x)/2)^3*(A*a*b^2 - 2*A*b^3 - A*a^3 + A*a^2*b
 + B*a*b^2))/(a^2*(a + b)*(a - b)) - (2*tan(c/2 + (d*x)/2)*(A*a^3 - 2*A*b^3
 - A*a*b^2 + A*a^2*b + B*a*b^2))/(a^2*(a + b)*(a - b)))/(d*(a + b - tan(c/2
 + (d*x)/2)^4*(a - b) - 2*b*tan(c/2 + (d*x)/2)^2)) + (b*atan(((b*((32*tan(c
/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2
*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B
^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2
- 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b
^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (b
((32(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a
^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*a^11*b + 2*B*a^11*b))/
(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*b*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a
 - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*(2*a^11*b - 2*a^6*
b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*
b^3 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(-(a + b)^3*(a - b
)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^
5*b^4 - 3*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a
^2*b - B*a*b^2)*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) + (b*((32*tan(c
/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2
*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B
^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2
- 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b
^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b
((32(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a
^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*a^11*b + 2*B*a^11*b))/
(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*b*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a
 - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*(2*a^11*b - 2*a^6*
b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*
b^3 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(-(a + b)^3*(a - b
)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^
5*b^4 - 3*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a
^2*b - B*a*b^2)*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))/((64*(8*A^3*b^
8 - 4*A^3*a*b^7 - 2*B^3*a^7*b - 20*A^3*a^2*b^6 + 6*A^3*a^3*b^5 + 12*A^3*a^4
*b^4 - B^3*a^3*b^5 + B^3*a^4*b^4 + 3*B^3*a^5*b^3 - 2*B^3*a^6*b^2 - 12*A^2*B
*a*b^7 + 6*A*B^2*a^2*b^6 - 5*A*B^2*a^3*b^5 - 17*A*B^2*a^4*b^4 + 9*A*B^2*a^5
*b^3 + 11*A*B^2*a^6*b^2 + 8*A^2*B*a^2*b^6 + 32*A^2*B*a^3*b^5 - 13*A^2*B*a^4
*b^4 - 20*A^2*B*a^5*b^3))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b*((32*tan(c
/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2
*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B
^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2

```
- 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b
^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (b
*((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a
^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*a^11*b + 2*B*a^11*b))/
(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*b*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a
 - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*(2*a^11*b - 2*a^6*
b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*
b^3 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(-(a + b)^3*(a - b
)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^
5*b^4 - 3*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a
^2*b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) - (b*((32*tan(c/2
+ (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2*b^
6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B^2*
a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2 - 8
*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b^4
- 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b*((
32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a^10
*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*a^11*b + 2*B*a^11*b))/(a^
8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*b*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a -
b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*(2*a^11*b - 2*a^6*b^6
 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3
 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(-(a + b)^3*(a - b)^3
)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^5*b
^4 - 3*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*
b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*(-(a + b)^3*(a - b)
^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*2i)/(d*(a^9 - a^3*b^6 +
 3*a^5*b^4 - 3*a^7*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**2, x)
```

3.264 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=270

$$\frac{\left(a^2A+2abB-3Ab^2\right)\tan(c+dx)\sec(c+dx)}{2a^2d\left(a^2-b^2\right)}+\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\left(a^2A-4abB+6Ab^2\right)\tanh^{-}}{2a^4d}$$

```
[Out] -2*b^2*(4*A*a^2*b-3*A*b^3-3*B*a^3+2*B*a*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x
+1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*(A*a^2+6*A*b^2-4*B*a
*b)*arctanh(sin(d*x+c))/a^4/d-(2*A*a^2*b-3*A*b^3-B*a^3+2*B*a*b^2)*tan(d*x+c
)/a^3/(a^2-b^2)/d+1/2*(A*a^2-3*A*b^2+2*B*a*b)*sec(d*x+c)*tan(d*x+c)/a^2/(a^
2-b^2)/d+b*(A*b-B*a)*sec(d*x+c)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.98, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b^2\left(4a^2Ab-3a^3B+2ab^2B-3Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{\left(2a^2Ab+a^3(-B)+2ab^2B-3Ab^3\right)\tan(c+dx)}{a^3d\left(a^2-b^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^2*(4*a^2*A*b - 3*A*b^3 - 3*a^3*B + 2*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((a^2*A
+ 6*A*b^2 - 4*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - ((2*a^2*A*b - 3*A*b
^3 - a^3*B + 2*a*b^2*B)*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2*A - 3*A*b
^2 + 2*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b*(A*b -
a*B)*Sec[c + d*x]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2A-3Ab^2+2abB-a(Ab-aB)\cos(c+dx)\right.}{a+b\cos(c}}{a\left(a^2-\right.}$$

$$= \frac{\left(a^2A-3Ab^2+2abB\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sec(c+dx}{a\left(a^2-b^2\right)d(a+b\,c}$$

$$= -\frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2A-3Ab^2+2abB\right)}{2a^2\left(a^2\right.}$$

$$= -\frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2A-3Ab^2+2abB\right)}{2a^2\left(a^2\right.}$$

$$= \frac{\left(a^2A+6Ab^2-4abB\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{\left(2a^2Ab-3Ab^3-a^3B+2\right.}{a^3\left(a^2-b^2\right.}$$

$$= -\frac{2b^2\left(4a^2Ab-3Ab^3-3a^3B+2ab^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\left(a^2A\right.}{}$$

Mathematica [A] time = 6.27, size = 438, normalized size = 1.62

$$\frac{Ab^4\sin(c+dx)-ab^3B\sin(c+dx)}{a^3d(a-b)(a+b)(a+b\cos(c+dx))} + \frac{aB\sin\left(\frac{1}{2}(c+dx)\right)-2Ab\sin\left(\frac{1}{2}(c+dx)\right)}{a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{aB\sin\left(\frac{1}{2}(c+dx)\right)-2Ab\sin\left(\frac{1}{2}\right.}{a^3d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^2*(-4*a^2*A*b + 3*A*b^3 + 3*a^3*B - 2*a*b^2*B)*ArcTanh[((a - b)*Tan[(
c + d*x)/2])/Sqrt[-a^2 + b^2]])/(a^4*(a^2 - b^2)*Sqrt[-a^2 + b^2]*d) + ((-(
a^2*A) - 6*A*b^2 + 4*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2*a^
4*d) + ((a^2*A + 6*A*b^2 - 4*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
])/(2*a^4*d) + A/(4*a^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - A/(4*a
^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (-2*A*b*Sin[(c + d*x)/2] +
a*B*Sin[(c + d*x)/2])/(a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (-2*A
```

```
*b*Sin[(c + d*x)/2] + a*B*Sin[(c + d*x)/2])/(a^3*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])) + (A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x])/(a^3*(a - b)*(
a + b)*d*(a + b*Cos[c + d*x]))
```

fricas [B] time = 33.86, size = 1329, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(2*((3*B*a^3*b^3 - 4*A*a^2*b^4 - 2*B*a*b^5 + 3*A*b^6)*cos(d*x + c)^3
+ (3*B*a^4*b^2 - 4*A*a^3*b^3 - 2*B*a^2*b^4 + 3*A*a*b^5)*cos(d*x + c)^2)*sqr
t(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sq
rt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*
x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - ((A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4*b
^3 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^3 + (A*
a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a^2*b^5 +
6*A*a*b^6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((A*a^6*b - 4*B*a^5*b^2
+ 4*A*a^4*b^3 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x +
 c)^3 + (A*a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B
*a^2*b^5 + 6*A*a*b^6)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(A*a^7 - 2
*A*a^5*b^2 + A*a^3*b^4 + 2*(B*a^6*b - 2*A*a^5*b^2 - 3*B*a^4*b^3 + 5*A*a^3*b
^4 + 2*B*a^2*b^5 - 3*A*a*b^6)*cos(d*x + c)^2 + (2*B*a^7 - 3*A*a^6*b - 4*B*a
^5*b^2 + 6*A*a^4*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c))*sin(d*x + c
))/((a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(d*x + c)^3 + (a^9 - 2*a^7*b^2 + a^5
*b^4)*d*cos(d*x + c)^2), 1/4*(4*((3*B*a^3*b^3 - 4*A*a^2*b^4 - 2*B*a*b^5 + 3
*A*b^6)*cos(d*x + c)^3 + (3*B*a^4*b^2 - 4*A*a^3*b^3 - 2*B*a^2*b^4 + 3*A*a*b
^5)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2
- b^2)*sin(d*x + c))) + ((A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4*b^3 + 8*B*a^3*b^4
 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^3 + (A*a^7 - 4*B*a^6*b
+ 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a^2*b^5 + 6*A*a*b^6)*cos(d
*x + c)^2)*log(sin(d*x + c) + 1) - ((A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4*b^3 +
8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^3 + (A*a^7 -
 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a^2*b^5 + 6*A*a
*b^6)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(A*a^7 - 2*A*a^5*b^2 + A*a
^3*b^4 + 2*(B*a^6*b - 2*A*a^5*b^2 - 3*B*a^4*b^3 + 5*A*a^3*b^4 + 2*B*a^2*b^5
 - 3*A*a*b^6)*cos(d*x + c)^2 + (2*B*a^7 - 3*A*a^6*b - 4*B*a^5*b^2 + 6*A*a^4
*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b - 2*a
^6*b^3 + a^4*b^5)*d*cos(d*x + c)^3 + (a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x
+ c)^2)]
```

giac [A] time = 1.23, size = 378, normalized size = 1.40

$$-\frac{\frac{4\left(3\,Ba^{3}b^{2}-4\,Aa^{2}b^{3}-2\,Bab^{4}+3\,Ab^{5}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{6}-a^{4}b^{2}\right)\sqrt{a^{2}-b^{2}}}+\frac{4\left(Bab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Ab^{4}\tan\left(\cdots\right.\right.}{\left(a^{5}-a^{3}b^{2}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\cdots\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*(3*B*a^3*b^2 - 4*A*a^2*b^3 - 2*B*a*b^4 + 3*A*b^5)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(
1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*sqrt(a^2 - b^2)) + 4*(
B*a*b^3*tan(1/2*d*x + 1/2*c) - A*b^4*tan(1/2*d*x + 1/2*c))/((a^5 - a^3*b^2)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - (A*a^2 -
4*B*a*b + 6*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 + (A*a^2 - 4*B*a*
b + 6*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 2*(A*a*tan(1/2*d*x +
1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 4*A*b*tan(1/2*d*x + 1/2*c)^3 + A*
a*tan(1/2*d*x + 1/2*c) + 2*B*a*tan(1/2*d*x + 1/2*c) - 4*A*b*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

maple [B] time = 0.19, size = 690, normalized size = 2.56

$$\frac{2b^{4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)A}{d\,a^{3}\left(a^{2}-b^{2}\right)\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b\right)}-\frac{2b^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)B}{d\,a^{2}\left(a^{2}-b^{2}\right)\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*b^4/a^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)*A-2/d*b^3/a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x
+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)*B-8/d/a^2/(a-b)/(a+b)/((a-b)*(a+b))^(
1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^3+6/d*b^5/a^4
/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))*A+6/d*b^2/a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1
/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-4/d*b^4/a^3/(a-b)/(a+b)/((a-b)*(a+b))^(1
/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+1/2/d/a^2*A/(tan
(1/2*d*x+1/2*c)-1)^2+1/2/d/a^2*A/(tan(1/2*d*x+1/2*c)-1)+2/d/a^3/(tan(1/2*d*
x+1/2*c)-1)*A*b-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B-1/2/d/a^2*A*ln(tan(1/2*d*x
```

```
+1/2*c)-1)-3/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*A*b^2+2/d/a^3*ln(tan(1/2*d*x+1/
2*c)-1)*B*b-1/2/d/a^2*A/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2*A/(tan(1/2*d*x+1
/2*c)+1)+2/d/a^3/(tan(1/2*d*x+1/2*c)+1)*A*b-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*
B+1/2/d/a^2*A*ln(tan(1/2*d*x+1/2*c)+1)+3/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*A*b
^2-2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.28, size = 6692, normalized size = 24.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^2),x)
```

```
[Out] (atan(-((((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 - 72*A^2*a*b^9 - 2*
A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5
*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 -
32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*
a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 17
6*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*
a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (((8*(2*A*a^
15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^11*b^4 - 16*A*a^12
*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a^11*b^4 + 12*B*a^1
2*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) -
 (4*tan(c/2 + (d*x)/2)*(A*a^2 + 6*A*b^2 - 4*B*a*b)*(8*a^13*b - 8*a^8*b^6 +
8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^
6*b^3 - a^7*b^2)))*(A*a^2 + 6*A*b^2 - 4*B*a*b))/(2*a^4))*(A*a^2 + 6*A*b^2 -
 4*B*a*b)*1i)/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 - 7
2*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*
b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 3
2*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a
^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*
A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a
```

^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2
) + (((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^11
*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a^1
1*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))/(a^11*b + a^12 - a^9*b
^3 - a^10*b^2) + (4*tan(c/2 + (d*x)/2)*(A*a^2 + 6*A*b^2 - 4*B*a*b)*(8*a^13*
b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(
a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*(A*a^2 + 6*A*b^2 - 4*B*a*b))/(2*a^4))*(A
*a^2 + 6*A*b^2 - 4*B*a*b)*1i)/(2*a^4))/((16*(108*A^3*b^11 - 54*A^3*a*b^10 -
216*A^3*a^2*b^9 + 81*A^3*a^3*b^8 + 63*A^3*a^4*b^7 - 9*A^3*a^5*b^6 + 41*A^3
*a^6*b^5 - 4*A^3*a^7*b^4 + 4*A^3*a^8*b^3 - 32*B^3*a^3*b^8 + 16*B^3*a^4*b^7
+ 80*B^3*a^5*b^6 - 24*B^3*a^6*b^5 - 48*B^3*a^7*b^4 - 216*A^2*B*a*b^10 + 144
*A*B^2*a^2*b^9 - 72*A*B^2*a^3*b^8 - 336*A*B^2*a^4*b^7 + 108*A*B^2*a^5*b^6 +
168*A*B^2*a^6*b^5 - 6*A*B^2*a^7*b^4 + 24*A*B^2*a^8*b^3 + 108*A^2*B*a^2*b^9
+ 468*A^2*B*a^3*b^8 - 162*A^2*B*a^4*b^7 - 186*A^2*B*a^5*b^6 + 15*A^2*B*a^6
*b^5 - 63*A^2*B*a^7*b^4 + 3*A^2*B*a^8*b^3 - 3*A^2*B*a^9*b^2))/(a^11*b + a^1
2 - a^9*b^3 - a^10*b^2) - (((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 -
72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^
4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 +
32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2
*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 9
6*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B
*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b
^2) - (((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^
11*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a
^11*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))/(a^11*b + a^12 - a^9
*b^3 - a^10*b^2) - (4*tan(c/2 + (d*x)/2)*(A*a^2 + 6*A*b^2 - 4*B*a*b)*(8*a^1
3*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4
*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*(A*a^2 + 6*A*b^2 - 4*B*a*b))/(2*a^4))*
(A*a^2 + 6*A*b^2 - 4*B*a*b))/(2*a^4) + (((8*tan(c/2 + (d*x)/2)*(A^2*a^10 +
72*A^2*b^10 - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^
7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*
A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5
*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*
A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5
*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^
6*b^3 - a^7*b^2) + (((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*
b^5 - 14*A*a^11*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10
*b^5 - 20*B*a^11*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))/(a^11*b
+ a^12 - a^9*b^3 - a^10*b^2) + (4*tan(c/2 + (d*x)/2)*(A*a^2 + 6*A*b^2 - 4*
B*a*b)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^
12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*(A*a^2 + 6*A*b^2 - 4*B*a*
b))/(2*a^4))*(A*a^2 + 6*A*b^2 - 4*B*a*b))/(2*a^4)))*(A*a^2 + 6*A*b^2 - 4*B*
a*b)*1i)/(a^4*d) - ((tan(c/2 + (d*x)/2)^5*(A*a^4 + 6*A*b^4 - 2*B*a^4 - 5*A*
a^2*b^2 + 2*B*a^2*b^2 - 3*A*a*b^3 + 3*A*a^3*b - 4*B*a*b^3 + 2*B*a^3*b))/((a
^3*b - a^4)*(a + b)) + (tan(c/2 + (d*x)/2)*(A*a^4 + 6*A*b^4 + 2*B*a^4 - 5*A

*a^2*b^2 - 2*B*a^2*b^2 + 3*A*a*b^3 - 3*A*a^3*b - 4*B*a*b^3 + 2*B*a^3*b))/((
a^3*b - a^4)*(a + b)) + (2*tan(c/2 + (d*x)/2)^3*(A*a^4 - 6*A*b^4 + 3*A*a^2*
b^2 + 4*B*a*b^3 - 2*B*a^3*b))/(a*(a^2*b - a^3)*(a + b)))/(d*(a + b - tan(c/
2 + (d*x)/2)^2*(a + 3*b) - tan(c/2 + (d*x)/2)^4*(a - 3*b) + tan(c/2 + (d*x)
/2)^6*(a - b))) - (b^2*atan(((b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2
+ (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a
^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4
 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B
^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*
b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A
*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*
b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b^2*((8*(2*A*a^15 - 12*A*a^8*b^7
 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^11*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b
^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a^11*b^4 + 12*B*a^12*b^3 + 12*B*a^13
*b^2 - 8*B*a^14*b))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (8*b^2*tan(c/2 +
 (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B
*a*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a
^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 -
3*a^8*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b -
2*B*a*b^2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*(3*A*b^3 + 3*B*a^3 -
4*A*a^2*b - 2*B*a*b^2)*1i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + (b^2*
(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10
 - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*
a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2
 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B
^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b +
 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A
*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7
b^2) - (b^2((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 1
4*A*a^11*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 -
20*B*a^11*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))/(a^11*b + a^12
 - a^9*b^3 - a^10*b^2) - (8*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(
1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*
a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3
 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(-(a + b)^3*(a - b)^3
)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2))/(a^10 - a^4*b^6 + 3*a^
6*b^4 - 3*a^8*b^2))*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2)*1i)/(a^10 -
 a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))/((16*(108*A^3*b^11 - 54*A^3*a*b^10 - 216
*A^3*a^2*b^9 + 81*A^3*a^3*b^8 + 63*A^3*a^4*b^7 - 9*A^3*a^5*b^6 + 41*A^3*a^6
*b^5 - 4*A^3*a^7*b^4 + 4*A^3*a^8*b^3 - 32*B^3*a^3*b^8 + 16*B^3*a^4*b^7 + 80
*B^3*a^5*b^6 - 24*B^3*a^6*b^5 - 48*B^3*a^7*b^4 - 216*A^2*B*a*b^10 + 144*A*B
^2*a^2*b^9 - 72*A*B^2*a^3*b^8 - 336*A*B^2*a^4*b^7 + 108*A*B^2*a^5*b^6 + 168
*A*B^2*a^6*b^5 - 6*A*B^2*a^7*b^4 + 24*A*B^2*a^8*b^3 + 108*A^2*B*a^2*b^9 + 4
68*A^2*B*a^3*b^8 - 162*A^2*B*a^4*b^7 - 186*A^2*B*a^5*b^6 + 15*A^2*B*a^6*b^5
 - 63*A^2*B*a^7*b^4 + 3*A^2*B*a^8*b^3 - 3*A^2*B*a^9*b^2))/(a^11*b + a^12 -

```
a^9*b^3 - a^10*b^2) + (b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)
/2)*(A^2*a^10 + 72*A^2*b^10 - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8
+ 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A
^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*
b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 9
6*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*
b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(
a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b^2*((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*
a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^11*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*
B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a^11*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 -
8*B*a^14*b))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (8*b^2*tan(c/2 + (d*x)/
2)*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2)
*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2
))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b
^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b
^2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*(3*A*b^3 + 3*B*a^3 - 4*A*a^2
*b - 2*B*a*b^2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) - (b^2*(-(a + b)^
3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 - 72*A^2*
a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 -
26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*
a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4
 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^
2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4
 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b
^2*((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^11*b
^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a^11*
b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))/(a^11*b + a^12 - a^9*b^3
 - a^10*b^2) - (8*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*
b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 +
16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)
*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(3
*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*
a^8*b^2))*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2))/(a^10 - a^4*b^6 + 3*
a^6*b^4 - 3*a^8*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*
A*a^2*b - 2*B*a*b^2)*2i)/(d*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{(a + b\cos(c + dx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x))**2, x)
```

3.265 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=398

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{x\left(-12a^2B+6aAb-b^2B\right)}{2b^5}+\frac{a\left(-4a^3B+2a^2Ab+7ab^2B-5Ab^3\right)\sin(c+d}{2b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx}$$

```
[Out] -1/2*(6*A*a*b-12*B*a^2-B*b^2)*x/b^5+a^2*(6*A*a^4*b-15*A*a^2*b^3+12*A*b^5-12
*B*a^5+29*B*a^3*b^2-20*B*a*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))/(a-b)^(5/2)/b^5/(a+b)^(5/2)/d+1/2*(6*A*a^4*b-11*A*a^2*b^3+2*A*b^5-1
2*B*a^5+21*B*a^3*b^2-6*B*a*b^4)*sin(d*x+c)/b^4/(a^2-b^2)^2/d-1/2*(3*A*a^3*b
-6*A*a*b^3-6*B*a^4+10*B*a^2*b^2-B*b^4)*cos(d*x+c)*sin(d*x+c)/b^3/(a^2-b^2)^
2/d+1/2*a*(A*b-B*a)*cos(d*x+c)^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^
2+1/2*a*(2*A*a^2*b-5*A*b^3-4*B*a^3+7*B*a*b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/(
a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 1.72, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2989, 3047, 3049, 3023, 2735, 2659, 205}

$$\frac{\left(-11a^2Ab^3+6a^4Ab+21a^3b^2B-12a^5B-6ab^4B+2Ab^5\right)\sin(c+dx)}{2b^4d\left(a^2-b^2\right)^2}+\frac{a^2\left(-15a^2Ab^3+6a^4Ab+29a^3b^2B-12a}{b^5d(a-}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((6*a*A*b - 12*a^2*B - b^2*B)*x)/(2*b^5) + (a^2*(6*a^4*A*b - 15*a^2*A*b^3
+ 12*A*b^5 - 12*a^5*B + 29*a^3*b^2*B - 20*a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*d) + ((6*a^4*A
*b - 11*a^2*A*b^3 + 2*A*b^5 - 12*a^5*B + 21*a^3*b^2*B - 6*a*b^4*B)*Sin[c +
d*x])/(2*b^4*(a^2 - b^2)^2*d) - ((3*a^3*A*b - 6*a*A*b^3 - 6*a^4*B + 10*a^2*
b^2*B - b^4*B)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d) + (a*(A*b
 - a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^2) + (a*(2*a^2*A*b - 5*A*b^3 - 4*a^3*B + 7*a*b^2*B)*Cos[c + d*x]^2*Sin[c
+ d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
```

```
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(-3a(Ab-aB)+2b(Ab-aB)\cos\ldots\right.}{(a+b\cos\ldots}}{2b\left(a\ldots\right.}$$

$$= \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(2a^2Ab-5Ab^3-4a^3B+7ab^2\ldots\right.}{2b^2\left(a^2-b^2\right)^2d(a+\ldots}$$

$$= -\frac{\left(3a^3Ab-6aAb^3-6a^4B+10a^2b^2B-b^4B\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)^2d} + \ldots$$

$$= \frac{\left(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6ab^4B\right)\sin(c+dx)}{2b^4\left(a^2-b^2\right)^2d} - \ldots$$

$$= -\frac{\left(6aAb-12a^2B-b^2B\right)x}{2b^5} + \frac{\left(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3\ldots\right.}{2b^4\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(6aAb-12a^2B-b^2B\right)x}{2b^5} + \frac{\left(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3\ldots\right.}{2b^4\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(6aAb-12a^2B-b^2B\right)x}{2b^5} + \frac{a^2\left(6a^4Ab-15a^2Ab^3+12Ab^5-12a^5B+\ldots\right.}{(a-b)^{5/2}b\ldots}$$

Mathematica [A] time = 3.60, size = 734, normalized size = 1.84

$$\frac{\frac{16a^2\left(12a^5B-6a^4Ab-29a^3b^2B+15a^2Ab^3+20ab^4B-12Ab^5\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{96a^8Bc+96a^8Bdx-48a^7Abc-48a^7Abdx-96a^7bB\sin(c+dx)+4\ldots}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((16*a^2*(-6*a^4*A*b + 15*a^2*A*b^3 - 12*A*b^5 + 12*a^5*B - 29*a^3*b^2*B +
20*a*b^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b
^2)^(5/2) + (-48*a^7*A*b*c + 72*a^5*A*b^3*c - 24*a*A*b^7*c + 96*a^8*B*c - 1
```

```
36*a^6*b^2*B*c - 12*a^4*b^4*B*c + 48*a^2*b^6*B*c + 4*b^8*B*c - 48*a^7*A*b*d
*x + 72*a^5*A*b^3*d*x - 24*a*A*b^7*d*x + 96*a^8*B*d*x - 136*a^6*b^2*B*d*x -
 12*a^4*b^4*B*d*x + 48*a^2*b^6*B*d*x + 4*b^8*B*d*x + 16*a*b*(a^2 - b^2)^2*(
-6*a*A*b + 12*a^2*B + b^2*B)*(c + d*x)*Cos[c + d*x] + 4*(-(a^2*b) + b^3)^2*
(-6*a*A*b + 12*a^2*B + b^2*B)*(c + d*x)*Cos[2*(c + d*x)] + 48*a^6*A*b^2*Sin
[c + d*x] - 84*a^4*A*b^4*Sin[c + d*x] + 8*a^2*A*b^6*Sin[c + d*x] + 4*A*b^8*
Sin[c + d*x] - 96*a^7*b*B*Sin[c + d*x] + 160*a^5*b^3*B*Sin[c + d*x] - 32*a^
3*b^5*B*Sin[c + d*x] - 8*a*b^7*B*Sin[c + d*x] + 36*a^5*A*b^3*Sin[2*(c + d*x
)] - 64*a^3*A*b^5*Sin[2*(c + d*x)] + 16*a*A*b^7*Sin[2*(c + d*x)] - 72*a^6*b
^2*B*Sin[2*(c + d*x)] + 130*a^4*b^4*B*Sin[2*(c + d*x)] - 48*a^2*b^6*B*Sin[2
*(c + d*x)] + 2*b^8*B*Sin[2*(c + d*x)] + 4*a^4*A*b^4*Sin[3*(c + d*x)] - 8*a
^2*A*b^6*Sin[3*(c + d*x)] + 4*A*b^8*Sin[3*(c + d*x)] - 8*a^5*b^3*B*Sin[3*(c
 + d*x)] + 16*a^3*b^5*B*Sin[3*(c + d*x)] - 8*a*b^7*B*Sin[3*(c + d*x)] + a^4
*b^4*B*Sin[4*(c + d*x)] - 2*a^2*b^6*B*Sin[4*(c + d*x)] + b^8*B*Sin[4*(c + d
*x)])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2))/(16*b^5*d)
```

fricas [B] time = 1.23, size = 1812, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(2*(12*B*a^8*b^2 - 6*A*a^7*b^3 - 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^
4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b^8 + 6*A*a*b^9 - B*b^10)*d*x*cos(d*x + c)^2
 + 4*(12*B*a^9*b - 6*A*a^8*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5
 - 18*A*a^4*b^6 - 9*B*a^3*b^7 + 6*A*a^2*b^8 - B*a*b^9)*d*x*cos(d*x + c) + 2
*(12*B*a^10 - 6*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A
*a^5*b^5 - 9*B*a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*d*x + (12*B*a^9 - 6*A*a^8
*b - 29*B*a^7*b^2 + 15*A*a^6*b^3 + 20*B*a^5*b^4 - 12*A*a^4*b^5 + (12*B*a^7*
b^2 - 6*A*a^6*b^3 - 29*B*a^5*b^4 + 15*A*a^4*b^5 + 20*B*a^3*b^6 - 12*A*a^2*b
^7)*cos(d*x + c)^2 + 2*(12*B*a^8*b - 6*A*a^7*b^2 - 29*B*a^6*b^3 + 15*A*a^5*
b^4 + 20*B*a^4*b^5 - 12*A*a^3*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*
b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d
*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + a^2)) - 2*(12*B*a^9*b - 6*A*a^8*b^2 - 33*B*a^7*b^3 + 17*A*a^6*b^4
+ 27*B*a^5*b^5 - 13*A*a^4*b^6 - 6*B*a^3*b^7 + 2*A*a^2*b^8 - (B*a^6*b^4 - 3*
B*a^4*b^6 + 3*B*a^2*b^8 - B*b^10)*cos(d*x + c)^3 + 2*(2*B*a^7*b^3 - A*a^6*b
^4 - 6*B*a^5*b^5 + 3*A*a^4*b^6 + 6*B*a^3*b^7 - 3*A*a^2*b^8 - 2*B*a*b^9 + A*
b^10)*cos(d*x + c)^2 + (18*B*a^8*b^2 - 9*A*a^7*b^3 - 50*B*a^6*b^4 + 25*A*a^
5*b^5 + 43*B*a^4*b^6 - 20*A*a^3*b^7 - 11*B*a^2*b^8 + 4*A*a*b^9)*cos(d*x + c
))*sin(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^
2 + 2*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5
 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d), 1/2*((12*B*a^8*b^2 - 6*A*a^7*b^3 -
```

```
 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b^8 +
6*A*a*b^9 - B*b^10)*d*x*cos(d*x + c)^2 + 2*(12*B*a^9*b - 6*A*a^8*b^2 - 35*B
*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B*a^3*b^7 + 6*A*a
^2*b^8 - B*a*b^9)*d*x*cos(d*x + c) + (12*B*a^10 - 6*A*a^9*b - 35*B*a^8*b^2
+ 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 - 9*B*a^4*b^6 + 6*A*a^3*b^7 -
B*a^2*b^8)*d*x - (12*B*a^9 - 6*A*a^8*b - 29*B*a^7*b^2 + 15*A*a^6*b^3 + 20*B
*a^5*b^4 - 12*A*a^4*b^5 + (12*B*a^7*b^2 - 6*A*a^6*b^3 - 29*B*a^5*b^4 + 15*A
*a^4*b^5 + 20*B*a^3*b^6 - 12*A*a^2*b^7)*cos(d*x + c)^2 + 2*(12*B*a^8*b - 6*
A*a^7*b^2 - 29*B*a^6*b^3 + 15*A*a^5*b^4 + 20*B*a^4*b^5 - 12*A*a^3*b^6)*cos(
d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin
(d*x + c))) - (12*B*a^9*b - 6*A*a^8*b^2 - 33*B*a^7*b^3 + 17*A*a^6*b^4 + 27*
B*a^5*b^5 - 13*A*a^4*b^6 - 6*B*a^3*b^7 + 2*A*a^2*b^8 - (B*a^6*b^4 - 3*B*a^4
*b^6 + 3*B*a^2*b^8 - B*b^10)*cos(d*x + c)^3 + 2*(2*B*a^7*b^3 - A*a^6*b^4 -
6*B*a^5*b^5 + 3*A*a^4*b^6 + 6*B*a^3*b^7 - 3*A*a^2*b^8 - 2*B*a*b^9 + A*b^10)
*cos(d*x + c)^2 + (18*B*a^8*b^2 - 9*A*a^7*b^3 - 50*B*a^6*b^4 + 25*A*a^5*b^5
 + 43*B*a^4*b^6 - 20*A*a^3*b^7 - 11*B*a^2*b^8 + 4*A*a*b^9)*cos(d*x + c))*si
n(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + 2
*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5 - 3*
a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d)]
```

giac [B] time = 2.32, size = 2712, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/2*((3*(2*a^5*b - a^4*b^2 - 4*a^3*b^3 + 2*a^2*b^4 + 2*a*b^5)*sqrt(a^2 - b^
2)*A*abs(a^4*b^5 - 2*a^2*b^7 + b^9)*abs(-a + b) - (12*a^6 - 6*a^5*b - 23*a^
4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 - a*b^5 + b^6)*sqrt(a^2 - b^2)*B*abs(a^4*b^
5 - 2*a^2*b^7 + b^9)*abs(-a + b) + 3*(4*a^10*b^5 - 2*a^9*b^6 - 17*a^8*b^7 +
 8*a^7*b^8 + 28*a^6*b^9 - 12*a^5*b^10 - 21*a^4*b^11 + 8*a^3*b^12 + 6*a^2*b^
13 - 2*a*b^14)*sqrt(a^2 - b^2)*A*abs(-a + b) - (24*a^11*b^4 - 12*a^10*b^5 -
 100*a^9*b^6 + 47*a^8*b^7 + 158*a^7*b^8 - 68*a^6*b^9 - 111*a^5*b^10 + 42*a^
4*b^11 + 28*a^3*b^12 - 8*a^2*b^13 + a*b^14 - b^15)*sqrt(a^2 - b^2)*B*abs(-a
 + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*tan(1/2*d*x + 1/2*c)/sq
rt((4*a^5*b^4 - 8*a^3*b^6 + 4*a*b^8 + sqrt(-16*(a^5*b^4 + a^4*b^5 - 2*a^3*b
^6 - 2*a^2*b^7 + a*b^8 + b^9)*(a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 +
a*b^8 - b^9) + 16*(a^5*b^4 - 2*a^3*b^6 + a*b^8)^2))/(a^5*b^4 - a^4*b^5 - 2*
a^3*b^6 + 2*a^2*b^7 + a*b^8 - b^9))))/((a^4*b^5 - 2*a^2*b^7 + b^9)^2*(a^2 -
 2*a*b + b^2) + (a^7*b^4 - 2*a^6*b^5 - a^5*b^6 + 4*a^4*b^7 - a^3*b^8 - 2*a^
2*b^9 + a*b^10)*abs(a^4*b^5 - 2*a^2*b^7 + b^9)) + (24*B*a^11*b^4 - 12*A*a^1
0*b^5 - 12*B*a^10*b^5 + 6*A*a^9*b^6 - 100*B*a^9*b^6 + 51*A*a^8*b^7 + 47*B*a
^8*b^7 - 24*A*a^7*b^8 + 158*B*a^7*b^8 - 84*A*a^6*b^9 - 68*B*a^6*b^9 + 36*A*
```

a^5*b^10 - 111*B*a^5*b^10 + 63*A*a^4*b^11 + 42*B*a^4*b^11 - 24*A*a^3*b^12 + 28*B*a^3*b^12 - 18*A*a^2*b^13 - 8*B*a^2*b^13 + 6*A*a*b^14 + B*a*b^14 - B*b^15 - 12*B*a^6*abs(a^4*b^5 - 2*a^2*b^7 + b^9) + 6*A*a^5*b*abs(a^4*b^5 - 2*a^2*b^7 + b^9) + 6*B*a^5*b*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - 3*A*a^4*b^2*abs(a^4*b^5 - 2*a^2*b^7 + b^9) + 23*B*a^4*b^2*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - 12*A*a^3*b^3*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - 10*B*a^3*b^3*abs(a^4*b^5 - 2*a^2*b^7 + b^9) + 6*A*a^2*b^4*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - 10*B*a^2*b^4*abs(a^4*b^5 - 2*a^2*b^7 + b^9) + 6*A*a*b^5*abs(a^4*b^5 - 2*a^2*b^7 + b^9) + B*a*b^5*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - B*b^6*abs(a^4*b^5 - 2*a^2*b^7 + b^9))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*tan(1/2*d*x + 1/2*c)/sqrt((4*a^5*b^4 - 8*a^3*b^6 + 4*a*b^8 - sqrt(-16*(a^5*b^4 + a^4*b^5 - 2*a^3*b^6 - 2*a^2*b^7 + a*b^8 + b^9)*(a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 + a*b^8 - b^9) + 16*(a^5*b^4 - 2*a^3*b^6 + a*b^8)^2))/(a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 + a*b^8 - b^9)))))/(a^5*b^4*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - 2*a^3*b^6*abs(a^4*b^5 - 2*a^2*b^7 + b^9) + a*b^8*abs(a^4*b^5 - 2*a^2*b^7 + b^9) - (a^4*b^5 - 2*a^2*b^7 + b^9)^2) - 2*(12*B*a^7*tan(1/2*d*x + 1/2*c)^7 - 6*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 18*B*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 17*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 9*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 2*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 + 2*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 13*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 4*B*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 2*A*b^7*tan(1/2*d*x + 1/2*c)^7 + B*b^7*tan(1/2*d*x + 1/2*c)^7 + 36*B*a^7*tan(1/2*d*x + 1/2*c)^5 - 18*A*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 18*B*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 67*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 35*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 29*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 26*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 10*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 5*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 4*B*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 2*A*b^7*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^7*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^7*tan(1/2*d*x + 1/2*c)^3 - 18*A*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 18*B*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 67*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 35*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 26*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 10*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 5*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^3 - 4*B*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^7*tan(1/2*d*x + 1/2*c)^3 + 3*B*b^7*tan(1/2*d*x + 1/2*c)^3 + 12*B*a^7*tan(1/2*d*x + 1/2*c) - 6*A*a^6*b*tan(1/2*d*x + 1/2*c) + 18*B*a^6*b*tan(1/2*d*x + 1/2*c) - 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c) - 17*B*a^5*b^2*tan(1/2*d*x + 1/2*c) + 9*A*a^4*b^3*tan(1/2*d*x + 1/2*c) - 33*B*a^4*b^3*tan(1/2*d*x + 1/2*c) + 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c) - 2*B*a^3*b^4*tan(1/2*d*x + 1/2*c) + 2*A*a^2*b^5*tan(1/2*d*x + 1/2*c) + 13*B*a^2*b^5*tan(1/2*d*x + 1/2*c) - 4*A*a*b^6*tan(1/2*d*x + 1/2*c) + 4*B*a*b^6*tan(1/2*d*x + 1/2*c) - 2*A*b^7*tan(1/2*d*x + 1/2*c) - B*b^7*tan(1/2*d*x + 1/2*c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*

```
x + 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

maple [B] time = 0.10, size = 1504, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -20/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*
c)*(a-b)/((a-b)*(a+b))^(1/2))*B+6/d*a^6/b^4/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b
))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-15/d*a^4/b^
2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/(
(a-b)*(a+b))^(1/2))*A-12/d*a^7/b^5/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*
arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+29/d*a^5/b^3/(a^4-2*
a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))*B+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))*B+2/d/b^3/(1+tan(1/2*d*x+1/
2*c)^2)^2*tan(1/2*d*x+1/2*c)*A+4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*
d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+1/d*a^4/b^2/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1
/2*c)*A+4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(
a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d*a^4/b^2/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)
^3*A-1/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b
)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+10/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-8/d*a^3/b/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/
2*c)*A-6/d*a^6/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a
+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-8/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*
d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-6/d*a^6/
b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3*B+1/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+10/d*a^4/b^
2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^
2)*tan(1/2*d*x+1/2*c)^3*B-1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/
2*c)^3*B+2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*A+1/d/b^3/
(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B-6/d/b^4*arctan(tan(1/2*d*x+
1/2*c))*A*a+12/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^2*B+12/d*a^2/(a^4-2*a^2*b
^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(
1/2))*A-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B*a-6/d/b^4
/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.01, size = 10598, normalized size = 26.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(3*B*b^7 - 36*B*a^7 - 2*A*b^7 + 10*A*a^2*b^5 + 16*A*
a^3*b^4 - 35*A*a^4*b^3 - 9*A*a^5*b^2 + 5*B*a^2*b^5 - 26*B*a^3*b^4 - 29*B*a^
4*b^3 + 67*B*a^5*b^2 - 4*A*a*b^6 + 18*A*a^6*b + 4*B*a*b^6 + 18*B*a^6*b))/((
a + b)^2*(b^6 - 2*a*b^5 + a^2*b^4)) - (tan(c/2 + (d*x)/2)^3*(2*A*b^7 + 36*B
*a^7 + 3*B*b^7 - 10*A*a^2*b^5 + 16*A*a^3*b^4 + 35*A*a^4*b^3 - 9*A*a^5*b^2 +
 5*B*a^2*b^5 + 26*B*a^3*b^4 - 29*B*a^4*b^3 - 67*B*a^5*b^2 - 4*A*a*b^6 - 18*
A*a^6*b - 4*B*a*b^6 + 18*B*a^6*b))/((a + b)^2*(b^6 - 2*a*b^5 + a^2*b^4)) +
(tan(c/2 + (d*x)/2)^7*(B*b^6 - 12*B*a^6 - 2*A*b^6 + 4*A*a^2*b^4 - 12*A*a^3*
b^3 - 3*A*a^4*b^2 - 8*B*a^2*b^4 - 10*B*a^3*b^3 + 23*B*a^4*b^2 + 2*A*a*b^5 +
 6*A*a^5*b + 5*B*a*b^5 + 6*B*a^5*b))/((a*b^4 - b^5)*(a + b)^2) + (tan(c/2 +
 (d*x)/2)*(2*A*b^6 - 12*B*a^6 + B*b^6 - 4*A*a^2*b^4 - 12*A*a^3*b^3 + 3*A*a^
4*b^2 - 8*B*a^2*b^4 + 10*B*a^3*b^3 + 23*B*a^4*b^2 + 2*A*a*b^5 + 6*A*a^5*b -
 5*B*a*b^5 - 6*B*a^5*b))/((a + b)*(b^6 - 2*a*b^5 + a^2*b^4)))/(d*(2*a*b + t
an(c/2 + (d*x)/2)^4*(6*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^2*(4*a*b + 4*a^2)
- tan(c/2 + (d*x)/2)^6*(4*a*b - 4*a^2) + tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b
+ b^2) + a^2 + b^2)) + (atan(((((8*tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B^2*b
^14 - 2*B^2*a*b^13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11 + 3
6*A^2*a^4*b^10 + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*
A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72*A^2
*a^12*b^2 + 21*B^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B^2*a
^5*b^9 + 18*B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*
b^5 + 1538*B^2*a^10*b^4 + 1104*B^2*a^11*b^3 - 1104*B^2*a^12*b^2 - 12*A*B*a*
b^13 - 288*A*B*a^13*b + 24*A*B*a^2*b^12 - 108*A*B*a^3*b^11 + 192*A*B*a^4*b^
10 - 72*A*B*a^5*b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6
 - 1650*A*B*a^9*b^5 - 1128*A*B*a^10*b^4 + 1128*A*B*a^11*b^3 + 288*A*B*a^12*
b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 -
a^6*b^9 - a^7*b^8) + (((4*(4*B*b^21 + 48*A*a^2*b^19 + 72*A*a^3*b^18 - 156*A
*a^4*b^17 - 84*A*a^5*b^16 + 192*A*a^6*b^15 + 48*A*a^7*b^14 - 108*A*a^8*b^13
```

- 12*A*a^9*b^12 + 24*A*a^10*b^11 + 28*B*a^2*b^19 - 80*B*a^3*b^18 - 120*B*a
^4*b^17 + 276*B*a^5*b^16 + 164*B*a^6*b^15 - 360*B*a^7*b^14 - 100*B*a^8*b^13
 + 212*B*a^9*b^12 + 24*B*a^10*b^11 - 48*B*a^11*b^10 - 24*A*a*b^20))/(a*b^18
 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^
7*b^12) - (4*tan(c/2 + (d*x)/2)*(B*a^2*12i + B*b^2*1i - A*a*b*6i)*(8*a*b^19
 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*
a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/(b^5*(a*b^14 + b^15 - 3
*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(B*
a^2*12i + B*b^2*1i - A*a*b*6i))/(2*b^5))*(B*a^2*12i + B*b^2*1i - A*a*b*6i)*
1i)/(2*b^5) + (((8*tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B^2*b^14 - 2*B^2*a*b^
13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11 + 36*A^2*a^4*b^10 +
 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 28
8*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72*A^2*a^12*b^2 + 21*B
^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B^2*a^5*b^9 + 18*B^2*
a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a
^10*b^4 + 1104*B^2*a^11*b^3 - 1104*B^2*a^12*b^2 - 12*A*B*a*b^13 - 288*A*B*a
^13*b + 24*A*B*a^2*b^12 - 108*A*B*a^3*b^11 + 192*A*B*a^4*b^10 - 72*A*B*a^5*
b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6 - 1650*A*B*a^9*
b^5 - 1128*A*B*a^10*b^4 + 1128*A*B*a^11*b^3 + 288*A*B*a^12*b^2))/(a*b^14 +
b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^
8) - (((4*(4*B*b^21 + 48*A*a^2*b^19 + 72*A*a^3*b^18 - 156*A*a^4*b^17 - 84*A
*a^5*b^16 + 192*A*a^6*b^15 + 48*A*a^7*b^14 - 108*A*a^8*b^13 - 12*A*a^9*b^12
 + 24*A*a^10*b^11 + 28*B*a^2*b^19 - 80*B*a^3*b^18 - 120*B*a^4*b^17 + 276*B*
a^5*b^16 + 164*B*a^6*b^15 - 360*B*a^7*b^14 - 100*B*a^8*b^13 + 212*B*a^9*b^1
2 + 24*B*a^10*b^11 - 48*B*a^11*b^10 - 24*A*a*b^20))/(a*b^18 + b^19 - 3*a^2*
b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) + (4*tan
(c/2 + (d*x)/2)*(B*a^2*12i + B*b^2*1i - A*a*b*6i)*(8*a*b^19 - 8*a^2*b^18 -
32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^
8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/(b^5*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^
3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(B*a^2*12i + B*b^2*
1i - A*a*b*6i))/(2*b^5))*(B*a^2*12i + B*b^2*1i - A*a*b*6i)*1i)/(2*b^5))/((8
*(1728*B^3*a^15 - 864*B^3*a^14*b - 432*A^3*a^4*b^11 - 432*A^3*a^5*b^10 + 14
04*A^3*a^6*b^9 + 756*A^3*a^7*b^8 - 1728*A^3*a^8*b^7 - 486*A^3*a^9*b^6 + 972
*A^3*a^10*b^5 + 108*A^3*a^11*b^4 - 216*A^3*a^12*b^3 + 20*B^3*a^3*b^12 - 20*
B^3*a^4*b^11 + 411*B^3*a^5*b^10 - 11*B^3*a^6*b^9 + 1314*B^3*a^7*b^8 + 2326*
B^3*a^8*b^7 - 7829*B^3*a^9*b^6 - 4770*B^3*a^10*b^5 + 11700*B^3*a^11*b^4 + 3
456*B^3*a^12*b^3 - 7344*B^3*a^13*b^2 - 2592*A*B^2*a^14*b - 12*A*B^2*a^2*b^1
3 + 12*A*B^2*a^3*b^12 - 489*A*B^2*a^4*b^11 + 9*A*B^2*a^5*b^10 - 2892*A*B^2*
a^6*b^9 - 3972*A*B^2*a^7*b^8 + 13347*A*B^2*a^8*b^7 + 7767*A*B^2*a^9*b^6 - 1
8594*A*B^2*a^10*b^5 - 5400*A*B^2*a^11*b^4 + 11232*A*B^2*a^12*b^3 + 1296*A*B
^2*a^13*b^2 + 144*A^2*B*a^3*b^12 + 1980*A^2*B*a^5*b^10 + 2268*A^2*B*a^6*b^9
 - 7524*A^2*B*a^7*b^8 - 4203*A^2*B*a^8*b^7 + 9828*A^2*B*a^9*b^6 + 2808*A^2*
B*a^10*b^5 - 5724*A^2*B*a^11*b^4 - 648*A^2*B*a^12*b^3 + 1296*A^2*B*a^13*b^2
))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6
*b^13 - a^7*b^12) - (((8*tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B^2*b^14 - 2*B^

2*a*b^13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11 + 36*A^2*a^4*b^10 + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72*A^2*a^12*b^2 + 21*B^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B^2*a^5*b^9 + 18*B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a^10*b^4 + 1104*B^2*a^11*b^3 - 1104*B^2*a^12*b^2 - 12*A*B*a*b^13 - 288*A*B*a^13*b + 24*A*B*a^2*b^12 - 108*A*B*a^3*b^11 + 192*A*B*a^4*b^10 - 72*A*B*a^5*b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6 - 1650*A*B*a^9*b^5 - 1128*A*B*a^10*b^4 + 1128*A*B*a^11*b^3 + 288*A*B*a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8) + (((4*(4*B*b^21 + 48*A*a^2*b^19 + 72*A*a^3*b^18 - 156*A*a^4*b^17 - 84*A*a^5*b^16 + 192*A*a^6*b^15 + 48*A*a^7*b^14 - 108*A*a^8*b^13 - 12*A*a^9*b^12 + 24*A*a^10*b^11 + 28*B*a^2*b^19 - 80*B*a^3*b^18 - 120*B*a^4*b^17 + 276*B*a^5*b^16 + 164*B*a^6*b^15 - 360*B*a^7*b^14 - 100*B*a^8*b^13 + 212*B*a^9*b^12 + 24*B*a^10*b^11 - 48*B*a^11*b^10 - 24*A*a*b^20))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) - (4*tan(c/2 + (d*x)/2)*(B*a^2*12i + B*b^2*1i - A*a*b*6i)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/(b^5*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(B*a^2*12i + B*b^2*1i - A*a*b*6i))/(2*b^5))*(B*a^2*12i + B*b^2*1i - A*a*b*6i))/(2*b^5) + (((8*tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B^2*b^14 - 2*B^2*a*b^13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11 + 36*A^2*a^4*b^10 + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72*A^2*a^12*b^2 + 21*B^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B^2*a^5*b^9 + 18*B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a^10*b^4 + 1104*B^2*a^11*b^3 - 1104*B^2*a^12*b^2 - 12*A*B*a*b^13 - 288*A*B*a^13*b + 24*A*B*a^2*b^12 - 108*A*B*a^3*b^11 + 192*A*B*a^4*b^10 - 72*A*B*a^5*b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6 - 1650*A*B*a^9*b^5 - 1128*A*B*a^10*b^4 + 1128*A*B*a^11*b^3 + 288*A*B*a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8) - (((4*(4*B*b^21 + 48*A*a^2*b^19 + 72*A*a^3*b^18 - 156*A*a^4*b^17 - 84*A*a^5*b^16 + 192*A*a^6*b^15 + 48*A*a^7*b^14 - 108*A*a^8*b^13 - 12*A*a^9*b^12 + 24*A*a^10*b^11 + 28*B*a^2*b^19 - 80*B*a^3*b^18 - 120*B*a^4*b^17 + 276*B*a^5*b^16 + 164*B*a^6*b^15 - 360*B*a^7*b^14 - 100*B*a^8*b^13 + 212*B*a^9*b^12 + 24*B*a^10*b^11 - 48*B*a^11*b^10 - 24*A*a*b^20))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) + (4*tan(c/2 + (d*x)/2)*(B*a^2*12i + B*b^2*1i - A*a*b*6i)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/(b^5*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(B*a^2*12i + B*b^2*1i - A*a*b*6i))/(2*b^5))*(B*a^2*12i + B*b^2*1i - A*a*b*6i))/(2*b^5)))*(B*a^2*12i + B*b^2*1i - A*a*b*6i)*1i)/(b^5*d) + (a^2*atan(((a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B^2*b^14 - 2*B^2*a*b^13 - 288*B^2*a^

13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11 + 36*A^2*a^4*b^10 + 288*A^2*a^5*b^
9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 288*A^2*a^9*b^5 -
288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72*A^2*a^12*b^2 + 21*B^2*a^2*b^12 - 4
0*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B^2*a^5*b^9 + 18*B^2*a^6*b^8 + 872*B
^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a^10*b^4 + 1104*
B^2*a^11*b^3 - 1104*B^2*a^12*b^2 - 12*A*B*a*b^13 - 288*A*B*a^13*b + 24*A*B*
a^2*b^12 - 108*A*B*a^3*b^11 + 192*A*B*a^4*b^10 - 72*A*B*a^5*b^9 - 1008*A*B*
a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6 - 1650*A*B*a^9*b^5 - 1128*A*B*
a^10*b^4 + 1128*A*B*a^11*b^3 + 288*A*B*a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^
13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8) + (a^2*((4*(
4*B*b^21 + 48*A*a^2*b^19 + 72*A*a^3*b^18 - 156*A*a^4*b^17 - 84*A*a^5*b^16 +
192*A*a^6*b^15 + 48*A*a^7*b^14 - 108*A*a^8*b^13 - 12*A*a^9*b^12 + 24*A*a^1
0*b^11 + 28*B*a^2*b^19 - 80*B*a^3*b^18 - 120*B*a^4*b^17 + 276*B*a^5*b^16 +
164*B*a^6*b^15 - 360*B*a^7*b^14 - 100*B*a^8*b^13 + 212*B*a^9*b^12 + 24*B*a^
10*b^11 - 48*B*a^11*b^10 - 24*A*a*b^20))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^
3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) - (4*a^2*tan(c/2 +
(d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 +
29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^1
7 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8
*a^9*b^11 - 8*a^10*b^10))/((b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 +
5*a^8*b^7 - a^10*b^5)*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11
+ 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5
- 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4))/(2*(b^
15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*(12*A*
b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4)*1i)/
(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)) +
(a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B
^2*b^14 - 2*B^2*a*b^13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11
+ 36*A^2*a^4*b^10 + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 +
441*A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72
*A^2*a^12*b^2 + 21*B^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B
^2*a^5*b^9 + 18*B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*
a^9*b^5 + 1538*B^2*a^10*b^4 + 1104*B^2*a^11*b^3 - 1104*B^2*a^12*b^2 - 12*A*
B*a*b^13 - 288*A*B*a^13*b + 24*A*B*a^2*b^12 - 108*A*B*a^3*b^11 + 192*A*B*a^
4*b^10 - 72*A*B*a^5*b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8
*b^6 - 1650*A*B*a^9*b^5 - 1128*A*B*a^10*b^4 + 1128*A*B*a^11*b^3 + 288*A*B*a
^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^1
0 - a^6*b^9 - a^7*b^8) - (a^2*((4*(4*B*b^21 + 48*A*a^2*b^19 + 72*A*a^3*b^18
- 156*A*a^4*b^17 - 84*A*a^5*b^16 + 192*A*a^6*b^15 + 48*A*a^7*b^14 - 108*A*
a^8*b^13 - 12*A*a^9*b^12 + 24*A*a^10*b^11 + 28*B*a^2*b^19 - 80*B*a^3*b^18 -
120*B*a^4*b^17 + 276*B*a^5*b^16 + 164*B*a^6*b^15 - 360*B*a^7*b^14 - 100*B*
a^8*b^13 + 212*B*a^9*b^12 + 24*B*a^10*b^11 - 48*B*a^11*b^10 - 24*A*a*b^20))
/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b
^13 - a^7*b^12) + (4*a^2*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(1
2*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4)*

(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b
^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/((b^15 - 5*a^2
*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)*(a*b^14 + b^15 - 3
*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(-(
a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^
2 + 6*A*a^4*b - 20*B*a*b^4))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b
^9 + 5*a^8*b^7 - a^10*b^5)))*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3
*b^2 + 6*A*a^4*b - 20*B*a*b^4)*1i)/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10
*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))/((8*(1728*B^3*a^15 - 864*B^3*a^14*b - 43
2*A^3*a^4*b^11 - 432*A^3*a^5*b^10 + 1404*A^3*a^6*b^9 + 756*A^3*a^7*b^8 - 17
28*A^3*a^8*b^7 - 486*A^3*a^9*b^6 + 972*A^3*a^10*b^5 + 108*A^3*a^11*b^4 - 21
6*A^3*a^12*b^3 + 20*B^3*a^3*b^12 - 20*B^3*a^4*b^11 + 411*B^3*a^5*b^10 - 11*
B^3*a^6*b^9 + 1314*B^3*a^7*b^8 + 2326*B^3*a^8*b^7 - 7829*B^3*a^9*b^6 - 4770
*B^3*a^10*b^5 + 11700*B^3*a^11*b^4 + 3456*B^3*a^12*b^3 - 7344*B^3*a^13*b^2
- 2592*A*B^2*a^14*b - 12*A*B^2*a^2*b^13 + 12*A*B^2*a^3*b^12 - 489*A*B^2*a^4
*b^11 + 9*A*B^2*a^5*b^10 - 2892*A*B^2*a^6*b^9 - 3972*A*B^2*a^7*b^8 + 13347*
A*B^2*a^8*b^7 + 7767*A*B^2*a^9*b^6 - 18594*A*B^2*a^10*b^5 - 5400*A*B^2*a^11
*b^4 + 11232*A*B^2*a^12*b^3 + 1296*A*B^2*a^13*b^2 + 144*A^2*B*a^3*b^12 + 19
80*A^2*B*a^5*b^10 + 2268*A^2*B*a^6*b^9 - 7524*A^2*B*a^7*b^8 - 4203*A^2*B*a^
8*b^7 + 9828*A^2*B*a^9*b^6 + 2808*A^2*B*a^10*b^5 - 5724*A^2*B*a^11*b^4 - 64
8*A^2*B*a^12*b^3 + 1296*A^2*B*a^13*b^2))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^
3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) - (a^2*(-(a + b)^5*
(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B^2*b^14 - 2*B^2*a*
b^13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11 + 36*A^2*a^4*b^10
+ 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*A^2*a^8*b^6 +
288*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72*A^2*a^12*b^2 + 21
*B^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B^2*a^5*b^9 + 18*B^
2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538*B^2
*a^10*b^4 + 1104*B^2*a^11*b^3 - 1104*B^2*a^12*b^2 - 12*A*B*a*b^13 - 288*A*B
*a^13*b + 24*A*B*a^2*b^12 - 108*A*B*a^3*b^11 + 192*A*B*a^4*b^10 - 72*A*B*a^
5*b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6 - 1650*A*B*a^
9*b^5 - 1128*A*B*a^10*b^4 + 1128*A*B*a^11*b^3 + 288*A*B*a^12*b^2))/(a*b^14
+ b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*
b^8) + (a^2*((4*(4*B*b^21 + 48*A*a^2*b^19 + 72*A*a^3*b^18 - 156*A*a^4*b^17
- 84*A*a^5*b^16 + 192*A*a^6*b^15 + 48*A*a^7*b^14 - 108*A*a^8*b^13 - 12*A*a^
9*b^12 + 24*A*a^10*b^11 + 28*B*a^2*b^19 - 80*B*a^3*b^18 - 120*B*a^4*b^17 +
276*B*a^5*b^16 + 164*B*a^6*b^15 - 360*B*a^7*b^14 - 100*B*a^8*b^13 + 212*B*a
^9*b^12 + 24*B*a^10*b^11 - 48*B*a^11*b^10 - 24*A*a*b^20))/(a*b^18 + b^19 -
3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) -
(4*a^2*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5
- 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4)*(8*a*b^19 - 8*a^2*
b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13
+ 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/((b^15 - 5*a^2*b^13 + 10*a^4*b^1
1 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*
b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(-(a + b)^5*(a - b)^5

```
)^(1/2)*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20
*B*a*b^4))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a
^10*b^5)))*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b -
 20*B*a*b^4))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7
- a^10*b^5)) + (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(28
8*B^2*a^14 + B^2*b^14 - 2*B^2*a*b^13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 7
2*A^2*a^3*b^11 + 36*A^2*a^4*b^10 + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*
A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2
*a^11*b^3 + 72*A^2*a^12*b^2 + 21*B^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^
4*b^10 - 108*B^2*a^5*b^9 + 18*B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b
^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a^10*b^4 + 1104*B^2*a^11*b^3 - 1104*B^2*a^
12*b^2 - 12*A*B*a*b^13 - 288*A*B*a^13*b + 24*A*B*a^2*b^12 - 108*A*B*a^3*b^1
1 + 192*A*B*a^4*b^10 - 72*A*B*a^5*b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7
+ 1632*A*B*a^8*b^6 - 1650*A*B*a^9*b^5 - 1128*A*B*a^10*b^4 + 1128*A*B*a^11*b
^3 + 288*A*B*a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^
11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8) - (a^2*((4*(4*B*b^21 + 48*A*a^2*b^19 +
 72*A*a^3*b^18 - 156*A*a^4*b^17 - 84*A*a^5*b^16 + 192*A*a^6*b^15 + 48*A*a^7
*b^14 - 108*A*a^8*b^13 - 12*A*a^9*b^12 + 24*A*a^10*b^11 + 28*B*a^2*b^19 - 8
0*B*a^3*b^18 - 120*B*a^4*b^17 + 276*B*a^5*b^16 + 164*B*a^6*b^15 - 360*B*a^7
*b^14 - 100*B*a^8*b^13 + 212*B*a^9*b^12 + 24*B*a^10*b^11 - 48*B*a^11*b^10 -
 24*A*a*b^20))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^
5*b^14 - a^6*b^13 - a^7*b^12) + (4*a^2*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a -
b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b
- 20*B*a*b^4)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b
^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/
((b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)*(a*b
^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 -
a^7*b^8)))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3
 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4))/(2*(b^15 - 5*a^2*b^13 + 10*a^4*b
^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5)))*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*
b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^4))/(2*(b^15 - 5*a^2*b^13 + 10*a^
4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*b^5))))*(-(a + b)^5*(a - b)^5)^(1/2)
*(12*A*b^5 - 12*B*a^5 - 15*A*a^2*b^3 + 29*B*a^3*b^2 + 6*A*a^4*b - 20*B*a*b^
4)*1i)/(d*(b^15 - 5*a^2*b^13 + 10*a^4*b^11 - 10*a^6*b^9 + 5*a^8*b^7 - a^10*
b^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.266 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=280

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\left(-3a^2B+aAb+2b^2B\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)}-\frac{a^2\left(-3a^3B+a^2Ab+6ab^2B-4Ab^3\right)\sin}{2b^3d\left(a^2-b^2\right)^2(a+b\cos(c+}$$

```
[Out] (A*b-3*B*a)*x/b^4-a*(2*A*a^4*b-5*A*a^2*b^3+6*A*b^5-6*B*a^5+15*B*a^3*b^2-12*
B*a*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^4
/(a+b)^(5/2)/d-1/2*(A*a*b-3*B*a^2+2*B*b^2)*sin(d*x+c)/b^3/(a^2-b^2)/d+1/2*a
*(A*b-B*a)*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/2*a^2
*(A*a^2*b-4*A*b^3-3*B*a^3+6*B*a*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*cos(
d*x+c))
```

Rubi [A] time = 1.22, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3031, 3023, 2735, 2659, 205}

$$-\frac{\left(-3a^2B+aAb+2b^2B\right)\sin(c+dx)}{2b^3d\left(a^2-b^2\right)}-\frac{a\left(-5a^2Ab^3+2a^4Ab+15a^3b^2B-6a^5B-12ab^4B+6Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan}{\sqrt{}}\right.}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((A*b - 3*a*B)*x)/b^4 - (a*(2*a^4*A*b - 5*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 1
5*a^3*b^2*B - 12*a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) - ((a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c
 + d*x])/(2*b^3*(a^2 - b^2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x]
)/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(a^2*A*b - 4*A*b^3 - 3*
a^3*B + 6*a*b^2*B)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]
))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(-2a(Ab-aB)+2b(Ab-aB)\,c\right.}{(a+b\,c}}{2b\,(}$$

$$= \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(a^2Ab-4Ab^3-3a^3B+6ab\right.}{2b^3\left(a^2-b^2\right)^2 d(a+b\cos}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx}$$

$$= \frac{(Ab-3aB)x}{b^4} - \frac{\left(aAb-3a^2B+2b^2B\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\cos^2(c}{2b\left(a^2-b^2\right)d(a}$$

$$= \frac{(Ab-3aB)x}{b^4} - \frac{\left(aAb-3a^2B+2b^2B\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\cos^2(c}{2b\left(a^2-b^2\right)d(a}$$

$$= \frac{(Ab-3aB)x}{b^4} - \frac{a\left(2a^4Ab-5a^2Ab^3+6Ab^5-6a^5B+15a^3b^2B-12ab^4\right.}{(a-b)^{5/2}b^4(a+b)^{5/2}d}$$

Mathematica [A] time = 2.16, size = 232, normalized size = 0.83

$$\frac{\frac{a^3b(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2} + \frac{a^2b\left(5a^3B-3a^2Ab-8ab^2B+6Ab^3\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} - \frac{2a\left(6a^5B-2a^4Ab-15a^3b^2B+5a^2Ab^3+12ab^4B-6Ab^5\right)\tanh^{-1}\left(\frac{(a-b)\tan}{\sqrt{b^2}}\right.}{\left(b^2-a^2\right)^{5/2}}}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*(A*b - 3*a*B)*(c + d*x) - (2*a*(-2*a^4*A*b + 5*a^2*A*b^3 - 6*A*b^5 + 6*a
^5*B - 15*a^3*b^2*B + 12*a*b^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]])/(-a^2 + b^2)^(5/2) + 2*b*B*Sin[c + d*x] + (a^3*b*(A*b - a*B)*S
```

```
in[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a^2*b*(-3*a^2*A*b
+ 6*A*b^3 + 5*a^3*B - 8*a*b^2*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*
Cos[c + d*x])))/(2*b^4*d)
```

fricas [B] time = 0.87, size = 1561, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(4*(3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6
 - 3*A*a^2*b^7 - 3*B*a*b^8 + A*b^9)*d*x*cos(d*x + c)^2 + 8*(3*B*a^8*b - A*a
^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^
7 + A*a*b^8)*d*x*cos(d*x + c) + 4*(3*B*a^9 - A*a^8*b - 9*B*a^7*b^2 + 3*A*a^
6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7)*d*x - (6*B*a^8
 - 2*A*a^7*b - 15*B*a^6*b^2 + 5*A*a^5*b^3 + 12*B*a^4*b^4 - 6*A*a^3*b^5 + (6
*B*a^6*b^2 - 2*A*a^5*b^3 - 15*B*a^4*b^4 + 5*A*a^3*b^5 + 12*B*a^2*b^6 - 6*A*
a*b^7)*cos(d*x + c)^2 + 2*(6*B*a^7*b - 2*A*a^6*b^2 - 15*B*a^5*b^3 + 5*A*a^4
*b^4 + 12*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*
b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d
*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + a^2)) - 2*(6*B*a^8*b - 2*A*a^7*b^2 - 17*B*a^6*b^3 + 7*A*a^5*b^4 +
13*B*a^4*b^5 - 5*A*a^3*b^6 - 2*B*a^2*b^7 + 2*(B*a^6*b^3 - 3*B*a^4*b^5 + 3*B
*a^2*b^7 - B*b^9)*cos(d*x + c)^2 + (9*B*a^7*b^2 - 3*A*a^6*b^3 - 25*B*a^5*b^
4 + 9*A*a^4*b^5 + 20*B*a^3*b^6 - 6*A*a^2*b^7 - 4*B*a*b^8)*cos(d*x + c))*sin
(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*
(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c) + (a^8*b^4 - 3*a^
6*b^6 + 3*a^4*b^8 - a^2*b^10)*d), -1/2*(2*(3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^
5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^2*b^7 - 3*B*a*b^8 + A*b^9)*d*x*co
s(d*x + c)^2 + 4*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9*B*a
^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*d*x*cos(d*x + c) + 2*(3*B*a^9
 - A*a^8*b - 9*B*a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^
3*b^6 + A*a^2*b^7)*d*x - (6*B*a^8 - 2*A*a^7*b - 15*B*a^6*b^2 + 5*A*a^5*b^3
+ 12*B*a^4*b^4 - 6*A*a^3*b^5 + (6*B*a^6*b^2 - 2*A*a^5*b^3 - 15*B*a^4*b^4 +
5*A*a^3*b^5 + 12*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x + c)^2 + 2*(6*B*a^7*b - 2*A
*a^6*b^2 - 15*B*a^5*b^3 + 5*A*a^4*b^4 + 12*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x
 + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*
x + c))) - (6*B*a^8*b - 2*A*a^7*b^2 - 17*B*a^6*b^3 + 7*A*a^5*b^4 + 13*B*a^4
*b^5 - 5*A*a^3*b^6 - 2*B*a^2*b^7 + 2*(B*a^6*b^3 - 3*B*a^4*b^5 + 3*B*a^2*b^7
 - B*b^9)*cos(d*x + c)^2 + (9*B*a^7*b^2 - 3*A*a^6*b^3 - 25*B*a^5*b^4 + 9*A*
a^4*b^5 + 20*B*a^3*b^6 - 6*A*a^2*b^7 - 4*B*a*b^8)*cos(d*x + c))*sin(d*x + c
))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^7*b^5
```

```
 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c) + (a^8*b^4 - 3*a^6*b^6 +
3*a^4*b^8 - a^2*b^10)*d)]
```

giac [B] time = 1.18, size = 543, normalized size = 1.94

$$-\frac{\frac{\left(6\,Ba^6-2\,Aa^5b-15\,Ba^4b^2+5\,Aa^3b^3+12\,Ba^2b^4-6\,Aab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^4-2\,a^2b^6+b^8\right)\sqrt{a^2-b^2}}-\frac{4\,Ba^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -((6*B*a^6 - 2*A*a^5*b - 15*B*a^4*b^2 + 5*A*a^3*b^3 + 12*B*a^2*b^4 - 6*A*a*
b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2
*b^6 + b^8)*sqrt(a^2 - b^2)) - (4*B*a^6*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^5*b*
tan(1/2*d*x + 1/2*c)^3 - 5*B*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^4*b^2*tan
(1/2*d*x + 1/2*c)^3 - 7*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^3*b^3*tan(
1/2*d*x + 1/2*c)^3 + 8*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^2*b^4*tan(1
/2*d*x + 1/2*c)^3 + 4*B*a^6*tan(1/2*d*x + 1/2*c) - 2*A*a^5*b*tan(1/2*d*x +
1/2*c) + 5*B*a^5*b*tan(1/2*d*x + 1/2*c) - 3*A*a^4*b^2*tan(1/2*d*x + 1/2*c)
- 7*B*a^4*b^2*tan(1/2*d*x + 1/2*c) + 5*A*a^3*b^3*tan(1/2*d*x + 1/2*c) - 8*B
*a^3*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^3
 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 +
a + b)^2) + (3*B*a - A*b)*(d*x + c)/b^4 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/
2*d*x + 1/2*c)^2 + 1)*b^3))/d
```

maple [B] time = 0.09, size = 1301, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -2/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+6/d*
a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3*A+4/d*a^5/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d*a^4/b^2
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2
```

```
)*tan(1/2*d*x+1/2*c)^3*B-8/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-2/d*a^4/b^2/(a*t
an(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x
+1/2*c)*A-1/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(
a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+6/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+4/d*a^5/b^3/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/
2*c)*B+1/d*a^4/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a
+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-8/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*
d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-2/d*a^5/b^3/(a^4-2
*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a
+b))^(1/2))*A+5/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(
1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a
-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+6/d
*a^6/b^4/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*
(a-b)/((a-b)*(a+b))^(1/2))*B-15/d*a^4/b^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))
^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+12/d*a^2/(a^4
-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*
(a+b))^(1/2))*B+2/d/b^3*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2/d/b
^3*A*arctan(tan(1/2*d*x+1/2*c))-6/d/b^4*B*arctan(tan(1/2*d*x+1/2*c))*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 7.66, size = 5542, normalized size = 19.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(6*B*a^5 - 2*B*b^5 + 6*A*a^2*b^3 + A*a^3*b^2 + 4*B*a
^2*b^3 - 12*B*a^3*b^2 - 2*A*a^4*b + 2*B*a*b^4 - 3*B*a^4*b))/((a*b^3 - b^4)*
(a + b)^2) + (tan(c/2 + (d*x)/2)*(6*B*a^5 + 2*B*b^5 + 6*A*a^2*b^3 - A*a^3*b
^2 - 4*B*a^2*b^3 - 12*B*a^3*b^2 - 2*A*a^4*b + 2*B*a*b^4 + 3*B*a^4*b))/((a +
```

b)*(b^5 - 2*a*b^4 + a^2*b^3)) + (2*tan(c/2 + (d*x)/2)^3*(6*B*a^6 - 2*B*b^6 + 5*A*a^3*b^3 + 6*B*a^2*b^4 - 13*B*a^4*b^2 - 2*A*a^5*b))/(b*(a*b^2 - b^3)*(a + b)^2*(a - b)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a*b + 3*a^2 - b^2) + tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - tan(c/2 + (d*x)/2)^4*(2*a*b - 3*a^2 + b^2))) + (log(tan(c/2 + (d*x)/2) + 1i)*(A*b - 3*B*a)*1i)/(b^4*d) - (log(tan(c/2 + (d*x)/2) - 1i)*(A*b*1i - B*a*3i))/(b^4*d) - (a*atan(((a*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^12 + 72*B^2*a^12 - 8*A^2*a*b^11 - 72*B^2*a^11*b + 24*A^2*a^2*b^10 + 32*A^2*a^3*b^9 - 52*A^2*a^4*b^8 - 48*A^2*a^5*b^7 + 57*A^2*a^6*b^6 + 32*A^2*a^7*b^5 - 32*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 8*A^2*a^10*b^2 + 36*B^2*a^2*b^10 - 72*B^2*a^3*b^9 + 36*B^2*a^4*b^8 + 288*B^2*a^5*b^7 - 288*B^2*a^6*b^6 - 432*B^2*a^7*b^5 + 441*B^2*a^8*b^4 + 288*B^2*a^9*b^3 - 288*B^2*a^10*b^2 - 24*A*B*a*b^11 - 48*A*B*a^11*b + 48*A*B*a^2*b^10 - 72*A*B*a^3*b^9 - 192*A*B*a^4*b^8 + 252*A*B*a^5*b^7 + 288*A*B*a^6*b^6 - 318*A*B*a^7*b^5 - 192*A*B*a^8*b^4 + 192*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (a*((8*(4*A*b^18 - 8*A*a^2*b^16 + 34*A*a^3*b^15 + 6*A*a^4*b^14 - 36*A*a^5*b^13 - 4*A*a^6*b^12 + 18*A*a^7*b^11 + 2*A*a^8*b^10 - 4*A*a^9*b^9 + 24*B*a^2*b^16 + 36*B*a^3*b^15 - 78*B*a^4*b^14 - 42*B*a^5*b^13 + 96*B*a^6*b^12 + 24*B*a^7*b^11 - 54*B*a^8*b^10 - 6*B*a^9*b^9 + 12*B*a^10*b^8 - 12*A*a*b^17 - 12*B*a*b^17))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4)*1i)/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)) + (a*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^12 + 72*B^2*a^12 - 8*A^2*a*b^11 - 72*B^2*a^11*b + 24*A^2*a^2*b^10 + 32*A^2*a^3*b^9 - 52*A^2*a^4*b^8 - 48*A^2*a^5*b^7 + 57*A^2*a^6*b^6 + 32*A^2*a^7*b^5 - 32*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 8*A^2*a^10*b^2 + 36*B^2*a^2*b^10 - 72*B^2*a^3*b^9 + 36*B^2*a^4*b^8 + 288*B^2*a^5*b^7 - 288*B^2*a^6*b^6 - 432*B^2*a^7*b^5 + 441*B^2*a^8*b^4 + 288*B^2*a^9*b^3 - 288*B^2*a^10*b^2 - 24*A*B*a*b^11 - 48*A*B*a^11*b + 48*A*B*a^2*b^10 - 72*A*B*a^3*b^9 - 192*A*B*a^4*b^8 + 252*A*B*a^5*b^7 + 288*A*B*a^6*b^6 - 318*A*B*a^7*b^5 - 192*A*B*a^8*b^4 + 192*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (a*((8*(4*A*b^18 - 8*A*a^2*b^16 + 34*A*a^3*b^15 + 6*A*a^4*b^14 - 36*A*a^5*b^13 - 4*A*a^6*b^12 + 18*A*a^7*b^11 + 2*A*a^8*b^10 - 4*A*a^9*b^9 + 24*B*a^2*b^16 + 36*B*a^3*b^15 - 78*B*a^4*b^14 - 42*B*a^5*b^13 + 96*B*a^6*b^12 + 24*B*a^7*b^11 - 54*B*a^8*b^10 - 6*B*a^9*b^9 + 12*B*a^10*b^8 - 12*A*a*b^17 - 12*B*a*b^17))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^1

1 - a^6*b^10 - a^7*b^9) + (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1
/2)*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^
4)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^
6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^14 - 5*a^
2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12 + b^13 -
3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*(-(a
 + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 +
2*A*a^4*b - 12*B*a*b^4))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 +
 5*a^8*b^6 - a^10*b^4)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 -
5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4)*1i)/(2*(b^14 - 5*a^2*b
^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)))/((16*(108*B^3*a^12
 - 12*A^3*a*b^11 - 54*B^3*a^11*b - 24*A^3*a^2*b^10 + 34*A^3*a^3*b^9 + 26*A^
3*a^4*b^8 - 36*A^3*a^5*b^7 - 13*A^3*a^6*b^6 + 18*A^3*a^7*b^5 + 2*A^3*a^8*b^
4 - 4*A^3*a^9*b^3 + 216*B^3*a^4*b^8 + 216*B^3*a^5*b^7 - 702*B^3*a^6*b^6 - 3
78*B^3*a^7*b^5 + 864*B^3*a^8*b^4 + 243*B^3*a^9*b^3 - 486*B^3*a^10*b^2 - 108
*A*B^2*a^11*b - 252*A*B^2*a^3*b^9 - 324*A*B^2*a^4*b^8 + 774*A*B^2*a^5*b^7 +
 486*A*B^2*a^6*b^6 - 900*A*B^2*a^7*b^5 - 279*A*B^2*a^8*b^4 + 486*A*B^2*a^9*
b^3 + 54*A*B^2*a^10*b^2 + 96*A^2*B*a^2*b^10 + 156*A^2*B*a^3*b^9 - 282*A^2*B
*a^4*b^8 - 198*A^2*B*a^5*b^7 + 312*A^2*B*a^6*b^6 + 105*A^2*B*a^7*b^5 - 162*
A^2*B*a^8*b^4 - 18*A^2*B*a^9*b^3 + 36*A^2*B*a^10*b^2))/(a*b^15 + b^16 - 3*a
^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (a*(
(8*tan(c/2 + (d*x)/2)*(4*A^2*b^12 + 72*B^2*a^12 - 8*A^2*a*b^11 - 72*B^2*a^1
1*b + 24*A^2*a^2*b^10 + 32*A^2*a^3*b^9 - 52*A^2*a^4*b^8 - 48*A^2*a^5*b^7 +
57*A^2*a^6*b^6 + 32*A^2*a^7*b^5 - 32*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 8*A^2*a^
10*b^2 + 36*B^2*a^2*b^10 - 72*B^2*a^3*b^9 + 36*B^2*a^4*b^8 + 288*B^2*a^5*b^
7 - 288*B^2*a^6*b^6 - 432*B^2*a^7*b^5 + 441*B^2*a^8*b^4 + 288*B^2*a^9*b^3 -
 288*B^2*a^10*b^2 - 24*A*B*a*b^11 - 48*A*B*a^11*b + 48*A*B*a^2*b^10 - 72*A*
B*a^3*b^9 - 192*A*B*a^4*b^8 + 252*A*B*a^5*b^7 + 288*A*B*a^6*b^6 - 318*A*B*a
^7*b^5 - 192*A*B*a^8*b^4 + 192*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a*b^12 + b^
13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) +
 (a*((8*(4*A*b^18 - 8*A*a^2*b^16 + 34*A*a^3*b^15 + 6*A*a^4*b^14 - 36*A*a^5*
b^13 - 4*A*a^6*b^12 + 18*A*a^7*b^11 + 2*A*a^8*b^10 - 4*A*a^9*b^9 + 24*B*a^2
*b^16 + 36*B*a^3*b^15 - 78*B*a^4*b^14 - 42*B*a^5*b^13 + 96*B*a^6*b^12 + 24*
B*a^7*b^11 - 54*B*a^8*b^10 - 6*B*a^9*b^9 + 12*B*a^10*b^8 - 12*A*a*b^17 - 12
*B*a*b^17))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b
^11 - a^6*b^10 - a^7*b^9) - (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^
(1/2)*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*
b^4)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*
a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^14 - 5*
a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12 + b^13
- 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*(-
(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2
+ 2*A*a^4*b - 12*B*a*b^4))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8
 + 5*a^8*b^6 - a^10*b^4)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5
- 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4))/(2*(b^14 - 5*a^2*b^

```
12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)) - (a*((8*tan(c/2 + (
d*x)/2)*(4*A^2*b^12 + 72*B^2*a^12 - 8*A^2*a*b^11 - 72*B^2*a^11*b + 24*A^2*a
^2*b^10 + 32*A^2*a^3*b^9 - 52*A^2*a^4*b^8 - 48*A^2*a^5*b^7 + 57*A^2*a^6*b^6
 + 32*A^2*a^7*b^5 - 32*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 8*A^2*a^10*b^2 + 36*B^
2*a^2*b^10 - 72*B^2*a^3*b^9 + 36*B^2*a^4*b^8 + 288*B^2*a^5*b^7 - 288*B^2*a^
6*b^6 - 432*B^2*a^7*b^5 + 441*B^2*a^8*b^4 + 288*B^2*a^9*b^3 - 288*B^2*a^10*
b^2 - 24*A*B*a*b^11 - 48*A*B*a^11*b + 48*A*B*a^2*b^10 - 72*A*B*a^3*b^9 - 19
2*A*B*a^4*b^8 + 252*A*B*a^5*b^7 + 288*A*B*a^6*b^6 - 318*A*B*a^7*b^5 - 192*A
*B*a^8*b^4 + 192*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^1
1 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (a*((8*(4*A*b
^18 - 8*A*a^2*b^16 + 34*A*a^3*b^15 + 6*A*a^4*b^14 - 36*A*a^5*b^13 - 4*A*a^6
*b^12 + 18*A*a^7*b^11 + 2*A*a^8*b^10 - 4*A*a^9*b^9 + 24*B*a^2*b^16 + 36*B*a
^3*b^15 - 78*B*a^4*b^14 - 42*B*a^5*b^13 + 96*B*a^6*b^12 + 24*B*a^7*b^11 - 5
4*B*a^8*b^10 - 6*B*a^9*b^9 + 12*B*a^10*b^8 - 12*A*a*b^17 - 12*B*a*b^17))/(a
*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10
 - a^7*b^9) + (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5
 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4)*(8*a*b^17
 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*
a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/((b^14 - 5*a^2*b^12 + 10*
a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4)*(a*b^12 + b^13 - 3*a^2*b^11 -
 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*(-(a + b)^5*(a -
 b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b -
12*B*a*b^4))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 -
 a^10*b^4)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 5*A*a^2*b^3
+ 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4))/(2*(b^14 - 5*a^2*b^12 + 10*a^4*b^
10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4))))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*
A*b^5 - 6*B*a^5 - 5*A*a^2*b^3 + 15*B*a^3*b^2 + 2*A*a^4*b - 12*B*a*b^4)*1i)/
(d*(b^14 - 5*a^2*b^12 + 10*a^4*b^10 - 10*a^6*b^8 + 5*a^8*b^6 - a^10*b^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.267 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=211

$$-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{a\left(-3a^3B+a^2Ab+6ab^2B-4Ab^3\right)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{\left(-2a^5B+5a^3b^2B+a^2Ab^3\right.}{b^3}$$

```
[Out] B*x/b^3+(A*a^2*b^3+2*A*b^5-2*B*a^5+5*B*a^3*b^2-6*B*a*b^4)*arctan((a-b)^(1/2
)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^3/(a+b)^(5/2)/d-1/2*a^2*(A*
b-B*a)*sin(d*x+c)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*a*(A*a^2*b-4*A*b^3
-3*B*a^3+6*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.56, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2988, 3021, 2735, 2659, 205}

$$\frac{\left(a^2Ab^3+5a^3b^2B-2a^5B-6ab^4B+2Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{a\left(a^2Ab\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (B*x)/b^3 + ((a^2*A*b^3 + 2*A*b^5 - 2*a^5*B + 5*a^3*b^2*B - 6*a*b^4*B)*ArcT
an[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^
(5/2)*d) - (a^2*(A*b - a*B)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) + (a*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sin[c + d*x])/(2
*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{2ab(Ab-aB)+\left(a^2-2b^2\right)(Ab-aB)\cos(c+d}{(a+b\cos(c+dx)}}{2b^2\left(a^2-b\right.}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2Ab-4Ab^3-3a^3B+6ab^2B\right.}{2b^2\left(a^2-b^2\right)^2d(a+b\cos(c\right.}$$

$$= \frac{Bx}{b^3} - \frac{a^2(Ab-aB)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2Ab-4Ab^3-3a^3B+6\right.}{2b^2\left(a^2-b^2\right)^2d(a+b\,c}$$

$$= \frac{Bx}{b^3} - \frac{a^2(Ab-aB)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2Ab-4Ab^3-3a^3B+6\right.}{2b^2\left(a^2-b^2\right)^2d(a+b\,c}$$

$$= \frac{Bx}{b^3} + \frac{\left(a^2Ab^3+2Ab^5-2a^5B+5a^3b^2B-6ab^4B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right.}{(a-b)^{5/2}b^3(a+b)^{5/2}d}$$

Mathematica [A] time = 1.36, size = 204, normalized size = 0.97

$$\frac{\frac{a^2b(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2} + \frac{ab\left(-3a^3B+a^2Ab+6ab^2B-4Ab^3\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} + \frac{2\left(2a^5B-5a^3b^2B-a^2Ab^3+6ab^4B-2Ab^5\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + 2}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*B*(c + d*x) + (2*(-(a^2*A*b^3) - 2*A*b^5 + 2*a^5*B - 5*a^3*b^2*B + 6*a*b
^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5
/2) + (a^2*b*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d
*x])^2) + (a*b*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sin[c + d*x])/((a
- b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*b^3*d)
```

fricas [B] time = 0.89, size = 1152, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(4*(B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8)*d*x*cos(d*x + c)^2
+ 8*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*d*x*cos(d*x + c) + 4*(B
*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*d*x + (2*B*a^7 - 5*B*a^5*b^2
- A*a^4*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5 + (2*B*a^5*b^2 - 5*B*a^3*b^4 - A*a^
2*b^5 + 6*B*a*b^6 - 2*A*b^7)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 5*B*a^4*b^3 -
A*a^3*b^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*
a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos
(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(
d*x + c) + a^2)) - 2*(2*B*a^7*b - 7*B*a^5*b^3 + 3*A*a^4*b^4 + 5*B*a^3*b^5 -
 3*A*a^2*b^6 + (3*B*a^6*b^2 - A*a^5*b^3 - 9*B*a^4*b^4 + 5*A*a^3*b^5 + 6*B*a
^2*b^6 - 4*A*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^6*b^5 - 3*a^4*b^7 + 3*a
^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^
10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d), 1/2*(2
*(B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8)*d*x*cos(d*x + c)^2 + 4*(B*
a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*d*x*cos(d*x + c) + 2*(B*a^8 -
3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*d*x - (2*B*a^7 - 5*B*a^5*b^2 - A*a^4
*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5 + (2*B*a^5*b^2 - 5*B*a^3*b^4 - A*a^2*b^5 +
 6*B*a*b^6 - 2*A*b^7)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 5*B*a^4*b^3 - A*a^3*b
^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(
d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*B*a^7*b - 7*B*a^5*b^3 +
3*A*a^4*b^4 + 5*B*a^3*b^5 - 3*A*a^2*b^6 + (3*B*a^6*b^2 - A*a^5*b^3 - 9*B*a^
4*b^4 + 5*A*a^3*b^5 + 6*B*a^2*b^6 - 4*A*a*b^7)*cos(d*x + c))*sin(d*x + c))/
((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3
*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^
4*b^7 - a^2*b^9)*d)]
```

giac [B] time = 1.21, size = 455, normalized size = 2.16

$$-\frac{\frac{\left(2\,Ba^{5}-5\,Ba^{3}b^{2}-Aa^{2}b^{3}+6\,Bab^{4}-2\,Ab^{5}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{4}b^{3}-2\,a^{2}b^{5}+b^{7}\right)\sqrt{a^{2}-b^{2}}}-\frac{(dx+c)B}{b^{3}}+\frac{2\,Ba^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -((2*B*a^5 - 5*B*a^3*b^2 - A*a^2*b^3 + 6*B*a*b^4 - 2*A*b^5)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(
1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 -
 b^2)) - (d*x + c)*B/b^3 + (2*B*a^5*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^4*b*tan(
```

```
1/2*d*x + 1/2*c)^3 + A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^3*b^2*tan(1/2
*d*x + 1/2*c)^3 + 3*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*b^3*tan(1/2*
d*x + 1/2*c)^3 - 4*A*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^5*tan(1/2*d*x + 1
/2*c) + 3*B*a^4*b*tan(1/2*d*x + 1/2*c) - A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5
*B*a^3*b^2*tan(1/2*d*x + 1/2*c) + 3*A*a^2*b^3*tan(1/2*d*x + 1/2*c) - 6*B*a^
2*b^3*tan(1/2*d*x + 1/2*c) + 4*A*a*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*
a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b
)^2))/d
```

maple [**B**] time = 0.10, size = 1023, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -1/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d*a^4/b^
2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^
2)*tan(1/2*d*x+1/2*c)^3*B+1/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+6/d/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*B+1/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-4/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2*a/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-2/d*a^4/b^2/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d
*x+1/2*c)*B-1/d*a^3/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2
/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)^2*a^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d*a^2/(a^4-2*a^
2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)
)^(1/2))*A+2/d*b^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d
*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a-
b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+5/d*
a^3/b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-
b)/((a-b)*(a+b))^(1/2))*B-6/d*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arc
tan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*a+2/d/b^3*arctan(tan(1/
2*d*x+1/2*c))*B
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.95, size = 6923, normalized size = 32.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] (2*B*atan(-((B*((B*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^12 +
 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11 -
36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*
A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b
^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (B*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a
^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^
9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)*8i)/(b^3*(a*b^10 + b^11 - 3*a^2*b^
9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*1i)/b^3 + (8*t
an(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 - 8*B
^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 -
52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*
a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(a
*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^
7*b^4)))/b^3 - (B*((B*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^1
2 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11
 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 -
 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^
4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (B*tan(c/2 + (d*x)/2)*(8*a*b^15 -
8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7
*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)*8i)/(b^3*(a*b^10 + b^11 - 3*a^2
*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*1i)/b^3 - (
8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 -
8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7
 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B
^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3))
/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 -
 a^7*b^4)))/b^3)/((B*((B*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*
b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b
^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^
6 - 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3
```

*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (B*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)*8i)/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*1i)/b^3 + (8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3 - (16*(4*B^3*a^9 - 4*A*B^2*b^9 + 4*A^2*B*b^9 + 12*B^3*a*b^8 - 2*B^3*a^8*b + 24*B^3*a^2*b^7 - 34*B^3*a^3*b^6 - 26*B^3*a^4*b^5 + 36*B^3*a^5*b^4 + 13*B^3*a^6*b^3 - 18*B^3*a^7*b^2 - 20*A*B^2*a*b^8 + 6*A*B^2*a^2*b^7 + 2*A*B^2*a^3*b^6 + 2*A*B^2*a^5*b^4 - 2*A*B^2*a^6*b^3 - 2*A*B^2*a^7*b^2 + 4*A^2*B*a^2*b^7 + A^2*B*a^4*b^5))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (B*((B*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (B*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)*8i)/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*1i)/b^3 - (8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3)))/(b^3*d) - ((tan(c/2 + (d*x)/2)^3*(2*B*a^4 + A*a^2*b^2 - 6*B*a^2*b^2 + 4*A*a*b^3 - B*a^3*b))/((a*b^2 - b^3)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*B*a^4 - A*a^2*b^2 - 6*B*a^2*b^2 + 4*A*a*b^3 + B*a^3*b))/((a + b)*(b^4 - 2*a*b^3 + a^2*b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (atan((((-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (4*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b

^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/((b^
13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)*(a*b^10 +
b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))
)*(2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4))/(2*(b^13 - 5*a
^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*A*b^5 - 2*B*
a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4)*1i)/(2*(b^13 - 5*a^2*b^11 + 10*a
^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)) + ((-(a + b)^5*(a - b)^5)^(1/2
)*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^
9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3
*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 -
32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b
^3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b
^5 - a^7*b^4) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*A*b^15 + 4*B*b^15 - 6*
A*a^2*b^13 + 6*A*a^3*b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B
*a^3*b^12 + 6*B*a^4*b^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B
*a^8*b^7 - 4*B*a^9*b^6 - 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*
b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (4*tan(c/2
+ (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5
*B*a^3*b^2 - 6*B*a*b^4)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12
+ 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*
b^6))/((b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)
*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 -
a^7*b^4)))*(2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4))/(2*(
b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*A*
b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4)*1i)/(2*(b^13 - 5*a^2*b
^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))/((16*(4*B^3*a^9 - 4
*A*B^2*b^9 + 4*A^2*B*b^9 + 12*B^3*a*b^8 - 2*B^3*a^8*b + 24*B^3*a^2*b^7 - 34
*B^3*a^3*b^6 - 26*B^3*a^4*b^5 + 36*B^3*a^5*b^4 + 13*B^3*a^6*b^3 - 18*B^3*a^
7*b^2 - 20*A*B^2*a*b^8 + 6*A*B^2*a^2*b^7 + 2*A*B^2*a^3*b^6 + 2*A*B^2*a^5*b^
4 - 2*A*B^2*a^6*b^3 - 2*A*B^2*a^7*b^2 + 4*A^2*B*a^2*b^7 + A^2*B*a^4*b^5))/(
a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 -
a^7*b^6) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^1
0 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A
^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*
b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A
*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3
*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) + ((-(a + b)^5*(a - b
)^5)^(1/2)*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^12 + 2*A*a^6
*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11 - 36*B*a^5
*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*A*a*b^14
- 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a
^5*b^8 - a^6*b^7 - a^7*b^6) - (4*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^
(1/2)*(2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4)*(8*a*b^15 -
8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^
7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/((b^13 - 5*a^2*b^11 + 10*a^4*

```
b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3
*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*(2*A*b^5 - 2*B*a^5 + A*
a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10
*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3
*b^2 - 6*B*a*b^4))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*
b^5 - a^10*b^3)) + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(4*
A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^2*
b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48*B
^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^
9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(a*b^10 + b^11 - 3*a^2*
b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4) - ((-(a + b)^5
*(a - b)^5)^(1/2)*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^12 +
2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11 - 3
6*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*A
*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^
9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (4*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a -
 b)^5)^(1/2)*(2*A*b^5 - 2*B*a^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4)*(8*a
*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10
- 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/((b^13 - 5*a^2*b^11 +
10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)*(a*b^10 + b^11 - 3*a^2*b^9
- 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*(2*A*b^5 - 2*B*a
^5 + A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b
^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3)))*(2*A*b^5 - 2*B*a^5 + A*a^2*b^3 +
5*B*a^3*b^2 - 6*B*a*b^4))/(2*(b^13 - 5*a^2*b^11 + 10*a^4*b^9 - 10*a^6*b^7 +
 5*a^8*b^5 - a^10*b^3))))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*A*b^5 - 2*B*a^5 +
 A*a^2*b^3 + 5*B*a^3*b^2 - 6*B*a*b^4)*1i)/(d*(b^13 - 5*a^2*b^11 + 10*a^4*b^
9 - 10*a^6*b^7 + 5*a^8*b^5 - a^10*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.268 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{\left(a^3B+a^2Ab-4ab^2B+\right.}{2bd\left(a^2-b^2\right)^2(a+}$$

```
[Out] -(3*A*a*b-B*a^2-2*B*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))
/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*co
s(d*x+c))^2+1/2*(A*a^2*b+2*A*b^3+B*a^3-4*B*a*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/
d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.29, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3021, 2754, 12, 2659, 205}

$$-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(a^2Ab+a^3B-4ab^2B+2Ab^3\right)\sin(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{a(Ab-a}{2bd\left(a^2-b^2\right)(}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -(((3*a*A*b - a^2*B - 2*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) + (a*(A*b - a*B)*Sin[c + d*x])/(2*
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((a^2*A*b + 2*A*b^3 + a^3*B - 4*a
*b^2*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{2b(Ab-aB)-\left(aAb+a^2B-2b^2B\right)\cos(c+dx)}{(a+b\cos(c+dx))^2} d}{2b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)\sin}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)\sin}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2Ab+2Ab^3+a^3B-4ab^2B\right)\sin}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= -\frac{\left(3aAb-a^2B-2b^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a(Ab-aB)\sin}{2b\left(a^2-b^2\right)d(a+b}$$

Mathematica [A] time = 0.86, size = 172, normalized size = 0.96

$$\frac{-\frac{2\left(a^2B-3aAb+2b^2B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{\left(a^3B+a^2Ab-4ab^2B+2Ab^3\right)\sin(c+dx)}{b(a-b)^2(a+b)^2(a+b\cos(c+dx))} + \frac{a(Ab-aB)\sin(c+dx)}{b(a-b)(a+b)(a+b\cos(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(-3*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]])/(-a^2 + b^2)^(5/2) + (a*(A*b - a*B)*Sin[c + d*x])/((a - b)*b*(
a + b)*(a + b*Cos[c + d*x])^2) + ((a^2*A*b + 2*A*b^3 + a^3*B - 4*a*b^2*B)*S
in[c + d*x])/((a - b)^2*b*(a + b)^2*(a + b*Cos[c + d*x])))/(2*d)
```

fricas [B] time = 0.78, size = 740, normalized size = 4.11

$$\left[-\frac{\left(Ba^4-3\,Aa^3b+2\,Ba^2b^2+\left(Ba^2b^2-3\,Aab^3+2\,Bb^4\right)\cos\left(dx+c\right)^2+2\left(Ba^3b-3\,Aa^2b^2+2\,Bab^3\right)\cos\left(dx+c\right.}{4\left(\left(a^6b^2-3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] [-1/4*((B*a^4 - 3*A*a^3*b + 2*B*a^2*b^2 + (B*a^2*b^2 - 3*A*a*b^3 + 2*B*b^4)
*cos(d*x + c)^2 + 2*(B*a^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqrt(
-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*A*a^5 - 3*B*a^4*b - A*a^3*b^2 +
3*B*a^2*b^3 - A*a*b^4 + (B*a^5 + A*a^4*b - 5*B*a^3*b^2 + A*a^2*b^3 + 4*B*a*
b^4 - 2*A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos
(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), 1/2*((B*a^4 - 3*A*a
^3*b + 2*B*a^2*b^2 + (B*a^2*b^2 - 3*A*a*b^3 + 2*B*b^4)*cos(d*x + c)^2 + 2*(
B*a^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a
*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (2*A*a^5 - 3*B*a^4*b -
 A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 + (B*a^5 + A*a^4*b - 5*B*a^3*b^2 + A*a^2
*b^3 + 4*B*a*b^4 - 2*A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

giac [B] time = 0.71, size = 391, normalized size = 2.17

$$\frac{\frac{\left(Ba^2-3\,Aab+2\,Bb^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{2\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa^2b\tan\left(\frac{1}{2}\,dx\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] ((B*a^2 - 3*A*a*b + 2*B*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*
b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^
2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (2*A*a^3*tan(1/2*d*x + 1/2
*c)^3 - B*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*B
*a^2*b*tan(1/2*d*x + 1/2*c)^3 + A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*B*a*b^2*
tan(1/2*d*x + 1/2*c)^3 - 2*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*A*a^3*tan(1/2*d
*x + 1/2*c) + B*a^3*tan(1/2*d*x + 1/2*c) + A*a^2*b*tan(1/2*d*x + 1/2*c) - 3
*B*a^2*b*tan(1/2*d*x + 1/2*c) + A*a*b^2*tan(1/2*d*x + 1/2*c) - 4*B*a*b^2*ta
n(1/2*d*x + 1/2*c) + 2*A*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

maple [B] time = 0.08, size = 886, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d*a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+1/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+2/d/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3*A*b^2-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^2*a^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-4/d/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c
)^3*B*a*b+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(
a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a^2*A-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A*a*b+2/d/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*
tan(1/2*d*x+1/2*c)*A*b^2+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*a^2*B-4/d/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2
*c)*B*a*b-3/d*a*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*
x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+1/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a
+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+2/d/(a^4-
2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(
a+b))^(1/2))*b^2*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 3.74, size = 248, normalized size = 1.38

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,A\,a^2+2\,A\,b^2-B\,a^2+A\,a\,b-4\,B\,a\,b\right)}{{\left(a+b\right)}^2\,\left(a-b\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A\,a^2+2\,A\,b^2+B\,a^2-A\,a\,b-4\,B\,a\,b\right)}{\left(a+b\right)\,\left(a^2-2\,a\,b+b^2\right)}}{d\,\left(2\,a\,b+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-2\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^2-2\,a\,b+b^2\right)+a^2+b^2\right)}+\frac{\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^2-2\right.}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{5/2}}\right.}{d\,\left(a+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(2*A*a^2 + 2*A*b^2 - B*a^2 + A*a*b - 4*B*a*b))/((a +
 b)^2*(a - b)) + (tan(c/2 + (d*x)/2)*(2*A*a^2 + 2*A*b^2 + B*a^2 - A*a*b - 4
*B*a*b))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2
*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (a
tan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2)*(
a - b)^(5/2)))*(B*a^2 + 2*B*b^2 - 3*A*a*b))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.269 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=164

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{(Ab-aB)\sin(c+d}{2d\left(a^2-b^2\right)(a+b\cos(c}$$

```
[Out] (2*A*a^2+A*b^2-3*B*a*b)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/
(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x
+c))^2-1/2*(3*A*a*b-B*a^2-2*B*b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c)
)
```

Rubi [A] time = 0.19, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2754, 12, 2659, 205}

$$\frac{\left(2a^2A-3abB+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{(Ab-aB)\sin(c+d}{2d\left(a^2-b^2\right)(a+b\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
 b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((A*b - a*B)*Sin[c + d*x])/(2*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^2) - ((3*a*A*b - a^2*B - 2*b^2*B)*Sin[c + d*x
])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{(Ab-aB)\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{-2(aA-bB)+(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(3aAb-a^2B-2b^2B\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{2a^2A+Ab^2}{a+b\cos(c}\cdots}{2\left(a^2-\cdots\right.}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(3aAb-a^2B-2b^2B\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\left(2a^2A+A\cdots\right.}{\cdots}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(3aAb-a^2B-2b^2B\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\left(2a^2A+A\cdots\right.}{\cdots}$$

$$= \frac{\left(2a^2A+Ab^2-3abB\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{(Ab-aB)\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \cdots$$

Mathematica [A] time = 0.67, size = 157, normalized size = 0.96

$$\frac{\frac{\left(a^2B-3aAb+2b^2B\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} - \frac{2\left(2a^2A-3abB+Ab^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a
^2 + b^2]])/(-a^2 + b^2)^(5/2) + ((-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a
+ b)*(a + b*Cos[c + d*x])^2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sin[c + d*x])/
((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*d)
```

fricas [B] time = 0.68, size = 742, normalized size = 4.52

$$\left[-\frac{\left(2\,Aa^4 - 3\,Ba^3b + Aa^2b^2 + \left(2\,Aa^2b^2 - 3\,Bab^3 + Ab^4\right)\cos\left(dx+c\right)^2 + 2\left(2\,Aa^3b - 3\,Ba^2b^2 + Aab^3\right)\cos\left(dx+\right.\right.}{4\left(\left(a^6b^2 - 3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*((2*A*a^4 - 3*B*a^3*b + A*a^2*b^2 + (2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)
*cos(d*x + c)^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(
-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt
(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*B*a^5 - 4*A*a^4*b - B*a^3*b^2 +
5*A*a^2*b^3 - B*a*b^4 - A*b^5 + (B*a^4*b - 3*A*a^3*b^2 + B*a^2*b^3 + 3*A*a*
b^4 - 2*B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^
6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos
(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), 1/2*((2*A*a^4 - 3*B
*a^3*b + A*a^2*b^2 + (2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*cos(d*x + c)^2 + 2*(
2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a
*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (2*B*a^5 - 4*A*a^4*b -
 B*a^3*b^2 + 5*A*a^2*b^3 - B*a*b^4 - A*b^5 + (B*a^4*b - 3*A*a^3*b^2 + B*a^2
*b^3 + 3*A*a*b^4 - 2*B*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
 a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

giac [B] time = 0.79, size = 390, normalized size = 2.38

$$\frac{\left(2\,Aa^2-3\,Bab+Ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,a-2\,b\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{2\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^2b\tan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*A*a^2 - 3*B*a*b + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*
b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^
2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (2*B*a^3*tan(1/2*d*x + 1/2
*c)^3 - 4*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)^3 +
 3*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + A*b^3*
tan(1/2*d*x + 1/2*c)^3 - 2*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d
*x + 1/2*c) - 4*A*a^2*b*tan(1/2*d*x + 1/2*c) + B*a^2*b*tan(1/2*d*x + 1/2*c)
 - 3*A*a*b^2*tan(1/2*d*x + 1/2*c) + B*a*b^2*tan(1/2*d*x + 1/2*c) + A*b^3*ta
n(1/2*d*x + 1/2*c) + 2*B*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

maple [B] time = 0.07, size = 886, normalized size = 5.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -4/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^2+2/d/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a^2/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3*B+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B*a*b+2/d/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3*b^2*B-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/
(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A*a*b+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*A*b^2+2/d/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)
*tan(1/2*d*x+1/2*c)*a^2*B-1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*B*a*b+2/d/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/
2*c)*b^2*B+2/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d
*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+1/d*b^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(
a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-3/d*b/(a
^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b
)*(a+b))^(1/2))*B*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 3.54, size = 248, normalized size = 1.51

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,B\,a^2-A\,b^2+2\,B\,b^2-4\,A\,a\,b+B\,a\,b\right)}{{\left(a+b\right)}^2\,\left(a-b\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A\,b^2+2\,B\,a^2+2\,B\,b^2-4\,A\,a\,b-B\,a\,b\right)}{\left(a+b\right)\,\left(a^2-2\,a\,b+b^2\right)}}{d\,\left(2\,a\,b+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-2\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^2-2\,a\,b+b^2\right)+a^2+b^2\right)}+\frac{\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^2-2\right.}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{5/2}}\right.}{d\,\left(a+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(2*B*a^2 - A*b^2 + 2*B*b^2 - 4*A*a*b + B*a*b))/((a +
 b)^2*(a - b)) + (tan(c/2 + (d*x)/2)*(A*b^2 + 2*B*a^2 + 2*B*b^2 - 4*A*a*b -
 B*a*b))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2
*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (a
tan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2)*(
a - b)^(5/2)))*(2*A*a^2 + A*b^2 - 3*B*a*b))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.270 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=214

$$\frac{A\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{b\left(-3a^3B+5a^2Ab-2Ab^3\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{\left(-2a^5B+6a\cdots\right.}{\cdots}$$

```
[Out] -(6*A*a^4*b-5*A*a^2*b^3+2*A*b^5-2*B*a^5-B*a^3*b^2)*arctan((a-b)^(1/2)*tan(1
/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+A*arctanh(sin(d*x+
c))/a^3/d+1/2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*b
*(5*A*a^2*b-2*A*b^3-3*B*a^3)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.71, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{\left(-5a^2Ab^3+6a^4Ab-a^3b^2B-2a^5B+2Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b\left(5a^2Ab-3a^3B-2Ab^3\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -(((6*a^4*A*b - 5*a^2*A*b^3 + 2*A*b^5 - 2*a^5*B - a^3*b^2*B)*ArcTan[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d))
+ (A*ArcTanh[Sin[c + d*x]])/(a^3*d) + (b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^
2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b*(5*a^2*A*b - 2*A*b^3 - 3*a^3*B)*Sin
[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^3}dx = \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2A\left(a^2-b^2\right)-2a(Ab-aB)\cos(c+dx)+b(Ab-a}{(a+b\cos(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(5a^2Ab-2Ab^3-3a^3B\right)\sin(c+d}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(5a^2Ab-2Ab^3-3a^3B\right)\sin(c+d}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(5a^2Ab\right.}{2a^2\left(a^2\right.}$$

$$= -\frac{\left(6a^4Ab-5a^2Ab^3+2Ab^5-2a^5B-a^3b^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} +$$

Mathematica [A] time = 1.33, size = 269, normalized size = 1.26

$$\frac{\cos(c+dx)(A\sec(c+dx)+B)\left(\frac{a^2b(Ab-aB)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2} + \frac{ab\left(-3a^3B+5a^2Ab-2Ab^3\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} - \frac{2\left(2a^5B-6a^4Ab+a^3b^2B+5a^2Ab^3-2\right.}{\left(b^2-a^2\right.}\right.}{2a^3d(A+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*((-2*(-6*a^4*A*b + 5*a^2*A*b^3 - 2*A*b^5
 + 2*a^5*B + a^3*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])/(-a^2 + b^2)^(5/2) - 2*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*A*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^2*b*(A*b - a*B)*Sin[c + d*x])
/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a*b*(5*a^2*A*b - 2*A*b^3 - 3*a
^3*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x]))))/(2*a^3*d*(
A + B*Cos[c + d*x]))
```

fricas [B] time = 37.51, size = 1400, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] [1/4*((2*B*a^7 - 6*A*a^6*b + B*a^5*b^2 + 5*A*a^4*b^3 - 2*A*a^2*b^5 + (2*B*a
^5*b^2 - 6*A*a^4*b^3 + B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7)*cos(d*x + c)^2 +
2*(2*B*a^6*b - 6*A*a^5*b^2 + B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*x +
 c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^
2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(A*a^8 - 3*A*a^6*b^2 + 3*
A*a^4*b^4 - A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*cos
(d*x + c)^2 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*cos(d*x + c
))*log(sin(d*x + c) + 1) - 2*(A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6
 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*cos(d*x + c)^2 + 2*(A*a^
7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*cos(d*x + c))*log(-sin(d*x + c)
+ 1) - 2*(4*B*a^7*b - 6*A*a^6*b^2 - 5*B*a^5*b^3 + 9*A*a^4*b^4 + B*a^3*b^5 -
 3*A*a^2*b^6 + (3*B*a^6*b^2 - 5*A*a^5*b^3 - 3*B*a^4*b^4 + 7*A*a^3*b^5 - 2*A
*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3
*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos
(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d), 1/2*((2*B*a^7 - 6*
A*a^6*b + B*a^5*b^2 + 5*A*a^4*b^3 - 2*A*a^2*b^5 + (2*B*a^5*b^2 - 6*A*a^4*b^
3 + B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 6*A*
a^5*b^2 + B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2
)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (A*a^8 - 3
*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b
^6 - A*b^8)*cos(d*x + c)^2 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b
^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - (A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^
4 - A*a^2*b^6 + (A*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*cos(d*x + c
)^2 + 2*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*cos(d*x + c))*log(-
sin(d*x + c) + 1) - (4*B*a^7*b - 6*A*a^6*b^2 - 5*B*a^5*b^3 + 9*A*a^4*b^4 +
B*a^3*b^5 - 3*A*a^2*b^6 + (3*B*a^6*b^2 - 5*A*a^5*b^3 - 3*B*a^4*b^4 + 7*A*a^
3*b^5 - 2*A*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3*a^
5*b^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4
*b^7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d)]
```

giac [B] time = 1.77, size = 481, normalized size = 2.25

$$\frac{\frac{\left(2\,Ba^5-6\,Aa^4b+Ba^3b^2+5\,Aa^2b^3-2\,Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{a^2-b^2}}+\frac{A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{A\log\left(\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] ((2*B*a^5 - 6*A*a^4*b + B*a^3*b^2 + 5*A*a^2*b^3 - 2*A*b^5)*(pi*floor(1/2*(d
*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 -
b^2)) + A*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - A*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^3 - (4*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^3*b^2*tan(1/2*
d*x + 1/2*c)^3 - 3*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^2*b^3*tan(1/2*d
*x + 1/2*c)^3 - B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 3*A*a*b^4*tan(1/2*d*x +
1/2*c)^3 - 2*A*b^5*tan(1/2*d*x + 1/2*c)^3 + 4*B*a^4*b*tan(1/2*d*x + 1/2*c)
- 6*A*a^3*b^2*tan(1/2*d*x + 1/2*c) + 3*B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*A
*a^2*b^3*tan(1/2*d*x + 1/2*c) - B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*A*a*b^4*
tan(1/2*d*x + 1/2*c) + 2*A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^
2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

maple [B] time = 0.16, size = 1045, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3*A*b^2+1/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^2*b^3/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d/a^2/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^4/(a-b)/(a^2+2*a*b+b
^2)*tan(1/2*d*x+1/2*c)^3*A-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B*a*b-1/d/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3*b^2*B+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b
^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-1/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)^2*b^3/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-2/d/a^2/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^4/(a+b)/(a-b)^2*tan(1/2*
d*x+1/2*c)*A-4/d*a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b/
(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)^2*b^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-6/d*a*b/(a^4-2*a^2
*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))
^(1/2))*A+5/d/a/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+
1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^3-2/d/a^3/(a^4-2*a^2*b^2+b^4)/((a-b)*
(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^5+2/d
*a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b
)/((a-b)*(a+b))^(1/2))*B+1/d/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan
(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*b^2*B-1/d/a^3*A*ln(tan(1/2*d
*x+1/2*c)-1)+1/d/a^3*A*ln(tan(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 9.63, size = 6913, normalized size = 32.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(2*A*b^4 - 6*A*a^2*b^2 + B*a^2*b^2 - A*a*b^3 + 4*B*a
^3*b))/((a^2*b - a^3)*(a + b)^2) - (tan(c/2 + (d*x)/2)*(2*A*b^4 - 6*A*a^2*b
^2 - B*a^2*b^2 + A*a*b^3 + 4*B*a^3*b))/((a + b)*(a^4 - 2*a^3*b + a^2*b^2)))
/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a
^2 - 2*a*b + b^2) + a^2 + b^2)) - (A*atan(((A*((8*tan(c/2 + (d*x)/2)*(4*A^2
*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^
8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*
A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b -
 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3))/(a^10*b + a^11 - a^4*b^7 -
 a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) + (A*((8*(4*A*a^1
5 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*
A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3 - 8*A*a^13*b^2 - 2*B*a^8*b^7 + 2*
B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 - 12*A*a^14*b - 4*B*a^14*b))/(a^12*
b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*
b^2) + (8*A*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*
b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 -
8*a^14*b^2))/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^
4 - 3*a^8*b^3 - 3*a^9*b^2))))/a^3)*1i)/a^3 + (A*((8*tan(c/2 + (d*x)/2)*(4*A
^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*
b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 3
2*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b
 - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3))/(a^10*b + a^11 - a^4*b^7
 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) - (A*((8*(4*A*a
^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9*b^6 - 3
```

6*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3 - 8*A*a^13*b^2 - 2*B*a^8*b^7 +
2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 - 12*A*a^14*b - 4*B*a^14*b))/(a^1
2*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^1
1*b^2) - (8*A*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^
8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3
- 8*a^14*b^2))/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*
b^4 - 3*a^8*b^3 - 3*a^9*b^2))))/a^3)*1i)/a^3)/((16*(4*A^3*b^9 + 4*A*B^2*a^9
 - 4*A^2*B*a^9 - 2*A^3*a*b^8 + 12*A^3*a^8*b - 18*A^3*a^2*b^7 + 13*A^3*a^3*b
^6 + 36*A^3*a^4*b^5 - 26*A^3*a^5*b^4 - 34*A^3*a^6*b^3 + 24*A^3*a^7*b^2 - 20
*A^2*B*a^8*b + A*B^2*a^5*b^4 + 4*A*B^2*a^7*b^2 - 2*A^2*B*a^2*b^7 - 2*A^2*B*
a^3*b^6 + 2*A^2*B*a^4*b^5 + 2*A^2*B*a^6*b^3 + 6*A^2*B*a^7*b^2))/(a^12*b + a
^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)
+ (A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a
*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*
A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^
4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^
7*b^3))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*
b^3 - 3*a^9*b^2) + (A*((8*(4*A*a^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8
+ 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3
 - 8*A*a^13*b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 -
 12*A*a^14*b - 4*B*a^14*b))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5
+ 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (8*A*tan(c/2 + (d*x)/2)*(8*a^15*b
- 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*
b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/(a^3*(a^10*b + a^11 - a^4*b^
7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))))/a^3))/a^3 -
 (A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*
b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A
^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4
 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7
*b^3))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b
^3 - 3*a^9*b^2) - (A*((8*(4*A*a^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 +
 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3
- 8*A*a^13*b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 -
12*A*a^14*b - 4*B*a^14*b))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 +
 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (8*A*tan(c/2 + (d*x)/2)*(8*a^15*b -
 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b
^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/(a^3*(a^10*b + a^11 - a^4*b^7
 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))))/a^3))/a^3))*
2i)/(a^3*d) - (atan((((-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(
4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a
^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4
+ 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^
9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3))/(a^10*b + a^11 - a^4*
b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) - ((-(a + b)
^5*(a - b)^5)^(1/2)*((8*(4*A*a^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 +

18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3 - 8*A*a^13*b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 - 12*A*a^14*b - 4*B*a^14*b))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (4*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6*A*a^4*b)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/((a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*(2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6*A*a^4*b))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*(2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6*A*a^4*b)*1i)/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)) + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*A*a^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3 - 8*A*a^13*b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 - 12*A*a^14*b - 4*B*a^14*b))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (4*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6*A*a^4*b)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/((a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*(2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6*A*a^4*b))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))*(2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6*A*a^4*b)*1i)/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))/((16*(4*A^3*b^9 + 4*A*B^2*a^9 - 4*A^2*B*a^9 - 2*A^3*a*b^8 + 12*A^3*a^8*b - 18*A^3*a^2*b^7 + 13*A^3*a^3*b^6 + 36*A^3*a^4*b^5 - 26*A^3*a^5*b^4 - 34*A^3*a^6*b^3 + 24*A^3*a^7*b^2 - 20*A^2*B*a^8*b + A*B^2*a^5*b^4 + 4*A*B^2*a^7*b^2 - 2*A^2*B*a^2*b^7 - 2*A^2*B*a^3*b^6 + 2*A^2*B*a^4*b^5 + 2*A^2*B*a^6*b^3 + 6*A^2*B*a^7*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(4*A*a^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3 - 8*A*a^13*b^2 - 2*B*a^8*b

```
^7 + 2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 - 12*A*a^14*b - 4*B*a^14*b))
/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 -
3*a^11*b^2) - (4*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*B*a^5 -
 2*A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6*A*a^4*b)*(8*a^15*b - 8*a^6*b^10 + 8*
a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4
 - 32*a^13*b^3 - 8*a^14*b^2))/((a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 +
10*a^9*b^4 - 5*a^11*b^2)*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3
*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*(2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^
3*b^2 - 6*A*a^4*b))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b
^4 - 5*a^11*b^2)))*(2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6*A*a^4*b
))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))
 + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^10 + 8*A^2
*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^
3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 +
 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^
7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*
a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) + ((-(a + b)^5*(a - b)^5)^(1/2
)*((8*(4*A*a^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A
*a^9*b^6 - 36*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3 - 8*A*a^13*b^2 - 2*
B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 - 12*A*a^14*b - 4*B*a
^14*b))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10
*b^3 - 3*a^11*b^2) + (4*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*
B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6*A*a^4*b)*(8*a^15*b - 8*a^6*b^
10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a
^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2))/((a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7
*b^6 + 10*a^9*b^4 - 5*a^11*b^2)*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*
b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*(2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3
 + B*a^3*b^2 - 6*A*a^4*b))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 1
0*a^9*b^4 - 5*a^11*b^2)))*(2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B*a^3*b^2 - 6*
A*a^4*b))/(2*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^1
1*b^2))))*(-(a + b)^5*(a - b)^5)^(1/2)*(2*B*a^5 - 2*A*b^5 + 5*A*a^2*b^3 + B
*a^3*b^2 - 6*A*a^4*b)*1i)/(d*(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10
*a^9*b^4 - 5*a^11*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral((A + B*cos(c + dx))*sec(c + d*x)/(a + b*cos(c + d*x))**3, x)
```

3.271 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=299

$$-\frac{(3Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{b\left(-4a^3B+6a^2Ab+ab^2B-3Ab^3\right)\tan(c}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}$$

```
[Out] b*(12*A*a^4*b-15*A*a^2*b^3+6*A*b^5-6*B*a^5+5*B*a^3*b^2-2*B*a*b^4)*arctan((a
-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(5/2)/(a+b)^(5/2)/d-(3*
A*b-B*a)*arctanh(sin(d*x+c))/a^4/d+1/2*(2*A*a^4-11*A*a^2*b^2+6*A*b^4+5*B*a^
3*b-2*B*a*b^3)*tan(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b*(A*b-B*a)*tan(d*x+c)/a/(a
^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*b*(6*A*a^2*b-3*A*b^3-4*B*a^3+B*a*b^2)*tan(
d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 1.76, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$\frac{b\left(-15a^2Ab^3+12a^4Ab+5a^3b^2B-6a^5B-2ab^4B+6Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(-11a^2Ab^2+2a^4A+5a^3b}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 5*a^3*b^2*B - 2*a*b^4*B
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a
 + b)^(5/2)*d) - ((3*A*b - a*B)*ArcTanh[Sin[c + d*x]])/(a^4*d) + ((2*a^4*A
- 11*a^2*A*b^2 + 6*A*b^4 + 5*a^3*b*B - 2*a*b^3*B)*Tan[c + d*x])/(2*a^3*(a^2
 - b^2)^2*d) + (b*(A*b - a*B)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) + (b*(6*a^2*A*b - 3*A*b^3 - 4*a^3*B + a*b^2*B)*Tan[c + d*x])/(2
*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2a^2A-3Ab^2+abB-2a(Ab-aB)\cos(c+dx\right.}{(a+b\cos(c}}{2a\left(a^2\right.}$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(6a^2Ab-3Ab^3-4a^3B+ab^2B\right)}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= \frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b(Ab}{2a\left(a^2-b\right.}$$

$$= \frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b(Ab}{2a\left(a^2-b\right.}$$

$$= -\frac{(3Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3b\right.}{2a^3\left(a^2-b^2\right)}$$

$$= \frac{b\left(12a^4Ab-15a^2Ab^3+6Ab^5-6a^5B+5a^3b^2B-2ab^4B\right)\tan^{-1}\left(\frac{\sqrt{a-b}}{}\right.}{a^4(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A] time = 5.97, size = 352, normalized size = 1.18

$$\frac{\frac{a^2b^2(aB-Ab)\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2} + \frac{ab^2\left(5a^3B-7a^2Ab-2ab^2B+4Ab^3\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} - \frac{2b\left(-6a^5B+12a^4Ab+5a^3b^2B-15a^2Ab^3-2ab^4B+6Ab^5\right)\tanh^{-1}\left(\frac{(a-b)\tan}{\sqrt{}}\right.}{\left(b^2-a^2\right)^{5/2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 5*a^3*b^2*B - 2*a*b
^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5
```

```
/2) + 2*(3*A*b - a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*(-3*A*b
+ a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a*A*Sin[(c + d*x)/2])/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + (2*a*A*Sin[(c + d*x)/2])/(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]) + (a^2*b^2*(-(A*b) + a*B)*Sin[c + d*x])/((a - b
)*(a + b)*(a + b*Cos[c + d*x])^2) + (a*b^2*(-7*a^2*A*b + 4*A*b^3 + 5*a^3*B
- 2*a*b^2*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*a
^4*d)
```

fricas [B] time = 76.80, size = 2100, normalized size = 7.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(((6*B*a^5*b^3 - 12*A*a^4*b^4 - 5*B*a^3*b^5 + 15*A*a^2*b^6 + 2*B*a*b^7
 - 6*A*b^8)*cos(d*x + c)^3 + 2*(6*B*a^6*b^2 - 12*A*a^5*b^3 - 5*B*a^4*b^4 +
15*A*a^3*b^5 + 2*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x + c)^2 + (6*B*a^7*b - 12*A*
a^6*b^2 - 5*B*a^5*b^3 + 15*A*a^4*b^4 + 2*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x +
 c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^
2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*((B*a^7*b^2 - 3*A*a^6*b^3
 - 3*B*a^5*b^4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^8 + 3*A*b^
9)*cos(d*x + c)^3 + 2*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b^4 +
3*B*a^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*cos(d*x + c)^2 + (B*a^9
- 3*A*a^8*b - 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 - B*a^3
*b^6 + 3*A*a^2*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - 2*((B*a^7*b^2 - 3
*A*a^6*b^3 - 3*B*a^5*b^4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^
8 + 3*A*b^9)*cos(d*x + c)^3 + 2*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*
a^5*b^4 + 3*B*a^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*cos(d*x + c)^2
 + (B*a^9 - 3*A*a^8*b - 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b
^5 - B*a^3*b^6 + 3*A*a^2*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(2*A
*a^9 - 6*A*a^7*b^2 + 6*A*a^5*b^4 - 2*A*a^3*b^6 + (2*A*a^7*b^2 + 5*B*a^6*b^3
 - 13*A*a^5*b^4 - 7*B*a^4*b^5 + 17*A*a^3*b^6 + 2*B*a^2*b^7 - 6*A*a*b^8)*cos
(d*x + c)^2 + (4*A*a^8*b + 6*B*a^7*b^2 - 20*A*a^6*b^3 - 9*B*a^5*b^4 + 25*A*
a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^10*b^2
 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d*cos(d*x + c)^3 + 2*(a^11*b - 3*a^9*b^
3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c)^2 + (a^12 - 3*a^10*b^2 + 3*a^8*b^4
- a^6*b^6)*d*cos(d*x + c)), -1/2*(((6*B*a^5*b^3 - 12*A*a^4*b^4 - 5*B*a^3*b^
5 + 15*A*a^2*b^6 + 2*B*a*b^7 - 6*A*b^8)*cos(d*x + c)^3 + 2*(6*B*a^6*b^2 - 1
2*A*a^5*b^3 - 5*B*a^4*b^4 + 15*A*a^3*b^5 + 2*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x
 + c)^2 + (6*B*a^7*b - 12*A*a^6*b^2 - 5*B*a^5*b^3 + 15*A*a^4*b^4 + 2*B*a^3*
b^5 - 6*A*a^2*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) +
b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((B*a^7*b^2 - 3*A*a^6*b^3 - 3*B*a^5*b^
```

```
4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^8 + 3*A*b^9)*cos(d*x +
c)^3 + 2*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b^4 + 3*B*a^4*b^5 -
 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*cos(d*x + c)^2 + (B*a^9 - 3*A*a^8*b -
 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 - B*a^3*b^6 + 3*A*a^
2*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((B*a^7*b^2 - 3*A*a^6*b^3 - 3*
B*a^5*b^4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^8 + 3*A*b^9)*co
s(d*x + c)^3 + 2*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b^4 + 3*B*a
^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*cos(d*x + c)^2 + (B*a^9 - 3*A
*a^8*b - 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 - B*a^3*b^6
+ 3*A*a^2*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) - (2*A*a^9 - 6*A*a^7*b^
2 + 6*A*a^5*b^4 - 2*A*a^3*b^6 + (2*A*a^7*b^2 + 5*B*a^6*b^3 - 13*A*a^5*b^4 -
 7*B*a^4*b^5 + 17*A*a^3*b^6 + 2*B*a^2*b^7 - 6*A*a*b^8)*cos(d*x + c)^2 + (4*
A*a^8*b + 6*B*a^7*b^2 - 20*A*a^6*b^3 - 9*B*a^5*b^4 + 25*A*a^4*b^5 + 3*B*a^3
*b^6 - 9*A*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^10*b^2 - 3*a^8*b^4 + 3*
a^6*b^6 - a^4*b^8)*d*cos(d*x + c)^3 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a
^5*b^7)*d*cos(d*x + c)^2 + (a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(
d*x + c))]
```

giac [B] time = 6.99, size = 574, normalized size = 1.92

$$\frac{\frac{\left(6\,Ba^{5}b-12\,Aa^{4}b^{2}-5\,Ba^{3}b^{3}+15\,Aa^{2}b^{4}+2\,Bab^{5}-6\,Ab^{6}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{8}-2\,a^{6}b^{2}+a^{4}b^{4}\right)\sqrt{a^{2}-b^{2}}}+\frac{6\,Ba^{4}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] ((6*B*a^5*b - 12*A*a^4*b^2 - 5*B*a^3*b^3 + 15*A*a^2*b^4 + 2*B*a*b^5 - 6*A*b
^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^8 - 2*a^6*b^2
+ a^4*b^4)*sqrt(a^2 - b^2)) + (6*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*A*a^3
*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 7*A*a^2*
b^4*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 + 5*A*a*b^5
*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*A*b^6*tan(1/
2*d*x + 1/2*c)^3 + 6*B*a^4*b^2*tan(1/2*d*x + 1/2*c) - 8*A*a^3*b^3*tan(1/2*d
*x + 1/2*c) + 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c) - 7*A*a^2*b^4*tan(1/2*d*x +
1/2*c) - 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c) + 5*A*a*b^5*tan(1/2*d*x + 1/2*c)
- 2*B*a*b^5*tan(1/2*d*x + 1/2*c) + 4*A*b^6*tan(1/2*d*x + 1/2*c))/((a^7 - 2*
a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a
 + b)^2) + (B*a - 3*A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - (B*a - 3*
A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 2*A*tan(1/2*d*x + 1/2*c)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)*a^3))/d
```

maple [B] time = 0.18, size = 1358, normalized size = 4.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -8/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a-b)/(a^2
+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*
d*x+1/2*c)^2*b+a+b)^2*b^4/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+4/d*
b^5/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2*B+1/d*b^3/a/(
a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*
tan(1/2*d*x+1/2*c)^3*B-2/d*b^4/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-8/d/a/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^3/(a+b)/(a-b)^2*tan(1/2*d*x+1
/2*c)*A+1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^4/(
a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+4/d*b^5/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+6/d/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/
2*c)*B-1/d*b^3/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b
)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-2/d*b^4/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*
d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+12/d*b^2/(a^4-2*a^
2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b)
)^(1/2))*A-15/d*b^4/a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(
1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+6/d*b^6/a^4/(a^4-2*a^2*b^2+b^4)
/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A
-6/d*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a
-b)/((a-b)*(a+b))^(1/2))*B*a+5/d*b^3/a/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1
/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-2/d*b^5/a^3/(a^4
-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*
(a+b))^(1/2))*B-1/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)+3/d/a^4*ln(tan(1/2*d*x+1/2
*c)-1)*A*b-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^3*A/(tan(1/2*d*x+1/2*c)
+1)-3/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*A*b+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.91, size = 9312, normalized size = 31.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(6*A*b^5 - 2*A*a^5 - 12*A*a^2*b^3 + 4*A*a^3*b^2 + B*
a^2*b^3 + 6*B*a^3*b^2 - 3*A*a*b^4 + 2*A*a^4*b - 2*B*a*b^4))/((a^3*b - a^4)*
(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*A*a^5 + 6*A*b^5 - 12*A*a^2*b^3 - 4*A*a^
3*b^2 - B*a^2*b^3 + 6*B*a^3*b^2 + 3*A*a*b^4 + 2*A*a^4*b - 2*B*a*b^4))/((a +
 b)*(a^5 - 2*a^4*b + a^3*b^2)) - (2*tan(c/2 + (d*x)/2)^3*(2*A*a^6 - 6*A*b^6
 + 13*A*a^2*b^4 - 6*A*a^4*b^2 - 5*B*a^3*b^3 + 2*B*a*b^5))/(a*(a^2*b - a^3)*
(a + b)^2*(a - b)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2*(2*a*b - a^2 + 3*b^2)
- tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - tan(c/2 + (d*x)/2)
^4*(2*a*b + a^2 - 3*b^2))) + (atan(((((8*tan(c/2 + (d*x)/2)*(72*A^2*b^12 +
4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 + 288*A^2*a^3*
b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 288*A^2*a^6*b^6 + 288*A^2*a^7*b^5
 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2*a^10*b^2 + 8*B^2*a^2*b^10 - 8*B
^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*
b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24*B^2*a^10*b^2 - 48*A*B*a*b^11 - 2
4*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^3*b^9 - 192*A*B*a^4*b^8 - 318*A*
B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8*b^4 - 72*A*B*a^
9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 +
3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (((8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A
*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 42*A*a^13*b^5 -
78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16*b^2 - 4*B*a^9*b^9 + 2*B*a^10*b^8
+ 18*B*a^11*b^7 - 4*B*a^12*b^6 - 36*B*a^13*b^5 + 6*B*a^14*b^4 + 34*B*a^15*b
^3 - 8*B*a^16*b^2 - 12*A*a^17*b - 12*B*a^17*b))/(a^15*b + a^16 - a^9*b^7 -
a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) + (8*tan(c/2
+ (d*x)/2)*(3*A*b - B*a)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 -
 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^
16*b^2))/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 -
3*a^10*b^3 - 3*a^11*b^2)))*(3*A*b - B*a))/a^4)*(3*A*b - B*a)*1i)/a^4 + (((8
*tan(c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*
b - 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7
- 288*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*
A^2*a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5
*b^7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 +
24*B^2*a^10*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B
```

*a^3*b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (((8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16*b^2 - 4*B*a^9*b^9 + 2*B*a^10*b^8 + 18*B*a^11*b^7 - 4*B*a^12*b^6 - 36*B*a^13*b^5 + 6*B*a^14*b^4 + 34*B*a^15*b^3 - 8*B*a^16*b^2 - 12*A*a^17*b - 12*B*a^17*b))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (8*tan(c/2 + (d*x)/2)*(3*A*b - B*a)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*(3*A*b - B*a))/a^4)*(3*A*b - B*a)*1i)/a^4)/((16*(108*A^3*b^12 - 54*A^3*a*b^11 - 12*B^3*a^11*b - 486*A^3*a^2*b^10 + 243*A^3*a^3*b^9 + 864*A^3*a^4*b^8 - 378*A^3*a^5*b^7 - 702*A^3*a^6*b^6 + 216*A^3*a^7*b^5 + 216*A^3*a^8*b^4 - 4*B^3*a^3*b^9 + 2*B^3*a^4*b^8 + 18*B^3*a^5*b^7 - 13*B^3*a^6*b^6 - 36*B^3*a^7*b^5 + 26*B^3*a^8*b^4 + 34*B^3*a^9*b^3 - 24*B^3*a^10*b^2 - 108*A^2*B*a*b^11 + 36*A*B^2*a^2*b^10 - 18*A*B^2*a^3*b^9 - 162*A*B^2*a^4*b^8 + 105*A*B^2*a^5*b^7 + 312*A*B^2*a^6*b^6 - 198*A*B^2*a^7*b^5 - 282*A*B^2*a^8*b^4 + 156*A*B^2*a^9*b^3 + 96*A*B^2*a^10*b^2 + 54*A^2*B*a^2*b^10 + 486*A^2*B*a^3*b^9 - 279*A^2*B*a^4*b^8 - 900*A^2*B*a^5*b^7 + 486*A^2*B*a^6*b^6 + 774*A^2*B*a^7*b^5 - 324*A^2*B*a^8*b^4 - 252*A^2*B*a^9*b^3))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (((8*tan(c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 288*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2*a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24*B^2*a^10*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^3*b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (((8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16*b^2 - 4*B*a^9*b^9 + 2*B*a^10*b^8 + 18*B*a^11*b^7 - 4*B*a^12*b^6 - 36*B*a^13*b^5 + 6*B*a^14*b^4 + 34*B*a^15*b^3 - 8*B*a^16*b^2 - 12*A*a^17*b - 12*B*a^17*b))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) + (8*tan(c/2 + (d*x)/2)*(3*A*b - B*a)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*(3*A*b - B*a))/a^4)*(3*A*b - B*a))/a^4 + (((8*tan(c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 288*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2*a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^

2*a^5*b^7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24*B^2*a^10*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^3*b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (((8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16*b^2 - 4*B*a^9*b^9 + 2*B*a^10*b^8 + 18*B*a^11*b^7 - 4*B*a^12*b^6 - 36*B*a^13*b^5 + 6*B*a^14*b^4 + 34*B*a^15*b^3 - 8*B*a^16*b^2 - 12*A*a^17*b - 12*B*a^17*b))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (8*tan(c/2 + (d*x)/2)*(3*A*b - B*a)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*(3*A*b - B*a))/a^4)*(3*A*b - B*a))/a^4))*(3*A*b - B*a)*2i)/(a^4*d) + (b*atan(((b*((8*tan(c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 288*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2*a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24*B^2*a^10*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^3*b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (b*((8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16*b^2 - 4*B*a^9*b^9 + 2*B*a^10*b^8 + 18*B*a^11*b^7 - 4*B*a^12*b^6 - 36*B*a^13*b^5 + 6*B*a^14*b^4 + 34*B*a^15*b^3 - 8*B*a^16*b^2 - 12*A*a^17*b - 12*B*a^17*b))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (4*b*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*1i)/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)) + (b*((8*tan(c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 288*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2*a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24*B^2*a^1

0*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^3*b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (b*((8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16*b^2 - 4*B*a^9*b^9 + 2*B*a^10*b^8 + 18*B*a^11*b^7 - 4*B*a^12*b^6 - 36*B*a^13*b^5 + 6*B*a^14*b^4 + 34*B*a^15*b^3 - 8*B*a^16*b^2 - 12*A*a^17*b - 12*B*a^17*b))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) + (4*b*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*1i)/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))/((16*(108*A^3*b^12 - 54*A^3*a*b^11 - 12*B^3*a^11*b - 486*A^3*a^2*b^10 + 243*A^3*a^3*b^9 + 864*A^3*a^4*b^8 - 378*A^3*a^5*b^7 - 702*A^3*a^6*b^6 + 216*A^3*a^7*b^5 + 216*A^3*a^8*b^4 - 4*B^3*a^3*b^9 + 2*B^3*a^4*b^8 + 18*B^3*a^5*b^7 - 13*B^3*a^6*b^6 - 36*B^3*a^7*b^5 + 26*B^3*a^8*b^4 + 34*B^3*a^9*b^3 - 24*B^3*a^10*b^2 - 108*A^2*B*a*b^11 + 36*A*B^2*a^2*b^10 - 18*A*B^2*a^3*b^9 - 162*A*B^2*a^4*b^8 + 105*A*B^2*a^5*b^7 + 312*A*B^2*a^6*b^6 - 198*A*B^2*a^7*b^5 - 282*A*B^2*a^8*b^4 + 156*A*B^2*a^9*b^3 + 96*A*B^2*a^10*b^2 + 54*A^2*B*a^2*b^10 + 486*A^2*B*a^3*b^9 - 279*A^2*B*a^4*b^8 - 900*A^2*B*a^5*b^7 + 486*A^2*B*a^6*b^6 + 774*A^2*B*a^7*b^5 - 324*A^2*B*a^8*b^4 - 252*A^2*B*a^9*b^3))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) + (b*((8*tan(c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 288*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2*a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24*B^2*a^10*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^3*b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (b*((8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16*b^2 - 4*B*a^9*b^9 + 2*B*a^10*b^8 + 18*B*a^11*b^7 - 4*B*a^12*b^6 - 36*B*a^13*b^5 + 6*B*a^14*b^4 + 34*B*a^15*b^3 - 8*B*a^16*b^2 - 12*A*a^17*b - 12*B*a^17*b))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (4*b*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A

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*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*(8*a^
17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 4
8*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a^14 - a^4*b^10 + 5
*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6*b^7
- a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*(-(a + b)^5*
(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4
*b - 2*B*a*b^4))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4
 - 5*a^12*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2
*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4))/(2*(a^14 - a^4*b^10 + 5*a^6*b
^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)) - (b*((8*tan(c/2 + (d*x)/2)*(7
2*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 +
 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 288*A^2*a^6*b^6 + 28
8*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2*a^10*b^2 + 8*B^2*a
^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^7 + 57*B^2*a^6*b^6
- 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24*B^2*a^10*b^2 - 48*A
*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^3*b^9 - 192*A*B*a^4
*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8*b^
4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 +
 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (b*((8*(4*B*a^18 + 12*A
*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 4
2*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16*b^2 - 4*B*a^9*b^9
+ 2*B*a^10*b^8 + 18*B*a^11*b^7 - 4*B*a^12*b^6 - 36*B*a^13*b^5 + 6*B*a^14*b^
4 + 34*B*a^15*b^3 - 8*B*a^16*b^2 - 12*A*a^17*b - 12*B*a^17*b))/(a^15*b + a^
16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2
) + (4*b*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5
 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*(8*a^17*b - 8*a^8*b
^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 3
2*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a^14 - a^4*b^10 + 5*a^6*b^8 - 10*
a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*
a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*(-(a + b)^5*(a - b)^5)^(1/
2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4
))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)
))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3
*b^2 + 12*A*a^4*b - 2*B*a*b^4))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^
6 + 10*a^10*b^4 - 5*a^12*b^2))))*(-(a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*
B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*1i)/(d*(a^14 -
 a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**3,x)
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**3, x)
```

3.272 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=402

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{\left(a^2A-6abB+12Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2a^5d}+\frac{b\left(-5a^3B+7a^2Ab+2ab^2B-\right.}{2a^2d\left(a^2-b^2\right)^2}$$

```
[Out] -b^2*(20*A*a^4*b-29*A*a^2*b^3+12*A*b^5-12*B*a^5+15*B*a^3*b^2-6*B*a*b^4)*arc
tan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(5/2)/(a+b)^(5/2)
/d+1/2*(A*a^2+12*A*b^2-6*B*a*b)*arctanh(sin(d*x+c))/a^5/d-1/2*(6*A*a^4*b-21
*A*a^2*b^3+12*A*b^5-2*B*a^5+11*B*a^3*b^2-6*B*a*b^4)*tan(d*x+c)/a^4/(a^2-b^2
)^2/d+1/2*(A*a^4-10*A*a^2*b^2+6*A*b^4+6*B*a^3*b-3*B*a*b^3)*sec(d*x+c)*tan(d
*x+c)/a^3/(a^2-b^2)^2/d+1/2*b*(A*b-B*a)*sec(d*x+c)*tan(d*x+c)/a/(a^2-b^2)/d
/(a+b*cos(d*x+c))^2+1/2*b*(7*A*a^2*b-4*A*b^3-5*B*a^3+2*B*a*b^2)*sec(d*x+c)*
tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 2.24, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{b^2\left(-29a^2Ab^3+20a^4Ab+15a^3b^2B-12a^5B-6ab^4B+12Ab^5\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\left(-21a^2Ab^3+6a^4Ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((b^2*(20*a^4*A*b - 29*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B - 6*
a*b^4*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(
5/2)*(a + b)^(5/2)*d)) + ((a^2*A + 12*A*b^2 - 6*a*b*B)*ArcTanh[Sin[c + d*x]
])/(2*a^5*d) - ((6*a^4*A*b - 21*a^2*A*b^3 + 12*A*b^5 - 2*a^5*B + 11*a^3*b^2
*B - 6*a*b^4*B)*Tan[c + d*x])/(2*a^4*(a^2 - b^2)^2*d) + ((a^4*A - 10*a^2*A*
b^2 + 6*A*b^4 + 6*a^3*b*B - 3*a*b^3*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*(a
^2 - b^2)^2*d) + (b*(A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*(a^2 - b^2)
*d*(a + b*Cos[c + d*x])^2) + (b*(7*a^2*A*b - 4*A*b^3 - 5*a^3*B + 2*a*b^2*B)
*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2\left(a^2A-2Ab^2+abB\right)-2a(Ab-aB)\cos(c}{(a+b\,\mathrm{cc}}}{2a\,(}$$

$$= \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(7a^2Ab-4Ab^3-5a^3B+2ab\right.}{2a^2\left(a^2-b^2\right)^2 d(a+}$$

$$= \frac{\left(a^4A-10a^2Ab^2+6Ab^4+6a^3bB-3ab^3B\right)\sec(c+dx)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} +$$

$$= -\frac{\left(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+11a^3b^2B-6ab^4B\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+11a^3b^2B-6ab^4B\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(a^2A+12Ab^2-6abB\right)\tanh^{-1}(\sin(c+dx))}{2a^5d} - \frac{\left(6a^4Ab-21a^2Ab^3+1\right.}{}$$

$$= -\frac{b^2\left(20a^4Ab-29a^2Ab^3+12Ab^5-12a^5B+15a^3b^2B-6ab^4B\right)\tan^{-1}}{a^5(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A] time = 2.96, size = 507, normalized size = 1.26

$$\frac{-8\left(a^2A-6abB+12Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+8\left(a^2A-6abB+12Ab^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((16*b^2*(20*a^4*A*b - 29*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B -
6*a*b^4*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^
2)^(5/2) - 8*(a^2*A + 12*A*b^2 - 6*a*b*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 8*(a^2*A + 12*A*b^2 - 6*a*b*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]] + (2*a*(4*a^7*A - 30*a^5*A*b^2 + 68*a^3*A*b^4 - 36*a*A*b^6 + 8*a^6*b
*B - 32*a^4*b^3*B + 18*a^2*b^5*B + (-16*a^6*A*b + 14*a^4*A*b^3 + 47*a^2*A*b
^5 - 36*A*b^7 + 8*a^7*B - 10*a^5*b^2*B - 25*a^3*b^4*B + 18*a*b^6*B)*Cos[c +
 d*x] + 2*a*b*(-11*a^4*A*b + 32*a^2*A*b^3 - 18*A*b^5 + 4*a^5*B - 16*a^3*b^2
*B + 9*a*b^4*B)*Cos[2*(c + d*x)] - 6*a^4*A*b^3*Cos[3*(c + d*x)] + 21*a^2*A*
b^5*Cos[3*(c + d*x)] - 12*A*b^7*Cos[3*(c + d*x)] + 2*a^5*b^2*B*Cos[3*(c + d
*x)] - 11*a^3*b^4*B*Cos[3*(c + d*x)] + 6*a*b^6*B*Cos[3*(c + d*x)])*Sec[c +
d*x]*Tan[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2))/(16*a^5*d)
```

fricas [B] time = 121.64, size = 2416, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(((12*B*a^5*b^4 - 20*A*a^4*b^5 - 15*B*a^3*b^6 + 29*A*a^2*b^7 + 6*B*a*b
^8 - 12*A*b^9)*cos(d*x + c)^4 + 2*(12*B*a^6*b^3 - 20*A*a^5*b^4 - 15*B*a^4*b
^5 + 29*A*a^3*b^6 + 6*B*a^2*b^7 - 12*A*a*b^8)*cos(d*x + c)^3 + (12*B*a^7*b^
2 - 20*A*a^6*b^3 - 15*B*a^5*b^4 + 29*A*a^4*b^5 + 6*B*a^3*b^6 - 12*A*a^2*b^7
)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*
cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + ((A*a^8*b^2 -
6*B*a^7*b^3 + 9*A*a^6*b^4 + 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*B*a^3*b^7 + 35
*A*a^2*b^8 + 6*B*a*b^9 - 12*A*b^10)*cos(d*x + c)^4 + 2*(A*a^9*b - 6*B*a^8*b
^2 + 9*A*a^7*b^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 18*B*a^4*b^6 + 35*A*a^3*b^
7 + 6*B*a^2*b^8 - 12*A*a*b^9)*cos(d*x + c)^3 + (A*a^10 - 6*B*a^9*b + 9*A*a^
8*b^2 + 18*B*a^7*b^3 - 33*A*a^6*b^4 - 18*B*a^5*b^5 + 35*A*a^4*b^6 + 6*B*a^3
*b^7 - 12*A*a^2*b^8)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((A*a^8*b^2 -
6*B*a^7*b^3 + 9*A*a^6*b^4 + 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*B*a^3*b^7 + 35
*A*a^2*b^8 + 6*B*a*b^9 - 12*A*b^10)*cos(d*x + c)^4 + 2*(A*a^9*b - 6*B*a^8*b
^2 + 9*A*a^7*b^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 18*B*a^4*b^6 + 35*A*a^3*b^
7 + 6*B*a^2*b^8 - 12*A*a*b^9)*cos(d*x + c)^3 + (A*a^10 - 6*B*a^9*b + 9*A*a^
8*b^2 + 18*B*a^7*b^3 - 33*A*a^6*b^4 - 18*B*a^5*b^5 + 35*A*a^4*b^6 + 6*B*a^3
*b^7 - 12*A*a^2*b^8)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(A*a^10 - 3
*A*a^8*b^2 + 3*A*a^6*b^4 - A*a^4*b^6 + (2*B*a^8*b^2 - 6*A*a^7*b^3 - 13*B*a^
6*b^4 + 27*A*a^5*b^5 + 17*B*a^4*b^6 - 33*A*a^3*b^7 - 6*B*a^2*b^8 + 12*A*a*b
^9)*cos(d*x + c)^3 + (4*B*a^9*b - 11*A*a^8*b^2 - 20*B*a^7*b^3 + 43*A*a^6*b^
```

```
4 + 25*B*a^5*b^5 - 50*A*a^4*b^6 - 9*B*a^3*b^7 + 18*A*a^2*b^8)*cos(d*x + c)^
2 + 2*(B*a^10 - 2*A*a^9*b - 3*B*a^8*b^2 + 6*A*a^7*b^3 + 3*B*a^6*b^4 - 6*A*a
^5*b^5 - B*a^4*b^6 + 2*A*a^3*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11*b^2 -
3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d*cos(d*x + c)^4 + 2*(a^12*b - 3*a^10*b^3
+ 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c)^3 + (a^13 - 3*a^11*b^2 + 3*a^9*b^4 -
a^7*b^6)*d*cos(d*x + c)^2), 1/4*(2*((12*B*a^5*b^4 - 20*A*a^4*b^5 - 15*B*a^3
*b^6 + 29*A*a^2*b^7 + 6*B*a*b^8 - 12*A*b^9)*cos(d*x + c)^4 + 2*(12*B*a^6*b^
3 - 20*A*a^5*b^4 - 15*B*a^4*b^5 + 29*A*a^3*b^6 + 6*B*a^2*b^7 - 12*A*a*b^8)*
cos(d*x + c)^3 + (12*B*a^7*b^2 - 20*A*a^6*b^3 - 15*B*a^5*b^4 + 29*A*a^4*b^5
 + 6*B*a^3*b^6 - 12*A*a^2*b^7)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*c
os(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + ((A*a^8*b^2 - 6*B*a^7*b^
3 + 9*A*a^6*b^4 + 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*B*a^3*b^7 + 35*A*a^2*b^8
 + 6*B*a*b^9 - 12*A*b^10)*cos(d*x + c)^4 + 2*(A*a^9*b - 6*B*a^8*b^2 + 9*A*a
^7*b^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 18*B*a^4*b^6 + 35*A*a^3*b^7 + 6*B*a^
2*b^8 - 12*A*a*b^9)*cos(d*x + c)^3 + (A*a^10 - 6*B*a^9*b + 9*A*a^8*b^2 + 18
*B*a^7*b^3 - 33*A*a^6*b^4 - 18*B*a^5*b^5 + 35*A*a^4*b^6 + 6*B*a^3*b^7 - 12*
A*a^2*b^8)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((A*a^8*b^2 - 6*B*a^7*b^
3 + 9*A*a^6*b^4 + 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*B*a^3*b^7 + 35*A*a^2*b^8
 + 6*B*a*b^9 - 12*A*b^10)*cos(d*x + c)^4 + 2*(A*a^9*b - 6*B*a^8*b^2 + 9*A*a
^7*b^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 18*B*a^4*b^6 + 35*A*a^3*b^7 + 6*B*a^
2*b^8 - 12*A*a*b^9)*cos(d*x + c)^3 + (A*a^10 - 6*B*a^9*b + 9*A*a^8*b^2 + 18
*B*a^7*b^3 - 33*A*a^6*b^4 - 18*B*a^5*b^5 + 35*A*a^4*b^6 + 6*B*a^3*b^7 - 12*
A*a^2*b^8)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(A*a^10 - 3*A*a^8*b^2
 + 3*A*a^6*b^4 - A*a^4*b^6 + (2*B*a^8*b^2 - 6*A*a^7*b^3 - 13*B*a^6*b^4 + 27
*A*a^5*b^5 + 17*B*a^4*b^6 - 33*A*a^3*b^7 - 6*B*a^2*b^8 + 12*A*a*b^9)*cos(d*
x + c)^3 + (4*B*a^9*b - 11*A*a^8*b^2 - 20*B*a^7*b^3 + 43*A*a^6*b^4 + 25*B*a
^5*b^5 - 50*A*a^4*b^6 - 9*B*a^3*b^7 + 18*A*a^2*b^8)*cos(d*x + c)^2 + 2*(B*a
^10 - 2*A*a^9*b - 3*B*a^8*b^2 + 6*A*a^7*b^3 + 3*B*a^6*b^4 - 6*A*a^5*b^5 - B
*a^4*b^6 + 2*A*a^3*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11*b^2 - 3*a^9*b^4
+ 3*a^7*b^6 - a^5*b^8)*d*cos(d*x + c)^4 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^
5 - a^6*b^7)*d*cos(d*x + c)^3 + (a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d
*cos(d*x + c)^2)]
```

giac [B] time = 1.92, size = 1395, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/2*(2*(12*B*a^5*b^2 - 20*A*a^4*b^3 - 15*B*a^3*b^4 + 29*A*a^2*b^5 + 6*B*a*
b^6 - 12*A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(
-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^9
- 2*a^7*b^2 + a^5*b^4)*sqrt(a^2 - b^2)) - 2*(A*a^7*tan(1/2*d*x + 1/2*c)^7 -
```

```
 2*B*a^7*tan(1/2*d*x + 1/2*c)^7 + 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 4*B*a^
6*b*tan(1/2*d*x + 1/2*c)^7 - 13*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 2*B*a^5*
b^2*tan(1/2*d*x + 1/2*c)^7 - 2*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 16*B*a^4*
b^3*tan(1/2*d*x + 1/2*c)^7 + 33*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^3*
b^4*tan(1/2*d*x + 1/2*c)^7 - 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^2*
b^5*tan(1/2*d*x + 1/2*c)^7 - 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 6*B*a*b^6*
tan(1/2*d*x + 1/2*c)^7 + 12*A*b^7*tan(1/2*d*x + 1/2*c)^7 + 3*A*a^7*tan(1/2*
d*x + 1/2*c)^5 - 2*B*a^7*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^6*b*tan(1/2*d*x + 1
/2*c)^5 - 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 5*A*a^5*b^2*tan(1/2*d*x + 1/2*
c)^5 + 10*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 26*A*a^4*b^3*tan(1/2*d*x + 1/2
*c)^5 + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 29*A*a^3*b^4*tan(1/2*d*x + 1/
2*c)^5 - 35*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 67*A*a^2*b^5*tan(1/2*d*x + 1
/2*c)^5 - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^6*tan(1/2*d*x + 1/2
*c)^5 + 18*B*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*A*b^7*tan(1/2*d*x + 1/2*c)^5
 + 3*A*a^7*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^
6*b*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^5*b^2
*tan(1/2*d*x + 1/2*c)^3 - 10*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 26*A*a^4*b^
3*tan(1/2*d*x + 1/2*c)^3 + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*A*a^3*b
^4*tan(1/2*d*x + 1/2*c)^3 + 35*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 67*A*a^2*
b^5*tan(1/2*d*x + 1/2*c)^3 - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*A*a*b^
6*tan(1/2*d*x + 1/2*c)^3 - 18*B*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^7*tan
(1/2*d*x + 1/2*c)^3 + A*a^7*tan(1/2*d*x + 1/2*c) + 2*B*a^7*tan(1/2*d*x + 1/
2*c) - 4*A*a^6*b*tan(1/2*d*x + 1/2*c) + 4*B*a^6*b*tan(1/2*d*x + 1/2*c) - 13
*A*a^5*b^2*tan(1/2*d*x + 1/2*c) - 2*B*a^5*b^2*tan(1/2*d*x + 1/2*c) + 2*A*a^
4*b^3*tan(1/2*d*x + 1/2*c) - 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c) + 33*A*a^3*b
^4*tan(1/2*d*x + 1/2*c) - 9*B*a^3*b^4*tan(1/2*d*x + 1/2*c) + 17*A*a^2*b^5*t
an(1/2*d*x + 1/2*c) + 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c) - 18*A*a*b^6*tan(1/2
*d*x + 1/2*c) + 6*B*a*b^6*tan(1/2*d*x + 1/2*c) - 12*A*b^7*tan(1/2*d*x + 1/2
*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x
 + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) - (A*a^2 - 6*B*a*b + 1
2*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 + (A*a^2 - 6*B*a*b + 12*A*b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5)/d
```

maple [B] time = 0.20, size = 1551, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*B+1/2/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)^2-1/d/
a^3/(tan(1/2*d*x+1/2*c)+1)*B-1/2/d/a^3*A/(tan(1/2*d*x+1/2*c)+1)^2-12/d*b^7/
a^5/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)
/((a-b)*(a+b))^(1/2))*A-15/d*b^4/a^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2
)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+6/d*b^6/a^4/(a^4-2
```

```
*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a
+b))^(1/2))*B+3/d/a^4/(tan(1/2*d*x+1/2*c)-1)*A*b+10/d/a^2/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b^4/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3*A+10/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*b
^4/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A-8/d*b^3/a/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-1/d
*b^4/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+1/d*b^5/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-20/d/a
/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((
a-b)*(a+b))^(1/2))*A*b^3+29/d/a^3/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*a
rctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^5+4/d*b^5/a^3/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(
1/2*d*x+1/2*c)^3*B-6/d*b^6/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+4/d*b^5/a^3/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B-6/d*
b^6/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-8/d*b^3/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*B+1/d*b^4/a^2/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a-b)^2*tan(1/2*d*x+1/
2*c)*B-1/d*b^5/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a
+b)/(a-b)^2*tan(1/2*d*x+1/2*c)*A+12/d/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/
2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*b^2*B+1/2/d/a^3*A*l
n(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^3*A*ln(tan(1/2*d*x+1/2*c)-1)-6/d/a^5*ln(tan
(1/2*d*x+1/2*c)-1)*A*b^2+3/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*B*b+3/d/a^4/(tan(
1/2*d*x+1/2*c)+1)*A*b+6/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)*A*b^2-3/d/a^4*ln(tan
(1/2*d*x+1/2*c)+1)*B*b+1/2/d/a^3*A/(tan(1/2*d*x+1/2*c)-1)+1/2/d/a^3*A/(tan(
1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.56, size = 10547, normalized size = 26.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(3*A*a^7 + 36*A*b^7 + 2*B*a^7 - 67*A*a^2*b^5 - 29*A*
a^3*b^4 + 26*A*a^4*b^3 + 5*A*a^5*b^2 - 9*B*a^2*b^5 + 35*B*a^3*b^4 + 16*B*a^
4*b^3 - 10*B*a^5*b^2 + 18*A*a*b^6 - 4*A*a^6*b - 18*B*a*b^6 - 4*B*a^6*b))/((
a + b)^2*(a^6 - 2*a^5*b + a^4*b^2)) + (tan(c/2 + (d*x)/2)^5*(3*A*a^7 - 36*A
*b^7 - 2*B*a^7 + 67*A*a^2*b^5 - 29*A*a^3*b^4 - 26*A*a^4*b^3 + 5*A*a^5*b^2 -
 9*B*a^2*b^5 - 35*B*a^3*b^4 + 16*B*a^4*b^3 + 10*B*a^5*b^2 + 18*A*a*b^6 + 4*
A*a^6*b + 18*B*a*b^6 - 4*B*a^6*b))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2)) -
(tan(c/2 + (d*x)/2)^7*(A*a^6 - 12*A*b^6 - 2*B*a^6 + 23*A*a^2*b^4 - 10*A*a^3
*b^3 - 8*A*a^4*b^2 - 3*B*a^2*b^4 - 12*B*a^3*b^3 + 4*B*a^4*b^2 + 6*A*a*b^5 +
 5*A*a^5*b + 6*B*a*b^5 + 2*B*a^5*b))/((a^4*b - a^5)*(a + b)^2) + (tan(c/2 +
 (d*x)/2)*(A*a^6 - 12*A*b^6 + 2*B*a^6 + 23*A*a^2*b^4 + 10*A*a^3*b^3 - 8*A*a
^4*b^2 + 3*B*a^2*b^4 - 12*B*a^3*b^3 - 4*B*a^4*b^2 - 6*A*a*b^5 - 5*A*a^5*b +
 6*B*a*b^5 + 2*B*a^5*b))/((a + b)*(a^6 - 2*a^5*b + a^4*b^2)))/(d*(2*a*b - t
an(c/2 + (d*x)/2)^4*(2*a^2 - 6*b^2) - tan(c/2 + (d*x)/2)^2*(4*a*b + 4*b^2)
+ tan(c/2 + (d*x)/2)^6*(4*a*b - 4*b^2) + tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b
+ b^2) + a^2 + b^2)) - (atan(((((8*tan(c/2 + (d*x)/2)*(A^2*a^14 + 288*A^2*b
^14 - 288*A^2*a*b^13 - 2*A^2*a^13*b - 1104*A^2*a^2*b^12 + 1104*A^2*a^3*b^11
 + 1538*A^2*a^4*b^10 - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7
 + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^10*b^4 - 40*A^2*a^11*b^3 + 2
1*A^2*a^12*b^2 + 72*B^2*a^2*b^12 - 72*B^2*a^3*b^11 - 288*B^2*a^4*b^10 + 288
*B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^
2*a^9*b^5 + 36*B^2*a^10*b^4 - 72*B^2*a^11*b^3 + 36*B^2*a^12*b^2 - 288*A*B*a
*b^13 - 12*A*B*a^13*b + 288*A*B*a^2*b^12 + 1128*A*B*a^3*b^11 - 1128*A*B*a^4
*b^10 - 1650*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^
8*b^6 - 72*A*B*a^9*b^5 + 192*A*B*a^10*b^4 - 108*A*B*a^11*b^3 + 24*A*B*a^12*
b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12
*b^3 - 3*a^13*b^2) - (((4*(4*A*a^21 - 48*A*a^10*b^11 + 24*A*a^11*b^10 + 212
*A*a^12*b^9 - 100*A*a^13*b^8 - 360*A*a^14*b^7 + 164*A*a^15*b^6 + 276*A*a^16
*b^5 - 120*A*a^17*b^4 - 80*A*a^18*b^3 + 28*A*a^19*b^2 + 24*B*a^11*b^10 - 12
*B*a^12*b^9 - 108*B*a^13*b^8 + 48*B*a^14*b^7 + 192*B*a^15*b^6 - 84*B*a^16*b
^5 - 156*B*a^17*b^4 + 72*B*a^18*b^3 + 48*B*a^19*b^2 - 24*B*a^20*b))/(a^18*b
 + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^
17*b^2) - (4*tan(c/2 + (d*x)/2)*(A*a^2 + 12*A*b^2 - 6*B*a*b)*(8*a^19*b - 8*
a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*
b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^
7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(A*a^2 +
 12*A*b^2 - 6*B*a*b))/(2*a^5))*(A*a^2 + 12*A*b^2 - 6*B*a*b)*1i)/(2*a^5) + (
((8*tan(c/2 + (d*x)/2)*(A^2*a^14 + 288*A^2*b^14 - 288*A^2*a*b^13 - 2*A^2*a^
13*b - 1104*A^2*a^2*b^12 + 1104*A^2*a^3*b^11 + 1538*A^2*a^4*b^10 - 1538*A^2
*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^6 - 108*A^2*a^9
*b^5 + 74*A^2*a^10*b^4 - 40*A^2*a^11*b^3 + 21*A^2*a^12*b^2 + 72*B^2*a^2*b^1
```

2 - 72*B^2*a^3*b^11 - 288*B^2*a^4*b^10 + 288*B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B^2*a^10*b^4 - 72*B^2*a^11*b^3 + 36*B^2*a^12*b^2 - 288*A*B*a*b^13 - 12*A*B*a^13*b + 288*A*B*a^2*b^12 + 1128*A*B*a^3*b^11 - 1128*A*B*a^4*b^10 - 1650*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^5 + 192*A*B*a^10*b^4 - 108*A*B*a^11*b^3 + 24*A*B*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) + (((4*(4*A*a^21 - 48*A*a^10*b^11 + 24*A*a^11*b^10 + 212*A*a^12*b^9 - 100*A*a^13*b^8 - 360*A*a^14*b^7 + 164*A*a^15*b^6 + 276*A*a^16*b^5 - 120*A*a^17*b^4 - 80*A*a^18*b^3 + 28*A*a^19*b^2 + 24*B*a^11*b^10 - 12*B*a^12*b^9 - 108*B*a^13*b^8 + 48*B*a^14*b^7 + 192*B*a^15*b^6 - 84*B*a^16*b^5 - 156*B*a^17*b^4 + 72*B*a^18*b^3 + 48*B*a^19*b^2 - 24*B*a^20*b))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) + (4*tan(c/2 + (d*x)/2)*(A*a^2 + 12*A*b^2 - 6*B*a*b)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(A*a^2 + 12*A*b^2 - 6*B*a*b))/(2*a^5))*(A*a^2 + 12*A*b^2 - 6*B*a*b)*1i)/(2*a^5))/((8*(1728*A^3*b^15 - 864*A^3*a*b^14 - 7344*A^3*a^2*b^13 + 3456*A^3*a^3*b^12 + 11700*A^3*a^4*b^11 - 4770*A^3*a^5*b^10 - 7829*A^3*a^6*b^9 + 2326*A^3*a^7*b^8 + 1314*A^3*a^8*b^7 - 11*A^3*a^9*b^6 + 411*A^3*a^10*b^5 - 20*A^3*a^11*b^4 + 20*A^3*a^12*b^3 - 216*B^3*a^3*b^12 + 108*B^3*a^4*b^11 + 972*B^3*a^5*b^10 - 486*B^3*a^6*b^9 - 1728*B^3*a^7*b^8 + 756*B^3*a^8*b^7 + 1404*B^3*a^9*b^6 - 432*B^3*a^10*b^5 - 432*B^3*a^11*b^4 - 2592*A^2*B*a*b^14 + 1296*A*B^2*a^2*b^13 - 648*A*B^2*a^3*b^12 - 5724*A*B^2*a^4*b^11 + 2808*A*B^2*a^5*b^10 + 9828*A*B^2*a^6*b^9 - 4203*A*B^2*a^7*b^8 - 7524*A*B^2*a^8*b^7 + 2268*A*B^2*a^9*b^6 + 1980*A*B^2*a^10*b^5 + 144*A*B^2*a^12*b^3 + 1296*A^2*B*a^2*b^13 + 11232*A^2*B*a^3*b^12 - 5400*A^2*B*a^4*b^11 - 18594*A^2*B*a^5*b^10 + 7767*A^2*B*a^6*b^9 + 13347*A^2*B*a^7*b^8 - 3972*A^2*B*a^8*b^7 - 2892*A^2*B*a^9*b^6 + 9*A^2*B*a^10*b^5 - 489*A^2*B*a^11*b^4 + 12*A^2*B*a^12*b^3 - 12*A^2*B*a^13*b^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (((8*tan(c/2 + (d*x)/2)*(A^2*a^14 + 288*A^2*b^14 - 288*A^2*a*b^13 - 2*A^2*a^13*b - 1104*A^2*a^2*b^12 + 1104*A^2*a^3*b^11 + 1538*A^2*a^4*b^10 - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^10*b^4 - 40*A^2*a^11*b^3 + 21*A^2*a^12*b^2 + 72*B^2*a^2*b^12 - 72*B^2*a^3*b^11 - 288*B^2*a^4*b^10 + 288*B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B^2*a^10*b^4 - 72*B^2*a^11*b^3 + 36*B^2*a^12*b^2 - 288*A*B*a*b^13 - 12*A*B*a^13*b + 288*A*B*a^2*b^12 + 1128*A*B*a^3*b^11 - 1128*A*B*a^4*b^10 - 1650*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^5 + 192*A*B*a^10*b^4 - 108*A*B*a^11*b^3 + 24*A*B*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) - (((4*(4*A*a^21 - 48*A*a^10*b^11 + 24*A*a^11*b^10 + 212*A*a^12*b^9 - 100*A*a^13*b^8 - 360*A*a^14*b^7 + 164*A*a^15*b^6 + 276*A*a^16*b^5 - 120*A*a^17*b^4 - 80*A*a^18*b^3 + 28*A*a^19*b^2 + 24*B*a^11*b^10 - 12*B*a^12*b^9 - 108*B*a^13*b^8 + 48*B*a^14

*b^7 + 192*B*a^15*b^6 - 84*B*a^16*b^5 - 156*B*a^17*b^4 + 72*B*a^18*b^3 + 48
*B*a^19*b^2 - 24*B*a^20*b))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b
^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4*tan(c/2 + (d*x)/2)*(A*a^2 +
 12*A*b^2 - 6*B*a*b)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 3
2*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18
b^2))/(a^5(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 -
3*a^12*b^3 - 3*a^13*b^2)))*(A*a^2 + 12*A*b^2 - 6*B*a*b))/(2*a^5))*(A*a^2 +
12*A*b^2 - 6*B*a*b))/(2*a^5) + (((8*tan(c/2 + (d*x)/2)*(A^2*a^14 + 288*A^2*
b^14 - 288*A^2*a*b^13 - 2*A^2*a^13*b - 1104*A^2*a^2*b^12 + 1104*A^2*a^3*b^1
1 + 1538*A^2*a^4*b^10 - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^
7 + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^10*b^4 - 40*A^2*a^11*b^3 +
21*A^2*a^12*b^2 + 72*B^2*a^2*b^12 - 72*B^2*a^3*b^11 - 288*B^2*a^4*b^10 + 28
8*B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B
^2*a^9*b^5 + 36*B^2*a^10*b^4 - 72*B^2*a^11*b^3 + 36*B^2*a^12*b^2 - 288*A*B*
a*b^13 - 12*A*B*a^13*b + 288*A*B*a^2*b^12 + 1128*A*B*a^3*b^11 - 1128*A*B*a^
4*b^10 - 1650*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a
^8*b^6 - 72*A*B*a^9*b^5 + 192*A*B*a^10*b^4 - 108*A*B*a^11*b^3 + 24*A*B*a^12
*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^1
2*b^3 - 3*a^13*b^2) + (((4*(4*A*a^21 - 48*A*a^10*b^11 + 24*A*a^11*b^10 + 21
2*A*a^12*b^9 - 100*A*a^13*b^8 - 360*A*a^14*b^7 + 164*A*a^15*b^6 + 276*A*a^1
6*b^5 - 120*A*a^17*b^4 - 80*A*a^18*b^3 + 28*A*a^19*b^2 + 24*B*a^11*b^10 - 1
2*B*a^12*b^9 - 108*B*a^13*b^8 + 48*B*a^14*b^7 + 192*B*a^15*b^6 - 84*B*a^16*
b^5 - 156*B*a^17*b^4 + 72*B*a^18*b^3 + 48*B*a^19*b^2 - 24*B*a^20*b))/(a^18*
b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a
^17*b^2) + (4*tan(c/2 + (d*x)/2)*(A*a^2 + 12*A*b^2 - 6*B*a*b)*(8*a^19*b - 8
*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15
*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b
^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(A*a^2
+ 12*A*b^2 - 6*B*a*b))/(2*a^5))*(A*a^2 + 12*A*b^2 - 6*B*a*b))/(2*a^5)))*(A*
a^2 + 12*A*b^2 - 6*B*a*b)*1i)/(a^5*d) - (b^2*atan(((b^2*(-(a + b)^5*(a - b)
^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(A^2*a^14 + 288*A^2*b^14 - 288*A^2*a*b^13
- 2*A^2*a^13*b - 1104*A^2*a^2*b^12 + 1104*A^2*a^3*b^11 + 1538*A^2*a^4*b^10
- 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^6 - 1
08*A^2*a^9*b^5 + 74*A^2*a^10*b^4 - 40*A^2*a^11*b^3 + 21*A^2*a^12*b^2 + 72*B
^2*a^2*b^12 - 72*B^2*a^3*b^11 - 288*B^2*a^4*b^10 + 288*B^2*a^5*b^9 + 441*B^
2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B^2*a^
10*b^4 - 72*B^2*a^11*b^3 + 36*B^2*a^12*b^2 - 288*A*B*a*b^13 - 12*A*B*a^13*b
 + 288*A*B*a^2*b^12 + 1128*A*B*a^3*b^11 - 1128*A*B*a^4*b^10 - 1650*A*B*a^5*
b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^
5 + 192*A*B*a^10*b^4 - 108*A*B*a^11*b^3 + 24*A*B*a^12*b^2))/(a^14*b + a^15
- a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) -
(b^2*((4*(4*A*a^21 - 48*A*a^10*b^11 + 24*A*a^11*b^10 + 212*A*a^12*b^9 - 100
*A*a^13*b^8 - 360*A*a^14*b^7 + 164*A*a^15*b^6 + 276*A*a^16*b^5 - 120*A*a^17
*b^4 - 80*A*a^18*b^3 + 28*A*a^19*b^2 + 24*B*a^11*b^10 - 12*B*a^12*b^9 - 108
*B*a^13*b^8 + 48*B*a^14*b^7 + 192*B*a^15*b^6 - 84*B*a^16*b^5 - 156*B*a^17*b

^4 + 72*B*a^18*b^3 + 48*B*a^19*b^2 - 24*B*a^20*b))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/((a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4)*1i)/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)) + (b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(A^2*a^14 + 288*A^2*b^14 - 288*A^2*a*b^13 - 2*A^2*a^13*b - 1104*A^2*a^2*b^12 + 1104*A^2*a^3*b^11 + 1538*A^2*a^4*b^10 - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^10*b^4 - 40*A^2*a^11*b^3 + 21*A^2*a^12*b^2 + 72*B^2*a^2*b^12 - 72*B^2*a^3*b^11 - 288*B^2*a^4*b^10 + 288*B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B^2*a^10*b^4 - 72*B^2*a^11*b^3 + 36*B^2*a^12*b^2 - 288*A*B*a*b^13 - 12*A*B*a^13*b + 288*A*B*a^2*b^12 + 1128*A*B*a^3*b^11 - 1128*A*B*a^4*b^10 - 1650*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^5 + 192*A*B*a^10*b^4 - 108*A*B*a^11*b^3 + 24*A*B*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) + (b^2*((4*(4*A*a^21 - 48*A*a^10*b^11 + 24*A*a^11*b^10 + 212*A*a^12*b^9 - 100*A*a^13*b^8 - 360*A*a^14*b^7 + 164*A*a^15*b^6 + 276*A*a^16*b^5 - 120*A*a^17*b^4 - 80*A*a^18*b^3 + 28*A*a^19*b^2 + 24*B*a^11*b^10 - 12*B*a^12*b^9 - 108*B*a^13*b^8 + 48*B*a^14*b^7 + 192*B*a^15*b^6 - 84*B*a^16*b^5 - 156*B*a^17*b^4 + 72*B*a^18*b^3 + 48*B*a^19*b^2 - 24*B*a^20*b))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) + (4*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/((a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4)*1i)/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))/((8*(1728*A^3*b^15 - 864*A^3*a*b^14 - 7344*A^3*a^2*b^13 + 3456*A^3*a^3*b^12 + 11700*A^3*a^4*b^11 - 4770*A^3*a^5*b^10 - 7829*A^3*a^6*b^9 + 2326*A^3*a^7*b^8 + 1314*A^3*a^8*b^7 - 11*A^3*a^9*b^6 + 411*A^3*a^10*b^5 - 20*A^3*a^11*b^4 + 20*A^3*a^12*b^3 - 216*B^3*a^3*b^12 + 108*B^3*a^4*b^11 + 972*B^3*a^5*b^10 - 486*B^3*a^6*b^9 - 1728*B^3*a^7*b^8 + 756*B^3*a^8*b^7 + 1404*B^3*a^9*b^6 - 432*B^3*a^10*b^5 - 432*B^3 *a^11*b^4 - 2592*A^2*B*a*b^14 + 1296*A*B^2*a^2*b^13 - 648*A*B^2*a^3*b^12 - 5724*A*B^2*a^4*b^11 + 2808*A*B^2*a^5*b^10 + 9828*A*B^2*a^6*b^9 - 4203*A*B^2*a^7*b^8 - 7524*A*B^2*a^8*b^7 + 2268*A*B^2*a^9*b^6 + 1980*A*B^2*a^10*b^5 + 144*A*B^2*a^12*b^3 + 1296*A^2*B*a^2*b^13 + 11232*A^2*B*a^3*b^12 - 5400*A^2*B*a^4*b^11 - 18594*A^2*B*a^5*b^10 + 7767*A^2*B*a^6*b^9 + 13347*A^2*B*a^7*b^8 - 3972*A^2*B*a^8*b^7 - 2892*A^2*B*a^9*b^6 + 9*A^2*B*a^10*b^5 - 489*A^2*B*a^11*b^4 + 12*A^2*B*a^12*b^3 - 12*A^2*B*a^13*b^2))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(A^2*a^14 + 288*A^2*b^14 - 288*A^2*a*b^13 - 2*A^2*a^13*b - 1104*A^2*a^2*b^12 + 1104*A^2*a^3*b^11 + 1538*A^2*a^4*b^10 - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^10*b^4 - 40*A^2*a^11*b^3 + 21*A^2*a^12*b^2 + 72*B^2*a^2*b^12 - 72*B^2*a^3*b^11 - 288*B^2*a^4*b^10 + 288*B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B^2*a^10*b^4 - 72*B^2*a^11*b^3 + 36*B^2*a^12*b^2 - 288*A*B*a*b^13 - 12*A*B*a^13*b + 288*A*B*a^2*b^12 + 1128*A*B*a^3*b^11 - 1128*A*B*a^4*b^10 - 1650*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^5 + 192*A*B*a^10*b^4 - 108*A*B*a^11*b^3 + 24*A*B*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) - (b^2*((4*(4*A*a^21 - 48*A*a^10*b^11 + 24*A*a^11*b^10 + 212*A*a^12*b^9 - 100*A*a^13*b^8 - 360*A*a^14*b^7 + 164*A*a^15*b^6 + 276*A*a^16*b^5 - 120*A*a^17*b^4 - 80*A*a^18*b^3 + 28*A*a^19*b^2 + 24*B*a^11*b^10 - 12*B*a^12*b^9 - 108*B*a^13*b^8 + 48*B*a^14*b^7 + 192*B*a^15*b^6 - 84*B*a^16*b^5 - 156*B*a^17*b^4 + 72*B*a^18*b^3 + 48*B*a^19*b^2 - 24*B*a^20*b))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/((a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4))/(2*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)) + (b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(A^2*a^14 + 288*A^2*b^14 - 288*A^2*a*b^13 - 2*A^2*a^13*b - 1104*A^2*a^2*b^12 + 1104*A^2*a^3*b^11 + 1538*A^2*a^4*b^10 - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^10*b^4 - 40*A^2*a^11*b^3 + 21*A^2*a^12*b^2 + 72*B^2*a^2*b^12 - 72*B^2*a^3*b^11 - 288*B^2*a^4*b^10 + 288*B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B^2*a^10*b^4 - 72*B^2*a^11*b^3 + 36*B^2*a^12*b^2 - 288*A*B*a*b^13 - 12*A*B*a^13*b + 288*A*B*a^2*b^12 + 1128*A*B*a^3*b^11 - 1128*A*B*a^4*b^10 - 1650*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^5 + 192*A*B*a^10*b^4 - 108

```
*A*B*a^11*b^3 + 24*A*B*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^
10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) + (b^2*((4*(4*A*a^21 - 48*A*
a^10*b^11 + 24*A*a^11*b^10 + 212*A*a^12*b^9 - 100*A*a^13*b^8 - 360*A*a^14*b
^7 + 164*A*a^15*b^6 + 276*A*a^16*b^5 - 120*A*a^17*b^4 - 80*A*a^18*b^3 + 28*
A*a^19*b^2 + 24*B*a^11*b^10 - 12*B*a^12*b^9 - 108*B*a^13*b^8 + 48*B*a^14*b^
7 + 192*B*a^15*b^6 - 84*B*a^16*b^5 - 156*B*a^17*b^4 + 72*B*a^18*b^3 + 48*B*
a^19*b^2 - 24*B*a^20*b))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5
+ 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) + (4*b^2*tan(c/2 + (d*x)/2)*(-(a +
b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 +
20*A*a^4*b - 6*B*a*b^4)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8
- 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a
^18*b^2))/((a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13
*b^2)*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12
*b^3 - 3*a^13*b^2)))*(-(a + b)^5*(a - b)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 29
*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4))/(2*(a^15 - a^5*b^10 +
5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2)))*(12*A*b^5 - 12*B*a^5 -
 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b - 6*B*a*b^4))/(2*(a^15 - a^5*b^10
 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4 - 5*a^13*b^2))))*(-(a + b)^5*(a - b
)^5)^(1/2)*(12*A*b^5 - 12*B*a^5 - 29*A*a^2*b^3 + 15*B*a^3*b^2 + 20*A*a^4*b
- 6*B*a*b^4)*1i)/(d*(a^15 - a^5*b^10 + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^11*b^4
 - 5*a^13*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^3(c + dx)}{(a + b\cos(c + dx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x))**3, x)
```

3.273 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=409

$$\frac{a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{a\left(-4a^3B+a^2Ab+9ab^2B-6Ab^3\right)\sin(c+dx)\cos^2(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{\left(-12a^4B+3a\right.}{}$$

```
[Out] (A*b-4*B*a)*x/b^5-a*(2*A*a^6*b-7*A*a^4*b^3+8*A*a^2*b^5-8*A*b^7-8*B*a^7+28*B
*a^5*b^2-35*B*a^3*b^4+20*B*a*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+
b)^(1/2))/(a-b)^(7/2)/b^5/(a+b)^(7/2)/d-1/6*(3*A*a^3*b-8*A*a*b^3-12*B*a^4+2
3*B*a^2*b^2-6*B*b^4)*sin(d*x+c)/b^4/(a^2-b^2)^2/d+1/3*a*(A*b-B*a)*cos(d*x+c
)^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*a*(A*a^2*b-6*A*b^3-4*B*
a^3+9*B*a*b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2
-1/2*a^2*(A*a^4*b-2*A*a^2*b^3+6*A*b^5-4*B*a^5+11*B*a^3*b^2-12*B*a*b^4)*sin(
d*x+c)/b^4/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 5.17, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2989, 3047, 3031, 3023, 2735, 2659, 205}

$$-\frac{\left(3a^3Ab+23a^2b^2B-12a^4B-8aAb^3-6b^4B\right)\sin(c+dx)}{6b^4d\left(a^2-b^2\right)^2}-\frac{a\left(-7a^4Ab^3+8a^2Ab^5+2a^6Ab+28a^5b^2B-35a^3b^4\right.}{b^5d(a-b)^{7/}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((A*b - 4*a*B)*x)/b^5 - (a*(2*a^6*A*b - 7*a^4*A*b^3 + 8*a^2*A*b^5 - 8*A*b^7
 - 8*a^7*B + 28*a^5*b^2*B - 35*a^3*b^4*B + 20*a*b^6*B)*ArcTan[(Sqrt[a - b]*
Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5*(a + b)^(7/2)*d) - ((3*a
^3*A*b - 8*a*A*b^3 - 12*a^4*B + 23*a^2*b^2*B - 6*b^4*B)*Sin[c + d*x])/(6*b^
4*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^3) + (a*(a^2*A*b - 6*A*b^3 - 4*a^3*B + 9*a*b^
2*B)*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x
])^2) - (a^2*(a^4*A*b - 2*a^2*A*b^3 + 6*A*b^5 - 4*a^5*B + 11*a^3*b^2*B - 12
*a*b^4*B)*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
```

```
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(-3a(Ab-aB)+3b(Ab-aB)\right.}{(a+b\,c}}{3b\left(\right.}$$

$$= \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2\right.}{6b^2\left(a^2-b^2\right)^2 d(a+}$$

$$= \frac{a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2\right.}{6b^2\left(a^2-b^2\right)^2 d(a+}$$

$$= -\frac{\left(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B\right)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} + \frac{a(Ab-}{3b\left(a^2\right.}$$

$$= \frac{(Ab-4aB)x}{b^5} - \frac{\left(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B\right)\sin(c+}{6b^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{(Ab-4aB)x}{b^5} - \frac{\left(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B\right)\sin(c+}{6b^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{(Ab-4aB)x}{b^5} - \frac{a\left(2a^6Ab-7a^4Ab^3+8a^2Ab^5-8Ab^7-8a^7B+28a^5b\right.}{(a-b)^{7/2}b^5(a}$$

Mathematica [B] time = 6.65, size = 1278, normalized size = 3.12

$$\frac{96B(c+dx)a^{10}-24Ab(c+dx)a^9+288bB(c+dx)\cos(c+dx)a^9-96bB\sin(c+dx)a^9-144b^2B(c+dx)a^8-72}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((a*(-2*a^6*A*b + 7*a^4*A*b^3 - 8*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 28*a^5*b
^2*B + 35*a^3*b^4*B - 20*a*b^6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]])/(b^5*(a^2 - b^2)^3*Sqrt[-a^2 + b^2]*d)) + (-24*a^9*A*b*(c + d*
x) + 36*a^7*A*b^3*(c + d*x) + 36*a^5*A*b^5*(c + d*x) - 84*a^3*A*b^7*(c + d*
```

```
x) + 36*a*A*b^9*(c + d*x) + 96*a^10*B*(c + d*x) - 144*a^8*b^2*B*(c + d*x) -
 144*a^6*b^4*B*(c + d*x) + 336*a^4*b^6*B*(c + d*x) - 144*a^2*b^8*B*(c + d*x
) - 72*a^8*A*b^2*(c + d*x)*Cos[c + d*x] + 198*a^6*A*b^4*(c + d*x)*Cos[c + d
*x] - 162*a^4*A*b^6*(c + d*x)*Cos[c + d*x] + 18*a^2*A*b^8*(c + d*x)*Cos[c +
 d*x] + 18*A*b^10*(c + d*x)*Cos[c + d*x] + 288*a^9*b*B*(c + d*x)*Cos[c + d*
x] - 792*a^7*b^3*B*(c + d*x)*Cos[c + d*x] + 648*a^5*b^5*B*(c + d*x)*Cos[c +
 d*x] - 72*a^3*b^7*B*(c + d*x)*Cos[c + d*x] - 72*a*b^9*B*(c + d*x)*Cos[c +
d*x] - 36*a^7*A*b^3*(c + d*x)*Cos[2*(c + d*x)] + 108*a^5*A*b^5*(c + d*x)*Co
s[2*(c + d*x)] - 108*a^3*A*b^7*(c + d*x)*Cos[2*(c + d*x)] + 36*a*A*b^9*(c +
 d*x)*Cos[2*(c + d*x)] + 144*a^8*b^2*B*(c + d*x)*Cos[2*(c + d*x)] - 432*a^6
*b^4*B*(c + d*x)*Cos[2*(c + d*x)] + 432*a^4*b^6*B*(c + d*x)*Cos[2*(c + d*x)
] - 144*a^2*b^8*B*(c + d*x)*Cos[2*(c + d*x)] - 6*a^6*A*b^4*(c + d*x)*Cos[3*
(c + d*x)] + 18*a^4*A*b^6*(c + d*x)*Cos[3*(c + d*x)] - 18*a^2*A*b^8*(c + d*
x)*Cos[3*(c + d*x)] + 6*A*b^10*(c + d*x)*Cos[3*(c + d*x)] + 24*a^7*b^3*B*(c
 + d*x)*Cos[3*(c + d*x)] - 72*a^5*b^5*B*(c + d*x)*Cos[3*(c + d*x)] + 72*a^3
*b^7*B*(c + d*x)*Cos[3*(c + d*x)] - 24*a*b^9*B*(c + d*x)*Cos[3*(c + d*x)] +
 24*a^8*A*b^2*Sin[c + d*x] - 57*a^6*A*b^4*Sin[c + d*x] + 72*a^4*A*b^6*Sin[c
 + d*x] + 36*a^2*A*b^8*Sin[c + d*x] - 96*a^9*b*B*Sin[c + d*x] + 228*a^7*b^3
*B*Sin[c + d*x] - 135*a^5*b^5*B*Sin[c + d*x] - 90*a^3*b^7*B*Sin[c + d*x] +
18*a*b^9*B*Sin[c + d*x] + 30*a^7*A*b^3*Sin[2*(c + d*x)] - 90*a^5*A*b^5*Sin[
2*(c + d*x)] + 120*a^3*A*b^7*Sin[2*(c + d*x)] - 120*a^8*b^2*B*Sin[2*(c + d*
x)] + 336*a^6*b^4*B*Sin[2*(c + d*x)] - 300*a^4*b^6*B*Sin[2*(c + d*x)] + 18*
a^2*b^8*B*Sin[2*(c + d*x)] + 6*b^10*B*Sin[2*(c + d*x)] + 11*a^6*A*b^4*Sin[3
*(c + d*x)] - 32*a^4*A*b^6*Sin[3*(c + d*x)] + 36*a^2*A*b^8*Sin[3*(c + d*x)]
 - 44*a^7*b^3*B*Sin[3*(c + d*x)] + 125*a^5*b^5*B*Sin[3*(c + d*x)] - 114*a^3
*b^7*B*Sin[3*(c + d*x)] + 18*a*b^9*B*Sin[3*(c + d*x)] - 3*a^6*b^4*B*Sin[4*(
c + d*x)] + 9*a^4*b^6*B*Sin[4*(c + d*x)] - 9*a^2*b^8*B*Sin[4*(c + d*x)] + 3
*b^10*B*Sin[4*(c + d*x)])/(24*b^5*(-a^2 + b^2)^3*d*(a + b*Cos[c + d*x])^3)
```

fricas [B] time = 1.40, size = 2567, normalized size = 6.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [-1/12*(12*(4*B*a^9*b^3 - A*a^8*b^4 - 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5
*b^7 - 6*A*a^4*b^8 - 16*B*a^3*b^9 + 4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12)*d*x
*cos(d*x + c)^3 + 36*(4*B*a^10*b^2 - A*a^9*b^3 - 16*B*a^8*b^4 + 4*A*a^7*b^5
 + 24*B*a^6*b^6 - 6*A*a^5*b^7 - 16*B*a^4*b^8 + 4*A*a^3*b^9 + 4*B*a^2*b^10 -
 A*a*b^11)*d*x*cos(d*x + c)^2 + 36*(4*B*a^11*b - A*a^10*b^2 - 16*B*a^9*b^3
+ 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^5*b^7 + 4*A*a^4*b^8 + 4
*B*a^3*b^9 - A*a^2*b^10)*d*x*cos(d*x + c) + 12*(4*B*a^12 - A*a^11*b - 16*B*
a^10*b^2 + 4*A*a^9*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6 + 4*A*a^
```

5*b^7 + 4*B*a^4*b^8 - A*a^3*b^9)*d*x - 3*(8*B*a^11 - 2*A*a^10*b - 28*B*a^9*
b^2 + 7*A*a^8*b^3 + 35*B*a^7*b^4 - 8*A*a^6*b^5 - 20*B*a^5*b^6 + 8*A*a^4*b^7
 + (8*B*a^8*b^3 - 2*A*a^7*b^4 - 28*B*a^6*b^5 + 7*A*a^5*b^6 + 35*B*a^4*b^7 -
 8*A*a^3*b^8 - 20*B*a^2*b^9 + 8*A*a*b^10)*cos(d*x + c)^3 + 3*(8*B*a^9*b^2 -
 2*A*a^8*b^3 - 28*B*a^7*b^4 + 7*A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20
*B*a^3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + 3*(8*B*a^10*b - 2*A*a^9*b^2 - 28
*B*a^8*b^3 + 7*A*a^7*b^4 + 35*B*a^6*b^5 - 8*A*a^5*b^6 - 20*B*a^4*b^7 + 8*A*
a^3*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
- a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(24*B*a
^11*b - 6*A*a^10*b^2 - 92*B*a^9*b^3 + 23*A*a^8*b^4 + 133*B*a^7*b^5 - 43*A*a
^6*b^6 - 71*B*a^5*b^7 + 26*A*a^4*b^8 + 6*B*a^3*b^9 + 6*(B*a^8*b^4 - 4*B*a^6
*b^6 + 6*B*a^4*b^8 - 4*B*a^2*b^10 + B*b^12)*cos(d*x + c)^3 + (44*B*a^9*b^3
- 11*A*a^8*b^4 - 169*B*a^7*b^5 + 43*A*a^6*b^6 + 239*B*a^5*b^7 - 68*A*a^4*b^
8 - 132*B*a^3*b^9 + 36*A*a^2*b^10 + 18*B*a*b^11)*cos(d*x + c)^2 + 3*(20*B*a
^10*b^2 - 5*A*a^9*b^3 - 77*B*a^8*b^4 + 20*A*a^7*b^5 + 110*B*a^6*b^6 - 35*A*
a^5*b^7 - 59*B*a^4*b^8 + 20*A*a^3*b^9 + 6*B*a^2*b^10)*cos(d*x + c))*sin(d*x
 + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^16)*d*cos(d*x +
 c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^15)*d*cos(d*
x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14)*d*
cos(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d
), -1/6*(6*(4*B*a^9*b^3 - A*a^8*b^4 - 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5
*b^7 - 6*A*a^4*b^8 - 16*B*a^3*b^9 + 4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12)*d*x
*cos(d*x + c)^3 + 18*(4*B*a^10*b^2 - A*a^9*b^3 - 16*B*a^8*b^4 + 4*A*a^7*b^5
 + 24*B*a^6*b^6 - 6*A*a^5*b^7 - 16*B*a^4*b^8 + 4*A*a^3*b^9 + 4*B*a^2*b^10 -
 A*a*b^11)*d*x*cos(d*x + c)^2 + 18*(4*B*a^11*b - A*a^10*b^2 - 16*B*a^9*b^3
+ 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^5*b^7 + 4*A*a^4*b^8 + 4
*B*a^3*b^9 - A*a^2*b^10)*d*x*cos(d*x + c) + 6*(4*B*a^12 - A*a^11*b - 16*B*a
^10*b^2 + 4*A*a^9*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6 + 4*A*a^5
*b^7 + 4*B*a^4*b^8 - A*a^3*b^9)*d*x - 3*(8*B*a^11 - 2*A*a^10*b - 28*B*a^9*b
^2 + 7*A*a^8*b^3 + 35*B*a^7*b^4 - 8*A*a^6*b^5 - 20*B*a^5*b^6 + 8*A*a^4*b^7
+ (8*B*a^8*b^3 - 2*A*a^7*b^4 - 28*B*a^6*b^5 + 7*A*a^5*b^6 + 35*B*a^4*b^7 -
8*A*a^3*b^8 - 20*B*a^2*b^9 + 8*A*a*b^10)*cos(d*x + c)^3 + 3*(8*B*a^9*b^2 -
2*A*a^8*b^3 - 28*B*a^7*b^4 + 7*A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*
B*a^3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + 3*(8*B*a^10*b - 2*A*a^9*b^2 - 28*
B*a^8*b^3 + 7*A*a^7*b^4 + 35*B*a^6*b^5 - 8*A*a^5*b^6 - 20*B*a^4*b^7 + 8*A*a
^3*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^
2 - b^2)*sin(d*x + c))) - (24*B*a^11*b - 6*A*a^10*b^2 - 92*B*a^9*b^3 + 23*A
*a^8*b^4 + 133*B*a^7*b^5 - 43*A*a^6*b^6 - 71*B*a^5*b^7 + 26*A*a^4*b^8 + 6*B
*a^3*b^9 + 6*(B*a^8*b^4 - 4*B*a^6*b^6 + 6*B*a^4*b^8 - 4*B*a^2*b^10 + B*b^12
)*cos(d*x + c)^3 + (44*B*a^9*b^3 - 11*A*a^8*b^4 - 169*B*a^7*b^5 + 43*A*a^6*
b^6 + 239*B*a^5*b^7 - 68*A*a^4*b^8 - 132*B*a^3*b^9 + 36*A*a^2*b^10 + 18*B*a
*b^11)*cos(d*x + c)^2 + 3*(20*B*a^10*b^2 - 5*A*a^9*b^3 - 77*B*a^8*b^4 + 20*
A*a^7*b^5 + 110*B*a^6*b^6 - 35*A*a^5*b^7 - 59*B*a^4*b^8 + 20*A*a^3*b^9 + 6*
B*a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12

```
 - 4*a^2*b^14 + b^16)*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^1
1 - 4*a^3*b^13 + a*b^15)*d*cos(d*x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6
*b^10 - 4*a^4*b^12 + a^2*b^14)*d*cos(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a
^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d)]
```

giac [B] time = 3.43, size = 966, normalized size = 2.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(8*B*a^8 - 2*A*a^7*b - 28*B*a^6*b^2 + 7*A*a^5*b^3 + 35*B*a^4*b^4 -
8*A*a^3*b^5 - 20*B*a^2*b^6 + 8*A*a*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*s
gn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(a^2 - b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*sqrt(a^2 - b^2)
) - (18*B*a^9*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^8*b*tan(1/2*d*x + 1/2*c)^5 - 4
2*B*a^8*b*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 - 24
*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 11
7*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 45*A*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 -
24*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 -
105*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 60*A*a^3*b^6*tan(1/2*d*x + 1/2*c)^5
+ 60*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^5
 + 36*B*a^9*tan(1/2*d*x + 1/2*c)^3 - 12*A*a^8*b*tan(1/2*d*x + 1/2*c)^3 - 15
2*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 + 56*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^3 +
236*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 - 116*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^3
 - 120*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 72*A*a^2*b^7*tan(1/2*d*x + 1/2*c)
^3 + 18*B*a^9*tan(1/2*d*x + 1/2*c) - 6*A*a^8*b*tan(1/2*d*x + 1/2*c) + 42*B*
a^8*b*tan(1/2*d*x + 1/2*c) - 15*A*a^7*b^2*tan(1/2*d*x + 1/2*c) - 24*B*a^7*b
^2*tan(1/2*d*x + 1/2*c) + 6*A*a^6*b^3*tan(1/2*d*x + 1/2*c) - 117*B*a^6*b^3*
tan(1/2*d*x + 1/2*c) + 45*A*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*B*a^5*b^4*tan
(1/2*d*x + 1/2*c) + 6*A*a^4*b^5*tan(1/2*d*x + 1/2*c) + 105*B*a^4*b^5*tan(1/
2*d*x + 1/2*c) - 60*A*a^3*b^6*tan(1/2*d*x + 1/2*c) + 60*B*a^3*b^6*tan(1/2*d
*x + 1/2*c) - 36*A*a^2*b^7*tan(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b^6 + 3*
a^2*b^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a +
b)^3) + 3*(4*B*a - A*b)*(d*x + c)/b^5 - 6*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*
d*x + 1/2*c)^2 + 1)*b^4))/d
```

maple [B] time = 0.10, size = 2787, normalized size = 6.81

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d/b^4*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-1/d*a^5/b^2/(a*tan(1/
2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3
)*tan(1/2*d*x+1/2*c)*A-4/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-2/d*a^7/b^
4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*
c)*(a-b)/((a-b)*(a+b))^(1/2))*A-116/3/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3*B-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a
-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+1/d*a^5/b^2/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)
*tan(1/2*d*x+1/2*c)^5*A-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2
*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+6/d*a^7
/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2
*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+5/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c
)^5*B+6/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)
/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+2/d*a^6/b^3/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)*B-18/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-18/d*a^5/b^2/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*
a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-5/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B
+12/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*
a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-4/d*a^6/b^3/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(
1/2*d*x+1/2*c)^3*A+44/3/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-24/d*a^2*
b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^
2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-12/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c
)*A+6/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(
a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-12/d*a^2*b/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1
/2*d*x+1/2*c)^5*A+6/d*a^7/b^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-2/d*a^6/b^3
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3
*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+8/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/
((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+
8/d*a^8/b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/
2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+7/d*a^5/b^2/(a^6-3*a^4*b^2+3*a^2*
b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^
(1/2))*A-28/d*a^6/b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arc
tan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+35/d*a^4/b/(a^6-3*a^4*b
^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b
```

```
)*(a+b))^(1/2))*B-20/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1
/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+20/d*a^3/(a*tan(
1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b
^3)*tan(1/2*d*x+1/2*c)*B+40/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+4/d*a^3
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3
*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+20/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d
*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B
-8/d/b^5*B*arctan(tan(1/2*d*x+1/2*c))*a-8/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^
6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
*A+2/d/b^4*A*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.51, size = 7823, normalized size = 19.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^4,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1i)*(A*b - 4*B*a)*1i)/(b^5*d) - ((tan(c/2 + (d*x)
/2)^7*(12*A*a^2*b^5 - 2*B*b^7 - 8*B*a^7 + 4*A*a^3*b^4 - 6*A*a^4*b^3 - A*a^5
*b^2 + 6*B*a^2*b^5 - 26*B*a^3*b^4 - 11*B*a^4*b^3 + 24*B*a^5*b^2 + 2*A*a^6*b
 + 2*B*a*b^6 + 4*B*a^6*b))/(b^4*(a + b)^3*(a - b)) - (tan(c/2 + (d*x)/2)^3*
(72*B*a^8 + 18*B*b^8 + 36*A*a^2*b^6 - 96*A*a^3*b^5 - 14*A*a^4*b^4 + 59*A*a^
5*b^3 + 3*A*a^6*b^2 - 72*B*a^2*b^6 - 60*B*a^3*b^5 + 273*B*a^4*b^4 + 47*B*a^
5*b^3 - 236*B*a^6*b^2 - 18*A*a^7*b - 12*B*a^7*b))/(3*b^4*(a + b)^2*(a - b)^
3) - (tan(c/2 + (d*x)/2)^5*(72*B*a^8 + 18*B*b^8 - 36*A*a^2*b^6 - 96*A*a^3*b
^5 + 14*A*a^4*b^4 + 59*A*a^5*b^3 - 3*A*a^6*b^2 - 72*B*a^2*b^6 + 60*B*a^3*b^
5 + 273*B*a^4*b^4 - 47*B*a^5*b^3 - 236*B*a^6*b^2 - 18*A*a^7*b + 12*B*a^7*b)
)/(3*b^4*(a + b)^3*(a - b)^2) + (tan(c/2 + (d*x)/2)*(2*B*b^7 - 8*B*a^7 + 12
*A*a^2*b^5 - 4*A*a^3*b^4 - 6*A*a^4*b^3 + A*a^5*b^2 - 6*B*a^2*b^5 - 26*B*a^3
*b^4 + 11*B*a^4*b^3 + 24*B*a^5*b^2 + 2*A*a^6*b + 2*B*a*b^6 - 4*B*a^6*b))/(b
```

^4*(a + b)*(a - b)^3))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2 + (d*x)/2)^4*(6*a*b^2 - 6*a^3) + tan(c/2 + (d*x)/2)^2*(6*a^2*b + 4*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)^6*(4*a^3 - 6*a^2*b + 2*b^3) + a^3 + b^3 + tan(c/2 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (log(tan(c/2 + (d*x)/2) - 1i)*(A*b*1i - B*a*4i))/(b^5*d) - (a*atan(((a*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^16 + 128*B^2*a^16 - 8*A^2*a*b^15 - 128*B^2*a^15*b + 44*A^2*a^2*b^14 + 48*A^2*a^3*b^13 - 92*A^2*a^4*b^12 - 120*A^2*a^5*b^11 + 156*A^2*a^6*b^10 + 160*A^2*a^7*b^9 - 164*A^2*a^8*b^8 - 120*A^2*a^9*b^7 + 117*A^2*a^10*b^6 + 48*A^2*a^11*b^5 - 48*A^2*a^12*b^4 - 8*A^2*a^13*b^3 + 8*A^2*a^14*b^2 + 64*B^2*a^2*b^14 - 128*B^2*a^3*b^13 + 80*B^2*a^4*b^12 + 768*B^2*a^5*b^11 - 824*B^2*a^6*b^10 - 1920*B^2*a^7*b^9 + 2025*B^2*a^8*b^8 + 2560*B^2*a^9*b^7 - 2600*B^2*a^10*b^6 - 1920*B^2*a^11*b^5 + 1920*B^2*a^12*b^4 + 768*B^2*a^13*b^3 - 768*B^2*a^14*b^2 - 32*A*B*a*b^15 - 64*A*B*a^15*b + 64*A*B*a^2*b^14 - 160*A*B*a^3*b^13 - 384*A*B*a^4*b^12 + 592*A*B*a^5*b^11 + 960*A*B*a^6*b^10 - 1128*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 + 1306*A*B*a^9*b^7 + 960*A*B*a^10*b^6 - 948*A*B*a^11*b^5 - 384*A*B*a^12*b^4 + 384*A*B*a^13*b^3 + 64*A*B*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) + (a*(-(a + b)^7*(a - b)^7)^(1/2)*((8*(4*A*b^24 - 12*A*a^2*b^22 + 64*A*a^3*b^21 + 20*A*a^4*b^20 - 110*A*a^5*b^19 - 30*A*a^6*b^18 + 110*A*a^7*b^17 + 30*A*a^8*b^16 - 70*A*a^9*b^15 - 14*A*a^10*b^14 + 26*A*a^11*b^13 + 2*A*a^12*b^12 - 4*A*a^13*b^11 + 40*B*a^2*b^22 + 72*B*a^3*b^21 - 190*B*a^4*b^20 - 146*B*a^5*b^19 + 386*B*a^6*b^18 + 174*B*a^7*b^17 - 434*B*a^8*b^16 - 126*B*a^9*b^15 + 286*B*a^10*b^14 + 50*B*a^11*b^13 - 104*B*a^12*b^12 - 8*B*a^13*b^11 + 16*B*a^14*b^10 - 16*A*a*b^23 - 16*B*a*b^23))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) - (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11 - 8*a^14*b^10))/((b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6))/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)))*(-(a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6)*1i)/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)) + (a*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^16 + 128*B^2*a^16 - 8*A^2*a*b^15 - 128*B^2*a^15*b + 44*A^2*a^2*b^14 + 48*A^2*a^3*b^13 - 92*A^2*a^4*b^12 - 120*A^2*a^5*b^11 + 156*A^2*a^6*b^10 + 160*A^2*a^7*b^9 - 164*A^2*a^8*b^8 - 120*A^2*a^9*b^7 + 117*A^2*a^10*b^6 + 48*A^2*a^11*b^5 - 48*A^2*a^12*b^4 - 8*A^2*a^13*b^3 + 8*A^2*a^14*b^2 + 64*B^2*a^2*b^14 - 128*B^2*a^3*b^13

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+ 80*B^2*a^4*b^12 + 768*B^2*a^5*b^11 - 824*B^2*a^6*b^10 - 1920*B^2*a^7*b^9
+ 2025*B^2*a^8*b^8 + 2560*B^2*a^9*b^7 - 2600*B^2*a^10*b^6 - 1920*B^2*a^11*b
^5 + 1920*B^2*a^12*b^4 + 768*B^2*a^13*b^3 - 768*B^2*a^14*b^2 - 32*A*B*a*b^1
5 - 64*A*B*a^15*b + 64*A*B*a^2*b^14 - 160*A*B*a^3*b^13 - 384*A*B*a^4*b^12 +
 592*A*B*a^5*b^11 + 960*A*B*a^6*b^10 - 1128*A*B*a^7*b^9 - 1280*A*B*a^8*b^8
+ 1306*A*B*a^9*b^7 + 960*A*B*a^10*b^6 - 948*A*B*a^11*b^5 - 384*A*B*a^12*b^4
 + 384*A*B*a^13*b^3 + 64*A*B*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3
*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11
+ 5*a^9*b^10 - a^10*b^9 - a^11*b^8) - (a*(-(a + b)^7*(a - b)^7)^(1/2)*((8*(
4*A*b^24 - 12*A*a^2*b^22 + 64*A*a^3*b^21 + 20*A*a^4*b^20 - 110*A*a^5*b^19 -
 30*A*a^6*b^18 + 110*A*a^7*b^17 + 30*A*a^8*b^16 - 70*A*a^9*b^15 - 14*A*a^10
*b^14 + 26*A*a^11*b^13 + 2*A*a^12*b^12 - 4*A*a^13*b^11 + 40*B*a^2*b^22 + 72
*B*a^3*b^21 - 190*B*a^4*b^20 - 146*B*a^5*b^19 + 386*B*a^6*b^18 + 174*B*a^7*
b^17 - 434*B*a^8*b^16 - 126*B*a^9*b^15 + 286*B*a^10*b^14 + 50*B*a^11*b^13 -
 104*B*a^12*b^12 - 8*B*a^13*b^11 + 16*B*a^14*b^10 - 16*A*a*b^23 - 16*B*a*b^
23))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 -
 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^1
2) + (4*a*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^
7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 2
0*B*a*b^6)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^1
9 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^
14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11 - 8*a^14*b^10))/((b^19 - 7*a
^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^
7 - a^14*b^5)*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a
^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 -
a^11*b^8)))*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 -
 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6))/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^1
5 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)))*(-(a
 + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 3
5*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6)*1i)/(2*(b^19 - 7*a^2*b
^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 -
a^14*b^5)))/((16*(256*B^3*a^16 - 16*A^3*a*b^15 - 128*B^3*a^15*b - 48*A^3*a^
2*b^14 + 64*A^3*a^3*b^13 + 64*A^3*a^4*b^12 - 110*A^3*a^5*b^11 - 66*A^3*a^6*
b^10 + 110*A^3*a^7*b^9 + 34*A^3*a^8*b^8 - 70*A^3*a^9*b^7 - 11*A^3*a^10*b^6
+ 26*A^3*a^11*b^5 + 2*A^3*a^12*b^4 - 4*A^3*a^13*b^3 + 640*B^3*a^4*b^12 + 96
0*B^3*a^5*b^11 - 3040*B^3*a^6*b^10 - 2560*B^3*a^7*b^9 + 6176*B^3*a^8*b^8 +
3204*B^3*a^9*b^7 - 6944*B^3*a^10*b^6 - 2176*B^3*a^11*b^5 + 4576*B^3*a^12*b^
4 + 800*B^3*a^13*b^3 - 1664*B^3*a^14*b^2 - 192*A*B^2*a^15*b - 576*A*B^2*a^3
*b^13 - 1104*A*B^2*a^4*b^12 + 2544*A*B^2*a^5*b^11 + 2376*A*B^2*a^6*b^10 - 4
848*A*B^2*a^7*b^9 - 2649*A*B^2*a^8*b^8 + 5232*A*B^2*a^9*b^7 + 1632*A*B^2*a^
10*b^6 - 3408*A*B^2*a^11*b^5 - 576*A*B^2*a^12*b^4 + 1248*A*B^2*a^13*b^3 + 9
6*A*B^2*a^14*b^2 + 168*A^2*B*a^2*b^14 + 408*A^2*B*a^3*b^13 - 702*A^2*B*a^4*
b^12 - 690*A^2*B*a^5*b^11 + 1266*A^2*B*a^6*b^10 + 726*A^2*B*a^7*b^9 - 1314*
A^2*B*a^8*b^8 - 408*A^2*B*a^9*b^7 + 846*A^2*B*a^10*b^6 + 138*A^2*B*a^11*b^5
 - 312*A^2*B*a^12*b^4 - 24*A^2*B*a^13*b^3 + 48*A^2*B*a^14*b^2))/(a*b^22 + b
```

^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 1
0*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) + (a*((8*tan(
c/2 + (d*x)/2)*(4*A^2*b^16 + 128*B^2*a^16 - 8*A^2*a*b^15 - 128*B^2*a^15*b +
 44*A^2*a^2*b^14 + 48*A^2*a^3*b^13 - 92*A^2*a^4*b^12 - 120*A^2*a^5*b^11 + 1
56*A^2*a^6*b^10 + 160*A^2*a^7*b^9 - 164*A^2*a^8*b^8 - 120*A^2*a^9*b^7 + 117
*A^2*a^10*b^6 + 48*A^2*a^11*b^5 - 48*A^2*a^12*b^4 - 8*A^2*a^13*b^3 + 8*A^2*
a^14*b^2 + 64*B^2*a^2*b^14 - 128*B^2*a^3*b^13 + 80*B^2*a^4*b^12 + 768*B^2*a
^5*b^11 - 824*B^2*a^6*b^10 - 1920*B^2*a^7*b^9 + 2025*B^2*a^8*b^8 + 2560*B^2
*a^9*b^7 - 2600*B^2*a^10*b^6 - 1920*B^2*a^11*b^5 + 1920*B^2*a^12*b^4 + 768*
B^2*a^13*b^3 - 768*B^2*a^14*b^2 - 32*A*B*a*b^15 - 64*A*B*a^15*b + 64*A*B*a^
2*b^14 - 160*A*B*a^3*b^13 - 384*A*B*a^4*b^12 + 592*A*B*a^5*b^11 + 960*A*B*a
^6*b^10 - 1128*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 + 1306*A*B*a^9*b^7 + 960*A*B*
a^10*b^6 - 948*A*B*a^11*b^5 - 384*A*B*a^12*b^4 + 384*A*B*a^13*b^3 + 64*A*B*
a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*
b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^1
1*b^8) + (a*(-(a + b)^7*(a - b)^7)^(1/2)*((8*(4*A*b^24 - 12*A*a^2*b^22 + 64
*A*a^3*b^21 + 20*A*a^4*b^20 - 110*A*a^5*b^19 - 30*A*a^6*b^18 + 110*A*a^7*b^
17 + 30*A*a^8*b^16 - 70*A*a^9*b^15 - 14*A*a^10*b^14 + 26*A*a^11*b^13 + 2*A*
a^12*b^12 - 4*A*a^13*b^11 + 40*B*a^2*b^22 + 72*B*a^3*b^21 - 190*B*a^4*b^20
- 146*B*a^5*b^19 + 386*B*a^6*b^18 + 174*B*a^7*b^17 - 434*B*a^8*b^16 - 126*B
*a^9*b^15 + 286*B*a^10*b^14 + 50*B*a^11*b^13 - 104*B*a^12*b^12 - 8*B*a^13*b
^11 + 16*B*a^14*b^10 - 16*A*a*b^23 - 16*B*a*b^23))/(a*b^22 + b^23 - 5*a^2*b
^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 +
5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) - (4*a*tan(c/2 + (d*x)/2)*
(-(a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3
 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6)*(8*a*b^23 - 8*a^2*
b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^
17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b
^12 + 8*a^13*b^11 - 8*a^14*b^10))/((b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^
6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)*(a*b^18 + b^19
- 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^
7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*(8*A*b^7 + 8*B*a^
7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 2
0*B*a*b^6))/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11
 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)))*(-(a + b)^7*(a - b)^7)^(1/2)*(8*A
*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 -
2*A*a^6*b - 20*B*a*b^6))/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13
+ 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)) - (a*((8*tan(c/2 + (d
x)/2)(4*A^2*b^16 + 128*B^2*a^16 - 8*A^2*a*b^15 - 128*B^2*a^15*b + 44*A^2*
a^2*b^14 + 48*A^2*a^3*b^13 - 92*A^2*a^4*b^12 - 120*A^2*a^5*b^11 + 156*A^2*a
^6*b^10 + 160*A^2*a^7*b^9 - 164*A^2*a^8*b^8 - 120*A^2*a^9*b^7 + 117*A^2*a^1
0*b^6 + 48*A^2*a^11*b^5 - 48*A^2*a^12*b^4 - 8*A^2*a^13*b^3 + 8*A^2*a^14*b^2
 + 64*B^2*a^2*b^14 - 128*B^2*a^3*b^13 + 80*B^2*a^4*b^12 + 768*B^2*a^5*b^11
- 824*B^2*a^6*b^10 - 1920*B^2*a^7*b^9 + 2025*B^2*a^8*b^8 + 2560*B^2*a^9*b^7
 - 2600*B^2*a^10*b^6 - 1920*B^2*a^11*b^5 + 1920*B^2*a^12*b^4 + 768*B^2*a^13

```
*b^3 - 768*B^2*a^14*b^2 - 32*A*B*a*b^15 - 64*A*B*a^15*b + 64*A*B*a^2*b^14 -
 160*A*B*a^3*b^13 - 384*A*B*a^4*b^12 + 592*A*B*a^5*b^11 + 960*A*B*a^6*b^10
- 1128*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 + 1306*A*B*a^9*b^7 + 960*A*B*a^10*b^6
 - 948*A*B*a^11*b^5 - 384*A*B*a^12*b^4 + 384*A*B*a^13*b^3 + 64*A*B*a^14*b^2
))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 1
0*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) -
 (a*(-(a + b)^7*(a - b)^7)^(1/2)*((8*(4*A*b^24 - 12*A*a^2*b^22 + 64*A*a^3*b
^21 + 20*A*a^4*b^20 - 110*A*a^5*b^19 - 30*A*a^6*b^18 + 110*A*a^7*b^17 + 30*
A*a^8*b^16 - 70*A*a^9*b^15 - 14*A*a^10*b^14 + 26*A*a^11*b^13 + 2*A*a^12*b^1
2 - 4*A*a^13*b^11 + 40*B*a^2*b^22 + 72*B*a^3*b^21 - 190*B*a^4*b^20 - 146*B*
a^5*b^19 + 386*B*a^6*b^18 + 174*B*a^7*b^17 - 434*B*a^8*b^16 - 126*B*a^9*b^1
5 + 286*B*a^10*b^14 + 50*B*a^11*b^13 - 104*B*a^12*b^12 - 8*B*a^13*b^11 + 16
*B*a^14*b^10 - 16*A*a*b^23 - 16*B*a*b^23))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*
a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^
15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) + (4*a*tan(c/2 + (d*x)/2)*(-(a + b
)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*
a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^6)*(8*a*b^23 - 8*a^2*b^22 - 4
8*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160
*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*
a^13*b^11 - 8*a^14*b^10))/((b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 +
 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5)*(a*b^18 + b^19 - 5*a^2*
b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 +
 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8)))*(8*A*b^7 + 8*B*a^7 - 8*A*
a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*b - 20*B*a*b^
6))/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8*b^11 - 21*a^
10*b^9 + 7*a^12*b^7 - a^14*b^5)))*(-(a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8
*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*b^2 - 2*A*a^6*
b - 20*B*a*b^6))/(2*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*a^6*b^13 + 35*a^8
*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5))))*(-(a + b)^7*(a - b)^7)^(1/2
)*(8*A*b^7 + 8*B*a^7 - 8*A*a^2*b^5 + 7*A*a^4*b^3 + 35*B*a^3*b^4 - 28*B*a^5*
b^2 - 2*A*a^6*b - 20*B*a*b^6)*1i)/(d*(b^19 - 7*a^2*b^17 + 21*a^4*b^15 - 35*
a^6*b^13 + 35*a^8*b^11 - 21*a^10*b^9 + 7*a^12*b^7 - a^14*b^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.274 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=301

$$\frac{a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{a^2\left(3a^3B-8ab^2B+5Ab^3\right)\sin(c+dx)}{6b^3d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{a\left(9a^5B-28a^3b^2B+a^2Ab^3+34a\right.}{6b^3d\left(a^2-b^2\right)^3(a+b}$$

```
[Out] B*x/b^4-(3*A*a^2*b^5+2*A*b^7+2*B*a^7-7*B*a^5*b^2+8*B*a^3*b^4-8*B*a*b^6)*arc
tan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^4/(a+b)^(7/2)
/d+1/3*a*(A*b-B*a)*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^3
+1/6*a^2*(5*A*b^3+3*B*a^3-8*B*a*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*cos(
d*x+c))^2-1/6*a*(A*a^2*b^3-16*A*b^5+9*B*a^5-28*B*a^3*b^2+34*B*a*b^4)*sin(d*
x+c)/b^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 1.21, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2989, 3031, 3021, 2735, 2659, 205}

$$-\frac{\left(3a^2Ab^5-7a^5b^2B+8a^3b^4B+2a^7B-8ab^6B+2Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a(Ab-aB)\sin(c+dx)\cos^2(c}{3bd\left(a^2-b^2\right)(a+b\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (B*x)/b^4 - ((3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B - 7*a^5*b^2*B + 8*a^3*b^4*B -
 8*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7
/2)*b^4*(a + b)^(7/2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (a^2*(5*A*b^3 + 3*a^3*B - 8*a*b^2*
B)*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - (a*(a^2*A
*b^3 - 16*A*b^5 + 9*a^5*B - 28*a^3*b^2*B + 34*a*b^4*B)*Sin[c + d*x])/(6*b^3
*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
```

```
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(-2a(Ab-aB)+3b(Ab-aB)\,c\right.}{(a+b\cos(c}}{3b\left(a^2\right.}$$

$$= \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2B\right)\sin(}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2B\right)\sin(}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+}$$

$$= \frac{Bx}{b^4} + \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2\right.}{6b^3\left(a^2-b^2\right)^2 d(a+b\,c}$$

$$= \frac{Bx}{b^4} + \frac{a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2\right.}{6b^3\left(a^2-b^2\right)^2 d(a+b\,c}$$

$$= \frac{Bx}{b^4} - \frac{\left(3a^2Ab^5+2Ab^7+2a^7B-7a^5b^2B+8a^3b^4B-8ab^6B\right)\tan^{-1}\left(\frac{\sqrt{a}}{}\right.}{(a-b)^{7/2}b^4(a+b)^{7/2}d}$$

Mathematica [B] time = 3.31, size = 717, normalized size = 2.38

$$\frac{24a^9Bc+24a^9Bdx-24a^8bB\sin(c+dx)-30a^7b^2B\sin(2(c+dx))-36a^7b^2Bc-36a^7b^2Bdx+57a^6b^3B\sin(c+dx)-11a^6b^3B\sin(3(c+dx))+6a^6b^3Bc\cos(3(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-24*(3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B - 7*a^5*b^2*B + 8*a^3*b^4*B - 8*a*b^
6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/
```

```
2) + (24*a^9*B*c - 36*a^7*b^2*B*c - 36*a^5*b^4*B*c + 84*a^3*b^6*B*c - 36*a*
b^8*B*c + 24*a^9*B*d*x - 36*a^7*b^2*B*d*x - 36*a^5*b^4*B*d*x + 84*a^3*b^6*B
*d*x - 36*a*b^8*B*d*x + 18*b*(a^2 - b^2)^3*(4*a^2 + b^2)*B*(c + d*x)*Cos[c
+ d*x] + 36*a*b^2*(a^2 - b^2)^3*B*(c + d*x)*Cos[2*(c + d*x)] + 6*a^6*b^3*B*
c*Cos[3*(c + d*x)] - 18*a^4*b^5*B*c*Cos[3*(c + d*x)] + 18*a^2*b^7*B*c*Cos[3
*(c + d*x)] - 6*b^9*B*c*Cos[3*(c + d*x)] + 6*a^6*b^3*B*d*x*Cos[3*(c + d*x)]
 - 18*a^4*b^5*B*d*x*Cos[3*(c + d*x)] + 18*a^2*b^7*B*d*x*Cos[3*(c + d*x)] -
6*b^9*B*d*x*Cos[3*(c + d*x)] + 18*a^5*A*b^4*Sin[c + d*x] + 39*a^3*A*b^6*Sin
[c + d*x] + 18*a*A*b^8*Sin[c + d*x] - 24*a^8*b*B*Sin[c + d*x] + 57*a^6*b^3*
B*Sin[c + d*x] - 72*a^4*b^5*B*Sin[c + d*x] - 36*a^2*b^7*B*Sin[c + d*x] + 6*
a^4*A*b^5*Sin[2*(c + d*x)] + 54*a^2*A*b^7*Sin[2*(c + d*x)] - 30*a^7*b^2*B*S
in[2*(c + d*x)] + 90*a^5*b^4*B*Sin[2*(c + d*x)] - 120*a^3*b^6*B*Sin[2*(c +
d*x)] + 2*a^5*A*b^4*Sin[3*(c + d*x)] - 5*a^3*A*b^6*Sin[3*(c + d*x)] + 18*a*
A*b^8*Sin[3*(c + d*x)] - 11*a^6*b^3*B*Sin[3*(c + d*x)] + 32*a^4*b^5*B*Sin[3
*(c + d*x)] - 36*a^2*b^7*B*Sin[3*(c + d*x)])/((a^2 - b^2)^3*(a + b*Cos[c +
d*x])^3))/(24*b^4*d)
```

fricas [B] time = 1.31, size = 1857, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(12*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11)*d*
x*cos(d*x + c)^3 + 36*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8
+ B*a*b^10)*d*x*cos(d*x + c)^2 + 36*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^5 -
 4*B*a^4*b^7 + B*a^2*b^9)*d*x*cos(d*x + c) + 12*(B*a^11 - 4*B*a^9*b^2 + 6*B
*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*d*x + 3*(2*B*a^10 - 7*B*a^8*b^2 + 8*B*a
^6*b^4 + 3*A*a^5*b^5 - 8*B*a^4*b^6 + 2*A*a^3*b^7 + (2*B*a^7*b^3 - 7*B*a^5*b
^5 + 8*B*a^3*b^7 + 3*A*a^2*b^8 - 8*B*a*b^9 + 2*A*b^10)*cos(d*x + c)^3 + 3*(
2*B*a^8*b^2 - 7*B*a^6*b^4 + 8*B*a^4*b^6 + 3*A*a^3*b^7 - 8*B*a^2*b^8 + 2*A*a
*b^9)*cos(d*x + c)^2 + 3*(2*B*a^9*b - 7*B*a^7*b^3 + 8*B*a^5*b^5 + 3*A*a^4*b
^6 - 8*B*a^3*b^7 + 2*A*a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*c
os(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x
+ c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x +
 c) + a^2)) - 2*(6*B*a^10*b - 23*B*a^8*b^3 - 4*A*a^7*b^4 + 43*B*a^6*b^5 - 7
*A*a^5*b^6 - 26*B*a^4*b^7 + 11*A*a^3*b^8 + (11*B*a^8*b^3 - 2*A*a^7*b^4 - 43
*B*a^6*b^5 + 7*A*a^5*b^6 + 68*B*a^4*b^7 - 23*A*a^3*b^8 - 36*B*a^2*b^9 + 18*
A*a*b^10)*cos(d*x + c)^2 + 3*(5*B*a^9*b^2 - 20*B*a^7*b^4 - A*a^6*b^5 + 35*B
*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^3*b^8 + 9*A*a^2*b^9)*cos(d*x + c))*sin(d*x
+ c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d*cos(d*x + c
)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x
+ c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos
```

```
(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 + a^3*b^12)*d),
1/6*(6*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11)*d*x*c
os(d*x + c)^3 + 18*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8 + B
*a*b^10)*d*x*cos(d*x + c)^2 + 18*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^5 - 4*
B*a^4*b^7 + B*a^2*b^9)*d*x*cos(d*x + c) + 6*(B*a^11 - 4*B*a^9*b^2 + 6*B*a^7
*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*d*x - 3*(2*B*a^10 - 7*B*a^8*b^2 + 8*B*a^6*b
^4 + 3*A*a^5*b^5 - 8*B*a^4*b^6 + 2*A*a^3*b^7 + (2*B*a^7*b^3 - 7*B*a^5*b^5 +
 8*B*a^3*b^7 + 3*A*a^2*b^8 - 8*B*a*b^9 + 2*A*b^10)*cos(d*x + c)^3 + 3*(2*B*
a^8*b^2 - 7*B*a^6*b^4 + 8*B*a^4*b^6 + 3*A*a^3*b^7 - 8*B*a^2*b^8 + 2*A*a*b^9
)*cos(d*x + c)^2 + 3*(2*B*a^9*b - 7*B*a^7*b^3 + 8*B*a^5*b^5 + 3*A*a^4*b^6 -
 8*B*a^3*b^7 + 2*A*a^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*
x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*B*a^10*b - 23*B*a^8*b^3 -
4*A*a^7*b^4 + 43*B*a^6*b^5 - 7*A*a^5*b^6 - 26*B*a^4*b^7 + 11*A*a^3*b^8 + (1
1*B*a^8*b^3 - 2*A*a^7*b^4 - 43*B*a^6*b^5 + 7*A*a^5*b^6 + 68*B*a^4*b^7 - 23*
A*a^3*b^8 - 36*B*a^2*b^9 + 18*A*a*b^10)*cos(d*x + c)^2 + 3*(5*B*a^9*b^2 - 2
0*B*a^7*b^4 - A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^3*b^8 + 9*A*a
^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*
a^2*b^13 + b^15)*d*cos(d*x + c)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4
*a^3*b^12 + a*b^14)*d*cos(d*x + c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9
- 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8
 - 4*a^5*b^10 + a^3*b^12)*d)]
```

giac [B] time = 2.19, size = 813, normalized size = 2.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(2*B*a^7 - 7*B*a^5*b^2 + 8*B*a^3*b^4 + 3*A*a^2*b^5 - 8*B*a*b^6 + 2*A
*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^4 - 3*a^
4*b^6 + 3*a^2*b^8 - b^10)*sqrt(a^2 - b^2)) + 3*(d*x + c)*B/b^4 - (6*B*a^8*t
an(1/2*d*x + 1/2*c)^5 - 15*B*a^7*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^6*b^2*tan
(1/2*d*x + 1/2*c)^5 - 6*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 45*B*a^5*b^3*tan
(1/2*d*x + 1/2*c)^5 + 3*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^4*b^4*tan(
1/2*d*x + 1/2*c)^5 - 6*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*B*a^3*b^5*tan(
1/2*d*x + 1/2*c)^5 + 27*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b^6*tan
(1/2*d*x + 1/2*c)^5 - 18*A*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^8*tan(1/2*
d*x + 1/2*c)^3 - 56*B*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^5*b^3*tan(1/2*
d*x + 1/2*c)^3 + 116*B*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 32*A*a^3*b^5*tan(1/
2*d*x + 1/2*c)^3 - 72*B*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*A*a*b^7*tan(1/2
*d*x + 1/2*c)^3 + 6*B*a^8*tan(1/2*d*x + 1/2*c) + 15*B*a^7*b*tan(1/2*d*x + 1
/2*c) - 6*B*a^6*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^5*b^3*tan(1/2*d*x + 1/2*c)
```

```
 - 45*B*a^5*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6
*B*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6*A*a^3*b^5*tan(1/2*d*x + 1/2*c) + 60*B*a
^3*b^5*tan(1/2*d*x + 1/2*c) - 27*A*a^2*b^6*tan(1/2*d*x + 1/2*c) + 36*B*a^2*
b^6*tan(1/2*d*x + 1/2*c) - 18*A*a*b^7*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c
)^2 + a + b)^3))/d
```

maple [B] time = 0.10, size = 2158, normalized size = 7.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d*B/b^4*arctan(tan(1/2*d*x+1/2*c))-2/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/
((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-
8/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/
2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*a^3+2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)
^5*A+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/
(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+6/d*a^4/b/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5*B-2/d*a^6/b^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+
a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-1/d*a^5/b^2/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b
^2-b^3)*tan(1/2*d*x+1/2*c)*B+1/d*a^5/b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+
6/d*a^4/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-
3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-3/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2
-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+
1/2*c)*A+3/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a
-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-2/d*a^6/b^3/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)
*tan(1/2*d*x+1/2*c)^5*B+12/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)
^2*b+a+b)^3*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/d/b^
3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^6/(a^2+2*a*b+b^2)
/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/
2*c)^5*A+44/3/d/b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^4
/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B-24/d*b/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^
2)*tan(1/2*d*x+1/2*c)^3*B-12/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)
^2*b+a+b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-12/d*b
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a-b)/(a^3+3*a^2
*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+4/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(
```

```
1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c
)*B+2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^
3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-4/d*a^3/(a*tan(1/2*d*x+1/2*c)^2
-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+
1/2*c)^5*B+4/3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/
(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-3/d*b/(a^6-3*a^4*b^2
+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*
(a+b))^(1/2))*A*a^2+7/d/b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/
2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*a^5+8/d*b^2/(a^6-
3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b
)/((a-b)*(a+b))^(1/2))*B*a-2/d/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+
b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*a^7
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 12.58, size = 9733, normalized size = 32.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^4,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(3*A*a^2*b^4 - 2*B*a^6 + 2*A*a^3*b^3 - 12*B*a^2*b^4
- 4*B*a^3*b^3 + 6*B*a^4*b^2 + 6*A*a*b^5 + B*a^5*b))/((a*b^3 - b^4)*(a + b)^
3) - (tan(c/2 + (d*x)/2)*(2*B*a^6 + 3*A*a^2*b^4 - 2*A*a^3*b^3 + 12*B*a^2*b^
4 - 4*B*a^3*b^3 - 6*B*a^4*b^2 - 6*A*a*b^5 + B*a^5*b))/((a + b)*(3*a*b^5 - b
^6 - 3*a^2*b^4 + a^3*b^3)) + (4*tan(c/2 + (d*x)/2)^3*(A*a^3*b^3 - 3*B*a^6 -
 18*B*a^2*b^4 + 11*B*a^4*b^2 + 9*A*a*b^5))/(3*(a + b)^2*(b^5 - 2*a*b^4 + a^
2*b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*
b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b +
 a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (2*B*
atan(((B*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B
^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^
12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8
```

+ 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^10*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) + (B*((8*(4*A*b^21 + 4*B*b^21 - 6*A*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*a^6*b^15 - 14*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64*B*a^3*b^18 + 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^14 + 30*B*a^8*b^13 - 70*B*a^9*b^12 - 14*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*a^12*b^9 - 4*B*a^13*b^8 - 4*A*a*b^20 - 16*B*a*b^20))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (B*tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8)*8i)/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*1i)/b^4))/b^4 + (B*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^10*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) - (B*((8*(4*A*b^21 + 4*B*b^21 - 6*A*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*a^6*b^15 - 14*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64*B*a^3*b^18 + 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^14 + 30*B*a^8*b^13 - 70*B*a^9*b^12 - 14*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*a^12*b^9 - 4*B*a^13*b^8 - 4*A*a*b^20 - 16*B*a*b^20))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (B*tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8)*8i)/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*1i)/b^4))/b^4)/((16*(4*B^3*a^13 - 4*A*B^2*b^13 + 4*A^2*B*b^13 + 16*B^3*a*b^12 - 2*B^3*a^12*b + 48*B^3*a^2*b^11 - 64*B^3*a^3*b^10 - 64*B^3*a^4*b^9 + 110*B^3*a^5*b^8 + 66*B^3*a^6*b^7 - 110*B^3*a^7*b^6 - 34*B^3*a^8*b^5 + 70*B^3*a^9*b^4 + 11*B^3*a^10*b^3 - 26*B^3*a^11*b^2 - 28*A*B^2*a*b^12 + 6*A*B^2*a^2*b^11 - 22*A*B^2*a^3*b^10 + 6*A*B^2*a^4*b^9 + 14*A*B^2*a^5*b^8 - 14*A*B^2*a^6*b^7 - 20*A*B^2*a^7*b^6 + 6*A*B^2*a^8*b^5 + 6*A*B^2*a^9*b^4 + 12*A^2*B*a^2*b^11 + 9*A^2*B*a^4*b^9))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (B*((8*tan(c/2 + (d*x)/2)*(4*A^2*b

^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^
12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 -
120*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 12
0*B^2*a^9*b^5 + 117*B^2*a^10*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A
*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*
b^5))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12
- 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)
+ (B*((8*(4*A*b^21 + 4*B*b^21 - 6*A*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17
+ 6*A*a^5*b^16 + 14*A*a^6*b^15 - 14*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^1
2 - 12*B*a^2*b^19 + 64*B*a^3*b^18 + 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*a
^6*b^15 + 110*B*a^7*b^14 + 30*B*a^8*b^13 - 70*B*a^9*b^12 - 14*B*a^10*b^11 +
26*B*a^11*b^10 + 2*B*a^12*b^9 - 4*B*a^13*b^8 - 4*A*a*b^20 - 16*B*a*b^20))/
(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a
^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (
B*tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 1
20*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 1
20*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8)*8i)/(
b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 -
10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))*
1i)/b^4)*1i)/b^4 + (B*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B
^2*b^14 - 8*B^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 +
44*B^2*a^2*b^12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156
*B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^
2*a^10*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3
*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/(a*b^16 + b^17 -
5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7
*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) - (B*((8*(4*A*b^21 + 4
*B*b^21 - 6*A*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*
a^6*b^15 - 14*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64
*B*a^3*b^18 + 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^
14 + 30*B*a^8*b^13 - 70*B*a^9*b^12 - 14*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*
a^12*b^9 - 4*B*a^13*b^8 - 4*A*a*b^20 - 16*B*a*b^20))/(a*b^19 + b^20 - 5*a^2
*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13
+ 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (B*tan(c/2 + (d*x)/2)*(
8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*
b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11
*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8)*8i)/(b^4*(a*b^16 + b^17 - 5
*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b
^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))*1i)/b^4)*1i)/b^4)))/(b
^4*d) + (atan(((((8*tan(c/2 + (d*x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*b^1
4 - 8*B^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44*B^2
*a^2*b^12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*B^2*a
^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^10
*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3*b^11
+ 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/(a*b^16 + b^17 - 5*a^2

```
*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10
+ 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) + (((8*(4*A*b^21 + 4*B*b^21
- 6*A*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*a^6*b^15
 - 14*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64*B*a^3*b
^18 + 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^14 + 30*
B*a^8*b^13 - 70*B*a^9*b^12 - 14*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*a^12*b^9
 - 4*B*a^13*b^8 - 4*A*a*b^20 - 16*B*a*b^20))/(a*b^19 + b^20 - 5*a^2*b^18 -
5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*
b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (4*tan(c/2 + (d*x)/2)*(-(a + b)
^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^
5*b^2 - 8*B*a*b^6)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120
*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120
*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/((b^18
 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a
^12*b^6 - a^14*b^4)*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13
+ 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^
7 - a^11*b^6)))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b
^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6))/(2*(b^18 - 7*a^2*b^16 + 21*a^4
*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)))*
(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4
 - 7*B*a^5*b^2 - 8*B*a*b^6)*1i)/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^
6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)) + (((8*tan(c/2
 + (d*x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B^2*a*b^13 - 8*B^2*a^
13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^12 + 48*B^2*a^3*b^11
 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160*B^2*a^7*b^7 -
164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^10*b^4 + 48*B^2*a^11*b^3 - 48
*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a
^7*b^7 + 12*A*B*a^9*b^5))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4
*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a
^10*b^7 - a^11*b^6) - (((8*(4*A*b^21 + 4*B*b^21 - 6*A*a^2*b^19 + 6*A*a^3*b^
18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*a^6*b^15 - 14*A*a^7*b^14 - 6*A*a^8*
b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64*B*a^3*b^18 + 20*B*a^4*b^17 - 110*B
*a^5*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^14 + 30*B*a^8*b^13 - 70*B*a^9*b^12
- 14*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*a^12*b^9 - 4*B*a^13*b^8 - 4*A*a*b^2
0 - 16*B*a*b^20))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 +
10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^
10 - a^11*b^9) + (4*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^
7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6)*(8*a*b^2
1 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 -
160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 +
 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/((b^18 - 7*a^2*b^16 + 21*a^4*b^14
 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)*(a*b^16
 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11
 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*(-(a + b)^7
*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*
```

b^2 - 8*B*a*b^6))/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^
8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)))*(-(a + b)^7*(a - b)^7)^(1/2
)*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6)
1i)/(2(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a
^10*b^8 + 7*a^12*b^6 - a^14*b^4)))/((16*(4*B^3*a^13 - 4*A*B^2*b^13 + 4*A^2*
B*b^13 + 16*B^3*a*b^12 - 2*B^3*a^12*b + 48*B^3*a^2*b^11 - 64*B^3*a^3*b^10 -
 64*B^3*a^4*b^9 + 110*B^3*a^5*b^8 + 66*B^3*a^6*b^7 - 110*B^3*a^7*b^6 - 34*B
^3*a^8*b^5 + 70*B^3*a^9*b^4 + 11*B^3*a^10*b^3 - 26*B^3*a^11*b^2 - 28*A*B^2*
a*b^12 + 6*A*B^2*a^2*b^11 - 22*A*B^2*a^3*b^10 + 6*A*B^2*a^4*b^9 + 14*A*B^2*
a^5*b^8 - 14*A*B^2*a^6*b^7 - 20*A*B^2*a^7*b^6 + 6*A*B^2*a^8*b^5 + 6*A*B^2*a
^9*b^4 + 12*A^2*B*a^2*b^11 + 9*A^2*B*a^4*b^9))/(a*b^19 + b^20 - 5*a^2*b^18
- 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^
8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) - (((8*tan(c/2 + (d*x)/2)*(4*A^
2*b^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2
*b^12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^1
0 - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 -
 120*B^2*a^9*b^5 + 117*B^2*a^10*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 3
2*A*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a
^9*b^5))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^
12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^
6) + (((8*(4*A*b^21 + 4*B*b^21 - 6*A*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17
 + 6*A*a^5*b^16 + 14*A*a^6*b^15 - 14*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^
12 - 12*B*a^2*b^19 + 64*B*a^3*b^18 + 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*
a^6*b^15 + 110*B*a^7*b^14 + 30*B*a^8*b^13 - 70*B*a^9*b^12 - 14*B*a^10*b^11
+ 26*B*a^11*b^10 + 2*B*a^12*b^9 - 4*B*a^13*b^8 - 4*A*a*b^20 - 16*B*a*b^20))
/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*
a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) -
(4*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 + 3*A
*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6)*(8*a*b^21 - 8*a^2*b^20 -
48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 16
0*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8
*a^13*b^9 - 8*a^14*b^8))/((b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 +
35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)*(a*b^16 + b^17 - 5*a^2*b
^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 +
5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6)))*(-(a + b)^7*(a - b)^7)^(1/2)
*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6))
/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*
b^8 + 7*a^12*b^6 - a^14*b^4)))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*
a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6))/(2*(b^18 - 7*a^
2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6
 - a^14*b^4)) + (((8*tan(c/2 + (d*x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*b^
14 - 8*B^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44*B^
2*a^2*b^12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*B^2*
a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^1
0*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3*b^11

```
 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/(a*b^16 + b^17 - 5*a^
2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10
 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) - (((8*(4*A*b^21 + 4*B*b^21
 - 6*A*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*a^6*b^1
5 - 14*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64*B*a^3*
b^18 + 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^14 + 30
*B*a^8*b^13 - 70*B*a^9*b^12 - 14*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*a^12*b^
9 - 4*B*a^13*b^8 - 4*A*a*b^20 - 16*B*a*b^20))/(a*b^19 + b^20 - 5*a^2*b^18 -
 5*a^3*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8
*b^12 + 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (4*tan(c/2 + (d*x)/2)*(-(a + b
)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a
^5*b^2 - 8*B*a*b^6)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 12
0*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 12
0*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/((b^1
8 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*
a^12*b^6 - a^14*b^4)*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13
 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b
^7 - a^11*b^6)))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*
b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2 - 8*B*a*b^6))/(2*(b^18 - 7*a^2*b^16 + 21*a^
4*b^14 - 35*a^6*b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4)))
*(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^
4 - 7*B*a^5*b^2 - 8*B*a*b^6))/(2*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*
b^12 + 35*a^8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4))))*(-(a + b)^7*(a
 - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 + 3*A*a^2*b^5 + 8*B*a^3*b^4 - 7*B*a^5*b^2
 - 8*B*a*b^6)*1i)/(d*(b^18 - 7*a^2*b^16 + 21*a^4*b^14 - 35*a^6*b^12 + 35*a^
8*b^10 - 21*a^10*b^8 + 7*a^12*b^6 - a^14*b^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.275 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=274

$$-\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{\left(a^3A-3a^2bB+4aAb^2-2b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a\left(-4a^3B+a^2A\right.}{6b^2d\left(a^2\right.}$$

```
[Out] (A*a^3+4*A*a*b^2-3*B*a^2*b-2*B*b^3)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(
a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/3*a^2*(A*b-B*a)*sin(d*x+c)/b^2/(a^2
-b^2)/d/(a+b*cos(d*x+c))^3+1/6*a*(A*a^2*b-6*A*b^3-4*B*a^3+9*B*a*b^2)*sin(d*
x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*(A*a^4*b-10*A*a^2*b^3-6*A*b^5
+2*B*a^5-5*B*a^3*b^2+18*B*a*b^4)*sin(d*x+c)/b^2/(a^2-b^2)^3/d/(a+b*cos(d*x+
c))
```

Rubi [A] time = 0.64, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2988, 3021, 2754, 12, 2659, 205}

$$\frac{\left(a^3A-3a^2bB+4aAb^2-2b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{a\left(a^2Ab-4a^3B\right.}{6b^2d\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (a^2*(A*b - a*B)*Sin[
c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (a*(a^2*A*b - 6*A*
b^3 - 4*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[
c + d*x])^2) + ((a^4*A*b - 10*a^2*A*b^3 - 6*A*b^5 + 2*a^5*B - 5*a^3*b^2*B +
 18*a*b^4*B)*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{3ab(Ab-aB)+\left(a^2-3b^2\right)(Ab-aB)\cos(c\cdot}{(a+b\cos(c+a}}{3b^2\left(a^2-\right.}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2\right.}{6b^2\left(a^2-b^2\right)^2d(a+b\cos\right.}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2\right.}{6b^2\left(a^2-b^2\right)^2d(a+b\cos\right.}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2\right.}{6b^2\left(a^2-b^2\right)^2d(a+b\cos\right.}$$

$$= -\frac{a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2Ab-6Ab^3-4a^3B+9ab^2\right.}{6b^2\left(a^2-b^2\right)^2d(a+b\cos\right.}$$

$$= \frac{\left(a^3A+4aAb^2-3a^2bB-2b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2(A\cdot}{3b^2\left(a^2-b\right.}$$

Mathematica [A] time = 1.32, size = 251, normalized size = 0.92

$$\frac{\frac{2\sin(c+dx)\left(10a^5B-25a^4Ab+17a^3b^2B-14a^2Ab^3+6a\left(a^4A+a^3bB-9a^2Ab^2+9ab^3B-2Ab^4\right)\cos(c+dx)+\left(2a^5B+a^4Ab-5a^3b^2B-10a^2Ab^3+18ab^4B-6Ab^5\right)\right.}{(a+b\cos(c+dx))^3}}{24d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-24*(a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTanh[((a - b)*Tan[(c + d
*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(-25*a^4*A*b - 14*a^2*A*b^
3 - 6*A*b^5 + 10*a^5*B + 17*a^3*b^2*B + 18*a*b^4*B + 6*a*(a^4*A - 9*a^2*A*b
^2 - 2*A*b^4 + a^3*b*B + 9*a*b^3*B)*Cos[c + d*x] + (a^4*A*b - 10*a^2*A*b^3
- 6*A*b^5 + 2*a^5*B - 5*a^3*b^2*B + 18*a*b^4*B)*Cos[2*(c + d*x)])*Sin[c + d
*x])/(a + b*Cos[c + d*x])^3)/(24*(a^2 - b^2)^3*d)
```

fricas [B] time = 0.87, size = 1220, normalized size = 4.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [-1/12*(3*(A*a^6 - 3*B*a^5*b + 4*A*a^4*b^2 - 2*B*a^3*b^3 + (A*a^3*b^3 - 3*B
*a^2*b^4 + 4*A*a*b^5 - 2*B*b^6)*cos(d*x + c)^3 + 3*(A*a^4*b^2 - 3*B*a^3*b^3
 + 4*A*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c)^2 + 3*(A*a^5*b - 3*B*a^4*b^2 + 4*A
*a^3*b^3 - 2*B*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x +
 c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b
)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^
2)) - 2*(4*B*a^7 - 13*A*a^6*b + 7*B*a^5*b^2 + 11*A*a^4*b^3 - 11*B*a^3*b^4 +
 2*A*a^2*b^5 + (2*B*a^7 + A*a^6*b - 7*B*a^5*b^2 - 11*A*a^4*b^3 + 23*B*a^3*b
^4 + 4*A*a^2*b^5 - 18*B*a*b^6 + 6*A*b^7)*cos(d*x + c)^2 + 3*(A*a^7 + B*a^6*
b - 10*A*a^5*b^2 + 8*B*a^4*b^3 + 7*A*a^3*b^4 - 9*B*a^2*b^5 + 2*A*a*b^6)*cos
(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^
11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b
^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2
*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)
*d), 1/6*(3*(A*a^6 - 3*B*a^5*b + 4*A*a^4*b^2 - 2*B*a^3*b^3 + (A*a^3*b^3 - 3
*B*a^2*b^4 + 4*A*a*b^5 - 2*B*b^6)*cos(d*x + c)^3 + 3*(A*a^4*b^2 - 3*B*a^3*b
^3 + 4*A*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c)^2 + 3*(A*a^5*b - 3*B*a^4*b^2 + 4
*A*a^3*b^3 - 2*B*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x
+ c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (4*B*a^7 - 13*A*a^6*b + 7*B*a^5
*b^2 + 11*A*a^4*b^3 - 11*B*a^3*b^4 + 2*A*a^2*b^5 + (2*B*a^7 + A*a^6*b - 7*B
*a^5*b^2 - 11*A*a^4*b^3 + 23*B*a^3*b^4 + 4*A*a^2*b^5 - 18*B*a*b^6 + 6*A*b^7
)*cos(d*x + c)^2 + 3*(A*a^7 + B*a^6*b - 10*A*a^5*b^2 + 8*B*a^4*b^3 + 7*A*a^
3*b^4 - 9*B*a^2*b^5 + 2*A*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*
a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a
^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a
^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^
2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

giac [B] time = 1.03, size = 689, normalized size = 2.51

$$-\frac{\frac{3\left(Aa^3-3Ba^2b+4Aab^2-2Bb^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{3Aa^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6Ba^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(A*a^3 - 3*B*a^2*b + 4*A*a*b^2 - 2*B*b^3)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 -
 b^2)) + (3*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^5*tan(1/2*d*x + 1/2*c)^5 +
 12*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 3*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*
A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*
A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*
A*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 18*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^
5*tan(1/2*d*x + 1/2*c)^5 - 4*B*a^5*tan(1/2*d*x + 1/2*c)^3 + 28*A*a^4*b*tan(
1/2*d*x + 1/2*c)^3 - 32*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*A*a^2*b^3*tan
(1/2*d*x + 1/2*c)^3 + 36*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^5*tan(1/2*
d*x + 1/2*c)^3 - 3*A*a^5*tan(1/2*d*x + 1/2*c) - 6*B*a^5*tan(1/2*d*x + 1/2*c
) + 12*A*a^4*b*tan(1/2*d*x + 1/2*c) - 3*B*a^4*b*tan(1/2*d*x + 1/2*c) + 27*A
*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*B*a^3*b^2*tan(1/2*d*x + 1/2*c) + 12*A*a^2
*b^3*tan(1/2*d*x + 1/2*c) - 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a*b^4*t
an(1/2*d*x + 1/2*c) - 18*B*a*b^4*tan(1/2*d*x + 1/2*c) + 6*A*b^5*tan(1/2*d*x
 + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

maple [B] time = 0.08, size = 1726, normalized size = 6.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -1/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3
*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-6/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x
+1/2*c)^5*A-2/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-2/d/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(
1/2*d*x+1/2*c)^5*A*b^3+2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+3/d*b/(a*t
an(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*
a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*a*b
^2-28/3/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+
2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/d/(a*tan(1/2*d*x+1/2*c)
^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3*A*b^3+4/3/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a
+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+12/d/(a*tan(1/
```

```
2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2
)*tan(1/2*d*x+1/2*c)^3*B*a*b^2+1/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+
1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-6/d*
a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^
2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+2/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*
c)*A-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3
*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^3+2/d*a^3/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x
+1/2*c)*B-3/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(
a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+6/d/(a*tan(1/2*d*x+1/2*
c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*
d*x+1/2*c)*B*a*b^2+1/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2
)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+4/d*a*b^2/(a^6-3*a
^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/(
(a-b)*(a+b))^(1/2))*A-3/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))
^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-2/d/(a^6-3*a^
4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((
a-b)*(a+b))^(1/2))*b^3*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 4.15, size = 440, normalized size = 1.61

$$\frac{\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}}\right)\,\left(A\,a^3-3\,B\,a^2\,b+4\,A\,a\,b^2-2\,B\,b^3\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}-\frac{\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(-B\,a^3+7\,A\,a^2\,b-9\right.}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right.}}{d\,\left(3\,a\,b^2-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-3\,a^3+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^4,x)
```

```
[Out] (atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(
a + b)^(1/2)*(a - b)^(7/2)))*(A*a^3 - 2*B*b^3 + 4*A*a*b^2 - 3*B*a^2*b))/(d*
(a + b)^(7/2)*(a - b)^(7/2)) - ((4*tan(c/2 + (d*x)/2)^3*(3*A*b^3 - B*a^3 +
7*A*a^2*b - 9*B*a*b^2))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x
)/2)^5*(A*a^3 + 2*A*b^3 - 2*B*a^3 + 2*A*a*b^2 + 6*A*a^2*b - 6*B*a*b^2 - 3*B
*a^2*b))/((a + b)^3*(a - b)) - (tan(c/2 + (d*x)/2)*(A*a^3 - 2*A*b^3 + 2*B*a
^3 + 2*A*a*b^2 - 6*A*a^2*b + 6*B*a*b^2 - 3*B*a^2*b))/((a + b)*(3*a*b^2 - 3*
a^2*b + a^3 - b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b
- 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3)
 + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^
3)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.276 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=263

$$\frac{a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}-\frac{\left(a^3(-B)+4a^2Ab-4ab^2B+Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(a^3B+2a^2Ab-6\right.}{6bd\left(a^2-b^2\right)}$$

```
[Out] -(4*A*a^2*b+A*b^3-B*a^3-4*B*a*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a
+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d+1/3*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)
/d/(a+b*cos(d*x+c))^3+1/6*(2*A*a^2*b+3*A*b^3+B*a^3-6*B*a*b^2)*sin(d*x+c)/b/
(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*(2*A*a^3*b+13*A*a*b^3+B*a^4-10*B*a^2*b
^2-6*B*b^4)*sin(d*x+c)/b/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.53, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2968, 3021, 2754, 12, 2659, 205}

$$-\frac{\left(4a^2Ab+a^3(-B)-4ab^2B+Ab^3\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(2a^3Ab-10a^2b^2B+a^4B+13aAb^3-6b^4B\right)\sin(c+}{6bd\left(a^2-b^2\right)^3(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -(((4*a^2*A*b + A*b^3 - a^3*B - 4*a*b^2*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) + (a*(A*b - a*B)*Sin[c
 + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + ((2*a^2*A*b + 3*A*b^3
 + a^3*B - 6*a*b^2*B)*Sin[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x
])^2) + ((2*a^3*A*b + 13*a*A*b^3 + a^4*B - 10*a^2*b^2*B - 6*b^4*B)*Sin[c +
d*x])/(6*b*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^4}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{3b(Ab-aB)-\left(2aAb+a^2B-3b^2B\right)\cos(c+dx)}{(a+b\cos(c+dx))^3}\,d}{3b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2Ab+3Ab^3+a^3B-6ab^2B\right)\sin}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2Ab+3Ab^3+a^3B-6ab^2B\right)\sin}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2Ab+3Ab^3+a^3B-6ab^2B\right)\sin}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2Ab+3Ab^3+a^3B-6ab^2B\right)\sin}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx}$$

$$= -\frac{\left(4a^2Ab+Ab^3-a^3B-4ab^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a(Ab-}{3b\left(a^2-b^2\right)}$$

Mathematica [A] time = 1.16, size = 252, normalized size = 0.96

$$\frac{\frac{2\sin(c+dx)\left(12a^5A-25a^4bB+22a^3Ab^2-14a^2b^3B+b\left(a^4B+2a^3Ab-10a^2b^2B+13aAb^3-6b^4B\right)\cos(2(c+dx))+6\left(a^5B+2a^4Ab-9a^3b^2B+9a^2Ab^3-2ab^4B-Ab^5\right.}{(a+b\cos(c+dx))^3}}{24d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-24*(-4*a^2*A*b - A*b^3 + a^3*B + 4*a*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*
x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(12*a^5*A + 22*a^3*A*b^2 +
11*a*A*b^4 - 25*a^4*b*B - 14*a^2*b^3*B - 6*b^5*B + 6*(2*a^4*A*b + 9*a^2*A*b
^3 - A*b^5 + a^5*B - 9*a^3*b^2*B - 2*a*b^4*B)*Cos[c + d*x] + b*(2*a^3*A*b +
```

```
 13*a*A*b^3 + a^4*B - 10*a^2*b^2*B - 6*b^4*B)*Cos[2*(c + d*x)])*Sin[c + d*x
])/(a + b*Cos[c + d*x])^3)/(24*(a^2 - b^2)^3*d)
```

fricas [B] time = 1.44, size = 1232, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] [-1/12*(3*(B*a^6 - 4*A*a^5*b + 4*B*a^4*b^2 - A*a^3*b^3 + (B*a^3*b^3 - 4*A*a
^2*b^4 + 4*B*a*b^5 - A*b^6)*cos(d*x + c)^3 + 3*(B*a^4*b^2 - 4*A*a^3*b^3 + 4
*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + 3*(B*a^5*b - 4*A*a^4*b^2 + 4*B*a^3*b
^3 - A*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2
*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*
x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*
(6*A*a^7 - 13*B*a^6*b + 4*A*a^5*b^2 + 11*B*a^4*b^3 - 11*A*a^3*b^4 + 2*B*a^2
*b^5 + A*a*b^6 + (B*a^6*b + 2*A*a^5*b^2 - 11*B*a^4*b^3 + 11*A*a^3*b^4 + 4*B
*a^2*b^5 - 13*A*a*b^6 + 6*B*b^7)*cos(d*x + c)^2 + 3*(B*a^7 + 2*A*a^6*b - 10
*B*a^5*b^2 + 7*A*a^4*b^3 + 7*B*a^3*b^4 - 10*A*a^2*b^5 + 2*B*a*b^6 + A*b^7)*
cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 +
 b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 +
a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 +
a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b
^8)*d), 1/6*(3*(B*a^6 - 4*A*a^5*b + 4*B*a^4*b^2 - A*a^3*b^3 + (B*a^3*b^3 -
4*A*a^2*b^4 + 4*B*a*b^5 - A*b^6)*cos(d*x + c)^3 + 3*(B*a^4*b^2 - 4*A*a^3*b^
3 + 4*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + 3*(B*a^5*b - 4*A*a^4*b^2 + 4*B*
a^3*b^3 - A*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c)
+ b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (6*A*a^7 - 13*B*a^6*b + 4*A*a^5*b^2
+ 11*B*a^4*b^3 - 11*A*a^3*b^4 + 2*B*a^2*b^5 + A*a*b^6 + (B*a^6*b + 2*A*a^5*
b^2 - 11*B*a^4*b^3 + 11*A*a^3*b^4 + 4*B*a^2*b^5 - 13*A*a*b^6 + 6*B*b^7)*cos
(d*x + c)^2 + 3*(B*a^7 + 2*A*a^6*b - 10*B*a^5*b^2 + 7*A*a^4*b^3 + 7*B*a^3*b
^4 - 10*A*a^2*b^5 + 2*B*a*b^6 + A*b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^
3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2
 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b
 - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*
a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

giac [B] time = 1.89, size = 722, normalized size = 2.75

$$\frac{3\left(Ba^3-4\,Aa^2b+4\,Bab^2-Ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2-b^2}}-\frac{6\,Aa^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}$$

$$-\frac{}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(B*a^3 - 4*A*a^2*b + 4*B*a*b^2 - A*b^3)*(pi*floor(1/2*(d*x + c)/pi
+ 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b
^2)) - (6*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^5*tan(1/2*d*x + 1/2*c)^5 - 6
*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*A*
a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*A
*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*
A*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^5
*tan(1/2*d*x + 1/2*c)^5 - 6*B*b^5*tan(1/2*d*x + 1/2*c)^5 + 12*A*a^5*tan(1/2
*d*x + 1/2*c)^3 - 28*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 16*A*a^3*b^2*tan(1/2*
d*x + 1/2*c)^3 + 16*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 28*A*a*b^4*tan(1/2*d
*x + 1/2*c)^3 + 12*B*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^5*tan(1/2*d*x + 1/2
*c) + 3*B*a^5*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b*tan(1/2*d*x + 1/2*c) - 12*B*
a^4*b*tan(1/2*d*x + 1/2*c) + 12*A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*B*a^3*b
^2*tan(1/2*d*x + 1/2*c) + 27*A*a^2*b^3*tan(1/2*d*x + 1/2*c) - 12*B*a^2*b^3*
tan(1/2*d*x + 1/2*c) + 12*A*a*b^4*tan(1/2*d*x + 1/2*c) - 6*B*a*b^4*tan(1/2*
d*x + 1/2*c) - 3*A*b^5*tan(1/2*d*x + 1/2*c) - 6*B*b^5*tan(1/2*d*x + 1/2*c))
/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2
*d*x + 1/2*c)^2 + a + b)^3))/d
```

maple [B] time = 0.08, size = 1883, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2
-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+
1/2*c)^5*A+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/
(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+1/d/(a*tan(1/2*d*x+1
/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1
/2*d*x+1/2*c)^5*A*b^3-1/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-6/d*b/(a*ta
n(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a
*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*a*b^
2-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^
2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^3*B+4/d/(a*tan(1/2*d*x+1/2*c)^2-tan
```

```
(1/2*d*x+1/2*c)^2*b+a+b)^3*a^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3*A+28/3/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-28/3/d*b/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a^2+2*a*b+b^2)/(a^2-2*a*
b+b^2)*tan(1/2*d*x+1/2*c)^3*B-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^3*B+2/d
*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2
*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c
)*A+6/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(
a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-1/d/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/
2*c)*A*b^3+1/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a
+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-6/d*b/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan
(1/2*d*x+1/2*c)*B+2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B*a*b^2-2/d/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*
tan(1/2*d*x+1/2*c)*b^3*B-4/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^
(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*a^2-1/d*b^3/(a
^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(
a-b)/((a-b)*(a+b))^(1/2))*A+1/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))
^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*a^3+4/d*b^2/(
a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*
(a-b)/((a-b)*(a+b))^(1/2))*B*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 4.03, size = 451, normalized size = 1.71

$$\frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\,A\,a^3-7\,B\,a^2\,b+7\,A\,a\,b^2-3\,B\,b^3\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A\,a^3-A\,b^3+B\,a^3-2\,B\,b^3+6\,A\,a\,b^2-2\,A\,a^2\,b+2\,B\,a\,b^2-6\,B\,a^2\,b\right)}{\left(a+b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d}{2}\right.\cdots}{\cdots}}{d\,\left(3\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)+3\,a^2\,\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^4,x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^3*(3*A*a^3 - 3*B*b^3 + 7*A*a*b^2 - 7*B*a^2*b))/(3*(a
 + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x)/2)*(2*A*a^3 - A*b^3 + B*a^3
 - 2*B*b^3 + 6*A*a*b^2 - 2*A*a^2*b + 2*B*a*b^2 - 6*B*a^2*b))/((a + b)*(3*a*
b^2 - 3*a^2*b + a^3 - b^3)) + (tan(c/2 + (d*x)/2)^5*(2*A*a^3 + A*b^3 - B*a^
3 - 2*B*b^3 + 6*A*a*b^2 + 2*A*a^2*b - 2*B*a*b^2 - 6*B*a^2*b))/((a + b)^3*(a
 - b)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b
^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b +
a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (atan(
(tan(c/2 + (d*x)/2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)
^(1/2)*(a - b)^(7/2)))*(A*b^3 - B*a^3 + 4*A*a^2*b - 4*B*a*b^2))/(d*(a + b)^
(7/2)*(a - b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.277 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=237

$$-\frac{\left(-2a^2B+5aAb-3b^2B\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tan^{-1}$$

```
[Out] (2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(
a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/3*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/
(a+b*cos(d*x+c))^3-1/6*(5*A*a*b-2*B*a^2-3*B*b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(
a+b*cos(d*x+c))^2-1/6*(11*A*a^2*b+4*A*b^3-2*B*a^3-13*B*a*b^2)*sin(d*x+c)/(a
^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.48, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2754, 12, 2659, 205}

$$\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(11a^2Ab-2a^3B-13ab^2B+4Ab^3\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}-\frac{(-2a}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - ((A*b - a*B)*Sin[c +
d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - ((5*a*A*b - 2*a^2*B - 3*b^
2*B)*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - ((11*a^2*A*
b + 4*A*b^3 - 2*a^3*B - 13*a*b^2*B)*Sin[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b
*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{-3(aA-bB)+2(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(5aAb-2a^2B-3b^2B\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\int \frac{2\left(3a^2A+\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(5aAb-2a^2B-3b^2B\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{\left(11a^2Ab-\cdots\right.}{6\left(\cdots\right.}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(5aAb-2a^2B-3b^2B\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{\left(11a^2Ab-\cdots\right.}{6\left(\cdots\right.}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\left(5aAb-2a^2B-3b^2B\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{\left(11a^2Ab-\cdots\right.}{6\left(\cdots\right.}$$

$$= \frac{\left(2a^3A+3aAb^2-4a^2bB-b^3B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{(Ab-aB)\sin(c+\cdots}{3\left(a^2-b^2\right)d(a+b\cos(c\cdots}$$

Mathematica [A] time = 2.37, size = 227, normalized size = 0.96

$$\frac{\frac{\left(2a^2B-5aAb+3b^2B\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2}+\frac{\left(2a^3B-11a^2Ab+13ab^2B-4Ab^3\right)\sin(c+dx)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))}+\frac{6\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}}+\frac{2(}{(a-b)(}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[((a - b)*Tan[(c + d*x
)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + (2*(-(A*b) + a*B)*Sin[c + d*x
])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3) + ((-5*a*A*b + 2*a^2*B + 3*b^2*
B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2) + ((-11*a^2*A
*b - 4*A*b^3 + 2*a^3*B + 13*a*b^2*B)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(a
+ b*Cos[c + d*x])))/(6*d)
```

fricas [B] time = 1.13, size = 1228, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*A*a^6 - 4*B*a^5*b + 3*A*a^4*b^2 - B*a^3*b^3 + (2*A*a^3*b^3 - 4
*B*a^2*b^4 + 3*A*a*b^5 - B*b^6)*cos(d*x + c)^3 + 3*(2*A*a^4*b^2 - 4*B*a^3*b
^3 + 3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + 3*(2*A*a^5*b - 4*B*a^4*b^2 + 3
*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x +
 c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b
)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^
2)) - 2*(6*B*a^7 - 18*A*a^6*b + 4*B*a^5*b^2 + 23*A*a^4*b^3 - 11*B*a^3*b^4 -
 7*A*a^2*b^5 + B*a*b^6 + 2*A*b^7 + (2*B*a^5*b^2 - 11*A*a^4*b^3 + 11*B*a^3*b
^4 + 7*A*a^2*b^5 - 13*B*a*b^6 + 4*A*b^7)*cos(d*x + c)^2 + 3*(2*B*a^6*b - 9*
A*a^5*b^2 + 7*B*a^4*b^3 + 8*A*a^3*b^4 - 10*B*a^2*b^5 + A*a*b^6 + B*b^7)*cos
(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^
11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b
^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2
*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)
*d), 1/6*(3*(2*A*a^6 - 4*B*a^5*b + 3*A*a^4*b^2 - B*a^3*b^3 + (2*A*a^3*b^3 -
 4*B*a^2*b^4 + 3*A*a*b^5 - B*b^6)*cos(d*x + c)^3 + 3*(2*A*a^4*b^2 - 4*B*a^3
*b^3 + 3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c)^2 + 3*(2*A*a^5*b - 4*B*a^4*b^2 +
 3*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x
+ c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + (6*B*a^7 - 18*A*a^6*b + 4*B*a^5
*b^2 + 23*A*a^4*b^3 - 11*B*a^3*b^4 - 7*A*a^2*b^5 + B*a*b^6 + 2*A*b^7 + (2*B
```

```
*a^5*b^2 - 11*A*a^4*b^3 + 11*B*a^3*b^4 + 7*A*a^2*b^5 - 13*B*a*b^6 + 4*A*b^7
)*cos(d*x + c)^2 + 3*(2*B*a^6*b - 9*A*a^5*b^2 + 7*B*a^4*b^3 + 8*A*a^3*b^4 -
 10*B*a^2*b^5 + A*a*b^6 + B*b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*
a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a
^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a
^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^
2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

giac [B] time = 1.22, size = 691, normalized size = 2.92

$$-\frac{\frac{3\left(2\,Aa^3-4\,Ba^2b+3\,Aab^2-Bb^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{a^2-b^2}}-\frac{6\,Ba^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-18\,Aa^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
+ 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 -
 b^2)) - (6*B*a^5*tan(1/2*d*x + 1/2*c)^5 - 18*A*a^4*b*tan(1/2*d*x + 1/2*c)^
5 - 6*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5
+ 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5
- 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 +
12*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*A*b^5*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^
5*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^5*tan(1/2*d*x + 1/2*c)^3 - 36*A*a^4*b*tan
(1/2*d*x + 1/2*c)^3 + 16*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 32*A*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^3 - 28*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*A*b^5*tan(1/2*
d*x + 1/2*c)^3 + 6*B*a^5*tan(1/2*d*x + 1/2*c) - 18*A*a^4*b*tan(1/2*d*x + 1/
2*c) + 6*B*a^4*b*tan(1/2*d*x + 1/2*c) - 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c) +
 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + 27*
B*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a*b^4*tan(1/2*d*x + 1/2*c) + 12*B*a*b^
4*tan(1/2*d*x + 1/2*c) - 6*A*b^5*tan(1/2*d*x + 1/2*c) - 3*B*b^5*tan(1/2*d*x
 + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

maple [B] time = 0.08, size = 1727, normalized size = 7.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -6/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-3/d*b^2/(a*tan(1/2*d*x+1/2*c)^
2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d
*x+1/2*c)^5*A-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-
b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^3+2/d*a^3/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*
tan(1/2*d*x+1/2*c)^5*B+2/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3*a^2/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+6/d/(a*t
an(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^
2+b^3)*tan(1/2*d*x+1/2*c)^5*B*a*b^2+1/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^3
*B-12/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*
a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-4/3/d/(a*tan(1/2*d*x+1/2*c)
^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3*A*b^3+4/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b
)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+28/3/d/(a*tan(1/
2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2
)*tan(1/2*d*x+1/2*c)^3*B*a*b^2-6/d*a^2*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+3/
d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*
a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A
*b^3+2/d*a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a
^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-2/d*b/(a*tan(1/2*d*x+1/2*c)^2-
tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d
*x+1/2*c)*B+6/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)
/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B*a*b^2-1/d/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/
2*d*x+1/2*c)*b^3*B+2/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2
)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+3/d*a*b^2/(a^6-3*a
^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/(
(a-b)*(a+b))^(1/2))*A-4/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))
^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B-1/d/(a^6-3*a^
4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((
a-b)*(a+b))^(1/2))*b^3*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 4.00, size = 440, normalized size = 1.86

$$\frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(2\,a-2\,b)\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,(a-b)^{7/2}}\right)\left(2\,A\,a^3-4\,B\,a^2\,b+3\,A\,a\,b^2-B\,b^3\right)}{d\,(a+b)^{7/2}\,(a-b)^{7/2}}-\frac{\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(-3\,B\,a^3+9\,A\,a^2\,b-\right.}{3\,(a+b)^2\,\left(a^2-2\,a\,b+b\right.}}{d\,\left(3\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-3\,a^3+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^4,x)
```

```
[Out] (atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(
a + b)^(1/2)*(a - b)^(7/2)))*(2*A*a^3 - B*b^3 + 3*A*a*b^2 - 4*B*a^2*b))/(d*
(a + b)^(7/2)*(a - b)^(7/2)) - ((4*tan(c/2 + (d*x)/2)^3*(A*b^3 - 3*B*a^3 +
9*A*a^2*b - 7*B*a*b^2))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) - (tan(c/2 + (d*x
)/2)^5*(2*B*a^3 - 2*A*b^3 + B*b^3 - 3*A*a*b^2 - 6*A*a^2*b + 6*B*a*b^2 + 2*B
*a^2*b))/((a + b)^3*(a - b)) + (tan(c/2 + (d*x)/2)*(2*A*b^3 - 2*B*a^3 + B*b
^3 - 3*A*a*b^2 + 6*A*a^2*b - 6*B*a*b^2 + 2*B*a^2*b))/((a + b)*(3*a*b^2 - 3*
a^2*b + a^3 - b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b
- 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3)
 + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^
3)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.278 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=301

$$\frac{A\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{b\left(-5a^3B+8a^2Ab-3Ab^3\right)\sin(c+dx)}{6a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{b\left(-11a^5B\right.}{}$$

```
[Out] -(8*A*a^6*b-8*A*a^4*b^3+7*A*a^2*b^5-2*A*b^7-2*B*a^7-3*B*a^5*b^2)*arctan((a-
b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(7/2)/(a+b)^(7/2)/d+A*ar
ctanh(sin(d*x+c))/a^4/d+1/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d
*x+c))^3+1/6*b*(8*A*a^2*b-3*A*b^3-5*B*a^3)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+
b*cos(d*x+c))^2+1/6*b*(26*A*a^4*b-17*A*a^2*b^3+6*A*b^5-11*B*a^5-4*B*a^3*b^2
)*sin(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 1.51, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{\left(-8a^4Ab^3+7a^2Ab^5+8a^6Ab-3a^5b^2B-2a^7B-2Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b\left(-17a^2Ab^3+26a^4Ab-4\right.}{6a^3d\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -(((8*a^6*A*b - 8*a^4*A*b^3 + 7*a^2*A*b^5 - 2*A*b^7 - 2*a^7*B - 3*a^5*b^2*B
)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a
 + b)^(7/2)*d)) + (A*ArcTanh[Sin[c + d*x]])/(a^4*d) + (b*(A*b - a*B)*Sin[c
+ d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (b*(8*a^2*A*b - 3*A*b^
3 - 5*a^3*B)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) +
 (b*(26*a^4*A*b - 17*a^2*A*b^3 + 6*A*b^5 - 11*a^5*B - 4*a^3*b^2*B)*Sin[c +
d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(3A\left(a^2-b^2\right)-3a(Ab-aB)\cos(c+dx)+2b(A\cdots}{(a+b\cos(c+dx)\cdots}}{3a\left(a^2-b^2\cdots}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(8a^2Ab-3Ab^3-5a^3B\right)\sin(c+\cdots}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx\cdots}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(8a^2Ab-3Ab^3-5a^3B\right)\sin(c+\cdots}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx\cdots}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(8a^2Ab-3Ab^3-5a^3B\right)\sin(c+\cdots}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx\cdots}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(8a^2A\cdots}{6a^2\left(a^2\cdots}$$

$$= -\frac{\left(8a^6Ab-8a^4Ab^3+7a^2Ab^5-2Ab^7-2a^7B-3a^5b^2B\right)\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\cdots}{\sqrt{\cdots}}\right.}{a^4(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A] time = 1.73, size = 368, normalized size = 1.22

$$\frac{\cos(c+dx)(A\sec(c+dx)+B)\left(\frac{24\left(2a^7B-8a^6Ab+3a^5b^2B+8a^4Ab^3-7a^2Ab^5+2Ab^7\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} - \frac{2ab\sin(c+dx)\left(36a^7B-\cdots}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*((24*(-8*a^6*A*b + 8*a^4*A*b^3 - 7*a^2*A
*b^5 + 2*A*b^7 + 2*a^7*B + 3*a^5*b^2*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/
Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - 24*A*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 24*A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (2*a*b*(-72*a^
6*A*b + 38*a^4*A*b^3 - 5*a^2*A*b^5 - 6*A*b^7 + 36*a^7*B + a^5*b^2*B + 8*a^3
*b^4*B + 6*a*b*(-20*a^4*A*b + 15*a^2*A*b^3 - 5*A*b^5 + 9*a^5*B + a^3*b^2*B)
*Cos[c + d*x] + b^2*(-26*a^4*A*b + 17*a^2*A*b^3 - 6*A*b^5 + 11*a^5*B + 4*a^
3*b^2*B)*Cos[2*(c + d*x)])*Sin[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x]
)^3)))/(24*a^4*d*(A + B*Cos[c + d*x]))
```

fricas [B] time = 136.53, size = 2269, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(3*(2*B*a^10 - 8*A*a^9*b + 3*B*a^8*b^2 + 8*A*a^7*b^3 - 7*A*a^5*b^5 +
2*A*a^3*b^7 + (2*B*a^7*b^3 - 8*A*a^6*b^4 + 3*B*a^5*b^5 + 8*A*a^4*b^6 - 7*A*
a^2*b^8 + 2*A*b^10)*cos(d*x + c)^3 + 3*(2*B*a^8*b^2 - 8*A*a^7*b^3 + 3*B*a^6
*b^4 + 8*A*a^5*b^5 - 7*A*a^3*b^7 + 2*A*a*b^9)*cos(d*x + c)^2 + 3*(2*B*a^9*b
 - 8*A*a^8*b^2 + 3*B*a^7*b^3 + 8*A*a^6*b^4 - 7*A*a^4*b^6 + 2*A*a^2*b^8)*cos
(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 6*(A*a^11 - 4*A*a^9*b
^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8 + (A*a^8*b^3 - 4*A*a^6*b^5 + 6*A
*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*cos(d*x + c)^3 + 3*(A*a^9*b^2 - 4*A*a^7*b^
4 + 6*A*a^5*b^6 - 4*A*a^3*b^8 + A*a*b^10)*cos(d*x + c)^2 + 3*(A*a^10*b - 4*
A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^2*b^9)*cos(d*x + c))*log(sin(d*
x + c) + 1) - 6*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b
^8 + (A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*cos(d*x
 + c)^3 + 3*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6 - 4*A*a^3*b^8 + A*a*b^10
)*cos(d*x + c)^2 + 3*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 +
A*a^2*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(18*B*a^10*b - 36*A*a^9
*b^2 - 23*B*a^8*b^3 + 68*A*a^7*b^4 + 7*B*a^6*b^5 - 43*A*a^5*b^6 - 2*B*a^4*b
^7 + 11*A*a^3*b^8 + (11*B*a^8*b^3 - 26*A*a^7*b^4 - 7*B*a^6*b^5 + 43*A*a^5*b
^6 - 4*B*a^4*b^7 - 23*A*a^3*b^8 + 6*A*a*b^10)*cos(d*x + c)^2 + 3*(9*B*a^9*b
^2 - 20*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 - B*a^5*b^6 - 20*A*a^4*b^7 +
 5*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^12*b^3 - 4*a^10*b^5 + 6*a^8*b
^7 - 4*a^6*b^9 + a^4*b^11)*d*cos(d*x + c)^3 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*
a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c)^2 + 3*(a^14*b - 4*a^12*b^3 +
 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c) + (a^15 - 4*a^13*b^2 + 6*
a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d), 1/6*(3*(2*B*a^10 - 8*A*a^9*b + 3*B*a^8*
b^2 + 8*A*a^7*b^3 - 7*A*a^5*b^5 + 2*A*a^3*b^7 + (2*B*a^7*b^3 - 8*A*a^6*b^4
```

```
+ 3*B*a^5*b^5 + 8*A*a^4*b^6 - 7*A*a^2*b^8 + 2*A*b^10)*cos(d*x + c)^3 + 3*(2
*B*a^8*b^2 - 8*A*a^7*b^3 + 3*B*a^6*b^4 + 8*A*a^5*b^5 - 7*A*a^3*b^7 + 2*A*a*
b^9)*cos(d*x + c)^2 + 3*(2*B*a^9*b - 8*A*a^8*b^2 + 3*B*a^7*b^3 + 8*A*a^6*b^
4 - 7*A*a^4*b^6 + 2*A*a^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos
(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) + 3*(A*a^11 - 4*A*a^9*b^2 +
6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8 + (A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^4*
b^7 - 4*A*a^2*b^9 + A*b^11)*cos(d*x + c)^3 + 3*(A*a^9*b^2 - 4*A*a^7*b^4 + 6
*A*a^5*b^6 - 4*A*a^3*b^8 + A*a*b^10)*cos(d*x + c)^2 + 3*(A*a^10*b - 4*A*a^8
*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^2*b^9)*cos(d*x + c))*log(sin(d*x + c
) + 1) - 3*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8 +
(A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*cos(d*x + c)
^3 + 3*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6 - 4*A*a^3*b^8 + A*a*b^10)*cos
(d*x + c)^2 + 3*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^2
*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - (18*B*a^10*b - 36*A*a^9*b^2 -
23*B*a^8*b^3 + 68*A*a^7*b^4 + 7*B*a^6*b^5 - 43*A*a^5*b^6 - 2*B*a^4*b^7 + 11
*A*a^3*b^8 + (11*B*a^8*b^3 - 26*A*a^7*b^4 - 7*B*a^6*b^5 + 43*A*a^5*b^6 - 4*
B*a^4*b^7 - 23*A*a^3*b^8 + 6*A*a*b^10)*cos(d*x + c)^2 + 3*(9*B*a^9*b^2 - 20
*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 - B*a^5*b^6 - 20*A*a^4*b^7 + 5*A*a^
2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*
a^6*b^9 + a^4*b^11)*d*cos(d*x + c)^3 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6
 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c)^2 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10
*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c) + (a^15 - 4*a^13*b^2 + 6*a^11*b^
4 - 4*a^9*b^6 + a^7*b^8)*d)]
```

giac [B] time = 2.40, size = 837, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] 1/3*(3*(2*B*a^7 - 8*A*a^6*b + 3*B*a^5*b^2 + 8*A*a^4*b^3 - 7*A*a^2*b^5 + 2*A
*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^10 - 3*a^8*b^2
 + 3*a^6*b^4 - a^4*b^6)*sqrt(a^2 - b^2)) + 3*A*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/a^4 - 3*A*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - (18*B*a^7*b*tan(1
/2*d*x + 1/2*c)^5 - 36*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*B*a^6*b^2*tan(
1/2*d*x + 1/2*c)^5 + 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^5*b^3*tan(
1/2*d*x + 1/2*c)^5 + 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^4*b^4*tan(1
/2*d*x + 1/2*c)^5 - 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^3*b^5*tan(1
/2*d*x + 1/2*c)^5 + 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 15*A*a*b^7*tan(1/2
*d*x + 1/2*c)^5 - 6*A*b^8*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^7*b*tan(1/2*d*x +
 1/2*c)^3 - 72*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 32*B*a^5*b^3*tan(1/2*d*x
+ 1/2*c)^3 + 116*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^3*b^5*tan(1/2*d*x
```

```
 + 1/2*c)^3 - 56*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^8*tan(1/2*d*x +
1/2*c)^3 + 18*B*a^7*b*tan(1/2*d*x + 1/2*c) - 36*A*a^6*b^2*tan(1/2*d*x + 1/2
*c) + 27*B*a^6*b^2*tan(1/2*d*x + 1/2*c) - 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c)
 + 6*B*a^5*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c) + 3*
B*a^4*b^4*tan(1/2*d*x + 1/2*c) + 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c) + 6*B*a^
3*b^5*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c) - 15*A*a*b^7*
tan(1/2*d*x + 1/2*c) - 6*A*b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*
a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a
 + b)^3))/d
```

maple [B] time = 0.21, size = 2180, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 8/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*
x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+2/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a
-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*a^3
-6/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a+b)/(a^3
-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-
tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^5/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d
*x+1/2*c)*A+2/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b
^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+3/d/(a*tan(1/2*d*x+
1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)*B*a*b^2-3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a
+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*a*b^2+12/d*b^2
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b
+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-6/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^
5*A-1/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^5/(a-b)
/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d/a^3/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*ta
n(1/2*d*x+1/2*c)^5*A-44/3/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3*b^4/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+4/d/a^3
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a^2+2*a*b+b^2)/
(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+24/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/
2*c)^3*A+12/d*b^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(
a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-12/d*b/(a*tan(1/2*d*x
+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)
*tan(1/2*d*x+1/2*c)^3*B-6/d*b/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b+a+b)^3*a^2/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-6/d*b/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a^2/(a-b)/(a^3+3*a^2*b+3
```

```
*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+1/d/a^4*A*ln(tan(1/2*d*x+1/2*c)+1)-1/d/a
^4*A*ln(tan(1/2*d*x+1/2*c)-1)-2/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^3*B+4/d
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3
*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^3-4/3/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3*b^3*B-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^
3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^3-2/d/(a*tan(1/2*d*x+1/2*c)^2
-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+
1/2*c)*b^3*B+2/d/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arct
an(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^7-7/d/a^2/(a^6-3*a^4*b
^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b
)*(a+b))^(1/2))*A*b^5-8/d*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/
2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*a^2+3/d*b^2/(a^6-
3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b
)/((a-b)*(a+b))^(1/2))*B*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 12.81, size = 9727, normalized size = 32.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^4),x)
```

```
[Out] (A*atan(-((A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 -
 8*A^2*a*b^13 - 8*A^2*a^13*b - 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*
a^4*b^10 - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^
8*b^6 - 120*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b
^2 + 9*B^2*a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34
*A*B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^11*b^3))/(a^16*b + a^17 - a^6*b^11
 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b
^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) + (A*((8*(4*A*a^21 + 4*B*a^21 -
```

4*A*a^8*b^13 + 2*A*a^9*b^12 + 26*A*a^10*b^11 - 14*A*a^11*b^10 - 70*A*a^12*b^9 + 30*A*a^13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 110*A*a^16*b^5 + 20*A*a^17*b^4 + 64*A*a^18*b^3 - 12*A*a^19*b^2 + 6*B*a^12*b^9 - 6*B*a^13*b^8 - 14*B*a^14*b^7 + 14*B*a^15*b^6 + 6*B*a^16*b^5 - 6*B*a^17*b^4 + 6*B*a^18*b^3 - 6*B*a^19*b^2 - 16*A*a^20*b - 4*B*a^20*b))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (8*A*tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))))/a^4)*1i)/a^4 + (A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 - 8*A^2*a*b^13 - 8*A^2*a^13*b - 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*a^4*b^10 - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b^6 - 120*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b^2 + 9*B^2*a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^11*b^3))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) - (A*((8*(4*A*a^21 + 4*B*a^21 - 4*A*a^8*b^13 + 2*A*a^9*b^12 + 26*A*a^10*b^11 - 14*A*a^11*b^10 - 70*A*a^12*b^9 + 30*A*a^13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 110*A*a^16*b^5 + 20*A*a^17*b^4 + 64*A*a^18*b^3 - 12*A*a^19*b^2 + 6*B*a^12*b^9 - 6*B*a^13*b^8 - 14*B*a^14*b^7 + 14*B*a^15*b^6 + 6*B*a^16*b^5 - 6*B*a^17*b^4 + 6*B*a^18*b^3 - 6*B*a^19*b^2 - 16*A*a^20*b - 4*B*a^20*b))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (8*A*tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))))/a^4)*1i)/a^4)/((16*(4*A^3*b^13 + 4*A*B^2*a^13 - 4*A^2*B*a^13 - 2*A^3*a*b^12 + 16*A^3*a^12*b - 26*A^3*a^2*b^11 + 11*A^3*a^3*b^10 + 70*A^3*a^4*b^9 - 34*A^3*a^5*b^8 - 110*A^3*a^6*b^7 + 66*A^3*a^7*b^6 + 110*A^3*a^8*b^5 - 64*A^3*a^9*b^4 - 64*A^3*a^10*b^3 + 48*A^3*a^11*b^2 - 28*A^2*B*a^12*b + 9*A*B^2*a^9*b^4 + 12*A*B^2*a^11*b^2 + 6*A^2*B*a^4*b^9 + 6*A^2*B*a^5*b^8 - 20*A^2*B*a^6*b^7 - 14*A^2*B*a^7*b^6 + 14*A^2*B*a^8*b^5 + 6*A^2*B*a^9*b^4 - 22*A^2*B*a^10*b^3 + 6*A^2*B*a^11*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 - 8*A^2*a*b^13 - 8*A^2*a^13*b - 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*a^4*b^10 - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b^6 - 120*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b^2 + 9*B^2*a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^1

1*b^3))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) + (A*((8*(4*A*a^21 + 4*B*a^21 - 4*A*a^8*b^13 + 2*A*a^9*b^12 + 26*A*a^10*b^11 - 14*A*a^11*b^10 - 70*A*a^12*b^9 + 30*A*a^13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 110*A*a^16*b^5 + 20*A*a^17*b^4 + 64*A*a^18*b^3 - 12*A*a^19*b^2 + 6*B*a^12*b^9 - 6*B*a^13*b^8 - 14*B*a^14*b^7 + 14*B*a^15*b^6 + 6*B*a^16*b^5 - 6*B*a^17*b^4 + 6*B*a^18*b^3 - 6*B*a^19*b^2 - 16*A*a^20*b - 4*B*a^20*b))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (8*A*tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))))/a^4))/a^4 - (A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 - 8*A^2*a*b^13 - 8*A^2*a^13*b - 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*a^4*b^10 - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b^6 - 120*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b^2 + 9*B^2*a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^11*b^3))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) - (A*((8*(4*A*a^21 + 4*B*a^21 - 4*A*a^8*b^13 + 2*A*a^9*b^12 + 26*A*a^10*b^11 - 14*A*a^11*b^10 - 70*A*a^12*b^9 + 30*A*a^13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 110*A*a^16*b^5 + 20*A*a^17*b^4 + 64*A*a^18*b^3 - 12*A*a^19*b^2 + 6*B*a^12*b^9 - 6*B*a^13*b^8 - 14*B*a^14*b^7 + 14*B*a^15*b^6 + 6*B*a^16*b^5 - 6*B*a^17*b^4 + 6*B*a^18*b^3 - 6*B*a^19*b^2 - 16*A*a^20*b - 4*B*a^20*b))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (8*A*tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))))/a^4))/a^4))*2i)/(a^4*d) - ((tan(c/2 + (d*x)/2)*(2*A*b^6 - 6*A*a^2*b^4 - 4*A*a^3*b^3 + 12*A*a^4*b^2 - 2*B*a^3*b^3 + 3*B*a^4*b^2 + A*a*b^5 - 6*B*a^5*b))/((a + b)*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)^5*(6*A*a^2*b^4 - 2*A*b^6 - 4*A*a^3*b^3 - 12*A*a^4*b^2 + 2*B*a^3*b^3 + 3*B*a^4*b^2 + A*a*b^5 + 6*B*a^5*b))/((a^3*b - a^4)*(a + b)^3) + (4*tan(c/2 + (d*x)/2)^3*(11*A*a^2*b^4 - 3*A*b^6 - 18*A*a^4*b^2 + B*a^3*b^3 + 9*B*a^5*b))/(3*(a + b)^2*(a^5 - 2*a^4*b + a^3*b^2)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (atan(((((8*tan(c/2 + (d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 - 8*A^2*a*b^13 - 8*A^2*a^13*b - 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*a^4*b^10 -

120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b^6 - 120
*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b^2 + 9*B^2*
a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34*A*B*a^7*b^
7 + 20*A*B*a^9*b^5 - 16*A*B*a^11*b^3))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10
 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^1
3*b^4 - 5*a^14*b^3 - 5*a^15*b^2) - (((8*(4*A*a^21 + 4*B*a^21 - 4*A*a^8*b^13
 + 2*A*a^9*b^12 + 26*A*a^10*b^11 - 14*A*a^11*b^10 - 70*A*a^12*b^9 + 30*A*a^
13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 110*A*a^16*b^5 + 20*A*a^17*b^4 +
64*A*a^18*b^3 - 12*A*a^19*b^2 + 6*B*a^12*b^9 - 6*B*a^13*b^8 - 14*B*a^14*b^7
 + 14*B*a^15*b^6 + 6*B*a^16*b^5 - 6*B*a^17*b^4 + 6*B*a^18*b^3 - 6*B*a^19*b^
2 - 16*A*a^20*b - 4*B*a^20*b))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^
11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4
 - 5*a^17*b^3 - 5*a^18*b^2) - (4*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^
(1/2)*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^
6*b)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 12
0*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 1
20*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/((a^18 - a^4*b^14 +
7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16
b^2)(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^1
0*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))
)*(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b
^3 + 3*B*a^5*b^2 - 8*A*a^6*b))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^
10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*(-(a + b)^7*(a
 - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2
 - 8*A*a^6*b)*1i)/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*
b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)) + (((8*tan(c/2 + (d*x)/2)*(4
*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 - 8*A^2*a*b^13 - 8*A^2*a^13*b - 48*A^2*
a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*a^4*b^10 - 120*A^2*a^5*b^9 - 164*A^2*a
^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b^6 - 120*A^2*a^9*b^5 - 92*A^2*a^10*
b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b^2 + 9*B^2*a^10*b^4 + 12*B^2*a^12*b^2
 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B
*a^11*b^3))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 -
10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15
b^2) + (((8(4*A*a^21 + 4*B*a^21 - 4*A*a^8*b^13 + 2*A*a^9*b^12 + 26*A*a^10
*b^11 - 14*A*a^11*b^10 - 70*A*a^12*b^9 + 30*A*a^13*b^8 + 110*A*a^14*b^7 - 3
0*A*a^15*b^6 - 110*A*a^16*b^5 + 20*A*a^17*b^4 + 64*A*a^18*b^3 - 12*A*a^19*b
^2 + 6*B*a^12*b^9 - 6*B*a^13*b^8 - 14*B*a^14*b^7 + 14*B*a^15*b^6 + 6*B*a^16
*b^5 - 6*B*a^17*b^4 + 6*B*a^18*b^3 - 6*B*a^19*b^2 - 16*A*a^20*b - 4*B*a^20*
b))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^
13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2)
 + (4*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 -
7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b)*(8*a^21*b - 8*a^8*b^14
 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9
+ 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 -
 48*a^19*b^3 - 8*a^20*b^2))/((a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 +

35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)*(a^16*b + a^17 - a^6*
b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^
12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*(-(a + b)^7*(a - b)^7)^(1
/2)*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*
b))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*
b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2
*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b)*1i)/(2*(a^18
- a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14
*b^4 - 7*a^16*b^2)))/((16*(4*A^3*b^13 + 4*A*B^2*a^13 - 4*A^2*B*a^13 - 2*A^3
*a*b^12 + 16*A^3*a^12*b - 26*A^3*a^2*b^11 + 11*A^3*a^3*b^10 + 70*A^3*a^4*b^
9 - 34*A^3*a^5*b^8 - 110*A^3*a^6*b^7 + 66*A^3*a^7*b^6 + 110*A^3*a^8*b^5 - 6
4*A^3*a^9*b^4 - 64*A^3*a^10*b^3 + 48*A^3*a^11*b^2 - 28*A^2*B*a^12*b + 9*A*B
^2*a^9*b^4 + 12*A*B^2*a^11*b^2 + 6*A^2*B*a^4*b^9 + 6*A^2*B*a^5*b^8 - 20*A^2
*B*a^6*b^7 - 14*A^2*B*a^7*b^6 + 14*A^2*B*a^8*b^5 + 6*A^2*B*a^9*b^4 - 22*A^2
*B*a^10*b^3 + 6*A^2*B*a^11*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*
a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b
^4 - 5*a^17*b^3 - 5*a^18*b^2) - (((8*tan(c/2 + (d*x)/2)*(4*A^2*a^14 + 8*A^2
*b^14 + 4*B^2*a^14 - 8*A^2*a*b^13 - 8*A^2*a^13*b - 48*A^2*a^2*b^12 + 48*A^2
*a^3*b^11 + 117*A^2*a^4*b^10 - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*
a^7*b^7 + 156*A^2*a^8*b^6 - 120*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11
*b^3 + 44*A^2*a^12*b^2 + 9*B^2*a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b +
12*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^11*b^3))/(a^16
*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*
a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) - (((8*(4*A
*a^21 + 4*B*a^21 - 4*A*a^8*b^13 + 2*A*a^9*b^12 + 26*A*a^10*b^11 - 14*A*a^11
*b^10 - 70*A*a^12*b^9 + 30*A*a^13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 11
0*A*a^16*b^5 + 20*A*a^17*b^4 + 64*A*a^18*b^3 - 12*A*a^19*b^2 + 6*B*a^12*b^9
- 6*B*a^13*b^8 - 14*B*a^14*b^7 + 14*B*a^15*b^6 + 6*B*a^16*b^5 - 6*B*a^17*b
^4 + 6*B*a^18*b^3 - 6*B*a^19*b^2 - 16*A*a^20*b - 4*B*a^20*b))/(a^19*b + a^2
0 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*
b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (4*tan(c/2 + (
d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A
*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 4
8*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 -
160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*
a^20*b^2))/((a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*
a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 +
5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*
b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*
B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b))/(2*(a^18 - a^
4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4
- 7*a^16*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*
b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b))/(2*(a^18 - a^4*b^14 + 7*a^6*b
^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2))
+ (((8*tan(c/2 + (d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 - 8*A^2*a*b

```
^13 - 8*A^2*a^13*b - 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*a^4*b^10 -
 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b^6 - 12
0*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b^2 + 9*B^2
*a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34*A*B*a^7*b
^7 + 20*A*B*a^9*b^5 - 16*A*B*a^11*b^3))/(a^16*b + a^17 - a^6*b^11 - a^7*b^1
0 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^
13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) + (((8*(4*A*a^21 + 4*B*a^21 - 4*A*a^8*b^1
3 + 2*A*a^9*b^12 + 26*A*a^10*b^11 - 14*A*a^11*b^10 - 70*A*a^12*b^9 + 30*A*a
^13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 110*A*a^16*b^5 + 20*A*a^17*b^4 +
 64*A*a^18*b^3 - 12*A*a^19*b^2 + 6*B*a^12*b^9 - 6*B*a^13*b^8 - 14*B*a^14*b^
7 + 14*B*a^15*b^6 + 6*B*a^16*b^5 - 6*B*a^17*b^4 + 6*B*a^18*b^3 - 6*B*a^19*b
^2 - 16*A*a^20*b - 4*B*a^20*b))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a
^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^
4 - 5*a^17*b^3 - 5*a^18*b^2) + (4*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)
^(1/2)*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a
^6*b)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 1
20*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 +
120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2))/((a^18 - a^4*b^14 +
 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^1
6*b^2)*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^
10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)
))*(-(a + b)^7*(a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*
b^3 + 3*B*a^5*b^2 - 8*A*a^6*b))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b
^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*(-(a + b)^7*(
a - b)^7)^(1/2)*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^
2 - 8*A*a^6*b))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^
8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2))))*(-(a + b)^7*(a - b)^7)^(1/2)
*(2*A*b^7 + 2*B*a^7 - 7*A*a^2*b^5 + 8*A*a^4*b^3 + 3*B*a^5*b^2 - 8*A*a^6*b)*
1i)/(d*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*
b^6 + 21*a^14*b^4 - 7*a^16*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(a + b\cos(c + dx))^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**4, x)
```

3.279 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=420

$$-\frac{(4Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^5d}+\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{b\left(-6a^3B+9a^2Ab+ab^2B-4Ab^3\right)\tan(c}{6a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}$$

```
[Out] b*(20*A*a^6*b-35*A*a^4*b^3+28*A*a^2*b^5-8*A*b^7-8*B*a^7+8*B*a^5*b^2-7*B*a^3
*b^4+2*B*a*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b
)^(7/2)/(a+b)^(7/2)/d-(4*A*b-B*a)*arctanh(sin(d*x+c))/a^5/d+1/6*(6*A*a^6-65
*A*a^4*b^2+68*A*a^2*b^4-24*A*b^6+26*B*a^5*b-17*B*a^3*b^3+6*B*a*b^5)*tan(d*x
+c)/a^4/(a^2-b^2)^3/d+1/3*b*(A*b-B*a)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x
+c))^3+1/6*b*(9*A*a^2*b-4*A*b^3-6*B*a^3+B*a*b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2
/d/(a+b*cos(d*x+c))^2+1/2*b*(12*A*a^4*b-11*A*a^2*b^3+4*A*b^5-6*B*a^5+2*B*a^
3*b^2-B*a*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 6.22, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$\frac{b\left(-35a^4Ab^3+28a^2Ab^5+20a^6Ab+8a^5b^2B-7a^3b^4B-8a^7B+2ab^6B-8Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{(-65}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (b*(20*a^6*A*b - 35*a^4*A*b^3 + 28*a^2*A*b^5 - 8*A*b^7 - 8*a^7*B + 8*a^5*b^
2*B - 7*a^3*b^4*B + 2*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
 + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d) - ((4*A*b - a*B)*ArcTanh[Sin[c
+ d*x]])/(a^5*d) + ((6*a^6*A - 65*a^4*A*b^2 + 68*a^2*A*b^4 - 24*A*b^6 + 26*
a^5*b*B - 17*a^3*b^3*B + 6*a*b^5*B)*Tan[c + d*x])/(6*a^4*(a^2 - b^2)^3*d) +
 (b*(A*b - a*B)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) +
(b*(9*a^2*A*b - 4*A*b^3 - 6*a^3*B + a*b^2*B)*Tan[c + d*x])/(6*a^2*(a^2 - b^
2)^2*d*(a + b*Cos[c + d*x])^2) + (b*(12*a^4*A*b - 11*a^2*A*b^3 + 4*A*b^5 -
6*a^5*B + 2*a^3*b^2*B - a*b^4*B)*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a +
b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(3a^2A-4Ab^2+abB-3a(Ab-aB)\cos(c+dx\right.}{(a+b\cos(c}}{3a\left(a^2\right.}$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(9a^2Ab-4Ab^3-6a^3B+ab^2B\right)}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(9a^2Ab-4Ab^3-6a^3B+ab^2B\right)}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c}$$

$$= \frac{\left(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5B\right)}{6a^4\left(a^2-b^2\right)^3 d}$$

$$= \frac{\left(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5B\right)}{6a^4\left(a^2-b^2\right)^3 d}$$

$$= -\frac{(4Ab-aB)\tanh^{-1}(\sin(c+dx))}{a^5 d} + \frac{\left(6a^6A-65a^4Ab^2+68a^2Ab^4-2\right.}{}$$

$$= \frac{b\left(20a^6Ab-35a^4Ab^3+28a^2Ab^5-8Ab^7-8a^7B+8a^5b^2B-7a^3b^4B\right.}{a^5(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A] time = 3.27, size = 549, normalized size = 1.31

$$-\frac{\frac{48b\left(8a^7B-20a^6Ab-8a^5b^2B+35a^4Ab^3+7a^3b^4B-28a^2Ab^5-2ab^6B+8Ab^7\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}}+\frac{2a\tan(c+dx)\left(24a^9A-36a^7Ab^2+6a^6Ab^3\cos(3\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-48*b*(-20*a^6*A*b + 35*a^4*A*b^3 - 28*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 8*
a^5*b^2*B + 7*a^3*b^4*B - 2*a*b^6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqr
t[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + 48*(4*A*b - a*B)*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]] + 48*(-4*A*b + a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + (2*a*(24*a^9*A - 36*a^7*A*b^2 - 246*a^5*A*b^4 + 318*a^3*A*b^6 - 120
*a*A*b^8 + 120*a^6*b^3*B - 90*a^4*b^5*B + 30*a^2*b^7*B + b*(72*a^8*A - 438*
a^6*A*b^2 + 305*a^4*A*b^4 + 28*a^2*A*b^6 - 72*A*b^8 + 144*a^7*b*B - 50*a^5*
b^3*B - 7*a^3*b^5*B + 18*a*b^7*B)*Cos[c + d*x] + 6*a*b^2*(6*a^6*A - 53*a^4*
A*b^2 + 57*a^2*A*b^4 - 20*A*b^6 + 20*a^5*b*B - 15*a^3*b^3*B + 5*a*b^5*B)*Co
s[2*(c + d*x)] + 6*a^6*A*b^3*Cos[3*(c + d*x)] - 65*a^4*A*b^5*Cos[3*(c + d*x
)] + 68*a^2*A*b^7*Cos[3*(c + d*x)] - 24*A*b^9*Cos[3*(c + d*x)] + 26*a^5*b^4
*B*Cos[3*(c + d*x)] - 17*a^3*b^6*B*Cos[3*(c + d*x)] + 6*a*b^8*B*Cos[3*(c +
d*x)])*Tan[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x])^3))/(48*a^5*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [B] time = 6.39, size = 996, normalized size = 2.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(8*B*a^7*b - 20*A*a^6*b^2 - 8*B*a^5*b^3 + 35*A*a^4*b^4 + 7*B*a^3*b^5
 - 28*A*a^2*b^6 - 2*B*a*b^7 + 8*A*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
```

```
n(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/s
qrt(a^2 - b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*sqrt(a^2 - b^2))
 + (36*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*A*a^6*b^3*tan(1/2*d*x + 1/2*c)
^5 - 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 105*A*a^5*b^4*tan(1/2*d*x + 1/2*
c)^5 - 6*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 24*A*a^4*b^5*tan(1/2*d*x + 1/2*
c)^5 + 45*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 - 117*A*a^3*b^6*tan(1/2*d*x + 1/
2*c)^5 - 6*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 24*A*a^2*b^7*tan(1/2*d*x + 1/
2*c)^5 - 15*B*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 + 42*A*a*b^8*tan(1/2*d*x + 1/2
*c)^5 + 6*B*a*b^8*tan(1/2*d*x + 1/2*c)^5 - 18*A*b^9*tan(1/2*d*x + 1/2*c)^5
+ 72*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^
3 - 116*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 + 236*A*a^4*b^5*tan(1/2*d*x + 1/2*
c)^3 + 56*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 - 152*A*a^2*b^7*tan(1/2*d*x + 1/
2*c)^3 - 12*B*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^9*tan(1/2*d*x + 1/2*c)^
3 + 36*B*a^7*b^2*tan(1/2*d*x + 1/2*c) - 60*A*a^6*b^3*tan(1/2*d*x + 1/2*c) +
 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c) - 105*A*a^5*b^4*tan(1/2*d*x + 1/2*c) - 6
*B*a^5*b^4*tan(1/2*d*x + 1/2*c) + 24*A*a^4*b^5*tan(1/2*d*x + 1/2*c) - 45*B*
a^4*b^5*tan(1/2*d*x + 1/2*c) + 117*A*a^3*b^6*tan(1/2*d*x + 1/2*c) - 6*B*a^3
*b^6*tan(1/2*d*x + 1/2*c) + 24*A*a^2*b^7*tan(1/2*d*x + 1/2*c) + 15*B*a^2*b^
7*tan(1/2*d*x + 1/2*c) - 42*A*a*b^8*tan(1/2*d*x + 1/2*c) + 6*B*a*b^8*tan(1/
2*d*x + 1/2*c) - 18*A*b^9*tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2 + 3*a^6*
b^4 - a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b
)^3) + 3*(B*a - 4*A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 3*(B*a - 4*
A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5 - 6*A*tan(1/2*d*x + 1/2*c)/((ta
n(1/2*d*x + 1/2*c)^2 - 1)*a^4))/d
```

maple [B] time = 0.21, size = 2844, normalized size = 6.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)*B-1/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)+1/d/a^
4*ln(tan(1/2*d*x+1/2*c)+1)*B-1/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)+4/d/a^5*ln(ta
n(1/2*d*x+1/2*c)-1)*A*b-6/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c
)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-6/d*b^7/a
^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b
+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A+5/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*
x+1/2*c)^2*b+a+b)^3*b^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*
A+18/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^5/(a+b)/
(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-2/d/a^3/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1
/2*d*x+1/2*c)*A-12/d*b^7/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-6/d*b^7/a^4/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*
```

a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+12/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B*a*b^2+12/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B*a*b^2+24/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B*a*b^2-5/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+18/d/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^5/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+2/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-6/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B-1/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+2/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+116/3/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-44/3/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+4/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+1/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B-4/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)*A*b+20/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-8/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^3*B-20/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A*b^3-40/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A*b^3-20/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A*b^3-4/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^3*B+8/d/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*b^3*B-8/d*b^8/a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-35/d*b^4/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A+28/d*b^6/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-7/d*b^5/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+2/d*b^7/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 18.11, size = 13119, normalized size = 31.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^4),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(18*A*a^8 + 72*A*b^8 - 236*A*a^2*b^6 + 47*A*a^3*b^5
+ 273*A*a^4*b^4 - 60*A*a^5*b^3 - 72*A*a^6*b^2 + 3*B*a^2*b^6 + 59*B*a^3*b^5
- 14*B*a^4*b^4 - 96*B*a^5*b^3 + 36*B*a^6*b^2 - 12*A*a*b^7 - 18*B*a*b^7))/(3
*a^4*(a + b)^2*(a - b)^3) - (tan(c/2 + (d*x)/2)^7*(24*A*a^2*b^5 - 8*A*b^7 -
 2*A*a^7 - 11*A*a^3*b^4 - 26*A*a^4*b^3 + 6*A*a^5*b^2 - B*a^2*b^5 - 6*B*a^3*
b^4 + 4*B*a^4*b^3 + 12*B*a^5*b^2 + 4*A*a*b^6 + 2*A*a^6*b + 2*B*a*b^6))/(a^4
*(a + b)^3*(a - b)) + (tan(c/2 + (d*x)/2)^5*(18*A*a^8 + 72*A*b^8 - 236*A*a^
2*b^6 - 47*A*a^3*b^5 + 273*A*a^4*b^4 + 60*A*a^5*b^3 - 72*A*a^6*b^2 - 3*B*a^
2*b^6 + 59*B*a^3*b^5 + 14*B*a^4*b^4 - 96*B*a^5*b^3 - 36*B*a^6*b^2 + 12*A*a*
b^7 - 18*B*a*b^7))/(3*a^4*(a + b)^3*(a - b)^2) + (tan(c/2 + (d*x)/2)*(2*A*a
^7 - 8*A*b^7 + 24*A*a^2*b^5 + 11*A*a^3*b^4 - 26*A*a^4*b^3 - 6*A*a^5*b^2 + B
*a^2*b^5 - 6*B*a^3*b^4 - 4*B*a^4*b^3 + 12*B*a^5*b^2 - 4*A*a*b^6 + 2*A*a^6*b
 + 2*B*a*b^6))/(a^4*(a + b)*(a - b)^3))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2 + (
d*x)/2)^4*(6*a^2*b - 6*b^3) - tan(c/2 + (d*x)/2)^2*(6*a*b^2 - 2*a^3 + 4*b^3
) - tan(c/2 + (d*x)/2)^6*(2*a^3 - 6*a*b^2 + 4*b^3) + a^3 + b^3 - tan(c/2 +
(d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (atan((((((4*A*b - B*a)*((8*
(4*B*a^24 + 16*A*a^10*b^14 - 8*A*a^11*b^13 - 104*A*a^12*b^12 + 50*A*a^13*b^
11 + 286*A*a^14*b^10 - 126*A*a^15*b^9 - 434*A*a^16*b^8 + 174*A*a^17*b^7 + 3
86*A*a^18*b^6 - 146*A*a^19*b^5 - 190*A*a^20*b^4 + 72*A*a^21*b^3 + 40*A*a^22
*b^2 - 4*B*a^11*b^13 + 2*B*a^12*b^12 + 26*B*a^13*b^11 - 14*B*a^14*b^10 - 70
*B*a^15*b^9 + 30*B*a^16*b^8 + 110*B*a^17*b^7 - 30*B*a^18*b^6 - 110*B*a^19*b
^5 + 20*B*a^20*b^4 + 64*B*a^21*b^3 - 12*B*a^22*b^2 - 16*A*a^23*b - 16*B*a^2
3*b))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10
*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b
```

^2) - (8*tan(c/2 + (d*x)/2)*(4*A*b - B*a)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*
b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^1
6*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*
b^3 - 8*a^22*b^2))/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 +
5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^1
6*b^3 - 5*a^17*b^2))))/a^5 - (8*tan(c/2 + (d*x)/2)*(128*A^2*b^16 + 4*B^2*a^
16 - 128*A^2*a*b^15 - 8*B^2*a^15*b - 768*A^2*a^2*b^14 + 768*A^2*a^3*b^13 +
1920*A^2*a^4*b^12 - 1920*A^2*a^5*b^11 - 2600*A^2*a^6*b^10 + 2560*A^2*a^7*b^
9 + 2025*A^2*a^8*b^8 - 1920*A^2*a^9*b^7 - 824*A^2*a^10*b^6 + 768*A^2*a^11*b
^5 + 80*A^2*a^12*b^4 - 128*A^2*a^13*b^3 + 64*A^2*a^14*b^2 + 8*B^2*a^2*b^14
- 8*B^2*a^3*b^13 - 48*B^2*a^4*b^12 + 48*B^2*a^5*b^11 + 117*B^2*a^6*b^10 - 1
20*B^2*a^7*b^9 - 164*B^2*a^8*b^8 + 160*B^2*a^9*b^7 + 156*B^2*a^10*b^6 - 120
*B^2*a^11*b^5 - 92*B^2*a^12*b^4 + 48*B^2*a^13*b^3 + 44*B^2*a^14*b^2 - 64*A*
B*a*b^15 - 32*A*B*a^15*b + 64*A*B*a^2*b^14 + 384*A*B*a^3*b^13 - 384*A*B*a^4
*b^12 - 948*A*B*a^5*b^11 + 960*A*B*a^6*b^10 + 1306*A*B*a^7*b^9 - 1280*A*B*a
^8*b^8 - 1128*A*B*a^9*b^7 + 960*A*B*a^10*b^6 + 592*A*B*a^11*b^5 - 384*A*B*a
^12*b^4 - 160*A*B*a^13*b^3 + 64*A*B*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 -
a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^
5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))*(4*A*b - B*a)*1i)/a^5 - ((((4*A
*b - B*a)*((8*(4*B*a^24 + 16*A*a^10*b^14 - 8*A*a^11*b^13 - 104*A*a^12*b^12
+ 50*A*a^13*b^11 + 286*A*a^14*b^10 - 126*A*a^15*b^9 - 434*A*a^16*b^8 + 174*
A*a^17*b^7 + 386*A*a^18*b^6 - 146*A*a^19*b^5 - 190*A*a^20*b^4 + 72*A*a^21*b
^3 + 40*A*a^22*b^2 - 4*B*a^11*b^13 + 2*B*a^12*b^12 + 26*B*a^13*b^11 - 14*B*
a^14*b^10 - 70*B*a^15*b^9 + 30*B*a^16*b^8 + 110*B*a^17*b^7 - 30*B*a^18*b^6
- 110*B*a^19*b^5 + 20*B*a^20*b^4 + 64*B*a^21*b^3 - 12*B*a^22*b^2 - 16*A*a^2
3*b - 16*B*a^23*b))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5
*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*
b^3 - 5*a^21*b^2) + (8*tan(c/2 + (d*x)/2)*(4*A*b - B*a)*(8*a^23*b - 8*a^10*
b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15
*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*
b^4 - 48*a^21*b^3 - 8*a^22*b^2))/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10
+ 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^
15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))))/a^5 + (8*tan(c/2 + (d*x)/2)*(128*A^2*b
^16 + 4*B^2*a^16 - 128*A^2*a*b^15 - 8*B^2*a^15*b - 768*A^2*a^2*b^14 + 768*A
^2*a^3*b^13 + 1920*A^2*a^4*b^12 - 1920*A^2*a^5*b^11 - 2600*A^2*a^6*b^10 + 2
560*A^2*a^7*b^9 + 2025*A^2*a^8*b^8 - 1920*A^2*a^9*b^7 - 824*A^2*a^10*b^6 +
768*A^2*a^11*b^5 + 80*A^2*a^12*b^4 - 128*A^2*a^13*b^3 + 64*A^2*a^14*b^2 + 8
*B^2*a^2*b^14 - 8*B^2*a^3*b^13 - 48*B^2*a^4*b^12 + 48*B^2*a^5*b^11 + 117*B^
2*a^6*b^10 - 120*B^2*a^7*b^9 - 164*B^2*a^8*b^8 + 160*B^2*a^9*b^7 + 156*B^2*
a^10*b^6 - 120*B^2*a^11*b^5 - 92*B^2*a^12*b^4 + 48*B^2*a^13*b^3 + 44*B^2*a^
14*b^2 - 64*A*B*a*b^15 - 32*A*B*a^15*b + 64*A*B*a^2*b^14 + 384*A*B*a^3*b^13
- 384*A*B*a^4*b^12 - 948*A*B*a^5*b^11 + 960*A*B*a^6*b^10 + 1306*A*B*a^7*b^
9 - 1280*A*B*a^8*b^8 - 1128*A*B*a^9*b^7 + 960*A*B*a^10*b^6 + 592*A*B*a^11*b
^5 - 384*A*B*a^12*b^4 - 160*A*B*a^13*b^3 + 64*A*B*a^14*b^2))/(a^18*b + a^19
- a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^

6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))*(4*A*b - B*a)*1i)
/a^5)/(((((4*A*b - B*a)*((8*(4*B*a^24 + 16*A*a^10*b^14 - 8*A*a^11*b^13 - 10
4*A*a^12*b^12 + 50*A*a^13*b^11 + 286*A*a^14*b^10 - 126*A*a^15*b^9 - 434*A*a
^16*b^8 + 174*A*a^17*b^7 + 386*A*a^18*b^6 - 146*A*a^19*b^5 - 190*A*a^20*b^4
 + 72*A*a^21*b^3 + 40*A*a^22*b^2 - 4*B*a^11*b^13 + 2*B*a^12*b^12 + 26*B*a^1
3*b^11 - 14*B*a^14*b^10 - 70*B*a^15*b^9 + 30*B*a^16*b^8 + 110*B*a^17*b^7 -
30*B*a^18*b^6 - 110*B*a^19*b^5 + 20*B*a^20*b^4 + 64*B*a^21*b^3 - 12*B*a^22*
b^2 - 16*A*a^23*b - 16*B*a^23*b))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 +
5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19
*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (8*tan(c/2 + (d*x)/2)*(4*A*b - B*a)*(8*a^
23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b
^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*
b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2))/(a^5*(a^18*b + a^19 - a^8*b^
11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^
14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))))/a^5 - (8*tan(c/2 + (d*x)
/2)*(128*A^2*b^16 + 4*B^2*a^16 - 128*A^2*a*b^15 - 8*B^2*a^15*b - 768*A^2*a^
2*b^14 + 768*A^2*a^3*b^13 + 1920*A^2*a^4*b^12 - 1920*A^2*a^5*b^11 - 2600*A^
2*a^6*b^10 + 2560*A^2*a^7*b^9 + 2025*A^2*a^8*b^8 - 1920*A^2*a^9*b^7 - 824*A
^2*a^10*b^6 + 768*A^2*a^11*b^5 + 80*A^2*a^12*b^4 - 128*A^2*a^13*b^3 + 64*A^
2*a^14*b^2 + 8*B^2*a^2*b^14 - 8*B^2*a^3*b^13 - 48*B^2*a^4*b^12 + 48*B^2*a^5
*b^11 + 117*B^2*a^6*b^10 - 120*B^2*a^7*b^9 - 164*B^2*a^8*b^8 + 160*B^2*a^9*
b^7 + 156*B^2*a^10*b^6 - 120*B^2*a^11*b^5 - 92*B^2*a^12*b^4 + 48*B^2*a^13*b
^3 + 44*B^2*a^14*b^2 - 64*A*B*a*b^15 - 32*A*B*a^15*b + 64*A*B*a^2*b^14 + 38
4*A*B*a^3*b^13 - 384*A*B*a^4*b^12 - 948*A*B*a^5*b^11 + 960*A*B*a^6*b^10 + 1
306*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 - 1128*A*B*a^9*b^7 + 960*A*B*a^10*b^6 +
592*A*B*a^11*b^5 - 384*A*B*a^12*b^4 - 160*A*B*a^13*b^3 + 64*A*B*a^14*b^2))/
(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^
7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))*(4*
A*b - B*a))/a^5 - (16*(256*A^3*b^16 - 128*A^3*a*b^15 - 16*B^3*a^15*b - 1664
*A^3*a^2*b^14 + 800*A^3*a^3*b^13 + 4576*A^3*a^4*b^12 - 2176*A^3*a^5*b^11 -
6944*A^3*a^6*b^10 + 3204*A^3*a^7*b^9 + 6176*A^3*a^8*b^8 - 2560*A^3*a^9*b^7
- 3040*A^3*a^10*b^6 + 960*A^3*a^11*b^5 + 640*A^3*a^12*b^4 - 4*B^3*a^3*b^13
+ 2*B^3*a^4*b^12 + 26*B^3*a^5*b^11 - 11*B^3*a^6*b^10 - 70*B^3*a^7*b^9 + 34*
B^3*a^8*b^8 + 110*B^3*a^9*b^7 - 66*B^3*a^10*b^6 - 110*B^3*a^11*b^5 + 64*B^3
*a^12*b^4 + 64*B^3*a^13*b^3 - 48*B^3*a^14*b^2 - 192*A^2*B*a*b^15 + 48*A*B^2
*a^2*b^14 - 24*A*B^2*a^3*b^13 - 312*A*B^2*a^4*b^12 + 138*A*B^2*a^5*b^11 + 8
46*A*B^2*a^6*b^10 - 408*A*B^2*a^7*b^9 - 1314*A*B^2*a^8*b^8 + 726*A*B^2*a^9*
b^7 + 1266*A*B^2*a^10*b^6 - 690*A*B^2*a^11*b^5 - 702*A*B^2*a^12*b^4 + 408*A
*B^2*a^13*b^3 + 168*A*B^2*a^14*b^2 + 96*A^2*B*a^2*b^14 + 1248*A^2*B*a^3*b^1
3 - 576*A^2*B*a^4*b^12 - 3408*A^2*B*a^5*b^11 + 1632*A^2*B*a^6*b^10 + 5232*A
^2*B*a^7*b^9 - 2649*A^2*B*a^8*b^8 - 4848*A^2*B*a^9*b^7 + 2376*A^2*B*a^10*b^
6 + 2544*A^2*B*a^11*b^5 - 1104*A^2*B*a^12*b^4 - 576*A^2*B*a^13*b^3))/(a^22*
b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 -
10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) + ((((4*
A*b - B*a)*((8*(4*B*a^24 + 16*A*a^10*b^14 - 8*A*a^11*b^13 - 104*A*a^12*b^12

+ 50*A*a^13*b^11 + 286*A*a^14*b^10 - 126*A*a^15*b^9 - 434*A*a^16*b^8 + 174
*A*a^17*b^7 + 386*A*a^18*b^6 - 146*A*a^19*b^5 - 190*A*a^20*b^4 + 72*A*a^21*
b^3 + 40*A*a^22*b^2 - 4*B*a^11*b^13 + 2*B*a^12*b^12 + 26*B*a^13*b^11 - 14*B
*a^14*b^10 - 70*B*a^15*b^9 + 30*B*a^16*b^8 + 110*B*a^17*b^7 - 30*B*a^18*b^6
- 110*B*a^19*b^5 + 20*B*a^20*b^4 + 64*B*a^21*b^3 - 12*B*a^22*b^2 - 16*A*a^
23*b - 16*B*a^23*b))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 +
5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20
*b^3 - 5*a^21*b^2) + (8*tan(c/2 + (d*x)/2)*(4*A*b - B*a)*(8*a^23*b - 8*a^10
*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^1
5*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20
*b^4 - 48*a^21*b^3 - 8*a^22*b^2))/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10
+ 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a
^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))))/a^5 + (8*tan(c/2 + (d*x)/2)*(128*A^2*
b^16 + 4*B^2*a^16 - 128*A^2*a*b^15 - 8*B^2*a^15*b - 768*A^2*a^2*b^14 + 768*
A^2*a^3*b^13 + 1920*A^2*a^4*b^12 - 1920*A^2*a^5*b^11 - 2600*A^2*a^6*b^10 +
2560*A^2*a^7*b^9 + 2025*A^2*a^8*b^8 - 1920*A^2*a^9*b^7 - 824*A^2*a^10*b^6 +
768*A^2*a^11*b^5 + 80*A^2*a^12*b^4 - 128*A^2*a^13*b^3 + 64*A^2*a^14*b^2 +
8*B^2*a^2*b^14 - 8*B^2*a^3*b^13 - 48*B^2*a^4*b^12 + 48*B^2*a^5*b^11 + 117*B
^2*a^6*b^10 - 120*B^2*a^7*b^9 - 164*B^2*a^8*b^8 + 160*B^2*a^9*b^7 + 156*B^2
*a^10*b^6 - 120*B^2*a^11*b^5 - 92*B^2*a^12*b^4 + 48*B^2*a^13*b^3 + 44*B^2*a
^14*b^2 - 64*A*B*a*b^15 - 32*A*B*a^15*b + 64*A*B*a^2*b^14 + 384*A*B*a^3*b^1
3 - 384*A*B*a^4*b^12 - 948*A*B*a^5*b^11 + 960*A*B*a^6*b^10 + 1306*A*B*a^7*b
^9 - 1280*A*B*a^8*b^8 - 1128*A*B*a^9*b^7 + 960*A*B*a^10*b^6 + 592*A*B*a^11*
b^5 - 384*A*B*a^12*b^4 - 160*A*B*a^13*b^3 + 64*A*B*a^14*b^2))/(a^18*b + a^1
9 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b
^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))*(4*A*b - B*a))/a
^5))*(4*A*b - B*a)*2i)/(a^5*d) + (b*atan(((b*((8*tan(c/2 + (d*x)/2)*(128*A^
2*b^16 + 4*B^2*a^16 - 128*A^2*a*b^15 - 8*B^2*a^15*b - 768*A^2*a^2*b^14 + 76
8*A^2*a^3*b^13 + 1920*A^2*a^4*b^12 - 1920*A^2*a^5*b^11 - 2600*A^2*a^6*b^10
+ 2560*A^2*a^7*b^9 + 2025*A^2*a^8*b^8 - 1920*A^2*a^9*b^7 - 824*A^2*a^10*b^6
+ 768*A^2*a^11*b^5 + 80*A^2*a^12*b^4 - 128*A^2*a^13*b^3 + 64*A^2*a^14*b^2
+ 8*B^2*a^2*b^14 - 8*B^2*a^3*b^13 - 48*B^2*a^4*b^12 + 48*B^2*a^5*b^11 + 117
*B^2*a^6*b^10 - 120*B^2*a^7*b^9 - 164*B^2*a^8*b^8 + 160*B^2*a^9*b^7 + 156*B
^2*a^10*b^6 - 120*B^2*a^11*b^5 - 92*B^2*a^12*b^4 + 48*B^2*a^13*b^3 + 44*B^2
*a^14*b^2 - 64*A*B*a*b^15 - 32*A*B*a^15*b + 64*A*B*a^2*b^14 + 384*A*B*a^3*b
^13 - 384*A*B*a^4*b^12 - 948*A*B*a^5*b^11 + 960*A*B*a^6*b^10 + 1306*A*B*a^7
*b^9 - 1280*A*B*a^8*b^8 - 1128*A*B*a^9*b^7 + 960*A*B*a^10*b^6 + 592*A*B*a^1
1*b^5 - 384*A*B*a^12*b^4 - 160*A*B*a^13*b^3 + 64*A*B*a^14*b^2))/(a^18*b + a
^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13
*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) - (b*(-(a + b)^
7*(a - b)^7)^(1/2)*((8*(4*B*a^24 + 16*A*a^10*b^14 - 8*A*a^11*b^13 - 104*A*a
^12*b^12 + 50*A*a^13*b^11 + 286*A*a^14*b^10 - 126*A*a^15*b^9 - 434*A*a^16*b
^8 + 174*A*a^17*b^7 + 386*A*a^18*b^6 - 146*A*a^19*b^5 - 190*A*a^20*b^4 + 72
*A*a^21*b^3 + 40*A*a^22*b^2 - 4*B*a^11*b^13 + 2*B*a^12*b^12 + 26*B*a^13*b^1
1 - 14*B*a^14*b^10 - 70*B*a^15*b^9 + 30*B*a^16*b^8 + 110*B*a^17*b^7 - 30*B*

a^18*b^6 - 110*B*a^19*b^5 + 20*B*a^20*b^4 + 64*B*a^21*b^3 - 12*B*a^22*b^2 - 16*A*a^23*b - 16*B*a^23*b))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (4*b*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2))/((a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6)*1i)/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)) + (b*((8*tan(c/2 + (d*x)/2)*(128*A^2*b^16 + 4*B^2*a^16 - 128*A^2*a*b^15 - 8*B^2*a^15*b - 768*A^2*a^2*b^14 + 768*A^2*a^3*b^13 + 1920*A^2*a^4*b^12 - 1920*A^2*a^5*b^11 - 2600*A^2*a^6*b^10 + 2560*A^2*a^7*b^9 + 2025*A^2*a^8*b^8 - 1920*A^2*a^9*b^7 - 824*A^2*a^10*b^6 + 768*A^2*a^11*b^5 + 80*A^2*a^12*b^4 - 128*A^2*a^13*b^3 + 64*A^2*a^14*b^2 + 8*B^2*a^2*b^14 - 8*B^2*a^3*b^13 - 48*B^2*a^4*b^12 + 48*B^2*a^5*b^11 + 117*B^2*a^6*b^10 - 120*B^2*a^7*b^9 - 164*B^2*a^8*b^8 + 160*B^2*a^9*b^7 + 156*B^2*a^10*b^6 - 120*B^2*a^11*b^5 - 92*B^2*a^12*b^4 + 48*B^2*a^13*b^3 + 44*B^2*a^14*b^2 - 64*A*B*a*b^15 - 32*A*B*a^15*b + 64*A*B*a^2*b^14 + 384*A*B*a^3*b^13 - 384*A*B*a^4*b^12 - 948*A*B*a^5*b^11 + 960*A*B*a^6*b^10 + 1306*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 - 1128*A*B*a^9*b^7 + 960*A*B*a^10*b^6 + 592*A*B*a^11*b^5 - 384*A*B*a^12*b^4 - 160*A*B*a^13*b^3 + 64*A*B*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) + (b*(-(a + b)^7*(a - b)^7)^(1/2)*((8*(4*B*a^24 + 16*A*a^10*b^14 - 8*A*a^11*b^13 - 104*A*a^12*b^12 + 50*A*a^13*b^11 + 286*A*a^14*b^10 - 126*A*a^15*b^9 - 434*A*a^16*b^8 + 174*A*a^17*b^7 + 386*A*a^18*b^6 - 146*A*a^19*b^5 - 190*A*a^20*b^4 + 72*A*a^21*b^3 + 40*A*a^22*b^2 - 4*B*a^11*b^13 + 2*B*a^12*b^12 + 26*B*a^13*b^11 - 14*B*a^14*b^10 - 70*B*a^15*b^9 + 30*B*a^16*b^8 + 110*B*a^17*b^7 - 30*B*a^18*b^6 - 110*B*a^19*b^5 + 20*B*a^20*b^4 + 64*B*a^21*b^3 - 12*B*a^22*b^2 - 16*A*a^23*b - 16*B*a^23*b))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) + (4*b*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2))/((a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13 *b^6 + 21*a^15*b^4 - 7*a^17*b^2)*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6)*1i)/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))/((16*(256*A^3*b^16 - 128*A^3*a*b^15 - 16*B^3*a^15*b - 1664*A^3*a^2*b^14 + 800*A^3*a^3*b^13 + 4576*A^3*a^4*b^12 - 2176*A^3*a^5*b^11 - 6944*A^3*a^6*b^10 + 3204*A^3*a^7*b^9 + 6176*A^3*a^8*b^8 - 2560*A^3*a^9*b^7 - 3040*A^3*a^10*b^6 + 960*A^3*a^11*b^5 + 640*A^3*a^12*b^4 - 4*B^3*a^3*b^13 + 2*B^3*a^4*b^12 + 26*B^3*a^5*b^11 - 11*B^3*a^6*b^10 - 70*B^3*a^7*b^9 + 34*B^3*a^8*b^8 + 110*B^3*a^9*b^7 - 66*B^3*a^10*b^6 - 110*B^3*a^11*b^5 + 64*B^3*a^12*b^4 + 64*B^3*a^13*b^3 - 48*B^3*a^14*b^2 - 192*A^2*B*a*b^15 + 48*A*B^2*a^2*b^14 - 24*A*B^2*a^3*b^13 - 312*A*B^2*a^4*b^12 + 138*A*B^2*a^5*b^11 + 846*A*B^2*a^6*b^10 - 408*A*B^2*a^7*b^9 - 1314*A*B^2*a^8*b^8 + 726*A*B^2*a^9*b^7 + 1266*A*B^2*a^10*b^6 - 690*A*B^2*a^11*b^5 - 702*A*B^2*a^12*b^4 + 408*A*B^2*a^13*b^3 + 168*A*B^2*a^14*b^2 + 96*A^2*B*a^2*b^14 + 1248*A^2*B*a^3*b^13 - 576*A^2*B*a^4*b^12 - 3408*A^2*B*a^5*b^11 + 1632*A^2*B*a^6*b^10 + 5232*A^2*B*a^7*b^9 - 2649*A^2*B*a^8*b^8 - 4848*A^2*B*a^9*b^7 + 2376*A^2*B*a^10*b^6 + 2544*A^2*B*a^11*b^5 - 1104*A^2*B*a^12*b^4 - 576*A^2*B*a^13*b^3))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) + (b*((8*tan(c/2 + (d*x)/2)*(128*A^2*b^16 + 4*B^2*a^16 - 128*A^2*a*b^15 - 8*B^2*a^15*b - 768*A^2*a^2*b^14 + 768*A^2*a^3*b^13 + 1920*A^2*a^4*b^12 - 1920*A^2*a^5*b^11 - 2600*A^2*a^6*b^10 + 2560*A^2*a^7*b^9 + 2025*A^2*a^8*b^8 - 1920*A^2*a^9*b^7 - 824*A^2*a^10*b^6 + 768*A^2*a^11*b^5 + 80*A^2*a^12*b^4 - 128*A^2*a^13*b^3 + 64*A^2*a^14*b^2 + 8*B^2*a^2*b^14 - 8*B^2*a^3*b^13 - 48*B^2*a^4*b^12 + 48*B^2*a^5*b^11 + 117*B^2*a^6*b^10 - 120*B^2*a^7*b^9 - 164*B^2*a^8*b^8 + 160*B^2*a^9*b^7 + 156*B^2*a^10*b^6 - 120*B^2*a^11*b^5 - 92*B^2*a^12*b^4 + 48*B^2*a^13*b^3 + 44*B^2*a^14*b^2 - 64*A*B*a*b^15 - 32*A*B*a^15*b + 64*A*B*a^2*b^14 + 384*A*B*a^3*b^13 - 384*A*B*a^4*b^12 - 948*A*B*a^5*b^11 + 960*A*B*a^6*b^10 + 1306*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 - 1128*A*B*a^9*b^7 + 960*A*B*a^10*b^6 + 592*A*B*a^11*b^5 - 384*A*B*a^12*b^4 - 160*A*B*a^13*b^3 + 64*A*B*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) - (b*(-(a + b)^7*(a - b)^7)^(1/2)*((8*(4*B*a^24 + 16*A*a^10*b^14 - 8*A*a^11*b^13 - 104*A*a^12*b^12 + 50*A*a^13*b^11 + 286*A*a^14*b^10 - 126*A*a^15*b^9 - 434*A*a^16*b^8 + 174*A*a^17*b^7 + 386*A*a^18*b^6 - 146*A*a^19*b^5 - 190*A*a^20*b^4 + 72*A*a^21*b^3 + 40*A*a^22*b^2 - 4*B*a^11*b^13 + 2*B*a^12*b^12 + 26*B*a^13*b^11 - 14*B*a^14*b^10 - 70*B*a^15*b^9 + 30*B*a^16*b^8 + 110*B*a^17*b^7 - 30*B*a^18*b^6 - 110*B*a^19*b^5 + 20*B*a^20*b^4 + 64*B*a^21*b^3 - 12*B*a^22*b^2 - 16*A*a^23*b - 16*B*a^23*b))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 -

10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (4*b*
tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 28*A*a
^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6)
*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*
a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120
*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2))/((a^19 - a^5*b^14 + 7*
a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b
^2)*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^1
2*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))
)*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B*a^5*
b^2 - 20*A*a^6*b - 2*B*a*b^6))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^
10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*(-(a + b)^7*(a
 - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b
^4 - 8*B*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6))/(2*(a^19 - a^5*b^14 + 7*a^7*b^1
2 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)) -
(b*((8*tan(c/2 + (d*x)/2)*(128*A^2*b^16 + 4*B^2*a^16 - 128*A^2*a*b^15 - 8*B
^2*a^15*b - 768*A^2*a^2*b^14 + 768*A^2*a^3*b^13 + 1920*A^2*a^4*b^12 - 1920*
A^2*a^5*b^11 - 2600*A^2*a^6*b^10 + 2560*A^2*a^7*b^9 + 2025*A^2*a^8*b^8 - 19
20*A^2*a^9*b^7 - 824*A^2*a^10*b^6 + 768*A^2*a^11*b^5 + 80*A^2*a^12*b^4 - 12
8*A^2*a^13*b^3 + 64*A^2*a^14*b^2 + 8*B^2*a^2*b^14 - 8*B^2*a^3*b^13 - 48*B^2
*a^4*b^12 + 48*B^2*a^5*b^11 + 117*B^2*a^6*b^10 - 120*B^2*a^7*b^9 - 164*B^2*
a^8*b^8 + 160*B^2*a^9*b^7 + 156*B^2*a^10*b^6 - 120*B^2*a^11*b^5 - 92*B^2*a^
12*b^4 + 48*B^2*a^13*b^3 + 44*B^2*a^14*b^2 - 64*A*B*a*b^15 - 32*A*B*a^15*b
+ 64*A*B*a^2*b^14 + 384*A*B*a^3*b^13 - 384*A*B*a^4*b^12 - 948*A*B*a^5*b^11
+ 960*A*B*a^6*b^10 + 1306*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 - 1128*A*B*a^9*b^7
 + 960*A*B*a^10*b^6 + 592*A*B*a^11*b^5 - 384*A*B*a^12*b^4 - 160*A*B*a^13*b^
3 + 64*A*B*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5
*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*
b^3 - 5*a^17*b^2) + (b*(-(a + b)^7*(a - b)^7)^(1/2)*((8*(4*B*a^24 + 16*A*a^
10*b^14 - 8*A*a^11*b^13 - 104*A*a^12*b^12 + 50*A*a^13*b^11 + 286*A*a^14*b^1
0 - 126*A*a^15*b^9 - 434*A*a^16*b^8 + 174*A*a^17*b^7 + 386*A*a^18*b^6 - 146
*A*a^19*b^5 - 190*A*a^20*b^4 + 72*A*a^21*b^3 + 40*A*a^22*b^2 - 4*B*a^11*b^1
3 + 2*B*a^12*b^12 + 26*B*a^13*b^11 - 14*B*a^14*b^10 - 70*B*a^15*b^9 + 30*B*
a^16*b^8 + 110*B*a^17*b^7 - 30*B*a^18*b^6 - 110*B*a^19*b^5 + 20*B*a^20*b^4
+ 64*B*a^21*b^3 - 12*B*a^22*b^2 - 16*A*a^23*b - 16*B*a^23*b))/(a^22*b + a^2
3 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17
*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) + (4*b*tan(c/2
+ (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 +
 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6)*(8*a^23
*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^1
0 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^
5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2))/((a^19 - a^5*b^14 + 7*a^7*b^12
 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)*(a^1
8*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 -
10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*(8*A*b

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^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B*a^5*b^2 - 20
*A*a^6*b - 2*B*a*b^6))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*
a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*(-(a + b)^7*(a - b)^7)
^(1/2)*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B
*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a
^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2))))*(-(a + b
)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B
*a^3*b^4 - 8*B*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6)*1i)/(d*(a^19 - a^5*b^14 +
7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17
*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{(a + b\cos(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**4, x)
```

3.280 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=547

$$\frac{b(Ab-aB)\tan(c+dx)\sec(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{\left(a^2A-8abB+20Ab^2\right)\tanh^{-1}(\sin(c+dx))}{2a^6d}+\frac{b\left(-7a^3B+10a^2Ab+2ab^2B\right.}{6a^2d\left(a^2-b^2\right)^2}$$

```
[Out] -b^2*(40*A*a^6*b-84*A*a^4*b^3+69*A*a^2*b^5-20*A*b^7-20*B*a^7+35*B*a^5*b^2-2
8*B*a^3*b^4+8*B*a*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a
^6/(a-b)^(7/2)/(a+b)^(7/2)/d+1/2*(A*a^2+20*A*b^2-8*B*a*b)*arctanh(sin(d*x+c
))/a^6/d-1/6*(24*A*a^6*b-146*A*a^4*b^3+167*A*a^2*b^5-60*A*b^7-6*B*a^7+65*B*
a^5*b^2-68*B*a^3*b^4+24*B*a*b^6)*tan(d*x+c)/a^5/(a^2-b^2)^3/d+1/2*(A*a^6-23
*A*a^4*b^2+27*A*a^2*b^4-10*A*b^6+12*B*a^5*b-11*B*a^3*b^3+4*B*a*b^5)*sec(d*x
+c)*tan(d*x+c)/a^4/(a^2-b^2)^3/d+1/3*b*(A*b-B*a)*sec(d*x+c)*tan(d*x+c)/a/(a
^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*b*(10*A*a^2*b-5*A*b^3-7*B*a^3+2*B*a*b^2)*s
ec(d*x+c)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*b*(48*A*a^4*b
-53*A*a^2*b^3+20*A*b^5-27*B*a^5+20*B*a^3*b^2-8*B*a*b^4)*sec(d*x+c)*tan(d*x+
c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 7.30, antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3000, 3055, 3001, 3770, 2659, 205}

$$-\frac{b^2\left(-84a^4Ab^3+69a^2Ab^5+40a^6Ab+35a^5b^2B-28a^3b^4B-20a^7B+8ab^6B-20Ab^7\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right.}{a^6d(a-b)^{7/2}(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((b^2*(40*a^6*A*b - 84*a^4*A*b^3 + 69*a^2*A*b^5 - 20*A*b^7 - 20*a^7*B + 35
*a^5*b^2*B - 28*a^3*b^4*B + 8*a*b^6*B)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a^6*(a - b)^(7/2)*(a + b)^(7/2)*d)) + ((a^2*A + 20*A*b^2 -
 8*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*a^6*d) - ((24*a^6*A*b - 146*a^4*A*b^3 +
 167*a^2*A*b^5 - 60*A*b^7 - 6*a^7*B + 65*a^5*b^2*B - 68*a^3*b^4*B + 24*a*b^
6*B)*Tan[c + d*x])/(6*a^5*(a^2 - b^2)^3*d) + ((a^6*A - 23*a^4*A*b^2 + 27*a^
2*A*b^4 - 10*A*b^6 + 12*a^5*b*B - 11*a^3*b^3*B + 4*a*b^5*B)*Sec[c + d*x]*Ta
n[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + (b*(A*b - a*B)*Sec[c + d*x]*Tan[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (b*(10*a^2*A*b - 5*A*b^3
- 7*a^3*B + 2*a*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a
```

```
 + b*Cos[c + d*x])^2) + (b*(48*a^4*A*b - 53*a^2*A*b^3 + 20*A*b^5 - 27*a^5*B
 + 20*a^3*b^2*B - 8*a*b^4*B)*Sec[c + d*x]*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^
3*d*(a + b*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
```

```
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(3a^2A-5Ab^2+2abB-3a(Ab-aB)\cos(c+d\right.}{(a+b\cos(}}{3a\left(a^2\right.}$$

$$= \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(10a^2Ab-5Ab^3-7a^3B+2ab^2\right.}{6a^2\left(a^2-b^2\right)^2 d(a+b}$$

$$= \frac{b(Ab-aB)\sec(c+dx)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b\left(10a^2Ab-5Ab^3-7a^3B+2ab^2\right.}{6a^2\left(a^2-b^2\right)^2 d(a+b}$$

$$= \frac{\left(a^6A-23a^4Ab^2+27a^2Ab^4-10Ab^6+12a^5bB-11a^3b^3B+4ab^5B\right)\sec}{2a^4\left(a^2-b^2\right)^3 d}$$

$$= -\frac{\left(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B+65a^5b^2B-68a^3\right.}{6a^5\left(a^2-b^2\right)^3 d}$$

$$= -\frac{\left(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B+65a^5b^2B-68a^3\right.}{6a^5\left(a^2-b^2\right)^3 d}$$

$$= \frac{\left(a^2A+20Ab^2-8abB\right)\tanh^{-1}(\sin(c+dx))}{2a^6d} - \frac{\left(24a^6Ab-146a^4Ab^3+\right.}{}$$

$$= -\frac{b^2\left(40a^6Ab-84a^4Ab^3+69a^2Ab^5-20Ab^7-20a^7B+35a^5b^2B-28a\right.}{a^6(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A] time = 5.29, size = 781, normalized size = 1.43

$$\frac{-48\left(a^2A-8abB+20Ab^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+48\left(a^2A-8abB+20Ab^2\right)\log\left(\sin\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((96*b^2*(-40*a^6*A*b + 84*a^4*A*b^3 - 69*a^2*A*b^5 + 20*A*b^7 + 20*a^7*B -
 35*a^5*b^2*B + 28*a^3*b^4*B - 8*a*b^6*B)*ArcTanh[((a - b)*Tan[(c + d*x)/2]
)/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - 48*(a^2*A + 20*A*b^2 - 8*a*b*B)*L
og[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 48*(a^2*A + 20*A*b^2 - 8*a*b*B)*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a*(24*a^10*A - 324*a^8*A*b^2 +
 1116*a^6*A*b^4 - 830*a^4*A*b^6 - 61*a^2*A*b^8 + 180*A*b^10 + 72*a^9*b*B -
438*a^7*b^3*B + 305*a^5*b^5*B + 28*a^3*b^7*B - 72*a*b^9*B + 6*a*(-20*a^8*A*
b - 9*a^6*A*b^3 + 309*a^4*A*b^5 - 400*a^2*A*b^7 + 150*A*b^9 + 8*a^9*B - 6*a
^7*b^2*B - 135*a^5*b^4*B + 163*a^3*b^6*B - 60*a*b^8*B)*Cos[c + d*x] + 12*b*
(-21*a^8*A*b + 85*a^6*A*b^3 - 55*a^4*A*b^5 - 19*a^2*A*b^7 + 20*A*b^9 + 6*a^
9*B - 36*a^7*b^2*B + 20*a^5*b^4*B + 8*a^3*b^6*B - 8*a*b^8*B)*Cos[2*(c + d*x
)] - 138*a^7*A*b^3*Cos[3*(c + d*x)] + 738*a^5*A*b^5*Cos[3*(c + d*x)] - 840*
a^3*A*b^7*Cos[3*(c + d*x)] + 300*a*A*b^9*Cos[3*(c + d*x)] + 36*a^8*b^2*B*Co
s[3*(c + d*x)] - 318*a^6*b^4*B*Cos[3*(c + d*x)] + 342*a^4*b^6*B*Cos[3*(c +
d*x)] - 120*a^2*b^8*B*Cos[3*(c + d*x)] - 24*a^6*A*b^4*Cos[4*(c + d*x)] + 14
6*a^4*A*b^6*Cos[4*(c + d*x)] - 167*a^2*A*b^8*Cos[4*(c + d*x)] + 60*A*b^10*C
os[4*(c + d*x)] + 6*a^7*b^3*B*Cos[4*(c + d*x)] - 65*a^5*b^5*B*Cos[4*(c + d*
x)] + 68*a^3*b^7*B*Cos[4*(c + d*x)] - 24*a*b^9*B*Cos[4*(c + d*x)])*Sec[c +
d*x]*Tan[c + d*x])/((a^2 - b^2)^3*(a + b*Cos[c + d*x])^3))/(96*a^6*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

giac [B] time = 1.83, size = 1090, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/6*(6*(20*B*a^7*b^2 - 40*A*a^6*b^3 - 35*B*a^5*b^4 + 84*A*a^4*b^5 + 28*B*a
^3*b^6 - 69*A*a^2*b^7 - 8*B*a*b^8 + 20*A*b^9)*(pi*floor(1/2*(d*x + c)/pi +
1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/
2*c))/sqrt(a^2 - b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*sqrt(a^2
 - b^2)) + 2*(60*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 - 90*A*a^6*b^4*tan(1/2*d*
x + 1/2*c)^5 - 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 + 162*A*a^5*b^5*tan(1/2
*d*x + 1/2*c)^5 - 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^4*b^6*tan(1/
```

```
2*d*x + 1/2*c)^5 + 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 213*A*a^3*b^7*tan
(1/2*d*x + 1/2*c)^5 - 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^2*b^8*ta
n(1/2*d*x + 1/2*c)^5 - 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 + 81*A*a*b^9*tan
(1/2*d*x + 1/2*c)^5 + 18*B*a*b^9*tan(1/2*d*x + 1/2*c)^5 - 36*A*b^10*tan(1/2
*d*x + 1/2*c)^5 + 120*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^3 - 180*A*a^6*b^4*tan(
1/2*d*x + 1/2*c)^3 - 236*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^3 + 392*A*a^4*b^6*t
an(1/2*d*x + 1/2*c)^3 + 152*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^3 - 284*A*a^2*b^
8*tan(1/2*d*x + 1/2*c)^3 - 36*B*a*b^9*tan(1/2*d*x + 1/2*c)^3 + 72*A*b^10*ta
n(1/2*d*x + 1/2*c)^3 + 60*B*a^7*b^3*tan(1/2*d*x + 1/2*c) - 90*A*a^6*b^4*tan
(1/2*d*x + 1/2*c) + 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c) - 162*A*a^5*b^5*tan(
1/2*d*x + 1/2*c) - 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c) + 48*A*a^4*b^6*tan(1/2
*d*x + 1/2*c) - 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c) + 213*A*a^3*b^7*tan(1/2*
d*x + 1/2*c) - 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c) + 48*A*a^2*b^8*tan(1/2*d*x
 + 1/2*c) + 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c) - 81*A*a*b^9*tan(1/2*d*x + 1/
2*c) + 18*B*a*b^9*tan(1/2*d*x + 1/2*c) - 36*A*b^10*tan(1/2*d*x + 1/2*c))/((
a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1
/2*d*x + 1/2*c)^2 + a + b)^3) - 3*(A*a^2 - 8*B*a*b + 20*A*b^2)*log(abs(tan(
1/2*d*x + 1/2*c) + 1))/a^6 + 3*(A*a^2 - 8*B*a*b + 20*A*b^2)*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^6 - 6*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x
 + 1/2*c)^3 + 8*A*b*tan(1/2*d*x + 1/2*c)^3 + A*a*tan(1/2*d*x + 1/2*c) + 2*B
*a*tan(1/2*d*x + 1/2*c) - 8*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c
)^2 - 1)^2*a^5))/d
```

maple [B] time = 0.26, size = 3042, normalized size = 5.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)+1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)-40/d*
b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/
2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A-6/d*b^7/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)
*B+5/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a
^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+3/d*b^7/a^4/(a*tan(1/2*d*x+1/2
*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c)*A+30/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*
b^4/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-6/d/a^2/(a*tan(1/2
*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^5/(a+b)/(a^3-3*a^2*b+3*a*b^2-
b^3)*tan(1/2*d*x+1/2*c)*A-34/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)^2*b+a+b)^3*b^6/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*A-3/d*
b^7/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+12/d*b^8/a^5/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d
```

```
*x+1/2*c)^5*A-6/d*b^7/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+
b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+24/d*b^8/a^5/(a
*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A+116/3/d*b^5/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3*B-12/d*b^7/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(
a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*B+12/d*b^8/a^5/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^
3)*tan(1/2*d*x+1/2*c)*A+30/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b+a+b)^3*b^4/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A+6/d/a^
2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^5/(a-b)/(a^3+3*a^
2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*A-34/d/a^3/(a*tan(1/2*d*x+1/2*c)^2-ta
n(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x
+1/2*c)^5*A+60/d/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^
4/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-212/3/d/a^3/(a*tan
(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^6/(a^2+2*a*b+b^2)/(a^2-2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3*A-2/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2
*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B
-5/d*b^4/a/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+18/d*b^5/a^2/(a*tan(1/2*d*x+1/
2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/
2*d*x+1/2*c)^5*B+2/d*b^6/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*B+18/d*b^5/a^2
/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3
*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*B+1/2/d/a^4*A*ln(tan(1/2*d*x+1/2*c)+1)-1/2/d
/a^4*A*ln(tan(1/2*d*x+1/2*c)-1)-20/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/
2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^3*B-
40/d/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/
(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^3*B-20/d/(a*tan(1/2*d*x+1/2*c)^2-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*
c)*b^3*B-10/d/a^6*ln(tan(1/2*d*x+1/2*c)-1)*A*b^2+4/d/a^5*ln(tan(1/2*d*x+1/2
*c)-1)*B*b+4/d/a^5/(tan(1/2*d*x+1/2*c)+1)*A*b+10/d/a^6*ln(tan(1/2*d*x+1/2*c
)+1)*A*b^2+4/d/a^5/(tan(1/2*d*x+1/2*c)-1)*A*b-4/d/a^5*ln(tan(1/2*d*x+1/2*c)
+1)*B*b-1/d/a^4/(tan(1/2*d*x+1/2*c)+1)*B-1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)+1)
^2-1/d/a^4/(tan(1/2*d*x+1/2*c)-1)*B+1/2/d/a^4*A/(tan(1/2*d*x+1/2*c)-1)^2-69
/d/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x
+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*A*b^7+84/d/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-
b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2
))*A*b^5+20/d*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(
tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B*a+20/d*b^9/a^6/(a^6-3*a^4*b
^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b
)*(a+b))^(1/2))*A-35/d*b^4/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1
/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B+28/d*b^6/a^3/(a^
6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a
-b)/((a-b)*(a+b))^(1/2))*B-8/d*b^8/a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)
```

```
*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 13.94, size = 14398, normalized size = 26.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^4),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(A*a^8 + 20*A*b^8 + 2*B*a^8 - 59*A*a^2*b^6 - 27*A*a^3*
b^5 + 57*A*a^4*b^4 + 21*A*a^5*b^3 - 11*A*a^6*b^2 - 4*B*a^2*b^6 + 24*B*a^3*b
^5 + 11*B*a^4*b^4 - 26*B*a^5*b^3 - 6*B*a^6*b^2 + 10*A*a*b^7 - 7*A*a^7*b - 8
*B*a*b^7 + 2*B*a^7*b))/(a^5*(a + b)*(a - b)^3) + (2*tan(c/2 + (d*x)/2)^5*(9
*A*a^10 + 180*A*b^10 - 611*A*a^2*b^8 + 740*A*a^4*b^6 - 324*A*a^6*b^4 + 36*A
*a^8*b^2 + 248*B*a^3*b^7 - 320*B*a^5*b^5 + 132*B*a^7*b^3 - 72*B*a*b^9 - 18*
B*a^9*b))/(3*a^5*(a + b)^3*(a - b)^3) + (tan(c/2 + (d*x)/2)^9*(A*a^8 + 20*A
*b^8 - 2*B*a^8 - 59*A*a^2*b^6 + 27*A*a^3*b^5 + 57*A*a^4*b^4 - 21*A*a^5*b^3
- 11*A*a^6*b^2 + 4*B*a^2*b^6 + 24*B*a^3*b^5 - 11*B*a^4*b^4 - 26*B*a^5*b^3 +
 6*B*a^6*b^2 - 10*A*a*b^7 + 7*A*a^7*b - 8*B*a*b^7 + 2*B*a^7*b))/(a^5*(a + b
)^3*(a - b)) + (2*tan(c/2 + (d*x)/2)^3*(6*A*a^9 - 120*A*b^9 + 6*B*a^9 + 364
*A*a^2*b^7 + 71*A*a^3*b^6 - 369*A*a^4*b^5 - 45*A*a^5*b^4 + 111*A*a^6*b^3 +
3*A*a^7*b^2 + 12*B*a^2*b^7 - 148*B*a^3*b^6 - 29*B*a^4*b^5 + 159*B*a^5*b^4 +
 18*B*a^6*b^3 - 30*B*a^7*b^2 - 30*A*a*b^8 - 21*A*a^8*b + 48*B*a*b^8 - 6*B*a
^8*b))/(3*a^5*(a + b)^2*(a - b)^3) + (2*tan(c/2 + (d*x)/2)^7*(6*A*a^9 + 120
*A*b^9 - 6*B*a^9 - 364*A*a^2*b^7 + 71*A*a^3*b^6 + 369*A*a^4*b^5 - 45*A*a^5*
b^4 - 111*A*a^6*b^3 + 3*A*a^7*b^2 + 12*B*a^2*b^7 + 148*B*a^3*b^6 - 29*B*a^4
*b^5 - 159*B*a^5*b^4 + 18*B*a^6*b^3 + 30*B*a^7*b^2 - 30*A*a*b^8 + 21*A*a^8*
b - 48*B*a*b^8 - 6*B*a^8*b))/(3*a^5*(a + b)^3*(a - b)^2))/(d*(tan(c/2 + (d*
x)/2)^4*(6*a*b^2 - 6*a^2*b - 2*a^3 + 10*b^3) - tan(c/2 + (d*x)/2)^2*(9*a*b^
2 + 3*a^2*b - a^3 + 5*b^3) + tan(c/2 + (d*x)/2)^6*(6*a*b^2 + 6*a^2*b - 2*a^
3 - 10*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^10*(3*a*b^
2 - 3*a^2*b + a^3 - b^3) + tan(c/2 + (d*x)/2)^8*(3*a^2*b - 9*a*b^2 + a^3 +
```

5*b^3))) + (atan(((((8*tan(c/2 + (d*x)/2)*(800*A^2*a*b^17 - 800*A^2*b^18 - A^2*a^18 + 2*A^2*a^17*b + 4720*A^2*a^2*b^16 - 4720*A^2*a^3*b^15 - 11522*A^2*a^4*b^14 + 11522*A^2*a^5*b^13 + 14837*A^2*a^6*b^12 - 14812*A^2*a^7*b^11 - 10385*A^2*a^8*b^10 + 10430*A^2*a^9*b^9 + 3325*A^2*a^10*b^8 - 3640*A^2*a^11*b^7 + 45*A^2*a^12*b^6 + 350*A^2*a^13*b^5 - 209*A^2*a^14*b^4 + 68*A^2*a^15*b^3 - 35*A^2*a^16*b^2 - 128*B^2*a^2*b^16 + 128*B^2*a^3*b^15 + 768*B^2*a^4*b^14 - 768*B^2*a^5*b^13 - 1920*B^2*a^6*b^12 + 1920*B^2*a^7*b^11 + 2600*B^2*a^8*b^10 - 2560*B^2*a^9*b^9 - 2025*B^2*a^10*b^8 + 1920*B^2*a^11*b^7 + 824*B^2*a^12*b^6 - 768*B^2*a^13*b^5 - 80*B^2*a^14*b^4 + 128*B^2*a^15*b^3 - 64*B^2*a^16*b^2 + 640*A*B*a*b^17 + 16*A*B*a^17*b - 640*A*B*a^2*b^16 - 3808*A*B*a^3*b^15 + 3808*A*B*a^4*b^14 + 9408*A*B*a^5*b^13 - 9408*A*B*a^6*b^12 - 12430*A*B*a^7*b^11 + 12320*A*B*a^8*b^10 + 9200*A*B*a^9*b^9 - 8960*A*B*a^10*b^8 - 3360*A*B*a^11*b^7 + 3360*A*B*a^12*b^6 + 144*A*B*a^13*b^5 - 448*A*B*a^14*b^4 + 240*A*B*a^15*b^3 - 32*A*B*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2) + (((4*(4*A*a^27 - 80*A*a^12*b^15 + 40*A*a^13*b^14 + 516*A*a^14*b^13 - 248*A*a^15*b^12 - 1404*A*a^16*b^11 + 640*A*a^17*b^10 + 2076*A*a^18*b^9 - 896*A*a^19*b^8 - 1764*A*a^20*b^7 + 724*A*a^21*b^6 + 816*A*a^22*b^5 - 316*A*a^23*b^4 - 160*A*a^24*b^3 + 52*A*a^25*b^2 + 32*B*a^13*b^14 - 16*B*a^14*b^13 - 208*B*a^15*b^12 + 100*B*a^16*b^11 + 572*B*a^17*b^10 - 252*B*a^18*b^9 - 868*B*a^19*b^8 + 348*B*a^20*b^7 + 772*B*a^21*b^6 - 292*B*a^22*b^5 - 380*B*a^23*b^4 + 144*B*a^24*b^3 + 80*B*a^25*b^2 - 32*B*a^26*b))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) - (4*tan(c/2 + (d*x)/2)*(A*a^2 + 20*A*b^2 - 8*B*a*b)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/(a^6*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*(A*a^2 + 20*A*b^2 - 8*B*a*b))/(2*a^6))*(A*a^2 + 20*A*b^2 - 8*B*a*b)*1i)/(2*a^6) + (((8*tan(c/2 + (d*x)/2)*(800*A^2*a*b^17 - 800*A^2*b^18 - A^2*a^18 + 2*A^2*a^17*b + 4720*A^2*a^2*b^16 - 4720*A^2*a^3*b^15 - 11522*A^2*a^4*b^14 + 11522*A^2*a^5*b^13 + 14837*A^2*a^6*b^12 - 14812*A^2*a^7*b^11 - 10385*A^2*a^8*b^10 + 10430*A^2*a^9*b^9 + 3325*A^2*a^10*b^8 - 3640*A^2*a^11*b^7 + 45*A^2*a^12*b^6 + 350*A^2*a^13*b^5 - 209*A^2*a^14*b^4 + 68*A^2*a^15*b^3 - 35*A^2*a^16*b^2 - 128*B^2*a^2*b^16 + 128*B^2*a^3*b^15 + 768*B^2*a^4*b^14 - 768*B^2*a^5*b^13 - 1920*B^2*a^6*b^12 + 1920*B^2*a^7*b^11 + 2600*B^2*a^8*b^10 - 2560*B^2*a^9*b^9 - 2025*B^2*a^10*b^8 + 1920*B^2*a^11*b^7 + 824*B^2*a^12*b^6 - 768*B^2*a^13*b^5 - 80*B^2*a^14*b^4 + 128*B^2*a^15*b^3 - 64*B^2*a^16*b^2 + 640*A*B*a*b^17 + 16*A*B*a^17*b - 640*A*B*a^2*b^16 - 3808*A*B*a^3*b^15 + 3808*A*B*a^4*b^14 + 9408*A*B*a^5*b^13 - 9408*A*B*a^6*b^12 - 12430*A*B*a^7*b^11 + 12320*A*B*a^8*b^10 + 9200*A*B*a^9*b^9 - 8960*A*B*a^10*b^8 - 3360*A*B*a^11*b^7 + 3360*A*B*a^12*b^6 + 144*A*B*a^13*b^5 - 448*A*B*a^14*b^4 + 240*A*B*a^15*b^3 - 32*A*B*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^

7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2) - ((
(4*(4*A*a^27 - 80*A*a^12*b^15 + 40*A*a^13*b^14 + 516*A*a^14*b^13 - 248*A*a^
15*b^12 - 1404*A*a^16*b^11 + 640*A*a^17*b^10 + 2076*A*a^18*b^9 - 896*A*a^19
*b^8 - 1764*A*a^20*b^7 + 724*A*a^21*b^6 + 816*A*a^22*b^5 - 316*A*a^23*b^4 -
160*A*a^24*b^3 + 52*A*a^25*b^2 + 32*B*a^13*b^14 - 16*B*a^14*b^13 - 208*B*a
^15*b^12 + 100*B*a^16*b^11 + 572*B*a^17*b^10 - 252*B*a^18*b^9 - 868*B*a^19*
b^8 + 348*B*a^20*b^7 + 772*B*a^21*b^6 - 292*B*a^22*b^5 - 380*B*a^23*b^4 + 1
44*B*a^24*b^3 + 80*B*a^25*b^2 - 32*B*a^26*b))/(a^25*b + a^26 - a^15*b^11 -
a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b
^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) + (4*tan(c/2 + (d*x)/2)*(A*a^2
+ 20*A*b^2 - 8*B*a*b)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12
- 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7
- 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/(
a^6*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a
^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2
)))*(A*a^2 + 20*A*b^2 - 8*B*a*b))/(2*a^6))*(A*a^2 + 20*A*b^2 - 8*B*a*b)*1i)
/(2*a^6))/((8*(8000*A^3*b^19 - 4000*A^3*a*b^18 - 50800*A^3*a^2*b^17 + 24400
*A^3*a^3*b^16 + 135260*A^3*a^4*b^15 - 62030*A^3*a^5*b^14 - 193689*A^3*a^6*b
^13 + 82337*A^3*a^7*b^12 + 155991*A^3*a^8*b^11 - 57345*A^3*a^9*b^10 - 64479
*A^3*a^10*b^9 + 16999*A^3*a^11*b^8 + 8281*A^3*a^12*b^7 + 204*A^3*a^13*b^6 +
1396*A^3*a^14*b^5 - 40*A^3*a^15*b^4 + 40*A^3*a^16*b^3 - 512*B^3*a^3*b^16 +
256*B^3*a^4*b^15 + 3328*B^3*a^5*b^14 - 1600*B^3*a^6*b^13 - 9152*B^3*a^7*b^
12 + 4352*B^3*a^8*b^11 + 13888*B^3*a^9*b^10 - 6408*B^3*a^10*b^9 - 12352*B^3
*a^11*b^8 + 5120*B^3*a^12*b^7 + 6080*B^3*a^13*b^6 - 1920*B^3*a^14*b^5 - 128
0*B^3*a^15*b^4 - 9600*A^2*B*a*b^18 + 3840*A*B^2*a^2*b^17 - 1920*A*B^2*a^3*b
^16 - 24768*A*B^2*a^4*b^15 + 11904*A*B^2*a^5*b^14 + 67392*A*B^2*a^6*b^13 -
31680*A*B^2*a^7*b^12 - 100368*A*B^2*a^8*b^11 + 45148*A*B^2*a^9*b^10 + 86512
*A*B^2*a^10*b^9 - 34567*A*B^2*a^11*b^8 - 40368*A*B^2*a^12*b^7 + 11960*A*B^2
*a^13*b^6 + 7440*A*B^2*a^14*b^5 + 80*A*B^2*a^15*b^4 + 320*A*B^2*a^16*b^3 +
4800*A^2*B*a^2*b^17 + 61440*A^2*B*a^3*b^16 - 29520*A^2*B*a^4*b^15 - 165384*
A^2*B*a^5*b^14 + 76812*A^2*B*a^6*b^13 + 241596*A^2*B*a^7*b^12 - 105755*A^2*
B*a^8*b^11 - 201479*A^2*B*a^9*b^10 + 77359*A^2*B*a^10*b^9 + 88721*A^2*B*a^1
1*b^8 - 24711*A^2*B*a^12*b^7 - 13929*A^2*B*a^13*b^6 - 255*A^2*B*a^14*b^5 -
1345*A^2*B*a^15*b^4 + 20*A^2*B*a^16*b^3 - 20*A^2*B*a^17*b^2))/(a^25*b + a^2
6 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20
*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) + (((8*tan(c/2
+ (d*x)/2)*(800*A^2*a*b^17 - 800*A^2*b^18 - A^2*a^18 + 2*A^2*a^17*b + 4720*
A^2*a^2*b^16 - 4720*A^2*a^3*b^15 - 11522*A^2*a^4*b^14 + 11522*A^2*a^5*b^13
+ 14837*A^2*a^6*b^12 - 14812*A^2*a^7*b^11 - 10385*A^2*a^8*b^10 + 10430*A^2*
a^9*b^9 + 3325*A^2*a^10*b^8 - 3640*A^2*a^11*b^7 + 45*A^2*a^12*b^6 + 350*A^2
*a^13*b^5 - 209*A^2*a^14*b^4 + 68*A^2*a^15*b^3 - 35*A^2*a^16*b^2 - 128*B^2*
a^2*b^16 + 128*B^2*a^3*b^15 + 768*B^2*a^4*b^14 - 768*B^2*a^5*b^13 - 1920*B^
2*a^6*b^12 + 1920*B^2*a^7*b^11 + 2600*B^2*a^8*b^10 - 2560*B^2*a^9*b^9 - 202
5*B^2*a^10*b^8 + 1920*B^2*a^11*b^7 + 824*B^2*a^12*b^6 - 768*B^2*a^13*b^5 -
80*B^2*a^14*b^4 + 128*B^2*a^15*b^3 - 64*B^2*a^16*b^2 + 640*A*B*a*b^17 + 16*

A*B*a^17*b - 640*A*B*a^2*b^16 - 3808*A*B*a^3*b^15 + 3808*A*B*a^4*b^14 + 940
8*A*B*a^5*b^13 - 9408*A*B*a^6*b^12 - 12430*A*B*a^7*b^11 + 12320*A*B*a^8*b^1
0 + 9200*A*B*a^9*b^9 - 8960*A*B*a^10*b^8 - 3360*A*B*a^11*b^7 + 3360*A*B*a^1
2*b^6 + 144*A*B*a^13*b^5 - 448*A*B*a^14*b^4 + 240*A*B*a^15*b^3 - 32*A*B*a^1
6*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 -
10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19
b^2) + (((4(4*A*a^27 - 80*A*a^12*b^15 + 40*A*a^13*b^14 + 516*A*a^14*b^13
- 248*A*a^15*b^12 - 1404*A*a^16*b^11 + 640*A*a^17*b^10 + 2076*A*a^18*b^9 -
896*A*a^19*b^8 - 1764*A*a^20*b^7 + 724*A*a^21*b^6 + 816*A*a^22*b^5 - 316*A*
a^23*b^4 - 160*A*a^24*b^3 + 52*A*a^25*b^2 + 32*B*a^13*b^14 - 16*B*a^14*b^13
 - 208*B*a^15*b^12 + 100*B*a^16*b^11 + 572*B*a^17*b^10 - 252*B*a^18*b^9 - 8
68*B*a^19*b^8 + 348*B*a^20*b^7 + 772*B*a^21*b^6 - 292*B*a^22*b^5 - 380*B*a^
23*b^4 + 144*B*a^24*b^3 + 80*B*a^25*b^2 - 32*B*a^26*b))/(a^25*b + a^26 - a^
15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 +
 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) - (4*tan(c/2 + (d*x)/
2)*(A*a^2 + 20*A*b^2 - 8*B*a*b)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*
a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 16
0*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^
24*b^2))/(a^6*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*
b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 -
5*a^19*b^2)))*(A*a^2 + 20*A*b^2 - 8*B*a*b))/(2*a^6))*(A*a^2 + 20*A*b^2 - 8*
B*a*b))/(2*a^6) - (((8*tan(c/2 + (d*x)/2)*(800*A^2*a*b^17 - 800*A^2*b^18 -
A^2*a^18 + 2*A^2*a^17*b + 4720*A^2*a^2*b^16 - 4720*A^2*a^3*b^15 - 11522*A^2
*a^4*b^14 + 11522*A^2*a^5*b^13 + 14837*A^2*a^6*b^12 - 14812*A^2*a^7*b^11 -
10385*A^2*a^8*b^10 + 10430*A^2*a^9*b^9 + 3325*A^2*a^10*b^8 - 3640*A^2*a^11*
b^7 + 45*A^2*a^12*b^6 + 350*A^2*a^13*b^5 - 209*A^2*a^14*b^4 + 68*A^2*a^15*b
^3 - 35*A^2*a^16*b^2 - 128*B^2*a^2*b^16 + 128*B^2*a^3*b^15 + 768*B^2*a^4*b^
14 - 768*B^2*a^5*b^13 - 1920*B^2*a^6*b^12 + 1920*B^2*a^7*b^11 + 2600*B^2*a^
8*b^10 - 2560*B^2*a^9*b^9 - 2025*B^2*a^10*b^8 + 1920*B^2*a^11*b^7 + 824*B^2
*a^12*b^6 - 768*B^2*a^13*b^5 - 80*B^2*a^14*b^4 + 128*B^2*a^15*b^3 - 64*B^2*
a^16*b^2 + 640*A*B*a*b^17 + 16*A*B*a^17*b - 640*A*B*a^2*b^16 - 3808*A*B*a^3
*b^15 + 3808*A*B*a^4*b^14 + 9408*A*B*a^5*b^13 - 9408*A*B*a^6*b^12 - 12430*A
*B*a^7*b^11 + 12320*A*B*a^8*b^10 + 9200*A*B*a^9*b^9 - 8960*A*B*a^10*b^8 - 3
360*A*B*a^11*b^7 + 3360*A*B*a^12*b^6 + 144*A*B*a^13*b^5 - 448*A*B*a^14*b^4
+ 240*A*B*a^15*b^3 - 32*A*B*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^
10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10
*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2) - (((4*(4*A*a^27 - 80*A*a^12*b^15 + 40
*A*a^13*b^14 + 516*A*a^14*b^13 - 248*A*a^15*b^12 - 1404*A*a^16*b^11 + 640*A
*a^17*b^10 + 2076*A*a^18*b^9 - 896*A*a^19*b^8 - 1764*A*a^20*b^7 + 724*A*a^2
1*b^6 + 816*A*a^22*b^5 - 316*A*a^23*b^4 - 160*A*a^24*b^3 + 52*A*a^25*b^2 +
32*B*a^13*b^14 - 16*B*a^14*b^13 - 208*B*a^15*b^12 + 100*B*a^16*b^11 + 572*B
*a^17*b^10 - 252*B*a^18*b^9 - 868*B*a^19*b^8 + 348*B*a^20*b^7 + 772*B*a^21*
b^6 - 292*B*a^22*b^5 - 380*B*a^23*b^4 + 144*B*a^24*b^3 + 80*B*a^25*b^2 - 32
*B*a^26*b))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^
8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*

a^24*b^2) + (4*tan(c/2 + (d*x)/2)*(A*a^2 + 20*A*b^2 - 8*B*a*b)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/(a^6*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*(A*a^2 + 20*A*b^2 - 8*B*a*b))/(2*a^6))*(A*a^2 + 20*A*b^2 - 8*B*a*b))/(2*a^6)))*(A*a^2 + 20*A*b^2 - 8*B*a*b)*1i)/(a^6*d) + (b^2*atan(((b^2*((8*tan(c/2 + (d*x)/2)*(800*A^2*a*b^17 - 800*A^2*b^18 - A^2*a^18 + 2*A^2*a^17*b + 4720*A^2*a^2*b^16 - 4720*A^2*a^3*b^15 - 11522*A^2*a^4*b^14 + 11522*A^2*a^5*b^13 + 14837*A^2*a^6*b^12 - 14812*A^2*a^7*b^11 - 10385*A^2*a^8*b^10 + 10430*A^2*a^9*b^9 + 3325*A^2*a^10*b^8 - 3640*A^2*a^11*b^7 + 45*A^2*a^12*b^6 + 350*A^2*a^13*b^5 - 209*A^2*a^14*b^4 + 68*A^2*a^15*b^3 - 35*A^2*a^16*b^2 - 128*B^2*a^2*b^16 + 128*B^2*a^3*b^15 + 768*B^2*a^4*b^14 - 768*B^2*a^5*b^13 - 1920*B^2*a^6*b^12 + 1920*B^2*a^7*b^11 + 2600*B^2*a^8*b^10 - 2560*B^2*a^9*b^9 - 2025*B^2*a^10*b^8 + 1920*B^2*a^11*b^7 + 824*B^2*a^12*b^6 - 768*B^2*a^13*b^5 - 80*B^2*a^14*b^4 + 128*B^2*a^15*b^3 - 64*B^2*a^16*b^2 + 640*A*B*a*b^17 + 16*A*B*a^17*b - 640*A*B*a^2*b^16 - 3808*A*B*a^3*b^15 + 3808*A*B*a^4*b^14 + 9408*A*B*a^5*b^13 - 9408*A*B*a^6*b^12 - 12430*A*B*a^7*b^11 + 12320*A*B*a^8*b^10 + 9200*A*B*a^9*b^9 - 8960*A*B*a^10*b^8 - 3360*A*B*a^11*b^7 + 3360*A*B*a^12*b^6 + 144*A*B*a^13*b^5 - 448*A*B*a^14*b^4 + 240*A*B*a^15*b^3 - 32*A*B*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2) + (b^2*(-(a + b)^7*(a - b)^7)^(1/2)*((4*(4*A*a^27 - 80*A*a^12*b^15 + 40*A*a^13*b^14 + 516*A*a^14*b^13 - 248*A*a^15*b^12 - 1404*A*a^16*b^11 + 640*A*a^17*b^10 + 2076*A*a^18*b^9 - 896*A*a^19*b^8 - 1764*A*a^20*b^7 + 724*A*a^21*b^6 + 816*A*a^22*b^5 - 316*A*a^23*b^4 - 160*A*a^24*b^3 + 52*A*a^25*b^2 + 32*B*a^13*b^14 - 16*B*a^14*b^13 - 208*B*a^15*b^12 + 100*B*a^16*b^11 + 572*B*a^17*b^10 - 252*B*a^18*b^9 - 868*B*a^19*b^8 + 348*B*a^20*b^7 + 772*B*a^21*b^6 - 292*B*a^22*b^5 - 380*B*a^23*b^4 + 144*B*a^24*b^3 + 80*B*a^25*b^2 - 32*B*a^26*b))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) - (4*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/((a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*(20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6))/(2*(a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6)*1i)/(2*(a^20 - a^6*b^14 +

7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)) + (b^2*((8*tan(c/2 + (d*x)/2)*(800*A^2*a*b^17 - 800*A^2*b^18 - A^2*a^18 + 2*A^2*a^17*b + 4720*A^2*a^2*b^16 - 4720*A^2*a^3*b^15 - 11522*A^2*a^4*b^14 + 11522*A^2*a^5*b^13 + 14837*A^2*a^6*b^12 - 14812*A^2*a^7*b^11 - 10385*A^2*a^8*b^10 + 10430*A^2*a^9*b^9 + 3325*A^2*a^10*b^8 - 3640*A^2*a^11*b^7 + 45*A^2*a^12*b^6 + 350*A^2*a^13*b^5 - 209*A^2*a^14*b^4 + 68*A^2*a^15*b^3 - 35*A^2*a^16*b^2 - 128*B^2*a^2*b^16 + 128*B^2*a^3*b^15 + 768*B^2*a^4*b^14 - 768*B^2*a^5*b^13 - 1920*B^2*a^6*b^12 + 1920*B^2*a^7*b^11 + 2600*B^2*a^8*b^10 - 2560*B^2*a^9*b^9 - 2025*B^2*a^10*b^8 + 1920*B^2*a^11*b^7 + 824*B^2*a^12*b^6 - 768*B^2*a^13*b^5 - 80*B^2*a^14*b^4 + 128*B^2*a^15*b^3 - 64*B^2*a^16*b^2 + 640*A*B*a*b^17 + 16*A*B*a^17*b - 640*A*B*a^2*b^16 - 3808*A*B*a^3*b^15 + 3808*A*B*a^4*b^14 + 9408*A*B*a^5*b^13 - 9408*A*B*a^6*b^12 - 12430*A*B*a^7*b^11 + 12320*A*B*a^8*b^10 + 9200*A*B*a^9*b^9 - 8960*A*B*a^10*b^8 - 3360*A*B*a^11*b^7 + 3360*A*B*a^12*b^6 + 144*A*B*a^13*b^5 - 448*A*B*a^14*b^4 + 240*A*B*a^15*b^3 - 32*A*B*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2) - (b^2*(-(a + b)^7*(a - b)^7)^(1/2)*((4*(4*A*a^27 - 80*A*a^12*b^15 + 40*A*a^13*b^14 + 516*A*a^14*b^13 - 248*A*a^15*b^12 - 1404*A*a^16*b^11 + 640*A*a^17*b^10 + 2076*A*a^18*b^9 - 896*A*a^19*b^8 - 1764*A*a^20*b^7 + 724*A*a^21*b^6 + 816*A*a^22*b^5 - 316*A*a^23*b^4 - 160*A*a^24*b^3 + 52*A*a^25*b^2 + 32*B*a^13*b^14 - 16*B*a^14*b^13 - 208*B*a^15*b^12 + 100*B*a^16*b^11 + 572*B*a^17*b^10 - 252*B*a^18*b^9 - 868*B*a^19*b^8 + 348*B*a^20*b^7 + 772*B*a^21*b^6 - 292*B*a^22*b^5 - 380*B*a^23*b^4 + 144*B*a^24*b^3 + 80*B*a^25*b^2 - 32*B*a^26*b))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) + (4*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/((a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*(20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6))/(2*(a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6)*1i)/(2*(a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)))/((8*(8000*A^3*b^19 - 4000*A^3*a*b^18 - 50800*A^3*a^2*b^17 + 24400*A^3*a^3*b^16 + 135260*A^3*a^4*b^15 - 62030*A^3*a^5*b^14 - 193689*A^3*a^6*b^13 + 82337*A^3*a^7*b^12 + 155991*A^3*a^8*b^11 - 57345*A^3*a^9*b^10 - 64479*A^3*a^10*b^9 + 16999*A^3*a^11*b^8 + 8281*A^3*a^12*b^7 + 204*A^3*a^13*b^6 + 1396*A^3*a^14*b^5 - 40*A^3*a^15*b^4 + 40*A^3*a^16*b^3 - 512*B^3*a^3*b^16 + 256*B^3*a^4*b^15

+ 3328*B^3*a^5*b^14 - 1600*B^3*a^6*b^13 - 9152*B^3*a^7*b^12 + 4352*B^3*a^8
*b^11 + 13888*B^3*a^9*b^10 - 6408*B^3*a^10*b^9 - 12352*B^3*a^11*b^8 + 5120*
B^3*a^12*b^7 + 6080*B^3*a^13*b^6 - 1920*B^3*a^14*b^5 - 1280*B^3*a^15*b^4 -
9600*A^2*B*a*b^18 + 3840*A*B^2*a^2*b^17 - 1920*A*B^2*a^3*b^16 - 24768*A*B^2
*a^4*b^15 + 11904*A*B^2*a^5*b^14 + 67392*A*B^2*a^6*b^13 - 31680*A*B^2*a^7*b
^12 - 100368*A*B^2*a^8*b^11 + 45148*A*B^2*a^9*b^10 + 86512*A*B^2*a^10*b^9 -
34567*A*B^2*a^11*b^8 - 40368*A*B^2*a^12*b^7 + 11960*A*B^2*a^13*b^6 + 7440*
A*B^2*a^14*b^5 + 80*A*B^2*a^15*b^4 + 320*A*B^2*a^16*b^3 + 4800*A^2*B*a^2*b^
17 + 61440*A^2*B*a^3*b^16 - 29520*A^2*B*a^4*b^15 - 165384*A^2*B*a^5*b^14 +
76812*A^2*B*a^6*b^13 + 241596*A^2*B*a^7*b^12 - 105755*A^2*B*a^8*b^11 - 2014
79*A^2*B*a^9*b^10 + 77359*A^2*B*a^10*b^9 + 88721*A^2*B*a^11*b^8 - 24711*A^2
*B*a^12*b^7 - 13929*A^2*B*a^13*b^6 - 255*A^2*B*a^14*b^5 - 1345*A^2*B*a^15*b
^4 + 20*A^2*B*a^16*b^3 - 20*A^2*B*a^17*b^2))/(a^25*b + a^26 - a^15*b^11 - a
^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^
5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) + (b^2*((8*tan(c/2 + (d*x)/2)*(8
00*A^2*a*b^17 - 800*A^2*b^18 - A^2*a^18 + 2*A^2*a^17*b + 4720*A^2*a^2*b^16
- 4720*A^2*a^3*b^15 - 11522*A^2*a^4*b^14 + 11522*A^2*a^5*b^13 + 14837*A^2*a
^6*b^12 - 14812*A^2*a^7*b^11 - 10385*A^2*a^8*b^10 + 10430*A^2*a^9*b^9 + 332
5*A^2*a^10*b^8 - 3640*A^2*a^11*b^7 + 45*A^2*a^12*b^6 + 350*A^2*a^13*b^5 - 2
09*A^2*a^14*b^4 + 68*A^2*a^15*b^3 - 35*A^2*a^16*b^2 - 128*B^2*a^2*b^16 + 12
8*B^2*a^3*b^15 + 768*B^2*a^4*b^14 - 768*B^2*a^5*b^13 - 1920*B^2*a^6*b^12 +
1920*B^2*a^7*b^11 + 2600*B^2*a^8*b^10 - 2560*B^2*a^9*b^9 - 2025*B^2*a^10*b^
8 + 1920*B^2*a^11*b^7 + 824*B^2*a^12*b^6 - 768*B^2*a^13*b^5 - 80*B^2*a^14*b
^4 + 128*B^2*a^15*b^3 - 64*B^2*a^16*b^2 + 640*A*B*a*b^17 + 16*A*B*a^17*b -
640*A*B*a^2*b^16 - 3808*A*B*a^3*b^15 + 3808*A*B*a^4*b^14 + 9408*A*B*a^5*b^1
3 - 9408*A*B*a^6*b^12 - 12430*A*B*a^7*b^11 + 12320*A*B*a^8*b^10 + 9200*A*B*
a^9*b^9 - 8960*A*B*a^10*b^8 - 3360*A*B*a^11*b^7 + 3360*A*B*a^12*b^6 + 144*A
*B*a^13*b^5 - 448*A*B*a^14*b^4 + 240*A*B*a^15*b^3 - 32*A*B*a^16*b^2))/(a^20
*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 -
10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2) + (b^2*
(-(a + b)^7*(a - b)^7)^(1/2)*((4*(4*A*a^27 - 80*A*a^12*b^15 + 40*A*a^13*b^1
4 + 516*A*a^14*b^13 - 248*A*a^15*b^12 - 1404*A*a^16*b^11 + 640*A*a^17*b^10
+ 2076*A*a^18*b^9 - 896*A*a^19*b^8 - 1764*A*a^20*b^7 + 724*A*a^21*b^6 + 816
*A*a^22*b^5 - 316*A*a^23*b^4 - 160*A*a^24*b^3 + 52*A*a^25*b^2 + 32*B*a^13*b
^14 - 16*B*a^14*b^13 - 208*B*a^15*b^12 + 100*B*a^16*b^11 + 572*B*a^17*b^10
- 252*B*a^18*b^9 - 868*B*a^19*b^8 + 348*B*a^20*b^7 + 772*B*a^21*b^6 - 292*B
*a^22*b^5 - 380*B*a^23*b^4 + 144*B*a^24*b^3 + 80*B*a^25*b^2 - 32*B*a^26*b))
/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19
*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) -
(4*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(20*A*b^7 + 20*B*a^
7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b
- 8*B*a*b^6)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15
*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^
20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/((a^20 - a
^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b

^4 - 7*a^18*b^2)*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*(20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6))/(2*(a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6))/(2*(a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)) - (b^2*((8*tan(c/2 + (d*x)/2)*(800*A^2*a*b^17 - 800*A^2*b^18 - A^2*a^18 + 2*A^2*a^17*b + 4720*A^2*a^2*b^16 - 4720*A^2*a^3*b^15 - 11522*A^2*a^4*b^14 + 11522*A^2*a^5*b^13 + 14837*A^2*a^6*b^12 - 14812*A^2*a^7*b^11 - 10385*A^2*a^8*b^10 + 10430*A^2*a^9*b^9 + 3325*A^2*a^10*b^8 - 3640*A^2*a^11*b^7 + 45*A^2*a^12*b^6 + 350*A^2*a^13*b^5 - 209*A^2*a^14*b^4 + 68*A^2*a^15*b^3 - 35*A^2*a^16*b^2 - 128*B^2*a^2*b^16 + 128*B^2*a^3*b^15 + 768*B^2*a^4*b^14 - 768*B^2*a^5*b^13 - 1920*B^2*a^6*b^12 + 1920*B^2*a^7*b^11 + 2600*B^2*a^8*b^10 - 2560*B^2*a^9*b^9 - 2025*B^2*a^10*b^8 + 1920*B^2*a^11*b^7 + 824*B^2*a^12*b^6 - 768*B^2*a^13*b^5 - 80*B^2*a^14*b^4 + 128*B^2*a^15*b^3 - 64*B^2*a^16*b^2 + 640*A*B*a*b^17 + 16*A*B*a^17*b - 640*A*B*a^2*b^16 - 3808*A*B*a^3*b^15 + 3808*A*B*a^4*b^14 + 9408*A*B*a^5*b^13 - 9408*A*B*a^6*b^12 - 12430*A*B*a^7*b^11 + 12320*A*B*a^8*b^10 + 9200*A*B*a^9*b^9 - 8960*A*B*a^10*b^8 - 3360*A*B*a^11*b^7 + 3360*A*B*a^12*b^6 + 144*A*B*a^13*b^5 - 448*A*B*a^14*b^4 + 240*A*B*a^15*b^3 - 32*A*B*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2) - (b^2*(-(a + b)^7*(a - b)^7)^(1/2)*((4*(4*A*a^27 - 80*A*a^12*b^15 + 40*A*a^13*b^14 + 516*A*a^14*b^13 - 248*A*a^15*b^12 - 1404*A*a^16*b^11 + 640*A*a^17*b^10 + 2076*A*a^18*b^9 - 896*A*a^19*b^8 - 1764*A*a^20*b^7 + 724*A*a^21*b^6 + 816*A*a^22*b^5 - 316*A*a^23*b^4 - 160*A*a^24*b^3 + 52*A*a^25*b^2 + 32*B*a^13*b^14 - 16*B*a^14*b^13 - 208*B*a^15*b^12 + 100*B*a^16*b^11 + 572*B*a^17*b^10 - 252*B*a^18*b^9 - 868*B*a^19*b^8 + 348*B*a^20*b^7 + 772*B*a^21*b^6 - 292*B*a^22*b^5 - 380*B*a^23*b^4 + 144*B*a^24*b^3 + 80*B*a^25*b^2 - 32*B*a^26*b))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) + (4*b^2*tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/((a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*(20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6))/(2*(a^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2)))*(-(a + b)^7*(a - b)^7)^(1/2)*(20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*A*a^4*b^3 + 28*B*a^

```
3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6))/(2*(a^20 - a^6*b^14 + 7*a^8
*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21*a^16*b^4 - 7*a^18*b^2
))))*(-(a + b)^7*(a - b)^7)^(1/2)*(20*A*b^7 + 20*B*a^7 - 69*A*a^2*b^5 + 84*
A*a^4*b^3 + 28*B*a^3*b^4 - 35*B*a^5*b^2 - 40*A*a^6*b - 8*B*a*b^6)*1i)/(d*(a
^20 - a^6*b^14 + 7*a^8*b^12 - 21*a^10*b^10 + 35*a^12*b^8 - 35*a^14*b^6 + 21
*a^16*b^4 - 7*a^18*b^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

3.281 $\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{B\sin(c+dx)}{d}-\frac{B\sin^3(c+dx)}{3d}$$

```
[Out] B*sin(d*x+c)/d-1/3*B*sin(d*x+c)^3/d
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {21, 2633}

$$\frac{B\sin(c+dx)}{d}-\frac{B\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*Sin[c + d*x])/d - (B*Sin[c + d*x]^3)/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx &= B\int \cos^3(c+dx)\,dx\\ &= -\frac{B\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\ &= \frac{B\sin(c+dx)}{d}-\frac{B\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$B\left(\frac{\sin(c+dx)}{d}-\frac{\sin^3(c+dx)}{3d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*(Sin[c + d*x]/d - Sin[c + d*x]^3/(3*d))
```

fricas [A] time = 0.69, size = 25, normalized size = 0.89

$$\frac{\left(B\cos\left(dx+c\right)^{2}+2\,B\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/3*(B*cos(d*x + c)^2 + 2*B)*sin(d*x + c)/d
```

giac [A] time = 0.42, size = 25, normalized size = 0.89

$$-\frac{B\sin\left(dx+c\right)^{3}-3\,B\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] -1/3*(B*sin(d*x + c)^3 - 3*B*sin(d*x + c))/d
```

maple [A] time = 0.08, size = 23, normalized size = 0.82

$$\frac{B\left(2+\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/3/d*B*(2+cos(d*x+c)^2)*sin(d*x+c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.48, size = 24, normalized size = 0.86

$$\frac{B\left(9\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] (B*(9*sin(c + d*x) + sin(3*c + 3*d*x)))/(12*d)
```

sympy [A] time = 1.26, size = 56, normalized size = 2.00

$$\begin{cases} \frac{2B\sin^3(c+dx)}{3d} + \frac{B\sin(c+dx)\cos^2(c+dx)}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\cos(c))\cos^3(c)}{a+b\cos(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((2*B*sin(c + d*x)**3/(3*d) + B*sin(c + d*x)*cos(c + d*x)**2/d, Ne
(d, 0)), (x*(B*a + B*b*cos(c))*cos(c)**3/(a + b*cos(c)), True))
```

3.282 $\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}$$

```
[Out] 1/2*B*x+1/2*B*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {21, 2635, 8}

$$\frac{B\sin(c+dx)\cos(c+dx)}{2d}+\frac{Bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*x)/2 + (B*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = B\int \cos^2(c+dx)\,dx$$
$$= \frac{B\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}B\int 1\,dx$$
$$= \frac{Bx}{2} + \frac{B\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.89

$$\frac{B(2(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
[Out] (B*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

fricas [A] time = 0.84, size = 24, normalized size = 0.89

$$\frac{Bdx + B\cos(dx+c)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
[Out] 1/2*(B*d*x + B*cos(d*x + c)*sin(d*x + c))/d
```

giac [A] time = 0.37, size = 33, normalized size = 1.22

$$\frac{(dx+c)B + \frac{B\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
[Out] 1/2*((d*x + c)*B + B*tan(d*x + c)/(tan(d*x + c)^2 + 1))/d
```

maple [A] time = 0.08, size = 28, normalized size = 1.04

$$\frac{B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.87, size = 50, normalized size = 1.85

$$\frac{B\,x}{2}+\frac{B\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-B\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] (B*x)/2 + (B*tan(c/2 + (d*x)/2) - B*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*
x)/2)^2 + 1)^2)
```

sympy [A] time = 0.89, size = 68, normalized size = 2.52

$$\begin{cases} \frac{Bx\sin^{2}{\left(c+dx \right)}}{2} + \frac{Bx\cos^{2}{\left(c+dx \right)}}{2} + \frac{B\sin{\left(c+dx \right)}\cos{\left(c+dx \right)}}{2d} & \text{for}\: d \neq 0 \\ \frac{x\left(Ba+Bb\cos{\left(c \right)}\right)\cos^{2}{\left(c \right)}}{a+b\cos{\left(c \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*x*sin(c + d*x)**2/2 + B*x*cos(c + d*x)**2/2 + B*sin(c + d*x)*c
os(c + d*x)/(2*d), Ne(d, 0)), (x*(B*a + B*b*cos(c))*cos(c)**2/(a + b*cos(c)
), True))
```

3.283 $\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=11

$$\frac{B\sin(c+dx)}{d}$$

```
[Out] B*sin(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {21, 2637}

$$\frac{B\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*Sin[c + d*x])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx &= B\int \cos(c+dx)\,dx\\ &= \frac{B\sin(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 0.01, size = 23, normalized size = 2.09

$$B\left(\frac{\sin(c)\cos(dx)}{d}+\frac{\cos(c)\sin(dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*((Cos[d*x]*Sin[c])/d + (Cos[c]*Sin[d*x])/d)
```

fricas [**A**] time = 0.93, size = 11, normalized size = 1.00

$$\frac{B\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] B*sin(d*x + c)/d
```

giac [**A**] time = 0.35, size = 11, normalized size = 1.00

$$\frac{B\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] B*sin(d*x + c)/d
```

maple [**A**] time = 0.06, size = 12, normalized size = 1.09

$$\frac{B\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] B*sin(d*x+c)/d
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.47, size = 11, normalized size = 1.00

$$\frac{B\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] (B*sin(c + d*x))/d
```

sympy [A] time = 0.60, size = 31, normalized size = 2.82

$$\begin{cases} \frac{B\sin\left(c+dx\right)}{d} & \text{for}\: d \neq 0 \\ \frac{x\left(Ba+Bb\cos\left(c\right)\right)\cos\left(c\right)}{a+b\cos\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*sin(c + d*x)/d, Ne(d, 0)), (x*(B*a + B*b*cos(c))*cos(c)/(a + b
*cos(c)), True))
```

3.284 $\int \frac{aB+bB\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=3

$$Bx$$

```
[Out] B*x
```

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {21, 8}

$$Bx$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{aB+bB\cos(c+dx)}{a+b\cos(c+dx)}\,dx &= B\int 1\,dx \\ &= Bx \end{aligned}$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*x
```

fricas [A] time = 0.51, size = 3, normalized size = 1.00

$$Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
[Out] B*x
```

giac [C] time = 0.33, size = 10, normalized size = 3.33

$$\frac{(dx+c)B}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
[Out] (d*x + c)*B/d
```

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
[Out] B*x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.45, size = 3, normalized size = 1.00

$$B\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x)),x)
```

```
[Out] B*x
```

sympy [A] time = 0.13, size = 2, normalized size = 0.67

$$Bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] B*x
```

3.285 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=12

$$\frac{B\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] B*arctanh(sin(d*x+c))/d
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {21, 3770}

$$\frac{B\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)}\,dx &= B\int \sec(c+dx)\,dx\\ &= \frac{B\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{B\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/d
```

fricas [B] time = 1.09, size = 31, normalized size = 2.58

$$\frac{B\log\left(\sin\left(dx+c\right)+1\right)-B\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*(B*log(sin(d*x + c) + 1) - B*log(-sin(d*x + c) + 1))/d
```

giac [B] time = 0.45, size = 47, normalized size = 3.92

$$\frac{B\log\left(\left|\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)+2\right|\right)-B\log\left(\left|\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)-2\right|\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/4*(B*log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - B*log(abs(1/sin(d*x +
c) + sin(d*x + c) - 2)))/d
```

maple [A] time = 0.07, size = 20, normalized size = 1.67

$$\frac{B\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*B*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.49, size = 16, normalized size = 1.33

$$\frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))),x)
```

```
[Out] (2*B*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [A] time = 3.81, size = 39, normalized size = 3.25

$$\begin{cases} \frac{B\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)}{d} & \text{for}\; d\neq 0 \\ \frac{x(Ba+Bb\cos\left(c\right))\sec\left(c\right)}{a+b\cos\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(B*a + B*b*c
os(c))*sec(c)/(a + b*cos(c)), True))
```

3.286 $\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=11

$$\frac{B\tan(c+dx)}{d}$$

[Out] B*tan(d*x+c)/d

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {21, 3767, 8}

$$\frac{B\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]

[Out] (B*Tan[c + d*x])/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(aB + bB\cos(c + dx))\sec^2(c + dx)}{a + b\cos(c + dx)}\,dx &= B\int \sec^2(c + dx)\,dx\\ &= -\frac{B\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c + dx))}{d}\\ &= \frac{B\tan(c + dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{B\tan(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*Tan[c + d*x])/d
```

fricas [A] time = 0.69, size = 19, normalized size = 1.73

$$\frac{B\sin(dx + c)}{d\cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] B*sin(d*x + c)/(d*cos(d*x + c))
```

giac [A] time = 0.42, size = 11, normalized size = 1.00

$$\frac{B\tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] B*tan(d*x + c)/d
```

maple [A] time = 0.08, size = 12, normalized size = 1.09

$$\frac{B\tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

```
[Out] B*tan(d*x+c)/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.47, size = 30, normalized size = 2.73

$$-\frac{2\,B\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))),x)
```

```
[Out] -(2*B*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

sympy [A] time = 3.10, size = 32, normalized size = 2.91

$$\begin{cases} \frac{B\tan\left(c+dx\right)}{d} & \text{for}\: d \neq 0 \\ \frac{x\left(Ba+Bb\cos\left(c\right)\right)\sec^{2}\left(c\right)}{a+b\cos\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*tan(c + d*x)/d, Ne(d, 0)), (x*(B*a + B*b*cos(c))*sec(c)**2/(a
+ b*cos(c)), True))
```

3.287 $\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{B\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 1/2*B*arctanh(sin(d*x+c))/d+1/2*B*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {21, 3768, 3770}

$$\frac{B\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(2*d) + (B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(aB + bB\cos(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx = B\int \sec^3(c+dx)\,dx$$
$$= \frac{B\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}B\int \sec(c+dx)\,dx$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))}{2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$B\left(\frac{\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\tan(c+dx)\sec(c+dx)}{2d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] B*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))
```

fricas [A] time = 0.65, size = 64, normalized size = 1.78

$$\frac{B\cos(dx+c)^2\log(\sin(dx+c)+1) - B\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2B\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/4*(B*cos(d*x + c)^2*log(sin(d*x + c) + 1) - B*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*B*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.59, size = 52, normalized size = 1.44

$$\frac{B\log(|\sin(dx+c)+1|) - B\log(|\sin(dx+c)-1|) - \frac{2B\sin(dx+c)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/4*(B*log(abs(sin(d*x + c) + 1)) - B*log(abs(sin(d*x + c) - 1)) - 2*B*sin(
d*x + c)/(sin(d*x + c)^2 - 1))/d
```

maple [A] time = 0.09, size = 40, normalized size = 1.11

$$\frac{B\sec(dx+c)\tan(dx+c)}{2d}+\frac{B\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/2*B*sec(d*x+c)*tan(d*x+c)/d+1/2/d*B*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.86, size = 73, normalized size = 2.03

$$\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{B\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))),x)
```

```
[Out] (B*tan(c/2 + (d*x)/2) + B*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4 -
2*tan(c/2 + (d*x)/2)^2 + 1)) + (B*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

```
[Out] B*Integral(sec(c + d*x)**3, x)
```

3.288 $\int \frac{(aB+bB\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{B\tan^3(c+dx)}{3d}+\frac{B\tan(c+dx)}{d}$$

```
[Out] B*tan(d*x+c)/d+1/3*B*tan(d*x+c)^3/d
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {21, 3767}

$$\frac{B\tan^3(c+dx)}{3d}+\frac{B\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*Tan[c + d*x])/d + (B*Tan[c + d*x]^3)/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(aB+bB\cos(c+dx))\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx &= B\int \sec^4(c+dx)\,dx\\
&= -\frac{B\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{d}\\
&= \frac{B\tan(c+dx)}{d}+\frac{B\tan^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 24, normalized size = 0.86

$$\frac{B\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^4)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

fricas [A] time = 1.15, size = 32, normalized size = 1.14

$$\frac{\left(2\,B\cos\left(dx+c\right)^2+B\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*B*cos(d*x + c)^2 + B)*sin(d*x + c)/(d*cos(d*x + c)^3)
```

giac [A] time = 0.57, size = 25, normalized size = 0.89

$$\frac{B\tan\left(dx+c\right)^3+3\,B\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] 1/3*(B*tan(d*x + c)^3 + 3*B*tan(d*x + c))/d
```

maple [A] time = 0.10, size = 25, normalized size = 0.89

$$-\frac{B\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

```
[Out] -1/d*B*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.52, size = 39, normalized size = 1.39

$$\frac{2\,B\,\sin\left(c+d\,x\right)\,{\cos\left(c+d\,x\right)}^2+B\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^4*(a + b*cos(c + d*x))),x)
```

```
[Out] (B*sin(c + d*x) + 2*B*cos(c + d*x)^2*sin(c + d*x))/(3*d*cos(c + d*x)^3)
```

sympy [A] time = 17.65, size = 42, normalized size = 1.50

$$\begin{cases} \frac{B\left(\frac{\tan^{3}{\left(c+dx \right)}}{3}+\tan{\left(c+dx \right)}\right)}{d} & \text{for}\: d \neq 0 \\ \frac{x\left(Ba+Bb\cos{\left(c \right)}\right)\sec^{4}{\left(c \right)}}{a+b\cos{\left(c \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*(tan(c + d*x)**3/3 + tan(c + d*x))/d, Ne(d, 0)), (x*(B*a + B*b
*cos(c))*sec(c)**4/(a + b*cos(c)), True))
```

3.289 $\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=114

$$-\frac{2a^3B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{Bx\left(2a^2+b^2\right)}{2b^3}-\frac{aB\sin(c+dx)}{b^2d}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

```
[Out] 1/2*(2*a^2+b^2)*B*x/b^3-a*B*sin(d*x+c)/b^2/d+1/2*B*cos(d*x+c)*sin(d*x+c)/b/
d-2*a^3*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^3/d/(a-b)^(1
/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {21, 2793, 3023, 2735, 2659, 205}

$$-\frac{2a^3B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{Bx\left(2a^2+b^2\right)}{2b^3}-\frac{aB\sin(c+dx)}{b^2d}+\frac{B\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*B*x)/(2*b^3) - (2*a^3*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*B*Sin[c + d*x])/(b^2*d
) + (B*Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{B\cos(c+dx)\sin(c+dx)}{2bd} + \frac{B\int \frac{a+b\cos(c+dx)-2a\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2b}$$
$$= -\frac{aB\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd} + \frac{B\int \frac{ab+(2a^2+b^2)\cos(c+}{a+b\cos(c+dx)}}{2b^2}$$
$$= \frac{(2a^2+b^2)Bx}{2b^3} - \frac{aB\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd} - \frac{(a^3B)}{}$$
$$= \frac{(2a^2+b^2)Bx}{2b^3} - \frac{aB\sin(c+dx)}{b^2d} + \frac{B\cos(c+dx)\sin(c+dx)}{2bd} - \frac{(2a^3}{}$$
$$= \frac{(2a^2+b^2)Bx}{2b^3} - \frac{2a^3B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d} - \frac{aB\sin(c+dx)}{b^2d} + \frac{}{}$$

Mathematica [A] time = 0.25, size = 98, normalized size = 0.86

$$\frac{B\left(2\left(2a^2+b^2\right)(c+dx)+\frac{8a^3\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-4ab\sin(c+dx)+b^2\sin(2(c+dx))\right)}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]

[Out] (B*(2*(2*a^2 + b^2)*(c + d*x) + (8*a^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/S
qrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 4*a*b*Sin[c + d*x] + b^2*Sin[2*(c + d*
x)]))/(4*b^3*d)
```

fricas [A] time = 0.59, size = 350, normalized size = 3.07

$$\left[-\frac{\sqrt{-a^2+b^2}\,Ba^3\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-\left(2\,Ba^4-Ba^2b^2-Bb\right.}{2\left(a^2b^3-b^5\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*a^3*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d
*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (2*B*a^4 - B*a^2*b^
2 - B*b^4)*d*x + (2*B*a^3*b - 2*B*a*b^3 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))
*sin(d*x + c))/((a^2*b^3 - b^5)*d), -1/2*(2*sqrt(a^2 - b^2)*B*a^3*arctan(-(
a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*B*a^4 - B*a^2*b^2
- B*b^4)*d*x + (2*B*a^3*b - 2*B*a*b^3 - (B*a^2*b^2 - B*b^4)*cos(d*x + c))*s
in(d*x + c))/((a^2*b^3 - b^5)*d)]
```

giac [A] time = 0.42, size = 185, normalized size = 1.62

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Ba^3}{\sqrt{a^2-b^2}\,b^3}-\frac{\left(2Ba^2+Bb^2\right)(dx+c)}{b^3}+\frac{2\left(2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2B\cdots\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\cdots\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*a^3/(sqrt(a^2
- b^2)*b^3) - (2*B*a^2 + B*b^2)*(d*x + c)/b^3 + 2*(2*B*a*tan(1/2*d*x + 1/2*
c)^3 + B*b*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*tan(1/2*d*x + 1/2*c) - B*b*tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

maple [B] time = 0.10, size = 229, normalized size = 2.01

$$-\frac{2a^3\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)B}{d\,b^3\sqrt{(a-b)(a+b)}}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)Ba}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)Ba}{d\,b^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d*a^3/b^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))*B-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B*a-1/
```

```
d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*B-2/d/b^2/(1+tan(1/2*d*
x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*B*a+1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1
/2*d*x+1/2*c)*B+2/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2*B+1/d/b*arctan(tan(1
/2*d*x+1/2*c))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.17, size = 173, normalized size = 1.52

$$\frac{B\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}+\frac{B\,\sin\left(2\,c+2\,d\,x\right)}{4\,b\,d}+\frac{2\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^3\,d}-\frac{B\,a\,\sin\left(c+d\,x\right)}{b^2\,d}-\frac{B\,a^3\,\mathrm{atan}\left(\frac{\left(a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\ldots\right.\right.}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2\ldots}}\right.}{b^3\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (B*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (B*sin(2*c + 2*d*x)
)/(4*b*d) + (2*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d) -
 (B*a*sin(c + d*x))/(b^2*d) - (B*a^3*atan(((a*sin(c/2 + (d*x)/2) - b*sin(c/
2 + (d*x)/2))*1i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)))*2i)/(b^3*d*(b^2 -
 a^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.290 $\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$\frac{2a^2B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{aBx}{b^2}+\frac{B\sin(c+dx)}{bd}$$

```
[Out] -a*B*x/b^2+B*sin(d*x+c)/b/d+2*a^2*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(
a+b)^(1/2))/b^2/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {21, 2746, 12, 2735, 2659, 205}

$$\frac{2a^2B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{aBx}{b^2}+\frac{B\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((a*B*x)/b^2) + (2*a^2*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + (B*Sin[c + d*x])/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx \\
&= \frac{B\sin(c+dx)}{bd} - \frac{B\int \frac{a\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b} \\
&= \frac{B\sin(c+dx)}{bd} - \frac{(aB)\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b} \\
&= -\frac{aBx}{b^2} + \frac{B\sin(c+dx)}{bd} + \frac{\left(a^2B\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2} \\
&= -\frac{aBx}{b^2} + \frac{B\sin(c+dx)}{bd} + \frac{\left(2a^2B\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c\right.\right.}{b^2d} \\
&= -\frac{aBx}{b^2} + \frac{2a^2B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d} + \frac{B\sin(c+dx)}{bd}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 73, normalized size = 0.92

$$\frac{B\left(-\frac{2a^2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-a(c+dx)+b\sin(c+dx)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] (B*(-(a*(c + d*x)) - (2*a^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 +
b^2]])/Sqrt[-a^2 + b^2] + b*Sin[c + d*x]))/(b^2*d)
```

fricas [A] time = 0.65, size = 281, normalized size = 3.56

$$\left[-\frac{\sqrt{-a^2+b^2}\,Ba^2\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+2\left(Ba^3-Bab^2\right)dx-2}{2\left(a^2b^2-b^4\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*a^2*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d
*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(B*a^3 - B*a*b^2)
*d*x - 2*(B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d), (sqrt(a^2 - b
^2)*B*a^2*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (B
*a^3 - B*a*b^2)*d*x + (B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d)]
```

giac [A] time = 0.46, size = 128, normalized size = 1.62

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)Ba^2}{\sqrt{a^2-b^2}\,b^2}-\frac{(dx+c)Ba}{b^2}+\frac{2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*a^2/(sqrt(a^2 - b^2
)*b^2) - (d*x + c)*B*a/b^2 + 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c
)^2 + 1)*b))/d
```

maple [A] time = 0.09, size = 105, normalized size = 1.33

$$\frac{2a^2 \arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right) B}{d\,b^2 \sqrt{(a-b)\,(a+b)}} + \frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} - \frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right) Ba}{d\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*a^2/b^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))*B+2/d/b*B*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/d/b^2*arc
tan(tan(1/2*d*x+1/2*c))*B*a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.89, size = 193, normalized size = 2.44

$$\frac{B\,\sin\left(c+d\,x\right)}{b\,d} - \frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d} - \frac{B\,a^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b - 2\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^2 + 1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(b^2-a^2\right)}^{3/2} + a^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2} - a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)\,2\mathrm{i}}{b^2\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (B*sin(c + d*x))/(b*d) - (2*B*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
)/(b^2*d) - (B*a^2*atan((b^3*sin(c/2 + (d*x)/2)*1i - a*b^2*sin(c/2 + (d*x)/
```

```
2)*2i + a^2*b*sin(c/2 + (d*x)/2)*1i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2)
+ a^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a*b*cos(c/2 + (d*x)/2)*(b^2 -
a^2)^(1/2)))*2i)/(b^2*d*(b^2 - a^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.291 $\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=61

$$\frac{Bx}{b}-\frac{2aB\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] B*x/b-2*a*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b/d/(a-b)^(1
/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {21, 2735, 2659, 205}

$$\frac{Bx}{b}-\frac{2aB\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*x)/b - (2*a*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[
a - b]*b*Sqrt[a + b]*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx\\ &= \frac{Bx}{b}-\frac{(aB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b}\\ &= \frac{Bx}{b}-\frac{(2aB)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\\ &= \frac{Bx}{b}-\frac{2aB\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [A] time = 0.08, size = 59, normalized size = 0.97

$$\frac{B\left(\frac{2a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}+c+dx\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*(c + d*x + (2*a*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sq
rt[-a^2 + b^2]))/(b*d)
```

fricas [A] time = 0.87, size = 231, normalized size = 3.79

$$\left[-\frac{\sqrt{-a^2+b^2}\,Ba\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-2\left(Ba^2-Bb^2\right)dx}{2\left(a^2b-b^3\right)d},-\sqrt{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*a*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(B*a^2 - B*b^2)*d*x
)/((a^2*b - b^3)*d), -(sqrt(a^2 - b^2)*B*a*arctan(-(a*cos(d*x + c) + b)/(sq
rt(a^2 - b^2)*sin(d*x + c))) - (B*a^2 - B*b^2)*d*x)/((a^2*b - b^3)*d)]
```

giac [B] time = 0.65, size = 245, normalized size = 4.02

$$-\frac{\frac{\left(\sqrt{a^2-b^2}\,B(2a-b)|a-b|+\sqrt{a^2-b^2}\,B|a-b||b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{\frac{2a+\sqrt{-4(a+b)(a-b)+4a^2}}{a-b}}}\right)\right)}{\left(a^2-2ab+b^2\right)b^2+\left(a^3-2a^2b+ab^2\right)|b|}+\frac{(2Ba-Bb-B|b|)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan \cdots}{\sqrt{2a-\sqrt{-4(a \cdots}}}\right.\right.}{b^2-a|b|}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -((sqrt(a^2 - b^2)*B*(2*a - b)*abs(a - b) + sqrt(a^2 - b^2)*B*abs(a - b)*ab
s(b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x +
1/2*c)/sqrt((2*a + sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/((a^2 - 2*a
*b + b^2)*b^2 + (a^3 - 2*a^2*b + a*b^2)*abs(b)) + (2*B*a - B*b - B*abs(b))*
(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)
/sqrt((2*a - sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/(b^2 - a*abs(b)))
/d
```

maple [A] time = 0.08, size = 69, normalized size = 1.13

$$-\frac{2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)aB}{db\sqrt{(a-b)(a+b)}}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d/b/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1
/2))*a*B+2/d/b*arctan(tan(1/2*d*x+1/2*c))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.80, size = 101, normalized size = 1.66

$$\frac{2\,B\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d}+\frac{2\,B\,a\,\mathrm{atanh}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)}{b\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*B*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (2*B*a*atanh((a*s
in(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(
1/2))))/(b*d*(b^2 - a^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.292 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=50

$$\frac{2B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] 2*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/d/(a-b)^(1/2)/(a+b)^
(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {21, 2659, 205}

$$\frac{2B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[
a + b]*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{1}{a+b\cos(c+dx)}\,dx$$
$$=\frac{(2B)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$
$$=\frac{2B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}\,d}$$

Mathematica [**A**] time = 0.04, size = 49, normalized size = 0.98

$$-\frac{2B\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{d\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*B*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2 + b^
2]*d)
```

fricas [**A**] time = 0.72, size = 177, normalized size = 3.54

$$\left[-\frac{\sqrt{-a^2+b^2}\,B\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\left(a^2-b^2\right)d}, \frac{B\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\,\sin(dx+c)}\right)}{\sqrt{a^2-b^2}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*B*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/
(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))/((a^2 - b^2)*d), B*arctan(
-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))/(sqrt(a^2 - b^2)*d)]
```

giac [**A**] time = 0.52, size = 78, normalized size = 1.56

$$\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)B}{\sqrt{a^2-b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
+ 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B/(sqrt(a^2 - b^2)*d)
```

maple [**A**] time = 0.05, size = 45, normalized size = 0.90

$$\frac{2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)B}{d\sqrt{(a-b)(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)
)*B
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [**B**] time = 0.50, size = 44, normalized size = 0.88

$$\frac{2\,B\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a-b)}{\sqrt{a^2-b^2}}\right)}{d\,\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*B*atan((tan(c/2 + (d*x)/2)*(a - b))/(a^2 - b^2)^(1/2)))/(d*(a^2 - b^2)^(
1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.293 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$\frac{B\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2bB\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\,\sqrt{a+b}}$$

```
[Out] B*arctanh(sin(d*x+c))/a/d-2*b*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))/a/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {21, 2747, 3770, 2659, 205}

$$\frac{B\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2bB\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*
Sqrt[a + b]*d) + (B*ArcTanh[Sin[c + d*x]])/(a*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx\\ &= \frac{B\int \sec(c+dx)\,dx}{a}-\frac{(bB)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a}\\ &= \frac{B\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(2bB)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+d\right.\right.}{ad}\\ &= -\frac{2bB\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}\,d}+\frac{B\tanh^{-1}(\sin(c+dx))}{ad}\end{aligned}$$

Mathematica [A] time = 0.08, size = 103, normalized size = 1.47

$$\frac{B\left(\frac{2b\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (B*((2*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
b^2] - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]))/(a*d)
```

fricas [A] time = 0.94, size = 292, normalized size = 4.17

$$\left[-\frac{\sqrt{-a^2+b^2}\,Bb\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-\left(Ba^2-Bb^2\right)\log(\sin(}{2\left(a^3-ab^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="
fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*b*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (B*a^2 - B*b^2)*log(s
in(d*x + c) + 1) + (B*a^2 - B*b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d
), -1/2*(2*sqrt(a^2 - b^2)*B*b*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2
)*sin(d*x + c))) - (B*a^2 - B*b^2)*log(sin(d*x + c) + 1) + (B*a^2 - B*b^2)*
log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d)]
```

giac [A] time = 0.50, size = 122, normalized size = 1.74

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)Bb}{\sqrt{a^2-b^2}\,a}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}+\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="
giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*b/(sqrt(a^2 - b^2)
*a) - B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + B*log(abs(tan(1/2*d*x + 1/2*
c) - 1))/a)/d
```

maple [A] time = 0.09, size = 91, normalized size = 1.30

$$-\frac{2b\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)B}{da\sqrt{(a-b)\,(a+b)}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)B}{ad}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)B}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d*b/a/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^
(1/2))*B-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.76, size = 101, normalized size = 1.44

$$\frac{2\,B\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}+\frac{2\,B\,b\,\mathrm{atanh}\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}\right)}{a\,d\,\sqrt{b^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] (2*B*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) + (2*B*b*atanh((a*
sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^
(1/2))))/(a*d*(b^2 - a^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int\frac{\sec\left(c+dx\right)}{a+b\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] B*Integral(sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

3.294 $\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=88

$$\frac{2b^2B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{bB\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{B\tan(c+dx)}{ad}$$

```
[Out] -b*B*arctanh(sin(d*x+c))/a^2/d+2*b^2*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c
)/(a+b)^(1/2))/a^2/d/(a-b)^(1/2)/(a+b)^(1/2)+B*tan(d*x+c)/a/d
```

Rubi [A] time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {21, 2802, 12, 2747, 3770, 2659, 205}

$$\frac{2b^2B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\,\sqrt{a+b}}-\frac{bB\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{B\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*b^2*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*Sqrt[a -
b]*Sqrt[a + b]*d) - (b*B*ArcTanh[Sin[c + d*x]])/(a^2*d) + (B*Tan[c + d*x])/
(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$
$$= \frac{B\tan(c+dx)}{ad} - \frac{B\int \frac{b\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$
$$= \frac{B\tan(c+dx)}{ad} - \frac{(bB)\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$
$$= \frac{B\tan(c+dx)}{ad} - \frac{(bB)\int \sec(c+dx)\,dx}{a^2} + \frac{\left(b^2B\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2}$$
$$= -\frac{bB\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{B\tan(c+dx)}{ad} + \frac{\left(2b^2B\right)\mathrm{Subst}\left(\int \frac{1}{a+b+(a$$
$$= \frac{2b^2B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\,\sqrt{a+b}\,d} - \frac{bB\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{B\tan(c$$
$$ad$$

Mathematica [A] time = 0.37, size = 116, normalized size = 1.32

$$\frac{B\left(-\frac{2b^2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + a\tan(c+dx) + b\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] (B*((-2*b^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2
 + b^2] + b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]]) + a*Tan[c + d*x]))/(a^2*d)
```

fricas [B] time = 1.10, size = 398, normalized size = 4.52

$$\left[-\frac{\sqrt{-a^2+b^2}\,Bb^2\cos(dx+c)\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2+2\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)+\left(Ba^2b-\right.}{2\left(a^4-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*B*b^2*cos(d*x + c)*log((2*a*b*cos(d*x + c) + (2*a^2
 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x +
c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (B*a^2
*b - B*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) - (B*a^2*b - B*b^3)*cos(d*x
+ c)*log(-sin(d*x + c) + 1) - 2*(B*a^3 - B*a*b^2)*sin(d*x + c))/((a^4 - a^2
*b^2)*d*cos(d*x + c)), 1/2*(2*sqrt(a^2 - b^2)*B*b^2*arctan(-(a*cos(d*x + c)
 + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (B*a^2*b - B*b^3)*cos(
d*x + c)*log(sin(d*x + c) + 1) + (B*a^2*b - B*b^3)*cos(d*x + c)*log(-sin(d*
x + c) + 1) + 2*(B*a^3 - B*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*x
+ c))]
```

giac [A] time = 0.79, size = 155, normalized size = 1.76

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)Bb^2}{\sqrt{a^2-b^2}\,a^2}-\frac{Bb\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}+\frac{Bb\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{2B\tan\left(\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}dx+\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*b^2/(sqrt(a^2 - b^2
)*a^2) - B*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 + B*b*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^2 - 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 -
 1)*a))/d
```

maple [A] time = 0.11, size = 139, normalized size = 1.58

$$\frac{2b^2\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)B}{d\,a^2\sqrt{(a-b)\,(a+b)}}-\frac{B}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)Bb}{d\,a^2}-\frac{B}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*b^2/a^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))*B-1/a/d/(tan(1/2*d*x+1/2*c)-1)*B+1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)
*B*b-1/a/d/(tan(1/2*d*x+1/2*c)+1)*B-1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)*B*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.06, size = 326, normalized size = 3.70

$$\frac{2B\left(\frac{a^3\sin(c+dx)}{2}-\frac{ab^2\sin(c+dx)}{2}\right)}{a^2 d\cos(c+dx)\left(a^2-b^2\right)}-\frac{2B\left(a^2 b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)+b^2\,\mathrm{atanh}\left(\frac{a^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^2),x)
```

```
[Out] (2*B*((a^3*sin(c + d*x))/2 - (a*b^2*sin(c + d*x))/2))/(a^2*d*cos(c + d*x)*(
a^2 - b^2)) - (2*B*(a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - b^
3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + b^2*atanh((a^5*sin(c/2 + (
d*x)/2)*(b^2 - a^2)^(1/2) + 2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 2*
b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*a^2*b^3*sin(c/2 + (d*x)/2)*(b^
2 - a^2)^(1/2) - a^3*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^4*b*sin(c
/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)^2))*(b^2
 - a^2)^(1/2)))/(a^2*d*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int\frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

```
[Out] B*Integral(sec(c + d*x)**2/(a + b*cos(c + d*x)), x)
```

3.295 $\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=123

$$-\frac{2b^3B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}-\frac{bB\tan(c+dx)}{a^2d}+\frac{B\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{B\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] 1/2*(a^2+2*b^2)*B*arctanh(sin(d*x+c))/a^3/d-2*b^3*B*arctan((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/d/(a-b)^(1/2)/(a+b)^(1/2)-b*B*tan(d*x+c)/a^
2/d+1/2*B*sec(d*x+c)*tan(d*x+c)/a/d
```

Rubi [A] time = 0.35, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {21, 2802, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b^3B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{B\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{bB\tan(c+dx)}{a^2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^3*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a -
 b]*Sqrt[a + b]*d) + ((a^2 + 2*b^2)*B*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (b
*B*Tan[c + d*x])/(a^2*d) + (B*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{B\sec(c+dx)\tan(c+dx)}{2ad} + \frac{B\int \frac{\left(-2b+a\cos(c+dx)+b\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2a}$$

$$= -\frac{bB\tan(c+dx)}{a^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2ad} + \frac{B\int \frac{\left(a^2+2b^2+ab\cos(c+dx)\right.}{a+b\cos(c+d}}{2a^2}$$

$$= -\frac{bB\tan(c+dx)}{a^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\left(b^3B\right)\int \frac{1}{a+b\cos(c+dx)}\,d}{a^3}$$

$$= \frac{\left(a^2+2b^2\right)B\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{bB\tan(c+dx)}{a^2d} + \frac{B\sec(c+dx)\,\mathrm{ta}}{2ad}$$

$$= -\frac{2b^3B\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}\,d} + \frac{\left(a^2+2b^2\right)B\tanh^{-1}(\sin(c+dx))}{2a^3d} -$$

Mathematica [A] time = 1.05, size = 239, normalized size = 1.94

$$B\left(\frac{8b^3\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + \frac{a^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{a^2}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} - 2a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \mathrm{sin}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] (B*((8*b^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2
+ b^2] - 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*b^2*Log[Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] + 4*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 4*a
*b*Tan[c + d*x]))/(4*a^3*d)
```

fricas [A] time = 0.89, size = 487, normalized size = 3.96

$$\left[-\frac{2\sqrt{-a^2+b^2}\,Bb^3\cos(dx+c)^2\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-\left(Ba^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] [-1/4*(2*sqrt(-a^2 + b^2)*B*b^3*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) + (2
*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*
x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (B
*a^4 + B*a^2*b^2 - 2*B*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (B*a^4 +
 B*a^2*b^2 - 2*B*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^4 - B*
a^2*b^2 - 2*(B*a^3*b - B*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2
)*d*cos(d*x + c)^2), -1/4*(4*sqrt(a^2 - b^2)*B*b^3*arctan(-(a*cos(d*x + c)
+ b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (B*a^4 + B*a^2*b^2 -
2*B*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (B*a^4 + B*a^2*b^2 - 2*B*b^
4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^4 - B*a^2*b^2 - 2*(B*a^3*
b - B*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)^2)
]
```

giac [B] time = 0.75, size = 221, normalized size = 1.80

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)Bb^3}{\sqrt{a^2-b^2}\,a^3}-\frac{\left(Ba^2+2\,Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}+\frac{\left(Ba^2+2\,Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\right.\right.\right.}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*B*b^3/(sqrt(a^2
- b^2)*a^3) - (B*a^2 + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 + (B
*a^2 + 2*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 2*(B*a*tan(1/2*d*x
 + 1/2*c)^3 + 2*B*b*tan(1/2*d*x + 1/2*c)^3 + B*a*tan(1/2*d*x + 1/2*c) - 2*B
*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

maple [B] time = 0.13, size = 273, normalized size = 2.22

$$-\frac{2b^{3}\arctan\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)B}{d\,a^{3}\sqrt{(a-b)\,(a+b)}}+\frac{B}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{B}{2ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{Bb}{d\,a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{\ln\left(\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d*b^3/a^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))*B+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2*B+1/2/a/d/(tan(1/2*d*x+1/2*c)
-1)*B+1/d/a^2/(tan(1/2*d*x+1/2*c)-1)*B*b-1/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)*B
-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)*B*b^2-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2*B+
1/2/a/d/(tan(1/2*d*x+1/2*c)+1)*B+1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*B*b+1/2/a/d
*ln(tan(1/2*d*x+1/2*c)+1)*B+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)*B*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.83, size = 1099, normalized size = 8.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^2),x)
```

```
[Out] ((B*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (B*b^2*sin(c + d*
x))/2 + (B*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x
))/2)/(a*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (a*((B*sin(c + d*x))/2
 + (B*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (B*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2))/(d*(a^2 - b^2)*(cos(2*c
+ 2*d*x)/2 + 1/2)) - (B*b*sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d
```

```
*x)/2 + 1/2)) - (B*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^3*d
*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B*b^3*sin(2*c + 2*d*x))/(2*a^2*
d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b^3*atan(((a^9*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*b^
9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2) + 3*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 3*a^5*b^4*s
in(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2) + 2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^8*b*sin(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(a^7 -
3*a^3*b^4 + 2*a^5*b^2)))*1i)/(a^3*d*(b^2 - a^2)^(1/2)*(cos(2*c + 2*d*x)/2 +
 1/2)) - (B*b^3*atan(((a^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin
(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*b^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1
/2) + 8*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 3*a^4*b^5*sin(c/2 +
(d*x)/2)*(b^2 - a^2)^(1/2) - 3*a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)
 - 2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 2*a^7*b^2*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2) - a^8*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))*1i)/(
cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(a^7 - 3*a^3*b^4 + 2*a^5*b^2)))*cos(2*c +
2*d*x)*1i)/(a^3*d*(b^2 - a^2)^(1/2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b^4*at
anh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/(a^3*d*(a^2 -
b^2)*(cos(2*c + 2*d*x)/2 + 1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

```
[Out] B*Integral(sec(c + d*x)**3/(a + b*cos(c + d*x)), x)
```

3.296 $\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=386

$$-\frac{2\left(-24a^2B+36aAb-49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^3d}+\frac{2\left(-16a^3B+24a^2Ab-36ab^2B+75Ab^3\right)\sin(c}{315b^3d}$$

```
[Out] -2/315*(36*A*a*b-24*B*a^2-49*B*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d
+2/21*(3*A*b-2*B*a)*cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/9*
B*cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/315*(24*A*a^2*b+75*A
*b^3-16*B*a^3-36*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/d+2/315*(24
*A*a^3*b+57*A*a*b^3-16*B*a^4-24*B*a^2*b^2+147*B*b^4)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1
/2))*(a+b*cos(d*x+c))^(1/2)/b^4/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/315*(a^2
-b^2)*(24*A*a^2*b+75*A*b^3-16*B*a^3-36*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*
((a+b*cos(d*x+c))/(a+b))^(1/2)/b^4/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.79, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2990, 3049, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-24a^2B+36aAb-49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^3d}+\frac{2\left(24a^2Ab-16a^3B-36ab^2B+75Ab^3\right)\sin(c+}{315b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(24*a^3*A*b + 57*a*A*b^3 - 16*a^4*B - 24*a^2*b^2*B + 147*b^4*B)*Sqrt[a +
 b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^4*d*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(24*a^2*A*b + 75*A*b^3 - 16*a^
3*B - 36*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(315*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(24*a^2*A*b + 75
*A*b^3 - 16*a^3*B - 36*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315
*b^3*d) - (2*(36*a*A*b - 24*a^2*B - 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*Si
n[c + d*x])/(315*b^3*d) + (2*(3*A*b - 2*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(21*b^2*d) + (2*B*Cos[c + d*x]^2*(a + b*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(9*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
```

```
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{2B\cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{9bd} +$$

$$= \frac{2(3Ab-2aB)\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin}{21b^2d}$$

$$= -\frac{2\left(36aAb-24a^2B-49b^2B\right)(a+b\cos(c+dx))^{3/2}}{315b^3d}$$

$$= \frac{2\left(24a^2Ab+75Ab^3-16a^3B-36ab^2B\right)\sqrt{a+b\cos}}{315b^3d}$$

$$= \frac{2\left(24a^2Ab+75Ab^3-16a^3B-36ab^2B\right)\sqrt{a+b\cos}}{315b^3d}$$

$$= \frac{2\left(24a^2Ab+75Ab^3-16a^3B-36ab^2B\right)\sqrt{a+b\cos}}{315b^3d}$$

$$= \frac{2\left(24a^3Ab+57aAb^3-16a^4B-24a^2b^2B+147b^4B\right.}{315b^4d\sqrt{\frac{a+b\cos(c}{a+b}}}$$

Mathematica [A] time = 1.57, size = 292, normalized size = 0.76

$$\frac{8\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b^2\left(-4a^3B+6a^2Ab+111ab^2B+75Ab^3\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(-16a^4B+24a^3Ab-24a^2b^2B+5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(6*a^2*A*b + 75*A*b^3 - 4*a^3*B
+ 111*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (24*a^3*A*b + 57*a*A
*b^3 - 16*a^4*B - 24*a^2*b^2*B + 147*b^4*B)*((a + b)*EllipticE[(c + d*x)/2,
 (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) - b*(a + b*Cos[
c + d*x])*(-2*(-48*a^2*A*b + 345*A*b^3 + 32*a^3*B + 57*a*b^2*B)*Sin[c + d*x
] - b*((36*a*A*b - 24*a^2*B + 266*b^2*B)*Sin[2*(c + d*x)] + 5*b*(2*(9*A*b +
 a*B)*Sin[3*(c + d*x)] + 7*b*B*Sin[4*(c + d*x)]))))/(1260*b^4*d*Sqrt[a + b*
Cos[c + d*x]])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^{4}+A\cos\left(dx+c\right)^{3}\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^4 + A*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

maple [B] time = 1.58, size = 1635, normalized size = 4.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B
*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b^5+640*B*a*b^4+2240*B
*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-432*A*a*b^4-1080*A*b^5+8*B*
a^2*b^3-960*B*a*b^4-2072*B*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-1
2*A*a^2*b^3+432*A*a*b^4+840*A*b^5+8*B*a^3*b^2-8*B*a^2*b^3+728*B*a*b^4+952*B
*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(24*A*a^3*b^2+6*A*a^2*b^3-258
*A*a*b^4-240*A*b^5-16*B*a^4*b-4*B*a^3*b^2-24*B*a^2*b^3-204*B*a*b^4-168*B*b^
5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-24*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-51*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^2*b^3+75*A*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))+24*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^4*b-24*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
```

```
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+
57*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3-57*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4+16*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5+20*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-36*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*b^4-16*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^5+16*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^4*b-24*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3
*b^2+24*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+14
7*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5)/b^4/(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/
2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^3\,\left(A+B\,\cos(c+d\,x)\right)\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.297 $\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=303

$$-\frac{2\left(-8a^2B+14aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d}+\frac{2\left(a^2-b^2\right)\left(-8a^2B+14aAb-25b^2B\right)\sqrt{\frac{a+b\cos(c-}{a+b}}}{105b^3d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2/35*(7*A*b-4*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*B*cos(d*x+c)
*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d-2/105*(14*A*a*b-8*B*a^2-25*B*b^2)*si
n(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d-2/105*(14*A*a^2*b-63*A*b^3-8*B*a^3-19
*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/d/((a+b*cos
(d*x+c))/(a+b))^(1/2)+2/105*(a^2-b^2)*(14*A*a*b-8*B*a^2-25*B*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/
2)
```

Rubi [A] time = 0.54, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2990, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-8a^2B+14aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d}+\frac{2\left(a^2-b^2\right)\left(-8a^2B+14aAb-25b^2B\right)\sqrt{\frac{a+b\cos(c-}{a+b}}}{105b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*(14*a^2*A*b - 63*A*b^3 - 8*a^3*B - 19*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) + (2*(a^2 - b^2)*(14*a*A*b - 8*a^2*B - 25*b^2*B)*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^3*d*Sqrt
[a + b*Cos[c + d*x]]) - (2*(14*a*A*b - 8*a^2*B - 25*b^2*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*b^2*d) + (2*(7*A*b - 4*a*B)*(a + b*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(35*b^2*d) + (2*B*Cos[c + d*x]*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(7*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
```

```
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{2B\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7bd} + $$

$$= \frac{2(7Ab-4aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35b^2d} + $$

$$= -\frac{2\left(14aAb-8a^2B-25b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin}{105b^2d}$$

$$= -\frac{2\left(14aAb-8a^2B-25b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin}{105b^2d}$$

$$= -\frac{2\left(14aAb-8a^2B-25b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin}{105b^2d}$$

$$= -\frac{2\left(14a^2Ab-63Ab^3-8a^3B-19ab^2B\right)\sqrt{a+b\cos}}{105b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.01, size = 232, normalized size = 0.77

$$\frac{b(a+b\cos(c+dx))\left(\left(-16a^2B+28aAb+115b^2B\right)\sin(c+dx)+3b(2(aB+7Ab)\sin(2(c+dx))+5bB\sin(3(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(49*a*A*b + 2*a^2*B + 25*b^2*B)*
EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B +
 19*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(
c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((28*a*A*b - 16*a^2*B
 + 115*b^2*B)*Sin[c + d*x] + 3*b*(2*(7*A*b + a*B)*Sin[2*(c + d*x)] + 5*b*B*
Sin[3*(c + d*x)])))/(210*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.64, size = 1305, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^4-144*B*a*b^3-360*B*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(112*A*a*b^3+168*A*b^4-4*B*a^2*b
^2+144*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A*a^
2*b^2-56*A*a*b^3-42*A*b^4+8*B*a^3*b+2*B*a^2*b^2-86*B*a*b^3-80*B*b^4)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+14*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
```

```
-2*b/(a-b))^(1/2))*a^3*b-14*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b^3-14*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^3*b+14*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+
63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-63*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-8*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-17*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^4-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3
*b+19*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-19*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b^3/(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2
*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\,(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.298 $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=231

$$-\frac{2\left(a^2-b^2\right)(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-2a^2B+5aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 2/5*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/15*(5*A*b-2*B*a)*sin(d*x+c)*(
a+b*cos(d*x+c))^(1/2)/b/d+2/15*(5*A*a*b-2*B*a^2+9*B*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b)
)^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/15*(
a^2-b^2)*(5*A*b-2*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^
(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2968, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-2a^2B+5aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(5*a*A*b - 2*a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a
^2 - b^2)*(5*A*b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(15*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*A*b -
2*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b*d) + (2*B*(a + b*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \int \sqrt{a+b\cos(c+dx)}\,\left(A\cos(c+dx)+B\cos^2(c+\ldots\right.$$

$$= \frac{2B(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{2\int\sqrt{a+b\,\mathrm{co}\ldots}}{\ldots}$$

$$= \frac{2(5Ab-2aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15bd} + \frac{2B(\ldots}{\ldots}$$

$$= \frac{2(5Ab-2aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15bd} + \frac{2B(\ldots}{\ldots}$$

$$= \frac{2(5Ab-2aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15bd} + \frac{2B(\ldots}{\ldots}$$

$$= \frac{2\left(5aAb-2a^2B+9b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c\ldots\right.}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.89, size = 179, normalized size = 0.77

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(\left(-2a^2B+5aAb+9b^2B\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+b^2(7aB+5Ab)F\left(\frac{1}{2}\ldots\right.\right.}{15b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(5*A*b + 7*a*B)*EllipticF[(c + d
*x)/2, (2*b)/(a + b)] + (5*a*A*b - 2*a^2*B + 9*b^2*B)*((a + b)*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + 2*b
*(a + b*Cos[c + d*x])*(5*A*b + a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x])/(15*
b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

maple [B] time = 1.64, size = 993, normalized size = 4.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b^3+16*B*a*b^2+24*B*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3-2*B*a^2*b-8*B*a
*b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a^2*b-5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a*b^2+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-
2*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
```

```
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+2*B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*b^3)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*cos(c + d*x), x)
```

3.299 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=171

$$-\frac{2B\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2(aB+3Ab)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sin(c+dx}{}$$

```
[Out] 2/3*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*(3*A*b+B*a)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b
))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(a^
2-b^2)*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b
*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2B\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2(aB+3Ab)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sin(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(3*A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(3*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*B*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d*S
qrt[a + b*Cos[c + d*x]]) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d
)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{1}{2}(3aA+bB)+\frac{1}{2}(3Ab}{\sqrt{a+b\cos(}}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d}-\frac{\left(\left(a^2-b^2\right)B\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}}{3b}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{\left((3Ab+aB)\sqrt{a+b\cos(c+\right.}{3b\sqrt{\frac{a+b}{}}}$$

$$= \frac{2(3Ab+aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(a^2-b^2\right)E}{3b}$$

Mathematica [A] time = 0.59, size = 146, normalized size = 0.85

$$\frac{-2B\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2(a+b)(aB+3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2bB\sin(c+}{3bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(a + b)*(3*A*b + a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)] - 2*(a^2 - b^2)*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*B*(a + b*Cos[c + d*x])*Sin[c +
 d*x])/(3*b*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a), x)
```

maple [B] time = 1.65, size = 600, normalized size = 3.51

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4B\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+}{a-b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*cos(1
/2*d*x+1/2*c)^5*b^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)
^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b
-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+2*B*cos(1/2*d*x+1/
2*c)^3*a*b-6*B*cos(1/2*d*x+1/2*c)^3*b^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2+B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*B*cos(1/2*d*x+1/2*c)*a*b+2*B*cos(1/2*d*x+1
/2*c)*b^2)/b/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx)) \sqrt{a + b\cos(c + dx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x)), x)
```

3.300 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=178

$$\frac{2Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2aA\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(
a+b))^(1/2)+2*A*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)
/d/(a+b*cos(d*x+c))^(1/2)+2*a*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x
+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3002, 2655, 2653, 2803, 2663, 2661, 2807, 2805}

$$\frac{2Ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2aA\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (2*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*
a*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
```

```
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec(c+dx)\,dx = A\int \sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\,dx + B\int \sqrt{a+$$

$$= (aA)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx + (Ab)\int \frac{1}{\sqrt{a+b\cos}}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left(aA\sqrt{\frac{a+}{}}\right.}{}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2Ab\sqrt{\frac{a+}{}}}{d}$$

Mathematica [A] time = 2.40, size = 107, normalized size = 0.60

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(A\left(bF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+B(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*B*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + A*(b*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*EllipticPi[2,
(c + d*x)/2, (2*b)/(a + b)])))/(d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

maple [A] time = 1.21, size = 247, normalized size = 1.39

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\,\left(Ab\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(A*b*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-a*A*EllipticPi(cos(1/2*d*x+1/2*c),
2,(-2*b/(a-b))^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-
B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c
)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))\sqrt{a + b\cos(c + dx)}\,\sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sec(c + d*x), x)
```

3.301 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=213

$$\frac{(aA+2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{(2aB+Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cdots}}{d}$$

```
[Out] -A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a
+b))^(1/2)+(A*a+2*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^
(1/2)/d/(a+b*cos(d*x+c))^(1/2)+(A*b+2*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((
a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+A*(a+b*cos(d*x+c))^(1
/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.61, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2999, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(aA+2bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{(2aB+Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cdots}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] -((A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])) + ((a*A + 2*b*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x
]]) + ((A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c
+ d*x]]*Tan[c + d*x])/d
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
```

```
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} + \int \frac{\left(\frac{1}{2}(Ab+2aB\right.}{}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} - \frac{1}{2}A\int \sqrt{a+b\,\mathrm{co}}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} - \frac{1}{2}(-Ab-2aB)$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{a+b}}{}$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{(aA+2b}{}$$

Mathematica [C] time = 10.52, size = 372, normalized size = 1.75

$$\frac{\frac{2(4aB+Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4A\tan(c+dx)\sqrt{a+b\cos(c+dx)}-\frac{2iA\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\Big(}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] ((8*b*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(A*b + 4*a*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*
x]] - ((2*I)*A*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c
+ d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] +
 b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d
*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*A*Sqrt[a + b*Cos[c
 + d*x]]*Tan[c + d*x])/(4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

maple [B] time = 2.25, size = 746, normalized size = 3.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))-2*(A*b+B*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2
))+2*a*A*(-1/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*
b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/
(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1
/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,\sqrt{a+b\cos(c+d\,x)}}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))\sqrt{a + b\cos(c + dx)}\sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

3.302 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=292

$$\frac{\left(4a^2A+4abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{(4aB+Ab)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4ad}+\frac{(4aB+3A}{}$$

```
[Out] -1/4*(A*b+4*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/d/((a+
b*cos(d*x+c))/(a+b))^(1/2)+1/4*(3*A*b+4*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a
+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*A*a^2-A*b^2+4*B
*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*
x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b
*cos(d*x+c))^(1/2)+1/4*(A*b+4*B*a)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d+1/
2*A*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.95, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {2999, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2A+4abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{(4aB+Ab)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4ad}+\frac{(4aB+3A}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] -((A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(4*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*A*b + 4*a*B)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt
[a + b*Cos[c + d*x]]) + ((4*a^2*A - A*b^2 + 4*a*b*B)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*C
os[c + d*x]]) + ((A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*
d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
```

```
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4ad} + A\sqrt{a}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4ad} + A\sqrt{a}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4ad} + A\sqrt{a}$$

$$= -\frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 4.26, size = 420, normalized size = 1.44

$$\frac{\frac{2(8a^2A+4abB-3Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\sqrt{a+b\cos(c+dx)}} - \frac{2i(4aB+Ab)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a}\right.\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] ((8*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A - 3*A*b^2 + 4*a*b*B)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*Sq
rt[a + b*Cos[c + d*x]]) - ((2*I)*(A*b + 4*a*B)*Sqrt[-((b*(-1 + Cos[c + d*x]
))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a -
b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Co
s[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a
```

```
 + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a^2*b*Sqrt[-(a
+ b)^(-1)]) + (4*Sqrt[a + b*Cos[c + d*x]]*(2*a*A + (A*b + 4*a*B)*Cos[c + d*
x])*Sec[c + d*x]*Tan[c + d*x])/a)/(16*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

maple [B] time = 3.46, size = 1290, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*B*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*(A*b+B*a)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2
```

```
*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
,(-2*b/(a-b))^(1/2)))+2*a*A*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b
/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)
^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}\,\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sec(c + d*x)**3, x)
```

3.303 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=378

$$\frac{\left(16a^2A+6abB-3Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24a^2d}+\frac{\left(16a^2A+18abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{24ad\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -1/24*(16*A*a^2-3*A*b^2+6*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c)
)^(1/2)/a^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/24*(16*A*a^2-A*b^2+18*B*a*b)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x
+c))^(1/2)+1/8*(4*A*a^2*b+A*b^3+8*B*a^3-2*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(
1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)+1/24*(16*
A*a^2-3*A*b^2+6*B*a*b)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a^2/d+1/12*(A*b+6*
B*a)*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d+1/3*A*sec(d*x+c)^2*(a
+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 1.34, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2999, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(16a^2A+6abB-3Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24a^2d}+\frac{\left(16a^2A+18abB-Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right.}{24ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] -((16*a^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(24*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*
a^2*A - A*b^2 + 18*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2*A*b +
A*b^3 + 8*a^3*B - 2*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/(8*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((16*a
^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a^2*d)
 + ((A*b + 6*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*a
*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
```

```
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\,\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}$$

$$= \frac{(Ab+6aB)\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+d}{12ad}$$

$$= \frac{\left(16a^2A-3Ab^2+6abB\right)\sqrt{a+b\cos(c+dx)}\,\tan(c+}{24a^2d}$$

$$= \frac{\left(16a^2A-3Ab^2+6abB\right)\sqrt{a+b\cos(c+dx)}\,\tan(c+}{24a^2d}$$

$$= \frac{\left(16a^2A-3Ab^2+6abB\right)\sqrt{a+b\cos(c+dx)}\,\tan(c+}{24a^2d}$$

$$= -\frac{\left(16a^2A-3Ab^2+6abB\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{24a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(16a^2A-3Ab^2+6abB\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{24a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.55, size = 635, normalized size = 1.68

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)\left(16a^2A\sin(c+dx)+6abB\sin(c+dx)-3Ab^2\sin(c+dx)\right)}{24a^2}+\frac{\sec^2(c+dx)(6aB\sin(c+dx)+Ab\sin(c+dx))}{12a}+\frac{1}{3}A\tan\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((2*(4*a*A*b^2 + 24*a^2*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A*b + 9*A*
b^3 + 48*a^3*B - 18*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-16*a^2*
A*b + 3*A*b^3 - 6*a*b^2*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b
```

```
*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[
Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*El
lipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
 - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt
[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*
Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*C
os[c + d*x])^2)))/(96*a^2*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^2*(
A*b*Sin[c + d*x] + 6*a*B*Sin[c + d*x]))/(12*a) + (Sec[c + d*x]*(16*a^2*A*Si
n[c + d*x] - 3*A*b^2*Sin[c + d*x] + 6*a*b*B*Sin[c + d*x]))/(24*a^2) + (A*Se
c[c + d*x]^2*Tan[c + d*x])/3))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^4, x)
```

maple [B] time = 4.47, size = 2213, normalized size = 5.85

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*A*(-1/3
/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^
2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
```

```
*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1
/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*b^
3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*B*b*(-1/a*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x
+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-
b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b
+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2,(-2*b/(a-b))^(1/2)))+2*(A*b+B*a)*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
```

```
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2
*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^4,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.304 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=378

$$-\frac{2\left(-8a^2B+18aAb-49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^2d}-\frac{2\left(-8a^3B+18a^2Ab-39ab^2B-75Ab^3\right)\sin(c+}{315b^2d}$$

```
[Out] -2/315*(18*A*a*b-8*B*a^2-49*B*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+
2/63*(9*A*b-4*B*a)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+2/9*B*cos(d*x+c)
*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d-2/315*(18*A*a^2*b-75*A*b^3-8*B*a^3-3
9*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d-2/315*(18*A*a^3*b-246*A*
a*b^3-8*B*a^4-33*B*a^2*b^2-147*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d
*x+c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/315*(a^2-b^2)*(18*A*a^2
*b-75*A*b^3-8*B*a^3-39*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c)
)/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2990, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-8a^2B+18aAb-49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^2d}-\frac{2\left(18a^2Ab-8a^3B-39ab^2B-75Ab^3\right)\sin(c+dx}{315b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*(18*a^3*A*b - 246*a*A*b^3 - 8*a^4*B - 33*a^2*b^2*B - 147*b^4*B)*Sqrt[a
+ b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(18*a^2*A*b - 75*A*b^3 - 8*a^
3*B - 39*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(315*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(18*a^2*A*b - 75
*A*b^3 - 8*a^3*B - 39*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*
b^2*d) - (2*(18*a*A*b - 8*a^2*B - 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[
c + d*x])/(315*b^2*d) + (2*(9*A*b - 4*a*B)*(a + b*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(63*b^2*d) + (2*B*Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c +
d*x])/(9*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
```

```
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2B\cos(c+dx)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{9bd} +$$

$$= \frac{2(9Ab-4aB)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63b^2d} +$$

$$= -\frac{2\left(18aAb-8a^2B-49b^2B\right)(a+b\cos(c+dx))^{3/2}}{315b^2d}$$

$$= -\frac{2\left(18a^2Ab-75Ab^3-8a^3B-39ab^2B\right)\sqrt{a+b\cos}}{315b^2d}$$

$$= -\frac{2\left(18a^2Ab-75Ab^3-8a^3B-39ab^2B\right)\sqrt{a+b\cos}}{315b^2d}$$

$$= -\frac{2\left(18a^2Ab-75Ab^3-8a^3B-39ab^2B\right)\sqrt{a+b\cos}}{315b^2d}$$

$$= -\frac{2\left(18a^3Ab-246aAb^3-8a^4B-33a^2b^2B-147b^4\right.}{315b^3d\sqrt{\frac{a+b\cos}{a+}}}$$

Mathematica [A] time = 1.53, size = 291, normalized size = 0.77

$$\frac{b(a+b\cos(c+dx))\left(b\left(2\left(6a^2B+144aAb+133b^2B\right)\sin(2(c+dx))+5b(2(10aB+9Ab)\sin(3(c+dx))+7bB\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(153*a^2*A*b + 75*A*b^3 + 2*a^3*
B + 186*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-18*a^3*A*b + 246
*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*((a + b)*EllipticE[(c + d*x)
/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*C
os[c + d*x])*((72*a^2*A*b + 690*A*b^3 - 32*a^3*B + 804*a*b^2*B)*Sin[c + d*x
] + b*(2*(144*a*A*b + 6*a^2*B + 133*b^2*B)*Sin[2*(c + d*x)] + 5*b*(2*(9*A*b
 + 10*a*B)*Sin[3*(c + d*x)] + 7*b*B*Sin[4*(c + d*x)]))))/(1260*b^3*d*Sqrt[a
 + b*Cos[c + d*x]])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(Bb\cos\left(dx+c\right)^4+Aa\cos\left(dx+c\right)^2+\left(Ba+Ab\right)\cos\left(dx+c\right)^3\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^4 + A*a*cos(d*x + c)^2 + (B*a + A*b)*cos(d*x + c
)^3)*sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x
)
```

maple [B] time = 1.98, size = 1635, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)

[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b^5+1360*B*a*b^4+2240*B*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-936*A*a*b^4-1080*A*b^5-424*B*a^2*b^3-2040*B*a*b^4-2072*B*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(324*A*a^2*b^3+936*A*a*b^4+840*A*b^5-4*B*a^3*b^2+424*B*a^2*b^3+1568*B*a*b^4+952*B*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A*a^3*b^2-162*A*a^2*b^3-384*A*a*b^4-240*A*b^5+8*B*a^4*b+2*B*a^3*b^2-282*B*a^2*b^3-444*B*a*b^4-168*B*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+18*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-93*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+75*A*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-18*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+18*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+246*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3-246*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-31*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+39*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+33*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-33*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5)/b^3/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^{2}\,\left(A+B\,\cos(c+d\,x)\right)\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.305 $\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=297

$$\frac{2\left(-6a^2B+21aAb+25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105bd}-\frac{2\left(a^2-b^2\right)\left(-6a^2B+21aAb+25b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{105b^2d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2/35*(7*A*b-2*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*B*(a+b*cos(d*x
+c))^(5/2)*sin(d*x+c)/b/d+2/105*(21*A*a*b-6*B*a^2+25*B*b^2)*sin(d*x+c)*(a+b
*cos(d*x+c))^(1/2)/b/d+2/105*(21*A*a^2*b+63*A*b^3-6*B*a^3+82*B*a*b^2)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))
^(1/2)-2/105*(a^2-b^2)*(21*A*a*b-6*B*a^2+25*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2
))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2968, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(-6a^2B+21aAb+25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105bd}-\frac{2\left(a^2-b^2\right)\left(-6a^2B+21aAb+25b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{105b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(21*a^2*A*b + 63*A*b^3 - 6*a^3*B + 82*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]) - (2*(a^2 - b^2)*(21*a*A*b - 6*a^2*B + 25*b^2*B)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b^2*d*Sqrt[
a + b*Cos[c + d*x]]) + (2*(21*a*A*b - 6*a^2*B + 25*b^2*B)*Sqrt[a + b*Cos[c
+ d*x]]*Sin[c + d*x])/(105*b*d) + (2*(7*A*b - 2*a*B)*(a + b*Cos[c + d*x])^(
3/2)*Sin[c + d*x])/(35*b*d) + (2*B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])
/(7*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^{3/2}\left(A\cos(c+dx)+B\cos^2(c+\right.$$

$$= \frac{2B(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{2\int(a+b\cos}{}$$

$$= \frac{2(7Ab-2aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35bd} + \frac{2l}{}$$

$$= \frac{2\left(21aAb-6a^2B+25b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(}{105bd}$$

$$= \frac{2\left(21aAb-6a^2B+25b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(}{105bd}$$

$$= \frac{2\left(21aAb-6a^2B+25b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(}{105bd}$$

$$= \frac{2\left(21a^2Ab+63Ab^3-6a^3B+82ab^2B\right)\sqrt{a+b\cos(c}}{105b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.07, size = 233, normalized size = 0.78

$$\frac{b(a+b\cos(c+dx))\left(\left(12a^2B+168aAb+115b^2B\right)\sin(c+dx)+3b(2(8aB+7Ab)\sin(2(c+dx))+5bB\sin(3(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(84*a*A*b + 51*a^2*B + 25*b^2*B)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (21*a^2*A*b + 63*A*b^3 - 6*a^3*B +
 82*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(
c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((168*a*A*b + 12*a^2*
B + 115*b^2*B)*Sin[c + d*x] + 3*b*(2*(7*A*b + 8*a*B)*Sin[2*(c + d*x)] + 5*b
*B*Sin[3*(c + d*x)])))/(210*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{3}+Aa\cos\left(dx+c\right)+\left(Ba+Ab\right)\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^3 + A*a*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)^
2)*sqrt(b*cos(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [B] time = 1.58, size = 1305, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^4-312*B*a*b^3-360*B*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(252*A*a*b^3+168*A*b^4+108*B*a^2
*b^2+312*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-84*A*
a^2*b^2-126*A*a*b^3-42*A*b^4-6*B*a^3*b-54*B*a^2*b^2-128*B*a*b^3-80*B*b^4)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-21*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^3*b+21*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
```

```
*b/(a-b))^(1/2))*b^3+21*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^3*b-21*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2
*b^2+63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-63*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+6*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-31*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))-6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^3*b+82*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2
-82*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b^2/(-2
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)\,{\left(a+b\,\cos(c+d\,x)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.306 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=225

$$-\frac{2\left(a^2-b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2B+20aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{15bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 2/5*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/15*(5*A*b+3*B*a)*sin(d*x+c)*(a+
b*cos(d*x+c))^(1/2)/d+2/15*(20*A*a*b+3*B*a^2+9*B*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(
1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/15*(a^2-b
^2)*(5*A*b+3*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)
/b/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)(3aB+5Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2B+20aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{15bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^
2 - b^2)*(5*A*b + 3*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/(15*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(5*A*b + 3*a
*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*B*(a + b*Cos[c + d*x
])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{2B(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{a+b\cos(c+dx)}$$

$$= \frac{2(5Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2B(a+b\cos(c}{}$$

$$= \frac{2(5Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2B(a+b\cos(c}{}$$

$$= \frac{2(5Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2B(a+b\cos(c}{}$$

$$= \frac{2\left(20aAb+3a^2B+9b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.78, size = 203, normalized size = 0.90

$$\frac{2\left(b\left(15a^2A+12abB+5Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(3a^2B+20aAb+9b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)E\right.\right.}{15bd\sqrt{a+b\cos(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(b*(15*a^2*A + 5*A*b^2 + 12*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)] + (20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)]
 - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)]) + b*(a + b*Cos[c + d*x])*(5*A*b
 + 6*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x]))/(15*b*d*Sqrt[a + b*Cos[c + d*
x]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^2+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2), x)
```

maple [B] time = 1.65, size = 993, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b^3+36*B*a*b^2+24*B*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3-12*B*a^2*b-18*B
*a*b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^2*b-20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a*b^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^3+3*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+3*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-3*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3)/b/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\ \cos(c + d\,x))\ (a + b\ \cos(c + d\,x))^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))\,(a + b\cos(c + dx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(3/2), x)
```

3.307 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=236

$$\frac{2\left(a^2(-B)+3aAb+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2a^2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2(4aB+3A}{}$$

```
[Out] 2/3*b*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*(3*A*b+4*B*a)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/
(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(
3*A*a*b-B*a^2+B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1
/2)/d/(a+b*cos(d*x+c))^(1/2)+2*a^2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*co
s(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2990, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2\left(a^2(-B)+3aAb+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2a^2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2(4aB+3A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*(3*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(3*a*A*b - a^2*B + b^
2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a^2*A*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]
]) + (2*b*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
```

```
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\left(\frac{3a^2A}{2}\right.}{}$$

$$=\frac{2bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d}-\frac{2\int \frac{\left(-\frac{3}{2}a^2Ab-\right.}{}}{}$$

$$=\frac{2bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d}+\left(a^2A\right)\int \frac{}{\sqrt{a}}$$

$$=\frac{2(3Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$=\frac{2(3Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 2.58, size = 406, normalized size = 1.72

$$\frac{\frac{4\left(3a^2B+6aAb+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2A+4abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i(4aB+3Ab)\csc(c+dx)\sqrt{-\frac{b(\cos(c\cdots}{a\cdots}}}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] ((4*(6*a*A*b + 3*a^2*B + b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^2*A + 3*A
*b^2 + 4*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(3*A*b + 4*a*B)*Sqrt[-
((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Cs
c[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b
*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)
/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b
)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*b*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(6*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

maple [B] time = 1.52, size = 738, normalized size = 3.13

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4B\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+3Aab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{a-b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*cos(1
/2*d*x+1/2*c)^5*b^2+3*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-3*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-3*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2,(-2*b/(a-b))^(1/2))+2*B*cos(1/2*d*x+1/2*c)^3*a*b-6*B*cos(1/2*d*x+1/2*c)^3
*b^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+B*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*
cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^2-4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-
2*B*cos(1/2*d*x+1/2*c)*a*b+2*B*cos(1/2*d*x+1/2*c)*b^2)/(-2*sin(1/2*d*x+1/2*
c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x),x)
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
[Out] Timed out
```

3.308 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=232

$$\frac{\left(a^2A+2abB+2Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a(2aB}{}$$

```
[Out] -(A*a-2*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(
d*x+c))/(a+b))^(1/2)+(A*a^2+2*A*b^2+2*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a
+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+a*(3*A*b+2*B*a)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2
^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(
1/2)+a*A*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.69, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2989, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(a^2A+2abB+2Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a(2aB}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] -(((a*A - 2*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + ((a^2*A + 2*A*b^2 + 2*a*b*B
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
/(d*Sqrt[a + b*Cos[c + d*x]]) + (a*(3*A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c +
 d*x]]) + (a*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
```

```
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} + \int \frac{\left(\frac{1}{2}a(3Ab\right.}{}$$

$$= \frac{aA\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} - \frac{\int \frac{\left(-\frac{1}{2}ab(3Ab+\right.}{}}{}$$

$$= \frac{aA\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{d} + \frac{1}{2}(a(3Ab+$$

$$= -\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(aA-2bB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 2.54, size = 398, normalized size = 1.72

$$\frac{\frac{2\left(4a^2B+5aAb+2b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{8b(2aB+Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i(2bB-aA)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\ \sqrt{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] ((8*b*(A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(5*a*A*b + 4*a^2*B + 2*b^2
*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-(a*A) + 2*b*B)*Sqrt[-((b*(-1 + C
os[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*
(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*
x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sq
rt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*
Sqrt[-(a + b)^(-1)]) + 4*a*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

fricas [F] time = 8.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^2+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x
)
```

maple [B] time = 1.60, size = 1167, normalized size = 5.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*a*b*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*A*a^2-2*A*a*b)*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)-2*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2+2*A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-A*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+A*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a*b-3*A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
1/2))*a*b+2*B*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+2*B*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*B*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*B*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a
-b))^(1/2))*a^2)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^2+2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-3*A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b+2*B*a*b*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a*b-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*b^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
*a^2)/(2*cos(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)
/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.309 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=295

$$\frac{\left(4a^2B+7aAb+8b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{\left(4a^2A+12abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\right.}{4d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -1/4*(5*A*b+4*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+
b*cos(d*x+c))/(a+b))^(1/2)+1/4*(7*A*a*b+4*B*a^2+8*B*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b)
)^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*A*a
^2+3*A*b^2+12*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))
^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*(5*A*b+4*B*a)*(a+b*cos(d*x+c))^(1/2)*ta
n(d*x+c)/d+1/2*a*A*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 1.06, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2989, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2B+7aAb+8b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{\left(4a^2A+12abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\right.}{4d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] -((5*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((7*a*A*b + 4*a^2*B + 8*b
^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2*A + 3*A*b^2 + 12*a*b*B)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4
*d*Sqrt[a + b*Cos[c + d*x]]) + ((5*A*b + 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Ta
n[c + d*x])/(4*d) + (a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x]
)/(2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
```

```
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}$$

$$= \frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d} + \frac{aA}{}$$

$$= \frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d} + \frac{aA}{}$$

$$= \frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d} + \frac{aA}{}$$

$$= -\frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{(5Ab+4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 4.95, size = 422, normalized size = 1.43

$$\frac{\frac{2(8a^2A+20abB+Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{8b(aA+4bB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + 4\tan(c+dx)\sec(c+dx)\sqrt{a+b}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] ((8*b*(a*A + 4*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A + A*b^2 + 20*a*b*
B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
 b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(5*A*b + 4*a*B)*Sqrt[-((b*(-1 + Cos
[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-
2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]
]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt
[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh
```

```
[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sq
rt[-(a + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*(2*a*A + (5*A*b + 4*a*B)*Co
s[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(16*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x
)
```

maple [B] time = 3.67, size = 1403, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*b*(A*b+2*B*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))+2*a*(2*A*b+B*a)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*
b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1
```

```
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos
(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))
)+2*a^2*A*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1
/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2
*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1
/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b
))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
[Out] Timed out
```

3.310 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=375

$$\frac{(16a^2A+30abB+3Ab^2)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24ad}+\frac{(16a^2A+42abB+17Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right|}{24d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -1/24*(16*A*a^2+3*A*b^2+30*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c
))^(1/2)/a/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/24*(16*A*a^2+17*A*b^2+42*B*a*
b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x
+c))^(1/2)+1/8*(12*A*a^2*b-A*b^3+8*B*a^3+6*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^
(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+1/24*(16*A
*a^2+3*A*b^2+30*B*a*b)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d+1/12*(7*A*b+6*
B*a)*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/3*a*A*sec(d*x+c)^2*(a
+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 1.45, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {2989, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(16a^2A+30abB+3Ab^2)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24ad}+\frac{(16a^2A+42abB+17Ab^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right|}{24d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] -((16*a^2*A + 3*A*b^2 + 30*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(24*a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*a
^2*A + 17*A*b^2 + 42*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) + ((12*a^2*A*b -
 A*b^3 + 8*a^3*B + 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*b)/(a + b)])/(8*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((16*a^
2*A + 3*A*b^2 + 30*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*a*d) +
 ((7*A*b + 6*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*d
) + (a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
```

```
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA\sqrt{a+b\cos(c+dx)}\,\sec^2(c+dx)\tan(c+dx)}{3d} +$$

$$= \frac{(7Ab+6aB)\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c$$

$$= \frac{\left(16a^2A+3Ab^2+30abB\right)\sqrt{a+b\cos(c+dx)}\,\tan}{24ad}$$

$$= \frac{\left(16a^2A+3Ab^2+30abB\right)\sqrt{a+b\cos(c+dx)}\,\tan}{24ad}$$

$$= \frac{\left(16a^2A+3Ab^2+30abB\right)\sqrt{a+b\cos(c+dx)}\,\tan}{24ad}$$

$$= -\frac{\left(16a^2A+3Ab^2+30abB\right)\sqrt{a+b\cos(c+dx)}\,E}{24ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(16a^2A+3Ab^2+30abB\right)\sqrt{a+b\cos(c+dx)}\,E}{24ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.71, size = 634, normalized size = 1.69

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)\left(16a^2A\sin(c+dx)+30abB\sin(c+dx)+3Ab^2\sin(c+dx)\right)}{24a}+\frac{1}{12}\sec^2(c+dx)(6aB\sin(c+dx)+7Ab\,s\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((2*(28*a*A*b^2 + 24*a^2*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(56*a^2*A*b - 9*
A*b^3 + 48*a^3*B + 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi
```

```
[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-16*a^2
*A*b - 3*A*b^3 - 30*a*b^2*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b +
 b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSin
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*
EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/
(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b
*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sq
rt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a +
b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b
*Cos[c + d*x])^2)))/(96*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^2*(
7*A*b*Sin[c + d*x] + 6*a*B*Sin[c + d*x]))/12 + (Sec[c + d*x]*(16*a^2*A*Sin[
c + d*x] + 3*A*b^2*Sin[c + d*x] + 30*a*b*B*Sin[c + d*x]))/(24*a) + (a*A*Sec
[c + d*x]^2*Tan[c + d*x])/3))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x
)
```

maple [B] time = 4.81, size = 2327, normalized size = 6.21

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b^2*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
```

```
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*a^2*A*(-1/3/a*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)
/a^3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*
b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/
a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*
b/(a-b))^(1/2)))+2*b*(A*b+2*B*a)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2)))+2*a*(2*A*b+B*a)*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a
^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
```

```
)+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+
a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^4,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.311 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=462

$$-\frac{2\left(-8a^2B+22aAb-81b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{693b^2d}-\frac{2\left(-40a^3B+110a^2Ab-335ab^2B-539Ab^3\right)\text{si}}{3465b^2d}$$

```
[Out] -2/3465*(110*A*a^2*b-539*A*b^3-40*B*a^3-335*B*a*b^2)*(a+b*cos(d*x+c))^(3/2)
*sin(d*x+c)/b^2/d-2/693*(22*A*a*b-8*B*a^2-81*B*b^2)*(a+b*cos(d*x+c))^(5/2)*
sin(d*x+c)/b^2/d+2/99*(11*A*b-4*B*a)*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^2/
d+2/11*B*cos(d*x+c)*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d-2/3465*(110*A*a^3
*b-1254*A*a*b^3-40*B*a^4-285*B*a^2*b^2-675*B*b^4)*sin(d*x+c)*(a+b*cos(d*x+c
))^(1/2)/b^2/d-2/3465*(110*A*a^4*b-3069*A*a^2*b^3-1617*A*b^5-40*B*a^5-255*B
*a^3*b^2-3705*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^
3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3465*(a^2-b^2)*(110*A*a^3*b-1254*A*a*b
^3-40*B*a^4-285*B*a^2*b^2-675*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d
*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2990, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-8a^2B+22aAb-81b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{693b^2d}-\frac{2\left(110a^2Ab-40a^3B-335ab^2B-539Ab^3\right)\text{sin}}{3465b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*(110*a^4*A*b - 3069*a^2*A*b^3 - 1617*A*b^5 - 40*a^5*B - 255*a^3*b^2*B -
 3705*a*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b
)])/(3465*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(a^2 - b^2)*(110*a
^3*A*b - 1254*a*A*b^3 - 40*a^4*B - 285*a^2*b^2*B - 675*b^4*B)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3465*b^3*d*Sq
rt[a + b*Cos[c + d*x]]) - (2*(110*a^3*A*b - 1254*a*A*b^3 - 40*a^4*B - 285*a
^2*b^2*B - 675*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*b^2*d) -
 (2*(110*a^2*A*b - 539*A*b^3 - 40*a^3*B - 335*a*b^2*B)*(a + b*Cos[c + d*x])
^(3/2)*Sin[c + d*x])/(3465*b^2*d) - (2*(22*a*A*b - 8*a^2*B - 81*b^2*B)*(a +
```

```
 b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*b^2*d) + (2*(11*A*b - 4*a*B)*(a +
 b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^2*d) + (2*B*Cos[c + d*x]*(a + b*
Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{2B\cos(c+dx)(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{11bd} +$$

$$= \frac{2(11Ab-4aB)(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{99b^2d} +$$

$$= -\frac{2\left(22aAb-8a^2B-81b^2B\right)(a+b\cos(c+dx))^{5/2}}{693b^2d}$$

$$= -\frac{2\left(110a^2Ab-539Ab^3-40a^3B-335ab^2B\right)(a+b}{3465b^2d}$$

$$= -\frac{2\left(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2B-6\right.}{3465b^2d}$$

$$= -\frac{2\left(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2B-6\right.}{3465b^2d}$$

$$= -\frac{2\left(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2B-6\right.}{3465b^2d}$$

$$= -\frac{2\left(110a^4Ab-3069a^2Ab^3-1617Ab^5-40a^5B-2\right.}{3465b}$$

<u>**Mathematica**</u> [**A**] time = 2.12, size = 357, normalized size = 0.77

$$\frac{b(a+b\cos(c+dx))\left(b\left(5b\left(\left(452a^2B+836aAb+513b^2B\right)\sin(3(c+dx))+7b((46aB+22Ab)\sin(4(c+dx))+9b\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (16*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(1705*a^3*A*b + 2871*a*A*b^3 +
10*a^4*B + 3315*a^2*b^2*B + 675*b^4*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)
] + (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B
+ 3705*a*b^4*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])) + b*(a + b*Cos[c + d*x])*((880*a^3*A*b + 32
```

```
868*a*A*b^3 - 320*a^4*B + 18660*a^2*b^2*B + 13050*b^4*B)*Sin[c + d*x] + b*(
4*(1650*a^2*A*b + 1463*A*b^3 + 30*a^3*B + 3095*a*b^2*B)*Sin[2*(c + d*x)] +
5*b*((836*a*A*b + 452*a^2*B + 513*b^2*B)*Sin[3*(c + d*x)] + 7*b*((22*A*b +
46*a*B)*Sin[4*(c + d*x)] + 9*b*B*Sin[5*(c + d*x)])))))/(27720*b^3*d*Sqrt[a
+ b*Cos[c + d*x]])
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos\left(dx+c\right)^5+Aa^2\cos\left(dx+c\right)^2+\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^4+\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)^3\right)\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^5 + A*a^2*cos(d*x + c)^2 + (2*B*a*b + A*b^2)*c
os(d*x + c)^4 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos\left(dx+c\right)+A)(b\cos\left(dx+c\right)+a)^{\frac{5}{2}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x
)
```

maple [B] time = 1.88, size = 1983, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-245*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-3069*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4+110*A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b^2-110*A*(sin(1/2*d*x+1/2
```

c)^2)^(1/2)(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b+110*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b-1364*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1254*A*a*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+255*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^4*b^2+3069*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^3*b^3+1617*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a*b^5-40*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5*b-25
5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^3+3705*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4-3705*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^5-390*a^2*b^4*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+20160*B*b^6*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*A*b^6-35840*B*a*b^5-50400*B*b^6)*sin(1/2
*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(22880*A*a*b^5+24640*A*b^6+21920*B*a^2*b^
4+71680*B*a*b^5+56880*B*b^6)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1496
0*A*a^2*b^4-34320*A*a*b^5-22792*A*b^6-4640*B*a^3*b^3-32880*B*a^2*b^4-66160*
B*a*b^5-34920*B*b^6)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(3520*A*a^3*b^
3+14960*A*a^2*b^4+26488*A*a*b^5+10472*A*b^6-20*B*a^4*b^2+4640*B*a^3*b^3+251
20*B*a^2*b^4+30320*B*a*b^5+13860*B*b^6)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+(-110*A*a^4*b^2-1760*A*a^3*b^3-7326*A*a^2*b^4-7524*A*a*b^5-1848*A*b^6+
40*B*a^5*b+10*B*a^4*b^2-3210*B*a^3*b^3-7080*B*a^2*b^4-6690*B*a*b^5-2790*B*b
^6)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-40*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^6-1617*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*b^6+675*b^6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))+40*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
^6)/b^3/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\,\left(A+B\,\cos(c+d\,x)\right)\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.312 $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=372

$$\frac{2\left(-10a^2B+45aAb+49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315bd}+\frac{2\left(-10a^3B+45a^2Ab+114ab^2B+75Ab^3\right)\sin(c}{315bd}$$

```
[Out] 2/315*(45*A*a*b-10*B*a^2+49*B*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/
63*(9*A*b-2*B*a)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+2/9*B*(a+b*cos(d*x+c
))^(7/2)*sin(d*x+c)/b/d+2/315*(45*A*a^2*b+75*A*b^3-10*B*a^3+114*B*a*b^2)*si
n(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d+2/315*(45*A*a^3*b+435*A*a*b^3-10*B*a^4+
279*B*a^2*b^2+147*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/
b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/315*(a^2-b^2)*(45*A*a^2*b+75*A*b^3-1
0*B*a^3+114*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1
/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2968, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(-10a^2B+45aAb+49b^2B\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315bd}+\frac{2\left(45a^2Ab-10a^3B+114ab^2B+75Ab^3\right)\sin(c+}{315bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(45*a^3*A*b + 435*a*A*b^3 - 10*a^4*B + 279*a^2*b^2*B + 147*b^4*B)*Sqrt[a
 + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*(45*a^2*A*b + 75*A*b^3 - 10*
a^3*B + 114*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(45*a^2*A*b +
 75*A*b^3 - 10*a^3*B + 114*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(315*b*d) + (2*(45*a*A*b - 10*a^2*B + 49*b^2*B)*(a + b*Cos[c + d*x])^(3/2)*
Sin[c + d*x])/(315*b*d) + (2*(9*A*b - 2*a*B)*(a + b*Cos[c + d*x])^(5/2)*Sin
[c + d*x])/(63*b*d) + (2*B*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \int (a+b\cos(c+dx))^{5/2}\left(A\cos(c+dx)+B\cos^2(c+\right.$$

$$= \frac{2B(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{2\int(a+b\cos}{}$$

$$= \frac{2(9Ab-2aB)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63bd} + \frac{2I}{}$$

$$= \frac{2\left(45aAb-10a^2B+49b^2B\right)(a+b\cos(c+dx))^{3/2}\,\mathrm{si}}{315bd}$$

$$= \frac{2\left(45a^2Ab+75Ab^3-10a^3B+114ab^2B\right)\sqrt{a+b\cos}}{315bd}$$

$$= \frac{2\left(45a^2Ab+75Ab^3-10a^3B+114ab^2B\right)\sqrt{a+b\cos}}{315bd}$$

$$= \frac{2\left(45a^2Ab+75Ab^3-10a^3B+114ab^2B\right)\sqrt{a+b\cos}}{315bd}$$

$$= \frac{2\left(45a^3Ab+435aAb^3-10a^4B+279a^2b^2B+147b^4\right.}{315b^2d\sqrt{\frac{a+b\cos(}{a+b}}}$$

Mathematica [A] time = 1.60, size = 291, normalized size = 0.78

$$\frac{b(a+b\cos(c+dx))\left(b\left(\left(300a^2B+540aAb+266b^2B\right)\sin(2(c+dx))+5b(2(19aB+9Ab)\sin(3(c+dx))+7bB\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (8*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(405*a^2*A*b + 75*A*b^3 + 155*a^
3*B + 261*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (45*a^3*A*b + 43
5*a*A*b^3 - 10*a^4*B + 279*a^2*b^2*B + 147*b^4*B)*((a + b)*EllipticE[(c + d
*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + b*(a +
b*Cos[c + d*x])*(2*(540*a^2*A*b + 345*A*b^3 + 20*a^3*B + 747*a*b^2*B)*Sin[c
 + d*x] + b*((540*a*A*b + 300*a^2*B + 266*b^2*B)*Sin[2*(c + d*x)] + 5*b*(2*
(9*A*b + 19*a*B)*Sin[3*(c + d*x)] + 7*b*B*Sin[4*(c + d*x)]))))/(1260*b^2*d*
Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.37, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^4+Aa^2\cos(dx+c)+\left(2\,Bab+Ab^2\right)\cos(dx+c)^3+\left(Ba^2+2\,Aab\right)\cos(dx+c)^2\right)\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^4 + A*a^2*cos(d*x + c) + (2*B*a*b + A*b^2)*cos
(d*x + c)^3 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a), x
)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [B] time = 1.81, size = 1635, normalized size = 4.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B
*b^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b^5+2080*B*a*b^4+2240*
B*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1440*A*a*b^4-1080*A*b^5-13
60*B*a^2*b^3-3120*B*a*b^4-2072*B*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1080*A*a^2*b^3+1440*A*a*b^4+840*A*b^5+320*B*a^3*b^2+1360*B*a^2*b^3+2408
*B*a*b^4+952*B*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-270*A*a^3*b^2
-540*A*a^2*b^3-510*A*a*b^4-240*A*b^5-10*B*a^4*b-160*B*a^3*b^2-666*B*a^2*b^3
-684*B*a*b^4-168*B*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+45*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-45*A*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+435*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3-435*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^4*b-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a^2*b^3+75*A*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))-10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5+10*B
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+279*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-279*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*b^5+10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^5-124*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a^3*b^2+114*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*b^4)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/si
n(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.313 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=288

$$\frac{2\left(15a^2B+56aAb+25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(a^2-b^2\right)\left(15a^2B+56aAb+25b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{105bd\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2/35*(7*A*b+5*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(a+b*cos(d*x+c
))^(5/2)*sin(d*x+c)/d+2/105*(56*A*a*b+15*B*a^2+25*B*b^2)*sin(d*x+c)*(a+b*co
s(d*x+c))^(1/2)/d+2/105*(161*A*a^2*b+63*A*b^3+15*B*a^3+145*B*a*b^2)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1/
2)-2/105*(a^2-b^2)*(56*A*a*b+15*B*a^2+25*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*
((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(15a^2B+56aAb+25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(a^2-b^2\right)\left(15a^2B+56aAb+25b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{105bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) - (2*(a^2 - b^2)*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(105*b*d*Sqrt[
a + b*Cos[c + d*x]]) + (2*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(105*d) + (2*(7*A*b + 5*a*B)*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(35*d) + (2*B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7
*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{2B(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2}{7}\int (a+b\cos(c+dx))^{3/}$$

$$= \frac{2(7Ab+5aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2B(a+b\cos(}{}$$

$$= \frac{2\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105d} + $$

$$= \frac{2\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105d} + $$

$$= \frac{2\left(56aAb+15a^2B+25b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105d} + $$

$$= \frac{2\left(161a^2Ab+63Ab^3+15a^3B+145ab^2B\right)\sqrt{a+b\cos(c+dx)}\,E}{105bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.08, size = 254, normalized size = 0.88

$$\frac{b\sin(c+dx)(a+b\cos(c+dx))\left(90a^2B+6b(15aB+7Ab)\cos(c+dx)+154aAb+15b^2B\cos(2(c+dx))+65b^2B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*b*(105*a^3*A + 119*a*A*b^2 + 135*a^2*b*B + 25*b^3*B)*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*(161*a^2*A*b + 63
*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b
)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a
 + b)]) + b*(a + b*Cos[c + d*x])*(154*a*A*b + 90*a^2*B + 65*b^2*B + 6*b*(7*
A*b + 15*a*B)*Cos[c + d*x] + 15*b^2*B*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*
b*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.47, size = 1305, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^4-480*B*a*b^3-360*B*b^
4)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(392*A*a*b^3+168*A*b^4+360*B*a^2
*b^2+480*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-154*A
*a^2*b^2-196*A*a*b^3-42*A*b^4-90*B*a^3*b-180*B*a^2*b^2-170*B*a*b^3-80*B*b^4
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+56*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*b^3+161*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a^3*b-161*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^2*b^2+63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3
-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4-15*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-10*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*B*b^4*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
```

```
,(-2*b/(a-b))^(1/2))*a^4-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a^3*b+145*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^2*b^2-145*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^
3)/b/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
[Out] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.314 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=292

$$\frac{2a^3A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(23a^2B+35aAb+9b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(\cdots}{}$$

```
[Out] 2/5*b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/15*b*(5*A*b+8*B*a)*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/d+2/15*(35*A*a*b+23*B*a^2+9*B*b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+
b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/15*(10
*A*a^2*b+5*A*b^3-8*B*a^3+8*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*
x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+2*a^3*A*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^
(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 1.01, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2990, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2\left(10a^2Ab-8a^3B+8ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(23a^2B+35aAb+9b^2B\right)\sqrt{a+b\cos(c-\cdots}}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*(35*a*A*b + 23*a^2*B + 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(10*
a^2*A*b + 5*A*b^3 - 8*a^3*B + 8*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d*Sqrt[a + b*Cos[c + d*x]]) + (
2*a^3*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)
/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(5*A*b + 8*a*B)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(5*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
```

```
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = \frac{2bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{a+b}$$

$$= \frac{2b(5Ab+8aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2b}{}$$

$$= \frac{2b(5Ab+8aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2b}{}$$

$$= \frac{2b(5Ab+8aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2b}{}$$

$$= \frac{2\left(35aAb+23a^2B+9b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}\right.}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2\left(35aAb+23a^2B+9b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}\right.}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 2.91, size = 453, normalized size = 1.55

$$\frac{\frac{2i\left(23a^2B+35aAb+9b^2B\right)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\cos(c+dx)+1)}{a-b}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+}\right.\right.\right.\right.}{ab\sqrt{-\frac{1}{a+b}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] ((4*(45*a^2*A*b + 5*A*b^3 + 15*a^3*B + 17*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]]
+ (2*(30*a^3*A + 35*a*A*b^2 + 23*a^2*b*B + 9*b^3*B)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d
*x]] + ((2*I)*(35*a*A*b + 23*a^2*B + 9*b^2*B)*Sqrt[-((b*(-1 + Cos[c + d*x])
)/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a -
 b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(
```

```
a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a +
 b)^(-1)]) + 4*b*Sqrt[a + b*Cos[c + d*x]]*(5*A*b + 11*a*B + 3*b*B*Cos[c + d
*x])*Sin[c + d*x])/(30*d)
```

fricas [F] time = 3.34, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c) + A)(b\cos(dx+c) + a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

maple [B] time = 1.47, size = 1067, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b^3+56*B*a*b^2+24*B*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3-22*B*a^2*b-28*B
*a*b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^2*b-35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
```

```
b))^(1/2))*a*b^2-15*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+
(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+8*a
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+23*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-23*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{5/2}}{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

3.315 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=296

$$-\frac{(3a^2A-14abB-6Ab^2)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a^2(2aB+5Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2}{a+}\right.}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -1/3*b*(3*A*a-2*B*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d-1/3*(3*A*a^2-6*A*b
^2-14*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(
d*x+c))/(a+b))^(1/2)+1/3*(3*A*a^3+12*A*a*b^2+4*B*a^2*b+2*B*b^3)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(
b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+a^2
*(5*A*b+2*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(s
in(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)
/d/(a+b*cos(d*x+c))^(1/2)+a*A*(a+b*cos(d*x+c))^(3/2)*tan(d*x+c)/d
```

Rubi [A] time = 1.11, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {2989, 3049, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(3a^3A+4a^2bB+12aAb^2+2b^3B)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}-\frac{(3a^2A-14abB-6Ab^2)\sqrt{a+b\cos(c+dx)}}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] -((3*a^2*A - 6*A*b^2 - 14*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*
x)/2, (2*b)/(a + b)])/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*a^3*A
+ 12*a*A*b^2 + 4*a^2*b*B + 2*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (a^2*(5
*A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - (b*(3*a*A - 2*b*B)*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (a*A*(a + b*Cos[c + d*x])^(3/2)*Tan[
c + d*x])/d
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
```

```
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^{3/2}\tan(c+dx)}{d} + \int \sqrt{a+b\,c}$$

$$= -\frac{b(3aA-2bB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{a}{-}$$

$$= -\frac{b(3aA-2bB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{a}{-}$$

$$= -\frac{b(3aA-2bB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{a}{-}$$

$$= -\frac{\left(3a^2A-6Ab^2-14abB\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(3a^2A-6Ab^2-14abB\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 3.94, size = 442, normalized size = 1.49

$$\frac{4\tan(c+dx)\sqrt{a+b\cos(c+dx)}\left(3a^2A+2b^2B\cos(c+dx)\right) + \frac{8b\left(9a^2B+9aAb+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i\left(-3a^2A\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] ((8*b*(9*a*A*b + 9*a^2*B + b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipt
icF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(27*a^2*A*b
+ 6*A*b^3 + 12*a^3*B + 14*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-3
*a^2*A + 6*A*b^2 + 14*a*b*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[
(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*Arc
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-
2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
```

```
 + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*Sqrt
[a + b*Cos[c + d*x]]*(3*a^2*A + 2*b^2*B*Cos[c + d*x])*Tan[c + d*x])/(12*d)
```

fricas [F] time = 6.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c) + A)(b\cos(dx+c) + a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x
)
```

maple [B] time = 1.79, size = 1563, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*B*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(12*A*a^2*b+8*B*a*b^2+16*B*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a^3-6*A*a^2*b-4*B*a*b^2-4*B*b^3
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(3*A*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+12*A*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a*b^2-3*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+
3*A*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+6*A*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-6*A*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b^3-15*A*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^
```

```
(1/2))*a^2*b+4*B*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+2*B
*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+14*B*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-14*B*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))*a*b^2-6*B*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1
/2))*a^3)*sin(1/2*d*x+1/2*c)^2+3*A*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*a^3+12*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))-3*A*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3+3*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+6*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-6*A*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3-15*A*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*b+4*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))+2*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))+14*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
2*b-14*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-6*B*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2,(-2*b/(a-b))^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3)/(-2*sin(1/2*d
*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/
sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.316 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=315

$$-\frac{\left(4a^2B+9aAb-8b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a\left(4a^2A+20abB+15Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\right.}{4d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -1/4*(9*A*a*b+4*B*a^2-8*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^
(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/4*(11*A*a^2*b+8*A*b^3+4*B*a^3+16*B
*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*co
s(d*x+c))^(1/2)+1/4*a*(4*A*a^2+15*A*b^2+20*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1
/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/2*a*A*(a+b*c
os(d*x+c))^(3/2)*sec(d*x+c)*tan(d*x+c)/d+1/4*a*(7*A*b+4*B*a)*(a+b*cos(d*x+c
))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 1.06, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2989, 3047, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(11a^2Ab+4a^3B+16ab^2B+8Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}-\frac{\left(4a^2B+9aAb-8b^2B\right)\sqrt{a+b\cos(c+dx)}\,E}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] -((9*a*A*b + 4*a^2*B - 8*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((11*a^2*A*
b + 8*A*b^3 + 4*a^3*B + 16*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(4*a
^2*A + 15*A*b^2 + 20*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2
, (c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(7*A*b +
 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a*A*(a + b*Cos[c +
d*x])^(3/2)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
```

```
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^{3/2}\sec(c+dx)\tan(c+dx)}{2d}$$

$$= \frac{a(7Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d} +$$

$$= \frac{a(7Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d} +$$

$$= \frac{a(7Ab+4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4d} +$$

$$= -\frac{\left(9aAb+4a^2B-8b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}\right.}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(9aAb+4a^2B-8b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}\right.}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 5.88, size = 451, normalized size = 1.43

$$\frac{\frac{8b\left(a^2A+12abB+4Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2i\left(-4a^2B-9aAb+8b^2B\right)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] ((8*b*(a^2*A + 4*A*b^2 + 12*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^3*A +
21*a*A*b^2 + 36*a^2*b*B + 8*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-9
*a*A*b - 4*a^2*B + 8*b^2*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(
b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2
*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
```

```
+ b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*a*Sqr
t[a + b*Cos[c + d*x]]*(2*a*A + (9*A*b + 4*a*B)*Cos[c + d*x])*Sec[c + d*x]*T
an[c + d*x])/(16*d)
```

fricas [F] time = 8.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+\dots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c) + A)(b\cos(dx+c) + a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x
)
```

maple [B] time = 3.89, size = 1742, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b^2*B*(a
-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2)))+2*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+6*B*a*b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
```

```
2*c),(-2*b/(a-b))^(1/2))-2*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-6*a*b*(A
*b+B*a)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*a^2*(3*A*b+B*a)*(-1/a*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*A*a^3*(-1/2/a*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x
+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2
))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-
b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2))/sin(1/2*d*x+1/2*c)/(-
2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.317 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=376

$$\frac{\left(16a^2A+54abB+33Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}-\frac{\left(16a^2A+54abB+33Ab^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}\right.}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] -1/24*(16*A*a^2+33*A*b^2+54*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+
c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/24*(16*A*a^3+59*A*a*b^2+66*B*a
^2*b+48*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/
(a+b*cos(d*x+c))^(1/2)+1/8*(20*A*a^2*b+5*A*b^3+8*B*a^3+30*B*a*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(
1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/
2)+1/3*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2*tan(d*x+c)/d+1/24*(16*A*a^2+
33*A*b^2+54*B*a*b)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/4*a*(3*A*b+2*B*a)*
sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 1.43, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2989, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(16a^2A+54abB+33Ab^2\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\left(16a^3A+66a^2bB+59aAb^2+48b^3B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{24d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] -((16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((16*a^
3*A + 59*a*A*b^2 + 66*a^2*b*B + 48*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) +
((20*a^2*A*b + 5*A*b^3 + 8*a^3*B + 30*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c + d
*x]]) + ((16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c +
d*x])/(24*d) + (a*(3*A*b + 2*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan
```

```
[c + d*x])/(4*d) + (a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2*Tan[c + d
*x])/(3*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
```

```
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = \frac{aA(a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\tan(c+dx)}{3d}$$

$$= \frac{a(3Ab+2aB)\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan}{4d}$$

$$= \frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+dx)}\,\text{ta}}{24d}$$

$$= \frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+dx)}\,\text{ta}}{24d}$$

$$= \frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+dx)}\,\text{ta}}{24d}$$

$$= -\frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+dx)}\,E}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(16a^2A+33Ab^2+54abB\right)\sqrt{a+b\cos(c+dx)}\,E}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.05, size = 486, normalized size = 1.29

$$\frac{\frac{8b\left(6a^2B+13aAb+24b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}\left(\left(8a^2A+27abB+\frac{33Ab^2}{2}\right)\sin(2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] ((8*b*(13*a*A*b + 6*a^2*B + 24*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*El
lipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(104*a^2
*A*b - 3*A*b^3 + 48*a^3*B + 126*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*
I)*(16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))
]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*Elliptic
```

```
E[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]
 + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]
], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]
*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) +
 4*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*(2*a*(13*A*b + 6*a*B)*Sin[c + d*
x] + (8*a^2*A + (33*A*b^2)/2 + 27*a*b*B)*Sin[2*(c + d*x)] + 8*a^2*A*Tan[c +
 d*x]))/(96*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x
)
```

maple [B] time = 4.61, size = 2438, normalized size = 6.48

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*b^2*(A*b+3*B*a)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-
b))^(1/2))+2*a^2*(3*A*b+B*a)*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*
```

```
b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-
b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2
*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*A*a^3*(-1/3/a*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2-1/24*(16*a^2+15*b^2)/
a^3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5/16*b^2/a^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+1/4/a
*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/16*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b
/(a-b))^(1/2)))+6*a*b*(A*b+B*a)*(-1/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1
```

```
/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/
a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))
^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^4,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.318 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=465

$$\frac{\left(36a^2A+104abB+59Ab^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{96d}+\frac{\left(128a^3B+284a^2Ab+264ab^2B+15A\right.}{192a}$$

```
[Out] -1/192*(284*A*a^2*b+15*A*b^3+128*B*a^3+264*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/
2))*(a+b*cos(d*x+c))^(1/2)/a/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/192*(356*A*
a^2*b+133*A*b^3+128*B*a^3+472*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos
(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/64*(48*A*a^4+120*A*a^2*b^2
-5*A*b^4+160*B*a^3*b+40*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*
x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+1/4*a*A*(a+b*cos(d*x+c))^(3/2
)*sec(d*x+c)^3*tan(d*x+c)/d+1/192*(284*A*a^2*b+15*A*b^3+128*B*a^3+264*B*a*b
^2)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d+1/96*(36*A*a^2+59*A*b^2+104*B*a*b
)*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/24*a*(11*A*b+8*B*a)*sec(
d*x+c)^2*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 1.85, antiderivative size = 465, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2989, 3047, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(284a^2Ab+128a^3B+264ab^2B+15Ab^3\right)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{192ad}+\frac{\left(356a^2Ab+128a^3B+472ab^2B+13\right.}{192d\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] -((284*a^2*A*b + 15*A*b^3 + 128*a^3*B + 264*a*b^2*B)*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(192*a*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) + ((356*a^2*A*b + 133*A*b^3 + 128*a^3*B + 472*a*b^2*B)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(192*d*Sq
rt[a + b*Cos[c + d*x]]) + ((48*a^4*A + 120*a^2*A*b^2 - 5*A*b^4 + 160*a^3*b*
B + 40*a*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*b)/(a + b)])/(64*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((284*a^2*A*b + 15*A
```

```
*b^3 + 128*a^3*B + 264*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(192
*a*d) + ((36*a^2*A + 59*A*b^2 + 104*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
 d*x]*Tan[c + d*x])/(96*d) + (a*(11*A*b + 8*a*B)*Sqrt[a + b*Cos[c + d*x]]*S
ec[c + d*x]^2*Tan[c + d*x])/(24*d) + (a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c
+ d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a + b\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^5(c + dx)\,dx = \frac{aA(a + b\cos(c + dx))^{3/2}\sec^3(c + dx)\tan(c + dx)}{4d}$$

$$= \frac{a(11Ab + 8aB)\sqrt{a + b\cos(c + dx)}\ \sec^2(c + dx)\,\mathrm{ta}}{24d}$$

$$= \frac{\left(36a^2A + 59Ab^2 + 104abB\right)\sqrt{a + b\cos(c + dx)}\ \mathrm{s}}{96d}$$

$$= \frac{\left(284a^2Ab + 15Ab^3 + 128a^3B + 264ab^2B\right)\sqrt{a + b}}{192ad}$$

$$= \frac{\left(284a^2Ab + 15Ab^3 + 128a^3B + 264ab^2B\right)\sqrt{a + b}}{192ad}$$

$$= \frac{\left(284a^2Ab + 15Ab^3 + 128a^3B + 264ab^2B\right)\sqrt{a + b}}{192ad}$$

$$= -\frac{\left(284a^2Ab + 15Ab^3 + 128a^3B + 264ab^2B\right)\sqrt{a +}}{192ad\sqrt{\frac{a+b\cos(c+dx}{a+b}}}$$

$$= -\frac{\left(284a^2Ab + 15Ab^3 + 128a^3B + 264ab^2B\right)\sqrt{a +}}{192ad\sqrt{\frac{a+b\cos(c+dx}{a+b}}}$$

Mathematica [C] time = 6.77, size = 729, normalized size = 1.57

$$\sqrt{a + b\cos(c + dx)}\left(\frac{1}{96}\sec^2(c + dx)\left(36a^2A\sin(c + dx) + 104abB\sin(c + dx) + 59Ab^2\sin(c + dx)\right) + \frac{1}{24}\sec^3(c\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] ((2*(144*a^3*A*b + 236*a*A*b^3 + 416*a^2*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (
```

```
2*(288*a^4*A + 436*a^2*A*b^2 - 45*A*b^4 + 832*a^3*b*B - 24*a*b^3*B)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt
[a + b*Cos[c + d*x]] - ((2*I)*(-284*a^2*A*b^2 - 15*A*b^4 - 128*a^3*b*B - 26
4*a*b^3*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(
a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSin
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*Ellip
ticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (
a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x
]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)
/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2))
)/(768*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]^3*(17*a*A*b*Sin[c +
d*x] + 8*a^2*B*Sin[c + d*x]))/24 + (Sec[c + d*x]^2*(36*a^2*A*Sin[c + d*x] +
 59*A*b^2*Sin[c + d*x] + 104*a*b*B*Sin[c + d*x]))/96 + (Sec[c + d*x]*(284*a
^2*A*b*Sin[c + d*x] + 15*A*b^3*Sin[c + d*x] + 128*a^3*B*Sin[c + d*x] + 264*
a*b^2*B*Sin[c + d*x]))/(192*a) + (a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/4))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^5, x
)
```

maple [B] time = 6.83, size = 3548, normalized size = 7.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b^3*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+6*a*b*(A*b+B*a)*(-1/2/a*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^
2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/
(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*a^2*(3*A*b+B*a)*(
-1/3/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3+5/12*b/a^2*cos(1/2*d*x+1/2*c)*(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2
*c)^2-1)^2-1/24*(16*a^2+15*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+5/48*b^
2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/3/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))-5/16*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+5/16/a^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*b^3+1/4/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+5/1
6*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
```

```
ticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*b^2*(A*b+3*B*a)*(-1/a*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/
2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2
/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))+2*A*a^3*(-1/4/a*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2-1)^4+7/24*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^3-1/96*(36*a^2
+35*b^2)/a^3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+5/192*b*(20*a^2+21*b^2)/a^4*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-7/96*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-
35/384*b^3/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+25/96/a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))-25/96*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+35/128/a^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-35/128*b^4/a^4*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))-3/8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a
-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/16/a^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2,(-2*b/(a-b))^(1/2))*b^2-35/128/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2
))*b^4))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^5, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{5/2}}{\cos(c+dx)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^5,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.319 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=320

$$-\frac{2\left(-24a^2B+28aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^3d}+\frac{2\left(-48a^3B+56a^2Ab-44ab^2B+63Ab^3\right)\sqrt{a+b}}{105b^4d\sqrt{\frac{a+b\cos(c+dx}{a+b}}}$$

```
[Out] -2/105*(28*A*a*b-24*B*a^2-25*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/d
+2/35*(7*A*b-6*B*a)*cos(d*x+c)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2/7*
B*cos(d*x+c)^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d+2/105*(56*A*a^2*b+63*A
*b^3-48*B*a^3-44*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)
/b^4/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/105*(56*A*a^3*b+49*A*a*b^3-48*B*a^4
-32*B*a^2*b^2-25*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))
^(1/2)/b^4/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2990, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-24a^2B+28aAb-25b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^3d}-\frac{2\left(56a^3Ab-32a^2b^2B-48a^4B+49aAb^3-25b^4\right.}{105b^4d\sqrt{a+b\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(56*a^2*A*b + 63*A*b^3 - 48*a^3*B - 44*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(105*b^4*d*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]) - (2*(56*a^3*A*b + 49*a*A*b^3 - 48*a^4*B - 32*a^2*b^2*B - 25*b^
4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(105*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(28*a*A*b - 24*a^2*B - 25*b^2
*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b^3*d) + (2*(7*A*b - 6*a*B)
*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*b^2*d) + (2*B*Cos[
c + d*x]^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2B\cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7bd} + \frac{2\int \frac{\cos(c+dx)\left(2aB+\frac{5}{2}bB\right.}{}}{}$$

$$= \frac{2(7Ab-6aB)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35b^2d} + \frac{2B\cos^2(c}{}$$

$$= -\frac{2\left(28aAb-24a^2B-25b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105b^3d} + \frac{2(7A}{}$$

$$= -\frac{2\left(28aAb-24a^2B-25b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105b^3d} + \frac{2(7A}{}$$

$$= -\frac{2\left(28aAb-24a^2B-25b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{105b^3d} + \frac{2(7A}{}$$

$$= \frac{2\left(56a^2Ab+63Ab^3-48a^3B-44ab^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{105b^4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.06, size = 230, normalized size = 0.72

$$\frac{2b\sin(c+dx)(a+b\cos(c+dx))\left(48a^2B+6b(7Ab-6aB)\cos(c+dx)-56aAb+15b^2B\cos(2(c+dx))+65b^2B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(14*a*A*b - 12*a^2*B + 25*b^2*B)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (-56*a^2*A*b - 63*A*b^3 + 48*a^3*B
 + 44*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])) + 2*b*(a + b*Cos[c + d*x])*(-56*a*A*b + 48*a
^2*B + 65*b^2*B + 6*b*(7*A*b - 6*a*B)*Cos[c + d*x] + 15*b^2*B*Cos[2*(c + d*
x)])*Sin[c + d*x])/(210*b^4*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)^4 + A\cos\left(dx+c\right)^3}{\sqrt{b\cos\left(dx+c\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^4 + A*cos(d*x + c)^3)/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

maple [B] time = 1.65, size = 1305, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b
^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^4+24*B*a*b^3-360*B*b^4
)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-28*A*a*b^3+168*A*b^4+24*B*a^2*b
^2-24*B*a*b^3+280*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(56*A*a^2*
b^2+14*A*a*b^3-42*A*b^4-48*B*a^3*b-12*B*a^2*b^2-44*B*a*b^3-80*B*b^4)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*a^3*b-49*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b^3+56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^3*b-56*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+
63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-63*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+48*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+32*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*
```

```
x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*B*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-48*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^4+48*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^3*b-44*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+4
4*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b^4/(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\,\left(A+B\,\cos(c+d\,x)\right)}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.320 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=246

$$-\frac{2\left(-8a^2B+10aAb-9b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\left(-8a^3B+10a^2Ab-7ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos}{a}}}{15b^3d\sqrt{a+b\cos(c+d}}$$

```
[Out] 2/15*(5*A*b-4*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2/5*B*cos(d*x+c)
*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d-2/15*(10*A*a*b-8*B*a^2-9*B*b^2)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b)
)^(1/2)+2/15*(10*A*a^2*b+5*A*b^3-8*B*a^3-7*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/
2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2990, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(10a^2Ab-8a^3B-7ab^2B+5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2B+10aAb-9b^2B\right)\sqrt{a+b\cos(c+}}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(10*a*A*b - 8*a^2*B - 9*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*
(10*a^2*A*b + 5*A*b^3 - 8*a^3*B - 7*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^3*d*Sqrt[a + b*Cos[c + d*
x]]) + (2*(5*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^2*d)
 + (2*B*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2B\cos(c+dx)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5bd} + \frac{2\int \frac{aB+\frac{3}{2}bB\cos(c+dx)+\frac{1}{2}(5}{\sqrt{a+b\cos}}}{5b}$$

$$= \frac{2(5Ab-4aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15b^2d} + \frac{2B\cos(c+dx)\sqrt{a+b\,c}}{5b}$$

$$= \frac{2(5Ab-4aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15b^2d} + \frac{2B\cos(c+dx)\sqrt{a+b\,c}}{5b}$$

$$= \frac{2(5Ab-4aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15b^2d} + \frac{2B\cos(c+dx)\sqrt{a+b\,c}}{5b}$$

$$= -\frac{2\left(10aAb-8a^2B-9b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(1\right.}{}$$

Mathematica [A] time = 0.91, size = 180, normalized size = 0.73

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(\left(8a^2B-10aAb+9b^2B\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)+b^2(2aB+5Ab)F\left(\frac{1}{2}(c\right.\right.}{15b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b^2*(5*A*b + 2*a*B)*EllipticF[(c + d
*x)/2, (2*b)/(a + b)] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*((a + b)*EllipticE[
(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])) + 2
*b*(a + b*Cos[c + d*x])*(5*A*b - 4*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x])/
(15*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos(dx+c)^3 + A\cos(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

maple [B] time = 1.73, size = 993, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b^3-4*B*a*b^2+24*B*b^3)*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*a*b^2-10*A*b^3+8*B*a^2*b+2*B*a*
b^2-6*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+5*A*b^3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^2*b+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a*b^2-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
3-7*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+8*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-8*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
```

```
x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*b^3)/b^3/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,\left(A+B\,\cos(c+d\,x)\right)}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.321 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=183

$$-\frac{2\left(-2a^2B+3aAb-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 2/3*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d+2/3*(3*A*b-2*B*a)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/
(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2
/3*(3*A*a*b-2*B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+
b))^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2968, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(-2a^2B+3aAb-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}+\frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(3*A*b - 2*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(3*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(3*a*A*b - 2*a^2*
B - b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*B*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(3*b*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{bB}{2}+\frac{1}{2}(3Ab-2aB)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{3b}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{(3Ab-2aB)\int \sqrt{a+b\cos(c+dx)}}{3b^2}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{\left((3Ab-2aB)\sqrt{a+b\cos(c+dx)}\right)}{3b^2\sqrt{\frac{a+b\cos(c-}{a+b}}}$$

$$= \frac{2(3Ab-2aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(3aAb-2a^2B\right.}{3b}$$

Mathematica [A] time = 0.69, size = 154, normalized size = 0.84

$$\frac{2\left(2a^2B-3aAb+b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2(a+b)(2aB-3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(a + b)*(-3*A*b + 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)] + 2*(-3*a*A*b + 2*a^2*B + b^2*B)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*B*(a + b*Co
s[c + d*x])*Sin[c + d*x])/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos(dx+c)^2 + A\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

maple [B] time = 1.75, size = 671, normalized size = 3.67

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-4B\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+3Aab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{a-b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1
/2*d*x+1/2*c)^5*b^2+3*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/
2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+3*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-2*B*cos(1/2*d*x+1/2*c)^3*a*b+6*B*cos(
1/2*d*x+1/2*c)^3*b^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c
)^2*b+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^
2-B*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+2*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*c
os(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a*b+2*B*cos(1/2*d*x+1/2*c)*a*b-2*B*cos(1/2*d*x+1/2*c)*b^2)/b^
2/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [B] time = 0.80, size = 199, normalized size = 1.09

$$\frac{2\,B\,\sin\left(c+d\,x\right)\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}{3\,b\,d}+\frac{2\,A\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\left(a+b\right)-a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\right)\,\sqrt{\frac{a+b\,\cos\left(c+d\,x\right)}{a+b}}}{b\,d\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}+2\,B\,\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*B*sin(c + d*x)*(a + b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*(ellipticE(c/2
 + (d*x)/2, (2*b)/(a + b))*(a + b) - a*ellipticF(c/2 + (d*x)/2, (2*b)/(a +
b)))*((a + b*cos(c + d*x))/(a + b))^(1/2))/(b*d*(a + b*cos(c + d*x))^(1/2))
 + (2*B*((a + b*cos(c + d*x))/(a + b))^(1/2)*(ellipticF(c/2 + (d*x)/2, (2*b
)/(a + b))*(2*a^2 + b^2) - 2*a*ellipticE(c/2 + (d*x)/2, (2*b)/(a + b))*(a +
 b)))/(3*b^2*d*(a + b*cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos{(c+dx)})\cos{(c+dx)}}{\sqrt{a+b\cos{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

3.322 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=130

$$\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))
/(a+b))^(1/2)+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b
))^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{B\int\sqrt{a+b\cos(c+dx)}\,dx}{b}+\frac{(Ab-aB)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b}$$

$$= \frac{\left(B\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left((Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \cdots}{b\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\cdots\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 3.29, size = 93, normalized size = 0.72

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((Ab-aB)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+B(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*B*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + (A*b - a*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(b*d*Sqrt[a + b*Cos[c + d*x]])

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c) + a), x)

maple [A] time = 1.22, size = 249, normalized size = 1.92

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\left(Ab\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)

[Out] -2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(A*b*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-B*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [B] time = 0.89, size = 135, normalized size = 1.04

$$\frac{2\,A\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\sqrt{\frac{a+b\,\cos(c+d\,x)}{a+b}}}{d\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}+\frac{2\,B\,\left(\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\,\left(a+b\right)-a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|\frac{2\,b}{a+b}\right)\right)\,\sqrt{\frac{a+b\,\cos(c+d\,x)}{a+b}}}{b\,d\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*A*ellipticF(c/2 + (d*x)/2, (2*b)/(a + b))*((a + b*cos(c + d*x))/(a + b))
^(1/2))/(d*(a + b*cos(c + d*x))^(1/2)) + (2*B*(ellipticE(c/2 + (d*x)/2, (2*
b)/(a + b))*(a + b) - a*ellipticF(c/2 + (d*x)/2, (2*b)/(a + b)))*((a + b*co
s(c + d*x))/(a + b))^(1/2))/(b*d*(a + b*cos(c + d*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

3.323 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*
x+c))^(1/2)+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2
)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3002, 2663, 2661, 2807, 2805}

$$\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = A\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx + B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{\left(A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}} + \frac{\left(B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos}{}}}}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\right.}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.20, size = 81, normalized size = 0.69

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(A\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+BF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(B*EllipticF[(c + d*x)/2, (2*b)/(a +
b)] + A*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]))/(d*Sqrt[a + b*Cos[c + d
*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

maple [A] time = 1.19, size = 194, normalized size = 1.64

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\,\left(A\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)*(A*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-B*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cos(c+d\,x)}{\cos(c+d\,x)\,\sqrt{a+b\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

3.324 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=216

$$-\frac{(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}+\frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\right.}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/d/((a+b*cos(d*x+c))/
(a+b))^(1/2)+A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/
(a+b*cos(d*x+c))^(1/2)-(A*b-2*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(
d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+A*(a+b*cos(d*x+c))^(1/2)*ta
n(d*x+c)/a/d
```

Rubi [A] time = 0.66, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3000, 3060, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{(Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}+\frac{A\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}+\frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\right.}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -((A*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)])) + (A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - ((A*
b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2
*b)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Sqrt[a + b*Cos[c + d*x]]*
Tan[c + d*x])/(a*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
```

```
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{ad} + \frac{\int \frac{\left(\frac{1}{2}(-Ab+2aB)-\frac{1}{2}Ab\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}}{a}$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{ad} - \frac{A\int\sqrt{a+b\cos(c+dx)}\,dx}{2a} - \int$$

$$= \frac{A\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{ad} + \frac{1}{2}A\int\frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx - $$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{a+b\cos(c+dx)}\,\tan(c}{ad}$$

$$= -\frac{A\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx\right.}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [C] time = 6.53, size = 320, normalized size = 1.48

$$\frac{\frac{2(4aB-3Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4A\tan(c+dx)\sqrt{a+b\cos(c+dx)}-\frac{2iA\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(\cdots\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((2*(-3*A*b + 4*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*A*Sqrt[-((b*(-1 +
 Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x
]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*Arc
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*
b*Sqrt[-(a + b)^(-1)]) + 4*A*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

maple [B] time = 1.94, size = 639, normalized size = 2.96

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\sqrt{-\frac{2b}{a-b}}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A*(-1/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*co
s(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1
/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-
b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^2\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sec^2(c + dx)}{\sqrt{a + b\cos(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/sqrt(a + b*cos(c + d*x)), x)
```

3.325 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=299

$$\frac{\left(4a^2A-4abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{(3Ab-}{}$$

```
[Out] 1/4*(3*A*b-4*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^2/d/(
(a+b*cos(d*x+c))/(a+b))^(1/2)-1/4*(A*b-4*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((
a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*A*a^2+3*A*b^
2-4*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2
/d/(a+b*cos(d*x+c))^(1/2)-1/4*(3*A*b-4*B*a)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+
c)/a^2/d+1/2*A*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d
```

Rubi [A] time = 0.95, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3000, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\left(4a^2A-4abB+3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{(3Ab-4aB)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{(3Ab-}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(4*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((A*b - 4*a*B)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*S
qrt[a + b*Cos[c + d*x]]) + ((4*a^2*A + 3*A*b^2 - 4*a*b*B)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[
a + b*Cos[c + d*x]]) - ((3*A*b - 4*a*B)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*
x])/(4*a^2*d) + (A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*a
*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
```

```
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{A\sqrt{a+b\cos(c+dx)}\,\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(\frac{1}{2}(-3Ab+4aB)+aA\cos(c\right.}{\sqrt{a}}}{}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos(c+dx)}}{}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos(c+dx)}}{}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\,\tan(c+dx)}{4a^2d} + \frac{A\sqrt{a+b\cos(c+dx)}}{}$$

$$= \frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{(3Ab-4aB)\sqrt{a-}}{}$$

$$= \frac{(3Ab-4aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{(Ab-4aB)\sqrt{\frac{a+b}{}}}{4ad\sqrt{a}}$$

Mathematica [C] time = 6.02, size = 420, normalized size = 1.40

$$\frac{\frac{2\left(8a^2A-12abB+9Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + 4\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}\,((4aB-3Ab)\cos(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((8*a*A*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2*A + 9*A*b^2 - 12*a*b*B)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sq
rt[a + b*Cos[c + d*x]] + ((2*I)*(3*A*b - 4*a*B)*Sqrt[-((b*(-1 + Cos[c + d*x
]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a -
 b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*C
os[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(
```

```
a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a +
 b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*(2*a*A + (-3*A*b + 4*a*B)*Cos[c + d
*x])*Sec[c + d*x]*Tan[c + d*x])/(16*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

maple [B] time = 3.28, size = 1182, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(-1/2/a
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1
)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1
```

```
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x
+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*
B*(-1/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^3\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**(1/2),x)
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/sqrt(a + b*cos(c + d*x)), x)
```

3.326 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=387

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-6a^2B+5aAb+b^2B\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d\left(a^2-b^2\right)}+\frac{2(}{}$$

```
[Out] 2*a*(A*b-B*a)*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+
2/15*(20*A*a^2*b-5*A*b^3-24*B*a^3+9*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1
/2)/b^3/(a^2-b^2)/d-2/5*(5*A*a*b-6*B*a^2+B*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*
cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d-2/15*(40*A*a^3*b-25*A*a*b^3-48*B*a^4+24*B
*a^2*b^2+9*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^4/(a^
2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/15*(40*A*a^2*b+5*A*b^3-48*B*a^3-1
2*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^4/d/
(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2989, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-6a^2B+5aAb+b^2B\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d\left(a^2-b^2\right)}+\frac{2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(40*a^3*A*b - 25*a*A*b^3 - 48*a^4*B + 24*a^2*b^2*B + 9*b^4*B)*Sqrt[a +
b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^4*(a^2 - b^2)*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(40*a^2*A*b + 5*A*b^3 - 48*a^3*B
 - 12*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/(15*b^4*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Cos[c
+ d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(20*
a^2*A*b - 5*A*b^3 - 24*a^3*B + 9*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(15*b^3*(a^2 - b^2)*d) - (2*(5*a*A*b - 6*a^2*B + b^2*B)*Cos[c + d*x]*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^2*(a^2 - b^2)*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\cos(c+dx)\left(-2a(Ab-aB)+\frac{1}{2}b(Ab-aB\right.}{\sqrt{a}}}{b}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\left(5aAb-6a^2B+b^2B\right)\cos(c+}{5b^2\left(\right.}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(20a^2Ab-5Ab^3-24a^3B+9\right.}{15b^3}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(20a^2Ab-5Ab^3-24a^3B+9\right.}{15b^3}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(20a^2Ab-5Ab^3-24a^3B+9\right.}{15b^3}$$

$$= -\frac{2\left(40a^3Ab-25aAb^3-48a^4B+24a^2b^2B+9b^4B\right)\sqrt{a+b\cos(c+dx)}}{15b^4\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.86, size = 304, normalized size = 0.79

$$\frac{\frac{30a^3b(aB-Ab)\sin(c+dx)}{b^2-a^2} + \frac{2b^2\left(12a^3B-10a^2Ab+3ab^2B-5Ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{(a-b)(a+b)} + \frac{2\left(48a^4B-40a^3Ab-24a^2b^2B+25aAb^3-9b^4B\right)\sqrt{\frac{a+b}{}}}{(a-b}}{15b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] ((2*b^2*(-10*a^2*A*b - 5*A*b^3 + 12*a^3*B + 3*a*b^2*B)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/((a - b)*(a + b)) + (
2*(-40*a^3*A*b + 25*a*A*b^3 + 48*a^4*B - 24*a^2*b^2*B - 9*b^4*B)*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a
*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/((a - b)*(a + b)) + (30*a^3*b*(-(A
*b) + a*B)*Sin[c + d*x])/(-a^2 + b^2) + 2*b*(5*A*b - 9*a*B)*(a + b*Cos[c +
d*x])*Sin[c + d*x] + 3*b^2*B*(a + b*Cos[c + d*x])*Sin[2*(c + d*x)])/(15*b^4
*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{4}+A\cos\left(dx+c\right)^{3}\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^4 + A*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x
)
```

maple [B] time = 5.10, size = 1308, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16/b*B*(-1/
10/b*cos(1/2*d*x+1/2*c)^3*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+8/b
^2*(A*b-B*a-3*B*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a
```

```
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/6/b*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))-1/12/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2/b^4*(A*a*b+2*A*b^2-B*a^2-2*B*a*b-
3*B*b^2)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)
/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2)))+2*(A*a^2*b+A*a*b^2+A*b^3-B*a^3-B*a^2*b-B*a*b^2-B*b
^3)/b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*a^3*(A*b-B*a)/b^4/sin(1/2*d*x+1
/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*
b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b
+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1
/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.327 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2B+6aAb-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-8a^3B+6a^2Ab\right.}{}$$

```
[Out] -2*a^2*(A*b-B*a)*sin(d*x+c)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2/3*B*si
n(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2/3*(6*A*a^2*b-3*A*b^3-8*B*a^3+5*B*a*
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)/d/((a+
b*cos(d*x+c))/(a+b))^(1/2)-2/3*(6*A*a*b-8*B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^
(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2988, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-8a^2B+6aAb-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2Ab-8a^3B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(6*a^2*A*b - 3*A*b^3 - 8*a^3*B + 5*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) - (2*(6*a*A*b - 8*a^2*B - b^2*B)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*d*Sqrt[a + b*Cos[c
 + d*x]]) - (2*a^2*(A*b - a*B)*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*
Cos[c + d*x]]) + (2*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}ab(Ab-aB)+\frac{1}{2}\left(2a^2-b^2\right)(Ab-aB)\cos(c}{\sqrt{a+b\cos(c+}}}{b^2\left(a^2-b\right.}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3b^2d}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3b^2d}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3b^2d}$$

$$= \frac{2\left(6a^2Ab-3Ab^3-8a^3B+5ab^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{3b^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.51, size = 189, normalized size = 0.72

$$\frac{2\left(b\sin(c+dx)\left(\frac{a\left(-4a^2B+3aAb+b^2B\right)}{b^2-a^2}+bB\cos(c+dx)\right)+\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a-b)\left(8a^2B-6aAb+b^2B\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(-8a^3B+6a^2Ab+5\right.\right.}{a-b}}{3b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (2*((Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((6*a^2*A*b - 3*A*b^3 - 8*a^3*B + 5
*a*b^2*B)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + (a - b)*(-6*a*A*b + 8*a^2
*B + b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b) + b*((a*(3*a*A*
b - 4*a^2*B + b^2*B))/(-a^2 + b^2) + b*B*Cos[c + d*x])*Sin[c + d*x]))/(3*b^
3*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^3+A\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/(b^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x
)
```

maple [B] time = 4.67, size = 954, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/b^3*(4*
B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*B*a*b-2*B*b^2)*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a*b-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+8*a^2*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^2*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)+2*a^2*(A*b-B*a)/b^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a
```

```
+b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.328 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=204

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-2a^2B+aAb+b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(Ab-2aB)\sqrt{\cdots}}{b^2d\cdots}$$

```
[Out] 2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*(A*a*b-2*B*
a^2+B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/
d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2*(A*b-2*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(
(a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2968, 3021, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-2a^2B+aAb+b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2(Ab-2aB)\sqrt{\cdots}}{b^2d\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a*A*b - 2*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*b)/(a + b)])/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
 (2*(A*b - 2*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{1}{2}b(Ab-aB)+\frac{1}{2}\left(aAb-2a^2B+b^2B\right)\cos(c+dx}{\sqrt{a+b\cos(c+dx)}}}{b\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{(Ab-2aB)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b^2} - \frac{(a}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left(\left(aAb-2a^2B+b^2B\right)\sqrt{a+b\cos(c}\right.}{b^2\left(a^2-b^2\right)\sqrt{\frac{a+}{}}}$$

$$= -\frac{2\left(aAb-2a^2B+b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(Ab-}{}$$

Mathematica [A] time = 0.82, size = 170, normalized size = 0.83

$$-\frac{2\left(\left(a^2-b^2\right)(2aB-Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-\left((a+b)\left(2a^2B-aAb-b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+\right.\right.}{b^2d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(-((a + b)*(-(a*A*b) + 2*a^2*B - b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]) + (a^2 - b^2)*(-(A*b) + 2*a*B)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a
*b*(-(A*b) + a*B)*Sin[c + d*x]))/((a - b)*b^2*(a + b)*d*Sqrt[a + b*Cos[c +
d*x]])
```

fricas [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^2+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)

maple [B] time = 4.19, size = 515, normalized size = 2.52

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(Ab\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)\cdots\right.}{b^2\sqrt{-\cdots}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*b*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)-2*a*(A*b-B*a)/b^2/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)\,(A+B\,\cos(c+dx))}{(a+b\,\cos(c+dx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.329 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx\right.}{bd\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -2*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*(A*b-B*a)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/(a^2-b^2)/d/((a+b*cos(d*x+c
))/(a+b))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/
2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx\right.}{bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b
)])/(b*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*B*Sqrt[(a + b
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a
+ b*Cos[c + d*x]]) - (2*(A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b
*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-aA+bB)-\frac{1}{2}(Ab-aB)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} + \frac{(Ab-aB)\int \sqrt{a+b\cos(}{b\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left((Ab-aB)\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(}{a+}}}{b\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{bd\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.56, size = 151, normalized size = 0.82

$$\frac{2\left(B\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+b(aB-Ab)\sin(c+dx)-\left((a+b)(aB-Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c\right.\right.\right.}{bd(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(-((a + b)*(-(A*b) + a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(
c + d*x)/2, (2*b)/(a + b)]) + (a^2 - b^2)*B*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(-(A*b) + a*B)*Sin[c + d*x]))
/((a - b)*b*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/(b^2*cos(d*x + c)^2
+ 2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [A] time = 3.45, size = 428, normalized size = 2.31

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)}{b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+2*(A*b-B*a)/b/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2
*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*
c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a
+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(3/2),x)
[Out] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.330 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c-\right.}{ad\sqrt{a+b\cos(c+d}}$$

```
[Out] 2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*(A*b-B*a)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/(a^2-b^2)/d/((a+b*cos(d*
x+c))/(a+b))^(1/2)+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b
))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [**A**] time = 0.51, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3000, 3059, 2655, 2653, 12, 2807, 2805}

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c-\right.}{ad\sqrt{a+b\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
b)])/(a*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*A*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sq
rt[a + b*Cos[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*S
qrt[a + b*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\left(\frac{1}{2}A\left(a^2-b^2\right)-\frac{1}{2}a(Ab-aB)\cos(c+dx)-\frac{1}{2}b(A}{\sqrt{a+b\cos(c+d}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int -\frac{Ab\left(a^2-b^2\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}\,dx}{ab\left(a^2-b^2\right)} - (Ab-a$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{A\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a} - ((Ab-aB)$$

$$= -\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b(Ab-aB)\,s}{a\left(a^2-b^2\right)d\sqrt{a+}}$$

$$= -\frac{2(Ab-aB)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi}{ad\sqrt{a+b}}$$

Mathematica [C] time = 3.98, size = 460, normalized size = 2.42

$$\frac{\cos(c+dx)(A\sec(c+dx)+B)\left(\frac{4b(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}} - \frac{\frac{2\left(2a^2A+abB-3Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{4a(aB-Ab)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}{\sqrt{a+b\cos(c+d}}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*(-(((4*a*(-(A*b) + a*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
+ d*x]] + (2*(2*a^2*A - 3*A*b^2 + a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*
```

```
I)*(A*b - a*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c
+ d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] +
 b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d
*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]))/((-a + b)*(a + b))) +
 (4*b*(A*b - a*B)*Sin[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]])))/(2
*a*d*(A + B*Cos[c + d*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [A] time = 3.07, size = 429, normalized size = 2.26

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2(-Ab+aB)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-\frac{2b(\cdots}{\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b+B*a
)/a/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2/a*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(a + b*cos(c + d*x))**(3/2), x)
```

3.331 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{b\left(a^2A+2abB-3Ab^2\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(3Ab-2aB)\sqrt{}}{a^2 d}$$

```
[Out] b*(A*a^2-3*A*b^2+2*B*a*b)*sin(d*x+c)/a^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
-(A*a^2-3*A*b^2+2*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/
a^2/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+A*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))
*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)-(3*A*b-2*B*a)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c)
,2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d
*x+c))^(1/2)+A*tan(d*x+c)/a/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.99, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3000, 3056, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(a^2A+2abB-3Ab^2\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(a^2A+2abB-3Ab^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{(3Ab-2aB)\sqrt{}}{a^2 d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] -(((a^2*A - 3*A*b^2 + 2*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]))
 + (A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) - ((3*A*b - 2*a*B)*Sqrt[(a + b*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b
*Cos[c + d*x]]) + (b*(a^2*A - 3*A*b^2 + 2*a*b*B)*Sin[c + d*x])/(a^2*(a^2 -
b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*Sqrt[a + b*Cos[c +
 d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
```

```
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{2}(-3Ab+2aB)+\frac{1}{2}Ab\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{a}$$

$$= \frac{b\left(a^2A-3Ab^2+2abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{2\int \left(- \right.}{}$$

$$= \frac{b\left(a^2A-3Ab^2+2abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} - \frac{2\int \left(\frac{1}{4}\right.}{}$$

$$= \frac{b\left(a^2A-3Ab^2+2abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{A\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{A\int -}{}$$

$$= -\frac{\left(a^2A-3Ab^2+2abB\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{b\left(a^2\right.}{a^2\left(\right.}$$

$$= -\frac{\left(a^2A-3Ab^2+2abB\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{A\sqrt{}}{}$$

Mathematica [C] time = 5.74, size = 482, normalized size = 1.59

$$\frac{4\tan(c+dx)\left(b\left(a^2A+2abB-3Ab^2\right)\cos(c+dx)+aA\left(a^2-b^2\right)\right)}{\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}} + \frac{\frac{2i\left(a^2A+2abB-3Ab^2\right)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\cos(c+dx)+1)}{a-b}}\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\right.\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (((-8*a*b*(-(A*b) + a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(-7*a^2*A*b + 9*A*b^3
 + 4*a^3*B - 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(a^2*A - 3*A*b
^2 + 2*a*b*B)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c
```

```
 + d*x]))/(a - b))]*Csc[c + d*x]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I
*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] -
b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*
x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]))/((a - b)*(a + b)) + (4
*(a*A*(a^2 - b^2) + b*(a^2*A - 3*A*b^2 + 2*a*b*B)*Cos[c + d*x])*Tan[c + d*x
])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))/(4*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x
)
```

maple [B] time = 4.30, size = 908, normalized size = 3.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)
*b/a^2/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(
a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*(-A*b+B
```

```
*a)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A/a*(-1/a*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*
x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a
-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*
b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+
a+b)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^2\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(a + b*cos(c + d*x))**(3/2),
x)
```

3.332 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=398

$$-\frac{(5Ab-4aB)\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{(5Ab-4aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}+\frac{\left(4a^2A-12abB+15Ab^2\right)\sqrt{\frac{a+b\cos(c+d}{a+b}}}{4a^3d\sqrt{a+b\cos(c+}}$$

```
[Out] -1/4*b*(7*A*a^2*b-15*A*b^3-4*B*a^3+12*B*a*b^2)*sin(d*x+c)/a^3/(a^2-b^2)/d/(
a+b*cos(d*x+c))^(1/2)+1/4*(7*A*a^2*b-15*A*b^3-4*B*a^3+12*B*a*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/((a+b*cos(d*x+c))/
(a+b))^(1/2)-1/4*(5*A*b-4*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))
/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*A*a^2+15*A*b^2-12*B*a*b)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*
c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*cos
(d*x+c))^(1/2)-1/4*(5*A*b-4*B*a)*tan(d*x+c)/a^2/d/(a+b*cos(d*x+c))^(1/2)+1/
2*A*sec(d*x+c)*tan(d*x+c)/a/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 1.43, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3000, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(7a^2Ab-4a^3B+12ab^2B-15Ab^3\right)\sin(c+dx)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(7a^2Ab-4a^3B+12ab^2B-15Ab^3\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}\right.}{4a^3d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]) - ((5*A*b - 4*a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((
4*a^2*A + 15*A*b^2 - 12*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticP
i[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^3*d*Sqrt[a + b*Cos[c + d*x]]) - (b*(
7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Sin[c + d*x])/(4*a^3*(a^2 - b^
2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b - 4*a*B)*Tan[c + d*x])/(4*a^2*d*Sq
rt[a + b*Cos[c + d*x]]) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sqrt[a + b*C
os[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
```

```
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{2}(-5Ab+4aB)+aA\cos(c+dx)+\frac{3}{2}Ab\cos^2(c+dx)\right)\sec}{(a+b\cos(c+dx))^{3/2}}}{2a}$$

$$= -\frac{(5Ab-4aB)\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{4}\left(4a^2A+15A\right.\right.}{}}{}$$

$$= -\frac{b\left(7a^2Ab-15Ab^3-4a^3B+12ab^2B\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{(5Ab-4aB)\tan(c}{4a^2d\sqrt{a+b\cos(c}}$$

$$= -\frac{b\left(7a^2Ab-15Ab^3-4a^3B+12ab^2B\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{(5Ab-4aB)\tan(c}{4a^2d\sqrt{a+b\cos(c}}$$

$$= -\frac{b\left(7a^2Ab-15Ab^3-4a^3B+12ab^2B\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{(5Ab-4aB)\tan(c}{4a^2d\sqrt{a+b\cos(c}}$$

$$= \frac{\left(7a^2Ab-15Ab^3-4a^3B+12ab^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{4a^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{\left(7a^2Ab-15Ab^3-4a^3B+12ab^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{4a^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 6.94, size = 678, normalized size = 1.70

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\sec(c+dx)(4aB\sin(c+dx)-7Ab\sin(c+dx))}{4a^3}+\frac{A\tan(c+dx)\sec(c+dx)}{2a^2}-\frac{2\left(ab^3B\sin(c+dx)-Ab^4\sin(c+dx)\right)}{a^3\left(a^2-b^2\right)(a+b\cos(c+dx))}\right)}{d} - \frac{2\left(4a^3Ab+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] -1/16*((2*(4*a^3*A*b - 20*a*A*b^3 + 16*a^2*b^2*B)*Sqrt[(a + b*Cos[c + d*x])
/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] +
 (2*(8*a^4*A + 29*a^2*A*b^2 - 45*A*b^4 - 28*a^3*b*B + 36*a*b^3*B)*Sqrt[(a +
 b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a
 + b*Cos[c + d*x]] - ((2*I)*(7*a^2*A*b^2 - 15*A*b^4 - 4*a^3*b*B + 12*a*b^3*
B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]
*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt
[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-
(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a
 + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(
a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqr
t[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(
2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(a^3*(
-a + b)*(a + b)*d) + (Sqrt[a + b*Cos[c + d*x]]*((Sec[c + d*x]*(-7*A*b*Sin[c
 + d*x] + 4*a*B*Sin[c + d*x]))/(4*a^3) - (2*(-(A*b^4*Sin[c + d*x]) + a*b^3*
B*Sin[c + d*x]))/(a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (A*Sec[c + d*x]*T
an[c + d*x])/(2*a^2)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x
)
```

maple [B] time = 5.08, size = 1564, normalized size = 3.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(A*b-B*a
)*b^2/a^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*(A*b
-B*a)/a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*A/a*(-1/2/a*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1
/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/
8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b
))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b
))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a
-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)+2*(-A*b+B*a)/a^
2*(-1/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b
)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\ \cos\left(c + d\,x\right)}{\cos\left(c + d\,x\right)^3\,\left(a + b\ \cos\left(c + d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B\cos\left(c + dx\right)\right)\sec^3\left(c + dx\right)}{\left(a + b\cos\left(c + dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(a + b*cos(c + d*x))**(3/2),
x)
```

3.333 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=550

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(-8a^3B+5a^2Ab+12ab^2B-9Ab^3\right)\sin(c+dx)\cos^2(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-48a^4\right.}{}$$

```
[Out] 2/3*a*(A*b-B*a)*cos(d*x+c)^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2
)+2/3*a*(5*A*a^2*b-9*A*b^3-8*B*a^3+12*B*a*b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/
(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/15*(40*A*a^4*b-65*A*a^2*b^3+5*A*b^5-
64*B*a^5+98*B*a^3*b^2-14*B*a*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^4/(a^
2-b^2)^2/d-2/15*(30*A*a^3*b-50*A*a*b^3-48*B*a^4+71*B*a^2*b^2-3*B*b^4)*cos(d
*x+c)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d-2/15*(80*A*a^5*b-
140*A*a^3*b^3+40*A*a*b^5-128*B*a^6+212*B*a^4*b^2-55*B*a^2*b^4-9*B*b^6)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^5/(a^2-b^2)^2/d/((a+b*cos(d
*x+c))/(a+b))^(1/2)+2/15*(80*A*a^4*b-80*A*a^2*b^3-5*A*b^5-128*B*a^5+116*B*a
^3*b^2+17*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2
)/b^5/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 1.19, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2989, 3047, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(5a^2Ab-8a^3B+12ab^2B-9Ab^3\right)\sin(c+dx)\cos^2(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(30a^3Ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(80*a^5*A*b - 140*a^3*A*b^3 + 40*a*A*b^5 - 128*a^6*B + 212*a^4*b^2*B -
55*a^2*b^4*B - 9*b^6*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)])/(15*b^5*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
(2*(80*a^4*A*b - 80*a^2*A*b^3 - 5*A*b^5 - 128*a^5*B + 116*a^3*b^2*B + 17*a*
b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a +
 b)])/(15*b^5*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Co
s[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) +
 (2*a*(5*a^2*A*b - 9*A*b^3 - 8*a^3*B + 12*a*b^2*B)*Cos[c + d*x]^2*Sin[c + d
*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(40*a^4*A*b - 65
```

```
*a^2*A*b^3 + 5*A*b^5 - 64*a^5*B + 98*a^3*b^2*B - 14*a*b^4*B)*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(15*b^4*(a^2 - b^2)^2*d) - (2*(30*a^3*A*b - 50*a*A
*b^3 - 48*a^4*B + 71*a^2*b^2*B - 3*b^4*B)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d
*x]]*Sin[c + d*x])/(15*b^3*(a^2 - b^2)^2*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
```

```
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos^2(c+dx)\left(-3a(Ab-aB)+\frac{3}{2}b(Ab-\right.}{(}}{}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(5a^2Ab-9Ab^3-8a^3B+1\right.}{3b^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(5a^2Ab-9Ab^3-8a^3B+1\right.}{3b^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(5a^2Ab-9Ab^3-8a^3B+1\right.}{3b^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(5a^2Ab-9Ab^3-8a^3B+1\right.}{3b^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2a(Ab-aB)\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(5a^2Ab-9Ab^3-8a^3B+1\right.}{3b^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{2\left(80a^5Ab-140a^3Ab^3+40aAb^5-128a^6B+212a^4b^2B-55a^2b^4B-\right.}{15b^5\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+}{a+b}}}$$

Mathematica [A] time = 4.43, size = 372, normalized size = 0.68

$$\frac{b\left(\frac{10a^4(aB-Ab)\sin(c+dx)}{a^2-b^2} - \frac{10a^3\left(11a^3B-8a^2Ab-15ab^2B+12Ab^3\right)\sin(c+dx)(a+b\cos(c+dx))}{\left(a^2-b^2\right)^2} + 2(5Ab-14aB)\sin(c+dx)(a+b\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] ((-2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(20*a^4*A*b - 35*a^2*A*b^3 -
 5*A*b^5 - 32*a^5*B + 44*a^3*b^2*B + 8*a*b^4*B)*EllipticF[(c + d*x)/2, (2*b
```

```
)/(a + b)] - (-80*a^5*A*b + 140*a^3*A*b^3 - 40*a*A*b^5 + 128*a^6*B - 212*a^
4*b^2*B + 55*a^2*b^4*B + 9*b^6*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b)) + b*
((10*a^4*(-(A*b) + a*B)*Sin[c + d*x])/(a^2 - b^2) - (10*a^3*(-8*a^2*A*b + 1
2*A*b^3 + 11*a^3*B - 15*a*b^2*B)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(a^2 -
b^2)^2 + 2*(5*A*b - 14*a*B)*(a + b*Cos[c + d*x])^2*Sin[c + d*x] + 3*b*B*(a
+ b*Cos[c + d*x])^2*Sin[2*(c + d*x)]))/(15*b^5*d*(a + b*Cos[c + d*x])^(3/2)
)
```

fricas [F] time = 1.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{5}+A\cos\left(dx+c\right)^{4}\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^5 + A*cos(d*x + c)^4)*sqrt(b*cos(d*x + c) + a)/(b^
3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{4}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x
)
```

maple [B] time = 8.74, size = 1746, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16/b^2*B*(-
1/10/b*cos(1/2*d*x+1/2*c)^3*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
```

```
)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+8
/b^3*(A*b-2*B*a-3*B*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*
b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/6/b*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))-1/12/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d
*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2/b^5*(2*A*a*b+2*A*b^2-3*B*a^2-
4*B*a*b-3*B*b^2)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+2*(3*A*a^2*b+2*A*a*b^2+A*b^3-4*B*a^3-3*B*a
^2*b-2*B*a*b^2-B*b^3)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*
c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*a^3/b^5*(4*A*
b-5*B*a)/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*a^4*(
A*b-B*a)/b^5*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8
/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/
2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*
a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(
a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^4 \left(A+B\cos(c+dx)\right)}{\left(a+b\cos(c+dx)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.334 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=413

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-2a^2B+aAb+b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3b^3d\left(a^2-b^2\right)}-\frac{2a^2\left(-6a^3B+3a\right.}{3b^3d\left(a\right.}$$

```
[Out] 2/3*a*(A*b-B*a)*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2
)-2/3*a^2*(3*A*a^2*b-7*A*b^3-6*B*a^3+10*B*a*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2
/d/(a+b*cos(d*x+c))^(1/2)-2/3*(A*a*b-2*B*a^2+B*b^2)*sin(d*x+c)*(a+b*cos(d*x
+c))^(1/2)/b^3/(a^2-b^2)/d+2/3*(8*A*a^4*b-15*A*a^2*b^3+3*A*b^5-16*B*a^5+28*
B*a^3*b^2-8*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^4/
(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(8*A*a^3*b-9*A*a*b^3-16*B*
a^4+16*B*a^2*b^2+B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))
^(1/2)/b^4/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2989, 3031, 3023, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2a^2\left(3a^2Ab-6a^3B+10ab^2B-7Ab^3\right)\sin(c+dx)}{3b^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2\left(-2a^2B+aAb+b^2\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(8*a^4*A*b - 15*a^2*A*b^3 + 3*A*b^5 - 16*a^5*B + 28*a^3*b^2*B - 8*a*b^4*
B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(
a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(8*a^3*A*b - 9*a*A*
b^3 - 16*a^4*B + 16*a^2*b^2*B + b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]]) + (2*a*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*
d*(a + b*Cos[c + d*x])^(3/2)) - (2*a^2*(3*a^2*A*b - 7*A*b^3 - 6*a^3*B + 10*
a*b^2*B)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (
2*(a*A*b - 2*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(
a^2 - b^2)*d)
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos(c+dx)\left(-2a(Ab-aB)+\frac{3}{2}b(Ab-aB\right.}{(a+b}}{3b}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2\left(3a^2Ab-7Ab^3-6a^3B+1\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\,c}}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2\left(3a^2Ab-7Ab^3-6a^3B+1\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\,c}}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2\left(3a^2Ab-7Ab^3-6a^3B+1\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\,c}}$$

$$= \frac{2a(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2a^2\left(3a^2Ab-7Ab^3-6a^3B+1\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\,c}}$$

$$= \frac{2\left(8a^4Ab-15a^2Ab^3+3Ab^5-16a^5B+28a^3b^2B-8ab^4B\right)\sqrt{a+b\cos(c}}{3b^4\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 2.89, size = 334, normalized size = 0.81

$$\frac{2\left(\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(b^2\left(-4a^4B+2a^3Ab+7a^2b^2B-6aAb^3+b^4B\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-\left(16a^5B-8a^4Ab-28a^3b^2B+15a^2Ab^3+8ab^4B-3Ab^5\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{}{a}\right.\right.\right.}{(a-b)^2(a+b)}}{3b^4d(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(2*a^3*A*b - 6*a*A*b^3 - 4*a
^4*B + 7*a^2*b^2*B + b^4*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - (-8*a^4
*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*((a +
b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])))/((a - b)^2*(a + b)) + (b*(-8*a^5*A*b + 16*a^3*A*b^3 + 16*a^6*B -
 25*a^4*b^2*B + b^6*B + 2*a*b*(-5*a^3*A*b + 9*a*A*b^3 + 10*a^4*B - 16*a^2*b
```

```
^2*B + 2*b^4*B)*Cos[c + d*x] + (-(a^2*b) + b^3)^2*B*Cos[2*(c + d*x)])*Sin[c
 + d*x])/(2*(a^2 - b^2)^2)))/(3*b^4*d*(a + b*Cos[c + d*x])^(3/2))
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{4}+A\cos\left(dx+c\right)^{3}\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^4 + A*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a)/(b^
3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x
)
```

maple [B] time = 7.26, size = 1389, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/b^4*(4*
B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*B*a*b-2*B*b^2)*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-9*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a
-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a*b-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2+17*a^2
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
```

```
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^2*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a^2+8*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*a^2/b^4*(3*A*b-4*B*a)/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^
2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2)-2*a^3*(A*b-B*a)/b^4*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2
*c)^2+1/2/b*(a-b))^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x
+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*
a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/
(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/
(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(
EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b
)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.335 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=331

$$-\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(-5a^3B+2a^2Ab+9ab^2B-6Ab^3\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-8a^3B+2a^2Ab+9\right.}{3b^3d}$$

```
[Out] -2/3*a^2*(A*b-B*a)*sin(d*x+c)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*a*
(2*A*a^2*b-6*A*b^3-5*B*a^3+9*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos
(d*x+c))^(1/2)-2/3*(2*A*a^3*b-6*A*a*b^3-8*B*a^4+15*B*a^2*b^2-3*B*b^4)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d/((a+b*cos(d*
x+c))/(a+b))^(1/2)+2/3*(2*A*a^2*b-3*A*b^3-8*B*a^3+9*B*a*b^2)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(
a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/(a^2-b^2)/d/(a+b*cos(d*x+c)
)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2988, 3021, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(2a^2Ab-5a^3B+9ab^2B-6Ab^3\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2Ab-8a^3B+9ab^2\right.}{3b^3d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(2*a^3*A*b - 6*a*A*b^3 - 8*a^4*B + 15*a^2*b^2*B - 3*b^4*B)*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)^2*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2*A*b - 3*A*b^3 - 8*a^3*B + 9
*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(
a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*(A*b - a*B
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*a*(2*
a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2
*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3}{2}ab(Ab-aB)+\frac{1}{2}\left(2a^2-3b^2\right)(Ab-aB)}{(a+b\cos(}}{3b^2\left(a^2\right.}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B+9a\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{c}}}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B+9a\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{c}}}$$

$$= -\frac{2a^2(Ab-aB)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B+9a\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{c}}}$$

$$= -\frac{2\left(2a^3Ab-6aAb^3-8a^4B+15a^2b^2B-3b^4B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}\right.}{3b^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 2.33, size = 274, normalized size = 0.83

$$\frac{2\left(\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(b^2\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(8a^4B-2a^3Ab-15a^2b^2B+6aAb^3+3b^4B\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.\right.\right.}{(a-b)^2(a+b)}\right.}{3b^3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(b^2*(a^2*A*b + 3*A*b^3 + 2*a^3*B
 - 6*a*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + (-2*a^3*A*b + 6*a*A*b
^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*((a + b)*EllipticE[(c + d*x)/2, (2*b
)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*(a + b))
 - (a*b*(a*(-(a^2*A*b) + 5*A*b^3 + 4*a^3*B - 8*a*b^2*B) + b*(-2*a^2*A*b + 6
*A*b^3 + 5*a^3*B - 9*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(
3*b^3*d*(a + b*Cos[c + d*x])^(3/2))
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)/(b^
3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x
)
```

maple [B] time = 5.83, size = 950, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*s
in(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(A*b*Elliptic
F(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3*B*EllipticF(cos(1/2*d*x+1/2*c),(
```

```
-2*b/(a-b))^(1/2))*a+B*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-B
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)-2*a/b^3*(2*A*b-3*B*a)/
sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*
d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*a^2*(A*b-B*a)/
b^3*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*b*sin(
1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2
*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b
^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-
2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1
/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.336 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=307

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(2a^2B+aAb-3b^2B\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^3B+a^2Ab-6\right.}{3bd\left(a^2-b^2\right)^{2}}$$

```
[Out] 2/3*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*(A*a^2*
b+3*A*b^3+2*B*a^3-6*B*a*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1
/2)-2/3*(A*a^2*b+3*A*b^3+2*B*a^3-6*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b
*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(A*
a*b+2*B*a^2-3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1
/2)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2968, 3021, 2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(a^2Ab+2a^3B-6ab^2B+3Ab^3\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(2a^2B+aAb-3b^2B\right)\sqrt{\frac{a+b}{}}}{3b^2d\left(a^2-b^2\right)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]) + (2*(a*A*b + 2*a^2*B - 3*b^2*B)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqr
t[a + b*Cos[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)) + (2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*
Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \int \frac{A\cos(c+dx)+B\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3}{2}b(Ab-aB)-\frac{1}{2}\left(aAb+2a^2B-3b^2B\right)\cos(c+}{(a+b\cos(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2Ab+3Ab^3+2a^3B-6ab^2B\right)s}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c-}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2Ab+3Ab^3+2a^3B-6ab^2B\right)s}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c-}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2Ab+3Ab^3+2a^3B-6ab^2B\right)s}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c-}}$$

$$= -\frac{2\left(a^2Ab+3Ab^3+2a^3B-6ab^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 2.03, size = 224, normalized size = 0.73

$$\frac{2\left(\frac{b\sin(c+dx)\left(b\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)\cos(c+dx)+a\left(a^3B+2a^2Ab-5ab^2B+2Ab^3\right)\right)}{\left(a^2-b^2\right)^2} - \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(2a^3B+a^2Ab-6ab^2B+3Ab^3\right)E\left(\frac{1}{2}(c+dx)|\right.\right.}{(a-b)^2}\right.}{3b^2d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((a^2*A*b + 3*A*b^3 + 2*a^3*B -
 6*a*b^2*B)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - (a - b)*(a*A*b + 2*a^2*
B - 3*b^2*B)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2) + (b*(a*(2*
a^2*A*b + 2*A*b^3 + a^3*B - 5*a*b^2*B) + b*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6
*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^2*d*(a + b*Cos[c
 + d*x])^(3/2))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/(b^3*
cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 5.28, size = 860, normalized size = 2.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))+2/b^2*(A*b-2*B*a)/sin(1/2*d*x+1/2*c)^2/(-2*s
in(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*
```

```
x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*a*(A*b-B*a)/b^2*(1/6/b/(a-b)/(a+b)*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+
b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2
*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*
b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+
1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*
x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.337 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=275

$$-\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{3bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+}}$$

```
[Out] -2/3*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-2/3*(4*A*a*b-B
*a^2-3*B*b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(4*A*a*b-
B*a^2-3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/(a^2-b^2
)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2
))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2(-B)+4aAb-3b^2B\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{3bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(4*a*A*b - a^2*B - 3*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*b)/(a + b)])/(3*b*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
) - (2*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(A*b -
a*B)*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*(4*a*A
*b - a^2*B - 3*b^2*B)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d
*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}(aA-bB)+\frac{1}{2}(Ab-aB)\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{4\int \ldots}{\ldots}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{(Ab \ldots}{\ldots}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{((4a \ldots}{\ldots}$$

$$= \frac{2\left(4aAb-a^2B-3b^2B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(Ab-aB)\sqrt{\frac{a+b\,c\ldots}{}}}{3b\left(a^2-b^2\right)\ldots}$$

Mathematica [A] time = 1.65, size = 193, normalized size = 0.70

$$\frac{2\left(\frac{\sin(c+dx)\left(2a^3B+b\left(a^2B-4aAb+3b^2B\right)\cos(c+dx)-5a^2Ab+2ab^2B+Ab^3\right)}{\left(a^2-b^2\right)^2} - \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(a^2B-4aAb+3b^2B\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-(a-b)(aB-Ab)F\ldots}{b(a-b)^2}\right.}{3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((-4*a*A*b + a^2*B + 3*b^2*B)*E
llipticE[(c + d*x)/2, (2*b)/(a + b)] - (a - b)*(-(A*b) + a*B)*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)]))/((a - b)^2*b)) + ((-5*a^2*A*b + A*b^3 + 2*a^3*B
+ 2*a*b^2*B + b*(-4*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(a
^2 - b^2)^2))/(3*d*(a + b*Cos[c + d*x])^(3/2))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/(b^3*cos(d*x + c)^3
+ 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 5.00, size = 750, normalized size = 2.73

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a+b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2B\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\cdots\right.\right.}{a-b}\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b/sin(1
/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*d*x+1
/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2
*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2*(A*b-B*a)/b*(1/6/b/
(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*b*sin(1/2*d*x+1/2
*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
```

```
1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(5/2),x)
[Out] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.338 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=349

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2A\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx\right.}{a^2d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*b*(7*A*
a^2*b-3*A*b^3-4*B*a^3)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-
2/3*(7*A*a^2*b-3*A*b^3-4*B*a^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))
^(1/2)/a^2/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(A*b-B*a)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/(a^2-b^2)/d/(a+b*cos
(d*x+c))^(1/2)+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
Pi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(
1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3000, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2b\left(7a^2Ab-4a^3B-3Ab^3\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2(Ab-aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+d\right.}{3ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(7*a^2*A*b - 3*A*b^3 - 4*a^3*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(
a + b)]) + (2*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*
A*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(7*a^2*A*b - 3*A*b^3 - 4*
a^3*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
```

```
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\left(\frac{3}{2}A\left(a^2-b^2\right)-\frac{3}{2}a(Ab-aB)\cos(c+dx)+\right.}{(a+b\cos(c}}{3a\left(a^2-\right.}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(7a^2Ab-3Ab^3-4a^3B\right)\sin(c}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(7a^2Ab-3Ab^3-4a^3B\right)\sin(c}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(7a^2Ab-3Ab^3-4a^3B\right)\sin(c}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+}}$$

$$= -\frac{2\left(7a^2Ab-3Ab^3-4a^3B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{}{3a}$$

$$= -\frac{2\left(7a^2Ab-3Ab^3-4a^3B\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2(}{}$$

Mathematica [C] time = 6.81, size = 743, normalized size = 2.13

$$\frac{\cos(c+dx)\sqrt{a+b\cos(c+dx)}\,(A\sec(c+dx)+B)\left(-\frac{2\left(abB\sin(c+dx)-Ab^2\sin(c+dx)\right)}{3a\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{2\left(4a^3bB\sin(c+dx)-7a^2Ab^2\sin(c+dx)+\right.}{3a^2\left(a^2-b^2\right)^2(a+b\cos(c+d}\right.}{d(A+B\cos(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]*(B + A*Sec[c + d*x])*((2*(-12*a^3*A*b + 4*a*A*b^3 + 6*a^4*B +
 2*a^2*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^4*A - 19*a^2*A*b^2 + 9*A*b^
4 + 4*a^3*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2
, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-7*a^2*A*b^2 + 3*A*b^4
 + 4*a^3*b*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x]
```

```
)/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)
^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*Arc
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*El
lipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]]
, (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c +
d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])
^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^
2))))/(6*a^2*(a - b)^2*(a + b)^2*d*(A + B*Cos[c + d*x])) + (Cos[c + d*x]*Sq
rt[a + b*Cos[c + d*x]]*(B + A*Sec[c + d*x])*((-2*(-(A*b^2*Sin[c + d*x]) + a
*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(-7*a^2*A
*b^2*Sin[c + d*x] + 3*A*b^4*Sin[c + d*x] + 4*a^3*b*B*Sin[c + d*x]))/(3*a^2*
(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/(d*(A + B*Cos[c + d*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [B] time = 5.60, size = 854, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*A*b/a^2/
sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*
d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1
```

```
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*A/a^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2,(-2*b/(a-b))^(1/2))+2*(-A*b+B*a)/a*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*co
s(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^
(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^
2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos(c+d\,x)}{\cos(c+d\,x)\,{\left(a+b\cos(c+d\,x)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.339 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=437

$$-\frac{(5Ab-2aB)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^3d\sqrt{a+b\cos(c+dx)}}+\frac{b\left(3a^2A+2abB-5Ab^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\left(3a^2A+2abB-5Ab^2\right.}{3a^2d\left(a^2-b\right.}$$

```
[Out] 1/3*b*(3*A*a^2-5*A*b^2+2*B*a*b)*sin(d*x+c)/a^2/(a^2-b^2)/d/(a+b*cos(d*x+c))
^(3/2)+1/3*b*(3*A*a^4-26*A*a^2*b^2+15*A*b^4+14*B*a^3*b-6*B*a*b^3)*sin(d*x+c
)/a^3/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-1/3*(3*A*a^4-26*A*a^2*b^2+15*A*b
^4+14*B*a^3*b-6*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/
a^3/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/3*(3*A*a^2-5*A*b^2+2*B*a
*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/(a^2-b^2)
/d/(a+b*cos(d*x+c))^(1/2)-(5*A*b-2*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b
*cos(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*cos(d*x+c))^(1/2)+A*tan(d*x+c)/a/d/(a+
b*cos(d*x+c))^(3/2)
```

Rubi [A] time = 1.48, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3000, 3056, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(-26a^2Ab^2+3a^4A+14a^3bB-6ab^3B+15Ab^4\right)\sin(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{b\left(3a^2A+2abB-5Ab^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\left(3a^2A+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] -((3*a^4*A - 26*a^2*A*b^2 + 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Sqrt[a + b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*a^3*(a^2 - b^2)^2*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*a^2*A - 5*A*b^2 + 2*a*b*B)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*a^2*
(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((5*A*b - 2*a*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^3*d*Sqrt[a
 + b*Cos[c + d*x]]) + (b*(3*a^2*A - 5*A*b^2 + 2*a*b*B)*Sin[c + d*x])/(3*a^2
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (b*(3*a^4*A - 26*a^2*A*b^2 + 1
```

```
5*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt
[a + b*Cos[c + d*x]]) + (A*Tan[c + d*x])/(a*d*(a + b*Cos[c + d*x])^(3/2))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
```

```
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{1}{2}(-5Ab+2aB)+\frac{3}{2}Ab\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx}{a}$$

$$= \frac{b\left(3a^2A-5Ab^2+2abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{A\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} + \frac{2\int}{}$$

$$= \frac{b\left(3a^2A-5Ab^2+2abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{b\left(3a^4A-26a^2Ab^2+15Ab^4\right.}{3a^3\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{b\left(3a^2A-5Ab^2+2abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{b\left(3a^4A-26a^2Ab^2+15Ab^4\right.}{3a^3\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{b\left(3a^2A-5Ab^2+2abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{b\left(3a^4A-26a^2Ab^2+15Ab^4\right.}{3a^3\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{\left(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B\right)\sqrt{a+b\cos(c+dx)}\,E}{3a^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{\left(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B\right)\sqrt{a+b\cos(c+dx)}\,E}{3a^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 7.26, size = 750, normalized size = 1.72

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{A\tan(c+dx)}{a^3} + \frac{2\left(ab^2B\sin(c+dx)-Ab^3\sin(c+dx)\right)}{3a^2\left(a^2-b^2\right)(a+b\cos(c+dx))^2} + \frac{2\left(7a^3b^2B\sin(c+dx)-10a^2Ab^3\sin(c+dx)-3ab^4B\sin(c+dx)+6Ab^5\right.}{3a^3\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(a + b*Cos[c + d*x])^(5/2),
x]

[Out] ((2*(36*a^3*A*b^2 - 20*a*A*b^4 - 24*a^4*b*B + 8*a^2*b^3*B)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
```

```
+ d*x]] + (2*(-33*a^4*A*b + 86*a^2*A*b^3 - 45*A*b^5 + 12*a^5*B - 38*a^3*b^2
*B + 18*a*b^4*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(-3*a^4*A*b + 26*a^2*
A*b^3 - 15*A*b^5 - 14*a^3*b^2*B + 6*a*b^4*B)*Sqrt[(b - b*Cos[c + d*x])/(a +
 b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*El
lipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
 - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c +
d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-
(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c
 + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c +
d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(12*a^3*(-a + b)^2*(a + b)^2*d) + (Sqrt
[a + b*Cos[c + d*x]]*((2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x]))/(3
*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-10*a^2*A*b^3*Sin[c + d*x] +
 6*A*b^5*Sin[c + d*x] + 7*a^3*b^2*B*Sin[c + d*x] - 3*a*b^4*B*Sin[c + d*x]))
/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (A*Tan[c + d*x])/a^3))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x
)
```

maple [B] time = 7.93, size = 1341, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*(2*A*b-
B*a)/a^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2
*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b
/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*(-2*A
*b+B*a)/a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a
-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2*(A*b-B*a)*b/a^2*(1/6/b/(
a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*b*sin(1/2*d*x+1/2*
c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2/a^2*A*(-1/a*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*
x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2
*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/
a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d
*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos(c+d\,x)}{\cos(c+d\,x)^2\,(a+b\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2)),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(a+b*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.340 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=532

$$-\frac{(7Ab-4aB)\tan(c+dx)}{4a^2d(a+b\cos(c+dx))^{3/2}}+\frac{\left(4a^2A-20abB+35Ab^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^4d\sqrt{a+b\cos(c+dx)}}-\frac{b\left(-12a^3B+27a^2A\right.}{12a^3d\left(a^2-\right.}$$

```
[Out] -1/12*b*(27*A*a^2*b-35*A*b^3-12*B*a^3+20*B*a*b^2)*sin(d*x+c)/a^3/(a^2-b^2)/
d/(a+b*cos(d*x+c))^(3/2)-1/12*b*(33*A*a^4*b-170*A*a^2*b^3+105*A*b^5-12*B*a^
5+104*B*a^3*b^2-60*B*a*b^4)*sin(d*x+c)/a^4/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(
1/2)+1/12*(33*A*a^4*b-170*A*a^2*b^3+105*A*b^5-12*B*a^5+104*B*a^3*b^2-60*B*a
*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/(
(a+b*cos(d*x+c))/(a+b))^(1/2)-1/12*(27*A*a^2*b-35*A*b^3-12*B*a^3+20*B*a*b^2
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^3/(a^2-b^2)/d
/(a+b*cos(d*x+c))^(1/2)+1/4*(4*A*a^2+35*A*b^2-20*B*a*b)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+
b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^4/d/(a+b*cos(d*x+c))^(1/2)-1/4*
(7*A*b-4*B*a)*tan(d*x+c)/a^2/d/(a+b*cos(d*x+c))^(3/2)+1/2*A*sec(d*x+c)*tan(
d*x+c)/a/d/(a+b*cos(d*x+c))^(3/2)
```

Rubi [A] time = 1.93, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3000, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b\left(-170a^2Ab^3+33a^4Ab+104a^3b^2B-12a^5B-60ab^4B+105Ab^5\right)\sin(c+dx)}{12a^4d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{b\left(27a^2Ab-12a^3B+20ab^2B\right.}{12a^3d\left(a^2-b^2\right)(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((33*a^4*A*b - 170*a^2*A*b^3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*
b^4*B)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(12*
a^4*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - ((27*a^2*A*b - 35
*A*b^3 - 12*a^3*B + 20*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/(12*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x
]]) + ((4*a^2*A + 35*A*b^2 - 20*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^4*d*Sqrt[a + b*Cos[c + d*x]]
```

```
) - (b*(27*a^2*A*b - 35*A*b^3 - 12*a^3*B + 20*a*b^2*B)*Sin[c + d*x])/(12*a^
3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (b*(33*a^4*A*b - 170*a^2*A*b^
3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B - 60*a*b^4*B)*Sin[c + d*x])/(12*a^
4*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - ((7*A*b - 4*a*B)*Tan[c + d*x]
)/(4*a^2*d*(a + b*Cos[c + d*x])^(3/2)) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a
*d*(a + b*Cos[c + d*x])^(3/2))
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{A\sec(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{1}{2}(-7Ab+4aB)+aA\cos(c+dx)+\frac{5}{2}Ab\cos^2(c+dx)\right)\sec}{(a+b\cos(c+dx))^{5/2}}}{2a}$$

$$= -\frac{(7Ab-4aB)\tan(c+dx)}{4a^2d(a+b\cos(c+dx))^{3/2}} + \frac{A\sec(c+dx)\tan(c+dx)}{2ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{1}{4}\left(4a^2A+3\right.\right.}{}}{}$$

$$= -\frac{b\left(27a^2Ab-35Ab^3-12a^3B+20ab^2B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{(7Ab-4aB)\,\mathrm{ta}}{4a^2d(a+b\cos(}$$

$$= -\frac{b\left(27a^2Ab-35Ab^3-12a^3B+20ab^2B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{b\left(33a^4Ab-17\right.}{}$$

$$= -\frac{b\left(27a^2Ab-35Ab^3-12a^3B+20ab^2B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{b\left(33a^4Ab-17\right.}{}$$

$$= -\frac{b\left(27a^2Ab-35Ab^3-12a^3B+20ab^2B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{b\left(33a^4Ab-17\right.}{}$$

$$= \frac{\left(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B\right)\sqrt{a+b}}{12a^4\left(a^2-b^2\right)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{\left(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B\right)\sqrt{a+b}}{12a^4\left(a^2-b^2\right)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 7.95, size = 820, normalized size = 1.54

$$\frac{2\left(12Aba^5+144b^2Ba^4-216Ab^3a^3-80b^4Ba^2+140Ab^5a\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{2\left(24Aa^6-132bBa^5+195Ab^2a^4+344b^3Ba^3-566Ab^4a^2-180b\right.}{\sqrt{a+b\cos(c+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(a + b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] ((2*(12*a^5*A*b - 216*a^3*A*b^3 + 140*a*A*b^5 + 144*a^4*b^2*B - 80*a^2*b^4*
B)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
)/Sqrt[a + b*Cos[c + d*x]] + (2*(24*a^6*A + 195*a^4*A*b^2 - 566*a^2*A*b^4 +
 315*A*b^6 - 132*a^5*b*B + 344*a^3*b^3*B - 180*a*b^5*B)*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c
 + d*x]] - ((2*I)*(33*a^4*A*b^2 - 170*a^2*A*b^4 + 105*A*b^6 - 12*a^5*b*B +
104*a^3*b^3*B - 60*a*b^5*B)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b +
b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh
[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*E
llipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(
a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*
Cos[c + d*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqr
t[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b
*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*
Cos[c + d*x])^2)))/(48*a^4*(a - b)^2*(a + b)^2*d) + (Sqrt[a + b*Cos[c + d*x
]]*((Sec[c + d*x]*(-11*A*b*Sin[c + d*x] + 4*a*B*Sin[c + d*x]))/(4*a^4) - (2
*(-(A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(a + b*
Cos[c + d*x])^2) - (2*(-13*a^2*A*b^4*Sin[c + d*x] + 9*A*b^6*Sin[c + d*x] +
10*a^3*b^3*B*Sin[c + d*x] - 6*a*b^5*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(
a + b*Cos[c + d*x])) + (A*Sec[c + d*x]*Tan[c + d*x])/(2*a^3)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm ="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x)

maple [B] time = 9.71, size = 2000, normalized size = 3.76

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b^2*(3*A*b-2*B*a)/a^4/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*b*(3*A*b-2*B*a)/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-2*(A*b-B*a)*b^2/a^3*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2/b*(a-b))^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*cos(1/2*d*x+1/2*c)^2*b-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))+2*A/a^2*(-1/2/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/8*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1

```
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2,(-2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*
b^2)+2*(-2*A*b+B*a)/a^3*(-1/a*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1
/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*cos(
1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))
)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^3\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.341 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*
x+c))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {21, 2663, 2661}

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{\left(B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}}$$
$$= \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 1.00

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{B}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas"
)
[Out] integral(B/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(3/2), x)
```

maple [C] time = 0.13, size = 76, normalized size = 1.31

$$\frac{2B\sqrt{\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a+b}}\,\mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle|\frac{\sqrt{2}\,\sqrt{b}}{\sqrt{a+b}}\right)}{d\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
[Out] 2*B/d/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/
(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)/(a+b)^(1/2)*b^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima"
)
[Out] integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,a+B\,b\,\cos(c+d\,x)}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(3/2),x)
[Out] int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] B*Integral(1/sqrt(a + b*cos(c + d*x)), x)
```

3.342 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=59

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos
(d*x+c))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {21, 2807, 2805}

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
 + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx &= B\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx \\
&= \frac{\left(B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}} \\
&= \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 59, normalized size = 1.00

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
 + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2),
 x)
```

maple [A] time = 1.13, size = 167, normalized size = 2.83

$$\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{\frac{2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a-b}{a-b}}\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a+b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*((2*cos(1/2*d*x+1/2*c)^2*b+a-b)/(a-b))^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2,(-2*b/(a-b))^(1/2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b
)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,a+B\,b\,\cos(c+d\,x)}{\cos(c+d\,x)\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{\sec{\left(c+dx \right)}}{\sqrt{a+b\cos{\left(c+dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
[Out] B*Integral(sec(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

3.343 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=108

$$\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2bB\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -2*b*B*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b)
)^(1/2))*(a+b*cos(d*x+c))^(1/2)/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {21, 2664, 2655, 2653}

$$\frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2bB\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/((a^2
- b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*b*B*Sin[c + d*x])/((a^2 -
 b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = B\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{2bB\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{(2B)\int \frac{-\frac{a}{2}-\frac{1}{2}b\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2bB\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{B\int \sqrt{a+b\cos(c+dx)}\,dx}{a^2-b^2}$$

$$= -\frac{2bB\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(B\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2B\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2bB\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.21, size = 84, normalized size = 0.78

$$\frac{B\left(2(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2b\sin(c+dx)\right)}{d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (B*(2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] - 2*b*Sin[c + d*x]))/((a - b)*(a + b)*d*Sqrt[a + b*Cos[c + d*x]
])
```

fricas [F] time = 2.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,B}{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*B/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos\left(dx+c\right)+Ba}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(5/2), x)
```

maple [A] time = 1.71, size = 218, normalized size = 2.02

$$-\frac{2B\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\;\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a-\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\cdots\right.\right.}{a-b}}\cdots\right.}{\left(a-b\right)\left(a+b\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\cdots\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2*B*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2
*d*x+1/2*c)^2)/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+
b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos\left(dx+c\right)+Ba}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/(b*cos(d*x + c) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.344 $\int \frac{(aB+bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=179

$$\frac{2b^2B\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2bB\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

```
[Out] 2*b^2*B*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*b*B*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/
(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))
^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d
/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {21, 2802, 3059, 2655, 2653, 12, 2807, 2805}

$$\frac{2b^2B\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2bB\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*
(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*B*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b
*Cos[c + d*x]]) + (2*b^2*B*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
+ d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(aB + bB\cos(c+dx))\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = B\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2 B\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{(2B)\int \frac{\left(\frac{1}{2}\left(a^2-b^2\right)-\frac{1}{2}ab\cos(c+dx)-\frac{1}{2}b^2\cos\cdots}{\sqrt{a+b\cos(c+dx)}\cdots}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2 B\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{(2B)\int -\frac{b\left(a^2-b^2\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}\,dx}{ab\left(a^2-b^2\right)} - \frac{(bB)\cdots}{\cdots}$$

$$= \frac{2b^2 B\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{B\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a} - \frac{\left(bB\sqrt{a+b\cdots}\right.}{\cdots}$$

$$= -\frac{2bB\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b^2 B\sin(c+dx\cdots}{a\left(a^2-b^2\right)d\sqrt{a+b\cos\cdots}}$$

$$= -\frac{2bB\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2B\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\cdots\right.}{ad\sqrt{a+b\cos(c\cdots}}$$

Mathematica [C] time = 4.87, size = 403, normalized size = 2.25

$$\frac{B\left(\frac{4b^2\sin(c+dx)}{\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}} - \frac{\frac{2\left(2a^2-3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{4ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{2i\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+\cdots}{b-\cdots}}\cdots}{\cdots}}{\cdots}\right.}{2ad\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x])/(a + b*Cos[c + d*x])^(5/2
),x]
```

```
[Out] (B*(-(((-4*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(2*a^2 - 3*b^2)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*C
os[c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 +
 Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Ell
ipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a
- b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Co
s[c + d*x]]], (a + b)/(a - b)])))/(a*Sqrt[-(a + b)^(-1)]))/((-a + b)*(a + b
))) + (4*b^2*Sin[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]])))/(2*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2),
 x)
```

maple [A] time = 1.81, size = 377, normalized size = 2.11

$$\frac{2B\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}\;b\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a-b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}\right.\right.}{a-b}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] 2*B*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-
2*b/(a-b))^(1/2))*b^2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/a/(a-b
)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.345 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=170

$$\frac{10(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(aB+Ab)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2(9aA+7b}{}$$

```
[Out] 2/15*(9*A*a+7*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+10/21*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*(9*A*a+7*
B*b)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*(A*b+B*a)*cos(d*x+c)^(5/2)*sin(d*x+c
)/d+2/9*b*B*cos(d*x+c)^(7/2)*sin(d*x+c)/d+10/21*(A*b+B*a)*sin(d*x+c)*cos(d*
x+c)^(1/2)/d
```

Rubi [A] time = 0.21, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3023, 2748, 2635, 2639, 2641}

$$\frac{10(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(aB+Ab)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2(9aA+7b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(9*a*A + 7*b*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*(A*b + a*B)*Elli
pticF[(c + d*x)/2, 2])/(21*d) + (10*(A*b + a*B)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(21*d) + (2*(9*a*A + 7*b*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) +
 (2*(A*b + a*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*b*B*Cos[c + d*x
]^(7/2)*Sin[c + d*x])/(9*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^{\frac{5}{2}}(c+dx)\left(aA+(Ab+aB)\cos(c+dx)+bB\,\mathrm{co}\right.$$

$$= \frac{2bB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{2}{9}\int \cos^{\frac{5}{2}}(c+dx)\left(\frac{1}{2}\right.$$

$$= \frac{2bB\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + (Ab+aB)\int \cos^{\frac{7}{2}}(c$$

$$= \frac{2(9aA+7bB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{2(Ab+aB}{}$$

$$= \frac{2(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10(Ab+aB)\sqrt{\cos(c}}{21}$$

$$= \frac{2(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10(Ab+aB)F\left(\frac{1}{2}(c\right.}{21d}$$

Mathematica [A] time = 1.29, size = 125, normalized size = 0.74

$$\frac{300(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)+84(9aA+7bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\,(7(36aA+43bB)\cos}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (84*(9*a*A + 7*b*B)*EllipticE[(c + d*x)/2, 2] + 300*(A*b + a*B)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*a*A + 43*b*B)*Cos[c + d*x] + 5*(
78*A*b + 78*a*B + 18*(A*b + a*B)*Cos[2*(c + d*x)] + 7*b*B*Cos[3*(c + d*x)])
)*Sin[c + d*x])/(630*d)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bb\cos\left(dx+c\right)^{4}+Aa\cos\left(dx+c\right)^{2}+\left(Ba+Ab\right)\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^4 + A*a*cos(d*x + c)^2 + (B*a + A*b)*cos(d*x + c
)^3)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

maple [B] time = 1.47, size = 451, normalized size = 2.65

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120Bb\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(720Ab+720aB+224}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*b*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b+720*B*a+2240*B*b)*sin(1/2*
d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A*a-1080*A*b-1080*B*a-2072*B*b)*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(504*A*a+840*A*b+840*B*a+952*B*b)*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*A*a-240*A*b-240*B*a-168*B*b)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a+75*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.35, size = 177, normalized size = 1.04

$$-\frac{2\,A\,a\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,A\,b\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4}\right.}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

```
[Out] - (2*A*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
+ d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*b*cos(c + d*x)^(9/2)*sin(c +
 d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1
/2)) - (2*B*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, c
os(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b*cos(c + d*x)^(11/2)*s
in(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*
x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.346 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{2(7aA+5bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2(7aA+5b}{}$$

```
[Out] 6/5*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(7*A*a+5*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*(A*b+B*a)*co
s(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*b*B*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/21*(7*A
*a+5*B*b)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.18, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3023, 2748, 2635, 2641, 2639}

$$\frac{2(7aA+5bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2(7aA+5b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a*A + 5*b*B)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*(7*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(21*d) + (2*(A*b + a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*
b*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \cos^{\frac{3}{2}}(c+dx)\left(aA+(Ab+aB)\cos(c+dx)+bB\,\mathrm{co}\right.$$

$$= \frac{2bB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}\right.$$

$$= \frac{2bB\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+(Ab+aB)\int \cos^{\frac{5}{2}}(c$$

$$= \frac{2(7aA+5bB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2(Ab+aB}{}$$

$$= \frac{6(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(7aA+5bB)F\left(\frac{1}{2}(c+\right.}{21d}$$

Mathematica [A] time = 0.86, size = 103, normalized size = 0.74

$$\frac{10(7aA+5bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+126(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\,(42(aB+Ab)\cos(c}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (126*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + 10*(7*a*A + 5*b*B)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*a*A + 65*b*B + 42*(A*b + a*B)*Cos[c
 + d*x] + 15*b*B*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{3}+Aa\cos\left(dx+c\right)+\left(Ba+Ab\right)\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^3 + A*a*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)^
2)*sqrt(cos(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

maple [B] time = 1.39, size = 413, normalized size = 2.95

$$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240Bb\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168Ab-168aB-360Bb}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b-168*B*a-360*B*b)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*a+168*A*b+168*B*a+280*B*b)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*a-42*A*b-42*B*a-80*B*b)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+35*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b+25*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

mupad [B] time = 1.16, size = 166, normalized size = 1.19

$$\frac{2\,A\,a\,\left(\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}-\frac{2\,A\,b\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+d\,x\right.}{7\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

```
[Out] (2*A*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) - (2*A*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos
(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a*cos(c + d*x)^(7/2)*sin(
c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)
^(1/2)) - (2*B*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4
, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.347 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=108

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2bB\sin(c+}{}$$

```
[Out] 2/5*(5*A*a+3*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b*B*cos(d*x+c
)^(3/2)*sin(d*x+c)/d+2/3*(A*b+B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3023, 2748, 2639, 2635, 2641}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2bB\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(5*a*A + 3*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(3*d) + (2*b*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))(A+B\cos(c+dx))\,dx = \int \sqrt{\cos(c+dx)}\,\left(aA+(Ab+aB)\cos(c+dx)+b\right.$$

$$= \frac{2bB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\cos(c+dx)}$$

$$= \frac{2bB\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + (Ab+aB)\int \cos^{\frac{3}{2}}$$

$$= \frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(Ab+aB)\sqrt{\cos}}{}$$

$$= \frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(Ab+aB)F\left(\frac{1}{2}(\right.}{3d}$$

Mathematica [A] time = 0.41, size = 86, normalized size = 0.80

$$\frac{2\left(5(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\,(5aB+5Ab+3bB\,\mathrm{co}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(3*(5*a*A + 3*b*B)*EllipticE[(c + d*x)/2, 2] + 5*(A*b + a*B)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A*b + 5*a*B + 3*b*B*Cos[c + d*x])*Si
n[c + d*x]))/(15*d)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos(dx+c)^2 + Aa + (Ba+Ab)\cos(dx+c)\right)\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(cos(d*x
 + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c) + A)(b\cos(dx+c) + a)\sqrt{\cos(dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

maple [B] time = 1.24, size = 371, normalized size = 3.44

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-24Bb\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20Ab+20aB+24Bb)\left(\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b+20*B*a+24*B*b)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b-10*B*a-6*B*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+5*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
```

```
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+
5*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

mupad [B] time = 1.01, size = 128, normalized size = 1.19

$$\frac{2\,A\,b\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x)),x)
```

```
[Out] (2*A*b*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) + (2*B*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2))
)/(3*d) + (2*A*a*ellipticE(c/2 + (d*x)/2, 2))/d - (2*B*b*cos(c + d*x)^(7/2)
*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*
x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.348 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/d+2/3*(3*A*a+B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*B*sin(d*x+c)*co
s(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.15, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3023, 2748, 2641, 2639}

$$\frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a*A + b*B)*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*b*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2bB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}(3aA+bB)+\frac{3}{2}(Ab+aB}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2bB\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} + (Ab+aB)\int \sqrt{\cos(c+dx)}\,dx$$

$$= \frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2}{}$$

Mathematica [A] time = 0.23, size = 67, normalized size = 0.89

$$\frac{2\left((3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)+bB\sin(c+dx)\sqrt{\cos(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(3*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + (3*a*A + b*B)*EllipticF[(c +
d*x)/2, 2] + b*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb\cos\left(dx+c\right)^2+Aa+(Ba+Ab)\cos\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))/sqrt(cos(d*x
 + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

maple [B] time = 1.28, size = 326, normalized size = 4.35

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4Bb\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3aA\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*b*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*b+B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a-2*B*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

mupad [B] time = 0.99, size = 85, normalized size = 1.13

$$\frac{2\,B\,b\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*B*b*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) + (2*A*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*b*ellipticE(c/2 + (d*x)/2
, 2))/d + (2*B*a*ellipticE(c/2 + (d*x)/2, 2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.349 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*(A*a-B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/d+2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*A*sin(d*x+c)/d/cos(d
*x+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2968, 3021, 2748, 2641, 2639}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a*A - b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(A*b + a*B)*EllipticF[(c
+ d*x)/2, 2])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}(Ab+aB)-\frac{1}{2}(aA-bB)\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,d$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + (Ab+aB)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + (-aA+bB$$

$$= -\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2}{d}$$

Mathematica [A] time = 0.36, size = 64, normalized size = 0.90

$$\frac{2\left((aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)+(bB-aA)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{aA\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*((-(a*A) + b*B)*EllipticE[(c + d*x)/2, 2] + (A*b + a*B)*EllipticF[(c + d
*x)/2, 2] + (a*A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/d
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))/cos(d*x + c)
^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

maple [B] time = 1.41, size = 244, normalized size = 3.44

$$-\frac{2\left(Ab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2*(A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-2*A*a*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

mupad [B] time = 1.44, size = 96, normalized size = 1.35

$$\frac{2\,A\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*A*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*B*b*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*a*sin(c + d*x)*hypergeom(
[-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(
1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.350 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=103

$$\frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aB+Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] -2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*a+3*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*A*sin(d*x+c)/d
/cos(d*x+c)^(3/2)+2*(A*b+B*a)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3021, 2748, 2636, 2639, 2641}

$$\frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aB+Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a*A + 3*b*B)*EllipticF[(
c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*
(A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{3}{2}(Ab+aB)+\frac{1}{2}(aA+3bB)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,d$$

$$= \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + (Ab+aB)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{3}(aA+3bB)$$

$$= \frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aA\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+aB)\sin}{d\sqrt{\cos(c-}}$$

$$= -\frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2}{3}$$

Mathematica [A] time = 0.48, size = 107, normalized size = 1.04

$$\frac{2\left((aA+3bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right) - 3(aB+Ab)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right) + aA\tan(c+dx) + 3aB\right.}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(-3*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (a*A + 3*
b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*A*b*Sin[c + d*x] + 3*
a*B*Sin[c + d*x] + a*A*Tan[c + d*x]))/(3*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb\cos(dx+c)^2 + Aa + (Ba+Ab)\cos(dx+c)}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))/cos(d*x + c)
^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

maple [B] time = 3.24, size = 428, normalized size = 4.16

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2Bb\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}+\frac{2(Ab+aB\cdots}{\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*(A*
b+B*a)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)
^2-1)+2*a*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

mupad [B] time = 1.97, size = 150, normalized size = 1.46

$$\frac{2\,B\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,A\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+\right.\right.}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*B*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a*sin(c + d*x)*hypergeom([-3/4
, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2
)) + (2*A*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*co
s(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a*sin(c + d*x)*hypergeom([-
1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.351 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(3aA+5bB)\sin(c+dx}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -2/5*(3*A*a+5*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*A*sin(d*x+
c)/d/cos(d*x+c)^(5/2)+2/3*(A*b+B*a)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*(3*A*
a+5*B*b)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2968, 3021, 2748, 2636, 2641, 2639}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(3aA+5bB)\sin(c+dx}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a*A + 5*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(3*a*A + 5*b*
B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{5}{2}(Ab+aB)+\frac{1}{2}(3aA+5bB)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + (Ab+aB)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + \frac{1}{5}(3aA+5bB$$

$$= \frac{2aA\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(3aA+5bB)\sin(c}{5d\sqrt{\cos(c+dx}}$$

$$= -\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} +$$

Mathematica [A] time = 0.83, size = 134, normalized size = 0.96

$$\frac{10(aB+Ab)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6(3aA+5bB)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+9aA\sin(2(c+dx))+}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*(3*a*A + 5*b*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*(A*b
+ a*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*A*b*Sin[c + d*x] +
 10*a*B*Sin[c + d*x] + 9*a*A*Sin[2*(c + d*x)] + 15*b*B*Sin[2*(c + d*x)] + 6
*a*A*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))/cos(d*x + c)
^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

maple [B] time = 4.02, size = 663, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(-(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*a*A/(8
*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin
(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*(A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

mupad [B] time = 2.39, size = 177, normalized size = 1.26

$$\frac{2\,A\,a\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos(c+d\,x)^2\right)}{5\,d\cos(c+d\,x)^{5/2}\sqrt{\sin(c+d\,x)^2}}+\frac{2\,A\,b\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\,\frac{1}{4};\,\cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\sqrt{\sin(c+d\,x)^2}}+\frac{2\,B\,a\,\sin(}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*A*a*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(
c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*b*sin(c + d*x)*hypergeom([-3/
4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/
```

```
2)) + (2*B*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d
*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*b*sin(c + d*x)*hypergeom
([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^
(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.352 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=264

$$\frac{2\left(9a^2A+14abB+7Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(9a^2A+14abB+7Ab^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{10\left(11a(aB+2\right.}{}$$

```
[Out] 2/15*(9*A*a^2+7*A*b^2+14*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+10/231*(9*b^2*B+11*a*(2*A*b+B*
a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/d+2/45*(9*A*a^2+7*A*b^2+14*B*a*b)*cos(d*x+c)^(3/2)*sin(d*x+c
)/d+2/77*(9*b^2*B+11*a*(2*A*b+B*a))*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/99*b*(1
1*A*b+13*B*a)*cos(d*x+c)^(7/2)*sin(d*x+c)/d+2/11*b*B*cos(d*x+c)^(7/2)*(a+b*
cos(d*x+c))*sin(d*x+c)/d+10/231*(9*b^2*B+11*a*(2*A*b+B*a))*sin(d*x+c)*cos(d
*x+c)^(1/2)/d
```

Rubi [A] time = 0.38, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2990, 3023, 2748, 2635, 2639, 2641}

$$\frac{2\left(9a^2A+14abB+7Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(9a^2A+14abB+7Ab^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{10\left(11a(aB+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(9*a^2*A + 7*A*b^2 + 14*a*b*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*(
9*b^2*B + 11*a*(2*A*b + a*B))*EllipticF[(c + d*x)/2, 2])/(231*d) + (10*(9*b
^2*B + 11*a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(9
*a^2*A + 7*A*b^2 + 14*a*b*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*(
9*b^2*B + 11*a*(2*A*b + a*B))*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*
b*(11*A*b + 13*a*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*b*B*Cos[c
+ d*x]^(7/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(11*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{11d} + \ldots$$

$$= \frac{2b(11Ab+13aB)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d} + \frac{2bB\cos\ldots}{}$$

$$= \frac{2b(11Ab+13aB)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d} + \frac{2bB\cos\ldots}{}$$

$$= \frac{2\left(9a^2A+7Ab^2+14abB\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d}\ldots$$

$$= \frac{2\left(9a^2A+7Ab^2+14abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10\left(9b\ldots\right.}{}$$

$$= \frac{2\left(9a^2A+7Ab^2+14abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10\left(9b\ldots\right.}{}$$

Mathematica [A] time = 1.77, size = 196, normalized size = 0.74

$$\frac{1200\left(11a^2B+22aAb+9b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3696\left(9a^2A+14abB+7Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)\sqrt{\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3696*(9*a^2*A + 7*A*b^2 + 14*a*b*B)*EllipticE[(c + d*x)/2, 2] + 1200*(22*a
*A*b + 11*a^2*B + 9*b^2*B)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]
*(154*(36*a^2*A + 43*A*b^2 + 86*a*b*B)*Cos[c + d*x] + 180*(22*a*A*b + 11*a^
2*B + 16*b^2*B)*Cos[2*(c + d*x)] + 770*b*(A*b + 2*a*B)*Cos[3*(c + d*x)] + 1
5*(1144*a*A*b + 572*a^2*B + 531*b^2*B + 21*b^2*B*Cos[4*(c + d*x)]))*Sin[c +
 d*x])/(27720*d)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^5 + Aa^2\cos(dx+c)^2 + \left(2Bab+Ab^2\right)\cos(dx+c)^4 + \left(Ba^2+2Aab\right)\cos(dx+c)^3\right)\sqrt{c\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^5 + A*a^2*cos(d*x + c)^2 + (2*B*a*b + A*b^2)*c
os(d*x + c)^4 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^3)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x
)
```

maple [B] time = 1.29, size = 666, normalized size = 2.52

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(20160B\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-12320A\,b^2-24640\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*B*b^
2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*A*b^2-24640*B*a*b-50400*
B*b^2)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(15840*A*a*b+24640*A*b^2+79
20*B*a^2+49280*B*a*b+56880*B*b^2)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(
-5544*A*a^2-23760*A*a*b-22792*A*b^2-11880*B*a^2-45584*B*a*b-34920*B*b^2)*si
n(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(5544*A*a^2+18480*A*a*b+10472*A*b^2+9
240*B*a^2+20944*B*a*b+13860*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-1386*A*a^2-5280*A*a*b-1848*A*b^2-2640*B*a^2-3696*B*a*b-2790*B*b^2)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2079*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1617*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*b^2+1650*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3234*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a*b+825*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+675*b^2*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x
)
```

mupad [B] time = 1.53, size = 275, normalized size = 1.04

$$-\frac{2\,A\,a^2\cos(c+d\,x)^{7/2}\,\sin(c+d\,x)\;{}_2F_1\left(\frac{1}{2},\frac{7}{4};\;\frac{11}{4};\;\cos(c+d\,x)^2\right)}{7\,d\,\sqrt{\sin(c+d\,x)^2}}-\frac{2\,B\,a^2\cos(c+d\,x)^{9/2}\,\sin(c+d\,x)\;{}_2F_1\left(\frac{1}{2},\frac{9}{4};\right.}{9\,d\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

```
[Out] - (2*A*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(
c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^2*cos(c + d*x)^(9/2)*sin
(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2
)^(1/2)) - (2*A*b^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4],
 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b^2*cos(c + d*
x)^(13/2)*sin(c + d*x)*hypergeom([1/2, 13/4], 17/4, cos(c + d*x)^2))/(13*d*
(sin(c + d*x)^2)^(1/2)) - (4*A*a*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeo
m([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (4*B*a*
b*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x
)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.353 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=223

$$\frac{2\left(7a^2A+10abB+5Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a^2A+10abB+5Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2\left(9a(aB+2\right.}{}$$

```
[Out] 2/15*(7*b^2*B+9*a*(2*A*b+B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(7*A*a^2+5*A*b^2+10*B*a*b)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2))/d+2/45*(7*b^2*B+9*a*(2*A*b+B*a))*cos(d*x+c)^(3/2)*sin(d*x+c)/d+
2/63*b*(9*A*b+11*B*a)*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*b*B*cos(d*x+c)^(5/2
)*(a+b*cos(d*x+c))*sin(d*x+c)/d+2/21*(7*A*a^2+5*A*b^2+10*B*a*b)*sin(d*x+c)*
cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.33, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2990, 3023, 2748, 2635, 2641, 2639}

$$\frac{2\left(7a^2A+10abB+5Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a^2A+10abB+5Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2\left(9a(aB+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(7*b^2*B + 9*a*(2*A*b + a*B))*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*(7*
a^2*A + 5*A*b^2 + 10*a*b*B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(7*a^2*A
 + 5*A*b^2 + 10*a*b*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(7*b^2*
B + 9*a*(2*A*b + a*B))*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(9*A*
b + 11*a*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b*B*Cos[c + d*x]^(
5/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{9d} +$$

$$= \frac{2b(9Ab+11aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2bB\,\mathrm{c}}{}$$

$$= \frac{2b(9Ab+11aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2bB\,\mathrm{c}}{}$$

$$= \frac{2\left(7a^2A+5Ab^2+10abB\right)\sqrt{\cos(c+dx)}\,\sin(c+d}{21d}$$

$$= \frac{2\left(7b^2B+9a(2Ab+aB)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2\left(7a\right.}{}$$

Mathematica [A] time = 1.41, size = 167, normalized size = 0.75

$$\frac{60\left(7a^2A+10abB+5Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+84\left(9a^2B+18aAb+7b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (84*(18*a*A*b + 9*a^2*B + 7*b^2*B)*EllipticE[(c + d*x)/2, 2] + 60*(7*a^2*A
+ 5*A*b^2 + 10*a*b*B)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(72
*a*A*b + 36*a^2*B + 43*b^2*B)*Cos[c + d*x] + 5*(84*a^2*A + 78*A*b^2 + 156*a
*b*B + 18*b*(A*b + 2*a*B)*Cos[2*(c + d*x)] + 7*b^2*B*Cos[3*(c + d*x)]))*Sin
[c + d*x])/(630*d)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bb^2\cos(dx+c)^4+Aa^2\cos(dx+c)+\left(2\,Bab+Ab^2\right)\cos(dx+c)^3+\left(Ba^2+2\,Aab\right)\cos(dx+c)^2\right)\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^4 + A*a^2*cos(d*x + c) + (2*B*a*b + A*b^2)*cos
(d*x + c)^3 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

maple [B] time = 1.31, size = 610, normalized size = 2.74

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120B\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(720A\,b^2+1440Bab+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*b^2
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b^2+1440*B*a*b+2240*B*b^2)
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1008*A*a*b-1080*A*b^2-504*B*a^2-
2160*B*a*b-2072*B*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a^2+1
008*A*a*b+840*A*b^2+504*B*a^2+1680*B*a*b+952*B*b^2)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-210*A*a^2-252*A*a*b-240*A*b^2-126*B*a^2-480*B*a*b-168*B*
b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*a^2*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+75*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-378*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+1
50*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-147*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

mupad [B] time = 1.35, size = 264, normalized size = 1.18

$$\frac{2\,A\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}-\frac{2\,B\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+\right.\right.}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(
3*d) - (2*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4,
 cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*b^2*cos(c + d*x)^(9/2
)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d
*x)^2)^(1/2)) - (2*B*b^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 1
1/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (4*A*a*b*cos(c
 + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*
d*(sin(c + d*x)^2)^(1/2)) - (4*B*a*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hyperg
eom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.354 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=182

$$\frac{2\left(5a^2A+6abB+3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(7a(aB+2Ab)+5b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a(aB+2Ab)+5b^2B\right)}{21}$$

```
[Out] 2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(5*b^2*B+7*a*(2*A*b+B*a))*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))/d+2/35*b*(7*A*b+9*B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*b*B*cos(
d*x+c)^(3/2)*(a+b*cos(d*x+c))*sin(d*x+c)/d+2/21*(5*b^2*B+7*a*(2*A*b+B*a))*s
in(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.32, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2990, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\left(5a^2A+6abB+3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(7a(aB+2Ab)+5b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a(aB+2Ab)+5b^2B\right)}{21}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*b
^2*B + 7*a*(2*A*b + a*B))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*b^2*B +
 7*a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(7*A*b +
 9*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) + (2*b*B*Cos[c + d*x]^(3/2)
*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{7d} +$$
$$= \frac{2b(7Ab+9aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2bB\cos$$
$$= \frac{2b(7Ab+9aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2bB\cos$$
$$= \frac{2\left(5a^2A+3Ab^2+6abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(5b^2\right.}{}$$
$$= \frac{2\left(5a^2A+3Ab^2+6abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(5b^2\right.}{}$$

Mathematica [A] time = 1.13, size = 139, normalized size = 0.76

$$\frac{10\left(7a^2B+14aAb+5b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+42\left(5a^2A+6abB+3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+}}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]),x]
```

```
[Out] (42*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2] + 10*(14*a*A*b
+ 7*a^2*B + 5*b^2*B)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(42*b*(
A*b + 2*a*B)*Cos[c + d*x] + 5*(28*a*A*b + 14*a^2*B + 13*b^2*B + 3*b^2*B*Cos
[2*(c + d*x)]))*Sin[c + d*x])/(105*d)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

maple [B] time = 1.44, size = 548, normalized size = 3.01

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240B\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-168A\,b^2-336Bab-3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^2-336*B*a*b-360*B*b^2)*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a*b+168*A*b^2+140*B*a^2+336*B*a
*b+280*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-140*A*a*b-42*A*b^2-
70*B*a^2-84*B*a*b-80*B*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*A*a*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-63*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*b^2+35*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^2*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-126*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

mupad [B] time = 1.34, size = 229, normalized size = 1.26

$$\frac{2\,B\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,b\,\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\right.}{3}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*B*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(
3*d) + (2*A*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*a*b*((2*cos(c + d*x)^
(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d - (2*A*b^2*co
s(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/
(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hyp
ergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (4
*B*a*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**2*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.355 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{2\left(3a^2A+2abB+Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(5a(aB+2Ab)+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(7aB+5Ab)\sin(c+dx}{15d}$$

```
[Out] 2/5*(3*b^2*B+5*a*(2*A*b+B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(3*A*a^2+A*b^2+2*B*a*b)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))/d+2/15*b*(5*A*b+7*B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+2/5*b*B*(a+b*co
s(d*x+c))*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.27, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2990, 3023, 2748, 2641, 2639}

$$\frac{2\left(3a^2A+2abB+Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(5a(aB+2Ab)+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(7aB+5Ab)\sin(c+dx}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(3*b^2*B + 5*a*(2*A*b + a*B))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(3*a
^2*A + A*b^2 + 2*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b*(5*A*b + 7*
a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*B*Sqrt[Cos[c + d*x]]*(a
 + b*Cos[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2bB\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{\frac{1}{2}a(5aA$$

$$= \frac{2b(5Ab+7aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2bB\sqrt{\cos(c+dx)}\,(a$$

$$= \frac{2b(5Ab+7aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2bB\sqrt{\cos(c+dx)}\,(a$$

$$= \frac{2\left(3b^2B+5a(2Ab+aB)\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(3a^2A+Ab^2+2$$

Mathematica [A] time = 0.60, size = 106, normalized size = 0.76

$$\frac{2\left(5\left(3a^2A+2abB+Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3\left(5a^2B+10aAb+3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin(c+dx)\sqrt{\cos(c+}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (2*(3*(10*a*A*b + 5*a^2*B + 3*b^2*B)*EllipticE[(c + d*x)/2, 2] + 5*(3*a^2*A
 + A*b^2 + 2*a*b*B)*EllipticF[(c + d*x)/2, 2] + b*Sqrt[Cos[c + d*x]]*(5*A*b
 + 10*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c) + A)(b\cos(dx+c) + a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

maple [B] time = 1.19, size = 487, normalized size = 3.48

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24B\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(20A\,b^2+40Bab+24b^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b^2+40*B*a*b+24*B*b^2)*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b^2-20*B*a*b-6*B*b^2)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+15*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*A*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+10*B*a*b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b
^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

mupad [B] time = 1.34, size = 177, normalized size = 1.26

$$\frac{A\,b^2\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,A\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,b\,\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^(1/2),x)
```

```
[Out] (A*b^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d + (2*A*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a^2*ellipticE(c
/2 + (d*x)/2, 2))/d + (2*B*a*b*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*
ellipticF(c/2 + (d*x)/2, 2))/3))/d + (4*A*a*b*ellipticE(c/2 + (d*x)/2, 2))/
d - (2*B*b^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, co
s(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.356 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{2\left(3a^2B+6aAb+b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2A-2abB-Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c}{}$$

```
[Out] -2*(A*a^2-A*b^2-2*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(6*A*a*b+3*B*a^2+B*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
)/d+2*a^2*A*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*b^2*B*sin(d*x+c)*cos(d*x+c)^(
1/2)/d
```

Rubi [A] time = 0.25, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2988, 3023, 2748, 2641, 2639}

$$\frac{2\left(3a^2B+6aAb+b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2A-2abB-Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*A*b +
3*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*A*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - 2\int \frac{-\frac{1}{2}a(2Ab+aB)+\frac{1}{2}\left(a^2A-Ab^2-2ab\right.}{\sqrt{\cos(c}}$$

$$= \frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} - \frac{4}{3}\int \frac{\frac{1}{4}\left(-\right.}{}$$

$$= \frac{2a^2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} - \left(a^2A-A\right.$$

$$= -\frac{2\left(a^2A-Ab^2-2abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(6aAb+3a^2B+\right.}{3}$$

Mathematica [A] time = 0.64, size = 102, normalized size = 0.84

$$\frac{2\left(\left(3a^2B+6aAb+b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\left(-3a^2A+6abB+3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)\left(3a^2A+b^2B\cos(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] (2*((-3*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2] + (6*a*A*b + 3
*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2] + ((3*a^2*A + b^2*B*Cos[c + d*x])
*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(3*d)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

maple [B] time = 1.39, size = 404, normalized size = 3.34

$$-\frac{2\left(4B\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6Aab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6*A*a*b*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b
^2-6*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*a^2*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*
B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

mupad [B] time = 1.57, size = 158, normalized size = 1.31

$$\frac{B\,b^2\left(\frac{2\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,A\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,A\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,B\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^(3/2),x)
```

```
[Out] (B*b^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d + (2*A*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*a^2*ellipticF(c
/2 + (d*x)/2, 2))/d + (4*A*a*b*ellipticF(c/2 + (d*x)/2, 2))/d + (4*B*a*b*el
lipticE(c/2 + (d*x)/2, 2))/d + (2*A*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2],
 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.357 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{2\left(a^2A+6abB+3Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2B+2aAb-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(aB+2}{d\sqrt{c}}$$

```
[Out] -2*(2*A*a*b+B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*a^2+3*A*b^2+6*B*a*b)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
)/d+2/3*a^2*A*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*a*(2*A*b+B*a)*sin(d*x+c)/d/co
s(d*x+c)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2988, 3021, 2748, 2641, 2639}

$$\frac{2\left(a^2A+6abB+3Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2B+2aAb-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(aB+2}{d\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2*A + 3*
A*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*A*Sin[c + d*x])/
(3*d*Cos[c + d*x]^(3/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c +
 d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{-\frac{3}{2}a(2Ab+aB)-\frac{1}{2}\left(a^2A+3Ab^2+6a\right.}{\cos^{\frac{3}{2}}(c}\\
&= \frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{4}{3}\int \frac{\frac{1}{4}\left(-a^2A-\right.}{}\\
&= \frac{2a^2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{3}\left(-a^2A-3Ab\right.\\
&= -\frac{2\left(2aAb+a^2B-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(a^2A+3Ab^2+6ab\right.}{3d}\end{aligned}$$

Mathematica [A] time = 1.18, size = 105, normalized size = 0.83

$$\frac{2\left(\left(a^2A+6abB+3Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-3\left(a^2B+2aAb-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{a\sin(c+dx)(3(aB+2Ab)\cos(c+dx)+}{\cos^{\frac{3}{2}}(c+dx)}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] (2*(-3*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2] + (a^2*A + 3*A*b
^2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2] + (a*(a*A + 3*(2*A*b + a*B)*Cos[c +
 d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x
)
```

maple [B] time = 3.02, size = 677, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*B*a*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^2*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
a*(2*A*b+B*a)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x
+1/2*c)^2-1)+2*a^2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x
)
```

mupad [B] time = 2.29, size = 194, normalized size = 1.54

$$\frac{2\,A\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,B\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\,\frac{1}{4};\,\cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*A*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*b^2*ellipticE(c/2 + (d*x)/2,
 2))/d + (4*B*a*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^2*sin(c + d*x)*hy
```

```
pergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c +
 d*x)^2)^(1/2)) + (2*B*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c +
 d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (4*A*a*b*sin(c +
d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(si
n(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.358 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2\left(a^2B+2aAb+3b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^2A+10abB+5Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(3a^2A+10abB+5Ab^2\right)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] -2/5*(3*A*a^2+5*A*b^2+10*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(2*A*a*b+B*a^2+3*B*b^2)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))/d+2/5*a^2*A*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/3*a*(2*A*b+B*a)*sin(d*
x+c)/d/cos(d*x+c)^(3/2)+2/5*(3*A*a^2+5*A*b^2+10*B*a*b)*sin(d*x+c)/d/cos(d*x
+c)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2988, 3021, 2748, 2636, 2639, 2641}

$$\frac{2\left(a^2B+2aAb+3b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^2A+10abB+5Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(3a^2A+10abB+5Ab^2\right)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(2
*a*A*b + a^2*B + 3*b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*A*Sin[c
 + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(3*d*C
os[c + d*x]^(3/2)) + (2*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Sin[c + d*x])/(5*d*S
qrt[Cos[c + d*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(2Ab+aB)-\frac{1}{2}\left(3a^2A+5Ab^2+1\right.}{\cos^{\frac{5}{2}}(c}$$

$$= \frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}\left(3a^2\right.}{}$$

$$= \frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{5}\left(-3a^2A-5A\right.$$

$$= \frac{2\left(2aAb+a^2B+3b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{}$$

$$= -\frac{2\left(3a^2A+5Ab^2+10abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(2aAb+a^2B+\right.}{}$$

Mathematica [A] time = 1.11, size = 175, normalized size = 1.02

$$\frac{10\left(a^2B+2aAb+3b^2B\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6\left(3a^2A+10abB+5Ab^2\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] (-6*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2
, 2] + 10*(2*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x
)/2, 2] + 20*a*A*b*Sin[c + d*x] + 10*a^2*B*Sin[c + d*x] + 9*a^2*A*Sin[2*(c
+ d*x)] + 15*A*b^2*Sin[2*(c + d*x)] + 30*a*b*B*Sin[2*(c + d*x)] + 6*a^2*A*T
an[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))/cos(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x
)
```

maple [B] time = 4.04, size = 750, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
*(A*b+2*B*a)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1)-2/5*a^2*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin
(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a*(2*A*b+B*a)*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+
cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x
)
```

mupad [B] time = 2.62, size = 227, normalized size = 1.32

$$\frac{6\,A\,a^2\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos(c+d\,x)^2\right)+30\,A\,b^2\cos(c+d\,x)^2\,\sin(c+d\,x)\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\,\frac{3}{4};\,\cos(c+d\,x\right.}{15\,d\cos(c+d\,x)^{5/2}\,\sqrt{1-\cos(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/cos(c + d*x)^(7/2),x)
```

```
[Out] (6*A*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*A*b
^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2)
+ 20*A*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*
x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (2*B*b^2*elli
pticF(c/2 + (d*x)/2, 2))/d + (2*B*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1
/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (4*B
*a*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d
*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.359 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=305

$$\frac{2b\left(26a^2B+33aAb+9b^2B\right)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{77d}+\frac{2\left(77a^3A+165a^2bB+165aAb^2+45b^3B\right)F\left(\frac{1}{2}(c+dx)\right|}{231d}$$

```
[Out] 2/15*(27*A*a^2*b+7*A*b^3+9*B*a^3+21*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/231*(77*A*a^3+1
65*A*a*b^2+165*B*a^2*b+45*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*(27*A*a^2*b+7*A*b^3+9*B*
a^3+21*B*a*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/77*b*(33*A*a*b+26*B*a^2+9*B
*b^2)*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/99*b^2*(11*A*b+15*B*a)*cos(d*x+c)^(7/
2)*sin(d*x+c)/d+2/11*b*B*cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+2
/231*(77*A*a^3+165*A*a*b^2+165*B*a^2*b+45*B*b^3)*sin(d*x+c)*cos(d*x+c)^(1/2
)/d
```

Rubi [A] time = 0.54, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2990, 3033, 3023, 2748, 2635, 2641, 2639}

$$\frac{2\left(77a^3A+165a^2bB+165aAb^2+45b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2\left(27a^2Ab+9a^3B+21ab^2B+7Ab^3\right)E\left(\frac{1}{2}(c+dx)\right|}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(27*a^2*A*b + 7*A*b^3 + 9*a^3*B + 21*a*b^2*B)*EllipticE[(c + d*x)/2, 2])
/(15*d) + (2*(77*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 45*b^3*B)*EllipticF[(c
 + d*x)/2, 2])/(231*d) + (2*(77*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 45*b^3*
B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (2*(27*a^2*A*b + 7*A*b^3 + 9*
a^3*B + 21*a*b^2*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b*(33*a*A*
b + 26*a^2*B + 9*b^2*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (2*b^2*(1
1*A*b + 15*a*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d) + (2*b*B*Cos[c + d*
x]^(5/2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(11*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
```

```
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{11d}$$

$$= \frac{2b^2(11Ab+15aB)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d} + \frac{2bB}{}$$

$$= \frac{2b\left(33aAb+26a^2B+9b^2B\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+}{77d}$$

$$= \frac{2b\left(33aAb+26a^2B+9b^2B\right)\cos^{\frac{5}{2}}(c+dx)\sin(c+}{77d}$$

$$= \frac{2\left(77a^3A+165aAb^2+165a^2bB+45b^3B\right)\sqrt{\cos(c}}{231d}$$

$$= \frac{2\left(27a^2Ab+7Ab^3+9a^3B+21ab^2B\right)E\left(\frac{1}{2}(c+dx)\right.}{15d}$$

Mathematica [A] time = 1.99, size = 235, normalized size = 0.77

$$\frac{240\left(77a^3A+165a^2bB+165aAb^2+45b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3696\left(9a^3B+27a^2Ab+21ab^2B+7Ab^3\right)E\left(\frac{1}{2}(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3696*(27*a^2*A*b + 7*A*b^3 + 9*a^3*B + 21*a*b^2*B)*EllipticE[(c + d*x)/2,
2] + 240*(77*a^3*A + 165*a*A*b^2 + 165*a^2*b*B + 45*b^3*B)*EllipticF[(c + d
*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(154*(108*a^2*A*b + 43*A*b^3 + 36*a^3*B +
129*a*b^2*B)*Cos[c + d*x] + 180*b*(33*a*A*b + 33*a^2*B + 16*b^2*B)*Cos[2*(c
 + d*x)] + 770*b^2*(A*b + 3*a*B)*Cos[3*(c + d*x)] + 15*(616*a^3*A + 1716*a*
A*b^2 + 1716*a^2*b*B + 531*b^3*B + 21*b^3*B*Cos[4*(c + d*x)]))*Sin[c + d*x]
)/(27720*d)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^3\cos(dx+c)^5 + Aa^3\cos(dx+c) + \left(3\,Bab^2 + Ab^3\right)\cos(dx+c)^4 + 3\left(Ba^2b + Aab^2\right)\cos(dx+c)^3 + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^5 + A*a^3*cos(d*x + c) + (3*B*a*b^2 + A*b^3)*c
os(d*x + c)^4 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^3 + (B*a^3 + 3*A*a^2*b)*
cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c) + A)(b\cos(dx+c) + a)^3 \cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x
)
```

maple [B] time = 1.40, size = 825, normalized size = 2.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*B*b^
3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-12320*A*b^3-36960*B*a*b^2-5040
0*B*b^3)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(23760*A*a*b^2+24640*A*b^
3+23760*B*a^2*b+73920*B*a*b^2+56880*B*b^3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x
+1/2*c)+(-16632*A*a^2*b-35640*A*a*b^2-22792*A*b^3-5544*B*a^3-35640*B*a^2*b-
68376*B*a*b^2-34920*B*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(4620*A*
a^3+16632*A*a^2*b+27720*A*a*b^2+10472*A*b^3+5544*B*a^3+27720*B*a^2*b+31416*
B*a*b^2+13860*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2310*A*a^3-4
158*A*a^2*b-7920*A*a*b^2-1848*A*b^3-1386*B*a^3-7920*B*a^2*b-5544*B*a*b^2-27
90*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-6237*A*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a^2*b-1617*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+1155*A*a^3*(sin(1/2*d*x+1/
```

```
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+2475*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2079*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*a^3-4851*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+2475*a^2*b*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+675*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x
)
```

mupad [B] time = 1.74, size = 364, normalized size = 1.19

$$\frac{A\,a^3\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}-\frac{2\,B\,a^3\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

```
[Out] (A*a^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d - (2*B*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4],
 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*b^3*cos(c + d*x
)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(
sin(c + d*x)^2)^(1/2)) - (2*B*b^3*cos(c + d*x)^(13/2)*sin(c + d*x)*hypergeo
m([1/2, 13/4], 17/4, cos(c + d*x)^2))/(13*d*(sin(c + d*x)^2)^(1/2)) - (6*A*
a^2*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d
*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a*b^2*cos(c + d*x)^(9/2)*sin(c
+ d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(
1/2)) - (2*B*a^2*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13
```

```
/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2)) - (6*B*a*b^2*cos(c + d*x)
^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(s
in(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.360 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=255

$$\frac{2b\left(22a^2B+27aAb+7b^2B\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2\left(7a^3B+21a^2Ab+15ab^2B+5Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+$$

```
[Out] 2/15*(15*A*a^3+27*A*a*b^2+27*B*a^2*b+7*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(21*A*a^2*b
+5*A*b^3+7*B*a^3+15*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*b*(27*A*a*b+22*B*a^2+7*B*b^2
)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/63*b^2*(9*A*b+13*B*a)*cos(d*x+c)^(5/2)*si
n(d*x+c)/d+2/9*b*B*cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2*sin(d*x+c)/d+2/21*(2
1*A*a^2*b+5*A*b^3+7*B*a^3+15*B*a*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.50, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2990, 3033, 3023, 2748, 2639, 2635, 2641}

$$\frac{2\left(21a^2Ab+7a^3B+15ab^2B+5Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(15a^3A+27a^2bB+27aAb^2+7b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*EllipticE[(c + d*x)/2, 2]
)/(15*d) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*EllipticF[(c +
d*x)/2, 2])/(21*d) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(27*a*A*b + 22*a^2*B + 7*b^2*B)*C
os[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (2*b^2*(9*A*b + 13*a*B)*Cos[c + d*
x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b*B*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^2*Sin[c + d*x])/(9*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
```

```
&& NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^3(A+B\cos(c+dx))\,dx = \frac{2bB\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{9d}$$

$$= \frac{2b^2(9Ab+13aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2bB}{}$$

$$= \frac{2b\left(27aAb+22a^2B+7b^2B\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+}{45d}$$

$$= \frac{2b\left(27aAb+22a^2B+7b^2B\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+}{45d}$$

$$= \frac{2\left(15a^3A+27aAb^2+27a^2bB+7b^3B\right)E\left(\frac{1}{2}(c+a\right.}{15d}$$

$$= \frac{2\left(15a^3A+27aAb^2+27a^2bB+7b^3B\right)E\left(\frac{1}{2}(c+a\right.}{15d}$$

Mathematica [A] time = 1.23, size = 197, normalized size = 0.77

$$\frac{60\left(7a^3B+21a^2Ab+15ab^2B+5Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+84\left(15a^3A+27a^2bB+27aAb^2+7b^3B\right)E\left(\frac{1}{2}(c+dx)\right|}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]),x]
```

```
[Out] (84*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*EllipticE[(c + d*x)/2, 2
] + 60*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*EllipticF[(c + d*x)/2,
 2] + Sqrt[Cos[c + d*x]]*(7*b*(108*a*A*b + 108*a^2*B + 43*b^2*B)*Cos[c + d*
x] + 5*(252*a^2*A*b + 78*A*b^3 + 84*a^3*B + 234*a*b^2*B + 18*b^2*(A*b + 3*a
*B)*Cos[2*(c + d*x)] + 7*b^3*B*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^3\cos(dx+c)^4+Aa^3+\left(3Bab^2+Ab^3\right)\cos(dx+c)^3+3\left(Ba^2b+Aab^2\right)\cos(dx+c)^2+\left(Ba^3+3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x
)
```

maple [B] time = 1.58, size = 745, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b^3+2160*B*a*b^2+2240*B*b^
3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*A*a*b^2-1080*A*b^3-1512*B
*a^2*b-3240*B*a*b^2-2072*B*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(12
60*A*a^2*b+1512*A*a*b^2+840*A*b^3+420*B*a^3+1512*B*a^2*b+2520*B*a*b^2+952*B
*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-630*A*a^2*b-378*A*a*b^2-240
*A*b^3-210*B*a^3-378*B*a^2*b-720*B*a*b^2-168*B*b^3)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*a*b^2+315*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+75*A*b^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-567*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-147*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*b^3+105*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*B*a*b^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
```

```
2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3*(A+B*cos(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x
)
```

mupad [B] time = 1.54, size = 328, normalized size = 1.29

$$\frac{2\left(A\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+A\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+A\,a^2\,b\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}+\frac{B\,a^3\,\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3,x)
```

```
[Out] (2*(A*a^3*ellipticE(c/2 + (d*x)/2, 2) + A*a^2*b*ellipticF(c/2 + (d*x)/2, 2)
 + A*a^2*b*cos(c + d*x)^(1/2)*sin(c + d*x)))/d + (B*a^3*((2*cos(c + d*x)^(1
/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d - (2*A*b^3*cos(
c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9
*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hype
rgeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (
6*A*a*b^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
 + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (6*B*a^2*b*cos(c + d*x)^(7/2)*si
n(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^
2)^(1/2)) - (2*B*a*b^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4]
, 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**3*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.361 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=205

$$\frac{2b\left(18a^2B+21aAb+5b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+$$

```
[Out] 2/5*(15*A*a^2*b+3*A*b^3+5*B*a^3+9*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(21*A*a^3+21*A
*a*b^2+21*B*a^2*b+5*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/35*b^2*(7*A*b+11*B*a)*cos(d*x+c)^
(3/2)*sin(d*x+c)/d+2/21*b*(21*A*a*b+18*B*a^2+5*B*b^2)*sin(d*x+c)*cos(d*x+c)
^(1/2)/d+2/7*b*B*(a+b*cos(d*x+c))^2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.48, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2990, 3033, 3023, 2748, 2641, 2639}

$$\frac{2\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(15a^2Ab+5a^3B+9ab^2B+3Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/
(5*d) + (2*(21*a^3*A + 21*a*A*b^2 + 21*a^2*b*B + 5*b^3*B)*EllipticF[(c + d*
x)/2, 2])/(21*d) + (2*b*(21*a*A*b + 18*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(21*d) + (2*b^2*(7*A*b + 11*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(35*d) + (2*b*B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2*Sin[c + d*x]
)/(7*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{2bB\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2\sin(c+dx)}{7d} + \frac{2}{7}\int \frac{(a+b}{\quad}$$

$$= \frac{2b^2(7Ab+11aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2bB\sqrt{\cos(c+dx)}\,(}{\quad}$$

$$= \frac{2b\left(21aAb+18a^2B+5b^2B\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2b^2(7A}{\quad}$$

$$= \frac{2b\left(21aAb+18a^2B+5b^2B\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2b^2(7A}{\quad}$$

$$= \frac{2\left(15a^2Ab+3Ab^3+5a^3B+9ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(21a^3A\right.}{\quad}$$

Mathematica [A] time = 1.30, size = 158, normalized size = 0.77

$$\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}\,\left(5\left(42a^2B+42aAb+3b^2B\cos(2(c+dx))+13b^2B\right)+42b(3aB+Ab)\cos(c+dx)\right)+1}{105}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (42*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2]
+ 10*(21*a^3*A + 21*a*A*b^2 + 21*a^2*b*B + 5*b^3*B)*EllipticF[(c + d*x)/2,
2] + b*Sqrt[Cos[c + d*x]]*(42*b*(A*b + 3*a*B)*Cos[c + d*x] + 5*(42*a*A*b +
42*a^2*B + 13*b^2*B + 3*b^2*B*Cos[2*(c + d*x)]))*Sin[c + d*x])/(105*d)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb^3\cos(dx+c)^4+Aa^3+\left(3\,Bab^2+Ab^3\right)\cos(dx+c)^3+3\left(Ba^2b+Aab^2\right)\cos(dx+c)^2+\left(Ba^3+3\,Aa\right.}{\sqrt{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x
)
```

maple [B] time = 1.50, size = 664, normalized size = 3.24

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240B\,b^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-168A\,b^3-504Ba\,b^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^3-504*B*a*b^2-360*B*b^3)*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a*b^2+168*A*b^3+420*B*a^2*b+5
04*B*a*b^2+280*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*A*a*b^2
-42*A*b^3-210*B*a^2*b-126*B*a*b^2-80*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)+105*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*A*a*b^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^
3+105*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10
5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x
)
```

mupad [B] time = 1.43, size = 275, normalized size = 1.34

$$\frac{2\left(B\,a^3\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+B\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+B\,a^2\,b\,\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)\right)}{d}+\frac{2\,A\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a^2\,b\,\mathrm{E}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*(B*a^3*ellipticE(c/2 + (d*x)/2, 2) + B*a^2*b*ellipticF(c/2 + (d*x)/2, 2)
 + B*a^2*b*cos(c + d*x)^(1/2)*sin(c + d*x)))/d + (2*A*a^3*ellipticF(c/2 + (
d*x)/2, 2))/d + (6*A*a^2*b*ellipticE(c/2 + (d*x)/2, 2))/d + (3*A*a*b^2*((2*
cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d
- (2*A*b^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(
c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*b^3*cos(c + d*x)^(9/2)*sin
(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2
)^(1/2)) - (6*B*a*b^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4],
 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.362 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=202

$$-\frac{2b\left(6a^2A-3abB-Ab^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2\left(3a^3B+9a^2Ab+3ab^2B+Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(5a^3\right.}{}$$

```
[Out] -2/5*(5*A*a^3-15*A*a*b^2-15*B*a^2*b-3*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(9*A*a^2*b+A*
b^3+3*B*a^3+3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d-2/5*b^2*(5*A*a-B*b)*cos(d*x+c)^(3/2)*si
n(d*x+c)/d+2*a*A*(a+b*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(1/2)-2/3*b*(6*
A*a^2-A*b^2-3*B*a*b)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.46, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2989, 3033, 3023, 2748, 2641, 2639}

$$\frac{2\left(9a^2Ab+3a^3B+3ab^2B+Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(5a^3A-15a^2bB-15aAb^2-3b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{2b\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*EllipticE[(c + d*x)/2, 2]
)/(5*d) + (2*(9*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B)*EllipticF[(c + d*x)/
2, 2])/(3*d) - (2*b*(6*a^2*A - A*b^2 - 3*a*b*B)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(3*d) - (2*b^2*(5*a*A - b*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) +
 (2*a*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a+b\cos(c+dx))}{}$$

$$= -\frac{2b^2(5aA-bB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2aA(a+b\cos(c+}{d\sqrt{\cos(}}$$

$$= -\frac{2b\left(6a^2A-Ab^2-3abB\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} - \frac{2b^2(5a}{}$$

$$= -\frac{2b\left(6a^2A-Ab^2-3abB\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d} - \frac{2b^2(5a}{}$$

$$= -\frac{2\left(5a^3A-15aAb^2-15a^2bB-3b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(9a\right.}{}$$

Mathematica [A] time = 1.15, size = 150, normalized size = 0.74

$$\frac{\frac{\sin(c+dx)\left(3\left(10a^3A+b^3B\cos(2(c+dx))+b^3B\right)+10b^2(3aB+Ab)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)}} + 10\left(3a^3B+9a^2Ab+3ab^2B+Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right) + }{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] ((-30*a^3*A + 90*a*A*b^2 + 90*a^2*b*B + 18*b^3*B)*EllipticE[(c + d*x)/2, 2]
 + 10*(9*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B)*EllipticF[(c + d*x)/2, 2] +
 ((10*b^2*(A*b + 3*a*B)*Cos[c + d*x] + 3*(10*a^3*A + b^3*B + b^3*B*Cos[2*(c
 + d*x)]))*Sin[c + d*x])/Sqrt[Cos[c + d*x]])/(15*d)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb^3\cos(dx+c)^4 + Aa^3 + \left(3\,Bab^2+Ab^3\right)\cos(dx+c)^3 + 3\left(Ba^2b+Aab^2\right)\cos(dx+c)^2 + \left(Ba^3+3\,A\right.}{\cos(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x
)
```

maple [B] time = 1.63, size = 867, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/15*(-24*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*b^2*(5*A*b+15*B*a+6*B*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*a^3+5*A*b
^3+15*B*a*b^2+3*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+45*A*a^2*b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)+5*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)+15*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*a^3-45*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2+15*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+15*B*a*b^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-45*B*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^
2*b-9*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(
```

```
1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x
)
```

mupad [B] time = 1.46, size = 248, normalized size = 1.23

$$\frac{A\,b^3\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,B\,a^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6\,A\,a^2\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^(3/2),x)
```

```
[Out] (A*b^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d + (2*B*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*A*a*b^2*ellipticE
(c/2 + (d*x)/2, 2))/d + (6*A*a^2*b*ellipticF(c/2 + (d*x)/2, 2))/d + (6*B*a^
2*b*ellipticE(c/2 + (d*x)/2, 2))/d + (3*B*a*b^2*((2*cos(c + d*x)^(1/2)*sin(
c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d + (2*A*a^3*sin(c + d*x)
*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c
+ d*x)^2)^(1/2)) - (2*B*b^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2,
 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.363 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=192

$$\frac{2a^2(3aB+7Ab)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)}{d}$$

```
[Out] -2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*a^3+9*A*a*b^2+9*B*
a^2*b+B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*A*(a+b*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c
)^(3/2)+2/3*a^2*(7*A*b+3*B*a)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-2/3*b^2*(A*a-B*
b)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.47, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2989, 3031, 3023, 2748, 2641, 2639}

$$\frac{2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^2Ab+a^3B-3ab^2B-Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2(3aB+}{3d\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/d +
(2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*B)*EllipticF[(c + d*x)/2, 2])/(3*d)
 + (2*a^2*(7*A*b + 3*a*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) - (2*b^2*(
a*A - b*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + b*Cos[c + d
*x])^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{(a+b\cos(c+dx))\big(}{}$$

$$= \frac{2a^2(7Ab+3aB)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2aA(a+b\cos(c+dx))^2\sin(c+}{3d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(7Ab+3aB)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} - \frac{2b^2(aA-bB)\sqrt{\cos(c+dx)}\,\sin(c}{3d}$$

$$= \frac{2a^2(7Ab+3aB)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} - \frac{2b^2(aA-bB)\sqrt{\cos(c+dx)}\,\sin(c}{3d}$$

$$= -\frac{2\left(3a^2Ab-Ab^3+a^3B-3ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(a^3A+9\right.}{}$$

Mathematica [A] time = 1.11, size = 165, normalized size = 0.86

$$\frac{2a^3A\tan(c+dx)+6a^3B\sin(c+dx)+18a^2Ab\sin(c+dx)+2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] (-6*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2] + 2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*B)*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + 18*a^2*A*b*Sin[c + d*x] + 6*a^3*B*Sin[c + d*x
] + b^3*B*Sin[2*(c + d*x)] + 2*a^3*A*Tan[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb^3\cos(dx+c)^4+Aa^3+\left(3\,Bab^2+Ab^3\right)\cos(dx+c)^3+3\left(Ba^2b+Aab^2\right)\cos(dx+c)^2+\left(Ba^3+3\,Aa\right.}{\cos(dx+c)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x
)
```

maple [B] time = 3.80, size = 1212, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(8*B*b^3*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*sin(1/2*d*
x+1/2*c)^2+18*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2+18*A*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2-6*A*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*b^3*sin(1/2*d*x+1/2*c)^2-36*A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+18*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+2*B*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2+6*B*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*s
in(1/2*d*x+1/2*c)^2-18*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^
2-12*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-8*B*b^3*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4-A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*a*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
```

```
/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b+3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*b^3+2*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+18*A*a^2*b*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^3*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+9*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a*b^2+6*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*B*b^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x
)
```

mupad [B] time = 2.34, size = 255, normalized size = 1.33

$$\frac{2\left(A\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\,b^3+3\,A\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\,b^2\right)}{d}+\frac{B\,b^3\,\left(\frac{2\,\sqrt{\cos(c+d\,x)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{6\,B\,a\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^(5/2),x)
```

```
[Out] (2*(A*b^3*ellipticE(c/2 + (d*x)/2, 2) + 3*A*a*b^2*ellipticF(c/2 + (d*x)/2,
2)))/d + (B*b^3*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 +
 (d*x)/2, 2))/3))/d + (6*B*a*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (6*B*a^2*
b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^3*sin(c + d*x)*hypergeom([-3/4, 1
/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
+ (2*B*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos
(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (6*A*a^2*b*sin(c + d*x)*hypergeom
```

```
([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^
(1/2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.364 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=204

$$\frac{2a\left(3a^2A+15abB+14Ab^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a^2(5aB+9Ab)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(a^3B+3a^2Ab+9ab^2B+3Ab^3\right)F\left(\frac{1}{2}(c\right.}{3d}$$

```
[Out] -2/5*(3*A*a^3+15*A*a*b^2+15*B*a^2*b-5*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(3*A*a^2*b+3*
A*b^3+B*a^3+9*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/15*a^2*(9*A*b+5*B*a)*sin(d*x+c)/d/cos
(d*x+c)^(3/2)+2/5*a*A*(a+b*cos(d*x+c))^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/5*
a*(3*A*a^2+14*A*b^2+15*B*a*b)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2989, 3031, 3021, 2748, 2641, 2639}

$$\frac{2\left(3a^2Ab+a^3B+9ab^2B+3Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(3a^3A+15a^2bB+15aAb^2-5b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*EllipticE[(c + d*x)/2, 2]
)/(5*d) + (2*(3*a^2*A*b + 3*A*b^3 + a^3*B + 9*a*b^2*B)*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*a^2*(9*A*b + 5*a*B)*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2
)) + (2*a*(3*a^2*A + 14*A*b^2 + 15*a*b*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*a*A*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)
)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\cos(c+dx))\Big(}{}$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+b\cos(c+dx))^2\sin(c+}{5d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\left(3a^2A+14Ab^2+15abB\right)\sin(}{5d\sqrt{\cos(c+dx)}}$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\left(3a^2A+14Ab^2+15abB\right)\sin(}{5d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2\left(3a^3A+15aAb^2+15a^2bB-5b^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(3a^2\right.}{}$$

Mathematica [A] time = 2.21, size = 176, normalized size = 0.86

$$\frac{6a^3A\tan(c+dx)+9a\left(a^2A+5abB+5Ab^2\right)\sin(2(c+dx))+10a^2(aB+3Ab)\sin(c+dx)+10\left(a^3B+3a^2Ab+9\right.}{15d\cos^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] (-6*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Cos[c + d*x]^(3/2)*Ellipt
icE[(c + d*x)/2, 2] + 10*(3*a^2*A*b + 3*A*b^3 + a^3*B + 9*a*b^2*B)*Cos[c +
d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a^2*(3*A*b + a*B)*Sin[c + d*x] +
9*a*(a^2*A + 5*A*b^2 + 5*a*b*B)*Sin[2*(c + d*x)] + 6*a^3*A*Tan[c + d*x])/(1
5*d*Cos[c + d*x]^(3/2))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb^3\cos(dx+c)^4+Aa^3+\left(3\,Bab^2+Ab^3\right)\cos(dx+c)^3+3\left(Ba^2b+Aab^2\right)\cos(dx+c)^2+\left(Ba^3+3\,Aa\right.}{\cos(dx+c)^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
/cos(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x
)
```

maple [B] time = 4.33, size = 997, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*a*b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^3*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
6*a*b*(A*b+B*a)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d
*x+1/2*c)^2-1)-2/5*A*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*
sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE
```

```
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^2*(3*A*b+B*a)*(-1/
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x
)
```

mupad [B] time = 3.59, size = 291, normalized size = 1.43

$$\frac{2\left(B\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)b^3+3\,B\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)b^2\right)}{d}+\frac{2\,A\,b^3\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a^3\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\,-\frac{1}{4};\,\cos(c-\right.}{5\,d\cos(c+d\,x)^{5/2}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*(B*b^3*ellipticE(c/2 + (d*x)/2, 2) + 3*B*a*b^2*ellipticF(c/2 + (d*x)/2,
2)))/d + (2*A*b^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^3*sin(c + d*x)*hy
pergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c
+ d*x)^2)^(1/2)) + (2*B*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c
+ d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (6*A*a*b^2*sin
(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2
)*(sin(c + d*x)^2)^(1/2)) + (2*A*a^2*b*sin(c + d*x)*hypergeom([-3/4, 1/2],
1/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (6*B*
a^2*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c +
d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
[Out] Timed out
```

3.365 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=182

$$-\frac{2a^3(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^4d(a+b)}+\frac{2\left(3a^2+b^2\right)(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d}-\frac{2\left(-5a^2B+5aAb-3b^2B\right)E\left(\frac{1}{2}(c\right.}{5b^3d}$$

```
[Out] -2/5*(5*A*a*b-5*B*a^2-3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d+2/3*(3*a^2+b^2)*(A*b-B*a)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))/b^4/d-2*a^3*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b^4/(a+b)/d+2/5*B*cos
(d*x+c)^(3/2)*sin(d*x+c)/b/d+2/3*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/
d
```

Rubi [A] time = 0.82, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2990, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(3a^2+b^2\right)(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d}-\frac{2\left(-5a^2B+5aAb-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}-\frac{2a^3(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+\right.}{b^4d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*EllipticE[(c + d*x)/2, 2])/(5*b^3*d) + (2
*(3*a^2 + b^2)*(A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(3*b^4*d) - (2*a^3*(A
*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^4*(a + b)*d) + (2*(
A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*B*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5bd} + \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3aB}{2}+\frac{3}{2}bB\cos(c+dx)+\frac{5}{2}(Ab-aB)\cos\right.}{a+b\cos(c+dx)}}{5b}$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5bd} +$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b^2d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5bd} -$$

$$= -\frac{2\left(5aAb-5a^2B-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d} + \frac{2(Ab-aB)\sqrt{\cos(c+dx)}}{3b^2d}$$

$$= -\frac{2\left(5aAb-5a^2B-3b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d} + \frac{2\left(3a^2+b^2\right)(Ab-aB)F\left(\right.}{3b^4d}$$

Mathematica [A] time = 2.47, size = 260, normalized size = 1.43

$$\frac{\frac{2b^2\left(5a^2B-5aAb+9b^2B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{6\left(5a^2B-5aAb+3b^2B\right)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx}\right.\right.\right.}{a\sqrt{\sin^2(c+dx)}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*b^2*(-5*a*A*b + 5*a^2*B + 9*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2])/(a + b) + 2*b^2*(5*A*b + 4*a*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*
EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + 4*b^2*Sqrt[Cos[c + d*
x]]*(5*A*b - 5*a*B + 3*b*B*Cos[c + d*x])*Sin[c + d*x] + (6*(-5*a*A*b + 5*a^
2*B + 3*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a +
 b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-
```

```
(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^
2]))/(30*b^4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

maple [B] time = 1.64, size = 1074, normalized size = 5.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*B*a*b^3
+24*B*b^4)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*a*b^3-20*A*b^4-20*
B*a^2*b^2+44*B*a*b^3-24*B*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10
*A*a*b^3+10*A*b^4+10*B*a^2*b^2-16*B*a*b^3+6*B*b^4)*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-15*A*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*a^2*b^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-5*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*b^4+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
```

```
*a*b^3-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^3*b-15*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^4+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-5*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*a^2*b^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-15*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a^3*b+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a*b^3+9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))*a^4)/b^4/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{5/2}\,\left(A+B\,\cos(c+d\,x)\right)}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.366 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=137

$$\frac{2a^2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2\left(-3a^2B+3aAb-b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}+\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2B\,\mathrm{s}}{}$$

```
[Out] 2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/b^2/d-2/3*(3*A*a*b-3*B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d+2
*a^2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(s
in(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b^3/(a+b)/d+2/3*B*sin(d*x+c)*cos(d*x+c
)^(1/2)/b/d
```

Rubi [A] time = 0.51, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2990, 3059, 2639, 3002, 2641, 2805}

$$-\frac{2\left(-3a^2B+3aAb-b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}+\frac{2a^2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}+\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2B}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b^2*d) - (2*(3*a*A*b - 3*a^2*B -
 b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*b^3*d) + (2*a^2*(A*b - a*B)*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*B*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3*b*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx = \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{aB}{2}+\frac{1}{2}bB\cos(c+dx)+\frac{3}{2}(Ab-aB)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}dx}{3b}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} - \frac{2\int \frac{-\frac{1}{2}abB+\frac{1}{2}\left(3aAb-3a^2B-b^2B\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}dx}{3b^2} +$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{\left(a^2(Ab-\right.}{}$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} - \frac{2\left(3aAb-3a^2B-b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}$$

Mathematica [A] time = 1.44, size = 207, normalized size = 1.51

$$\frac{\frac{3(Ab-aB)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab^2\sqrt{\sin^2(c+dx)}} + \frac{(3Ab-aB)\Pi\left(\frac{2}{a}\right.}{a+}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (((3*A*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + B*(2*E
llipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
(a + b)) + 2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x] + (3*(A*b - a*B)*(-2*a*b*Ell
ipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c +
 d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/(3*b*d)
```

fricas [F] time = 114.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^2+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c))/(b*cos(d*x
 + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

maple [B] time = 1.64, size = 786, normalized size = 5.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-4*B*a*b^2+4*
B*b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(2*B*a*b^2-2*B*b^3)*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*a*b^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*b^3-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2*b-3*a^3*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+3*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*a*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a^2*b+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))*a^3)/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.367 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=89

$$\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/b/d+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d-2*a*(A*b-B*a)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),
2^(1/2))/b^2/(a+b)/d
```

Rubi [A] time = 0.21, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3002, 2639, 2803, 2641, 2805}

$$\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(b*d) + (2*(A*b - a*B)*EllipticF[(c + d*x)/
2, 2])/(b^2*d) - (2*a*(A*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(b^2*(a + b)*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
```

```
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{a+b\cos(c+dx)}\,dx &= \frac{B\int\sqrt{\cos(c+dx)}\,dx}{b}-\frac{(-Ab+aB)\int\frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx}{b}\\ &= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{(Ab-aB)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2}-\frac{(a(Ab-aB))\int\frac{}{\sqrt{\cos}}}{}\\ &= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\Pi\left(\frac{2}{a}\right.}{b^2(a+}\end{aligned}$$

Mathematica [A] time = 0.91, size = 128, normalized size = 1.44

$$\frac{Ab\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)-\frac{2B\sin(c+dx)\left(-(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+a\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+bE\left(\sin\right.\right.}{\sqrt{\sin^2(c+dx)}}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (A*b*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(a + b)) - (2*B*(b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a
```

```
+ b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + a*EllipticPi[-(b/a), ArcSi
n[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/Sqrt[Sin[c + d*x]^2])/(b^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

maple [A] time = 1.41, size = 295, normalized size = 3.31

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-A*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a*b-B*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*a*b+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+B*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2)/b^2/(a-b)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.368 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=61

$$\frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/b/d+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b/(a+b)/d
```

Rubi [A] time = 0.14, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3002, 2641, 2805}

$$\frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(b*d) + (2*(A*b - a*B)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(b*(a + b)*d)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
```

```
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx = \frac{B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b} - \frac{(-Ab+aB)\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{b}$$

$$= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} + \frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d}$$

Mathematica [A] time = 0.21, size = 58, normalized size = 0.95

$$\frac{2\left((Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)+B(a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{bd(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*((a + b)*B*EllipticF[(c + d*x)/2, 2] + (A*b - a*B)*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]))/(b*(a + b)*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

maple [A] time = 1.36, size = 217, normalized size = 3.56

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(A\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{(a-b)\,b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))*b+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))*a)/(a-b)/b/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,\left(a+b\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.369 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=86

$$-\frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] -2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a/d-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(a+b)/d+2*A*sin(d*x
+c)/a/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3000, 3059, 2639, 12, 2805}

$$-\frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (-2*A*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2])/(a*(a + b)*d) + (2*A*Sin[c + d*x])/(a*d*Sqrt[Cos[c
+ d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}(-Ab+aB)-\frac{1}{2}aA\cos(c+dx)-\frac{1}{2}Ab\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{a}$$

$$= \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{A\int\sqrt{\cos(c+dx)}\,dx}{a} - \frac{2\int \frac{b(Ab-aB)}{2\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,d}{ab}$$

$$= -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{2A\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(Ab-aB)\int\frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos}}{a}$$

$$= -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d} + \frac{2A\sin(c+}{ad\sqrt{\cos(c+}}$$

Mathematica [B] time = 2.48, size = 206, normalized size = 2.40

$$\frac{-\frac{2A\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}}+\frac{2(2aB-3Ab)\Pi\left(\frac{2}{a+}\right.}{a+b}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] ((2*(-3*A*b + 2*a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (
2*a*A*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*
x)/2, 2])/(a + b)))/b + (4*A*Sin[c + d*x])/Sqrt[Cos[c + d*x]] - (2*A*(-2*a*
b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[
Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Co
s[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(2*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

maple [B] time = 2.67, size = 327, normalized size = 3.80

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4(-Ab+aB)b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{\ldots}\right.}{a\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}\ldots\right.}{\sin\left(\frac{dx}{2}+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(-A*b+B*a)/a
/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d
*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^{3/2}\,\left(a+b\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

3.370 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=150

$$\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{}{3a}$$

```
[Out] 2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/a^2/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+2*b*(A*b-B*a)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b
),2^(1/2))/a^2/(a+b)/d+2/3*A*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-2*(A*b-B*a)*si
n(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*A*EllipticF[(c + d*x
)/2, 2])/(3*a*d) + (2*b*(A*b - a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a^2*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*(A
*b - a*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{-\frac{3}{2}(Ab-aB)+\frac{1}{2}aA\cos(c+dx)+\frac{1}{2}Ab\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{4\int \frac{\frac{1}{4}\left(a^2A+3Ab^2-3abB\right)+\frac{1}{4}a(4A\cdots}{\sqrt{\cos(c\cdots}}\cdots}{\cdots}$$

$$= \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{4\int \frac{-\frac{1}{4}b\left(a^2A+3Ab^2-3abB\right)-\frac{1}{4}aA\cdots}{\sqrt{\cos(c+dx)}\,(a+b\cos(\cdots}\cdots}{3a^2b}$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2A\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+d\cdots}{a^2d\sqrt{\cos(c+dx)}}\cdots$$

$$= \frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{2b(Ab-aB)\Pi\left(\frac{2b}{a+b}\cdots\right.}{a^2(a+\cdots}\cdots$$

Mathematica [A] time = 2.29, size = 260, normalized size = 1.73

$$\frac{\frac{2a\left(2a^2A-9abB+9Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{6(Ab-aB)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2ab\cdots\right.}{b\sqrt{\sin^2(c+dx)}}\cdots}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] ((2*a*(2*a^2*A + 9*A*b^2 - 9*a*b*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a + b) + (a*(8*a*A*b - 6*a^2*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (4*a^2*A*Sin[c + d*x
])/Cos[c + d*x]^(3/2) + (12*a*(-(A*b) + a*B)*Sin[c + d*x])/Sqrt[Cos[c + d*x
]] + (6*(A*b - a*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a
*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*Ellipti
cPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c +
d*x]^2]))/(6*a^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

maple [B] time = 3.72, size = 468, normalized size = 3.12

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4(Ab-aB)b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\right.}{a^2\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(A*b-B*a)*b^
2/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b+B*a)/a^2*(-(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*A/a*(-1/6*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2
*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^{5/2}\,\left(a+b\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.371 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=303

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)}+\frac{\left(-5a^3B+3a^2Ab+4a\right.}{b^3}$$

```
[Out] (3*A*a^2*b-2*A*b^3-5*B*a^3+4*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)/d-1/3*(9*A*a
^3*b-12*A*a*b^3-15*B*a^4+16*B*a^2*b^2+2*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^4/(a^2-b^2)/d+a
^2*(3*A*a^2*b-5*A*b^3-5*B*a^3+7*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)/b^4/(a
+b)^2/d+a*(A*b-B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+
c))-1/3*(3*A*a*b-5*B*a^2+2*B*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)
/d
```

Rubi [A] time = 0.93, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2989, 3049, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(9a^3Ab+16a^2b^2B-15a^4B-12aAb^3+2b^4B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\left(a^2-b^2\right)}+\frac{\left(3a^2Ab-5a^3B+4ab^2B-2Ab^3\right)E\left(\frac{1}{2}(c+dx\right.}{b^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b^
3*(a^2 - b^2)*d) - ((9*a^3*A*b - 12*a*A*b^3 - 15*a^4*B + 16*a^2*b^2*B + 2*b
^4*B)*EllipticF[(c + d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d) + (a^2*(3*a^2*A*b -
5*A*b^3 - 5*a^3*B + 7*a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(
(a - b)*b^4*(a + b)^2*d) - ((3*a*A*b - 5*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) + (a*(A*b - a*B)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(Ab-aB)+b(Ab-aB\cdots\right.}{a\cdots}}{b\cdots}$$

$$= -\frac{\left(3aAb-5a^2B+2b^2B\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\cos^{\frac{3}{2}}\cdots}{b\left(a^2-b^2\right)d(\cdots}$$

$$= -\frac{\left(3aAb-5a^2B+2b^2B\right)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\cos^{\frac{3}{2}}\cdots}{b\left(a^2-b^2\right)d(\cdots}$$

$$= \frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} - \frac{\left(3aAb-5a^2B+\cdots\right.}{\cdots}$$

$$= \frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} - \frac{\left(9a^3Ab-12aAb\cdots\right.}{\cdots}$$

Mathematica [A] time = 3.24, size = 318, normalized size = 1.05

$$\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(\frac{3a^2(aB-Ab)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}+2B\right) - \frac{\frac{8\left(2a^2B-3aAb+b^2B\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{2\left(5a^3B-3a^2Ab-8ab\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(2*B + (3*a^2*(-(A*b) + a*B))/((a^2 - b^2)*(a + b*Cos
[c + d*x])))*Sin[c + d*x] - ((2*(-3*a^2*A*b + 6*A*b^3 + 5*a^3*B - 8*a*b^2*B
)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(-3*a*A*b + 2*a^2
*B + b^2*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]))/(a + b) + (6*(-3*a^2*A*b + 2*A*b^3 + 5*a^3*B - 4*a*b^2*
B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Elliptic
F[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSi
n[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a
 - b)*(a + b)))/(12*b^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x
)
```

maple [B] time = 4.85, size = 1066, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^4/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*b^2*cos(1/2*d*x+1/2*c)
```

```
*sin(1/2*d*x+1/2*c)^4+6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^2-9*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^2*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+2*B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)-4*a^2/b^3*(3*A*b-4*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b-
B*a)/b^4*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*
b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{5/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

3.372 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=224

$$-\frac{\left(-3a^2B+aAb+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\left(-3a^3B+a^2Ab+4ab^2B-2A\right.}{b^3d\left(a^2-b\right.}$$

```
[Out] -(A*a*b-3*B*a^2+2*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d+(A*a^2*b-2*A*b^3-3*B*a^
3+4*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)/d-a*(A*a^2*b-3*A*b^3-3*B*a^3+5*B*a*b^
2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1
/2*c),2*b/(a+b),2^(1/2))/(a-b)/b^3/(a+b)^2/d+a*(A*b-B*a)*sin(d*x+c)*cos(d*x
+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.63, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2989, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^2Ab-3a^3B+4ab^2B-2Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}-\frac{\left(-3a^2B+aAb+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{a\left(a^2Ab-3a^3B+5\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -(((a*A*b - 3*a^2*B + 2*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*
d)) + ((a^2*A*b - 2*A*b^3 - 3*a^3*B + 4*a*b^2*B)*EllipticF[(c + d*x)/2, 2])
/(b^3*(a^2 - b^2)*d) - (a*(a^2*A*b - 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*Ellipti
cPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) + (a*(A*b - a
*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}a(Ab-aB)+b(Ab-aB)\cos(c+dx)+}{\sqrt{\cos(c+dx)}\,(a+}}{b\left(a^2-\right.}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}ab(Ab-aB)+\frac{1}{2}\left(a^2Ab-2Ab^3-3a^3B-\right.}{\sqrt{\cos(c+dx)}\,(a+b\cos(c}}{b^2\left(a^2-b^2\right)}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\mathrm{s}}{b\left(a^2-b^2\right)d(a+b\cos(c\right.}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} + \frac{\left(a^2Ab-2Ab^3-3a^3B+4a\right.}{b^3\left(a^2-b\right.}$$

Mathematica [A] time = 2.74, size = 280, normalized size = 1.25

$$\frac{\frac{2\left(a^2B+aAb-2b^2B\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{2\left(3a^2B-aAb-2b^2B\right)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx}\right.\right.\right.}{ab^2\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)} \Bigg/ 4bd$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] ((-4*a*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*
Cos[c + d*x])) + ((2*(a*A*b + a^2*B - 2*b^2*B)*EllipticPi[(2*b)/(a + b), (c
 + d*x)/2, 2])/(a + b) + (8*(-(A*b) + a*B)*((a + b)*EllipticF[(c + d*x)/2,
2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (2*(-(a*A*b) +
 3*a^2*B - 2*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a
*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*Ellipti
cPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[
c + d*x]^2]))/((a - b)*(a + b)))/(4*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x
)
```

maple [B] time = 3.96, size = 849, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-2
*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*b)+4*a/b^2*(2*A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(A*b-
B*a)/b^3*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*
b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
```

```
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.373 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=198

$$\frac{\left(a^2B+aAb-2b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\left(a^3B+a\right.}{}$$

```
[Out] (A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)/d+(A*a*b+B*a^2-2*B*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^
2-b^2)/d-(A*a^2*b+A*b^3+B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)/b^2/(a
+b)^2/d-(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.54, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2999, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^2B+aAb-2b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{\left(a^2Ab+a^3B-3ab^2B+Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\right.}{b^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((a*A*b + a^2*B
 - 2*b^2*B)*EllipticF[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - ((a^2*A*b + A*
b^3 + a^3*B - 3*a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b
)*b^2*(a + b)^2*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 -
b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}(Ab-aB)-(aA-bB)\cos(c+dx)-\frac{1}{2}(A}{\sqrt{\cos(c+dx)}\,(a+b\cos(c}}{-a^2+b^2}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{1}{2}b(Ab-aB)+\frac{1}{2}\left(aAb+a^2B-2b^2B\right)\text{co}}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}}{b\left(a^2-b^2\right)}$$

$$= \frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} - \frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{(a}{}$$

$$= \frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{\left(aAb+a^2B-2b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{(}{}$$

Mathematica [A] time = 2.38, size = 260, normalized size = 1.31

$$\frac{\frac{4(aB-Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a+b\cos(c+dx))} - \frac{\frac{2(Ab-aB)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab^2\sqrt{\sin^2(c+dx)}}+\cdots}{(b-a)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] ((4*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos
[c + d*x])) - ((2*(-(A*b) + a*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/(a + b) + ((4*a*A - 4*b*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(A*b - a*B)*(-2*a*b*Ellipt
icE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos
[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*
x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)))/
(4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x
)
```

maple [B] time = 3.53, size = 808, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a*(A*b-B*a)/b^2*(-b^2/a/(a^2-
b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*
b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b
^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.374 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\left(a^3(-B)+3a^2A\right.}{}$$

```
[Out] -(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)/d-(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)/d+(3*A
*a^2*b-A*b^3-B*a^3-B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(a-b)/b/(a+b)^2/d+b*(A*
b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.62, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3000, 3059, 2639, 3002, 2641, 2805}

$$-\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}+\frac{\left(3a^2Ab+a^3(-B)-ab^2B-Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] -(((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d)) - ((A*b - a*B)
*EllipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((3*a^2*A*b - A*b^3 - a^3*B
 - a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a*(a - b)*b*(a + b)
^2*d) + (b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2A-Ab^2-abB\right)-a(Ab-aB)\cos}{\sqrt{\cos(c+dx)}\,(a+}}{a\left(a^2-\right.}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}b\left(2a^2A-Ab^2-abB\right)+\frac{1}{2}ab(Ab-a}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+d}}{ab\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))}$$

$$= -\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} - \frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{\left(3a^2Ab\right.}{}$$

Mathematica [A] time = 2.68, size = 274, normalized size = 1.37

$$\frac{\frac{4b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a+b\cos(c+dx))} + \frac{\frac{2\left(4a^2A-abB-3Ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2(aB-Ab)\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(\right.\right.\right.}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),
x]
```

```
[Out] ((4*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[
c + d*x])) + ((2*(4*a^2*A - 3*A*b^2 - a*b*B)*EllipticPi[(2*b)/(a + b), (c +
 d*x)/2, 2])/(a + b) + (4*a*(-(A*b) + a*B)*(2*EllipticF[(c + d*x)/2, 2] - (
2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(-(A*b) + a
*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Ellipti
cF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcS
in[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a
- b)*(a + b)))/(4*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

maple [B] time = 3.36, size = 721, normalized size = 3.60

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}+\frac{2(A\cdots}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/(-2*a*b+2*
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+2*(A*b-B*a)/b*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b
+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-
b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
```

```
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.375 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=256

$$\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\left(2a^2A+abB-3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{\left(2a^2A+abB-3Ab^2\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}+\frac{}{ad\left(a^2-\right.}$$

```
[Out] -(2*A*a^2-3*A*b^2+B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d+(A*b-B*a)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/
(a^2-b^2)/d-(5*A*a^2*b-3*A*b^3-3*B*a^3+B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^2/(
a-b)/(a+b)^2/d+(2*A*a^2-3*A*b^2+B*a*b)*sin(d*x+c)/a^2/(a^2-b^2)/d/cos(d*x+c
)^(1/2)+b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))/cos(d*x+c)^(1
/2)
```

Rubi [A] time = 0.92, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\left(2a^2A+abB-3Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}-\frac{\left(5a^2Ab-3a^3B+ab^2B-3Ab^3\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c\right.}{a^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] -(((2*a^2*A - 3*A*b^2 + a*b*B)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*
d)) + ((A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) - ((5*a^2*A
*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(a^2*(a - b)*(a + b)^2*d) + ((2*a^2*A - 3*A*b^2 + a*b*B)*Sin[c + d*x])/(a
^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2A-3Ab^2+abB\right)-a(Ab\cdots}{\cos^{\frac{3}{2}}(c\cdots}}{\cdots}$$

$$= \frac{\left(2a^2A-3Ab^2+abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\,\mathrm{co}\cdots}$$

$$= \frac{\left(2a^2A-3Ab^2+abB\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\,\mathrm{co}\cdots}$$

$$= -\frac{\left(2a^2A-3Ab^2+abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{\left(2a^2A-3Ab^2+abB\right)\sin(c\cdots}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+d\cdots}}$$

$$= -\frac{\left(2a^2A-3Ab^2+abB\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} - \cdots$$

Mathematica [A] time = 4.22, size = 316, normalized size = 1.23

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{b^2(Ab-aB)\sin(c+dx)}{\left(b^2-a^2\right)(a+b\cos(c+dx))}+2A\tan(c+dx)\right)-\frac{-\frac{8a\left(a^2A+abB-2Ab^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{b(a+b)}-\frac{2\left(2a^2A+abB-3Ab^2\cdots}{\cdots}}{\cdots}}{4a\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),
x]
```

```
[Out] (-(((2*(-10*a^2*A*b + 9*A*b^3 + 4*a^3*B - 3*a*b^2*B)*EllipticPi[(2*b)/(a +
b), (c + d*x)/2, 2])/(a + b) - (8*a*(a^2*A - 2*A*b^2 + a*b*B)*((a + b)*Elli
pticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a
 + b)) - (2*(2*a^2*A - 3*A*b^2 + a*b*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c
 + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-
2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*
x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b))) + 4*Sqrt[Cos[c + d*x]]*
((b^2*(A*b - a*B)*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x])) + 2*A*T
an[c + d*x]))/(4*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)),
 x)
```

maple [B] time = 4.25, size = 883, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^2/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b+B*a)/a*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3
*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*
b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2)))+2*A/a^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2
/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

3.376 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=345

$$\frac{\left(2a^2A+3abB-5Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\left(a^2-b^2\right)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}+\frac{\left(2a^2A+3abB-5Ab^2\right)\text{si}}{3a^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c}$$

```
[Out] (4*A*a^2*b-5*A*b^3-2*B*a^3+3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)/d+1/3*(2*A*a
^2-5*A*b^2+3*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d+b*(7*A*a^2*b-5*A*b^3-5*B*a^3
+3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(
1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^3/(a-b)/(a+b)^2/d+1/3*(2*A*a^2-5*A*b^2+
3*B*a*b)*sin(d*x+c)/a^2/(a^2-b^2)/d/cos(d*x+c)^(3/2)+b*(A*b-B*a)*sin(d*x+c)
/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))-(4*A*a^2*b-5*A*b^3-2*B*a^3
+3*B*a*b^2)*sin(d*x+c)/a^3/(a^2-b^2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.29, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(2a^2A+3abB-5Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\left(a^2-b^2\right)}+\frac{\left(4a^2Ab-2a^3B+3ab^2B-5Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\left(a^2-b^2\right)}+\frac{b\left(7a^2Ab-5a^3B+\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(a^
3*(a^2 - b^2)*d) + ((2*a^2*A - 5*A*b^2 + 3*a*b*B)*EllipticF[(c + d*x)/2, 2]
)/(3*a^2*(a^2 - b^2)*d) + (b*(7*a^2*A*b - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*El
lipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) + ((2*a^
2*A - 5*A*b^2 + 3*a*b*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3
/2)) - ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Sin[c + d*x])/(a^3*(a^2
 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)
*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2A-5Ab^2+3abB\right)-a(A\cdots}{\cos^{\frac{5}{2}}(c\cdots}}{\cdots}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{b(Ab-aB)\sin(c+dx\cdots}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\,\mathrm{c}\cdots}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\cdots}{a^3\left(a^2-b^2\right)d\sqrt{\cos(c+a\cdots}}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\cdots}{a^3\left(a^2-b^2\right)d\sqrt{\cos(c+a\cdots}}$$

$$= \frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A-5Ab^2+3\cdots\right.}{3a^2\left(a^2-b^2\right)d\cdots}$$

$$= \frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A-5Ab^2+3\cdots\right.}{3a^2\left(a\cdots\right.}$$

Mathematica [A] time = 6.92, size = 427, normalized size = 1.24

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{2\sec(c+dx)(aB\sin(c+dx)-2Ab\sin(c+dx))}{a^3}+\frac{2A\tan(c+dx)\sec(c+dx)}{3a^2}+\frac{Ab^4\sin(c+dx)-ab^3B\sin(c+dx)}{a^3(a^2-b^2)(a+b\cos(c+dx))}\right)}{d}+\frac{2\left(-6a^3bB+12a^2A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),
x]
```

```
[Out] ((2*(4*a^4*A + 44*a^2*A*b^2 - 45*A*b^4 - 30*a^3*b*B + 27*a*b^3*B)*EllipticP
i[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + ((28*a^3*A*b - 40*a*A*b^3 - 12*
a^4*B + 24*a^2*b^2*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/
(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(12*a^2*A*b^2 - 15*A*b^4 - 6*a^3
*b*B + 9*a*b^3*B)*Cos[2*(c + d*x)]*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*
x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2
 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(
a*b^2*Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)))/(12*a^3*(a - b)*(a
 + b)*d) + (Sqrt[Cos[c + d*x]]*((2*Sec[c + d*x]*(-2*A*b*Sin[c + d*x] + a*B*
Sin[c + d*x]))/a^3 + (A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x])/(a^3*(a^2
- b^2)*(a + b*Cos[c + d*x])) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(3*a^2)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)),
 x)
```

maple [B] time = 6.82, size = 1031, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/
2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4*b^2*(2*A*b-B*a)/a^3/(-2*a*b+2*b^2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))+2*(A*b-B*a)*b/a^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b
)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)
/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2
^(1/2)))+2*(-2*A*b+B*a)/a^3*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/
(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,\cos\left(c + d\,x\right)}{{\cos\left(c + d\,x\right)}^{5/2}\,{\left(a + b\,\cos\left(c + d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2),x)
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

3.377 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=367

$$\frac{a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{a\left(-5a^3B+a^2Ab+11ab^2B-7Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{\left(-15a^4B+\right.}{}$$

```
[Out] -1/4*(3*A*a^3*b-9*A*a*b^3-15*B*a^4+29*B*a^2*b^2-8*B*b^4)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^
2-b^2)^2/d+1/4*(3*A*a^4*b-5*A*a^2*b^3+8*A*b^5-15*B*a^5+33*B*a^3*b^2-24*B*a*
b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))/b^4/(a^2-b^2)^2/d-1/4*a*(3*A*a^4*b-6*A*a^2*b^3+15*A*b^5-15*
B*a^5+38*B*a^3*b^2-35*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)^2/b^4/(a+b)^3/d+
1/2*a*(A*b-B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^
2+1/4*a*(A*a^2*b-7*A*b^3-5*B*a^3+11*B*a*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^
2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 1.01, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2989, 3047, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-5a^2Ab^3+3a^4Ab+33a^3b^2B-15a^5B-24ab^4B+8Ab^5\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{\left(3a^3Ab+29a^2b^2B-15a^4B-9aAb^3\right.}{4b^3d\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*EllipticE[(c
+ d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((3*a^4*A*b - 5*a^2*A*b^3 + 8*A*b^5
 - 15*a^5*B + 33*a^3*b^2*B - 24*a*b^4*B)*EllipticF[(c + d*x)/2, 2])/(4*b^4*
(a^2 - b^2)^2*d) - (a*(3*a^4*A*b - 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*a
^3*b^2*B - 35*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b
)^2*b^4*(a + b)^3*d) + (a*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b
*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (a*(a^2*A*b - 7*A*b^3 - 5*a^3*B +
11*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*
Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
```

```
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(Ab-aB)+2b(Ab-aB)\cdots\right.}{(a+b\cdots}}{2b\cdots}$$

$$= \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2Ab-7Ab^3-5a^3B+11ab^2\cdots\right.}{4b^2\left(a^2-b^2\right)^2 d(a+\cdots}$$

$$= \frac{a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{a\left(a^2Ab-7Ab^3-5a^3B+11ab^2\cdots\right.}{4b^2\left(a^2-b^2\right)^2 d(a+\cdots}$$

$$= -\frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d} + \frac{a(A\cdots}{2b\cdots}$$

$$= -\frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d} + \frac{\left(3a^4\cdots\right.}{\cdots}$$

Mathematica [A] time = 5.06, size = 390, normalized size = 1.06

$$\frac{\frac{\frac{8(a^3B+a^2Ab-4ab^2B+2Ab^3)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{(5a^4B-a^3Ab-7a^2b^2B-5aAb^3+8b^4B)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{(15a^4B-3a^3Ab-29a^2b^2B+9aAb^3+8b^4B)}{}}{(a-b)^2(a+b)^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] ((-2*a*Sqrt[Cos[c + d*x]]*(a*(-(a^2*A*b) + 7*A*b^3 + 5*a^3*B - 11*a*b^2*B)
+ b*(-3*a^2*A*b + 9*A*b^3 + 7*a^3*B - 13*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x
])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (((-(a^3*A*b) - 5*a*A*b^3 + 5*a
^4*B - 7*a^2*b^2*B + 8*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a
 + b) + (8*(a^2*A*b + 2*A*b^3 + a^3*B - 4*a*b^2*B)*((a + b)*EllipticF[(c +
d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + ((-3*a
^3*A*b + 9*a*A*b^3 + 15*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*(-2*a*b*EllipticE[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]],
 -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8
*b^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x
)
```

maple [B] time = 6.78, size = 1977, normalized size = 5.39

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^4/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-3
*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*b)-2*a^3*(A*b-B*a)/b^4*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b
+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8
/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b
^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+12/b^3*a*(A*b-2*B*a)/(-2*a*b+2*b^
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))+2*a^2/b^4*(3*A*b-4*B*a)*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
```

```
2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3
*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*
b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{5/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.378 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=344

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{\left(3a^3B+a^2Ab-9ab^2B\right.}{4bd\left(a^2-b^2\right.}$$

```
[Out] -1/4*(A*a^2*b+5*A*b^3+3*B*a^3-9*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d+1/4*(
A*a^3*b-7*A*a*b^3+3*B*a^4-5*B*a^2*b^2+8*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)^2/d
-1/4*(A*a^4*b-10*A*a^2*b^3-3*A*b^5+3*B*a^5-6*B*a^3*b^2+15*B*a*b^4)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(
a+b),2^(1/2))/(a-b)^2/b^3/(a+b)^3/d+1/2*a*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(
1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/4*(A*a^2*b+5*A*b^3+3*B*a^3-9*B*a*b^
2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.99, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2989, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^3Ab-5a^2b^2B+3a^4B-7aAb^3+8b^4B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}-\frac{\left(a^2Ab+3a^3B-9ab^2B+5Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{\left(-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*b
^2*(a^2 - b^2)^2*d) + ((a^3*A*b - 7*a*A*b^3 + 3*a^4*B - 5*a^2*b^2*B + 8*b^4
*B)*EllipticF[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) - ((a^4*A*b - 10*a^2
*A*b^3 - 3*A*b^5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) + (a*(A*b - a*B)*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((
a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
4*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{-\frac{1}{2}a(Ab-aB)+2b(Ab-aB)\cos(c+dx)-\ldots}{\sqrt{\cos(c+dx)}\,(a+b\ldots}}{2b\left(a^2-\ldots\right.}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\ldots\right.}{4b\left(a^2-b^2\right)^2 d(a+\ldots}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\ldots\right.}{4b\left(a^2-b^2\right)^2 d(a+\ldots}$$

$$= -\frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{a(Ab-aB)\sqrt{\cos(\ldots}}{2b\left(a^2-b^2\right)d(a\ldots}$$

$$= -\frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{\left(a^3Ab-7aAb^3+\ldots\right.}{\ldots}$$

Mathematica [A] time = 3.73, size = 360, normalized size = 1.05

$$\frac{\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\left(b\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)\cos(c+dx)+a\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{-\frac{8\left(a^2B-3aAb+2b^2B\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}\right.\right.}{a+b}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] ((2*Sqrt[Cos[c + d*x]]*(a*(3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B) + b*(a^
2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2 -
b^2)^2*(a + b*Cos[c + d*x])^2) - (((-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)
*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (8*(-3*a*A*b + a^2*B
+ 2*b^2*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b),
 (c + d*x)/2, 2]))/(a + b) + ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*(-2
*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt
[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^
2*(a + b)^2))/(8*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x
)
```

maple [B] time = 6.06, size = 1937, normalized size = 5.63

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a
^2*(A*b-B*a)/b^3*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^
2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a
^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/
(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+
2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2)))-4/b^2*(A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)
)-2*a/b^3*(2*A*b-3*B*a)*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(
a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
```

```
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*
b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.379 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=337

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{\left(a^3(-B)+5a^2Ab-5ab\right.}{4abd\left(\right.}$$

```
[Out] 1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/b/(a^2-b^2)^2/d+1/4*(3*A
*a^2*b+3*A*b^3+B*a^3-7*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d-1/4*(3*A*a^4*b
+10*A*a^2*b^3-A*b^5+B*a^5-10*B*a^3*b^2-3*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(
a-b)^2/b^2/(a+b)^3/d-1/2*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/
(a+b*cos(d*x+c))^2-1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*sin(d*x+c)*cos(d*x
+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 0.92, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2999, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(3a^2Ab+a^3B-7ab^2B+3Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{\left(5a^2Ab+a^3(-B)-5ab^2B+Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}-\frac{\left(10a^2Ab^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*a*b*
(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)*EllipticF[(c
+ d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) - ((3*a^4*A*b + 10*a^2*A*b^3 - A*b^5
+ a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(4*a*(a - b)^2*b^2*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - ((5*a^2*A*b + A*b^3 - a
^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(a
+ b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^3}\,dx = -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\frac{1}{2}(Ab-aB)-2(aA-bB)\cos(c+dx)+\frac{1}{2}(}{\sqrt{\cos(c+dx)}\,(a+b\cos(c}}{2\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)}{4a\left(a^2-b^2\right)^2 d(a+}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)}{4a\left(a^2-b^2\right)^2 d(a+}$$

$$= \frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\left(a^2-b^2\right)^2 d} - \frac{(Ab-aB)\sqrt{\cos(c+}}{2\left(a^2-b^2\right)d(a+b}$$

$$= \frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\left(a^2-b^2\right)^2 d} + \frac{\left(3a^2Ab+3Ab^3+a\right.}{4b^2}$$

Mathematica [A] time = 4.55, size = 365, normalized size = 1.08

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(b\left(a^3B-5a^2Ab+5ab^2B-Ab^3\right)\cos(c+dx)+a\left(3a^3B-7a^2Ab+3ab^2B+Ab^3\right)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{\frac{16a\left(2a^2A-3abB+Ab^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+\right.\right.}{b(a+b)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(a*(-7*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B) + b*(-
5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2 -
b^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^
2*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*a*(2*a^2*A +
A*b^2 - 3*a*b*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]))/(b*(a + b)) - (2*(-5*a^2*A*b - A*b^3 + a^3*B + 5*a
*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Ell
ipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a),
ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2])
)/((a - b)^2*(a + b)^2))/(16*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x
)
```

maple [B] time = 5.82, size = 1850, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(A*b-B*a)/
b^2*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)
/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a
+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(
-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2)))-4*B/b/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(A*b-2*B*a)/b^2*(-b^2
/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/
a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
```

```
ipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.380 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=345

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}-\frac{\left(-5a^3B+9a^2Ab-ab^2\right.}{4a^2d\,(}$$

```
[Out] -1/4*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)^2/d-1/4*(
7*A*a^2*b-A*b^3-3*B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/b/(a^2-b^2)^2/d+1/4*(15*A*a
^4*b-6*A*a^2*b^3+3*A*b^5-3*B*a^5-10*B*a^3*b^2+B*a*b^4)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2)
)/a^2/(a-b)^2/b/(a+b)^3/d+1/2*b*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^
2-b^2)/d/(a+b*cos(d*x+c))^2+1/4*b*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*sin(d
*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] time = 1.06, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{\left(7a^2Ab-3a^3B-3ab^2B-Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}-\frac{\left(9a^2Ab-5a^3B-ab^2B-3Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{\left(-6a^2Ab^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] -((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*a
^2*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*EllipticF[
(c + d*x)/2, 2])/(4*a*b*(a^2 - b^2)^2*d) + ((15*a^4*A*b - 6*a^2*A*b^3 + 3*A
*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(4*a^2*(a - b)^2*b*(a + b)^3*d) + (b*(A*b - a*B)*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b*(9*a^2*A*b
 - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2
 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2A-3Ab^2-abB\right)-2a(Ab-aB)\cos\ldots}{\sqrt{\cos(c+dx)}\,(a+b\ldots}}{2a\left(a^2-\ldots\right.}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(9a^2Ab-3Ab^3-5a^3B-ab^2\ldots\right.}{4a^2\left(a^2-b^2\right)^2d(a\ldots}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b\left(9a^2Ab-3Ab^3-5a^3B-ab^2\ldots\right.}{4a^2\left(a^2-b^2\right)^2d(a\ldots}$$

$$= -\frac{\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2d} + \frac{b(Ab-aB)\sqrt{\cos(\ldots}}{2a\left(a^2-b^2\right)d(a\ldots}$$

$$= -\frac{\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2d} - \frac{\left(7a^2Ab-Ab^3-\ldots\right.}{4\ldots}$$

Mathematica [A] time = 4.89, size = 383, normalized size = 1.11

$$\frac{\frac{\frac{8a(2a^3B-4a^2Ab+ab^2B+Ab^3)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{b(a+b)}+\frac{(5a^3B-9a^2Ab+ab^2B+3Ab^3)\sin(c+dx)\left((b^2-2a^2)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\right.\right.\right.}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),
x]
```

```
[Out] ((-2*b*Sqrt[Cos[c + d*x]]*(a*(-11*a^2*A*b + 5*A*b^3 + 7*a^3*B - a*b^2*B) +
b*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((
a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (((16*a^4*A - 19*a^2*A*b^2 + 9*A*b^4
 - 9*a^3*b*B + 3*a*b^3*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b
) + (8*a*(-4*a^2*A*b + A*b^3 + 2*a^3*B + a*b^2*B)*((a + b)*EllipticF[(c + d
*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) + ((-
9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[
c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (
-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d
*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))),
 x)
```

<u>**maple**</u> **[B]** time = 5.77, size = 1744, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)/b*(
-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/
(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(
a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*
b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1
5/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*
b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),
2^(1/2)))+2*B/b*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
```

```
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)
*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.381 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=420

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}+\frac{\left(-7a^3B+11a^2Ab+ab^2B-5Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{b\left(-7a^3B+\right.}{4a^2d\left(a^2-\right.}$$

```
[Out] -1/4*(8*A*a^4-29*A*a^2*b^2+15*A*b^4+9*B*a^3*b-3*B*a*b^3)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^
2-b^2)^2/d+1/4*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2
)^2/d-1/4*(35*A*a^4*b-38*A*a^2*b^3+15*A*b^5-15*B*a^5+6*B*a^3*b^2-3*B*a*b^4)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2
*c),2*b/(a+b),2^(1/2))/a^3/(a-b)^2/(a+b)^3/d+1/4*(8*A*a^4-29*A*a^2*b^2+15*A
*b^4+9*B*a^3*b-3*B*a*b^3)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)+1/2
*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2)+1
/4*b*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b
*cos(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.47, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(11a^2Ab-7a^3B+ab^2B-5Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(-29a^2Ab^2+8a^4A+9a^3bB-3ab^3B+15Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] -((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*EllipticE[(c
+ d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) + ((11*a^2*A*b - 5*A*b^3 - 7*a^3*B +
a*b^2*B)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) - ((35*a^4*A*b
- 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*a^3*b^2*B - 3*a*b^4*B)*EllipticPi[
(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^3*(a - b)^2*(a + b)^3*d) + ((8*a^4*A -
 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sin[c + d*x])/(4*a^3*(a^2
 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^2 -
b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2) + (b*(11*a^2*A*b - 5*A*b^
3 - 7*a^3*B + a*b^2*B)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*
x]]*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}dx = \frac{b(Ab-aB)\sin(c+dx)}{2a(a^2-b^2)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}(4a^2A-5Ab^2+abB)-}{\text{co}}}{}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a(a^2-b^2)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} + \frac{b(11a^2Ab-5Ab^3}{4a^2(a^2-b^2)^2d\sqrt{\text{co}}}$$

$$= \frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)\sin(c+dx)}{4a^3(a^2-b^2)^2d\sqrt{\cos(c+dx)}} + \frac{}{2a(a^2-}$$

$$= \frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)\sin(c+dx)}{4a^3(a^2-b^2)^2d\sqrt{\cos(c+dx)}} + \frac{}{2a(a^2-}$$

$$= -\frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3(a^2-b^2)^2d} + \frac{(8}{}$$

$$= -\frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3(a^2-b^2)^2d} + \frac{(1}{}$$

Mathematica [A] time = 5.51, size = 458, normalized size = 1.09

$$\frac{\frac{\sqrt{\cos(c+dx)}\left(16A(a^3-ab^2)^2\tan(c+dx)+b^2(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4)\sin(2(c+dx))+2ab(16a^4A+11a^3bB-47a^2Ab^2-5ab^3B+25Ab^4)\sin\right.}{(a^2-b^2)^2(a+b\cos(c+dx))^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),
x]
```

```
[Out] (-((((56*a^4*A*b - 95*a^2*A*b^3 + 45*A*b^5 - 16*a^5*B + 19*a^3*b^2*B - 9*a*
b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*a*(2*a^4*A -
 10*a^2*A*b^2 + 5*A*b^4 + 4*a^3*b*B - a*b^3*B)*((a + b)*EllipticF[(c + d*x)
/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(b*(a + b)) + ((8*a^
4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*(-2*a*b*EllipticE[Ar
```

```
cSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d
*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]],
-1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) + (Sq
rt[Cos[c + d*x]]*(2*a*b*(16*a^4*A - 47*a^2*A*b^2 + 25*A*b^4 + 11*a^3*b*B -
5*a*b^3*B)*Sin[c + d*x] + b^2*(8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*
B - 3*a*b^3*B)*Sin[2*(c + d*x)] + 16*A*(a^3 - a*b^2)^2*Tan[c + d*x]))/((a^2
 - b^2)^2*(a + b*Cos[c + d*x])^2))/(8*a^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)),
 x)
```

maple [B] time = 7.37, size = 2002, normalized size = 4.77

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b+B*a)/a*
(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2
/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/
```

```
(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8
*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-
15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a
*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2)))+4*A*b^2/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*A*b/a^2*(-b^2/a/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2
-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2/a^3*A*(-(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos(c+d\,x)}{\cos(c+d\,x)^{3/2}\,(a+b\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.382 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=523

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}+\frac{b\left(-9a^3B+13a^2Ab+3ab^2B-7Ab^3\right)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}+\frac{\left(8a^4A+33a\right.}{}$$

```
[Out] 1/4*(24*A*a^4*b-65*A*a^2*b^3+35*A*b^5-8*B*a^5+29*B*a^3*b^2-15*B*a*b^4)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2))/a^4/(a^2-b^2)^2/d+1/12*(8*A*a^4-61*A*a^2*b^2+35*A*b^4+33*B*a^3*b-15*
B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)^2/d+1/4*b*(63*A*a^4*b-86*A*a^2*b^3+35*A*b
^5-35*B*a^5+38*B*a^3*b^2-15*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^4/(a-b)^2/(a+b
)^3/d+1/12*(8*A*a^4-61*A*a^2*b^2+35*A*b^4+33*B*a^3*b-15*B*a*b^3)*sin(d*x+c)
/a^3/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)+1/2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/
d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2+1/4*b*(13*A*a^2*b-7*A*b^3-9*B*a^3+3*B
*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))-1/4*
(24*A*a^4*b-65*A*a^2*b^3+35*A*b^5-8*B*a^5+29*B*a^3*b^2-15*B*a*b^4)*sin(d*x+
c)/a^4/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 1.95, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3000, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-61a^2Ab^2+8a^4A+33a^3bB-15ab^3B+35Ab^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^3d\left(a^2-b^2\right)^2}+\frac{\left(-65a^2Ab^3+24a^4Ab+29a^3b^2B-8a^5B-\right.}{4a^4d\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] ((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^4*
B)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^4*A - 61*a^2*
A*b^2 + 35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B)*EllipticF[(c + d*x)/2, 2])/(12*
a^3*(a^2 - b^2)^2*d) + (b*(63*a^4*A*b - 86*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B
+ 38*a^3*b^2*B - 15*a*b^4*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*
a^4*(a - b)^2*(a + b)^3*d) + ((8*a^4*A - 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3*b
*B - 15*a*b^3*B)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2))
 - ((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a*b^
4*B)*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b*(A*b - a
```

```
*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]
)^2) + (b*(13*a^2*A*b - 7*A*b^3 - 9*a^3*B + 3*a*b^2*B)*Sin[c + d*x])/(4*a^2
*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2A-7Ab^2+3abB\right)-2}{\cos^{\frac{5}{2}}}}{}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} + \frac{b\left(13a^2Ab-7Ab^3-\right.}{4a^2\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} + \frac{}{2a\left(a^2-\right.}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(24a^4Al\right.}{}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(24a^4Al\right.}{}$$

$$= \frac{\left(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B\right)E\left(\frac{1}{2}(c+dx\right.}{4a^4\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B\right)E\left(\frac{1}{2}(c+dx\right.}{4a^4\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 7.27, size = 570, normalized size = 1.09

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{2\sec(c+dx)(aB\sin(c+dx)-3Ab\sin(c+dx))}{a^4} + \frac{2A\tan(c+dx)\sec(c+dx)}{3a^3} + \frac{Ab^4\sin(c+dx)-ab^3B\sin(c+dx)}{2a^3\left(a^2-b^2\right)(a+b\cos(c+dx))^2} + \frac{-13a^3b^3B\sin(c+dx)}{}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),
x]
```

```
[Out] ((2*(16*a^6*A + 328*a^4*A*b^2 - 641*a^2*A*b^4 + 315*A*b^6 - 168*a^5*b*B + 2
85*a^3*b^3*B - 135*a*b^5*B)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a +
 b) + ((160*a^5*A*b - 512*a^3*A*b^3 + 280*a*A*b^5 - 48*a^6*B + 240*a^4*b^2*
B - 120*a^2*b^4*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a
+ b), (c + d*x)/2, 2])/(a + b)))/b + (2*(72*a^4*A*b^2 - 195*a^2*A*b^4 + 105
*A*b^6 - 24*a^5*b*B + 87*a^3*b^3*B - 45*a*b^5*B)*Cos[2*(c + d*x)]*(-2*a*b*E
llipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqr
t[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c
 + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c
 + d*x]^2)))/(48*a^4*(a - b)^2*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*((2*Sec[c
 + d*x]*(-3*A*b*Sin[c + d*x] + a*B*Sin[c + d*x]))/a^4 + (A*b^4*Sin[c + d*x]
 - a*b^3*B*Sin[c + d*x])/(2*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (17*a
^2*A*b^4*Sin[c + d*x] - 11*A*b^6*Sin[c + d*x] - 13*a^3*b^3*B*Sin[c + d*x] +
 7*a*b^5*B*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*
Sec[c + d*x]*Tan[c + d*x])/(3*a^3)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)),
 x)
```

maple [B] time = 11.61, size = 2158, normalized size = 4.13

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)*b/a
^2*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/
a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+
b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-
2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2)))+2*A/a^3*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4*b^2*(3*A*
b-B*a)/a^4/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*b*(2*A*b-B*a)/a^3*(-b^2/a/(a^2-
b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*
b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/
```

```
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b
^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2*(-3*A*b+B*a)/a^4*(-(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos(c+dx)}{\cos(c+dx)^{5/2}\,(a+b\cos(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.383 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

```
[Out] 6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/5*B*cos(d*x+c)^(3/2)*sin(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 2635, 2639}

$$\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*Cos[c + d*x]^(3/2)*Sin[c + d*x
])/(5*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx = B\int \cos^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}(3B)\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.93

$$\frac{B\left(6E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\sqrt{\cos(c+dx)}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (B*(6*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[2*(c + d*x)]))/(5*
d)
```

fricas [F] time = 2.56, size = 0, normalized size = 0.00

$$\text{integral}\left(B\cos\left(dx+c\right)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B*cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos\left(dx+c\right)+Ba)\cos\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a),
 x)
```

maple [B] time = 1.18, size = 203, normalized size = 4.61

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\;\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(-8*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)^{5/2}\,\left(B\,a+B\,b\,\cos(c+d\,x)\right)}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

3.384 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*B*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 2635, 2641}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx &= B\int \cos^{\frac{3}{2}}(c+dx)\,dx\\ &= \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{1}{3}B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 37, normalized size = 0.84

$$\frac{2B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(B\cos\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B*cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos\left(dx+c\right)+Ba)\cos\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a),
 x)
```

maple [B] time = 1.10, size = 180, normalized size = 4.09

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(c\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(B\,a+B\,b\,\cos(c+d\,x)\right)}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.385 $\int \frac{\sqrt{\cos(c+dx)}\,(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=17

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {21, 2639}

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}\,(aB+bB\cos(c+dx))}{a+b\cos(c+dx)}\,dx &= B\int \sqrt{\cos(c+dx)}\,dx \\
&= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d
```

fricas [F] time = 1.65, size = 0, normalized size = 0.00

$$\text{integral}\left(B\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bb\cos\left(dx+c\right)+Ba\right)\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a),
 x)
```

maple [B] time = 0.96, size = 134, normalized size = 7.88

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(B\,a+B\,b\,\cos(c+d\,x)\right)}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.386 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=17

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/d
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {21, 2641}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx &= B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])
),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/d
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

maple [C] time = 0.01, size = 19, normalized size = 1.12

$$\frac{2B\,\mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}|\sqrt{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2*B/d*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{B\,a+B\,b\,\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,\left(a+b\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.387 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=40

$$\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] -2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d+2*B*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 2636, 2639}

$$\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x
]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{aB + bB\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b\cos(c + dx))}\,dx = B\int \frac{1}{\cos^{\frac{3}{2}}(c + dx)}\,dx$$
$$= \frac{2B\sin(c + dx)}{d\sqrt{\cos(c + dx)}} - B\int \sqrt{\cos(c + dx)}\,dx$$
$$= -\frac{2BE\left(\frac{1}{2}(c + dx)\middle|2\right)}{d} + \frac{2B\sin(c + dx)}{d\sqrt{\cos(c + dx)}}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 1.00

$$B\left(\frac{2\sin(c + dx)}{d\sqrt{\cos(c + dx)}} - \frac{2E\left(\frac{1}{2}(c + dx)\middle|2\right)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
),x]
```

```
[Out] B*((-2*EllipticE[(c + d*x)/2, 2])/d + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]
]))
```

fricas [F] time = 2.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B}{\cos\left(dx + c\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos\left(dx + c\right) + Ba}{\left(b\cos\left(dx + c\right) + a\right)\cos\left(dx + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

maple [A] time = 1.17, size = 102, normalized size = 2.55

$$-\frac{2B\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*B*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,a+B\,b\,\cos(c+d\,x)}{\cos(c+d\,x)^{3/2}\,\left(a+b\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.388 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=44

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))/d+2/3*B*sin(d*x+c)/d/cos(d*x+c)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 2636, 2641}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*B*Sin[c + d*x])/(3*d*Cos[c + d*x
]^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx &= B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{1}{3}B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 37, normalized size = 0.84

$$\frac{2B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])
),x]
```

```
[Out] (2*B*(EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]/Cos[c + d*x]^(3/2)))/(3*d)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B}{\cos\left(dx+c\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos\left(dx+c\right)+Ba}{\left(b\cos\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

maple [B] time = 1.29, size = 214, normalized size = 4.86

$$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2}\cdots}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*B*((2*cos(1/2*d*x+1/2*c)^2-
1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,a+B\,b\,\cos(c+d\,x)}{\cos(c+d\,x)^{5/2}\,\left(a+b\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

3.389 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$-\frac{2a^3B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}+\frac{2B\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2aBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

```
[Out] -2*a*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/b^2/d+2/3*(3*a^2+b^2)*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d-2*a^3*B*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(
a+b),2^(1/2))/b^3/(a+b)/d+2/3*B*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d
```

Rubi [A] time = 0.40, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {21, 2793, 3059, 2639, 3002, 2641, 2805}

$$\frac{2B\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a^3B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2aBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*a*B*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*(3*a^2 + b^2)*B*EllipticF[(
c + d*x)/2, 2])/(3*b^3*d) - (2*a^3*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2])/(b^3*(a + b)*d) + (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx = B\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} + \frac{(2B)\int \frac{\frac{a}{2}+\frac{1}{2}b\cos(c+dx)-\frac{3}{2}a\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{3b}$$

$$= \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} - \frac{(2B)\int \frac{-\frac{ab}{2}-\frac{1}{2}(3a^2+b^2)\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{3b^2} - \frac{(a$$

$$= -\frac{2aBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3bd} - \frac{(a^3B)\int \frac{}{\sqrt{\cos}}$$

$$= -\frac{2aBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2(3a^2+b^2)BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d} - \frac{2a^3B\Pi\left(\frac{2b}{a+}\right.}{b^3(}$$

Mathematica [A] time = 1.58, size = 159, normalized size = 1.37

$$\frac{B\left(-\frac{6\sin(c+dx)\left((b^2-2a^2)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{b^2\sqrt{\sin^2(c+dx)}} - \frac{6a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\right.}{a+b}\right.}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^2,x]
```

```
[Out] (B*(4*EllipticF[(c + d*x)/2, 2] - (6*a*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2])/(a + b) + 4*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (6*(-2*a*b*EllipticE[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]],
 -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2])))/(6*b*d)
```

fricas [F] time = 97.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{B\cos\left(dx+c\right)^{\frac{5}{2}}}{b\cos\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] integral(B*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^
2, x)
```

maple [B] time = 1.51, size = 517, normalized size = 4.46

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\left(\left(4b^2a-4b^3\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2b^2a+2b^3\right)\left(\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*((4*a*b^2-4*
b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a*b^2+2*b^3)*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)+3*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^2*b+b^2*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^3
+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*a^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^
2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.390 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=78

$$\frac{2a^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2aBF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/b/d-2*a*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d+2*a^2*B*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b^2
/(a+b)/d
```

Rubi [A] time = 0.17, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {21, 2804, 2639, 2803, 2641, 2805}

$$\frac{2a^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2aBF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(b*d) - (2*a*B*EllipticF[(c + d*x)/2, 2])/(
b^2*d) + (2*a^2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^2*(a + b)*d
)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2804

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*Sin[e + f*x]], x], x]
 - Dist[(b*c - a*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx \\
&= \frac{B\int \sqrt{\cos(c+dx)}\,dx}{b} - \frac{(aB)\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx}{b} \\
&= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{(aB)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2} + \frac{\left(a^2B\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos}}{b^2} \\
&= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2aBF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2a^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\right|}{b^2(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 82, normalized size = 1.05

$$-\frac{2B\sin(c+dx)\left(-(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+a\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+bE\left(\sin^{-1}\left(\sqrt{c\right.\right.\right.}{b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^2,x]
```

```
[Out] (-2*B*(b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + a*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]
]], -1])*Sin[c + d*x])/(b^2*d*Sqrt[Sin[c + d*x]^2])
```

fricas [F] time = 94.56, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{B\cos\left(dx+c\right)^{\frac{3}{2}}}{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] integral(B*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bb\cos\left(dx+c\right)+Ba\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
2, x)
```

maple [A] time = 1.35, size = 228, normalized size = 2.92

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\operatorname{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{b^2\left(a-b\right)\sqrt{-2\left(\sin^4\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a^2-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-a^2*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^2/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(B\,a+B\,b\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.391 $\int \frac{\sqrt{\cos(c+dx)}\,(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2aB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/b/d-2*a*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/b/(a+b)/d
```

Rubi [A] time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 2803, 2641, 2805}

$$\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2aB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*B*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a*B*EllipticPi[(2*b)/(a + b), (c
 + d*x)/2, 2])/(b*(a + b)*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
```

```
b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}\,(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^2}\,dx &= B\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx\\ &= \frac{B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b}-\frac{(aB)\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{b}\\ &= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2aB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 0.89

$$\frac{B\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^2,x]
```

```
[Out] (B*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/
2, 2])/(a + b)))/(b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^
2, x)
```

maple [A] time = 1.35, size = 189, normalized size = 3.44

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{(a-b)\,b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-a*EllipticPi(cos(1/2
*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/(a-b)/b/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^
2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\,(B\,a+B\,b\,\cos(c+d\,x))}{(a+b\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^2, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

3.392 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=30

$$\frac{2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a+b)/d

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {21, 2805}

$$\frac{2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

[In] Int[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]

[Out] (2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^2}\,dx = B\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx$$
$$= \frac{2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}$$

Mathematica [A] time = 0.07, size = 30, normalized size = 1.00

$$\frac{2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])
^2),x]
```

```
[Out] (2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c
))), x)
```

maple [B] time = 0.99, size = 151, normalized size = 5.03

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{B\,a+B\,b\,\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,\left(a+b\,\cos(c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.393 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=80

$$-\frac{2bB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] -2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a/d-2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(a+b)/d+2*B*sin(d*x+c)/a/
d/cos(d*x+c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {21, 2802, 3059, 2639, 12, 2805}

$$-\frac{2bB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*b*B*EllipticPi[(2*b)/(a + b), (
c + d*x)/2, 2])/(a*(a + b)*d) + (2*B*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$

$$= \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{(2B)\int \frac{-\frac{b}{2}-\frac{1}{2}a\cos(c+dx)-\frac{1}{2}b\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{a}$$

$$= \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{B\int\sqrt{\cos(c+dx)}\,dx}{a} - \frac{(2B)\int \frac{b^2}{2\sqrt{\cos(c+dx)}\,(a+b\cos(c+d}}{ab}$$

$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{(bB)\int \frac{1}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx)}}{a}$$

$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2bB\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d} + \frac{2B\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Mathematica [B] time = 2.68, size = 196, normalized size = 2.45

$$-\frac{B\left(\frac{2\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}}+\frac{6b\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\right.}{a+b}\right.}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
^2),x]
```

```
[Out] -1/2*(B*((6*b*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (2*a*(2*
EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/(a + b)))/b - (4*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*(-2*a*b*EllipticE[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c +
d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]],
 -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2])))/(a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/
2)), x)
```

maple [B] time = 1.48, size = 355, normalized size = 4.44

$$-\frac{2B\left(-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2*B*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/a/(a-b)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.394 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{2b^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^2/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+2*b^2*B*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a
^2/(a+b)/d+2/3*B*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)-2*b*B*sin(d*x+c)/a^2/d/cos
(d*x+c)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {21, 2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2b^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] (2*b*B*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*B*EllipticF[(c + d*x)/2, 2])
/(3*a*d) + (2*b^2*B*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a + b)
*d) + (2*B*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*b*B*Sin[c + d*x])/
(a^2*d*Sqrt[Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$$

$$= \frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{(2B)\int \frac{-\frac{3b}{2}+\frac{1}{2}a\cos(c+dx)+\frac{1}{2}b\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx}{3a}$$

$$= \frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{(4B)\int \frac{\frac{1}{4}\left(a^2+3b^2\right)+ab\cos(c+dx)+\frac{3}{4}}{\sqrt{\cos(c+dx)}\,(a+b\cos(}}{3a^2}$$

$$= \frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{(4B)\int \frac{-\frac{1}{4}b\left(a^2+3b^2\right)-\frac{1}{4}ab^2\cos(c+d}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx)}}{3a^2b}$$

$$= \frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2B\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2bB\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{B\int -}{}$$

$$= \frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{2b^2B\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right.}{a^2(a+b)d}$$

Mathematica [A] time = 4.12, size = 211, normalized size = 1.59

$$\frac{B\left(\frac{2(2a^2+9b^2)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{6\sin(c+dx)\left((b^2-2a^2)\Pi\left(-\frac{b}{a};\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c\right.\right.}{a\sqrt{\sin^2(c+dx)}}\right.}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])
^2),x]
```

```
[Out] (B*((2*(2*a^2 + 9*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) +
 8*a*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x
)/2, 2])/(a + b)) + (4*(a - 3*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3
/2) + (6*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*El
lipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a),
 ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2])))/
(6*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/
2)), x)
```

maple [B] time = 3.34, size = 452, normalized size = 3.40

$$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\left(-\frac{2b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\operatorname{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{a^2\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(-2*b^3/a^2/
(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),-2*b/(a-b),2^(1/2))-1/a^2*b*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+1/a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+
1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos\left(dx+c\right)+Ba}{\left(b\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algor
ithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.395 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=560

$$\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24b^2d\sqrt{\cos(c+dx)}}-\frac{(a-b)\sqrt{a+b}\,\left(-3a^2B+6aAb+16b^2B\right)\cot(c+dx)}{}$$

```
[Out] 1/3*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d+1/24*(6*A*a*b-
3*B*a^2+16*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)+
1/4*(2*A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/b/d-1/24
*(a-b)*(6*A*a*b-3*B*a^2+16*B*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d+1/24*(a+2*b)*(
6*A*b-3*B*a+8*B*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+1/8*(2*A*a^2*b-8*A*b^3-B*a^3-4*
B*a*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d
```

Rubi [A] time = 1.51, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24b^2d\sqrt{\cos(c+dx)}}-\frac{(a-b)\sqrt{a+b}\,\left(-3a^2B+6aAb+16b^2B\right)\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(6*a*A*b - 3*a^2*B + 16*b^2*B)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(24*a*b^2*d) + (Sqrt[a + b]*(a + 2*b)*(6*A*b - 3*a*B + 8*b*B)*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b^2*d) + (Sqrt[a + b]*(2*a^2*A*b - 8
*A*b^3 - a^3*B - 4*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
```

```
(8*b^3*d) + ((6*a*A*b - 3*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(24*b^2*d*Sqrt[Cos[c + d*x]]) + ((2*A*b - a*B)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d) + (B*Sqrt[Cos[c + d*x]]*(a +
b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{B\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3bd} + $$

$$= \frac{(2Ab-aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,\sin(c+}{4bd}$$

$$= \frac{\left(6aAb-3a^2B+16b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+}{24b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(6aAb-3a^2B+16b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+}{24b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{a+b}\,\left(2a^2Ab-8Ab^3-a^3B-4ab^2B\right)\cot(c+dx)}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\left(6aAb-3a^2B+16b^2B\right)\cot(c+dx)}{}$$

Mathematica [C] time = 6.33, size = 1224, normalized size = 2.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] -1/48*((-4*a*(-18*a*A*b + a^2*B - 16*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4
*a*(-24*A*b^2 - 28*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4
)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Co
```

```
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])) + 2*(-6*a*A*b + 3*a^2*B - 16*b^2*B)*((I*Cos[(c + d*x)/2]*Sq
rt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*
x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x
]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(b*d) + (Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]]*(((6*A*b + a*B)*Sin[c + d*x])/(12*b) + (B*Sin[2
*(c + d*x)])/6))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.44, size = 2949, normalized size = 5.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)

[Out] -1/24/d/(a+b*cos(d*x+c))^(1/2)*(-12*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+6*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+6*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-28*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+12*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+12*A*cos(d*x+c)^4*b^3-12*A*cos(d*x+c)^2*b^3+8*B*cos(d*x+c)^5*b^3+8*B*cos(d*x+c)^3*b^3-3*B*cos(d*x+c)^2*a^3-16*B*cos(d*x+c)^2*b^3+3*B*cos(d*x+c)*a^3+18*A*cos(d*x+c)^3*a*b^2+6*A*cos(d*x+c)^2*a^2*b-6*A*cos(d*x+c)^2*a*b^2-6*A*cos(d*x+c)*a^2*b-12*A*cos(d*x+c)*a*b^2+10*B*cos(d*x+c)^4*a*b^2-B*cos(d*x+c)^3*a^2*b+3*B*cos(d*x+c)^2*a^2*b+6*B*cos(d*x+c)^2*a*b^2-2*B*cos(d*x+c)*a^2*b-16*B*cos(d*x+c)*a*b^2+48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+12*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-12*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+6*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+6*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos

```
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b^2+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b-28*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+16*B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-24*A*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*b^3-24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*b^3+48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-3*B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3)/sin(d*x+c
)/b^2/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.396 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=473

$$-\frac{\sqrt{a+b}\left(a^2(-B)+4aAb+4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+}{a-}\right.}{4b^2d}$$

```
[Out] 1/4*(4*A*b+B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)+1/2*
B*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/4*(a-b)*(4*A*b+B*a
)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(
d*x+c))/(a-b))^(1/2)/a/b/d+1/4*(4*A*b+(a+2*b)*B)*cot(d*x+c)*EllipticF((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^
(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d-1/4
*(4*A*a*b-B*a^2+4*B*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A] time = 1.04, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3003, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(a^2(-B)+4aAb+4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+}{a-}\right.}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*a
*b*d) + (Sqrt[a + b]*(4*A*b + (a + 2*b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(4*b*d) - (Sqrt[a + b]*(4*a*A*b - a^2*B + 4*b^2*B)*Cot[c + d*x]*Elliptic
Pi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(4*b^2*d) + ((4*A*b + a*B)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
```

```
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\,dx = \frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{4}$$

$$= \frac{(4Ab+aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{B\sqrt{\mathrm{co}}}{}$$

$$= \frac{(4Ab+aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{B\sqrt{\mathrm{co}}}{}$$

$$= -\frac{\sqrt{a+b}\,\left(4aAb-a^2B+4b^2B\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(4Ab+aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.\right.}{4}$$

Mathematica [C] time = 21.11, size = 1175, normalized size = 2.48

$$-\frac{4a(4Ab+3aB)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\,\mathrm{cs}}{a}}}{(a+b)\sqrt{\cos(c+dx)}}$$

$$\frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d} + \frac{}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),
x]
```

```
[Out] (B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-4*a
*(4*A*b + 3*a*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
```

```
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*a*A + 4*b*B)*((Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*A*b + a*B)*
((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]
) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Co
s[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8
*d)
```

fricas [F] time = 3.76, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

maple [B] time = 0.26, size = 2052, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)^2*a^2-B*cos(d*x+c)*a^2+4*A*cos(
d*x+c)^3*b^2-4*A*cos(d*x+c)^2*b^2+2*B*cos(d*x+c)^4*b^2-2*B*cos(d*x+c)^2*b^2
+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*cos(d*x+c)*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+4*A*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
a*b-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)*a*b+4*A*cos(d*x+c)^2*a*b-4*A*cos(d*x+c)*a*b+3*B*cos(d*x+
c)^3*a*b-B*cos(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*b+4*A*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-2*B*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*b^2+4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*a*b-8*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
```

```
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-2*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)*a^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2-4*B*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2+8
*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*a*b)/sin(d*x+c)/b/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x)),
x)
```

3.397 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=385

$$\frac{\sqrt{a+b}\,(2A+B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}-\frac{\sqrt{a+b}\,(aB+2Ab}{}$$

```
[Out] B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(a-b)*B*cot(d*x+c)*E
llipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/a/d+(2*A+B)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-(2*A*b+B*a)*cot(d*x+c)*EllipticPi((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/b/d
```

Rubi [A] time = 0.71, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3003, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\,(2A+B)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}-\frac{\sqrt{a+b}\,(aB+2Ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)) + (Sqrt[
a + b]*(2*A + B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (Sqrt[a + b]*(2*A*b
 + a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (B*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
```

```
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
```

```
[n^2, 1/4]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\int \frac{-aB+2aA\cos(c+dx}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\int \frac{-aB+2aA\cos(c}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{co}}}$$

$$= -\frac{\sqrt{a+b}\,(2Ab+aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\right.}{bd}$$

$$= -\frac{(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

Mathematica [A] time = 11.36, size = 408, normalized size = 1.06

$$\frac{\sqrt{\cos(c+dx)}\left(-4(a(B-A)+Ab)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+8Ab\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\,\Gamma\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(a + b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(A*
```

```
b + a*(-A + B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 8*A*b*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)] + 4*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b
*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))
/2] + 2*a*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*B*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(2*d*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)),
 x)
```

maple [B] time = 0.41, size = 1693, normalized size = 4.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/d*(2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a-2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+4*A*sin(d*x+c)*cos(d*x+c)^2*(co
```

```
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b+4*A*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a-4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*b+8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b+2*A*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*A*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+4*A*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b-2*B*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*a+2*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+B*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*B*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*a+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*a+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+B*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+B*cos(d*x+c)
^4*b+B*cos(d*x+c)^3*a-B*cos(d*x+c)^3*b-B*cos(d*x+c)^2*a)/(a+b*cos(d*x+c))^(
1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/sqrt(cos(c + d*x)),
x)
```

3.398 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=351

$$\frac{2\sqrt{a+b}\,(Ab-a(A-B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+\frac{2A(a-b)}{}$$

```
[Out] 2*A*(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+2*(A*b-a*(A-B))*cot(d*x+c)*EllipticF((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^
(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-2*B
*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] time = 0.50, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2991, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\,(Ab-a(A-B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+\frac{2A(a-b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) + (2*Sqrt
[a + b]*(A*b - a*(A - B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqr
t[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
```

```
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2991

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)]])/((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(B*d
)/b^2, Int[Sqrt[b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Int[(A*c
 + (B*c + A*d)*Sin[e + f*x])/((b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = (bB)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx + \int \frac{aA+(Ab+aB)\cos(c+}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c}$$

$$= -\frac{2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}$$

$$= \frac{2A(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

Mathematica [A] time = 12.74, size = 273, normalized size = 0.78

$$\frac{2(a(A+B)+b(A-B))\sqrt{\cos(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+\frac{2A\tan\left(\frac{1}{2}(c+dx)\right)(a+b}{\sqrt{\cos(c+dx)}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] (-2*A*(a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2
*(b*(A - B) + a*(A + B))*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] + 4*b*B*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)] + (2*A*(a + b*Cos[c + d*x])*Tan[(c + d*x)/2])/Sqrt[Cos[c + d*x]])/(d*Sq
rt[a + b*Cos[c + d*x]])
```

fricas [F] time = 69.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

maple [B] time = 0.29, size = 1687, normalized size = 4.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/d/(a+b*cos(d*x+c))^(1/2)*(B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-B*sin(d*x+c)*cos(d*x+c)^2*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*B*sin(d*x+
c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+4*B*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b+A*sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a+A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*b-A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
```

```
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-B*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*
b+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*b-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+A*cos(d*x+c)^
3*b+A*cos(d*x+c)^2*a-A*cos(d*x+c)^2*b-A*cos(d*x+c)*a)/cos(d*x+c)^(3/2)/sin(
d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,\sqrt{a+b\cos(c+d\,x)}}{\cos(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(3/2),
 x)
```

3.399 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=284

$$\frac{2(a-b)\sqrt{a+b}\,(3aB+Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}+\frac{2(a-b)}{}$$

```
[Out] 2/3*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/3*(a-b)*(A*b+3
*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/a^2/d+2/3*(a-b)*(A-3*B)*cot(d*x+c)*EllipticF((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^
(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.51, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2999, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\,(3aB+Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}+\frac{2(a-b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b + 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*d) + (2*(a - b)*Sqrt[a + b]*(A - 3*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(3*a*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(
3/2))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{1}{2}(Ab+3aB)+\frac{1}{2}(aA}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}((a-b)(A-3B))\int \frac{}{\sqrt{c}}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(Ab+3aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{3a^2d}$$

Mathematica [A] time = 13.50, size = 407, normalized size = 1.43

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)(3aB\sin(c+dx)+Ab\sin(c+dx))}{3a}+\frac{2}{3}A\tan(c+dx)\sec(c+dx)\right)}{d}+\frac{4\left(\frac{\cos(c+dx)}{\cos(c+dx)+}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] (4*(Cos[(c + d*x)/2]^2)^(5/2)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[
1 + Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(a + b)*(A*b +
3*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)] + 2*a*(a + b)*(A + 3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)] - (A*b + 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x]
)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a*d*Cos[c + d*x]^(5/2)*Sqrt[a +
b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c +
 d*x]*(A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(3*a) + (2*A*Sec[c + d*x]*Ta
n[c + d*x])/3))/d
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2),
 x)
```

maple [B] time = 0.23, size = 1727, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(3*B*cos(d*x+c)^2*a^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*a*b-3*B*cos(d*x+c)*a^2+A*cos(
d*x+c)^3*b^2-A*cos(d*x+c)^2*b^2-a^2*A-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+A*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^2
-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)^2*b^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*a^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+A*sin(d*x+c)*
cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
a^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*cos(d*x+c)*a^2+A*cos(d*x+c)^2*a^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+3*B*sin(d*x+c)
```

```
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
*a*b-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+A*cos(d*x+c)^3*a*b-3*B*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*
a*b+A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*a*b+A*cos(d*x+c)^2*a*b-2*A*cos(d*x+c)*a*b+3*B*cos(d*x+c)
^3*a*b-3*B*cos(d*x+c)^2*a*b-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2)/(a+b*cos(d
*x+c))^(1/2)/a/sin(d*x+c)/cos(d*x+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}}{{\cos(c+d\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{a + b\cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(5/2),
 x)
```

3.400 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=350

$$-\frac{2(a-b)\sqrt{a+b}\,(9aA-5aB+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^2d}$$

```
[Out] 2/5*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/15*(A*b+5*B*a)
*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9*A*a^2
-2*A*b^2+5*B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d-2/15*(a-b)*(9*A*a+2*A*b-5*B*a)*c
ot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/a^2/d
```

Rubi [A] time = 0.82, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2999, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\,\left(9a^2A+5abB-2Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{15a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(9*a*A + 2*A*b - 5*a*B)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(A*b + 5*a*B)*Sqrt[a + b*Cos[c
 + d*x]]*Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
```

```
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}(Ab+5aB)+\frac{1}{2}(3a}{\cos^{\frac{5}{2}}(c}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+b\cos(c}{15ad\cos^{\frac{3}{2}}(c}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+b\cos(c}{15ad\cos^{\frac{3}{2}}(c}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(9a^2A-2Ab^2+5abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.\right.}{15a^3d}$$

Mathematica [C] time = 6.37, size = 1315, normalized size = 3.76

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2
),x]
```

```
[Out] -1/15*((-4*a*(2*a^2*A*b - 2*A*b^3 - 5*a^3*B + 5*a*b^2*B)*Sqrt[((a + b)*Cot[
(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/
a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
F[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)
/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]]) - 4*a*(9*a^3*A - 2*a*A*b^2 + 5*a^2*b*B)*((Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
```

```
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b - 2*A*b^3 + 5*a*b^2*
B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c
 + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a +
b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b
), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))
/(a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*
(A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/(15*a) + (2*Sec[c + d*x]*(9*a^2*A*
Sin[c + d*x] - 2*A*b^2*Sin[c + d*x] + 5*a*b*B*Sin[c + d*x]))/(15*a^2) + (2*
A*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

fricas [F] time = 1.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2),
 x)
```

maple [B] time = 0.33, size = 2481, normalized size = 7.09

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] 2/15/d*(3*A*a^3-9*A*cos(d*x+c)^3*a^3-2*A*cos(d*x+c)^3*b^3+6*A*cos(d*x+c)^2*
a^3-5*B*cos(d*x+c)^3*a^3-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*cos(d*x+c)^4*b^3+5*B*co
s(d*x+c)*a^3+2*A*cos(d*x+c)^3*a*b^2-A*cos(d*x+c)^2*a*b^2+4*A*cos(d*x+c)*a^2
*b-5*B*cos(d*x+c)^4*a*b^2-5*B*cos(d*x+c)^3*a^2*b+10*B*cos(d*x+c)^2*a^2*b+9*
A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-7*A*sin(d*x+c)*cos(d*x+c)^2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(
d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b^2+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-5*B*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d
*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2*b+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2+9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*
cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
^2-7*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+5*B*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*
```

```
x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+5*B
*cos(d*x+c)^3*a*b^2-9*A*cos(d*x+c)^4*a^2*b-A*cos(d*x+c)^4*a*b^2+5*A*cos(d*x
+c)^3*a^2*b-5*B*cos(d*x+c)^4*a^2*b-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)
*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*b^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-5*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^
3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3-2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^3-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3)/(a+b*cos(d*x+c))^(1/2
)/a^2/sin(d*x+c)/cos(d*x+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}}{\cos(c+d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(7/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.401 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=433

$$\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,\left(a^2(25A-63B)+2ab(3A-7B)+8A\right.}{}$$

[Out] 2/7*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/35*(A*b+7*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+2/105*(25*A*a^2-4*A*b^2+7*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(3/2)+2/105*(a-b)*(19*A*a^2*b+8*A*b^3+63*B*a^3-14*B*a*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/105*(a-b)*(8*A*b^2+a^2*(25*A-63*B)+2*a*b*(3*A-7*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d

Rubi [A] time = 1.18, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2999, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,\left(a^2(25A-63B)+2ab(3A-7B)+8A\right.}{}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]

[Out] (2*(a - b)*Sqrt[a + b]*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]*(8*A*b^2 + a^2*(25*A - 63*B) + 2*a*b*(3*A - 7*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*(A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*a*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2*A - 4*A*b^2 + 7*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*a^2*d*Cos[c + d*x]^(3/2))

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}(Ab+7aB)+\frac{1}{2}(5aA}{\cos^{\frac{7}{2}}(c}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b\cos(c-}}{35ad\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b\cos(c-}}{35ad\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b\cos(c-}}{35ad\cos^{\frac{5}{2}}(c+}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(19a^2Ab+8Ab^3+63a^3B-14ab^2B\right)\cot(c+dx)}{105a}$$

Mathematica [C] time = 6.46, size = 1408, normalized size = 3.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2
),x]
```

```
[Out] ((-4*a*(25*a^4*A - 17*a^2*A*b^2 - 8*A*b^4 - 14*a^3*b*B + 14*a*b^3*B)*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - 4*a*(-19*a^3*A*b - 8*a*A*b^3 - 63*a^4*B + 14*a^2*b^
2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(
-19*a^2*A*b^2 - 8*A*b^4 - 63*a^3*b*B + 14*a*b^3*B)*((I*Cos[(c + d*x)/2]*Sqr
t[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x
]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a^3*d) + (Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(A*b*Sin[c + d*x] + 7*a
*B*Sin[c + d*x]))/(35*a) + (2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x] - 4*A*b
^2*Sin[c + d*x] + 7*a*b*B*Sin[c + d*x]))/(105*a^2) + (2*Sec[c + d*x]*(19*a^
2*A*b*Sin[c + d*x] + 8*A*b^3*Sin[c + d*x] + 63*a^3*B*Sin[c + d*x] - 14*a*b^
2*B*Sin[c + d*x]))/(105*a^3) + (2*A*Sec[c + d*x]^3*Tan[c + d*x])/7))/d
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2),
 x)
```

maple [B] time = 0.44, size = 3428, normalized size = 7.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] 2/105/d*(-63*B*cos(d*x+c)^4*a^4+42*B*cos(d*x+c)^3*a^4+21*B*cos(d*x+c)*a^4-2
5*A*cos(d*x+c)^4*a^4+10*A*cos(d*x+c)^2*a^4-8*A*cos(d*x+c)^5*b^4+8*A*cos(d*x
+c)^4*b^4+28*B*cos(d*x+c)^2*a^3*b-25*A*cos(d*x+c)^5*a^3*b-19*A*cos(d*x+c)^5
*a^2*b^2+4*A*cos(d*x+c)^5*a*b^3-19*A*cos(d*x+c)^4*a^3*b+20*A*cos(d*x+c)^4*a
^2*b^2-8*A*cos(d*x+c)^4*a*b^3+26*A*cos(d*x+c)^3*a^3*b+4*A*cos(d*x+c)^3*a*b^
3-A*cos(d*x+c)^2*a^2*b^2+18*A*cos(d*x+c)*a^3*b-63*B*cos(d*x+c)^5*a^3*b-7*B*
cos(d*x+c)^5*a^2*b^2+14*B*cos(d*x+c)^5*a*b^3+35*B*cos(d*x+c)^4*a^3*b+14*B*c
os(d*x+c)^4*a^2*b^2-14*B*cos(d*x+c)^4*a*b^3-7*B*cos(d*x+c)^3*a^2*b^2-8*A*si
n(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b^3+8*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-25*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+63*B*sin(d*x+c)
*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^4-63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^4+8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-25*A*sin(d*x+c)*cos(d*x+c
```

)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+63*
B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^4-63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+15*A*a^4+63*B*sin(d*x+c)*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-1
4*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b^2-14*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-49*B*sin(d*x+c)*cos(d*x+c)
^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+14
*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b^2+19*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+19*A*sin(d*x+c)*cos(d*x+c)^
3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8
*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^3-19*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-2*A*sin(d*x+c)*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*A*
sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^3+63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-14*B*sin(d*x+c)*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-14*B*s
in(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^3-49*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+14*B*sin(d*x+c)*cos(d*x+c)^3*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+19*A*si
n(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(

```
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^3*b+19*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*sin(d*x+c)*cos(d*x+c)^4*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-19*A*sin(
d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^3*b-2*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2)/(a+b*cos(d*x+c))^(1/2)/a^3/sin(d*x
+c)/cos(d*x+c)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\cos(c+dx)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/cos(c + d*x)^(9/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.402 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=670

$$\frac{\left(-3a^2B+8aAb+12b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32bd}+\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab\right.}{192b^2d\sqrt{\cos(}}$$

```
[Out] 1/24*(8*A*b-3*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d+1
/4*B*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d+1/192*(24*A*a^2
*b+128*A*b^3-9*B*a^3+156*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d/c
os(d*x+c)^(1/2)+1/32*(8*A*a*b-3*B*a^2+12*B*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)
*(a+b*cos(d*x+c))^(1/2)/b/d-1/192*(a-b)*(24*A*a^2*b+128*A*b^3-9*B*a^3+156*B
*a*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d-1/192*(9*a^3*B-6*a^2*b*(4*A+B)-8*b^3*(16
*A+9*B)-4*a*b^2*(28*A+39*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a
+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+1/64*(8*A*a^3*b-96*A*
a*b^3-3*B*a^4-24*B*a^2*b^2-48*B*b^4)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/
2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d
```

Rubi [A] time = 2.10, antiderivative size = 670, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(-3a^2B+8aAb+12b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{32bd}+\frac{\left(24a^2Ab-9a^3B+156ab^2B+128Ab^3\right)}{192b^2d\sqrt{\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^2*d) - (Sqrt[a + b]*(9*a^3*B - 6*a^
2*b*(4*A + B) - 8*b^3*(16*A + 9*B) - 4*a*b^2*(28*A + 39*B))*Cot[c + d*x]*El
lipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(192*b^2*d) + (Sqrt[a + b]*(8*a^3*A*b - 96*a*A*b^3 - 3*
```

```
a^4*B - 24*a^2*b^2*B - 48*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(64*b^3*d) + ((24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(192*b^2*d*Sqrt[Cos[c + d*x]]) + ((8*a*A*b
- 3*a^2*B + 12*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(32*b*d) + ((8*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3
/2)*Sin[c + d*x])/(24*b*d) + (B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/
2)*Sin[c + d*x])/(4*b*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
```

```
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{B\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4bd} +$$

$$= \frac{(8Ab-3aB)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}\,\mathrm{si}}{24bd}$$

$$= \frac{\left(8aAb-3a^2B+12b^2B\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(}}{32bd}$$

$$= \frac{\left(24a^2Ab+128Ab^3-9a^3B+156ab^2B\right)\sqrt{a+b\cos}}{192b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(24a^2Ab+128Ab^3-9a^3B+156ab^2B\right)\sqrt{a+b\cos}}{192b^2d\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{a+b}\,\left(8a^3Ab-96aAb^3-3a^4B-24a^2b^2B-48b\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\left(24a^2Ab+128Ab^3-9a^3B+156a\right.}{}$$

Mathematica [C] time = 6.41, size = 1284, normalized size = 1.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] -1/384*((-4*a*(-136*a^2*A*b - 128*A*b^3 + 3*a^3*B - 228*a*b^2*B)*Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
```

```
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Cos[c + d*x]]) - 4*a*(-416*a*A*b^2 - 228*a^2*b*B - 144*b^3*B)*((Sqrt[((a
 + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-24*a^2*A*b - 1
28*A*b^3 + 9*a^3*B - 156*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a -
 b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Co
s[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(b*Sqrt[Cos[c + d*x]])))/(b*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]*(((56*a*A*b + 3*a^2*B + 42*b^2*B)*Sin[c + d*x])/(96*b) + ((8*A*b
+ 9*a*B)*Sin[2*(c + d*x)])/48 + (b*B*Sin[3*(c + d*x)])/16))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.63, size = 4048, normalized size = 6.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -1/192/d/(a+b*cos(d*x+c))^(1/2)*(136*A*cos(d*x+c)^3*a^2*b^2-3*B*cos(d*x+c)^
3*a^3*b+108*B*cos(d*x+c)^3*a*b^3+78*B*cos(d*x+c)^2*a^2*b^2-156*B*cos(d*x+c)
^2*a*b^3-6*B*cos(d*x+c)*a^3*b-156*B*cos(d*x+c)*a^2*b^2-72*B*cos(d*x+c)*a*b^
3+24*A*cos(d*x+c)^2*a^3*b-48*A*cos(d*x+c)^2*a*b^3-112*A*cos(d*x+c)*a^2*b^2-
128*A*cos(d*x+c)*a*b^3+128*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-9*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+18*B*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4+288*B*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-144*B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^4+48*B*cos(d*x+c)^6*b^4+64*A*cos(d*x+c)^3*b^4-128*A*cos(d*x+c)^2*b^4+24*B*
cos(d*x+c)^4*b^4-72*B*cos(d*x+c)^2*b^4-9*B*cos(d*x+c)^2*a^4+9*B*cos(d*x+c)*
a^4+64*A*cos(d*x+c)^5*b^4+9*B*cos(d*x+c)^2*a^3*b+176*A*cos(d*x+c)^4*a*b^3-2
4*A*cos(d*x+c)^2*a^2*b^2-24*A*cos(d*x+c)*a^3*b+120*B*cos(d*x+c)^5*a*b^3+78*
B*cos(d*x+c)^4*a^2*b^2+24*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b-9*B*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+18*B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a^4+288*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-144*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+24*A*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+24*A*
```

```
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b^2+128*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^3-48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b+576*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^3+112*A*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-416*A*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
3-9*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^3*b+156*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^2+156*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+144*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*B*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-228*B*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b^2+72*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^3+128*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+24*A*sin(d*x+c)*cos(d*x+c)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^2+128*A*sin(d*x+
c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*a*b^3-48*A*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b+576*A*sin(d*x+c)*cos(d*x+c)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^3+112*A*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b^2-416*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-9*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
```

```
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+156*B*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b^2+156*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b^3+144*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*B*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3*b-228*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b^2+72*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3)/sin(d*x+c)/b^2/cos(d*x+
c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.403 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=566

$$\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\,\left(3a^2B+30aAb+14abB+12Ab^2+16b^2B\right)\text{co}}{}$$

```
[Out] 1/3*b*B*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+1/24*(30*A*a*b
+3*B*a^2+16*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)+1
/12*(6*A*b+7*B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/24
*(a-b)*(30*A*a*b+3*B*a^2+16*B*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+1/24*(30*A*a*b+
12*A*b^2+3*B*a^2+14*B*a*b+16*B*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d-1/8*(6*A*a^2*b+8
*A*b^3-B*a^3+12*B*a*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A] time = 1.66, antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\,\left(3a^2B+30aAb+14abB+12Ab^2+16b^2B\right)\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(30*a*A*b + 3*a^2*B + 16*b^2*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(24*a*b*d) + (Sqrt[a + b]*(30*a*A*b + 12*A*b^2 + 3*a^2*B + 14
*a*b*B + 16*b^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d) - (Sqrt[a +
b]*(6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a +
b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
```

```
 d*x]))/(a - b)])/(8*b^2*d) + ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x]]) + ((6*A*b + 7*a*B)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d) + (b*B*Cos[
c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{bB\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

$$= \frac{(6Ab+7aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,\mathrm{s}}{12d}$$

$$= \frac{\left(30aAb+3a^2B+16b^2B\right)\sqrt{a+b\cos(c+dx)}\,\mathrm{si}}{24bd\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(30aAb+3a^2B+16b^2B\right)\sqrt{a+b\cos(c+dx)}\,\mathrm{si}}{24bd\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\,\left(6a^2Ab+8Ab^3-a^3B+12ab^2B\right)\cot(c}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\left(30aAb+3a^2B+16b^2B\right)\cot(c}{}$$

Mathematica [C] time = 6.32, size = 1227, normalized size = 2.17

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] ((-4*a*(42*a*A*b + 17*a^2*B + 16*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(
48*a^2*A + 24*A*b^2 + 52*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b
)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a +
```

```
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])) + 2*(30*a*A*b + 3*a^2*B + 16*b^2*B)*((I*Cos[(c + d*x)
/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[
c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(48*d) + (Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*(((6*A*b + 7*a*B)*Sin[c + d*x])/12 + (b*
B*Sin[2*(c + d*x)])/6))/d
```

fricas [F] time = 176.21, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)
), x)
```

maple [B] time = 0.39, size = 3139, normalized size = 5.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)

[Out] 1/24/d/(a+b*cos(d*x+c))^(1/2)*(-36*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b-30*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-30*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-14*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+52*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-72*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-12*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-12*A*cos(d*x+c)^4*b^3+12*A*cos(d*x+c)^2*b^3-8*B*cos(d*x+c)^5*b^3-8*B*cos(d*x+c)^3*b^3-3*B*cos(d*x+c)^2*a^3+16*B*cos(d*x+c)^2*b^3+3*B*cos(d*x+c)*a^3-42*A*cos(d*x+c)^3*a*b^2-30*A*cos(d*x+c)^2*a^2*b+30*A*cos(d*x+c)^2*a*b^2+30*A*cos(d*x+c)*a^2*b+12*A*cos(d*x+c)*a*b^2-22*B*cos(d*x+c)^4*a*b^2-17*B*cos(d*x+c)^3*a^2*b+3*B*cos(d*x+c)^2*a^2*b+6*B*cos(d*x+c)^2*a*b^2+14*B*cos(d*x+c)*a^2*b+16*B*cos(d*x+c)*a*b^2-48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-12*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-36*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))

```
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*a^2*b-30*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-30*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-14*B*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+52*B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-72*B*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*a*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b-16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b^2+24*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+48*A*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+24*A*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-48*A*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*b^3+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*a^3-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3-16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3)/sin(d*x+c)/b/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos (c+d\,x)}\ (A+B\ \cos (c+d\,x))\ (a+b\ \cos (c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.404 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=472

$$-\frac{\sqrt{a+b}\,\left(3a^2B+12aAb+4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}$$

```
[Out] 1/4*(4*A*b+5*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*
b*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/4*(a-b)*(4*A*b+5
*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/a/d+1/4*(8*A*a+4*A*b+5*B*a+2*B*b)*cot(d*x+c)*Ellip
ticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/d-1/4*(12*A*a*b+3*B*a^2+4*B*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^
(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2
)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A] time = 1.15, antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2990, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\,\left(3a^2B+12aAb+4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + 5*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*a*d) + (Sqrt[a + b]*(8*a*A + 4*A*b + 5*a*B + 2*b*B)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(4*d) - (Sqrt[a + b]*(12*a*A*b + 3*a^2*B + 4*b^2*B)*Cot[c + d*
x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + ((4*A*b + 5*a*B)*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
```

```
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{bB\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\frac{1}{2}a(4a$$

$$= \frac{(4Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{\cos(c+d}{}$$

$$= \frac{(4Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{\cos(c+d}{}$$

$$= -\frac{\sqrt{a+b}\,\left(12aAb+3a^2B+4b^2B\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\right.\right.}{4bd}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(4Ab+5aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\,\sqrt{\cos(c+}}\right.\right.}{4ad}$$

Mathematica [C] time = 6.35, size = 1198, normalized size = 2.54

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (b*B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-4
*a*(8*a^2*A + 4*A*b^2 + 7*a*b*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(16*a*A*
b + 8*a^2*B + 4*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
```

```
[c + d*x]])) + 2*(4*A*b^2 + 5*a*b*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
+ d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-
a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b
*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8*d)
```

fricas [F] time = 7.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/sqrt(cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)
), x)
```

maple [B] time = 0.39, size = 2430, normalized size = 5.15

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(5*B*cos(d*x+c)^2*a^2-8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-5*B*cos(d*x+c)*a^2+4*A*cos(d*x+c)^3*b^
2-4*A*cos(d*x+c)^2*b^2+2*B*cos(d*x+c)^4*b^2-2*B*cos(d*x+c)^2*b^2+8*A*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+24
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)*a*b+8*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-8*B*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*a^2+5*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*a*b+4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-16*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+4*A*cos(d*x+c
)^2*a*b-4*A*cos(d*x+c)*a*b+7*B*cos(d*x+c)^3*a*b-5*B*cos(d*x+c)^2*a*b-2*B*co
s(d*x+c)*a*b+4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*b^2+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+5*B*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+4*A*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-16*A*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a
*b+5*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b+4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
```

```
(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2+8*B*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*b^2+5*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)*a^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2+24*A*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b)/(a+b*cos(d*x+c
))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{3/2}}{\sqrt{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{\frac{3}{2}}}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(3/2)/sqrt(cos(c + d*x)
), x)
```

3.405 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=449

$$-\frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\,(2a(A-B)-b(4A+B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+}{a-}}}{d}$$

```
[Out] 2*a*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(2*A*a-B*b)*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+(a-b)*(2*A*a-B*b)*cot(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a
-b))^(1/2)/a/d-(2*a*(A-B)-b*(4*A+B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^
(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-
sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-(2*A*b+3*B*a)*cot
(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b
)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+
sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] time = 1.18, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2989, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{(2aA-bB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\,(2a(A-B)-b(4A+B))\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+}{a-}}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(2*a*A - b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
 - (Sqrt[a + b]*(2*a*(A - B) - b*(4*A + B))*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/d - (Sqrt[a + b]*(2*A*b + 3*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/d + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]) - ((2*a*A - b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[
c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
```

```
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}a(2Ab+aB)-\frac{1}{2}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(2aA-bB)\sqrt{a+b\cos}}{d\sqrt{\cos(c}}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(2aA-bB)\sqrt{a+b\cos}}{d\sqrt{\cos(c}}$$

$$- \frac{\sqrt{a+b}\,(2Ab+3aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\,\sqrt{\cos(c+d}}\right.\right.}{d}$$

$$+ \frac{(a-b)\sqrt{a+b}\,(2aA-bB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{ad}$$

Mathematica [C] time = 6.35, size = 1196, normalized size = 2.66

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((4*
a*(-2*a*A*b - 2*a^2*B - b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 4*a*(2*a^2*A -
 2*A*b^2 - 4*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
+ b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-
(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
```

```
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
+ d*x]])) - 2*(2*a*A*b - b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b
)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[
c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(b*Sqrt[Cos[c + d*x]])))/(2*d)
```

fricas [F] time = 3.31, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2
), x)
```

maple [B] time = 0.24, size = 2185, normalized size = 4.87

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)

[Out] -1/d*(2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-B*cos(d*x+c)^2*b^2-2*a^2*A+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+2*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2+2*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*a^2+B*cos(d*x+c)^3*b^2+2*A*cos(d*x+c)*a^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b-2*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+2*A*cos(d*x+c)^2*a*b-2*A*cos(d*x+c)*a*b+B*cos(d*x+c)^2*a*b-B*cos(d*x+c)*a*b-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s

```
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-2*A*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+4*A*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+B*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-4*B*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\ \cos(c+d\,x))\ (a+b\ \cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(3/2)/cos(c + d*x)**(3/
2), x)
```

3.406 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=419

$$\frac{2\sqrt{a+b}\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{3ad}$$

```
[Out] 2/3*a*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/3*(a-b)*(4*A
*b+3*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+2/3*(3*A*b^2+a^2*(A-3*B)-a*(4*A*b-6*B*b))*
cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((
-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*
x+c))/(a-b))^(1/2)/a/d-2*b*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(
a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] time = 0.86, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2989, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(4*A*b + 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
3*a*d) + (2*Sqrt[a + b]*(3*A*b^2 + a^2*(A - 3*B) - a*(4*A*b - 6*b*B))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) - (2*b*Sqrt[a + b]*B*Cot[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\frac{1}{2}a(4Ab+3aB)+}{}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\frac{1}{2}a(4Ab+3aB)+}{\cos^{\frac{3}{2}}(c}$$

$$= -\frac{2b\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+}{a-}\right.}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(4Ab+3aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\,\sqrt{\cos(c+}}\right.\right.}{3ad}$$

Mathematica [C] time = 6.36, size = 1236, normalized size = 2.95

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] ((-4*a*(a^2*A - A*b^2 + 3*a*b*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
```

```
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-4*a*A*
b - 3*a^2*B + 3*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])) + 2*(-4*A*b^2 - 3*a*b*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
 + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(
-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a +
b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
+ d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*((2*Sec[c + d*x]*(4*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/3
 + (2*a*A*Sec[c + d*x]*Tan[c + d*x])/3))/d
```

fricas [F] time = 2.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2
), x)
```

maple [B] time = 0.40, size = 2318, normalized size = 5.53

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(3*B*cos(d*x+c)^2*a^2+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*cos(d*x+c)
^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-
3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^2+6*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*a*b-3*B*cos(d*x+c)*a^2+4*A*cos(d*x+c)
^3*b^2-4*A*cos(d*x+c)^2*b^2-a^2*A-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+A*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^2-4
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)^2*b^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*a^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+A*sin(d*x+c)*
cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
a^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*cos(d*x+c)*a^2+A*cos(d*x+c)^2*a^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+6*B*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
```

```
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
*a*b-4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+A*cos(d*x+c)^3*a*b-3*B*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)^2*a*b+4*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*a*b+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2+4*A*cos(d*x+c)^2*a*b-5*A*co
s(d*x+c)*a*b+3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*b-4*A*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+6
*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2)/(a+b*cos(d*x+c
))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))(a + b\cos(c + dx))^{\frac{3}{2}}}{\cos^{\frac{5}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(3/2)/cos(c + d*x)**(5/
2), x)
```

3.407 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=353

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2A+20abB+3Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{15a^2d}$$

```
[Out] 2/5*a*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/15*(6*A*b+5*
B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9*A*a
^2+3*A*b^2+20*B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2
)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-2/15*(a-b)*(9*A*a-3*A*b-5*B*a
+15*B*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)
^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.92, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2989, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2A+20abB+3Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{15a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^2*d) - (2*(a - b)*Sqrt[a + b]*(9*a*A - 3*A*b - 5*a*B +
15*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d) + (2*a*A*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(6*A*b + 5*a*B)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{5}\int \frac{\frac{1}{2}a(6Ab+5aB)+\frac{1}{2}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(6Ab+5aB)\sqrt{a+b\cos}}{15d\cos^{\frac{3}{2}}(}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(6Ab+5aB)\sqrt{a+b\cos}}{15d\cos^{\frac{3}{2}}(}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(9a^2A+3Ab^2+20abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\right.\right.}{15a^2d}$$

Mathematica [C] time = 6.45, size = 1314, normalized size = 3.72

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] -1/15*((-4*a*(-3*a^2*A*b + 3*A*b^3 - 5*a^3*B + 5*a*b^2*B)*Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - 4*a*(9*a^3*A + 3*a*A*b^2 + 20*a^2*b*B)*((Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
```

```
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[A
rcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-
a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b + 3*A*b^3 + 20*a*b
^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin
[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a
 + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b
)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(
a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]
)))/(a*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^2
*(6*A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/15 + (2*Sec[c + d*x]*(9*a^2*A*S
in[c + d*x] + 3*A*b^2*Sin[c + d*x] + 20*a*b*B*Sin[c + d*x]))/(15*a) + (2*a*
A*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2
), x)
```

maple [B] time = 0.32, size = 2666, normalized size = 7.55

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] 2/15/d*(3*A*a^3-9*A*cos(d*x+c)^3*a^3+3*A*cos(d*x+c)^3*b^3+6*A*cos(d*x+c)^2*
a^3-5*B*cos(d*x+c)^3*a^3-20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-15*B*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d
*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-3*
A*cos(d*x+c)^4*b^3+5*B*cos(d*x+c)*a^3-3*A*cos(d*x+c)^3*a*b^2+9*A*cos(d*x+c)
^2*a*b^2+9*A*cos(d*x+c)*a^2*b-20*B*cos(d*x+c)^4*a*b^2-20*B*cos(d*x+c)^3*a^2
*b+25*B*cos(d*x+c)^2*a^2*b+9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+3*A*sin(d*x+c)*cos(d*x+c)
^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-12
*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b-3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+20*B*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c
)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+20*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^2-20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+20*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+9*A*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-12*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+
```

```
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-3*A*sin(d*x+c)
*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b^2+20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+20*B*cos(d*x+c)^3*
a*b^2-9*A*cos(d*x+c)^4*a^2*b-6*A*cos(d*x+c)^4*a*b^2-5*B*cos(d*x+c)^4*a^2*b-
5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-9*A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin
(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+3*A*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-9*A
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^3)/(a+b*cos(d*x+c))^(1/2)/a/sin(d*x+c)/cos(d*x+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{3/2}}{\cos(c+dx)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(7/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(7/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.408 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=433

$$\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\,\left(-\left(a^2(25A-63B)\right)+3ab(19A-7\right.}{}$$

```
[Out] 2/7*a*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/35*(8*A*b+7*
B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/105*(25*A*a^2+3
*A*b^2+42*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)+2/1
05*(a-b)*(82*A*a^2*b-6*A*b^3+63*B*a^3+21*B*a*b^2)*cot(d*x+c)*EllipticE((a+b
*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d-
2/105*(a-b)*(6*A*b^2-a^2*(25*A-63*B)+3*a*b*(19*A-7*B))*cot(d*x+c)*EllipticF
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a
^2/d
```

Rubi [A] time = 1.35, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2989, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\,\left(a^2(-(25A-63B))+3ab(19A-7B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d) - (2*(a - b)*Sqrt[a + b]*(6*A*b^2
 - a^2*(25*A - 63*B) + 3*a*b*(19*A - 7*B))*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(105*a^2*d) + (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c +
 d*x]^(7/2)) + (2*(8*A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3
5*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b*Cos
[c + d*x]]*Sin[c + d*x])/(105*a*d*Cos[c + d*x]^(3/2))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}a(8Ab+7aB)+$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a+b\,\mathrm{c}}}{35d\cos^{\frac{5}{2}}}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a+b\,\mathrm{c}}}{35d\cos^{\frac{5}{2}}}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{a+b\,\mathrm{c}}}{35d\cos^{\frac{5}{2}}}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(82a^2Ab-6Ab^3+63a^3B+21ab^2B\right)\cot(c+}{1}$$

Mathematica [C] time = 6.54, size = 1407, normalized size = 3.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9
/2),x]
```

```
[Out] ((-4*a*(25*a^4*A - 31*a^2*A*b^2 + 6*A*b^4 + 21*a^3*b*B - 21*a*b^3*B)*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - 4*a*(-82*a^3*A*b + 6*a*A*b^3 - 63*a^4*B - 21*a^2*b^
2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(
-82*a^2*A*b^2 + 6*A*b^4 - 63*a^3*b*B - 21*a*b^3*B)*((I*Cos[(c + d*x)/2]*Sqr
t[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x
]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]
]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
 (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((
a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[
c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a^2*d) + (Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(8*A*b*Sin[c + d*x] + 7
*a*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x] + 3*A*b^2
*Sin[c + d*x] + 42*a*b*B*Sin[c + d*x]))/(105*a) + (2*Sec[c + d*x]*(82*a^2*A
*b*Sin[c + d*x] - 6*A*b^3*Sin[c + d*x] + 63*a^3*B*Sin[c + d*x] + 21*a*b^2*B
*Sin[c + d*x]))/(105*a^2) + (2*a*A*Sec[c + d*x]^3*Tan[c + d*x])/7))/d
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2
), x)
```

maple [B] time = 0.53, size = 3413, normalized size = 7.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] 2/105/d*(-63*B*cos(d*x+c)^4*a^4+42*B*cos(d*x+c)^3*a^4+21*B*cos(d*x+c)*a^4-2
5*A*cos(d*x+c)^4*a^4+10*A*cos(d*x+c)^2*a^4+6*A*cos(d*x+c)^5*b^4-6*A*cos(d*x
+c)^4*b^4+63*B*cos(d*x+c)^2*a^3*b-25*A*cos(d*x+c)^5*a^3*b-82*A*cos(d*x+c)^5
*a^2*b^2-3*A*cos(d*x+c)^5*a*b^3-82*A*cos(d*x+c)^4*a^3*b+55*A*cos(d*x+c)^4*a
^2*b^2+6*A*cos(d*x+c)^4*a*b^3+68*A*cos(d*x+c)^3*a^3*b-3*A*cos(d*x+c)^3*a*b^
3+27*A*cos(d*x+c)^2*a^2*b^2+39*A*cos(d*x+c)*a^3*b-63*B*cos(d*x+c)^5*a^3*b-4
2*B*cos(d*x+c)^5*a^2*b^2-21*B*cos(d*x+c)^5*a*b^3-21*B*cos(d*x+c)^4*a^2*b^2+
21*B*cos(d*x+c)^4*a*b^3+63*B*cos(d*x+c)^3*a^2*b^2+6*A*sin(d*x+c)*cos(d*x+c)
^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-6*
A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*b^4-25*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*sin(d*x
+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
```

1/2))*a^4-6*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*b^4-25*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+63*B*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-
63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^4+15*A*a^4+63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+21*B*sin(d*x+c)*cos(
d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b^2+21*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^3-84*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-21*B*sin(d*x+c)*cos(d
x+c)^4(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b^2+82*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3*b+82*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-6*A*sin(d*x+c)*cos(d
x+c)^3(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
^3-82*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3*b-51*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*A*sin(d*x+c)*cos(d*x
+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3
+63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^3*b+21*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+21*B*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-
84*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3*b-21*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x

```
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+82*A*sin(d*x+c)*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+8
2*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b^2-6*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-82*A*sin(d*x+c)*cos(d*x+c)^
4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-51*
A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b^2)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(7
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(9/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
[Out] Timed out
```

3.409 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=522

$$\frac{2\left(49a^2A+72abB+3Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{5}{2}}(c+dx)}+\frac{2\left(75a^3B+88a^2Ab+9ab^2B-4Ab^3\right)\sin(c+dx)\sqrt{}}{315a^2d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/9*a*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+2/63*(10*A*b+9
*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/315*(49*A*a^2+
3*A*b^2+72*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+2/
315*(88*A*a^2*b-4*A*b^3+75*B*a^3+9*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/
2)/a^2/d/cos(d*x+c)^(3/2)+2/315*(a-b)*(147*A*a^4+33*A*a^2*b^2+8*A*b^4+246*B
*a^3*b-18*B*a*b^3)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/315*(a-b)*(8*A*b^3-a^3*(147*A
-75*B)+3*a^2*b*(13*A-57*B)+6*a*b^2*(A-3*B))*cot(d*x+c)*EllipticF((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)
*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d
```

Rubi [A] time = 1.88, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2989, 3055, 2998, 2816, 2994}

$$\frac{2\left(88a^2Ab+75a^3B+9ab^2B-4Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(49a^2A+72abB+3Ab^2\right)\sin(c+dx)\sqrt{}}{315ad\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x
]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B -
18*a*b^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*d) + (2*(a - b)*Sqr
t[a + b]*(8*A*b^3 - a^3*(147*A - 75*B) + 3*a^2*b*(13*A - 57*B) + 6*a*b^2*(A
 - 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) + (2*a*A*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (2*(10*A*b + 9*a*B)
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*(49*
```

```
a^2*A + 3*A*b^2 + 72*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d
*Cos[c + d*x]^(5/2)) + (2*(88*a^2*A*b - 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqr
t[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\frac{1}{2}a(10Ab+9aB)+}{}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\,\mathrm{c}}}{63d\cos^{\frac{7}{2}}(}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\,\mathrm{c}}}{63d\cos^{\frac{7}{2}}(}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\,\mathrm{c}}}{63d\cos^{\frac{7}{2}}(}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{a+b\,\mathrm{c}}}{63d\cos^{\frac{7}{2}}(}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(147a^4A+33a^2Ab^2+8Ab^4+246a^3bB-18ab\right.}{}$$

Mathematica [C] time = 6.63, size = 1515, normalized size = 2.90

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
1/2),x]
```

```
[Out] -1/315*((-4*a*(-39*a^4*A*b + 31*a^2*A*b^3 + 8*A*b^5 - 75*a^5*B + 93*a^3*b^2
*B - 18*a*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(147*a^5*A + 33*a^3*A*b^
2 + 8*a*A*b^4 + 246*a^4*b*B - 18*a^2*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2
]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sq
```

```
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
 (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*A*b + 33*a^2*A*b^3 + 8*A*b^5
 + 246*a^3*b^2*B - 18*a*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]
]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)
]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c
 + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (
a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(b*Sqrt[Cos[c + d*x]]))))/(a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]]*((2*Sec[c + d*x]^4*(10*A*b*Sin[c + d*x] + 9*a*B*Sin[c + d*x]))/63
+ (2*Sec[c + d*x]^3*(49*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 72*a*b*
B*Sin[c + d*x]))/(315*a) + (2*Sec[c + d*x]^2*(88*a^2*A*b*Sin[c + d*x] - 4*A
*b^3*Sin[c + d*x] + 75*a^3*B*Sin[c + d*x] + 9*a*b^2*B*Sin[c + d*x]))/(315*a
^2) + (2*Sec[c + d*x]*(147*a^4*A*Sin[c + d*x] + 33*a^2*A*b^2*Sin[c + d*x] +
 8*A*b^4*Sin[c + d*x] + 246*a^3*b*B*Sin[c + d*x] - 18*a*b^3*B*Sin[c + d*x])
)/(315*a^3) + (2*a*A*Sec[c + d*x]^4*Tan[c + d*x])/9))/d
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)/cos(d*x + c)^(11/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.71, size = 4392, normalized size = 8.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] 2/315/d*(-33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3-8*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)
^4*a*b^4+246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^4*a^4*b+246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b^2-18*B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^4*a^2*b^3-18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^4-246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^4*b-153*B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)^4*a^3*b^2+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3+147*A*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b+33*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^5*a^3*b^2+33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^2*b^3+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
```

/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a*b^4-186*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)^5*a^4*b-33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^3*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^2*b^3-8*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^5*a*b^4+246*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b+246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^3*b^2-18*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)^5*a^2*b^3-18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a*b^4-246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^4*b-153*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^5*a^3*b^2+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^2*b^3+147*A*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^4*b-186*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)^4*a^4*b+117*B*cos(d*x+c)^2*a^4*b-75*B*cos(d*x+c)^6*a^4*b-246*B*co
s(d*x+c)^6*a^3*b^2-9*B*cos(d*x+c)^6*a^2*b^3+18*B*cos(d*x+c)^6*a*b^4-246*B*c
os(d*x+c)^5*a^4*b+165*B*cos(d*x+c)^5*a^3*b^2+18*B*cos(d*x+c)^5*a^2*b^3-18*B
*cos(d*x+c)^5*a*b^4+204*B*cos(d*x+c)^4*a^4*b+35*A*a^5-147*A*cos(d*x+c)^6*a^
4*b-88*A*cos(d*x+c)^6*a^3*b^2-33*A*cos(d*x+c)^6*a^2*b^3+4*A*cos(d*x+c)^6*a*
b^4+10*A*cos(d*x+c)^5*a^4*b-33*A*cos(d*x+c)^5*a^3*b^2+34*A*cos(d*x+c)^5*a^2
*b^3-8*A*cos(d*x+c)^5*a*b^4+68*A*cos(d*x+c)^4*a^3*b^2+4*A*cos(d*x+c)^4*a*b^
4+52*A*cos(d*x+c)^3*a^4*b-A*cos(d*x+c)^3*a^2*b^3+53*A*cos(d*x+c)^2*a^3*b^2+
85*A*cos(d*x+c)*a^4*b-9*B*cos(d*x+c)^4*a^2*b^3+81*B*cos(d*x+c)^3*a^3*b^2-8*
A*cos(d*x+c)^6*b^5-147*A*cos(d*x+c)^5*a^5+8*A*cos(d*x+c)^5*b^5+98*A*cos(d*x
+c)^4*a^5+14*A*cos(d*x+c)^2*a^5-75*B*cos(d*x+c)^5*a^5+30*B*cos(d*x+c)^3*a^5
+45*B*cos(d*x+c)*a^5+147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b

```
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5+8*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*b^5-147*A*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*a^5-75*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5+147*A*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5+8*A*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
b^5-147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)^4*a^5-75*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^5+33*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b^2+
33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)^4*a^2*b^3+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^4)/(a+b*cos(d*x+c))^(1/2)/a^
3/sin(d*x+c)/cos(d*x+c)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/
2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(11/2),x
)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/cos(c + d*x)^(11/2),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

3.410 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=779

$$\frac{\left(-15a^2B+50aAb+64b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}{240bd}+\frac{\left(-15a^3B+50a^2Ab+172ab^2B+120\right.}{}$$

```
[Out] 1/240*(50*A*a*b-15*B*a^2+64*B*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*
x+c)^(1/2)/b/d+1/40*(10*A*b-3*B*a)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)*cos(d*
x+c)^(1/2)/b/d+1/5*B*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/d
+1/1920*(150*A*a^3*b+2840*A*a*b^3-45*B*a^4+1692*B*a^2*b^2+1024*B*b^4)*sin(d
*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)+1/320*(50*A*a^2*b+120*A
*b^3-15*B*a^3+172*B*a*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/
2)/b/d-1/1920*(a-b)*(150*A*a^3*b+2840*A*a*b^3-45*B*a^4+1692*B*a^2*b^2+1024*
B*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(
1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1
+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d-1/1920*(45*a^4*B-30*a^3*b*(5*A+B)-16*b^4*
(45*A+64*B)-8*a*b^3*(355*A+193*B)-4*a^2*b^2*(295*A+423*B))*cot(d*x+c)*Ellip
ticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/b^2/d+1/128*(10*A*a^4*b-240*A*a^2*b^3-96*A*b^5-3*B*a^5-40*B*a^3*b^2-240*
B*a*b^4)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d
```

Rubi [A] time = 3.08, antiderivative size = 779, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(-15a^2B+50aAb+64b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}{240bd}+\frac{\left(50a^2Ab-15a^3B+172ab^2B+120A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2
*B + 1024*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*a*b^2*d) - (Sqrt[
```

```
a + b]*(45*a^4*B - 30*a^3*b*(5*A + B) - 16*b^4*(45*A + 64*B) - 8*a*b^3*(355
*A + 193*B) - 4*a^2*b^2*(295*A + 423*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(1920*b^2*d) + (Sqrt[a + b]*(10*a^4*A*b - 240*a^2*A*b^3 - 96*A*b^5 - 3*a^5
*B - 40*a^3*b^2*B - 240*a*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(128*b^3*d) + ((150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2*B
 + 1024*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(1920*b^2*d*Sqrt[Cos[
c + d*x]]) + ((50*a^2*A*b + 120*A*b^3 - 15*a^3*B + 172*a*b^2*B)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(320*b*d) + ((50*a*A*b - 15*
a^2*B + 64*b^2*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x
])/(240*b*d) + ((10*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5
/2)*Sin[c + d*x])/(40*b*d) + (B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(7/
2)*Sin[c + d*x])/(5*b*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
```

```
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{B\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{5bd}$$

$$= \frac{(10Ab-3aB)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}}{40bd}$$

$$= \frac{\left(50aAb-15a^2B+64b^2B\right)\sqrt{\cos(c+dx)}\,(a+b\,\mathrm{co}}{240bd}$$

$$= \frac{\left(50a^2Ab+120Ab^3-15a^3B+172ab^2B\right)\sqrt{\cos(c}}{320bd}$$

$$= \frac{\left(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2B+1\right.}{1920b^2d\sqrt{\cos(c}}$$

$$= \frac{\left(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2B+1\right.}{1920b^2d\sqrt{\cos(c}}$$

$$= \frac{\sqrt{a+b}\,\left(10a^4Ab-240a^2Ab^3-96Ab^5-3a^5B-\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\left(150a^3Ab+2840aAb^3-45a^4B\right.}{}$$

Mathematica [C] time = 6.53, size = 1353, normalized size = 1.74

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] -1/3840*((-4*a*(-1330*a^3*A*b - 3560*a*A*b^3 + 15*a^4*B - 3236*a^2*b^2*B -
1024*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos
[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-6440*a^2*A*b^2 - 1440*A*b^4
 - 2292*a^3*b*B - 4624*a*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b
)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])) + 2*(-150*a^3*A*b - 2840*a*A*b^3 + 45*a^4*B - 1692*a^
2*b^2*B - 1024*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ellipti
cE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c +
 d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*
Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sq
rt[Cos[c + d*x]])))/(b*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((
(590*a^2*A*b + 420*A*b^3 + 15*a^3*B + 898*a*b^2*B)*Sin[c + d*x])/(960*b) +
((170*a*A*b + 93*a^2*B + 88*b^2*B)*Sin[2*(c + d*x)])/480 + (b*(10*A*b + 21*
a*B)*Sin[3*(c + d*x)])/160 + (b^2*B*Sin[4*(c + d*x)])/40))/d
```

fricas [F] time = 9.12, size = 0, normalized size = 0.00

integral $\left(\left(Bb^2\cos\left(dx+c\right)^4 + Aa^2\cos\left(dx+c\right) + \left(2\,Bab + Ab^2\right)\cos\left(dx+c\right)^3 + \left(Ba^2 + 2\,Aab\right)\cos\left(dx+c\right)^2\right)\sqrt{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^4 + A*a^2*cos(d*x + c) + (2*B*a*b + A*b^2)*cos
(d*x + c)^3 + (B*a^2 + 2*A*a*b)*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sq
rt(cos(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.97, size = 5164, normalized size = 6.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
[Out] Timed out
```

3.411 $\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=664

$$\frac{\left(5a^2B+24aAb+12b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{32d}+\frac{\left(15a^3B+264a^2Ab+284ab^2B+128Ab\right.}{192bd\sqrt{\cos(}}$$

```
[Out] 1/4*b*B*cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+1/24*(8*A*b+11
*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+1/192*(264*A*a^2
*b+128*A*b^3+15*B*a^3+284*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/co
s(d*x+c)^(1/2)+1/32*(24*A*a*b+5*B*a^2+12*B*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)
*(a+b*cos(d*x+c))^(1/2)/d-1/192*(a-b)*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B
*a*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+1/192*(15*a^3*B+8*b^3*(16*A+9*B)+2*a^2*b*(
132*A+59*B)+4*a*b^2*(52*A+71*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d-1/64*(40*A*a^3*b+16
0*A*a*b^3-5*B*a^4+120*B*a^2*b^2+48*B*b^4)*cot(d*x+c)*EllipticPi((a+b*cos(d*
x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b
)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A] time = 2.21, antiderivative size = 664, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(5a^2B+24aAb+12b^2B\right)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{32d}+\frac{\left(264a^2Ab+15a^3B+284ab^2B+128Ab\right.}{192bd\sqrt{\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b*d) + (Sqrt[a + b]*(15*a^3*B + 8*b
^3*(16*A + 9*B) + 2*a^2*b*(132*A + 59*B) + 4*a*b^2*(52*A + 71*B))*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(192*b*d) - (Sqrt[a + b]*(40*a^3*A*b + 160*a*A*b^
```

```
3 - 5*a^4*B + 120*a^2*b^2*B + 48*b^4*B)*Cot[c + d*x]*EllipticPi[(a + b)/b,
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(64*b^2*d) + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(192*b*d*Sqrt[Cos[c + d*x]]) + ((2
4*a*A*b + 5*a^2*B + 12*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(32*d) + ((8*A*b + 11*a*B)*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d
*x])^(3/2)*Sin[c + d*x])/(24*d) + (b*B*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
```

```
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{bB\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d}$$

$$= \frac{(8Ab+11aB)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}{24d}$$

$$= \frac{\left(24aAb+5a^2B+12b^2B\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\,\text{cc}}}{32d}$$

$$= \frac{\left(264a^2Ab+128Ab^3+15a^3B+284ab^2B\right)\sqrt{a+b}}{192bd\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(264a^2Ab+128Ab^3+15a^3B+284ab^2B\right)\sqrt{a+b}}{192bd\sqrt{\cos(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\,\left(40a^3Ab+160aAb^3-5a^4B+120a^2b^2B\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\left(264a^2Ab+128Ab^3+15a^3B+2\right.}{}$$

Mathematica [C] time = 6.42, size = 1287, normalized size = 1.94

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]
),x]
```

```
[Out] ((-4*a*(472*a^2*A*b + 128*A*b^3 + 133*a^3*B + 356*a*b^2*B)*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
```

```
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]) - 4*a*(384*a^3*A + 608*a*A*b^2 + 644*a^2*b*B + 144*b^3*B)*((Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(264*a^2*A*
b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos
[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)
/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a
+ b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(384*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]*(((104*a*A*b + 59*a^2*B + 42*b^2*B)*Sin[c + d*x])/96 + (b
*(8*A*b + 17*a*B)*Sin[2*(c + d*x)])/48 + (b^2*B*Sin[3*(c + d*x)])/16))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.58, size = 4238, normalized size = 6.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/192/d/(a+b*cos(d*x+c))^(1/2)*(-472*A*cos(d*x+c)^3*a^2*b^2-133*B*cos(d*x+c
)^3*a^3*b-172*B*cos(d*x+c)^3*a*b^3-30*B*cos(d*x+c)^2*a^2*b^2+284*B*cos(d*x+
c)^2*a*b^3+118*B*cos(d*x+c)*a^3*b+284*B*cos(d*x+c)*a^2*b^2+72*B*cos(d*x+c)*
a*b^3-264*A*cos(d*x+c)^2*a^3*b+144*A*cos(d*x+c)^2*a*b^3+208*A*cos(d*x+c)*a^
2*b^2+128*A*cos(d*x+c)*a*b^3-128*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-15*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+30*B*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4-288*B*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+
144*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^4-48*B*cos(d*x+c)^6*b^4-64*A*cos(d*x+c)^3*b^4+128*A*cos(d*x+c)^2*b^
4-24*B*cos(d*x+c)^4*b^4+72*B*cos(d*x+c)^2*b^4-15*B*cos(d*x+c)^2*a^4+15*B*co
s(d*x+c)*a^4-64*A*cos(d*x+c)^5*b^4+15*B*cos(d*x+c)^2*a^3*b-272*A*cos(d*x+c)
^4*a*b^3+264*A*cos(d*x+c)^2*a^2*b^2+264*A*cos(d*x+c)*a^3*b-184*B*cos(d*x+c)
^5*a*b^3-254*B*cos(d*x+c)^4*a^2*b^2-264*A*sin(d*x+c)*cos(d*x+c)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b-15*B*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^4+30*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*a^4-288*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+144*B*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-2
64*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
```

/2))*a^3*b-264*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b^2-128*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-240*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b-960*A*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^3-208*A*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b^2+608*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^3-15*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^3*b-284*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-284*B*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-720*B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2-
118*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^3*b+644*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^2-72*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-128*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-264*A*sin(d*x+c)*
cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
a^2*b^2-128*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*a*b^3-240*A*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b-960*A*sin(d*x+c)*
cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*a*b^3-208*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+608*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-15*B*sin(d*x+c)*c

```
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3*b-284*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b^2-284*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-720*B*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a
^2*b^2-118*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^3*b+644*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-72*B*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
3+384*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3*b+384*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b)/sin(d*x+c)/b/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c+d\,x)}\;\left(A+B\,\cos(c+d\,x)\right)\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.412 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=564

$$\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\,\left(a^2(48A+33B)+a(54Ab+26bB)+4b^2(3A\right.}{}$$

```
[Out] 1/3*b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+1/24*(54*A*a*b
+33*B*a^2+16*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/
4*b*(2*A*b+3*B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/24
*(a-b)*(54*A*a*b+33*B*a^2+16*B*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+1/24*(4*b^2*(3*A
+4*B)+a^2*(48*A+33*B)+a*(54*A*b+26*B*b))*cot(d*x+c)*EllipticF((a+b*cos(d*x+
c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a
*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/8*(30*A*a^2
*b+8*A*b^3+5*B*a^3+20*B*a*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)
/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A] time = 1.70, antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\,\left(a^2(48A+33B)+a(54Ab+26bB)+4b^2(3A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(54*a*A*b + 33*a^2*B + 16*b^2*B)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(24*a*d) + (Sqrt[a + b]*(4*b^2*(3*A + 4*B) + a^2*(48*A + 33*
B) + a*(54*A*b + 26*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d) - (Sqrt
[a + b]*(30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Cot[c + d*x]*Elliptic
Pi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(8*b*d) + ((54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[
```

```
a + b*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + (b*(2*A*b + 3
*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (b*
B*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{bB\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\int \frac{\sqrt{a}}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d} +$$

$$= \frac{\left(54aAb+33a^2B+16b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{24d\sqrt{\cos(c+dx)}} +$$

$$= \frac{\left(54aAb+33a^2B+16b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{24d\sqrt{\cos(c+dx)}} +$$

$$= -\frac{\sqrt{a+b}\,\left(30a^2Ab+8Ab^3+5a^3B+20ab^2B\right)\cot(c+dx)\Pi\left(\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\left(54aAb+33a^2B+16b^2B\right)\cot(c+dx)E\left(\sin\right.}{24a}$$

Mathematica [C] time = 6.51, size = 1251, normalized size = 2.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] ((-4*a*(48*a^3*A + 66*a*A*b^2 + 59*a^2*b*B + 16*b^3*B)*Sqrt[((a + b)*Cot[(c
 + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(
-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]]) - 4*a*(144*a^2*A*b + 24*A*b^3 + 48*a^3*B + 76*a*b^2*B)*((Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a +
```

```
b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
 + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(54*a*A*b^2 + 33*a
^2*b*B + 16*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[
I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*
x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec
[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[
Cos[c + d*x]])))/(48*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((b*
(6*A*b + 13*a*B)*Sin[c + d*x])/12 + (b^2*B*Sin[2*(c + d*x)])/6))/d
```

fricas [F] time = 106.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+}}{\sqrt{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)
), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.55, size = 3512, normalized size = 6.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/24/d*(-48*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3+48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^3+180*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+54*A*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b+54*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^2+26*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-76*B*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+120*B*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/
(a+b))^(1/2))*a*b^2+33*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+12*A*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+16*B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b^2-144*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+12*A*cos(d*x+c)^4*b^3-12*A*cos(d*x+c)^2*b
^3+8*B*cos(d*x+c)^5*b^3+8*B*cos(d*x+c)^3*b^3+33*B*cos(d*x+c)^2*a^3-16*B*cos
(d*x+c)^2*b^3-33*B*cos(d*x+c)*a^3+66*A*cos(d*x+c)^3*a*b^2+54*A*cos(d*x+c)^2
*a^2*b-54*A*cos(d*x+c)^2*a*b^2-54*A*cos(d*x+c)*a^2*b-12*A*cos(d*x+c)*a*b^2+
34*B*cos(d*x+c)^4*a*b^2+59*B*cos(d*x+c)^3*a^2*b-33*B*cos(d*x+c)^2*a^2*b-18*
B*cos(d*x+c)^2*a*b^2-26*B*cos(d*x+c)*a^2*b-16*B*cos(d*x+c)*a*b^2+48*A*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*b^3+30*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
```

n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+33*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*B*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^3+12*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^2+180*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*a^2*b+54*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+54*A*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+26*B*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-76*B*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+1
20*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a*b^2+33*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b+16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-24*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-144*A*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-24*A*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^3+48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2))*b^3+30*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*a^3+33*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+48*A*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48*B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3)/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}}{\sqrt{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.413 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=547

$$-\frac{\left(8a^2A-9abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\,\left(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+B)\right)\mathrm{co}}{}$$

```
[Out] 2*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)-1/4*(8*A*a^2-4*A
*b^2-9*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/2*b*(4
*A*a-B*b)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d+1/4*(a-b)*(8
*A*a^2-4*A*b^2-9*B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-1/4*(8*a^2*(A-B)-2*b^2*(2*A+
B)-3*a*b*(8*A+3*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b)
)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(20*A*a*b+15*B*a^2+4*B*b^2)*co
t(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+
b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1
+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] time = 1.67, antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2989, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(8a^2A-9abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\,\left(8a^2(A-B)-3ab(8A+3B)-2b^2(2A+B)\right)\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(8*a^2*A - 4*A*b^2 - 9*a*b*B)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(4*a*d) - (Sqrt[a + b]*(8*a^2*(A - B) - 2*b^2*(2*A + B) - 3*a*b*
(8*A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (Sqrt[a + b]*(20*a*
A*b + 15*a^2*B + 4*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
```

```
*d) - ((8*a^2*A - 4*A*b^2 - 9*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(4*d*Sqrt[Cos[c + d*x]]) - (b*(4*a*A - b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\sqrt{a+b\cos(c+}}{}$$

$$= -\frac{b(4aA-bB)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} +$$

$$= -\frac{\left(8a^2A-4Ab^2-9abB\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} - \frac{b(}{}$$

$$= -\frac{\left(8a^2A-4Ab^2-9abB\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} - \frac{b(}{}$$

$$= -\frac{\sqrt{a+b}\left(20aAb+15a^2B+4b^2B\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\right.\right.}{4d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(8a^2A-4Ab^2-9abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{4ad}$$

Mathematica [C] time = 6.49, size = 1241, normalized size = 2.27

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] ((4*a*(-16*a^2*A*b - 4*A*b^3 - 8*a^3*B - 11*a*b^2*B)*Sqrt[((a + b)*Cot[(c +
 d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]) + 4*a*(8*a^3*A - 24*a*A*b^2 - 24*a^2*b*B - 4*b^3*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
```

```
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(8*a^2*A*b - 4*A*b^3 -
9*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSin
h[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x
])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((
a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[
(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/
a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
Pi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c +
d*x]])))/(8*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((b^2*B*Sin[c
 + d*x])/2 + 2*a^2*A*Tan[c + d*x]))/d
```

fricas [F] time = 4.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c) +}}{\cos(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2
), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

[Out] Timed out

maple [B] time = 0.31, size = 3270, normalized size = 5.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)

[Out] 1/4/d*(8*A*a^3-8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3-8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3-40*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-40*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-4*A*cos(d*x+c)^3*b^3-8*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+24*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c

```
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-24*A*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b+4*A*cos(d*x+c)^2*b^3+2*B*cos(d*x+c)^2*b^3-8*A*cos(d*x+c)^2*a^2*b-
4*A*cos(d*x+c)^2*a*b^2+8*A*cos(d*x+c)*a^2*b+4*A*cos(d*x+c)*a*b^2-9*B*cos(d*
x+c)^2*a^2*b+9*B*cos(d*x+c)^2*a*b^2+9*B*cos(d*x+c)*a^2*b+2*B*cos(d*x+c)*a*b
^2-8*A*cos(d*x+c)*a^3-2*B*cos(d*x+c)^4*b^3+24*A*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+8*A*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-4*A*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2
4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^2-9*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b-9*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-11*B*cos(d*x+c)^3*a*b^2-
8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3-8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-4*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+4*B*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b^3-8*B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3)/
(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{5/2}}{\cos(c+dx)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.414 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=536

$$-\frac{\left(6a^2B+14aAb-3b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\,\left(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6A+B)\right)}{}$$

```
[Out] 2/3*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*a*(2*A*b+B*a
)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*(14*A*a*b+6*B*a^
2-3*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/3*(a-b)*(
14*A*a*b+6*B*a^2-3*B*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))
/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-1/3*(2*a*b*(7*A-9*B)-2*a^2
*(A-3*B)-3*b^2*(6*A+B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-b*(2*A*b+5*B*a)*cot(d*x+c)*Ell
ipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)
/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/d
```

Rubi [A] time = 1.67, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2989, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(6a^2B+14aAb-3b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\,\left(-2a^2(A-3B)+2ab(7A-9B)-3b^2(6A+B)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(14*a*A*b + 6*a^2*B - 3*b^2*B)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3*a*d) - (Sqrt[a + b]*(2*a*b*(7*A - 9*B) - 2*a^2*(A - 3*B) - 3
*b^2*(6*A + B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d) - (b*Sqrt[a + b]*(
2*A*b + 5*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c +
 d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a*(2*A
```

```
*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (
(14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*
Sqrt[Cos[c + d*x]]) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*
Cos[c + d*x]^(3/2))
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\sqrt{a+b\cos(c+}}{}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2aA(a+b\,\mathrm{cc}}{3a}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\left(14aAb+6a\right.}{}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\left(14aAb+6a\right.}{}$$

$$= -\frac{b\sqrt{a+b}\,(2Ab+5aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\,\sqrt{\cos(c+}}\right.\right.}{d}$$

$$= \frac{(a-b)\sqrt{a+b}\,\left(14aAb+6a^2B-3b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\right.\right.}{3ad}$$

Mathematica [C] time = 6.51, size = 1269, normalized size = 2.37

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] ((-4*a*(2*a^3*A + 4*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - 4*a*(-14*a^2*A*b + 6*A*b^3 - 6*a^3*B + 18*a*b^2*B)*((Sqrt[((a + b)*C
```

```
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-14*a*A*b^2 - 6*a^2*b*
B + 3*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcS
inh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d
*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c
+ d*x]])))/(6*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c +
 d*x]*(7*a*A*b*Sin[c + d*x] + 3*a^2*B*Sin[c + d*x]))/3 + (2*a^2*A*Sec[c + d
*x]*Tan[c + d*x])/3))/d
```

fricas [F] time = 2.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bb^2\cos\left(dx+c\right)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2
), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.32, size = 3204, normalized size = 5.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/3/d*(-2*A*a^3+2*A*cos(d*x+c)^2*a^3-14*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-14*A*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b^2+18*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2*b-18*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+30*B*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^
2-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b+18*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+3*B*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-18*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b^2+14*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-3*B*cos(d*x+c)^3*b^3+6*B*cos(d*x+c)^2*
a^3-6*B*cos(d*x+c)*a^3+14*A*cos(d*x+c)^3*a*b^2+14*A*cos(d*x+c)^2*a^2*b-14*A
*cos(d*x+c)^2*a*b^2-16*A*cos(d*x+c)*a^2*b+6*B*cos(d*x+c)^3*a^2*b-6*B*cos(d*
x+c)^2*a^2*b-3*B*cos(d*x+c)^2*a*b^2+3*B*cos(d*x+c)^4*b^3+12*A*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*b^3-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
```

```
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-6*A*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-14*A*sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^2*b-14*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+14*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+18*A*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b^2-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+18*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)
*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b+30*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*a*b^2+3*B*cos(d*x+c)^3*a*b^2+2*A*cos(d*x+c)^3*a^2
*b+2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+3*B*sin(d*x+c
)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^3-6*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*b^3+12*A*sin(d*x+c)*cos(d*x+c)^2*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^3+6*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*
cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
+2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{5/2}}{\cos(c+dx)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.415 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=493

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2A+35abB+23Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\right.}{15ad}$$

```
[Out] 2/5*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/15*a*(8*A*b+
5*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9*A
*a^2+23*A*b^2+35*B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+2/15*(15*A*b^3-a*b^2*(23*A-4
5*B)+a^2*b*(17*A-35*B)-a^3*(9*A-5*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-2*b^2*B*cot(d*
x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b
,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] time = 1.25, antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2989, 3047, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(a^2b(17A-35B)+a^3(-(9A-5B))-ab^2(23A-45B)+15Ab^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1}{a-b}}}{15ad}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(15*a*d) + (2*Sqrt[a + b]*(15*A*b^3 - a*b^2*(23*A - 45*B) +
a^2*b*(17*A - 35*B) - a^3*(9*A - 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
15*a*d) - (2*b^2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/d + (2*a*(8*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Co
```

```
s[c + d*x]^(3/2)) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Co
s[c + d*x]^(5/2))
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\sqrt{a+b\cos(c+}}{}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+b\,c}{5}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+b\,c}{5}$$

$$= -\frac{2b^2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\right| - \frac{a}{a}}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(9a^2A+23Ab^2+35abB\right)\cot(c+dx)E\left(\sin^{-1}\right.}{15ad}$$

Mathematica [C] time = 6.57, size = 1319, normalized size = 2.68

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7
/2),x]
```

```
[Out] ((4*a*(-8*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) + 4*a*(9*a^3*A + 23*a*A*b^2 + 35*a^2*b*B - 15*b^3*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(9*a^2*A*b + 23*A*b^3 +
```

```
35*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcS
inh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d
*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/
((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Co
t[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2
)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipt
icPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c
+ d*x]])))/(15*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c
+ d*x]^2*(11*a*A*b*Sin[c + d*x] + 5*a^2*B*Sin[c + d*x]))/15 + (2*Sec[c + d*
x]*(9*a^2*A*Sin[c + d*x] + 23*A*b^2*Sin[c + d*x] + 35*a*b*B*Sin[c + d*x]))/
15 + (2*a^2*A*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.38, size = 3274, normalized size = 6.64

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] 2/15/d*(3*A*a^3-9*A*cos(d*x+c)^3*a^3+23*A*cos(d*x+c)^3*b^3+6*A*cos(d*x+c)^2
*a^3-5*B*cos(d*x+c)^3*a^3-35*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-45*B*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(
d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-2
3*A*cos(d*x+c)^4*b^3+5*B*cos(d*x+c)*a^3-23*A*cos(d*x+c)^3*a*b^2+34*A*cos(d*
x+c)^2*a*b^2+14*A*cos(d*x+c)*a^2*b-35*B*cos(d*x+c)^4*a*b^2-35*B*cos(d*x+c)^
3*a^2*b+40*B*cos(d*x+c)^2*a^2*b+9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+23*A*sin(d*x+c)*cos(
d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^2-17*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2*b-23*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+35*B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*co
s(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
+35*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b^2-35*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+35*B*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d
*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+9*
A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b+23*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-17*A*sin(d*x+c)*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-23*A*s
in(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^2+35*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-45*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+35*B*cos(
d*x+c)^3*a*b^2-9*A*cos(d*x+c)^4*a^2*b-11*A*cos(d*x+c)^4*a*b^2-5*A*cos(d*x+c
)^3*a^2*b-5*B*cos(d*x+c)^4*a^2*b-15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+15*B*sin(d*x+c)*cos(
```

```
d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^
3-30*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*b^3+15*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-30*B*sin(d*x+c)*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3
-15*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*b^3-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^
3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+23*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*sin(d*x+c)*cos(d*
x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+
23*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*b^3-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+
c)/cos(d*x+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))^{5/2}}{\cos(c+dx)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2),x)
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(7/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
[Out] Timed out
```

3.416 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=434

$$\frac{2\left(25a^2A+77abB+45Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,\left(a^2(25A-63B)-8ab(15A-7B)+\right.}{}$$

```
[Out] 2/7*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/35*a*(10*A*b
+7*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/105*(25*A*a^
2+45*A*b^2+77*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2
/105*(a-b)*(145*A*a^2*b+15*A*b^3+63*B*a^3+161*B*a*b^2)*cot(d*x+c)*EllipticE
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a
^2/d+2/105*(a-b)*(a^2*(25*A-63*B)+15*b^2*(A-7*B)-8*a*b*(15*A-7*B))*cot(d*x+
c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a
-b))^(1/2)/a/d
```

Rubi [A] time = 1.37, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2A+77abB+45Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\,\left(a^2(25A-63B)-8ab(15A-7B)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Co
t[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d) + (2*(a - b)*Sqrt[a + b]*(a^2*
(25*A - 63*B) + 15*b^2*(A - 7*B) - 8*a*b*(15*A - 7*B))*Cot[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(105*a*d) + (2*a*(10*A*b + 7*a*B)*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2*A + 45*A*b^2 + 77*a*b*B
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)) + (2*a*
A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\sqrt{a+b\cos(c+}}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+b}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a^2A\right.}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a^2A\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(145a^2Ab+15Ab^3+63a^3B+161ab^2B\right)\cot(}{}$$

Mathematica [C] time = 6.63, size = 1409, normalized size = 3.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9
/2),x]
```

```
[Out] ((-4*a*(25*a^4*A - 10*a^2*A*b^2 - 15*A*b^4 + 56*a^3*b*B - 56*a*b^3*B)*Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - 4*a*(-145*a^3*A*b - 15*a*A*b^3 - 63*a^4*B - 161*a^
2*b^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[
c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) +
```

```
2*(-145*a^2*A*b^2 - 15*A*b^4 - 63*a^3*b*B - 161*a*b^3*B)*((I*Cos[(c + d*x)
/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[
c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a*d) + (Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(15*a*A*b*Sin[c +
d*x] + 7*a^2*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]^2*(25*a^2*A*Sin[c + d*x]
 + 45*A*b^2*Sin[c + d*x] + 77*a*b*B*Sin[c + d*x]))/105 + (2*Sec[c + d*x]*(1
45*a^2*A*b*Sin[c + d*x] + 15*A*b^3*Sin[c + d*x] + 63*a^3*B*Sin[c + d*x] + 1
61*a*b^2*B*Sin[c + d*x]))/(105*a) + (2*a^2*A*Sec[c + d*x]^3*Tan[c + d*x])/7
))/d
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bb^2\cos\left(dx+c\right)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2
), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.45, size = 3628, normalized size = 8.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(105*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+63*B*cos(d*x+c)^4*a^4-42*B*cos(d*x+c)^3*
a^4-21*B*cos(d*x+c)*a^4+25*A*cos(d*x+c)^4*a^4-10*A*cos(d*x+c)^2*a^4+15*A*co
s(d*x+c)^5*b^4-15*A*cos(d*x+c)^4*b^4-98*B*cos(d*x+c)^2*a^3*b+25*A*cos(d*x+c
)^5*a^3*b+145*A*cos(d*x+c)^5*a^2*b^2+45*A*cos(d*x+c)^5*a*b^3+145*A*cos(d*x+
c)^4*a^3*b-55*A*cos(d*x+c)^4*a^2*b^2+15*A*cos(d*x+c)^4*a*b^3-110*A*cos(d*x+
c)^3*a^3*b-60*A*cos(d*x+c)^3*a*b^3-90*A*cos(d*x+c)^2*a^2*b^2-60*A*cos(d*x+c
)*a^3*b+63*B*cos(d*x+c)^5*a^3*b+77*B*cos(d*x+c)^5*a^2*b^2+161*B*cos(d*x+c)^
5*a*b^3+35*B*cos(d*x+c)^4*a^3*b+161*B*cos(d*x+c)^4*a^2*b^2-161*B*cos(d*x+c)
^4*a*b^3-238*B*cos(d*x+c)^3*a^2*b^2+15*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+105*B*sin(d*x+c
)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^3-15*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*sin(d*x+c)*cos(d
*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4
+63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^4-15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*sin(d*x+c)*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*sin
(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^4+63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-15*A*a^4-63*B*sin(d*x+c)*cos(d*x+c)^4*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-161*B*
sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
```

```
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b^2-161*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+119*B*sin(d*x+c)*cos(d*x+c)^4
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+161*
B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b^2-145*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-145*A*sin(d*x+c)*cos(d*x+c)
^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-
15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^3+145*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+135*A*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2
+15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b^3-63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-161*B*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2
-161*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^3+119*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+161*B*sin(d*x+c)*cos(d*x
+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
^2-145*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3*b-145*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-15*A*sin(d*x+c)*cos(
d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^3+145*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3*b+135*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
```

```
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2)/(a+b*cos(d*x+c))^(
1/2)/a/sin(d*x+c)/cos(d*x+c)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(9/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(9/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.417 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=522

$$\frac{2\left(49a^2A+135abB+75Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315d\cos^{\frac{5}{2}}(c+dx)}+\frac{2\left(75a^3B+163a^2Ab+135ab^2B+5Ab^3\right)\sin(c+}{315ad\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/9*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(9/2)+2/21*a*(4*A*b+
3*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/315*(49*A*a^2
+75*A*b^2+135*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2
/315*(163*A*a^2*b+5*A*b^3+75*B*a^3+135*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))
^(1/2)/a/d/cos(d*x+c)^(3/2)+2/315*(a-b)*(147*A*a^4+279*A*a^2*b^2-10*A*b^4+4
35*B*a^3*b+45*B*a*b^3)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a
+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d-2/315*(a-b)*(10*A*b^3-6*a^2
*b*(19*A-60*B)+3*a^3*(49*A-25*B)+15*a*b^2*(11*A-3*B))*cot(d*x+c)*EllipticF(
(a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(
a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^
2/d
```

Rubi [A] time = 1.94, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(163a^2Ab+75a^3B+135ab^2B+5Ab^3\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(49a^2A+135abB+75Ab^2\right)\sin(c+}{315d\cos^{\frac{5}{2}}(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x
]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B
+ 45*a*b^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) - (2*(a - b)*S
qrt[a + b]*(10*A*b^3 - 6*a^2*b*(19*A - 60*B) + 3*a^3*(49*A - 25*B) + 15*a*b
^2*(11*A - 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d) + (2*a*(4*A
*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(7/2)
```

```
) + (2*(49*a^2*A + 75*A*b^2 + 135*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(315*d*Cos[c + d*x]^(5/2)) + (2*(163*a^2*A*b + 5*A*b^3 + 75*a^3*B + 13
5*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(3/
2)) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/
2))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\sqrt{a+b\cos(c+}}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2aA(a+b\,c}{9}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2A+\right.}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2A+\right.}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2A+\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(147a^4A+279a^2Ab^2-10Ab^4+435a^3bB+\right.}{}$$

Mathematica [C] time = 6.73, size = 1517, normalized size = 2.91

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
1/2),x]
```

```
[Out] -1/315*((-4*a*(-114*a^4*A*b + 124*a^2*A*b^3 - 10*A*b^5 - 75*a^5*B + 30*a^3*
b^2*B + 45*a*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(147*a^5*A + 279*a^3*
A*b^2 - 10*a*A*b^4 + 435*a^4*b*B + 45*a^2*b^3*B)*((Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
```

```
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*A*b + 279*a^2*A*b^3 - 1
0*A*b^5 + 435*a^3*b^2*B + 45*a*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
 + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(
-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a +
b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
+ d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin
[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*((2*Sec[c + d*x]^4*(19*a*A*b*Sin[c + d*x] + 9*a^2*B*Sin[c +
 d*x]))/63 + (2*Sec[c + d*x]^3*(49*a^2*A*Sin[c + d*x] + 75*A*b^2*Sin[c + d*
x] + 135*a*b*B*Sin[c + d*x]))/315 + (2*Sec[c + d*x]^2*(163*a^2*A*b*Sin[c +
d*x] + 5*A*b^3*Sin[c + d*x] + 75*a^3*B*Sin[c + d*x] + 135*a*b^2*B*Sin[c + d
*x]))/(315*a) + (2*Sec[c + d*x]*(147*a^4*A*Sin[c + d*x] + 279*a^2*A*b^2*Sin
[c + d*x] - 10*A*b^4*Sin[c + d*x] + 435*a^3*b*B*Sin[c + d*x] + 45*a*b^3*B*S
in[c + d*x]))/(315*a^2) + (2*a^2*A*Sec[c + d*x]^4*Tan[c + d*x])/9))/d
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bb^2\cos\left(dx+c\right)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2+\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+}}{\cos\left(dx+c\right)^{\frac{11}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(11/
2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.54, size = 4392, normalized size = 8.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*(279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b^2+155*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3-10*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^4*a*b^4-435*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^4*b-435*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b^2-45*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)^4*a^2*b^3-45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^4+435*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^4*b+405*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^4*a^3*b^2+45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3-147*A*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b-279*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)^5*a^3*b^2-279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^2*b^3+10*A*(cos(d*x+c)/(1+cos(d*x+
```

c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a*b^4+261*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^5*a^4*b+279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^3*b^2+155*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^2*b^3-
10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)^5*a*b^4-435*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b-435*B*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^3*b^2
-45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)^5*a^2*b^3-45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a*b^4+435*B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^4*b
+405*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)^5*a^3*b^2+45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^2*b^3-147*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^
4*b+261*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)^4*a^4*b-180*B*cos(d*x+c)^2*a^4*b+75*B*cos(d*x+c)^6*a^4
*b+435*B*cos(d*x+c)^6*a^3*b^2+135*B*cos(d*x+c)^6*a^2*b^3+45*B*cos(d*x+c)^6*
a*b^4+435*B*cos(d*x+c)^5*a^4*b-165*B*cos(d*x+c)^5*a^3*b^2+45*B*cos(d*x+c)^5
*a^2*b^3-45*B*cos(d*x+c)^5*a*b^4-330*B*cos(d*x+c)^4*a^4*b-35*A*a^5+147*A*co
s(d*x+c)^6*a^4*b+163*A*cos(d*x+c)^6*a^3*b^2+279*A*cos(d*x+c)^6*a^2*b^3+5*A*
cos(d*x+c)^6*a*b^4+65*A*cos(d*x+c)^5*a^4*b+279*A*cos(d*x+c)^5*a^3*b^2-199*A
*cos(d*x+c)^5*a^2*b^3-10*A*cos(d*x+c)^5*a*b^4-272*A*cos(d*x+c)^4*a^3*b^2+5*
A*cos(d*x+c)^4*a*b^4-82*A*cos(d*x+c)^3*a^4*b-80*A*cos(d*x+c)^3*a^2*b^3-170*
A*cos(d*x+c)^2*a^3*b^2-130*A*cos(d*x+c)*a^4*b-180*B*cos(d*x+c)^4*a^2*b^3-27
0*B*cos(d*x+c)^3*a^3*b^2-10*A*cos(d*x+c)^6*b^5+147*A*cos(d*x+c)^5*a^5+10*A*
cos(d*x+c)^5*b^5-98*A*cos(d*x+c)^4*a^5-14*A*cos(d*x+c)^2*a^5+75*B*cos(d*x+c
)^5*a^5-30*B*cos(d*x+c)^3*a^5-45*B*cos(d*x+c)*a^5-147*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+

```
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5+10
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)^5*b^5+147*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5+75*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5-147*A*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*a^5+10*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*b^5+147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^5+75*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4
*a^5-279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*cos(d*x+c)^4*a^3*b^2-279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3+10*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^
4*a*b^4)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/
2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2),x
)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(11/2),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

3.418 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=622

$$\frac{2\left(81a^2A+209abB+113Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{693d\cos^{\frac{7}{2}}(c+dx)}+\frac{2\left(539a^3B+1145a^2Ab+825ab^2B+15Ab^3\right)\sin}{3465ad\cos^{\frac{5}{2}}(c+dx}$$

```
[Out] 2/11*a*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(11/2)+2/99*a*(14*A
*b+11*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+2/693*(81*A
*a^2+113*A*b^2+209*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7
/2)+2/3465*(1145*A*a^2*b+15*A*b^3+539*B*a^3+825*B*a*b^2)*sin(d*x+c)*(a+b*co
s(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+2/3465*(675*A*a^4+1025*A*a^2*b^2-20*A*
b^4+1793*B*a^3*b+55*B*a*b^3)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*
x+c)^(3/2)+2/3465*(a-b)*(3705*A*a^4*b+255*A*a^2*b^3+40*A*b^5+1617*B*a^5+306
9*B*a^3*b^2-110*B*a*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/3465*(a-b)*(40*A*b^4+3*
a^4*(225*A-539*B)-6*a^3*b*(505*A-209*B)+15*a^2*b^2*(19*A-121*B)+10*a*b^3*(3
*A-11*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d
```

Rubi [A] time = 2.62, antiderivative size = 622, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(1025a^2Ab^2+675a^4A+1793a^3bB+55ab^3B-20Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3465a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(1145a^2Ab+539a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2),x
]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*
B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3465*
a^4*d) + (2*(a - b)*Sqrt[a + b]*(40*A*b^4 + 3*a^4*(225*A - 539*B) - 6*a^3*b
*(505*A - 209*B) + 15*a^2*b^2*(19*A - 121*B) + 10*a*b^3*(3*A - 11*B))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
```

```
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(3465*a^3*d) + (2*a*(14*A*b + 11*a*B)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*(81*a^2*A +
113*A*b^2 + 209*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*d*Cos[c
+ d*x]^(7/2)) + (2*(1145*a^2*A*b + 15*A*b^3 + 539*a^3*B + 825*a*b^2*B)*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3465*a*d*Cos[c + d*x]^(5/2)) + (2*(675*
a^4*A + 1025*a^2*A*b^2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(3465*a^2*d*Cos[c + d*x]^(3/2)) + (2*a*A*(a + b*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
```

```
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2aA(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2}{11}\int \frac{\sqrt{a+b\cos(c+dx)}}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2aA(a+b}{11}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81a^2A+\right.}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81a^2A+\right.}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81a^2A+\right.}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81a^2A+\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(3705a^4Ab+255a^2Ab^3+40Ab^5+1617a^5B+\right.}{}$$

Mathematica [C] time = 6.85, size = 1640, normalized size = 2.64

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(1
3/2),x]
```

```
[Out] ((-4*a*(675*a^6*A - 390*a^4*A*b^2 - 245*a^2*A*b^4 - 40*A*b^6 + 1254*a^5*b*B
 - 1364*a^3*b^3*B + 110*a*b^5*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
```

```
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-3705*a
^5*A*b - 255*a^3*A*b^3 - 40*a*A*b^5 - 1617*a^6*B - 3069*a^4*b^2*B + 110*a^2
*b^4*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) +
2*(-3705*a^4*A*b^2 - 255*a^2*A*b^4 - 40*A*b^6 - 1617*a^5*b*B - 3069*a^3*b^3
*B + 110*a*b^5*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I
*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x
])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[
c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a +
b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)
/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*E
llipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[C
os[c + d*x]])))/(3465*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
*((2*Sec[c + d*x]^5*(23*a*A*b*Sin[c + d*x] + 11*a^2*B*Sin[c + d*x]))/99 + (
2*Sec[c + d*x]^4*(81*a^2*A*Sin[c + d*x] + 113*A*b^2*Sin[c + d*x] + 209*a*b*
B*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^3*(1145*a^2*A*b*Sin[c + d*x] + 15*A*
b^3*Sin[c + d*x] + 539*a^3*B*Sin[c + d*x] + 825*a*b^2*B*Sin[c + d*x]))/(346
5*a) + (2*Sec[c + d*x]^2*(675*a^4*A*Sin[c + d*x] + 1025*a^2*A*b^2*Sin[c + d
*x] - 20*A*b^4*Sin[c + d*x] + 1793*a^3*b*B*Sin[c + d*x] + 55*a*b^3*B*Sin[c
+ d*x]))/(3465*a^2) + (2*Sec[c + d*x]*(3705*a^4*A*b*Sin[c + d*x] + 255*a^2*
A*b^3*Sin[c + d*x] + 40*A*b^5*Sin[c + d*x] + 1617*a^5*B*Sin[c + d*x] + 3069
*a^3*b^2*B*Sin[c + d*x] - 110*a*b^4*B*Sin[c + d*x]))/(3465*a^3) + (2*a^2*A*
Sec[c + d*x]^5*Tan[c + d*x])/11))/d
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{13}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
```

```
rithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(13/
2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.43, size = 5373, normalized size = 8.64

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/
2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,(a+b\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(13/2),x
)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/cos(c + d*x)^(13/2),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

3.419 $\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=418

$$-\frac{B(a-3b)\sqrt{a+b}\,\left(2a^2-ab+3b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+$$

```
[Out] b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*(a-b)*(a^2+3*b^2
)*B*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a/d-(a-3*b)*(2*a^2-a*b+3*b^2)*B*cot(d*x+c)*EllipticF
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a
/d-b*(5*a+3*b^2/a)*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/
2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x
+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] time = 0.95, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {2989, 2991, 2809, 2998, 2816, 2994}

$$-\frac{B(a-3b)\sqrt{a+b}\,\left(2a^2-ab+3b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*((3*b*B)/(2*a) + B*Cos[c + d*x]))/Cos[c + d
*x]^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(a^2 + 3*b^2)*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
a*d) - ((a - 3*b)*Sqrt[a + b]*(2*a^2 - a*b + 3*b^2)*B*Cot[c + d*x]*Elliptic
F[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(a*d) - (b*Sqrt[a + b]*(5*a + (3*b^2)/a)*B*Cot[c + d*x]*Ellip
ticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/d + (b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*
x])/(d*Cos[c + d*x]^(3/2))
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2991

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)]])/((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(B*d
)/b^2, Int[Sqrt[b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Int[(A*c
 + (B*c + A*d)*Sin[e + f*x])/((b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\cos(c+dx)\right)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\sqrt{a+b\cos(c+}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{\frac{3}{2}a\left(a^2+3b^2\right)B}{}$$

$$= -\frac{b\sqrt{a+b}\left(5a+\frac{3b^2}{a}\right)B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(a^2+3b^2\right)B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{ad}$$

Mathematica [C] time = 19.45, size = 1236, normalized size = 2.96

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*((3*b*B)/(2*a) + B*Cos[c + d*x]))/Cos
[c + d*x]^(5/2),x]
```

```
[Out] -1/2*(B*((-4*a*(-5*a^3*b - 3*a*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
 b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[
```

```
c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d
*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(2*a^
4 + a^2*b^2 - 3*b^4)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
 + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[
-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c
 + d*x]])) + 2*(2*a^3*b + 6*a*b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c +
d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a
- b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*C
os[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)
/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
 - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d
*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(b*Sqrt[Cos[c + d*x]]))))/(a*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]*(Sec[c + d*x]*(2*a^2*B*Sin[c + d*x] + 7*b^2*B*Sin[c + d*x]) + a
*b*B*Sec[c + d*x]*Tan[c + d*x]))/d
```

fricas [F] time = 39.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(2\,Bab^{2}\cos\left(dx+c\right)^{3}+3\,Ba^{2}b+\left(4\,Ba^{2}b+3\,Bb^{3}\right)\cos\left(dx+c\right)^{2}+2\left(Ba^{3}+3\,Bab^{2}\right)\cos\left(dx+c\right)\right)\sqrt{b\,\mathrm{co}}}{2\,a\cos\left(dx+c\right)^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),
x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*B*a*b^2*cos(d*x + c)^3 + 3*B*a^2*b + (4*B*a^2*b + 3*B*b^3)*
cos(d*x + c)^2 + 2*(B*a^3 + 3*B*a*b^2)*cos(d*x + c))*sqrt(b*cos(d*x + c) +
a)/(a*cos(d*x + c)^(5/2)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),
x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.51, size = 2346, normalized size = 5.61

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -B/a/d*(-2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^4+6*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+cos(d*x+c)*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+9*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^3-2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-6*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^2-6*cos(d*x+c)^2*si
n(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*
b^3+10*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+7*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+cos(d*x+c)^2*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+9
*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^3-2*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b-6*cos(d*x+c)*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^2-6*cos(d*x+c)*si
n(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*
```

```
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*
b^3+10*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a^2*b^2+7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-cos(d*x+c)^2*a^3*b-8*cos(d
*x+c)*a^2*b^2+2*cos(d*x+c)^3*a^2*b^2-7*cos(d*x+c)^2*a*b^3-b*a^3+cos(d*x+c)^
4*a*b^3+2*cos(d*x+c)^3*a^3*b+6*cos(d*x+c)^3*a*b^3+6*cos(d*x+c)^2*a^2*b^2+2*
cos(d*x+c)^2*a^4-2*a^4*cos(d*x+c)+2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*cos(d*x+c)^2*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-2*c
os(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*a^4+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3*cos(d*x+c)*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4)/(a+b*
cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\int \frac{\left(2\,B\cos\left(dx+c\right)+\frac{3\,Bb}{a}\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)^(5/2),
x, algorithm="maxima")
```

```
[Out] 1/2*integrate((2*B*cos(d*x + c) + 3*B*b/a)*(b*cos(d*x + c) + a)^(5/2)/cos(d
*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,\cos\left(c+d\,x\right)+\frac{3\,B\,b}{2\,a}\right)\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((B*cos(c + d*x) + (3*B*b)/(2*a))*(a + b*cos(c + d*x))^(5/2))/cos(c + d
*x)^(5/2),x)
```

```
[Out] int(((B*cos(c + d*x) + (3*B*b)/(2*a))*(a + b*cos(c + d*x))^(5/2))/cos(c + d
*x)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(3/2*b*B/a+B*cos(d*x+c))/cos(d*x+c)**(5/2
),x)
```

```
[Out] Timed out
```

3.420 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=479

$$\frac{\sqrt{a+b}\left(-3a^2B+4aAb-4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{4b^3d}$$

```
[Out] 1/4*(4*A*b-3*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)+
1/2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/b/d-1/4*(a-b)*(4*A
*b-3*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d+1/4*(4*A*b-3*B*a+2*B*b)*cot(d*x+c)*Ell
ipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/b^2/d+1/4*(4*A*a*b-3*B*a^2-4*B*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x
+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d
```

Rubi [A] time = 1.08, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2990, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(-3a^2B+4aAb-4b^2B\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b - 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4
*a*b^2*d) + (Sqrt[a + b]*(4*A*b - 3*a*B + 2*b*B)*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(4*b^2*d) + (Sqrt[a + b]*(4*a*A*b - 3*a^2*B - 4*b^2*B)*Cot[c + d*x
]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d) + ((4*A*b - 3*a*B)*Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd} + \frac{\int \frac{\frac{aB}{2}+bB\cos(c+dx)+\frac{1}{2}(4A}{\sqrt{\cos(c+dx)}\sqrt{a+b}}}{2b}$$

$$= \frac{(4Ab-3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\,\mathrm{c}}}{2b}$$

$$= \frac{(4Ab-3aB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\,\mathrm{c}}}{2b}$$

$$= \frac{\sqrt{a+b}\left(4aAb-3a^2B-4b^2B\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{4b^3d}$$

$$= -\frac{(a-b)\sqrt{a+b}(4Ab-3aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+}{a-}\right.}{4ab^2d}$$

Mathematica [C] time = 12.43, size = 1175, normalized size = 2.45

$$-\frac{4a(4Ab-aB)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc}{a}}}{(a+b)\sqrt{\cos(c+dx)}}$$

$$\frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd} + \frac{}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] (B*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d) + ((-4
*a*(4*A*b - a*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
```

```
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 16*a*b*B*((Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*A*b - 3*a*B)*((I*Cos[(
c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/S
qrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*
((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(8*b*d)
```

fricas [F] time = 2.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c))/sqrt(b*cos(
d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

maple [B] time = 0.38, size = 1871, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(-3*B*cos(d*x+c)^2*a^2+3*B*cos(d*x+c)*a^2+4*A
*cos(d*x+c)^3*b^2-4*A*cos(d*x+c)^2*b^2+2*B*cos(d*x+c)^4*b^2-2*B*cos(d*x+c)^
2*b^2-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)*a*b-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+4*A*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*a*b+4*A*cos(d*x+c)^2*a*b-4*A*cos(d*x+c)*a*b-B*cos(d*x+c)^3*a*b+3*B*c
os(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*b+4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+6*B*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-3*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-
4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^2+4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*a*b-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+6*B*sin(d*x+c)*(cos(d*x+
```

```
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2+8
*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2-8*A*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b)/si
n(d*x+c)/b^2/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.421 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=427

$$-\frac{\sqrt{a+b}\,(2Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{B\sin(c+d}{d\sqrt{a+b}}$$

```
[Out] a*B*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos
(d*x+c)^(1/2)/d/(a+b*cos(d*x+c))^(1/2)-(a-b)*B*cot(d*x+c)*EllipticE((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1
/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+B*c
ot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/b/d-(2*A*b-B*a)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A] time = 1.09, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3003, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\,(2Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{B\sin(c+d}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) - (Sqrt[a + b]*(2*A*b -
 a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (a*B*Sin[c +
 d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + (B*Sqrt[Cos[c +
d*x]]*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
```

```
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}} + \frac{1}{2}\int \frac{aB+2aA\cos(c+dx)+(2Ab-aB}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+}$$

$$= \frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{abB+(2aAb-a(2Ab-aB))\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx}{2b} + \frac{(2A}{}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{b^2d}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\mathrm{se}}{}}}{abd}$$

Mathematica [C] time = 17.36, size = 4017, normalized size = 9.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] ((1 + Cos[c + d*x])^(3/2)*((A*Sqrt[Cos[c + d*x]])/Sqrt[a + b*Cos[c + d*x]]
+ (B*Cos[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*x]])*Sec[(c + d*x)/2]^2*((2*I
)*(a - b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] +
 (4*I)*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a -
 b))] - (8*I)*A*b*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]
], -((a + b)/(a - b))] + (4*I)*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*
Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)
/2] - b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(
c + d*x)/2]))/(4*b*Sqrt[(a - b)/(a + b)]*d*Sqrt[a + b*Cos[c + d*x]]*(((1 +
Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*Sin[c + d*x]*((2*I)*(a - b)*B*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[
(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*(A*b - a*B)
*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSin
h[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] - (8*I)*A*b*
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/
(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a -
b))] + (4*I)*a*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]]
, -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a +
b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a -
b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(8*S
qrt[(a - b)/(a + b)]*(a + b*Cos[c + d*x])^(3/2)) - (3*Sqrt[1 + Cos[c + d*x]
]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*((2*I)*(a - b)*B*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*T
an[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*(A*b - a*B)*Sqrt[(a + b*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a
+ b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] - (8*I)*A*b*Sqrt[(a + b*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh
[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*a*B*S
qrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(
a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b
))] + b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(
c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c +
```

d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(a + b)]*B*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(8*b*Sqrt[(a - b)/(a +
b)]*Sqrt[a + b*Cos[c + d*x]]) + ((1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]*((2*I)*(a - b)*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2
]], -((a + b)/(a - b))] + (4*I)*(A*b - a*B)*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c +
d*x)/2]], -((a + b)/(a - b))] - (8*I)*A*b*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/
(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + (4*I)*a*B*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcS
inh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(
a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Si
n[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(4*b*Sqrt[(a - b)/(a + b)]*Sqrt[a +
b*Cos[c + d*x]]) + ((1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]^2*((3*b*Sqrt[
(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Cos[(3*(c + d*x))/
2]*Sec[(c + d*x)/2])/2 + a*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sec[(c + d*x)/2]^2 - (b*Sqrt[(a - b)/(a + b)]*B*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^2)/2 + (I*(a - b)*B*EllipticE[I*A
rcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*(-((b*S
in[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c +
d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))] + ((2*I)*(A*b - a*B)*EllipticF[I*ArcSinh[Sqrt[(a - b)/(
a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*(-((b*Sin[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + C
os[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] -
((4*I)*A*b*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan
[(c + d*x)/2]], -((a + b)/(a - b))]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]
)^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + ((2*I)*a*B
*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/
2]], -((a + b)/(a - b))]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])))
+ ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (b*Sqrt[(a - b)/(a +
b)]*B*Sec[(c + d*x)/2]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 -
Sin[c + d*x]/(1 + Cos[c + d*x]))*Sin[(3*(c + d*x))/2])/(2*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]) + (a*Sqrt[(a - b)/(a + b)]*B*((Cos[c + d*x]*Sin[c + d
*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*Tan[(c + d*x)/
2])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*Sqrt[(a - b)/(a + b)]*B*((Co
s[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d
*x]))*Tan[(c + d*x)/2])/(2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) + (b*Sqrt
[(a - b)/(a + b)]*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*
Sin[(3*(c + d*x))/2]*Tan[(c + d*x)/2])/2 - (2*Sqrt[(a - b)/(a + b)]*(A*b -

```
a*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/
2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 + ((a - b)*Tan[(c + d*x)/2]^2)/(
a + b)]) - ((a - b)*Sqrt[(a - b)/(a + b)]*B*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2])/S
qrt[1 + ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (4*A*b*Sqrt[(a - b)/(a + b)
]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^
2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 + ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)]) - (2*a*Sqrt[(a - b)/(a + b)]*B*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[1 + ((a - b)*Tan[(c + d*x)/
2]^2)/(a + b)])))/(4*b*Sqrt[(a - b)/(a + b)]*Sqrt[a + b*Cos[c + d*x]]))
```

fricas [F] time = 1.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

maple [B] time = 0.49, size = 1002, normalized size = 2.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d/(a+b*cos(d*x+c))^(1/2)*(4*A*sin(d*x+c)*cos(d*x+c)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b-2*A*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*b-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+4*A*sin(d*x+c)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b-2*A*sin(d*x+c)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+B*sin(d*x+c)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+B*sin(d*x+c)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*B*sin(
d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
a+B*cos(d*x+c)^3*b+B*cos(d*x+c)^2*a-b*B*cos(d*x+c)^2-B*cos(d*x+c)*a)/sin(d*
x+c)/b/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(1/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{\cos(c + dx)}}{\sqrt{a + b\cos(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/sqrt(a + b*cos(c + d*x)),
x)
```

3.422 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=228

$$\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(}{}}}{}$$

```
[Out] 2*A*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a/d-2*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)
/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A] time = 0.27, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {3006, 2809, 2816}

$$\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*Sqrt[a + b]*
B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 3006

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[B/d, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Dist
[(B*c - A*d)/d, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = A\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx + B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2A\sqrt{a+b}\,\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{ad}$$

Mathematica [A] time = 1.48, size = 144, normalized size = 0.63

$$\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left((A-B)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+2B\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{d\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*((A - B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] +
 2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2])
```

fricas [F] time = 1.37, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^2+a\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^2 + a*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

maple [A] time = 0.28, size = 197, normalized size = 0.86

$$\frac{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\left(\sin^2(dx+c)\right)\left(A\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)-B\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\right.\right.}{d\sqrt{a+b\cos(dx+c)}\,\left(-1+\cos(dx+c)\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(a+b*cos(d*x+c))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)^2*(A*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))-B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))+2*B*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2)))/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}\,\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x)))
, x)
```

3.423 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=230

$$\frac{2A(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}\,(A-B)\,\mathrm{co}}{}$$

```
[Out] 2*A*(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-2*(A-B)*cot(d*x+c)*EllipticF((a+b*cos(d*
x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.32, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2998, 2816, 2994}

$$\frac{2A(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}\,(A-B)\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sq
rt[a + b]*(A - B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
```

```
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = A\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx + (-A+B)\int \frac{}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2A(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\cdots}{}}}{a^2d}$$

Mathematica [A] time = 13.00, size = 299, normalized size = 1.30

$$\frac{2\left(A\sin(c+dx)(a+b\cos(c+dx)) - \frac{2\sqrt{2}\cos^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\left(-2a(A+B)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\cdots\right.\right.\right.\right.}{}\right.}{ad\sqrt{\cos(c+dx)}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]
),x]
```

```
[Out] (2*(A*(a + b*Cos[c + d*x])*Sin[c + d*x] - (2*Sqrt[2]*(Cos[(c + d*x)/2]^2)^(
3/2)*(2*A*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(A + B)*Cos[(c + d*x)/2]^2*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
```

```
c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + A*Cos[c
 + d*x]*(a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(1 + Cos[c + d*x])^(3/2)))/
(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^3+a\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^3 + a*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

maple [B] time = 0.27, size = 935, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d/(a+b*cos(d*x+c))^(1/2)*(B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+2*B*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+A*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a-A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
```

```
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*b+A*cos(d*x+c)^3*b+A*cos(d*x+c)^2*a-A*cos(d*x+c)^2*b-A*cos(d*x+c
)*a)/a/cos(d*x+c)^(3/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^{3/2}\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3/2)
), x)
```

3.424 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=290

$$-\frac{2(a-b)\sqrt{a+b}\,(2Ab-3aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d}+\frac{2\sqrt{a}}{}$$

```
[Out] 2/3*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)-2/3*(a-b)*(2*A
*b-3*B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d+2/3*(2*A*b+a*(A-3*B))*cot(d*x+c)*Ellipti
cF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2)
)*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)
/a^2/d
```

Rubi [A] time = 0.52, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3000, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\,(a(A-3B)+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}-\frac{2(a-b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b - 3*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(3*a^3*d) + (2*Sqrt[a + b]*(2*A*b + a*(A - 3*B))*Cot[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(3*a^2*d) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos
[c + d*x]^(3/2))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{\frac{1}{2}(-2Ab+3aB)+\frac{1}{2}aA\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{(2Ab+a(A-3B))\int \frac{}{\sqrt{\cos(c+dx)}\,\sqrt{}}}{3a}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(2Ab-3aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\right.}{3a^3d}$$

Mathematica [A] time = 15.73, size = 416, normalized size = 1.43

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)(3aB\sin(c+dx)-2Ab\sin(c+dx))}{3a^2}+\frac{2A\tan(c+dx)\sec(c+dx)}{3a}\right)}{d}+\frac{8\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\,\cos^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]
),x]
```

```
[Out] (8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Co
s[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(a + b)*(-2*A*b + 3*a*B)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(-2*A*b + a
*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] + (2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])^(3/
2)*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
*((2*Sec[c + d*x]*(-2*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(3*a^2) + (2*
A*Sec[c + d*x]*Tan[c + d*x])/(3*a)))/d
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b\cos(dx+c)^4+a\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^4 + a*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

maple [B] time = 0.27, size = 1536, normalized size = 5.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(3*B*cos(d*x+c)^2*a^2-3*B*cos(d*x+c)*a^2-2*A*cos(d*x+c)^3*b^2+2*A*co
s(d*x+c)^2*b^2-a^2*A+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+A*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^2+2*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)
^2*b^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*cos(d*x+c)^2*a^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+A*sin(d*x+c)*cos(d*x+c)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2+3*B*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d
*x+c)*a^2+A*cos(d*x+c)^2*a^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
```

```
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+2*A*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-2*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)*a*b+A*cos(d*x+c)^3*a*b-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a*b-2*A*sin(d*x+c)*cos(d*x+c
)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-2*A
*cos(d*x+c)^2*a*b+A*cos(d*x+c)*a*b+3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*
b+2*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2)/(a+b*cos(d*x+c))^(1/2)/a^2/sin(d*x
+c)/cos(d*x+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\,\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{{\cos(c+d\,x)}^{5/2}\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(1/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.425 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=363

$$-\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{15a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(9a^2A-10abB+8Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{15a^4d}$$

```
[Out] 2/5*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)-2/15*(4*A*b-5*
B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9
*A*a^2+8*A*b^2-10*B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d-2/15*(8*A*b^2+a^2*(9*A-5*
B)-2*a*b*(A+5*B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d
```

Rubi [A] time = 0.86, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3000, 3055, 2998, 2816, 2994}

$$-\frac{2\sqrt{a+b}\left(a^2(9A-5B)-2ab(A+5B)+8Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+d}}{\sqrt{a+b}\sqrt{\cos(c+d}}\right.\right.}{15a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(15*a^4*d) - (2*Sqrt[a + b]*(8*A*b^2 + a^2*(9*A - 5*B) - 2*a*
b*(A + 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d) + (2*A*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*Cos[c + d*x]^(5/2)) - (2*(4*A*b - 5*a*
B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{\frac{1}{2}(-4Ab+5aB)+\frac{3}{2}aA\cos(c+dx)+Ab\,c}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{5a}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}}{15a^2d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{5ad\cos^{\frac{5}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}}{15a^2d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2A+8Ab^2-10abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\,\sqrt{\cos}}\right.\right.}{15a^4d}$$

Mathematica [C] time = 6.41, size = 1319, normalized size = 3.63

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Cos[c + d*x]]
),x]
```

```
[Out] -1/15*((-4*a*(7*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
```

```
c + d*x]]) - 4*a*(9*a^3*A + 8*a*A*b^2 - 10*a^2*b*B)*((Sqrt[((a + b)*Cot[(c
+ d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]
*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[A
rcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-
a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt
[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(9*a^2*A*b + 8*A*b^3 - 10*a*b
^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin
[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a
 + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b
)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(
a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]
)))/(a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]
^2*(-4*A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/(15*a^2) + (2*Sec[c + d*x]*(
9*a^2*A*Sin[c + d*x] + 8*A*b^2*Sin[c + d*x] - 10*a*b*B*Sin[c + d*x]))/(15*a
^3) + (2*A*Sec[c + d*x]^2*Tan[c + d*x])/(5*a)))/d
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b\cos(dx+c)^5+a\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b*cos(d*x + c)^5 + a*cos(d*x + c)^4), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.44, size = 2480, normalized size = 6.83

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(-3*A*a^3+9*A*cos(d*x+c)^3*a^3-8*A*cos(d*x+c)^3*b^3-6*A*cos(d*x+c)^
2*a^3+5*B*cos(d*x+c)^3*a^3-10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*cos(d*x+c)^4*b^3-5*B
*cos(d*x+c)*a^3+8*A*cos(d*x+c)^3*a*b^2-4*A*cos(d*x+c)^2*a*b^2+A*cos(d*x+c)*
a^2*b-10*B*cos(d*x+c)^4*a*b^2-10*B*cos(d*x+c)^3*a^2*b+5*B*cos(d*x+c)^2*a^2*
b-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*A*sin(d*x+c)*cos(d*x+c)^2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^2+10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+10*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^
2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-10*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b+10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-8*A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin
(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b^2+2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
```

```
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+10*B*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+
c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b+10*B*cos(d*x+c)^3*a*b^2+9*A*cos(d*x+c)^4*a^2*b-4*A*cos(d*x+c)^4*a*b^2
-10*A*cos(d*x+c)^3*a^2*b+5*B*cos(d*x+c)^4*a^2*b+5*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x
+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+5*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c
)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9*A*sin(d*x+c)*cos(d*x+c)^2*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3)/(a+b*cos
(d*x+c))^(1/2)/a^3/sin(d*x+c)/cos(d*x+c)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\,\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{7/2}\,\sqrt{a+b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b*cos(c + d*x))^(1/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.426 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=500

$$-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(-3a^2B+2aA\right.}{}$$

```
[Out] 2*a*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1
/2)-(2*A*a*b-3*B*a^2+B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)
/d/cos(d*x+c)^(1/2)+(2*A*a*b-3*B*a^2+B*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d
*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d/(a+b)^(1/2)-(2*A
*b-(3*a+b)*B)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d
*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(
d*x+c))/(a-b))^(1/2)/b^2/d/(a+b)^(1/2)-(2*A*b-3*B*a)*cot(d*x+c)*EllipticPi(
(a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^
(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^
(1/2)/b^3/d
```

Rubi [A] time = 1.29, antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2989, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(-3a^2B+2aA\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((2*a*A*b - 3*a^2*B + b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*Sqrt
[a + b]*d) - ((2*A*b - (3*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2
*Sqrt[a + b]*d) - (Sqrt[a + b]*(2*A*b - 3*a*B)*Cot[c + d*x]*EllipticPi[(a +
 b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(b^3*d) + (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*
```

```
B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d
*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}a(Ab-aB)+\frac{1}{2}b(Ab-aB)\cos(c\cdots}{\sqrt{\cos(c+dx}\cdots}}{b\left(a\cdots\right.}$$

$$= \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{a+\cdots}}{b^2\left(a^2-b^2\right)d\sqrt{\cdots}}$$

$$= \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{a+\cdots}}{b^2\left(a^2-b^2\right)d\sqrt{\cdots}}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-3aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+\cdots}{a-\cdots}\right.}{b^3d}$$

$$= \frac{\left(2aAb-3a^2B+b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\cdots}}{ab^2\sqrt{a+b}\,d}$$

Mathematica [C] time = 6.42, size = 1234, normalized size = 2.47

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(-(a*A*b*Sin[c + d*x]) + a^2*B*Sin[c + d*x]))/(b*(-a^
2 + b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*B - b^2*B)*Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - 4*a*(-2*A*b^2 + 2*a*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
```

```
[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-2*a*A*b + 3*a^2*B - b^2*B)*((I*Cos[(c
 + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sq
rt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*(
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
 b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(2*(a - b)*b
*(a + b)*d)
```

fricas [F] time = 99.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

maple [B] time = 0.35, size = 2885, normalized size = 5.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/d*(4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-2*
B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+3*B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-B*cos(d*x+c)^3*b^3+3*B*c
os(d*x+c)^2*a^3+B*cos(d*x+c)^2*b^3-3*B*cos(d*x+c)*a^3-2*A*cos(d*x+c)^2*a^2*
b+2*A*cos(d*x+c)^2*a*b^2+2*A*cos(d*x+c)*a^2*b-2*A*cos(d*x+c)*a*b^2+B*cos(d*
x+c)^3*a^2*b-3*B*cos(d*x+c)^2*a^2*b-B*cos(d*x+c)^2*a*b^2+2*B*cos(d*x+c)*a^2
*b+B*cos(d*x+c)*a*b^2-4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-6*B*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+3*B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*b^3+2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
```

```
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-2*
B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^2+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*a*b^2+3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*A*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+2*A*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^
3-4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*b^3-6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a^3+3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/
b^2/(a^2-b^2)/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.427 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=416

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a-}}\right.\right.}{abd\sqrt{a+b}}$$

```
[Out] 2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1
/2)-2*(A*b-B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos
(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a/b/d/(a+b)^(1/2)+2*(A*b-B*a)*cot(d*x+c)*EllipticF((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/(a+b)^(1/
2)-2*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A] time = 0.61, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2992, 2809, 2794, 2795, 2816, 2994}

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a}}{\sqrt{a-}}\right.\right.}{abd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) + (2
*(A*b - a*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d) - (2*Sq
rt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (2*a*(
A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])
```

Rule 2794

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[(-2*a*d*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2992

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> D
ist[B/b, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Di
st[(A*b - a*B)/b, Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} + \frac{(Ab-aB)\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx}{b}$$

$$= -\frac{2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a}}}{b^2d}$$

$$= -\frac{2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a}}}{b^2d}$$

$$= -\frac{2(Ab-aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{ab\sqrt{a+b}\,d}$$

Mathematica [C] time = 17.99, size = 1012, normalized size = 2.43

$$\frac{2\sqrt{\cos(c+dx)}\,(aB\sin(c+dx)-Ab\sin(c+dx))}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{2(Ab-aB)\left(\frac{i\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\cos(c+dx)}\,E\left(i\sinh^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)|-\frac{2a}{-a-b}\right)\mathrm{s}}{b\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,\sqrt{\frac{(a+b\cos(c+dx))\sec(c+dx)}{a+b}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(-(A*b*Sin[c + d*x]) + a*B*Sin[c + d*x]))/((a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]]) - (-4*a*(a*A - b*B)*((Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(A*b - a*B)*((I*Cos[(c + d*x)/2
]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c
+ d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d
*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/((-a + b)*(a + b)*d)
```

fricas [F] time = 2.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2
), x)
```

maple [B] time = 0.33, size = 2013, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/d/(a+b*cos(d*x+c))^(1/2)*(B*cos(d*x+c)^2*a^2-B*cos(d*x+c)*a^2+A*cos(d*x+c
)^2*b^2-A*cos(d*x+c)*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b-A*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b
+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2-A*cos(d*x+c)^2*a*b+A*cos(d*x+c)*a*b-B*co
s(d*x+c)^2*a*b+B*cos(d*x+c)*a*b-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+2*B*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-B*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*b^2-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*a*b+A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
```

```
(a+b))^(1/2)*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a*b-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-2*B*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2+2*B*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2)/sin(d*x+c)/b/(a^2-b^2)/
cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(3/2
), x)
```

3.428 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\right.\right.}{a^2d\sqrt{a+b}}$$

```
[Out] -2*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)
+2*(A*b-B*a)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)+2*(A+B)*cot(d*x+c)*EllipticF((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2993, 2998, 2816, 2994}

$$-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\right.\right.}{a^2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d) + (2*
(A + B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) - (2*(A*b - a*
B)*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{Ab-aB+(aA-bB)\cos(c}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c}}}{a^2-b^2}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}} + \frac{(A+B)\int \frac{}{\sqrt{\cos(c+dx)}}}{a+b}$$

$$= \frac{2(Ab-aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{a^2\sqrt{a+b}\,d}$$

Mathematica [C] time = 6.36, size = 1223, normalized size = 4.31

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c + d*x]))/(a*(a^
2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2*A - A*b^2)*Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (
-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]) - 4*a*(-(a*A*b) + a^2*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[
(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-(A*b^2) + a*b*B)*((I*Cos[(c + d*x)/2]*S
qrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d
*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a*(a - b)*(a + b)*d)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^3+2\,ab\cos(dx+c)^2+a^2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^3 + 2*a*b*cos(d*x + c)^2 + a^2*cos(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.41, size = 1633, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/d/(a+b*cos(d*x+c))^(1/2)*(A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+A*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-A*sin(d*x+c)*
cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
a^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)*a*b-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+B*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*a^2+B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)*a*b+A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*a*b+A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
```

```
(d*x+c))/(a+b))^(1/2)*b^2-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2-A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+B*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+A*cos(d*x+c)^
2*a*b-A*cos(d*x+c)^2*b^2-B*cos(d*x+c)^2*a^2+B*cos(d*x+c)^2*a*b-A*cos(d*x+c)
*a*b+A*cos(d*x+c)*b^2+B*cos(d*x+c)*a^2-B*cos(d*x+c)*a*b)/(a^2-b^2)/a/sin(d*
x+c)/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x
))), x)
```

3.429 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=305

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2(a(A-B)+2Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\Big(\mathrm{s}}{a^2d\sqrt{a+b}}$$

```
[Out] 2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1
/2)+2*(A*a^2-2*A*b^2+B*a*b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)-2*(2*A*b+a*(A-B))*cot(
d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a^2/d/(a+b)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3000, 2998, 2816, 2994}

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2A+abB-2Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*(a^2*A - 2*A*b^2 + a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3*Sqrt[a
 + b]*d) - (2*(2*A*b + a*(A - B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*S
qrt[a + b]*d) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(a^2A-2Ab^2+abB\right)\cdots}{\cos^{\frac{3}{2}}(c+dx)\cdots}}{a\left(a^2\cdots\right.}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} - \frac{((a-b)(2Ab+a(A\cdots}{\cdots}$$

$$= \frac{2\left(a^2A-2Ab^2+abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\cdots}{a^3\sqrt{a+b}\,d}$$

Mathematica [C] time = 6.51, size = 1281, normalized size = 4.20

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] ((-4*a*(2*a^2*A*b - 2*A*b^3 - a^3*B + a*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- 4*a*(a^3*A - 2*a*A*b^2 + a^2*b*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]])) + 2*(a^2*A*b - 2*A*b^3 + a*b^2*B)*((I*Cos[(c + d*
x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Co
s[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec
[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
```

```
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a^2*(-a + b)*(a
+ b)*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(A*b^3*Sin[c +
 d*x]) + a*b^2*B*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2
*A*Tan[c + d*x])/a^2))/d
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^4+2\,ab\cos(dx+c)^3+a^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.35, size = 2280, normalized size = 7.48

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/d/(a+b*cos(d*x+c))^(1/2)*(A*a^3-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
```

```
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3-2*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3
-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^3-A*a*b^2+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-B*sin(d*x+c)*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+B
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+A*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b+2*A*cos(d*x+c)^2*b^3-2*A*cos(d*x+c)*b^3-A*cos(d*x+c)^2*a^2*b-A*cos(d*x+c)
^2*a*b^2+A*cos(d*x+c)*a^2*b+2*A*cos(d*x+c)*a*b^2+B*cos(d*x+c)^2*a^2*b-B*cos
(d*x+c)^2*a*b^2-B*cos(d*x+c)*a^2*b+B*cos(d*x+c)*a*b^2-A*cos(d*x+c)*a^3+2*A*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^2+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+A*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-A*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-B*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^3+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
```

```
1/2))*sin(d*x+c)*a^3-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*b^3)/a^2/(a^2-b^2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^{3/2}\,(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.430 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=393

$$\frac{2(a+2b)(a(A-3B)+4Ab)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3d\sqrt{a+b}}+\frac{2\left(a^2A\right.}{}$$

```
[Out] 2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1
/2)+2/3*(A*a^2-4*A*b^2+3*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-
b^2)/d/cos(d*x+c)^(3/2)-2/3*(5*A*a^2*b-8*A*b^3-3*B*a^3+6*B*a*b^2)*cot(d*x+c
)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-
b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^
4/d/(a+b)^(1/2)+2/3*(a+2*b)*(4*A*b+a*(A-3*B))*cot(d*x+c)*EllipticF((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)
```

Rubi [A] time = 0.98, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3000, 3055, 2998, 2816, 2994}

$$\frac{2\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(5a^2A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (-2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(3*a^4*Sqrt[a + b]*d) + (2*(a + 2*b)*(4*A*b + a*(A - 3*B))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d) + (2*b*(A*b - a*B)*Sin[c +
 d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(
a^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a
^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(a^2A-4Ab^2+3abB\right)\ldots}{\ldots}}{\ldots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3a\ldots\right.}{3a^2\left(a\ldots\right.}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3a\ldots\right.}{3a^2\left(a\ldots\right.}$$

$$= -\frac{2\left(5a^2Ab-8Ab^3-3a^3B+6ab^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+\ldots}}{\sqrt{a+b}\sqrt{\cos(c\ldots}}\right.\right.}{3a^4\sqrt{a+b}\,d}$$

Mathematica [C] time = 6.71, size = 1357, normalized size = 3.45

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/
2)),x]
```

```
[Out] ((-4*a*(a^4*A + 7*a^2*A*b^2 - 8*A*b^4 - 6*a^3*b*B + 6*a*b^3*B)*Sqrt[((a + b
)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]],
```

```
(-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Cos[c + d*x]]) - 4*a*(5*a^3*A*b - 8*a*A*b^3 - 3*a^4*B + 6*a^2*b^2*B)*((Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqr
t[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(5*a^2*A*b^
2 - 8*A*b^4 - 3*a^3*b*B + 6*a*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c
+ d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-
a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b
*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[
c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a^3*(a - b)*(a + b)*d) + (Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]*(-5*A*b*Sin[c + d*x] + 3*
a*B*Sin[c + d*x]))/(3*a^3) - (2*(-(A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*
x]))/(a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*A*Sec[c + d*x]*Tan[c + d*x
])/(3*a^2)))/d
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^5+2\,ab\cos(dx+c)^4+a^2\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^2*cos(d*x + c)^5 + 2*a*b*cos(d*x + c)^4 + a^2*cos(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

maple [B] time = 0.35, size = 3334, normalized size = 8.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(-5*A*cos(d*x+c)^3*a^2*b^2+3*B*cos(d*x+c)^3*a^3*b-6*B*cos(d*x+c)^3*a
*b^3-6*B*cos(d*x+c)^2*a^2*b^2+6*B*cos(d*x+c)^2*a*b^3+3*B*cos(d*x+c)*a^2*b^2
-5*A*cos(d*x+c)^2*a^3*b+8*A*cos(d*x+c)^2*a*b^3-4*A*cos(d*x+c)*a*b^3+A*a^2*b
^2+8*A*cos(d*x+c)^3*b^4-8*A*cos(d*x+c)^2*b^4+3*B*cos(d*x+c)^2*a^4-3*B*cos(d
*x+c)*a^4+A*cos(d*x+c)^2*a^4-3*B*cos(d*x+c)^2*a^3*b+A*cos(d*x+c)^3*a^3*b-4*
A*cos(d*x+c)^3*a*b^3+4*A*cos(d*x+c)^2*a^2*b^2+4*A*cos(d*x+c)*a^3*b+3*B*cos(
d*x+c)^3*a^2*b^2+5*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-8*A*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4
-A*a^4+5*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*a^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^3+2*A*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8
*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^3-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*B*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^3-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
```

```
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^3*b-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+5*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*
b+5*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-5*A*sin(d*x+c)*cos(d*x+c)
^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*a^2*b^2+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^3-3*B*sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+6*B*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b^2+6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-6*B*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^2*b^2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+A*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)^2*a^4+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^4+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4-5*A*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
```

```
3*b)/(a+b*cos(d*x+c))^(1/2)/a^3/(a^2-b^2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos(c+d\,x)}{\cos(c+d\,x)^{5/2}\,(a+b\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(3/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.431 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=674

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(-5a^3B+2a^2Ab+9ab^2B-6Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{\left(-15a^3B\right.}{}$$

```
[Out] 2/3*a*(A*b-B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^
(3/2)+2/3*a*(2*A*a^2*b-6*A*b^3-5*B*a^3+9*B*a*b^2)*sin(d*x+c)*cos(d*x+c)^(1/
2)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-1/3*(6*A*a^3*b-14*A*a*b^3-15*B*
a^4+26*B*a^2*b^2-3*B*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)^2
/d/cos(d*x+c)^(1/2)+1/3*(6*A*a^3*b-14*A*a*b^3-15*B*a^4+26*B*a^2*b^2-3*B*b^4
)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b
))^(1/2)/a/(a-b)/b^3/(a+b)^(3/2)/d-1/3*(6*A*a^2*b+2*A*a*b^2-12*A*b^3-15*B*a
^3-5*B*a^2*b+21*B*a*b^2+3*B*b^3)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^3/(a+b)^(3/2)/d-(2*A*b-5*B*
a)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d
```

Rubi [A] time = 2.19, antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2989, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{2a(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(2a^2Ab-5a^3B+9ab^2B-6Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{\left(6a^3Ab+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d) - ((6*a^2*A*b + 2*a
*A*b^2 - 12*A*b^3 - 15*a^3*B - 5*a^2*b*B + 21*a*b^2*B + 3*b^3*B)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
```

```
Sec[c + d*x]))/(a - b)])/(3*(a - b)*b^3*(a + b)^(3/2)*d) - (Sqrt[a + b]*(2*
A*b - 5*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^4*d) + (2*a*
(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
- ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
```

```
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
```

```
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(-\frac{3}{2}a(Ab-aB)+\frac{3}{2}b(A}{}}{}$$

$$= \frac{2a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B+\right.}{3b^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B+\right.}{3b^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2a(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2a\left(2a^2Ab-6Ab^3-5a^3B+\right.}{3b^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-5aB)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+}{a-}\right.}{b^4d}$$

$$= \frac{\left(6a^3Ab-14aAb^3-15a^4B+26a^2b^2B-3b^4B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{}{\sqrt{}}\right.\right.}{3a(a-b)b^3(a+b)^{3/}}$$

Mathematica [C] time = 6.70, size = 1396, normalized size = 2.07

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(-(a^2*A*b*Sin[c + d*x])
+ a^3*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) - (2*(-3
*a^3*A*b*Sin[c + d*x] + 7*a*A*b^3*Sin[c + d*x] + 6*a^4*B*Sin[c + d*x] - 10*
a^2*b^2*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d +
((-4*a*(-2*a^3*A*b + 2*a*A*b^3 + 5*a^4*B - 8*a^2*b^2*B + 3*b^4*B)*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]
], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]]) - 4*a*(2*a^2*A*b^2 + 6*A*b^4 + 4*a^3*b*B - 12*a*b^3*B)*(
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-6*a^3
*A*b + 14*a*A*b^3 + 15*a^4*B - 26*a^2*b^2*B + 3*b^4*B)*((I*Cos[(c + d*x)/2]
*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c +
 d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(6*(a - b)^2*b^2*(a +
b)^2*d)
```

fricas [F] time = 3.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{3}+A\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a)*sqr
t(cos(d*x + c))/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(
d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

maple [B] time = 0.70, size = 8611, normalized size = 12.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{5/2}\,\left(A+B\,\cos(c+d\,x)\right)}{{\left(a+b\,\cos(c+d\,x)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.432 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=545

$$\frac{2\left(3a^3B-7ab^2B+4Ab^3\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3ab^2d(a-b)(a+b)^{3/2}}+\frac{2a(Ab-}{3bd\left(a^2\right.}$$

```
[Out] 2/3*a*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^
(3/2)-2/3*a*(4*A*b^3+3*B*a^3-7*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/cos(d*
x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2/3*(4*A*b^3+3*B*a^3-7*B*a*b^2)*cot(d*x+c
)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-
b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/
(a-b)/b^2/(a+b)^(3/2)/d+2/3*(A*a*b^2-3*A*b^3-3*B*a^3-B*a^2*b+6*B*a*b^2)*cot
(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/a/(a-b)/b^2/(a+b)^(3/2)/d-2*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d
```

Rubi [A] time = 1.40, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2989, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-a^2bB-3a^3B+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*
(a - b)*b^2*(a + b)^(3/2)*d) + (2*(a*A*b^2 - 3*A*b^3 - 3*a^3*B - a^2*b*B +
6*a*b^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2
)*d) - (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3
```

```
*d) + (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)) - (2*a*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Sin[c +
d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{1}{2}a(Ab-aB)+\frac{3}{2}b(Ab-aB)\cos(c+a}{\sqrt{\cos(c+dx)}\,(a+b\cos}}{3b\left(a^2-b\right.}$$

$$= \frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{1}{2}ab(Ab-aB)+\left(\frac{3}{2}a\left(a^2-b^2\right)B+\frac{3}{2}b^2}{\sqrt{\cos(c+dx)}\,(a+b\cos(}}{3b^2\left(a^2-b^2\right.}$$

$$= -\frac{2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a+}}}{b^3d}$$

$$= -\frac{2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec}{a+}}}{b^3d}$$

$$= \frac{2\left(4Ab^3+3a^3B-7ab^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a(a-b)b^2(a+b)^{3/2}d}$$

Mathematica [C] time = 6.53, size = 1342, normalized size = 2.46

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(a*A*b*Sin[c + d*x]) + a
^2*B*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(4*A*b^3
*Sin[c + d*x] + 3*a^3*B*Sin[c + d*x] - 7*a*b^2*B*Sin[c + d*x]))/(3*b*(-a^2
+ b^2)^2*(a + b*Cos[c + d*x]))))/d - ((-4*a*(-(a^2*A*b) + A*b^3 + a^3*B - a
*b^2*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(4*a*A*b^2 - a^2*b*B - 3*b^3*B)*(
(Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*C
sc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x
```

```
]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]
*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(4*A*b^
3 + 3*a^3*B - 7*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Elli
pticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[
c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x
])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b
*Sqrt[Cos[c + d*x]])))/(3*(a - b)^2*b*(a + b)^2*d)
```

fricas [F] time = 1.37, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\cos\left(dx+c\right)}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*
x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

maple [B] time = 0.51, size = 5749, normalized size = 10.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)**(3/2)/(a + b*cos(c + d*x))**(5/
2), x)
```

3.433 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=391

$$\frac{2\left(3a^2A-4abB+Ab^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2A-4abB+Ab^2\right)}{}$$

```
[Out] -2/3*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/
2)+2/3*(3*A*a^2+A*b^2-4*B*a*b)*sin(d*x+c)/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a
+b*cos(d*x+c))^(1/2)-2/3*(3*A*a^2+A*b^2-4*B*a*b)*cot(d*x+c)*EllipticE((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-
sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/
2)/d+2/3*(3*A*a-A*b+B*a-3*B*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d
```

Rubi [A] time = 0.87, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2999, 2993, 2998, 2816, 2994}

$$\frac{2\left(3a^2A-4abB+Ab^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2A-4abB+Ab^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(3*a^2*A + A*b^2 - 4*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*(
a - b)*(a + b)^(3/2)*d) + (2*(3*a*A - A*b + a*B - 3*b*B)*Cot[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d) - (2*(A*b - a*B)*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(3*
a^2*A + A*b^2 - 4*a*b*B)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
```

```
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(Ab-aB)-\frac{3}{2}(aA-bB)\cos(c+d}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/}}}{3\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3a^2A+Ab^2-4ab\right.}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3a^2A+Ab^2-4ab\right.}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}}$$

$$= -\frac{2\left(3a^2A+Ab^2-4abB\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+}{a-}\right.}{3a^2(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.45, size = 1335, normalized size = 3.41

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*(-(A*b*Sin[c + d*x]) + a*B
*Sin[c + d*x]))/(3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(3*a^2*A*b*Sin[
c + d*x] + A*b^3*Sin[c + d*x] - 4*a*b^2*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2
*(a + b*Cos[c + d*x]))))/d + ((-4*a*(-(a^2*A*b) + A*b^3 + a^3*B - a*b^2*B)*
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^3*A + a*A*b^2 - 4*a^2*b*B)*((Sqrt[((
a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[
```

```
2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(
((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^2*A*b + A*
b^3 - 4*a*b^2*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*
ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x]
)/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c
 + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b
)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*El
lipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Co
s[c + d*x]])))/(3*a*(a - b)^2*(a + b)^2*d)
```

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2
), x)
```

maple [B] time = 0.39, size = 4237, normalized size = 10.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/(a+b*cos(d*x+c))^(3/2)*(-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+3*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4+3*B*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a*b^
3-3*A*cos(d*x+c)^3*a^2*b^2+4*B*cos(d*x+c)^3*a*b^3+8*B*cos(d*x+c)^2*a^2*b^2-
4*B*cos(d*x+c)^2*a*b^3+4*B*cos(d*x+c)*a^3*b-3*B*cos(d*x+c)*a^2*b^2-6*A*cos(
d*x+c)^2*a^3*b-2*A*cos(d*x+c)^2*a*b^3-A*cos(d*x+c)*a^2*b^2-A*cos(d*x+c)^3*b
^4+A*cos(d*x+c)^2*b^4+B*cos(d*x+c)^3*a^4-B*cos(d*x+c)*a^4+3*A*cos(d*x+c)^2*
a^4-4*B*cos(d*x+c)^2*a^3*b+2*A*cos(d*x+c)^3*a^3*b+2*A*cos(d*x+c)^3*a*b^3+4*
A*cos(d*x+c)^2*a^2*b^2+4*A*cos(d*x+c)*a^3*b-5*B*cos(d*x+c)^3*a^2*b^2+6*A*si
n(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*a^3*b+3*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^3*b+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-4*B*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-4*B*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+
4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^3*b+3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b^2+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
```

```
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-3*A*cos(d*x+c)*a^4+4*A*sin(d*x+c)*co
s(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^
2*b^2+2*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*a*b^3-5*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-A*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-4*B*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^3*b-8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+5*B*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3*b+7*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b^2+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+3*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b+3*A
*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b^2+A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-3*A*sin(d*x+c)*cos(d*x+c)^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-4*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)^2*a^2*b^2-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^3-4*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-4*B*sin(d*x+c
)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^3+B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
```

```
c),(-(a-b)/(a+b))^(1/2))*a^3*b+4*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4-4*
A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^3*b+A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*b^4-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4-7*A*sin(d*x+c)*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+3*A*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*a^4)/sin(d*x+c)/a/(a-b)^2/(a+b)^2/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(A+B\,\cos(c+d\,x)\right)}{\left(a+b\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(a + b*cos(c + d*x))^(5/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(5/2
), x)
```

3.434 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=429

$$\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-3a^2(A+B)+ab(3A+B)+2Ab^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec}{}}}{3a^2d\sqrt{a+b}\left(a^2-b^2\right)}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^
(3/2)-2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/co
s(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^
2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-
b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d-2/3*(2*A*b^2-3*a^2*(A+B)+a*b*(3*A+B))*cot
(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/a^2/(a^2-b^2)/d/(a+b)^(1/2)
```

Rubi [A] time = 0.99, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3000, 2993, 2998, 2816, 2994}

$$-\frac{2\left(6a^2Ab-3a^3B-ab^2B-2Ab^3\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-3a^2(A+B)+ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d) - (2*(2*A*b^2 - 3*a^2*(A + B) + a*b*(3
*A + B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b
]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*
d) + (2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(
a + b*Cos[c + d*x])^(3/2)) - (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*S
in[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
```

```
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(3a^2A-2Ab^2-abB\right)-\frac{3}{2}a(Ab}{\sqrt{\cos(c+dx)}\,(a+b\cos(}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(6a^2Ab-2Ab^3-3a^3B\right.}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+d}}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\left(6a^2Ab-2Ab^3-3a^3B\right.}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+d}}$$

$$= \frac{2\left(6a^2Ab-2Ab^3-3a^3B-ab^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\,\sqrt{\cos(c+d}}\right.\right.}{3a^3(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.58, size = 1384, normalized size = 3.23

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*(-(A*b^2*Sin[c + d*x]) +
a*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(-6*a^2*
A*b^2*Sin[c + d*x] + 2*A*b^4*Sin[c + d*x] + 3*a^3*b*B*Sin[c + d*x] + a*b^3*
B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(3
*a^4*A - 5*a^2*A*b^2 + 2*A*b^4 - a^3*b*B + a*b^3*B)*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a
+ b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
```

```
x]]) - 4*a*(-6*a^3*A*b + 2*a*A*b^3 + 3*a^4*B + a^2*b^2*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-6*a^2*A*b^2 + 2*A*b^4
 + 3*a^3*b*B + a*b^3*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ellip
ticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c
 + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x]
)*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*
Sqrt[Cos[c + d*x]])))/(3*a^2*(a - b)^2*(a + b)^2*d)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^4+3\,ab^2\cos(dx+c)^3+3\,a^2b\cos(dx+c)^2+a^3\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^4 + 3*a*b^2*cos(d*x + c)^3 + 3*a^2*b*cos(d*x + c)^2 + a^3*
cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

maple [B] time = 0.90, size = 5203, normalized size = 12.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.435 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=456

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}+\frac{2b\left(-5a^3B+8a^2Ab+ab^2B-4Ab^3\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-3a^3(A\right.}{}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^
(1/2)+2/3*b*(8*A*a^2*b-4*A*b^3-5*B*a^3+B*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/
d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2/3*(3*A*a^4-15*A*a^2*b^2+8*A*b^4
+6*B*a^3*b-2*B*a*b^3)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/
2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d+2/3*(8*A*b^3-3*a^3*(A-
B)+2*a*b^2*(3*A-B)-3*a^2*b*(3*A+B))*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a^2-b^2)/d/(a+b)^(1/2)
```

Rubi [A] time = 1.16, antiderivative size = 456, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3000, 3055, 2998, 2816, 2994}

$$\frac{2b\left(8a^2Ab-5a^3B+ab^2B-4Ab^3\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-3a^2b(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d) + (2*(8*A*b^3 - 3*a^3
*(A - B) + 2*a*b^2*(3*A - B) - 3*a^2*b*(3*A + B))*Cot[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(3*a^3*Sqrt[a + b]*(a^2 - b^2)*d) + (2*b*(A*b - a*B)*Sin[c + d*x]
)/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) + (2*b*
(8*a^2*A*b - 4*A*b^3 - 5*a^3*B + a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^
2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(3a^2A-4Ab^2+ab\right.}{}}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(8a^2Ab-4A\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(8a^2Ab-4A\right.}{3a^2\left(a^2-b^2\right)^2d\sqrt{}}$$

$$= \frac{2\left(3a^4A-15a^2Ab^2+8Ab^4+6a^3bB-2ab^3B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{3a^4(a-b)(a+b)^{3/2}}$$

Mathematica [C] time = 6.72, size = 1431, normalized size = 3.14

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] -1/3*((-4*a*(9*a^4*A*b - 17*a^2*A*b^3 + 8*A*b^5 - 3*a^5*B + 5*a^3*b^2*B - 2
*a*b^4*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c
```

```
 + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*
x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^5*A - 15*a^3*A*b^2 + 8*a*A
*b^4 + 6*a^4*b*B - 2*a^2*b^3*B)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b
)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a +
 b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x
)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*
EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Cos[c + d*x]])) + 2*(3*a^4*A*b - 15*a^2*A*b^3 + 8*A*b^5 + 6*a^3*b^2*B
 - 2*a*b^4*B)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*Arc
Sinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(
b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c +
d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c
 + d*x]])))/(a^3*(a - b)^2*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]*((2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x]))/(3*a^2*(a^2
 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-9*a^2*A*b^3*Sin[c + d*x] + 5*A*b^5*S
in[c + d*x] + 6*a^3*b^2*B*Sin[c + d*x] - 2*a*b^4*B*Sin[c + d*x]))/(3*a^3*(a
^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/a^3))/d
```

fricas [F] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^5+3\,ab^2\cos(dx+c)^4+3\,a^2b\cos(dx+c)^3+a^3\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^5 + 3*a*b^2*cos(d*x + c)^4 + 3*a^2*b*cos(d*x + c)^3 + a^3*
cos(d*x + c)^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 1.42, size = 6498, normalized size = 14.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\cos(c+d\,x)^{3/2}\,(a+b\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(5/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.436 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=567

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}+\frac{2b\left(-7a^3B+10a^2Ab+3ab^2B-6Ab^3\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^4A+8\right.}{}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^
(3/2)+2/3*b*(10*A*a^2*b-6*A*b^3-7*B*a^3+3*B*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)
^2/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2)+2/3*(A*a^4-13*A*a^2*b^2+8*A*b^
4+8*B*a^3*b-4*B*a*b^3)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d/
cos(d*x+c)^(3/2)-2/3*(8*A*a^4*b-28*A*a^2*b^3+16*A*b^5-3*B*a^5+15*B*a^3*b^2-
8*B*a*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/a^5/(a-b)/(a+b)^(3/2)/d-2/3*(16*A*b^4-a^4*(A-3*B)+4*a*b^3
*(3*A-2*B)-9*a^3*b*(A-B)-2*a^2*b^2*(8*A+3*B))*cot(d*x+c)*EllipticF((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec
(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a^2-b^2)/d/(a+b)^
(1/2)
```

Rubi [A] time = 1.88, antiderivative size = 567, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3000, 3055, 2998, 2816, 2994}

$$\frac{2\left(-13a^2Ab^2+a^4A+8a^3bB-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)}+\frac{2b\left(10a^2Ab-7a^3B+3ab^2B-6A\right.}{3a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (-2*(8*a^4*A*b - 28*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4
*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5*(a - b)*(a + b)^(3/2)*d) - (2
*(16*A*b^4 - a^4*(A - 3*B) + 4*a*b^3*(3*A - 2*B) - 9*a^3*b*(A - B) - 2*a^2*
b^2*(8*A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2
 - b^2)*d) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]
```

```
^(3/2)*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B +
 3*a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a
+ b*Cos[c + d*x]]) + (2*(a^4*A - 13*a^2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*b
^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Cos[c +
 d*x]^(3/2))
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
```

```
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3}{2}\left(a^2A-2Ab^2+abB\right.}{}}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab\right.}{3a^2\left(a^2-b^2\right)^2 d\,\mathrm{co}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab\right.}{3a^2\left(a^2-b^2\right)^2 d\,\mathrm{co}}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab\right.}{3a^2\left(a^2-b^2\right)^2 d\,\mathrm{co}}$$

$$= -\frac{2\left(8a^4Ab-28a^2Ab^3+16Ab^5-3a^5B+15a^3b^2B-8ab^4B\right)\cot(c+d}{3a^5(a-b)(a}$$

Mathematica [C] time = 6.95, size = 1499, normalized size = 2.64

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/
2)),x]
```

```
[Out] ((-4*a*(a^6*A + 15*a^4*A*b^2 - 32*a^2*A*b^4 + 16*A*b^6 - 9*a^5*b*B + 17*a^3
*b^3*B - 8*a*b^5*B)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*a^5*A*b - 28*a^3*A
*b^3 + 16*a*A*b^5 - 3*a^6*B + 15*a^4*b^2*B - 8*a^2*b^4*B)*((Sqrt[((a + b)*C
ot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^
2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Co
s[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(8*a^4*A*b^2 - 28*a^2*A
*b^4 + 16*A*b^6 - 3*a^5*b*B + 15*a^3*b^3*B - 8*a*b^5*B)*((I*Cos[(c + d*x)/2
]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c
+ d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[(
(a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c +
 d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt
[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d
*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b
*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a^4*(a - b)^2*(a +
 b)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]*(-
8*A*b*Sin[c + d*x] + 3*a*B*Sin[c + d*x]))/(3*a^4) - (2*(-(A*b^4*Sin[c + d*x
]) + a*b^3*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2
*(-12*a^2*A*b^4*Sin[c + d*x] + 8*A*b^6*Sin[c + d*x] + 9*a^3*b^3*B*Sin[c + d
*x] - 5*a*b^5*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) +
 (2*A*Sec[c + d*x]*Tan[c + d*x])/(3*a^3)))/d
```

fricas [F] time = 1.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^6+3\,ab^2\cos(dx+c)^5+3\,a^2b\cos(dx+c)^4+a^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(
b^3*cos(d*x + c)^6 + 3*a*b^2*cos(d*x + c)^5 + 3*a^2*b*cos(d*x + c)^4 + a^3*
cos(d*x + c)^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.50, size = 8093, normalized size = 14.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(5/2)), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.437 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=419

$$\frac{aB\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{B\sin(c+dx)\sqrt{\cos(c}{d\sqrt{a+b\cos(c+a}}$$

```
[Out] a*B*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+B*sin(d*x+c)*cos
(d*x+c)^(1/2)/d/(a+b*cos(d*x+c))^(1/2)-(a-b)*B*cot(d*x+c)*EllipticE((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1
/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+B*c
ot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/b/d+a*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b
)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A] time = 0.79, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {21, 2820, 2809, 3003, 2993, 12, 2801, 2816, 2994}

$$\frac{aB\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{B\sin(c+dx)\sqrt{\cos(c}{d\sqrt{a+b\cos(c+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqr
t[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (a*Sqrt[a + b]*B*Cot[
c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (a*B*Sin[c + d*x])/(b*
d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2820

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> -Dist[(a*d)/(2*b), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a
 + b*Sin[e + f*x]], x], x] + Dist[d/(2*b), Int[(Sqrt[d*Sin[e + f*x]]*(a + 2
*b*Sin[e + f*x]))/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{B\int \frac{\sqrt{\cos(c+dx)}\,(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx}{2b} - \frac{(aB)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{2b}$$

$$= \frac{a\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-s}{}}}{b^2d}$$

$$= \frac{a\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-s}{}}}{b^2d}$$

$$= \frac{a\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-s}{}}}{b^2d}$$

$$= \frac{a\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-s}{}}}{b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{abd}$$

Mathematica [C] time = 1.44, size = 480, normalized size = 1.15

$$\frac{B\sqrt{\cos(c+dx)}\left(2a\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\tan\left(\frac{1}{2}(c+dx)\right)-b\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\tan\left(\frac{1}{2}(c+dx)\right)+b\sqrt{\frac{a-b}{a+b}}\sin\left(\frac{3}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^(3/2),x]
```

```
[Out] (B*Sqrt[Cos[c + d*x]]*((2*I)*(a - b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2
]], -((a + b)/(a - b))] - (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]],
 -((a + b)/(a - b))] + (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan
```

```
[(c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(
a - b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*
Sqrt[(a - b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2
]))/(2*b*Sqrt[(a - b)/(a + b)]*d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[a + b*Cos[c + d*x]])
```

fricas [F] time = 53.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bb\cos\left(dx+c\right)+Ba\right)\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
(3/2), x)
```

maple [A] time = 0.34, size = 623, normalized size = 1.49

$$-\frac{B\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)a+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -B/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
```

```
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*a*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)+cos(d*x+c)^3
*b+a*cos(d*x+c)^2-cos(d*x+c)^2*b-a*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2)/sin(d
*x+c)/cos(d*x+c)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c) + Ba)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^{3/2}\,\left(B\,a + B\,b\,\cos(c+d\,x)\right)}{\left(a + b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2
),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a + b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
[Out] B*Integral(cos(c + d*x)**(3/2)/sqrt(a + b*cos(c + d*x)), x)
```

3.438 $\int \frac{\sqrt{\cos(c+dx)}\,(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{2B\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}$$

```
[Out] -2*B*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {21, 2809}

$$-\frac{2B\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (-2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(aB+bB\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= -\frac{2\sqrt{a+b}\,B\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{bd}$$

Mathematica [A] time = 0.14, size = 131, normalized size = 1.12

$$-\frac{2B\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-2\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)\right)}{d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(a*B + b*B*Cos[c + d*x]))/(a + b*Cos[c + d*x]
)^(3/2),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*EllipticPi
[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 1.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^
(3/2), x)
```

maple [A] time = 0.33, size = 160, normalized size = 1.37

$$-\frac{2B\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\sqrt{-\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}}{d\sqrt{a+b\cos\left(dx+c\right)}\,\left(-1+\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*B/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-2*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)^2
/(a+b*cos(d*x+c))^(1/2)/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bb\cos\left(dx+c\right)+Ba\right)\sqrt{\cos\left(dx+c\right)}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^
(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2
),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^(3/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{\sqrt{\cos (c+dx)}}{\sqrt{a+b\cos (c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2),x)
[Out] B*Integral(sqrt(cos(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

3.439 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=110

$$\frac{2B\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] 2*B*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+se
c(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {21, 2816}

$$\frac{2B\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)
),x]
```

```
[Out] (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b}\,B\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{ad}$$

Mathematica [A] time = 0.89, size = 171, normalized size = 1.55

$$-\frac{4B(a+b)\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{-\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{a-b}}\sqrt{\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{a}}F\left(\sin^{-1}\left(\sqrt{-\frac{a+b\cos(c+dx)}{a(\cos(c+dx)-1)}}\right)|\frac{2}{a-}\right.}{ad\sqrt{a+b\cos(c+dx)}\left(-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])
^(3/2)),x]
```

```
[Out] (-4*(a + b)*B*Cos[c + d*x]^(3/2)*Sqrt[-(((a + b)*Cot[(c + d*x)/2]^2)/(a - b
))]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Elliptic
F[ArcSin[Sqrt[-((a + b*Cos[c + d*x])/(a*(-1 + Cos[c + d*x])))]], (2*a)/(a -
 b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]*(-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/
2]^2)/a))^(3/2))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,B\sqrt{\cos(dx+c)}}{b\cos(dx+c)^2+a\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))/(b*cos(d*x + c)^2 +
a*cos(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x
 + c))), x)
```

maple [A] time = 0.23, size = 124, normalized size = 1.13

$$-\frac{2B\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\left(\sin^4(dx+c)\right)}{d\sqrt{a+b\cos(dx+c)}\,\cos(dx+c)^{\frac{3}{2}}\left(-1+\cos(dx+c)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*B/d*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(a+b*cos(d*x+c))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*sin(d*x+c)^4/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x
 + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,a+B\,b\,\cos(c+d\,x)}{\sqrt{\cos(c+d\,x)}\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{1}{\sqrt{a+b\cos(c+dx)}\,\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
[Out] B*Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.440 $\int \frac{aB+bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=226

$$\frac{2B(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2B\sqrt{a+b}\,\cot(c+$$

```
[Out] 2*(a-b)*B*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-2*B*cot(d*x+c)*EllipticF((a+b*cos(d*x+c)
)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] time = 0.27, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {21, 2801, 2816, 2994}

$$\frac{2B(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2B\sqrt{a+b}\,\cot(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)
),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sq
rt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
```

```
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\left(B\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}\,dx\right) + B\int \frac{1+\cos(}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= \frac{2(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)| -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\mathrm{s}}{}}}{a^2d}$$

Mathematica [A] time = 2.19, size = 212, normalized size = 0.94

$$\frac{2B\left(\tan\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx)) + a\sqrt{\cos(c+dx)}\,\sqrt{\cos(c+dx)+1}\,\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\,F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+\right.\right.\right.}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])
^(3/2)),x]
```

```
[Out] (2*B*(-((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]) + a*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a +
b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)] + (a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(a*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 2.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\,B\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))/(b*cos(d*x + c)^3 +
a*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos\left(dx+c\right)+Ba}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)
^(3/2)), x)
```

maple [B] time = 0.26, size = 613, normalized size = 2.71

$$-\frac{2B\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\;\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)a-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*B/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+cos(d*x+c)^2*b+a*cos(d
*x+c)-b*cos(d*x+c)-a)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)
^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,a+B\,b\,\cos(c+d\,x)}{\cos(c+d\,x)^{3/2}\,(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{1}{\sqrt{a+b\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
[Out] B*Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3/2)), x)
```

3.441 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=72

$$-\frac{\cot(c+dx)\sqrt{-\sec(c+dx)-1}\,\sqrt{1-\sec(c+dx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\,\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{d}$$

```
[Out] -cot(d*x+c)*EllipticE(1/5*(2+3*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/2),5
^(1/2))*(-1-sec(d*x+c))^(1/2)*(1-sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {2994}

$$-\frac{\cot(c+dx)\sqrt{-\sec(c+dx)-1}\,\sqrt{1-\sec(c+dx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\,\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

```
[Out] -((Cot[c + d*x]*EllipticE[ArcSin[Sqrt[2 + 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos
[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/d)
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx = -\frac{\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\,\sqrt{\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(c+dx)}\,\sqrt{1-\sec(c}{d}$$

Mathematica [F] time = 35.24, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{2+3\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[2 + 3*Cos[c + d*x]]),
x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[2 + 3*Cos[c + d*x]]),
 x]
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{3\,\cos\left(dx+c\right)+2}\,(\cos\left(dx+c\right)+1)\sqrt{\cos\left(dx+c\right)}}{3\,\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(3*cos(d*x + c) + 2)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(3*
cos(d*x + c)^3 + 2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)+1}{\sqrt{3\,\cos\left(dx+c\right)+2}\,\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2)),
 x)
```

maple [B] time = 0.45, size = 658, normalized size = 9.14

$$-\frac{2\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)+4\sqrt{2}\,\left(\frac{\cos(d}{1+\cos(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/10/d/(2+3*cos(d*x+c))^(1/2)*(2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))+4*2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))+2*2^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*10^(1/2)*((2+3*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))-5*sin(d
*x+c)*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),1/5
*5^(1/2))+2*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),1/5*5^(1/2))-5*sin(d*x+c)*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))+2*sin(d*x+c)*cos(d*x+c)*2^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))+30*cos(d*x+c)^3-10
*cos(d*x+c)^2-20*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)+1}{\sqrt{3\cos(dx+c)+2}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2+3*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\,\sqrt{3\,\cos(c+d\,x)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3*cos(c + d*x) + 2)^(1/2)),x)
```

```
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3*cos(c + d*x) + 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)+1}{\sqrt{3\cos(c+dx)+2}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(3*cos(c + d*x) + 2)*cos(c + d*x)**(3/2)),
 x)
```

3.442 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{5}\cot(c+dx)\sqrt{\sec(c+dx)-1}\,\sqrt{\sec(c+dx)+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{d}$$

```
[Out] -cot(d*x+c)*EllipticE((-2+3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),1/5*5^(1/2))
*5^(1/2)*(-1+sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {2994}

$$-\frac{\sqrt{5}\cot(c+dx)\sqrt{\sec(c+dx)-1}\,\sqrt{\sec(c+dx)+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[5]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[-2 + 3*Cos[c + d*x]]/Sqrt[Cos
[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d)
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx = -\frac{\sqrt{5}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\,\sqrt{1+}}{d}$$

Mathematica [F] time = 38.26, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-2+3\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-2 + 3*Cos[c + d*x]])
,x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-2 + 3*Cos[c + d*x]])
, x]
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{3\cos(dx+c)-2}\,(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{3\cos(dx+c)^3-2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(sqrt(3*cos(d*x + c) - 2)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(3*
cos(d*x + c)^3 - 2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)+1}{\sqrt{3\cos(dx+c)-2}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2)),
 x)
```

maple [B] time = 0.42, size = 600, normalized size = 8.57

$$-\frac{2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin(dx+c)\left(\cos^2(dx+c)\right)\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}+4\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d/(-2+3*cos(d*x+c))^(1/2)*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c
)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*((-2+3*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)+4*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*c
os(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*((-2+3*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*((-2+3
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(
1/2))+2*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2
))+sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))+2*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))+sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))-3*cos(d*x+c)^3
+5*cos(d*x+c)^2-2*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)+1}{\sqrt{3\,\cos\left(dx+c\right)-2}\,\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)+1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{3\,\cos\left(c+d\,x\right)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3*cos(c + d*x) - 2)^(1/2)),x)
```

```
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3*cos(c + d*x) - 2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(c+dx\right)+1}{\sqrt{3\cos\left(c+dx\right)-2}\,\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(3*cos(c + d*x) - 2)*cos(c + d*x)**(3/2)),
 x)
```

3.443 $\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\sec(c+dx)-1}\,\sqrt{\sec(c+dx)+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{d}$$

```
[Out] csc(d*x+c)*EllipticE((2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),1/5*5^(1/2)
)*5^(1/2)*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+sec(d*x+c))^(1/2)*(1+sec
(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.21, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2995, 2994}

$$\frac{\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\sec(c+dx)-1}\,\sqrt{\sec(c+dx)+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Sqrt[2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] (Sqrt[5]*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcS
in[Sqrt[2 - 3*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*
x]]*Sqrt[1 + Sec[c + d*x]])/d
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2995

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt
[-(b*Sin[e + f*x])]/Sqrt[b*Sin[e + f*x]], Int[(A + B*Sin[e + f*x])/((-(b*Si
n[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\sqrt{-\cos(c+dx)}\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}(-\cos(c+dx))^{3/2}}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)\right.}{d}$$

Mathematica [F] time = 33.19, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\sqrt{2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Sqrt[2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),
x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Sqrt[2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),
 x]
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(\cos(dx+c)+1)\sqrt{-3\cos(dx+c)+2}\sqrt{\cos(dx+c)}}{3\cos(dx+c)^3-2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2-3*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(cos(d*x + c) + 1)*sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(
3*cos(d*x + c)^3 - 2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)+1}{\sqrt{-3\cos(dx+c)+2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2-3*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2))
, x)
```

maple [B] time = 0.42, size = 614, normalized size = 6.60

$$-\frac{\sqrt{2-3\cos\left(dx+c\right)}\,\left(-2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2-3*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(2-3*cos(d*x+c))^(1/2)*(-2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c
)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*((-2+3*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*c
os(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*((-2+3*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)-2*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),5^(1/2))-sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),5^(1/2))-2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*((-2+3*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))-2*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))-sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))+3*cos(d*x+c)^3
-5*cos(d*x+c)^2+2*cos(d*x+c))/(-2+3*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)+1}{\sqrt{-3\,\cos\left(dx+c\right)+2}\,\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(2-3*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) + 2)*cos(d*x + c)^(3/2))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)+1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{2-3\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2 - 3*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2 - 3*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(c + dx\right) + 1}{\sqrt{2 - 3\cos\left(c + dx\right)}\cos^{\frac{3}{2}}\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(2 - 3*cos(c + d*x))*cos(c + d*x)**(3/2)),
 x)
```

3.444 $\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{-\sec(c+dx)-1}\,\sqrt{1-\sec(c+dx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{d}$$

```
[Out] csc(d*x+c)*EllipticE(1/5*(-2-3*cos(d*x+c))^(1/2)*5^(1/2)/(-cos(d*x+c))^(1/2
),5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1-sec(d*x+c))^(1/2)*(1-se
c(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2995, 2994}

$$\frac{\sqrt{-\cos(c+dx)}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{-\sec(c+dx)-1}\,\sqrt{1-\sec(c+dx)}\,E\left(\sin^{-1}\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\,\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Sqrt[-2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] (Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[
-2 - 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]])/d
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2995

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt
[-(b*Sin[e + f*x])]/Sqrt[b*Sin[e + f*x]], Int[(A + B*Sin[e + f*x])/((-(b*Si
n[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
```

```
f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\sqrt{-\cos(c+dx)}\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}(-\cos(c+dx))^{3/2}}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\right|}{d}$$

Mathematica [F] time = 30.16, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\sqrt{-2-3\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Sqrt[-2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
,x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Sqrt[-2 - 3*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
, x]
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(\cos(dx+c)+1)\sqrt{-3\cos(dx+c)-2}\sqrt{\cos(dx+c)}}{3\cos(dx+c)^3+2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2-3*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(-(cos(d*x + c) + 1)*sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(
3*cos(d*x + c)^3 + 2*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)+1}{\sqrt{-3\cos(dx+c)-2}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2-3*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2))
, x)
```

maple [B] time = 0.39, size = 703, normalized size = 7.40

$$-\frac{\sqrt{-2-3\cos(dx+c)}\left(-2\sqrt{2}\,\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{10}\,\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{EllipticF}\left(\frac{\sqrt{5}\,(-1+\cos}{5\sin(d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2-3*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/10/d*(-2-3*cos(d*x+c))^(1/2)*(-2*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*10^(1/2
)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
*EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*5^(1/2)-4*2^(1/2
)*sin(d*x+c)*cos(d*x+c)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d
*x+c),5^(1/2))*5^(1/2)+10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(1/5*5^(1/2)*(-1+cos(d*x+c)
)/sin(d*x+c),5^(1/2))*sin(d*x+c)*cos(d*x+c)^2*5^(1/2)+2*10^(1/2)*((2+3*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ell
ipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)*cos(d*x+c
)^2*5^(1/2)-2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*5^(1/2)*10^(1/2)*((
2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))
/sin(d*x+c),5^(1/2))*sin(d*x+c)+10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(1/5*5^(1/2)*(-1+c
os(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)*cos(d*x+c)*5^(1/2)+2*10^(1/2)*((2
+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)*co
s(d*x+c)*5^(1/2)-30*cos(d*x+c)^3+10*cos(d*x+c)^2+20*cos(d*x+c))/(2+3*cos(d*
x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)+1}{\sqrt{-3\cos(dx+c)-2}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-2-3*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-3*cos(d*x + c) - 2)*cos(d*x + c)^(3/2))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c+d\,x)+1}{{\cos (c+d\,x)}^{3/2}\,\sqrt{-3\,\cos (c+d\,x)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(- 3*cos(c + d*x) - 2)^(1/2)),x)
```

```
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(- 3*cos(c + d*x) - 2)^(1/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (c+dx)+1}{\sqrt{-3\cos (c+dx)-2}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(-3*cos(c + d*x) - 2)*cos(c + d*x)**(3/2))
, x)
```

3.445 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{2\cot(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\,\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{3d}$$

```
[Out] 2/3*cot(d*x+c)*EllipticE(1/5*(3+2*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/2
),I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {2994}

$$\frac{2\cot(c+dx)\sqrt{1-\sec(c+dx)}\,\sqrt{\sec(c+dx)+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\,\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos
[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx = \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\,\sqrt{\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec(}}{3d}$$

Mathematica [F] time = 37.99, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{3+2\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[3 + 2*Cos[c + d*x]]),
x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[3 + 2*Cos[c + d*x]]),
 x]
```

fricas [F] time = 1.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{2\cos\left(dx+c\right)+3}\left(\cos\left(dx+c\right)+1\right)\sqrt{\cos\left(dx+c\right)}}{2\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(sqrt(2*cos(d*x + c) + 3)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(2*
cos(d*x + c)^3 + 3*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)+1}{\sqrt{2\cos\left(dx+c\right)+3}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2)),
 x)
```

maple [B] time = 0.42, size = 665, normalized size = 9.24

$$-3\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{i\sqrt{5}}{5}\right)\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}-6\sin\left(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d/(3+2*cos(d*x+c))^(1/2)*(-3*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)-6*sin(d*x+c)*cos(d*x+c)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))*2^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(3/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)+5*sin(d
*x+c)*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),1/5
*I*5^(1/2))-3*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),1/5*I*5^(1/2))-3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*
10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),1/5*I*5^(1/2))*sin(d*x+c)+5*sin(d*x+c)*cos(d*x+c)*2^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))-3*sin(d*x+c)*cos(d*x+
c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))-20*c
os(d*x+c)^3-10*cos(d*x+c)^2+30*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)+1}{\sqrt{2\cos(dx+c)+3}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3+2*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\,\sqrt{2\,\cos(c+d\,x)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2*cos(c + d*x) + 3)^(1/2)),x)
```

```
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2*cos(c + d*x) + 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)+1}{\sqrt{2\cos(c+dx)+3}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(2*cos(c + d*x) + 3)*cos(c + d*x)**(3/2)),
 x)
```

3.446 $\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{2\sqrt{5}\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{3d}$$

```
[Out] 2/3*cot(d*x+c)*EllipticE((3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),1/5*I*5^(1
/2))*5^(1/2)*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {2994}

$$\frac{2\sqrt{5}\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Sqrt[3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] (2*Sqrt[5]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[
c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{5}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\sqrt{1+s}}{3d}$$

Mathematica [F] time = 38.38, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Sqrt[3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),
x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Sqrt[3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),
 x]
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(\cos(dx+c)+1)\sqrt{-2\cos(dx+c)+3}\,\sqrt{\cos(dx+c)}}{2\cos(dx+c)^3-3\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3-2*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-(cos(d*x + c) + 1)*sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(
2*cos(d*x + c)^3 - 3*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)+1}{\sqrt{-2\cos(dx+c)+3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3-2*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2))
, x)
```

maple [B] time = 0.42, size = 663, normalized size = 8.96

$$\frac{\sqrt{3-2\cos(dx+c)}\left(3\sqrt{2}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\sin(dx+c)\left(\cos^2(dx+c)\right)\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3-2*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3/d*(3-2*cos(d*x+c))^(1/2)*(3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(
-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))+6*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))+3*2^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),I*5^(1/2))*sin(d*x+c)-2^(1/2)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),I*5^(1/2))*sin(d*x+c)*cos(d*x+c)^2+3*2^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*sin(d*x+c)*cos(d*x+c)^2-2^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*sin(d*x+c)*cos(d*x+c)+3*2^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*sin(d*x+c)*cos(d*x+c
)-4*cos(d*x+c)^3+10*cos(d*x+c)^2-6*cos(d*x+c))/(-3+2*cos(d*x+c))/cos(d*x+c)
^(3/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)+1}{\sqrt{-2\cos(dx+c)+3}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(3-2*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) + 3)*cos(d*x + c)^(3/2))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\,\sqrt{3-2\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3 - 2*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(3 - 2*cos(c + d*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)+1}{\sqrt{3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(3 - 2*cos(c + d*x))*cos(c + d*x)**(3/2)),
 x)
```

3.447 $\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{3d}$$

```
[Out] -2/3*csc(d*x+c)*EllipticE((-3+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),1/5*I
*5^(1/2))*5^(1/2)*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(1-sec(d*x+c))^(1/2)
*(1+sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2995, 2994}

$$-\frac{2\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[5]*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2995

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt
[-(b*Sin[e + f*x])]/Sqrt[b*Sin[e + f*x]], Int[(A + B*Sin[e + f*x])/((-(b*Si
n[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx = -\frac{\sqrt{-\cos(c+dx)}\int \frac{1+\cos(c+dx)}{(-\cos(c+dx))^{3/2}\sqrt{-3+2\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{2\sqrt{5}\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{-3+2\cos(c}}{\sqrt{-\cos(c+d}}\right.\right.}{3d}$$

Mathematica [F] time = 41.06, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{-3+2\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]])
,x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]])
, x]
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{2\cos(dx+c)-3}\,(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{2\cos(dx+c)^{3}-3\cos(dx+c)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(sqrt(2*cos(d*x + c) - 3)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/(2*
cos(d*x + c)^3 - 3*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)+1}{\sqrt{2\cos(dx+c)-3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2)),
 x)
```

maple [B] time = 0.33, size = 714, normalized size = 7.29

$$3i \sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{5}\,\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\,\sqrt{2}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}} \mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)}, \frac{i\sqrt{5}}{5}\right) + 6i \sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d/(-3+2*cos(d*x+c))^(1/2)*(3*I*sin(d*x+c)*cos(d*x+c)^2*5^(1/2)*(-2*(-3
+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)*EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))+6*I*sin(d
*x+c)*cos(d*x+c)*5^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin
(d*x+c),1/5*I*5^(1/2))-3*I*sin(d*x+c)*cos(d*x+c)^2*5^(1/2)*(-2*(-3+2*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellip
ticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))+5*I*sin(d*x+c)*cos
(d*x+c)^2*5^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c)
,1/5*I*5^(1/2))+3*I*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*sin(d*x+c)*5^
(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c
))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))-3*I*sin(d*x+c)*cos(d*x+c)*5^(1/2)*(-2*
(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))+5*I*si
n(d*x+c)*cos(d*x+c)*5^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))*5^(1/2)/
sin(d*x+c),1/5*I*5^(1/2))+20*cos(d*x+c)^3-50*cos(d*x+c)^2+30*cos(d*x+c))/co
s(d*x+c)^(3/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)+1}{\sqrt{2\cos(dx+c)-3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)+1}{\cos(c+d\,x)^{3/2}\,\sqrt{2\,\cos(c+d\,x)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2*cos(c + d*x) - 3)^(1/2)),x)
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(2*cos(c + d*x) - 3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)+1}{\sqrt{2\cos(c+dx)-3}\,\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-3+2*cos(d*x+c))**(1/2),x)
[Out] Integral((cos(c + d*x) + 1)/(sqrt(2*cos(c + d*x) - 3)*cos(c + d*x)**(3/2)),
 x)
```

3.448 $\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{3d}$$

```
[Out] -2/3*csc(d*x+c)*EllipticE(1/5*(-3-2*cos(d*x+c))^(1/2)*5^(1/2)/(-cos(d*x+c))
^(1/2),I*5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(1-sec(d*x+c))^(1/2)
*(1+sec(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2995, 2994}

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,E\left(\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[c + d*x])/(Sqrt[-3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sq
rt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*d)
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2995

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt
[-(b*Sin[e + f*x])]/Sqrt[b*Sin[e + f*x]], Int[(A + B*Sin[e + f*x])/((-(b*Si
n[e + f*x]))^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{b, c, d, e,
```

```
f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && NegQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\sqrt{-\cos(c+dx)}\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\,(-\cos(c+dx))^{3/2}}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right.\right.}{3d}$$

Mathematica [F] time = 30.58, size = 0, normalized size = 0.00

$$\int \frac{1+\cos(c+dx)}{\sqrt{-3-2\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 + Cos[c + d*x])/(Sqrt[-3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
,x]
```

```
[Out] Integrate[(1 + Cos[c + d*x])/(Sqrt[-3 - 2*Cos[c + d*x]]*Cos[c + d*x]^(3/2))
, x]
```

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{(\cos(dx+c)+1)\sqrt{-2\cos(dx+c)-3}\sqrt{\cos(dx+c)}}{2\cos(dx+c)^3+3\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3-2*cos(d*x+c))^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(-(cos(d*x + c) + 1)*sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(
2*cos(d*x + c)^3 + 3*cos(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)+1}{\sqrt{-2\cos(dx+c)-3}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3-2*cos(d*x+c))^(1/2),x, algori
thm="giac")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2))
, x)
```

maple [B] time = 0.34, size = 740, normalized size = 7.71

$$-\frac{\sqrt{-3-2\cos(dx+c)}\,\left(3i\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{5}\,\sqrt{10}\,\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx}{5\sin(dx}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3-2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/15/d*(-3-2*cos(d*x+c))^(1/2)*(3*I*sin(d*x+c)*cos(d*x+c)^2*5^(1/2)*10^(1/
2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*2^(1/2)+6*I
*sin(d*x+c)*cos(d*x+c)*5^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1
/2)/sin(d*x+c),I*5^(1/2))*2^(1/2)+I*sin(d*x+c)*cos(d*x+c)^2*10^(1/2)*((3+2*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*EllipticE(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*5^(1/2)-3*I*
sin(d*x+c)*cos(d*x+c)^2*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(
1/2)/sin(d*x+c),I*5^(1/2))*5^(1/2)+3*I*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)*sin(d*x+c)*5^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))+I*sin(d*x+c)*c
os(d*x+c)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c
),I*5^(1/2))*5^(1/2)-3*I*sin(d*x+c)*cos(d*x+c)*10^(1/2)*((3+2*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(1/
5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*5^(1/2)-20*cos(d*x+c)^3-1
0*cos(d*x+c)^2+30*cos(d*x+c))/(3+2*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)+1}{\sqrt{-2\cos(dx+c)-3}\,\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)^(3/2)/(-3-2*cos(d*x+c))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((cos(d*x + c) + 1)/(sqrt(-2*cos(d*x + c) - 3)*cos(d*x + c)^(3/2))
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)+1}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{-2\,\cos\left(c+d\,x\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(- 2*cos(c + d*x) - 3)^(1/2)),x)
```

```
[Out] int((cos(c + d*x) + 1)/(cos(c + d*x)^(3/2)*(- 2*cos(c + d*x) - 3)^(1/2)), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(c+dx \right)}+1}{\sqrt{-2\cos{\left(c+dx \right)}-3}\,\cos^{\frac{3}{2}}{\left(c+dx \right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(d*x+c))/cos(d*x+c)**(3/2)/(-3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((cos(c + d*x) + 1)/(sqrt(-2*cos(c + d*x) - 3)*cos(c + d*x)**(3/2))
, x)
```

3.449 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=36

$$\text{Int}\left((A+B\cos(e+fx))(c\cos(e+fx))^m(a+b\cos(e+fx))^n,x\right)$$

```
[Out] Unintegrable((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]),x]
```

```
[Out] Defer[Int][(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]),
x]
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx=\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos$$

Mathematica [A] time = 7.98, size = 0, normalized size = 0.00

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^n(A+B\cos(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]),x]
```

```
[Out] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x]), x
]
```

fricas [A] time = 1.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^n\left(c\cos\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*cos(f*x + e))^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{n}\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*cos(f*x + e))^m, x
)
```

maple [A] time = 3.39, size = 0, normalized size = 0.00

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(a+b\cos\left(fx+e\right)\right)^{n}\left(A+B\cos\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{n}\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^n*(A+B*cos(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*cos(f*x + e))^m, x
)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(c\,\cos\left(e+f\,x\right)\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^n,x)
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**n*(A+B*cos(f*x+e)),x)
[Out] Timed out
```

3.450 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^4(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=595

$$\frac{b^2\sin(e+fx)\cos(e+fx)\left(a^2B\left(m^2+11m+36\right)+2aAb(m+5)^2+b^2B(m+4)^2\right)(c\cos(e+fx))^{m+1}}{cf(m+3)(m+4)(m+5)}+\frac{b\sin(e+$$

```
[Out] b*(A*b^3*(m^2+8*m+15)+4*a*b^2*B*(m^2+8*m+15)+2*a^3*B*(m^2+10*m+28)+a^2*A*b*
(5*m^2+47*m+110))*(c*cos(f*x+e))^(1+m)*sin(f*x+e)/c/f/(5+m)/(m^2+6*m+8)+b^2
*(b^2*B*(4+m)^2+2*a*A*b*(5+m)^2+a^2*B*(m^2+11*m+36))*cos(f*x+e)*(c*cos(f*x+
e))^(1+m)*sin(f*x+e)/c/f/(3+m)/(4+m)/(5+m)+b*(A*b*(5+m)+a*B*(8+m))*(c*cos(f
*x+e))^(1+m)*(a+b*cos(f*x+e))^2*sin(f*x+e)/c/f/(4+m)/(5+m)+b*B*(c*cos(f*x+e
))^(1+m)*(a+b*cos(f*x+e))^3*sin(f*x+e)/c/f/(5+m)-(A*b^4*(m^2+4*m+3)+4*a*b^3
*B*(m^2+4*m+3)+6*a^2*A*b^2*(m^2+5*m+4)+4*a^3*b*B*(m^2+5*m+4)+a^4*A*(m^2+6*m
+8))*(c*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)
^2)*sin(f*x+e)/c/f/(4+m)/(m^2+3*m+2)/(sin(f*x+e)^2)^(1/2)-(b^4*B*(m^2+6*m+8
)+4*a*A*b^3*(m^2+7*m+10)+6*a^2*b^2*B*(m^2+7*m+10)+4*a^3*A*b*(m^2+8*m+15)+a^
4*B*(m^2+8*m+15))*(c*cos(f*x+e))^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c
os(f*x+e)^2)*sin(f*x+e)/c^2/f/(2+m)/(3+m)/(5+m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 1.98, antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2990, 3049, 3033, 3023, 2748, 2643}

$$-\frac{\sin(e+fx)\left(4a^3Ab\left(m^2+8m+15\right)+6a^2b^2B\left(m^2+7m+10\right)+a^4B\left(m^2+8m+15\right)+4aAb^3\left(m^2+7m+10\right.\right.}{c^2f(m+2)(m+3)(m+5)\sqrt{\sin^2(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x]),x]
```

```
[Out] (b*(A*b^3*(15 + 8*m + m^2) + 4*a*b^2*B*(15 + 8*m + m^2) + 2*a^3*B*(28 + 10*
m + m^2) + a^2*A*b*(110 + 47*m + 5*m^2))*(c*Cos[e + f*x])^(1 + m)*Sin[e + f
*x])/(c*f*(2 + m)*(4 + m)*(5 + m)) + (b^2*(b^2*B*(4 + m)^2 + 2*a*A*b*(5 + m
)^2 + a^2*B*(36 + 11*m + m^2))*Cos[e + f*x]*(c*Cos[e + f*x])^(1 + m)*Sin[e
+ f*x])/(c*f*(3 + m)*(4 + m)*(5 + m)) + (b*(A*b*(5 + m) + a*B*(8 + m))*(c*C
os[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^2*Sin[e + f*x])/(c*f*(4 + m)*(5 +
 m)) + (b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^3*Sin[e + f*x])/(
c*f*(5 + m)) - ((A*b^4*(3 + 4*m + m^2) + 4*a*b^3*B*(3 + 4*m + m^2) + 6*a^2*
A*b^2*(4 + 5*m + m^2) + 4*a^3*b*B*(4 + 5*m + m^2) + a^4*A*(8 + 6*m + m^2))*
```

```
(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e
 + f*x]^2]*Sin[e + f*x])/(c*f*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[e + f*x]^2])
 - ((b^4*B*(8 + 6*m + m^2) + 4*a*A*b^3*(10 + 7*m + m^2) + 6*a^2*b^2*B*(10 +
 7*m + m^2) + 4*a^3*A*b*(15 + 8*m + m^2) + a^4*B*(15 + 8*m + m^2))*(c*Cos[e
 + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^
2]*Sin[e + f*x])/(c^2*f*(2 + m)*(3 + m)*(5 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int (c\cos(e+fx))^m (a+b\cos(e+fx))^4 (A+B\cos(e+fx))\,dx = \frac{bB(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))^3\sin(e}{cf(5+m)}$$

$$= \frac{b(Ab(5+m)+aB(8+m))(c\cos(e+fx))^{1+m}(a}{cf(4+m)(5+m)}$$

$$= \frac{b^2\left(b^2B(4+m)^2+2aAb(5+m)^2+a^2B\left(36+1\right.\right.}{cf(3+m)}$$

$$= \frac{b\left(Ab^3\left(15+8m+m^2\right)+4ab^2B\left(15+8m+m^2\right.\right.}{}$$

$$= \frac{b\left(Ab^3\left(15+8m+m^2\right)+4ab^2B\left(15+8m+m^2\right.\right.}{}$$

$$= \frac{b\left(Ab^3\left(15+8m+m^2\right)+4ab^2B\left(15+8m+m^2\right.\right.}{}$$

Mathematica [A] time = 6.20, size = 487, normalized size = 0.82

$$-\frac{a^4 A \sin(e+fx)\cos(e+fx)(c\cos(e+fx))^m {}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{f(m+1)\sqrt{\sin^2(e+fx)}} - \frac{a^3(aB+4Ab)\sin(e+fx)\cos^2(e-}{f(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x]),x]
```

```
[Out] -((a^4*A*Cos[e + f*x]*(c*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (1 + m)/2,
(3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 + m)*Sqrt[Sin[e + f*x]^2]))
- (a^3*(4*A*b + a*B)*Cos[e + f*x]^2*(c*Cos[e + f*x])^m*Hypergeometric2F1[1/
2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(2 + m)*Sqrt[Sin[
e + f*x]^2]) - (2*a^2*b*(3*A*b + 2*a*B)*Cos[e + f*x]^3*(c*Cos[e + f*x])^m*H
ypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(
f*(3 + m)*Sqrt[Sin[e + f*x]^2]) - (2*a*b^2*(2*A*b + 3*a*B)*Cos[e + f*x]^4*(
c*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[e + f*x]
^2]*Sin[e + f*x])/(f*(4 + m)*Sqrt[Sin[e + f*x]^2]) - (b^3*(A*b + 4*a*B)*Cos
[e + f*x]^5*(c*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2,
 Cos[e + f*x]^2]*Sin[e + f*x])/(f*(5 + m)*Sqrt[Sin[e + f*x]^2]) - (b^4*B*Co
s[e + f*x]^6*(c*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (6 + m)/2, (8 + m)/2
, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(6 + m)*Sqrt[Sin[e + f*x]^2])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^4\cos\left(fx+e\right)^5 + Aa^4 + \left(4\,Bab^3 + Ab^4\right)\cos\left(fx+e\right)^4 + 2\left(3\,Ba^2b^2 + 2\,Aab^3\right)\cos\left(fx+e\right)^3 + 2\left(2\,B\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^4*cos(f*x + e)^5 + A*a^4 + (4*B*a*b^3 + A*b^4)*cos(f*x + e)^4
 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*cos(f*x + e)^3 + 2*(2*B*a^3*b + 3*A*a^2*b^2)
*cos(f*x + e)^2 + (B*a^4 + 4*A*a^3*b)*cos(f*x + e))*(c*cos(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^4\left(c\cos\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*cos(f*x + e))^m, x
)
```

maple [F] time = 2.88, size = 0, normalized size = 0.00

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(a+b\cos\left(fx+e\right)\right)^{4}\left(A+B\cos\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{4}\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^4*(A+B*cos(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*cos(f*x + e))^m, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,\cos\left(e+f\,x\right)\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^4,x)
```

```
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**4*(A+B*cos(f*x+e)),x)
```

```
[Out] Timed out
```

3.451 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=406

$$-\frac{\sin(e+fx)\left(b(m+1)\left(2a^2B(m+5)+3aAb(m+4)+b^2B(m+3)\right)+a^2(m+2)(aA(m+4)+bB(m+1))\right)(c\cos}{cf(m+1)(m+2)(m+4)\sqrt{\sin^2(e+fx)}}$$

```
[Out] b*(b^2*B*(3+m)+3*a*A*b*(4+m)+2*a^2*B*(5+m))*(c*cos(f*x+e))^(1+m)*sin(f*x+e)
/c/f/(2+m)/(4+m)+b^2*(A*b*(4+m)+a*B*(6+m))*cos(f*x+e)*(c*cos(f*x+e))^(1+m)*
sin(f*x+e)/c/f/(3+m)/(4+m)+b*B*(c*cos(f*x+e))^(1+m)*(a+b*cos(f*x+e))^2*sin(
f*x+e)/c/f/(4+m)-(a^2*(2+m)*(b*B*(1+m)+a*A*(4+m))+b*(1+m)*(b^2*B*(3+m)+3*a*
A*b*(4+m)+2*a^2*B*(5+m)))*(c*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[
3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c/f/(1+m)/(2+m)/(4+m)/(sin(f*x+e)^2)^(1
/2)-(A*b^3*(2+m)+3*a*b^2*B*(2+m)+3*a^2*A*b*(3+m)+a^3*B*(3+m))*(c*cos(f*x+e)
)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c^2/f/(
2+m)/(3+m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 1.05, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2990, 3033, 3023, 2748, 2643}

$$-\frac{\sin(e+fx)\left(3a^2Ab(m+3)+a^3B(m+3)+3ab^2B(m+2)+Ab^3(m+2)\right)(c\cos(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\text{c}\right.}{c^2f(m+2)(m+3)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x]),x]
```

```
[Out] (b*(b^2*B*(3 + m) + 3*a*A*b*(4 + m) + 2*a^2*B*(5 + m))*(c*Cos[e + f*x])^(1
+ m)*Sin[e + f*x])/(c*f*(2 + m)*(4 + m)) + (b^2*(A*b*(4 + m) + a*B*(6 + m))
*Cos[e + f*x]*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(c*f*(3 + m)*(4 + m))
+ (b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])^2*Sin[e + f*x])/(c*f*(
4 + m)) - ((a^2*(2 + m)*(b*B*(1 + m) + a*A*(4 + m)) + b*(1 + m)*(b^2*B*(3 +
 m) + 3*a*A*b*(4 + m) + 2*a^2*B*(5 + m)))*(c*Cos[e + f*x])^(1 + m)*Hypergeo
metric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c*f*(1
+ m)*(2 + m)*(4 + m)*Sqrt[Sin[e + f*x]^2]) - ((A*b^3*(2 + m) + 3*a*b^2*B*(2
 + m) + 3*a^2*A*b*(3 + m) + a^3*B*(3 + m))*(c*Cos[e + f*x])^(2 + m)*Hyperge
ometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c^2*f*
(2 + m)*(3 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^3(A+B\cos(e+fx))\,dx = \frac{bB(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))^2\sin(e+}{cf(4+m)}$$

$$= \frac{b^2(Ab(4+m)+aB(6+m))\cos(e+fx)(c\cos(e}{cf(3+m)(4+m)}$$

$$= \frac{b\left(b^2B(3+m)+3aAb(4+m)+2a^2B(5+m)\right)(c}{cf(2+m)(4+m)}$$

$$= \frac{b\left(b^2B(3+m)+3aAb(4+m)+2a^2B(5+m)\right)(c}{cf(2+m)(4+m)}$$

$$= \frac{b\left(b^2B(3+m)+3aAb(4+m)+2a^2B(5+m)\right)(c}{cf(2+m)(4+m)}$$

Mathematica [A] time = 2.81, size = 269, normalized size = 0.66

$$\frac{\sin(e+fx)\cos(e+fx)(c\cos(e+fx))^m\left(\cos(e+fx)\left(b\cos(e+fx)\left(b\cos(e+fx)\left(-\frac{(3aB+Ab)\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};\cos^2(e+f}{m+4}\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*(c*Cos[e + f*x])^m*(-((a^3*A*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Cos[e + f*x]^2])/(1 + m)) + Cos[e + f*x]*(-((a^2*(3*A*b + a*B)
*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2])/(2 + m)) + b
*Cos[e + f*x]*((-3*a*(A*b + a*B)*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/
2, Cos[e + f*x]^2])/(3 + m) + b*Cos[e + f*x]*(-(((A*b + 3*a*B)*Hypergeometr
ic2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[e + f*x]^2])/(4 + m)) - (b*B*Cos[e + f
*x]*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, Cos[e + f*x]^2])/(5 + m)))
))*Sin[e + f*x])/(f*Sqrt[Sin[e + f*x]^2])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^3\cos\left(fx+e\right)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\cos\left(fx+e\right)^3 + 3\left(Ba^2b + Aab^2\right)\cos\left(fx+e\right)^2 + \left(Ba^3 + 3\,\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(f*x + e)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(f*x + e)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(f*x + e)^2 + (B*a^3 + 3*A*a^2*b)*cos(f*x + e))
*(c*cos(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{3}\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*cos(f*x + e))^m, x
)
```

maple [F] time = 3.00, size = 0, normalized size = 0.00

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(a+b\cos\left(fx+e\right)\right)^{3}\left(A+B\cos\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{3}\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^3*(A+B*cos(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*cos(f*x + e))^m, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,\cos\left(e+f\,x\right)\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^3,x)
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**3*(A+B*cos(f*x+e)),x)
[Out] Timed out
```

3.452 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=287

$$-\frac{\sin(e+fx)\left(a^2A(m+2)+2abB(m+1)+Ab^2(m+1)\right)(c\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{cf(m+1)(m+2)\sqrt{\sin^2(e+fx)}}-\text{si}$$

```
[Out] b*(A*b*(3+m)+a*B*(4+m))*(c*cos(f*x+e))^(1+m)*sin(f*x+e)/c/f/(2+m)/(3+m)+b*B
*(c*cos(f*x+e))^(1+m)*(a+b*cos(f*x+e))*sin(f*x+e)/c/f/(3+m)-(A*b^2*(1+m)+2*
a*b*B*(1+m)+a^2*A*(2+m))*(c*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3
/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c/f/(1+m)/(2+m)/(sin(f*x+e)^2)^(1/2)-(b^
2*B*(2+m)+a*(2*A*b+B*a)*(3+m))*(c*cos(f*x+e))^(2+m)*hypergeom([1/2, 1+1/2*m
],[2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c^2/f/(2+m)/(3+m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2990, 3023, 2748, 2643}

$$-\frac{\sin(e+fx)\left(a^2A(m+2)+2abB(m+1)+Ab^2(m+1)\right)(c\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{cf(m+1)(m+2)\sqrt{\sin^2(e+fx)}}-\text{si}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x]),x]
```

```
[Out] (b*(A*b*(3 + m) + a*B*(4 + m))*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(c*f*
(2 + m)*(3 + m)) + (b*B*(c*Cos[e + f*x])^(1 + m)*(a + b*Cos[e + f*x])*Sin[e
 + f*x])/(c*f*(3 + m)) - ((A*b^2*(1 + m) + 2*a*b*B*(1 + m) + a^2*A*(2 + m))
*(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[
e + f*x]^2]*Sin[e + f*x])/(c*f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*x]^2]) - ((b^
2*B*(2 + m) + a*(2*A*b + a*B)*(3 + m))*(c*Cos[e + f*x])^(2 + m)*Hypergeomet
ric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c^2*f*(2 +
 m)*(3 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^2(A+B\cos(e+fx))\,dx = \frac{bB(c\cos(e+fx))^{1+m}(a+b\cos(e+fx))\sin(e+}{cf(3+m)}$$

$$= \frac{b(Ab(3+m)+aB(4+m))(c\cos(e+fx))^{1+m}\sin}{cf(2+m)(3+m)}$$

$$= \frac{b(Ab(3+m)+aB(4+m))(c\cos(e+fx))^{1+m}\sin}{cf(2+m)(3+m)}$$

$$= \frac{b(Ab(3+m)+aB(4+m))(c\cos(e+fx))^{1+m}\sin}{cf(2+m)(3+m)}$$

Mathematica [A] time = 1.70, size = 217, normalized size = 0.76

$$\frac{\sin(e+fx)\cos(e+fx)(c\cos(e+fx))^m\left(\cos(e+fx)\left(b\cos(e+fx)\left(-\frac{(2aB+Ab)\,{}_2F_1\left(\frac{1}{2},\frac{m+3}{2};\frac{m+5}{2};\cos^2(e+fx)\right)}{m+3}-\frac{bB\cos(e+\right.\right.\right.}{f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*(c*Cos[e + f*x])^m*(-((a^2*A*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Cos[e + f*x]^2])/(1 + m)) + Cos[e + f*x]*(-((a*(2*A*b + a*B)*H
ypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2])/(2 + m)) + b*C
os[e + f*x]*(-(((A*b + 2*a*B)*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2,
Cos[e + f*x]^2])/(3 + m)) - (b*B*Cos[e + f*x]*Hypergeometric2F1[1/2, (4 + m
)/2, (6 + m)/2, Cos[e + f*x]^2])/(4 + m))))*Sin[e + f*x])/(f*Sqrt[Sin[e + f
*x]^2])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos\left(fx+e\right)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos\left(fx+e\right)^2+\left(Ba^2+2\,Aab\right)\cos\left(fx+e\right)\right)\left(c\cos\left(fx+e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(f*x + e)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(f*x + e)^2 +
 (B*a^2 + 2*A*a*b)*cos(f*x + e))*(c*cos(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^2\left(c\cos\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*cos(f*x + e))^m, x
)
```

maple [F] time = 1.88, size = 0, normalized size = 0.00

$$\int\left(c\cos\left(fx+e\right)\right)^m\left(a+b\cos\left(fx+e\right)\right)^2\left(A+B\cos\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{2}\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^2*(A+B*cos(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*cos(f*x + e))^m, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,\cos\left(e+f\,x\right)\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^2,x)
```

```
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**2*(A+B*cos(f*x+e)),x)
```

```
[Out] Timed out
```

3.453 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=196

$$-\frac{(aB+Ab)\sin(e+fx)(c\cos(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(e+fx)\right)}{c^2f(m+2)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+fx)(aA(m+2)+bB(m+1)}{cf(m+1}$$

```
[Out] b*B*(c*cos(f*x+e))^(1+m)*sin(f*x+e)/c/f/(2+m)-(b*B*(1+m)+a*A*(2+m))*(c*cos(
f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+
e)/c/f/(1+m)/(2+m)/(sin(f*x+e)^2)^(1/2)-(A*b+B*a)*(c*cos(f*x+e))^(2+m)*hype
rgeom([1/2, 1+1/2*m],[2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/c^2/f/(2+m)/(sin(f*
x+e)^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2968, 3023, 2748, 2643}

$$-\frac{(aB+Ab)\sin(e+fx)(c\cos(e+fx))^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(e+fx)\right)}{c^2f(m+2)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+fx)(aA(m+2)+bB(m+1)}{cf(m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x]),x]
```

```
[Out] (b*B*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(c*f*(2 + m)) - ((b*B*(1 + m) +
 a*A*(2 + m))*(c*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3
 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c*f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*
x]^2]) - ((A*b + a*B)*(c*Cos[e + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 +
m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(c^2*f*(2 + m)*Sqrt[Sin[e +
f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))(A+B\cos(e+fx))\,dx = \int (c\cos(e+fx))^m\left(aA+(Ab+aB)\cos(e+fx)\right.$$

$$=\frac{bB(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)}+\frac{\int (c\cos(e+f}{}$$

$$=\frac{bB(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)}+\frac{(Ab+aB)\int(}{}$$

$$=\frac{bB(c\cos(e+fx))^{1+m}\sin(e+fx)}{cf(2+m)}-\frac{\left(aA+\frac{bB(1+m}{2+m}\right.}{}$$

Mathematica [A] time = 0.34, size = 151, normalized size = 0.77

$$-\frac{\sin(e+fx)\cos(e+fx)(c\cos(e+fx))^m\left((aA(m+2)+bB(m+1))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)+(m+1)\left((a\right.\right.}{f(m+1)(m+2)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(c*Cos[e + f*x])^m*Sin[e + f*x]*((b*B*(1 + m) + a*A*(2 + m)
)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2] + (1 + m)*((
A*b + a*B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e
+ f*x]^2] - b*B*Sqrt[Sin[e + f*x]^2])))/(f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*x
]^2]))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(fx+e\right)^{2}+Aa+(Ba+Ab)\cos\left(fx+e\right)\right)\left(c\cos\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*(c*cos(f*x +
 e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m, x)
```

maple [F] time = 1.99, size = 0, normalized size = 0.00

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(a+b\cos\left(fx+e\right)\right)\left(A+B\cos\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,\cos\left(e+f\,x\right)\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)),x)
```

```
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\cos\left(e+fx\right)\right)^m \left(A+B\cos\left(e+fx\right)\right)\left(a+b\cos\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))*(A+B*cos(f*x+e)),x)
```

```
[Out] Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)), x)
```

3.454 $\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx$

Optimal. Leaf size=286

$$\frac{ac(Ab-aB)\sin(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)}{bf\left(a^2-b^2\right)}-\frac{(Ab-a}{}$$

```
[Out] a*(A*b-B*a)*c*AppellF1(1/2,1/2-1/2*m,1,3/2,sin(f*x+e)^2,-b^2*sin(f*x+e)^2/(
a^2-b^2))*(c*cos(f*x+e))^(-1+m)*(cos(f*x+e)^2)^(1/2-1/2*m)*sin(f*x+e)/b/(a^
2-b^2)/f-(A*b-B*a)*AppellF1(1/2,-1/2*m,1,3/2,sin(f*x+e)^2,-b^2*sin(f*x+e)^2
/(a^2-b^2))*(c*cos(f*x+e))^m*sin(f*x+e)/(a^2-b^2)/f/((cos(f*x+e)^2)^(1/2*m)
)-B*(c*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^
2)*sin(f*x+e)/b/c/f/(1+m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3002, 2643, 2823, 3189, 429}

$$\frac{ac(Ab-aB)\sin(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)}{bf\left(a^2-b^2\right)}-\frac{(Ab-a}{}$$

Antiderivative was successfully verified.

```
[In] Int[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x]),x]
```

```
[Out] (a*(A*b - a*B)*c*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Si
n[e + f*x]^2)/(a^2 - b^2))]*(c*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1
- m)/2)*Sin[e + f*x])/(b*(a^2 - b^2)*f) - ((A*b - a*B)*AppellF1[1/2, -m/2,
1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*x]^2)/(a^2 - b^2))]*(c*Cos[e + f*x
])^m*Sin[e + f*x])/((a^2 - b^2)*f*(Cos[e + f*x]^2)^(m/2)) - (B*(c*Cos[e + f
*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*S
in[e + f*x])/(b*c*f*(1 + m)*Sqrt[Sin[e + f*x]^2])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] && !IntegerQ[m]
```

Rubi steps

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{a+b\cos(e+fx)}\,dx = \frac{B\int (c\cos(e+fx))^m\,dx}{b} - \frac{(-Ab+aB)\int \frac{(c\cos(e+fx))^m}{a+b\cos(e+fx)}\,dx}{b}$$

$$= -\frac{B(c\cos(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin(e+fx)}{bcf(1+m)\sqrt{\sin^2(e+fx)}}$$

$$= -\frac{B(c\cos(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin(e+fx)}{bcf(1+m)\sqrt{\sin^2(e+fx)}}$$

$$= \frac{a(Ab-aB)cF_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)(c\cos(e+}{b\left(a^2-b^2\right)f}$$

Mathematica [**B**] time = 26.94, size = 10482, normalized size = 36.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x]),x]
[Out] Result too large to show
```

fricas [**F**] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^m}{b\cos\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x, algorithm="
fricas")
[Out] integral((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/t_nostep/2)>(-2*
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,1,0]%%%}+%%%{-1,[0,0,0,1]%%%} Error: Bad Argument Value

maple [F] time = 1.48, size = 0, normalized size = 0.00

$$\int \frac{\left(c \cos\left(f x + e\right)\right)^{m} \left(A + B \cos\left(f x + e\right)\right)}{a + b \cos\left(f x + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B \cos\left(f x + e\right) + A\right) \left(c \cos\left(f x + e\right)\right)^{m}}{b \cos\left(f x + e\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,\cos\left(e + f\,x\right)\right)^{m}\,\left(A + B\,\cos\left(e + f\,x\right)\right)}{a + b\,\cos\left(e + f\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x)),x)
```

```
[Out] int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e)),x)
```

```
[Out] Timed out
```

3.455 $\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=181

$$\frac{2\mathrm{Int}\left(\frac{(c\cos(e+fx))^m\left(\frac{1}{2}c\cos(e+fx)\left(a(2m+5)(aB+2Ab)+b^2B(2m+3)\right)+\frac{1}{2}bc\cos^2(e+fx)(2aB(m+3)+Ab(2m+5))+\frac{1}{2}ac\left(2aA\left(m+\frac{5}{2}\right)+2bB(m+1)\right)\right)}{\sqrt{a+b\cos(e+fx)}},x\right)}{c(2m+5)}+$$

```
[Out] 2*b*B*(c*cos(f*x+e))^(1+m)*sin(f*x+e)*(a+b*cos(f*x+e))^(1/2)/c/f/(5+2*m)+2*
Unintegrable((c*cos(f*x+e))^m*(1/2*a*c*(2*b*B*(1+m)+2*a*A*(5/2+m))+1/2*c*(b
^2*B*(3+2*m)+a*(2*A*b+B*a)*(5+2*m))*cos(f*x+e)+1/2*b*c*(2*a*B*(3+m)+A*b*(5+
2*m))*cos(f*x+e)^2)/(a+b*cos(f*x+e))^(1/2),x)/c/(5+2*m)
```

Rubi [A] time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x]),x]
```

```
[Out] (2*b*B*(c*Cos[e + f*x])^(1 + m)*Sqrt[a + b*Cos[e + f*x]]*Sin[e + f*x])/(c*f
*(5 + 2*m)) + (2*Defer[Int][((c*Cos[e + f*x])^m*((a*c*(2*b*B*(1 + m) + 2*a*
A*(5/2 + m)))/2 + (c*(b^2*B*(3 + 2*m) + a*(2*A*b + a*B)*(5 + 2*m))*Cos[e +
f*x])/2 + (b*c*(2*a*B*(3 + m) + A*b*(5 + 2*m))*Cos[e + f*x]^2)/2))/Sqrt[a +
 b*Cos[e + f*x]], x])/(c*(5 + 2*m))
```

Rubi steps

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx=\frac{2bB(c\cos(e+fx))^{1+m}\sqrt{a+b\cos(e+fx)}\,\sin(e}{cf(5+2m)}$$

Mathematica [A] time = 66.75, size = 0, normalized size = 0.00

$$\int (c\cos(e+fx))^m(a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x]
),x]
```

```
[Out] Integrate[(c*Cos[e + f*x])^m*(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x]
), x]
```

fricas [A] time = 1.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(fx+e\right)^2+Aa+\left(Ba+Ab\right)\cos\left(fx+e\right)\right)\sqrt{b\cos\left(fx+e\right)+a}\,\left(c\cos\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*sqrt(b*cos(f
*x + e) + a)*(c*cos(f*x + e))^m, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \left(c\cos\left(fx+e\right)\right)^m \left(a+b\cos\left(fx+e\right)\right)^{\frac{3}{2}} \left(A+B\cos\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x)
```

```
[Out] int((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}} \left(c\cos\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*cos(f*x + e))^
m, x)
```

mupad **[A]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,\cos\left(e+f\,x\right)\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(3/2),x)
```

```
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(a+b*cos(f*x+e))**(3/2)*(A+B*cos(f*x+e)),x)
```

```
[Out] Timed out
```

3.456 $\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))(c\cos(e+fx))^m,x\right)$$

```
[Out] Unintegrable((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x]),x]
```

```
[Out] Defer[Int][(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])
, x]
```

Rubi steps

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))\,dx = \int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}\,(A+B\cos$$

Mathematica [A] time = 9.67, size = 0, normalized size = 0.00

$$\int (c\cos(e+fx))^m\sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x]),
x]
```

```
[Out] Integrate[(c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x]),
 x]
```

fricas [A] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\,\left(c\cos\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m,
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\,\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m,
 x)
```

maple [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \left(c\cos\left(fx+e\right)\right)^{m}\left(A+B\cos\left(fx+e\right)\right)\sqrt{a+b\cos\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x)
```

```
[Out] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\,\left(c\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m,
 x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(c\,\cos\left(e+f\,x\right)\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\sqrt{a+b\,\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(1/2),x)
```

```
[Out] int((c*cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\cos\left(e+fx\right)\right)^{m}\left(A+B\cos\left(e+fx\right)\right)\sqrt{a+b\cos\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))*(a+b*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))*sqrt(a + b*cos(e + f*x)),
 x)
```

3.457 $\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\frac{(A+B\cos(e+fx))(c\cos(e+fx))^m}{\sqrt{a+b\cos(e+fx)}},x\right)$$

```
[Out] Unintegrable((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x)
```

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x]],x]
```

```
[Out] Defer[Int][((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x
]], x]
```

Rubi steps

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx=\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Mathematica [A] time = 8.10, size = 0, normalized size = 0.00

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x]
],x]
```

```
[Out] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/Sqrt[a + b*Cos[e + f*x]
], x]
```

fricas [A] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^{m}}{\sqrt{b\cos\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^{m}}{\sqrt{b\cos\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

maple [A] time = 0.32, size = 0, normalized size = 0.00

$$\int\frac{\left(c\cos\left(fx+e\right)\right)^{m}\left(A+B\cos\left(fx+e\right)\right)}{\sqrt{a+b\cos\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x)
```

```
[Out] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^{m}}{\sqrt{b\cos\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c\,\cos\left(e+f\,x\right)\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)}{\sqrt{a+b\,\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(1/2),x)
```

```
[Out] int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(1/2), x
)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\cos\left(e+fx\right)\right)^m\left(A+B\cos\left(e+fx\right)\right)}{\sqrt{a+b\cos\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))/sqrt(a + b*cos(e + f*x)),
 x)
```

3.458 $\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=191

$$\frac{2\text{Int}\left(\frac{(c\cos(e+fx))^m\left(-\frac{1}{2}bc(2m+3)(Ab-aB)\cos^2(e+fx)-\frac{1}{2}ac(Ab-aB)\cos(e+fx)+\frac{1}{2}c\left(2b\left(m+\frac{1}{2}\right)(Ab-aB)+a(aA-bB)\right)\right)}{\sqrt{a+b\cos(e+fx)}},x\right)}{ac\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin}{acf\left(a^2-b^2\right.}$$

```
[Out] 2*b*(A*b-B*a)*(c*cos(f*x+e))^(1+m)*sin(f*x+e)/a/(a^2-b^2)/c/f/(a+b*cos(f*x+
e))^(1/2)+2*Unintegrable((c*cos(f*x+e))^m*(1/2*c*(a*(A*a-B*b)+2*b*(A*b-B*a)
*(1/2+m))-1/2*a*(A*b-B*a)*c*cos(f*x+e)-1/2*b*(A*b-B*a)*c*(3+2*m)*cos(f*x+e)
^2)/(a+b*cos(f*x+e))^(1/2),x)/a/(a^2-b^2)/c
```

Rubi [A] time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x])^(3/2),x]
```

```
[Out] (2*b*(A*b - a*B)*(c*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*(a^2 - b^2)*c*f*
Sqrt[a + b*Cos[e + f*x]]) + (2*Defer[Int][((c*Cos[e + f*x])^m*((c*(a*(a*A -
 b*B) + 2*b*(A*b - a*B)*(1/2 + m)))/2 - (a*(A*b - a*B)*c*Cos[e + f*x])/2 -
(b*(A*b - a*B)*c*(3 + 2*m)*Cos[e + f*x]^2)/2))/Sqrt[a + b*Cos[e + f*x]], x]
)/(a*(a^2 - b^2)*c)
```

Rubi steps

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx=\frac{2b(Ab-aB)(c\cos(e+fx))^{1+m}\sin(e+fx)}{a\left(a^2-b^2\right)cf\sqrt{a+b\cos(e+fx)}}+\frac{2\int\frac{(c\cos(e+fx))^m\left(\frac{1}{2}c\left(a\right.\right.}{}}{}$$

Mathematica [A] time = 10.72, size = 0, normalized size = 0.00

$$\int \frac{(c\cos(e+fx))^m(A+B\cos(e+fx))}{(a+b\cos(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x])^(3
/2),x]
```

```
[Out] Integrate[((c*Cos[e + f*x])^m*(A + B*Cos[e + f*x]))/(a + b*Cos[e + f*x])^(3
/2), x]
```

fricas [A] time = 1.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\,\left(c\cos\left(fx+e\right)\right)^{m}}{b^{2}\cos\left(fx+e\right)^{2}+2\,ab\cos\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*cos(f*x + e))^m/(
b^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^{m}}{\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a)^(3/2
), x)
```

maple [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\left(c\cos\left(fx+e\right)\right)^{m}\left(A+B\cos\left(fx+e\right)\right)}{\left(a+b\cos\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x)
```

```
[Out] int((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\cos\left(fx+e\right)\right)^{m}}{\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*cos(f*x + e))^m/(b*cos(f*x + e) + a)^(3/2
), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,\cos\left(e+f\,x\right)\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)}{{\left(a+b\,\cos\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(3/2),x)
```

```
[Out] int(((c*cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(3/2), x
)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\cos\left(e+fx\right)\right)^{m}\left(A+B\cos\left(e+fx\right)\right)}{\left(a+b\cos\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(A+B*cos(f*x+e))/(a+b*cos(f*x+e))**(3/2),x)
```

```
[Out] Integral((c*cos(e + f*x))**m*(A + B*cos(e + f*x))/(a + b*cos(e + f*x))**(3/
2), x)
```

3.459 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=172

$$\frac{2a(A+B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(3A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d}}{3d}$$

```
[Out] 2/3*a*(A+B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*A*sec(d*x+c)^(5/2)*sin(d*x+
c)/d+2/5*a*(3*A+5*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*a*(3*A+5*B)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{2a(A+B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(3A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-2*a*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*d) + (2*a*(3*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*d) + (2*a*(A + B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*A*Sec[
c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rubi steps

$$\int (a + a\cos(c+dx))(A + B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sec^{\frac{3}{2}}(c+dx)(a + a\sec(c+dx))(B + A\sec(c+dx)$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sec^{\frac{3}{2}}(c+dx)\Big(\frac{1}{2}$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + (a(A+B))\int \sec^{\frac{5}{2}}(c$$

$$= \frac{2a(3A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a(A+B)\,s}{}$$

$$= \frac{2a(3A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a(A+B)\,s}{}$$

$$= -\frac{2a(3A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}{5d}$$

Mathematica [C] time = 1.93, size = 292, normalized size = 1.70

$$\frac{ae^{-ic}\left(-1+e^{2ic}\right)\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\left((3A+5B)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\,{}_2F_1\Big(\frac{1}{2},}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*(-1 + E^((2*I)*c))*(1 + Cos[c + d*x])*Csc[c]*(5*A + 5*B - 3*A*E^(I*(c +
d*x)) - 15*B*E^(I*(c + d*x)) - 24*A*E^((3*I)*(c + d*x)) - 30*B*E^((3*I)*(c
+ d*x)) - 5*A*E^((4*I)*(c + d*x)) - 5*B*E^((4*I)*(c + d*x)) - 9*A*E^((5*I)*
(c + d*x)) - 15*B*E^((5*I)*(c + d*x)) - (5*I)*(A + B)*(1 + E^((2*I)*(c + d*
x)))^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (3*A + 5*B)*E^(I*(c +
 d*x))*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))])*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]])/(30*d*E^(I*c)*(1
 + E^((2*I)*(c + d*x)))^2)
```

fricas [F] time = 2.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba\cos\left(dx+c\right)^2 + (A+B)a\cos\left(dx+c\right) + Aa\right)\sec\left(dx+c\right)^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)*sec(d*x + c)^(
7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

maple [B] time = 4.28, size = 661, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/10*A/(8*
sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(
1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)+1/2*B*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)
^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(1/2*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^
2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}\,(a+a\,\cos(c+d\,x))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x)),x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.460 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=135

$$\frac{2a(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B)\sqrt{\cos(}}{}$$

```
[Out] 2/3*a*A*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*(A+B)*sin(d*x+c)*sec(d*x+c)^(1/2)
/d-2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(A+3*B)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2a(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B)\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a*(A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a
*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rubi steps

$$\int (a + a\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx = \int \sqrt{\sec(c + dx)}\,(a + a\sec(c + dx))(B + A\sec(c + d$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c + dx)\sin(c + dx)}{3d} + \frac{2}{3}\int \sqrt{\sec(c + dx)}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c + dx)\sin(c + dx)}{3d} + (a(A + B))\int \sec^{\frac{3}{2}}($$

$$= \frac{2a(A + B)\sqrt{\sec(c + dx)}\,\sin(c + dx)}{d} + \frac{2aA\sec^{\frac{3}{2}}(c -}{}$$

$$= \frac{2a(A + 3B)\sqrt{\cos(c + dx)}\,F\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{\sec(c +}{3d}$$

$$= -\frac{2a(A + B)\sqrt{\cos(c + dx)}\,E\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{\sec(c +}{d}$$

Mathematica [C] time = 1.23, size = 225, normalized size = 1.67

$$\frac{a(\cos(c + dx) + 1)\sec^2\left(\frac{1}{2}(c + dx)\right)\sqrt{\sec(c + dx)}\left(i\left((A + B)e^{i(c+dx)}\left(1 + e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) - 3A\right.\right.}{3d\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*((A + 3*B)*(1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] + I*(A - 3*A*E^(I*(c + d*x)) - 3*B*E^(I*(c + d
*x)) - A*E^((2*I)*(c + d*x)) - 3*A*E^((3*I)*(c + d*x)) - 3*B*E^((3*I)*(c +
d*x)) + (A + B)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(3/2)*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))*Sec[(c + d*x)/2]^2*Sqrt[Sec[c
 + d*x]])/(3*d*(1 + E^((2*I)*(c + d*x))))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba\cos\left(dx + c\right)^2 + (A + B)a\cos\left(dx + c\right) + Aa\right)\sec\left(dx + c\right)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)*sec(d*x + c)^(
5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

maple [B] time = 3.57, size = 426, normalized size = 3.16

$$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(\frac{B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\;\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}+\frac{\left(\frac{A}{2}+\frac{B}{2}\right)\left(-\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*B*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(
1/2*A+1/2*B)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1)+1/2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{5/2}(a+a\cos(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x)),x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.461 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=106

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2}{}$$

```
[Out] 2*a*A*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.18, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2960, 3997, 3787, 3771, 2639, 2641}

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx=\int \frac{(a+a\sec(c+dx))(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$=\frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+2\int \frac{-\frac{1}{2}a(A-B)+$$

$$=\frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}-(a(A-B))\int \frac{}{\sqrt{\sec}}$$

$$=\frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}-\Big(a(A-B)\sqrt{\cos(c+$$

$$=-\frac{2a(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}{d}$$

Mathematica [C] time = 1.06, size = 157, normalized size = 1.48

$$\frac{2ae^{-idx}\sqrt{\sec(c+dx)}\left(\cos(dx)+i\sin(dx)\right)\left(i(A-B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+3(A+B)\sqrt{\cos(}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((-3*I)*A*Cos[c + d*x] + (3
*I)*B*Cos[c + d*x] + 3*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
 + I*(A - B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F
1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 3*A*Sin[c + d*x]))/(3*d*E^(I*d*x))
```

fricas [F] time = 2.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba\cos\left(dx+c\right)^{2}+(A+B)a\cos\left(dx+c\right)+Aa\right)\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)*sec(d*x + c)^(
3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

maple [A] time = 1.66, size = 240, normalized size = 2.26

$$-\frac{2a\left(A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -2*a*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}\,(a+a\,\cos(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x)),x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.462 $\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{2a(3A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+$$

```
[Out] 2/3*a*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+2/3*a*(3*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d
```

Rubi [A] time = 0.18, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2960, 3996, 3787, 3771, 2639, 2641}

$$\frac{2a(3A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}a(A+B)-\frac{1}{2}a(3A+B)}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + (a(A+B))\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2aB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \left(a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{2a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{d}$$

Mathematica [C] time = 1.27, size = 148, normalized size = 1.35

$$\frac{2ae^{-idx}\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(-i(A+B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((3*A + B)*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] - I*(A + B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*
(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c
+ d*x]*((3*I)*(A + B) + B*Sin[c + d*x])))/(3*d*E^(I*d*x))
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(Ba\cos\left(dx+c\right)^2+(A+B)a\cos\left(dx+c\right)+Aa\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)*sqrt(sec(d*x +
 c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

maple [B] time = 1.37, size = 321, normalized size = 2.92

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;a\left(4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\sqrt{2\left(\sin^2\left(\frac{d}{2}\right.\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*B*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*
```

```
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
maxima")
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}\,(a+a\cos(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x)),x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sqrt{\sec(c+dx)}\,dx+\int A\cos(c+dx)\sqrt{\sec(c+dx)}\,dx+\int B\cos(c+dx)\sqrt{\sec(c+dx)}\,dx+\int B\cos^2(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
[Out] a*(Integral(A*sqrt(sec(c + d*x)), x) + Integral(A*cos(c + d*x)*sqrt(sec(c +
 d*x)), x) + Integral(B*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(B*co
s(c + d*x)**2*sqrt(sec(c + d*x)), x))
```

3.463 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(\ldots}}{5d}$$

```
[Out] 2/5*a*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*a*(A+B)*sin(d*x+c)/d/sec(d*x+c)^(
1/2)+2/5*a*(5*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(
A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(\ldots}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*d) + (2*a*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(A+B) - \frac{1}{2}a(5A+3B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,d$$

$$= \frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{5}(a(5A+3B$$

$$= \frac{2aB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(a(A+B))\int \sqrt{\sec$$

$$= \frac{2a(5A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2aB\,\mathrm{s}}{5d\,\mathrm{se}}$$

$$= \frac{2a(5A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a(A}{}$$

Mathematica [C] time = 1.60, size = 148, normalized size = 1.05

$$\frac{a\sqrt{\sec(c+dx)}\left(-2i(5A+3B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right) + \cos(c+dx)(10(A+B)\sin(c+dx) + \right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(10*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2] - (2*I)*(5*A + 3*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyper
geometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((6*I)*(5*
A + 3*B) + 10*(A + B)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)])))/(15*d)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{Ba\cos(dx+c)^2 + (A+B)a\cos(dx+c) + Aa}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)/sqrt(sec(d*x +
 c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

maple [B] time = 1.31, size = 355, normalized size = 2.52

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+44B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+44*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-10*A-16*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-15*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{A}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{A\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{B\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(A*cos(c + d*x)/sqrt(sec(c +
 d*x)), x) + Integral(B*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(B*co
s(c + d*x)**2/sqrt(sec(c + d*x)), x))
```

3.464 $\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(7A+5B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a}{}$$

```
[Out] 2/7*a*B*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*a*(A+B)*sin(d*x+c)/d/sec(d*x+c)^(
3/2)+2/21*a*(7*A+5*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*a*(A+B)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*a*(7*A+5*B)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(7A+5B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(7A+5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (6*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(5*d) + (2*a*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(21*d) + (2*a*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) +
(2*a*(A + B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(7*A + 5*B)*Sin[
c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(A+B) - \frac{1}{2}a(7A+5B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}$$
$$= \frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \frac{1}{7}(a(7A+5$$
$$= \frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(7A+5B)\sin(c}{21d\sqrt{\sec(c+dx}$$
$$= \frac{2aB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(7A+5B)\sin(c}{21d\sqrt{\sec(c+dx}$$
$$= \frac{6a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a(7A}{}$$

Mathematica [C] time = 2.18, size = 182, normalized size = 1.06

$$\frac{ae^{-idx}\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(-84i(A+B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(20*(7*A + 5*B)*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2] - (84*I)*(A + B)*E^(I*(c + d*x))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
+ Cos[c + d*x]*((252*I)*(A + B) + 5*(28*A + 23*B)*Sin[c + d*x] + 42*(A + B)
*Sin[2*(c + d*x)] + 15*B*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x))
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{Ba\cos\left(dx+c\right)^{2} + (A+B)a\cos\left(dx+c\right) + Aa}{\sec\left(dx+c\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*a*cos(d*x + c)^2 + (A + B)*a*cos(d*x + c) + A*a)/sec(d*x + c)^(
3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

maple [A] time = 1.55, size = 383, normalized size = 2.23

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-528B)\left(\sin^6\left(\frac{dx}{2}+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-528*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(308*A+448*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-11
2*A-122*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\ \cos\left(c+d\,x\right))\ (a+a\ \cos\left(c+d\,x\right))}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x)))/(1/cos(c + d*x))^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{A}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int \frac{A\cos\left(c+dx\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int \frac{B\cos\left(c+dx\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int \frac{B\cos^{2}\left(c+dx\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(A*cos(c + d*x)/sec(c + d*x
)**(3/2), x) + Integral(B*cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(B
*cos(c + d*x)**2/sec(c + d*x)**(3/2), x))
```

3.465 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=199

$$\frac{2a^2(7A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{4a^2(4A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec}}{3d}$$

```
[Out] 2/15*a^2*(7*A+5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*A*sec(d*x+c)^(3/2)*(a^
2+a^2*sec(d*x+c))*sin(d*x+c)/d+4/5*a^2*(4*A+5*B)*sin(d*x+c)*sec(d*x+c)^(1/2
)/d-4/5*a^2*(4*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a^
2*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.35, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4018, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2a^2(7A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{4a^2(4A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-4*a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^2*(4*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(5*d) + (2*a^2*(7*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d
) + (2*A*Sec[c + d*x]^(3/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2(B+A\sec(c+$$

$$= \frac{2A\sec^{\frac{3}{2}}(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{5d} +$$

$$= \frac{2a^2(7A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2A\sec^{\frac{3}{2}}(c}{}$$

$$= \frac{2a^2(7A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2A\sec^{\frac{3}{2}}(c}{}$$

$$= \frac{4a^2(4A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2a^2(7A+}{}$$

$$= \frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{3d}$$

$$= -\frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{5d}$$

Mathematica [C] time = 2.99, size = 299, normalized size = 1.50

$$\frac{a^2e^{-ic}\left(-1+e^{2ic}\right)\csc(c)(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(2(4A+5B)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}{}_2F_1(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*(-1 + E^((2*I)*c))*(1 + Cos[c + d*x])^2*Csc[c]*(10*A + 5*B - 18*A*E^(I
*(c + d*x)) - 30*B*E^(I*(c + d*x)) - 54*A*E^((3*I)*(c + d*x)) - 60*B*E^((3*
I)*(c + d*x)) - 10*A*E^((4*I)*(c + d*x)) - 5*B*E^((4*I)*(c + d*x)) - 24*A*E
^((5*I)*(c + d*x)) - 30*B*E^((5*I)*(c + d*x)) - (10*I)*(A + 2*B)*(1 + E^((2
*I)*(c + d*x)))^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(4*A + 5
*B)*E^(I*(c + d*x))*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]])/(60
*d*E^(I*c)*(1 + E^((2*I)*(c + d*x)))^2)
```

fricas [F] time = 1.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^2\cos(dx+c)^3+(A+2B)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2\right)\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x
)
```

maple [B] time = 4.39, size = 741, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/20*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c
)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/4*A+1/2*B)*(-(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(1/2*A+1/4*B)*(-1/6*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos
(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
```

```
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}(a+a\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.466 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{2a^2(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{4a^2(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+}{}$$

```
[Out] 2/3*a^2*(5*A+3*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/3*A*(a^2+a^2*sec(d*x+c))*
sin(d*x+c)*sec(d*x+c)^(1/2)/d-4*a^2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+4/3*a^2*(2*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.32, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{2a^2(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{4a^2(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
d + (4*a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a^2*(5*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*A*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{\sec(c+dx)}\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{3d}$$

$$= \frac{2a^2(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A\sqrt{\mathrm{sec}}}{}$$

$$= \frac{2a^2(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A\sqrt{\mathrm{sec}}}{}$$

$$= \frac{2a^2(5A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2A\sqrt{\mathrm{sec}}}{}$$

$$= -\frac{4a^2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

Mathematica [C] time = 2.28, size = 279, normalized size = 1.74

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}\,(3\csc(c)\cos(dx)(4A-B\cos(2c)+B)+2A\tan(c+dx)+6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((-4*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(3*A*E^(I*
c)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E^(I*d*x)*((2*
A + 3*B)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c +
 d*x))] + A*E^(I*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))])))/(E^(I*d*x)*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(3*(4*A + B
- B*Cos[2*c])*Cos[d*x]*Csc[c] + 6*B*Cos[c]*Sin[d*x] + 2*A*Tan[c + d*x])))/(
12*d)
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Ba^2\cos(dx+c)^3+(A+2B)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x
)
```

maple [B] time = 1.62, size = 513, normalized size = 3.21

$$-\frac{4\left(6\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(2A+B)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -4/3*(6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A+B)*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(7*A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2)))*sin(1/2*d*x+1/2*c)^2+2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))*a^2/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*
x+1/2*c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}(a+a\,\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.467 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{4a^2(3A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx}{3d\sqrt{}}$$

```
[Out] 2/3*B*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/3*a^2*(3*A-B)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d+4*a^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/d+4/3*a^2*(3*A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.32, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4017, 3997, 3787, 3771, 2639, 2641}

$$\frac{2a^2(3A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{4a^2(3A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx}{3d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (4*a^2*(3*A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a^2*(3*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d)
 + (2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int (a + a\cos(c+dx))^2 (A + B\cos(c+dx)) \sec^{\frac{3}{2}}(c+dx)\, dx = \int \frac{(a + a\sec(c+dx))^2 (B + A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\, dx$$

$$= \frac{2B\left(a^2 + a^2\sec(c+dx)\right)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a + a\,\mathrm{s}}{}$$

$$= \frac{2a^2(3A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2B\left(a^2 + a^2\right.}{3}$$

$$= \frac{2a^2(3A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2B\left(a^2 + a^2\right.}{3}$$

$$= \frac{2a^2(3A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2B\left(a^2 + a^2\right.}{3}$$

$$= \frac{4a^2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} +$$

Mathematica [C] time = 1.87, size = 302, normalized size = 1.89

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}\,(6(A+2B)\cos(c)\sin(dx) - 3\csc(c)\cos(dx)((A+2B)\cos(2c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*Sqrt[E^(I*(c +
 d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(3*B*E^(I*c
)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E^(I*d*x)*(-((3
*A + 2*B)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c
+ d*x))]) + B*E^(I*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))])))/(E^(I*d*x)*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(-3*(-A +
2*B + (A + 2*B)*Cos[2*c])*Cos[d*x]*Csc[c] + B*Cos[2*d*x]*Sin[2*c] + 6*(A +
2*B)*Cos[c]*Sin[d*x] + B*Cos[2*c]*Sin[2*d*x])))/(12*d)
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Ba^2\cos(dx+c)^3 + (A+2B)a^2\cos(dx+c)^2 + (2A+B)a^2\cos(dx+c) + Aa^2\right)\sec(dx+c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)*sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

maple [A] time = 1.50, size = 388, normalized size = 2.42

$$-\frac{4a^2\left(2B\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -4/3*a^2*(2*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*(3*A+B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-3*B
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.468 $\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=166

$$\frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}}{}$$

```
[Out] 2/5*B*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/15*a^2*(5*A+7*B)
*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4/5*a^2*(5*A+4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+4/3*a^2*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d
```

Rubi [A] time = 0.34, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4017, 3996, 3787, 3771, 2639, 2641}

$$\frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a^2*(5*A + 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*(5*A + 7*B)*Sin[c + d*x])/(15*d*Sqrt
[Sec[c + d*x]]) + (2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c +
d*x]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2B\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a}{}$$

$$= \frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2B\left(a^2+a^2\sec(c+d\right.}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2B\left(a^2+a^2\sec(c+d\right.}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2a^2(5A+7B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2B\left(a^2+a^2\sec(c+d\right.}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{5d}$$

Mathematica [C] time = 1.63, size = 153, normalized size = 0.92

$$\frac{a^2\sqrt{\sec(c+dx)}\left(-4i(5A+4B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(10(A+2B)\sin(c+dx\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(20*(2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2] - (4*I)*(5*A + 4*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((60*I
)*A + (48*I)*B + 10*(A + 2*B)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)])))/(15*d)
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^2\cos(dx+c)^3+(A+2B)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

maple [A] time = 1.24, size = 357, normalized size = 2.15

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-12B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(10A+32B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(10*A+32*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-5*A-13*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-12*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2,x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sqrt{\sec\left(c+dx\right)}\,dx+\int 2A\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}\,dx+\int A\cos^2\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}\,dx+\int B\,\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
[Out] a**2*(Integral(A*sqrt(sec(c + d*x)), x) + Integral(2*A*cos(c + d*x)*sqrt(se
c(c + d*x)), x) + Integral(A*cos(c + d*x)**2*sqrt(sec(c + d*x)), x) + Integ
ral(B*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(2*B*cos(c + d*x)**2*sq
rt(sec(c + d*x)), x) + Integral(B*cos(c + d*x)**3*sqrt(sec(c + d*x)), x))
```

3.469 $\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=201

$$\frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(7A+6B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(7A+6B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

```
[Out] 2/35*a^2*(7*A+9*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*B*(a^2+a^2*sec(d*x+c))
*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/21*a^2*(7*A+6*B)*sin(d*x+c)/d/sec(d*x+c)^(
1/2)+4/5*a^2*(4*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*
a^2*(7*A+6*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.37, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4017, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(7A+6B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(7A+6B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a^2*(4*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(7*A + 6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(7*A + 9*B)*Sin[c + d*x])/(35*d*S
ec[c + d*x]^(3/2)) + (4*a^2*(7*A + 6*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*
x]]) + (2*B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx=\int \frac{(a+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$=\frac{2B\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2}{7}\int \frac{(a+a\sec(c+dx))\Big(}{}$$

$$=\frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2B\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$=\frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2B\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$=\frac{2a^2(7A+9B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(7A+6B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2B\left(a^2\right.}{}$$

$$=\frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a^2}{}$$

$$=\frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2}{}$$

Mathematica [C] time = 2.27, size = 193, normalized size = 0.96

$$\frac{a^2e^{-idx}\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(-56i(4A+3B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(7*A + 6*B)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(4*A + 3*B)*E^(I*(c + d*x))*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))] + Cos[c + d*x]*((672*I)*A + (504*I)*B + 5*(56*A + 51*B)*Sin[c + d*x]
+ 42*(A + 2*B)*Sin[2*(c + d*x)] + 15*B*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x)
)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^2\cos(dx+c)^3+(A+2B)a^2\cos(dx+c)^2+(2A+B)a^2\cos(dx+c)+Aa^2}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^2*cos(d*x + c)^3 + (A + 2*B)*a^2*cos(d*x + c)^2 + (2*A + B)*a
^2*cos(d*x + c) + A*a^2)/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

maple [A] time = 1.28, size = 385, normalized size = 1.92

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-84A-348B)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-348*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(224*A+378*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-9
1*A-117*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-84*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+30*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin
```

```
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+a\,\cos(c+d\,x)\right)^2}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{A}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{2A\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{A\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{2B\cos^2(c+}{\sqrt{\sec(c+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(2*A*cos(c + d*x)/sqrt(se
c(c + d*x)), x) + Integral(A*cos(c + d*x)**2/sqrt(sec(c + d*x)), x) + Integ
ral(B*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(2*B*cos(c + d*x)**2/sq
rt(sec(c + d*x)), x) + Integral(B*cos(c + d*x)**3/sqrt(sec(c + d*x)), x))
```

3.470 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=244

$$\frac{4a^3(41A+42B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(11A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d}+\frac{4a^3(7A$$

```
[Out] 4/105*a^3*(41*A+42*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a*A*sec(d*x+c)^(3/2
)*(a+a*sec(d*x+c))^2*sin(d*x+c)/d+2/35*(11*A+7*B)*sec(d*x+c)^(3/2)*(a^3+a^3
*sec(d*x+c))*sin(d*x+c)/d+4/5*a^3*(7*A+9*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4
/5*a^3*(7*A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*a^3*(1
3*A+21*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.51, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2960, 4018, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{4a^3(41A+42B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(11A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d}+\frac{4a^3(7A$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (-4*a^3*(7*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(13*A + 21*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(7*A + 9*B)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(5*d) + (4*a^3*(41*A + 42*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(105*d) + (2*a*A*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/
(7*d) + (2*(11*A + 7*B)*Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c +
 d*x])/(35*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
```

```
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3(B+A\sec(c+$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{7d}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{7d}$$

$$= \frac{4a^3(41A+42B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2aA\,\mathrm{s}}{}$$

$$= \frac{4a^3(41A+42B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \frac{2aA\,\mathrm{s}}{}$$

$$= \frac{4a^3(7A+9B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{4a^3(41A}{}$$

$$= \frac{4a^3(13A+21B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{21d}$$

$$= -\frac{4a^3(7A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{sec}}}{5d}$$

Mathematica [C] time = 4.16, size = 435, normalized size = 1.78

$$\frac{a^3\csc(c)e^{-idx}(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(7\sqrt{2}\left(-1+e^{2ic}\right)(7A+9B)e^{2idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Csc[c]*Sec[(c + d*x)/2]^6*(7*Sqrt[2]*(7*A + 9*B)*
E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*
```

```
x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))] - ((-1 + E^((2*I)*c))*(21*B*(-5 + 16*E^(I*(c + d*x)) - 5*E^
((2*I)*(c + d*x)) + 54*E^((3*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 56*E^(
(5*I)*(c + d*x)) + 5*E^((6*I)*(c + d*x)) + 18*E^((7*I)*(c + d*x))) + 2*A*(-
65 + 84*E^(I*(c + d*x)) - 95*E^((2*I)*(c + d*x)) + 441*E^((3*I)*(c + d*x))
+ 95*E^((4*I)*(c + d*x)) + 504*E^((5*I)*(c + d*x)) + 65*E^((6*I)*(c + d*x))
 + 147*E^((7*I)*(c + d*x))) + (10*I)*(13*A + 21*B)*(1 + E^((2*I)*(c + d*x))
)^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[Sec[c + d*x]])/(2*E^
(I*(c - d*x))*(1 + E^((2*I)*(c + d*x)))^3)))/(420*d*E^(I*d*x))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^3\cos\left(dx+c\right)^4 + (A+3\,B)a^3\cos\left(dx+c\right)^3 + 3\,(A+B)a^3\cos\left(dx+c\right)^2 + (3\,A+B)a^3\cos\left(dx+c\right) + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(9/2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)^3\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x
)
```

maple [B] time = 5.51, size = 929, normalized size = 3.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(1/8*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
```

```
)-1/5*(3/8*A+1/8*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/
2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/8*A+3/8*B)*(-(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+(3/8*A+3/8*B)*
(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/8*A*(-1/56*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c
)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}(a+a\cos(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.471 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=211

$$\frac{4a^3(21A+20B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\,\left(a^3\sec(c+dx)+a^3\right)}{15d}+\frac{4a^3(3A$$

```
[Out] 4/15*a^3*(21*A+20*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/5*a*A*(a+a*sec(d*x+c))
^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/15*(9*A+5*B)*(a^3+a^3*sec(d*x+c))*sin(d*
x+c)*sec(d*x+c)^(1/2)/d-4/5*a^3*(9*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d+4/3*a^3*(3*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A] time = 0.49, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(21A+20B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(9A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}\,\left(a^3\sec(c+dx)+a^3\right)}{15d}+\frac{4a^3(3A$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-4*a^3*(9*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(21*A + 20*B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(15*d) + (2*a*A*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(5*d) + (2*(9*A + 5*B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x
])*Sin[c + d*x])/(15*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2\sin(c+dx)}{5d}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2\sin(c+dx)}{5d}$$

$$= \frac{4a^3(21A+20B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d} + \frac{2aA\sqrt{}}{}$$

$$= \frac{4a^3(21A+20B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d} + \frac{2aA\sqrt{}}{}$$

$$= \frac{4a^3(21A+20B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d} + \frac{2aA\sqrt{}}{}$$

$$= -\frac{4a^3(9A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{5d}$$

Mathematica [C] time = 3.17, size = 268, normalized size = 1.27

$$\frac{a^3\csc(c)\sec(c)e^{-idx}\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(2\left(-1+e^{4ic}\right)(9A+5B)e^{-i(c-dx)}\sqrt{1+e^{2i(c+dx)}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^3*Csc[c]*Sec[c]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((2*(9*A + 5*
B)*(-1 + E^((4*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c - d*x)) + (Sec[c + d*x]^2*Sin[2*c]
*((-18*I)*(9*A + 5*B)*Cos[c + d*x] - (54*I)*A*Cos[3*(c + d*x)] - (30*I)*B*C
os[3*(c + d*x)] + 40*(3*A + 5*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2,
2] + 66*A*Sin[c + d*x] + 45*B*Sin[c + d*x] + 30*A*Sin[2*(c + d*x)] + 10*B*S
in[2*(c + d*x)] + 54*A*Sin[3*(c + d*x)] + 45*B*Sin[3*(c + d*x)]))/2))/(30*d
*E^(I*d*x))
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^3\cos(dx+c)^4 + (A+3B)a^3\cos(dx+c)^3 + 3(A+B)a^3\cos(dx+c)^2 + (3A+B)a^3\cos(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(7/2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x
)
```

maple [B] time = 4.27, size = 916, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] 4/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^3*(60*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+108*A*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-216*A*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^6+100*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+60*B*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-180*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^6-60*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-108*A*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+246*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4-100*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-60*B*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x
```

```
+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+190*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^4+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-72*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-50*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}\,(a+a\,\cos(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.472 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=199

$$\frac{4a^3(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}\,\left(a^3\sec(c+dx)+a^3\right)}{3d}+\frac{20a^3(A+B)\sqrt{c}}{}$$

```
[Out] 2/3*a*B*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4/3*a^3*(4*A+B)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d+2/3*(A-B)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)*sec(d
*x+c)^(1/2)/d-4*a^3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+20
/3*a^3*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.49, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4017, 4018, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(4A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}\,\left(a^3\sec(c+dx)+a^3\right)}{3d}+\frac{20a^3(A+B)\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (20*a^3*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (4*a^3*(4*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
3*d) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
 + (2*(A - B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(3*
d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
```

```
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a+a\,\mathrm{se}}{}$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2(A-B)\sqrt{\mathrm{se}}}{}$$

$$= \frac{4a^3(4A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aB(a+a}{3}$$

$$= \frac{4a^3(4A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aB(a+a}{3}$$

$$= \frac{4a^3(4A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aB(a+a}{3}$$

$$= -\frac{4a^3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{d}$$

Mathematica [C] time = 1.94, size = 202, normalized size = 1.02

$$\frac{a^3e^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(4i(A-B)\left(1+e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+40(A+B)\cos^{\frac{3}{2}}(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^3*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((-12*I)*A + (12*I)*B - (12
*I)*A*Cos[2*(c + d*x)] + (12*I)*B*Cos[2*(c + d*x)] + 40*(A + B)*Cos[c + d*x
]^(3/2)*EllipticF[(c + d*x)/2, 2] + (4*I)*(A - B)*(1 + E^((2*I)*(c + d*x)))
^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 4*A*Sin[c +
 d*x] + B*Sin[c + d*x] + 18*A*Sin[2*(c + d*x)] + 6*B*Sin[2*(c + d*x)] + B*S
in[3*(c + d*x)]))/(6*d*E^(I*d*x))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^3\cos\left(dx+c\right)^4+(A+3B)a^3\cos\left(dx+c\right)^3+3(A+B)a^3\cos\left(dx+c\right)^2+(3A+B)a^3\cos\left(dx+c\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(5/2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)^3\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x
)
```

maple [B] time = 1.60, size = 654, normalized size = 3.29

$$-\frac{4\left(-4B\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -4/3*(-4*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(9*A+5*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A+2*B)*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
```

```
*sin(1/2*d*x+1/2*c)^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-3*B*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*a^3/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d
*x+1/2*c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}\,{(a+a\,\cos(c+d\,x))}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.473 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=211

$$\frac{4a^3(5A-6B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(5A+9B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^3(5A+3B)\sqrt{\cos(c+}}{}$$

```
[Out] 2/5*a*B*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/15*(5*A+9*B)*(a^
3+a^3*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4/15*a^3*(5*A-6*B)*sin(d*x+
c)*sec(d*x+c)^(1/2)/d+4/5*a^3*(5*A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+4/3*a^3*(5*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d
```

Rubi [A] time = 0.49, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4017, 3997, 3787, 3771, 2639, 2641}

$$\frac{4a^3(5A-6B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(5A+9B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^3(5A+3B)\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^3*(5*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(5*A - 6*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c
+ d*x]^(3/2)) + (2*(5*A + 9*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d
*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int (a + a\cos(c + dx))^3(A + B\cos(c + dx))\sec^{\frac{3}{2}}(c + dx)\,dx = \int \frac{(a + a\sec(c + dx))^3(B + A\sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)}\,dx$$

$$= \frac{2aB(a + a\sec(c + dx))^2\sin(c + dx)}{5d\sec^{\frac{3}{2}}(c + dx)} + \frac{2}{5}\int \frac{(a + a\,s}{}$$

$$= \frac{2aB(a + a\sec(c + dx))^2\sin(c + dx)}{5d\sec^{\frac{3}{2}}(c + dx)} + \frac{2(5A + 9B)}{}$$

$$= \frac{4a^3(5A - 6B)\sqrt{\sec(c + dx)}\,\sin(c + dx)}{15d} + \frac{2aB(a +}{}$$

$$= \frac{4a^3(5A - 6B)\sqrt{\sec(c + dx)}\,\sin(c + dx)}{15d} + \frac{2aB(a +}{}$$

$$= \frac{4a^3(5A - 6B)\sqrt{\sec(c + dx)}\,\sin(c + dx)}{15d} + \frac{2aB(a +}{}$$

$$= \frac{4a^3(5A + 9B)\sqrt{\cos(c + dx)}\,E\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{\sec(}}{5d}$$

Mathematica [C] time = 1.64, size = 207, normalized size = 0.98

$$\frac{a^3e^{-idx}\sqrt{\sec(c + dx)}\,(\cos(dx) + i\sin(dx))\left(-8i(5A + 9B)e^{i(c+dx)}\sqrt{1 + e^{2i(c+dx)}}\ {}_2F_1\left(\frac{1}{2}, \frac{3}{4}; \frac{7}{4}; -e^{2i(c+dx)}\right) + 40(5A +\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((120*I)*A*Cos[c + d*x] + (
216*I)*B*Cos[c + d*x] + 40*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2] - (8*I)*(5*A + 9*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*
Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 60*A*Sin[c + d*x]
+ 3*B*Sin[c + d*x] + 10*A*Sin[2*(c + d*x)] + 30*B*Sin[2*(c + d*x)] + 3*B*Si
n[3*(c + d*x)]))/(30*d*E^(I*d*x))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^3\cos(dx + c)^4 + (A + 3B)a^3\cos(dx + c)^3 + 3(A + B)a^3\cos(dx + c)^2 + (3A + B)a^3\cos(dx + c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)*sec(d*x + c)^(3/2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

maple [B] time = 1.70, size = 519, normalized size = 2.46

$$-\frac{4a^3\left(-12B\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -4/15*a^3*(-12*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(5*A+21*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*A+9*B)*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+25*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)-15*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-27*B*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*
```

```
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos{(dx+c)}+A)(a\cos{(dx+c)}+a)^3\sec{(dx+c)}^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos{(c+d\,x)})\,{\left(\frac{1}{\cos{(c+d\,x)}}\right)}^{3/2}\,{(a+a\,\cos{(c+d\,x)})}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.474 $\int (a+a\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=211

$$\frac{2(7A+11B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^3(21A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec}}{21d}$$

```
[Out] 2/7*a*B*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/35*(7*A+11*B)*(a
^3+a^3*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/105*a^3*(42*A+41*B)*sin(
d*x+c)/d/sec(d*x+c)^(1/2)+4/5*a^3*(9*A+7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+4/21*a^3*(21*A+13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d
```

Rubi [A] time = 0.51, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4017, 3996, 3787, 3771, 2639, 2641}

$$\frac{2(7A+11B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^3(21A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a^3*(9*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(21*A + 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(42*A + 41*B)*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*
Sec[c + d*x]^(5/2)) + (2*(7*A + 11*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x]
)/(35*d*Sec[c + d*x]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int (a + a\cos(c+dx))^3 (A + B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\,\mathrm{s}}{}$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(7A+11B)}{}$$

$$= \frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aB(a+a\sec(c+dx}{7d\sec^{\frac{5}{2}}(c}$$

$$= \frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aB(a+a\sec(c+dx}{7d\sec^{\frac{5}{2}}(c}$$

$$= \frac{4a^3(42A+41B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aB(a+a\sec(c+dx}{7d\sec^{\frac{5}{2}}(c}$$

$$= \frac{4a^3(9A+7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}{5d}$$

Mathematica [C] time = 2.41, size = 194, normalized size = 0.92

$$\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(-56i(9A+7B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(21*A + 13*B)*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(9*A + 7*B)*E^(I*(c + d*x))*Sq
rt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))] + Cos[c + d*x]*((168*I)*(9*A + 7*B) + 5*(84*A + 107*B)*Sin[c + d*x]
 + 42*(A + 3*B)*Sin[2*(c + d*x)] + 15*B*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x
))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^3\cos(dx+c)^4 + (A+3B)a^3\cos(dx+c)^3 + 3(A+B)a^3\cos(dx+c)^2 + (3A+B)a^3\cos(dx+c) + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)*sqrt(sec(d*x + c)),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

maple [A] time = 1.44, size = 385, normalized size = 1.82

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(120B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-84A-432B)\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-84*A-432*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(294*A+602*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
26*A-208*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+65*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos\left(c+d\,x\right))\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\sqrt{\sec\left(c+dx\right)}\,dx+\int 3A\cos\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}\,dx+\int 3A\cos^2\left(c+dx\right)\sqrt{\sec\left(c+dx\right)}\,dx+\int A\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] a**3*(Integral(A*sqrt(sec(c + d*x)), x) + Integral(3*A*cos(c + d*x)*sqrt(se
c(c + d*x)), x) + Integral(3*A*cos(c + d*x)**2*sqrt(sec(c + d*x)), x) + Int
egral(A*cos(c + d*x)**3*sqrt(sec(c + d*x)), x) + Integral(B*cos(c + d*x)*sq
rt(sec(c + d*x)), x) + Integral(3*B*cos(c + d*x)**2*sqrt(sec(c + d*x)), x)
+ Integral(3*B*cos(c + d*x)**3*sqrt(sec(c + d*x)), x) + Integral(B*cos(c +
d*x)**4*sqrt(sec(c + d*x)), x))
```

3.475 $\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=244

$$\frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(9A+13B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(13A+11B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^3(}{}$$

```
[Out] 4/105*a^3*(24*A+23*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/9*a*B*(a+a*sec(d*x+c)
)^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2/63*(9*A+13*B)*(a^3+a^3*sec(d*x+c))*sin(
d*x+c)/d/sec(d*x+c)^(5/2)+4/21*a^3*(13*A+11*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2
)+4/15*a^3*(21*A+17*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*
a^3*(13*A+11*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.54, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4017, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(9A+13B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(13A+11B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a^3*(21*A + 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(13*A + 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(24*A + 23*B)*Sin[c + d*x])/
(105*d*Sec[c + d*x]^(3/2)) + (4*a^3*(13*A + 11*B)*Sin[c + d*x])/(21*d*Sqrt[
Sec[c + d*x]]) + (2*a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d
*x]^(7/2)) + (2*(9*A + 13*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(63*d*S
ec[c + d*x]^(5/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4017

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+a\sec(c+dx))^2}{}$$

$$= \frac{2aB(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(9A+13B)\left(a^3+a^3\,\mathrm{se}\right.}{63d\sec^{\frac{5}{2}}(}$$

$$= \frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aB(a+a\sec(c+dx))^2\sin(c+}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aB(a+a\sec(c+dx))^2\sin(c+}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{4a^3(24A+23B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^3(13A+11B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2}{}$$

$$= \frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} +$$

$$= \frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} +$$

Mathematica [C] time = 2.75, size = 196, normalized size = 0.80

$$\frac{a^3\sqrt{\sec(c+dx)}\left(-112i(21A+17B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(30(107A+97B)\,\mathrm{s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(240*(13*A + 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2] - (112*I)*(21*A + 17*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*
```

```
x]*((7056*I)*A + (5712*I)*B + 30*(107*A + 97*B)*Sin[c + d*x] + 14*(54*A + 7
3*B)*Sin[2*(c + d*x)] + 90*A*Sin[3*(c + d*x)] + 270*B*Sin[3*(c + d*x)] + 35
*B*Sin[4*(c + d*x)])))/(1260*d)
```

fricas [F] time = 2.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Ba^3\cos(dx+c)^4 + (A+3B)a^3\cos(dx+c)^3 + 3(A+B)a^3\cos(dx+c)^2 + (3A+B)a^3\cos(dx+c) + A}{\sqrt{\sec(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(d*x + c)^4 + (A + 3*B)*a^3*cos(d*x + c)^3 + 3*(A + B)*a
^3*cos(d*x + c)^2 + (3*A + B)*a^3*cos(d*x + c) + A*a^3)/sqrt(sec(d*x + c)),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

maple [A] time = 1.39, size = 413, normalized size = 1.69

$$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(-560B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(360A+2200B)\left(\sin^8\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(360*A+2200*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-1296*A-3412*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1806*A+2702*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-624*A-738*B)*s
```

```
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+195*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-441*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))+165*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(a\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,(a+a\,\cos(c+d\,x))^3}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{A}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{3A\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{3A\cos^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{A\cos^3(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{B\cos(}{\sqrt{\sec(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a**3*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(3*A*cos(c + d*x)/sqrt(se
c(c + d*x)), x) + Integral(3*A*cos(c + d*x)**2/sqrt(sec(c + d*x)), x) + Int
egral(A*cos(c + d*x)**3/sqrt(sec(c + d*x)), x) + Integral(B*cos(c + d*x)/sq
```

```
rt(sec(c + d*x)), x) + Integral(3*B*cos(c + d*x)**2/sqrt(sec(c + d*x)), x)
+ Integral(3*B*cos(c + d*x)**3/sqrt(sec(c + d*x)), x) + Integral(B*cos(c +
d*x)**4/sqrt(sec(c + d*x)), x))
```

3.476 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=193

$$-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(5A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{(5$$

```
[Out] 1/3*(5*A-3*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-(A-B)*sec(d*x+c)^(5/2)*sin(d*
x+c)/d/(a+a*sec(d*x+c))-3*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d+3*(A-B)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+1/3*(5*A-3*B)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.30, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 3787, 3768, 3771, 2639, 2641}

$$-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(5A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{(5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) + ((5*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*a*d) - (3*(A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (
(5*A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) - ((A - B)*Sec[c + d*x
]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}a(A-B)+\frac{1}{2}\right.}{a^2}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{(5A-3B)\int \sec^{\frac{5}{2}}(c+dx)\,dx}{2a} -$$

$$= -\frac{3(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} + \frac{(5A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{3ad}$$

$$= -\frac{3(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} + \frac{(5A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{3ad}$$

$$= \frac{3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(5A-3B)\sqrt{c}}{}$$

Mathematica [C] time = 7.32, size = 650, normalized size = 3.37

$$\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{2\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{d} - \frac{3(A-B)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos(dx)}{d} + \frac{2\tan\left(\frac{c}{2}\right)\sec(c)(5A\cos(c)+}{3d}\right.}{a\cos(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] -((A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E
^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])*Sec[c/2])/(Sqrt[2]*d*E^(I*d*x)*(a + a*Cos[c + d*x]))) + (B*Sqr
t[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*
Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*
d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])*Sec[c/2])/(Sqrt[2]*d*E^(I*d*x)*(a + a*Cos[c + d*x])) + (5*A*Cos[c/2 +
(d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*S
qrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])) - (B*Cos[c/2 + (d*x)/2
]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec
[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Se
c[c + d*x]]*((-3*(A - B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (2*Sec[c/2]*Sec[c/
```

```
2 + (d*x)/2]*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d + (4*A*Sec[c]*Sec[c + d*x
]*Sin[d*x])/(3*d) + (2*(2*A + 5*A*Cos[c] - 3*B*Cos[c])*Sec[c]*Tan[c/2])/(3*
d)))/(a + a*Cos[c + d*x])
```

fricas [F] time = 1.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

maple [B] time = 4.06, size = 493, normalized size = 2.55

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{(-2A+2B)\left(-\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\cdots}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)^2(\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((-2*A+2*B)*(-
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*A
```

```
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(A-B)*(cos(1/2*d*x+1/2*c)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x)),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.477 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=159

$$-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(3A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}\right.}{ad}$$

```
[Out] -(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))+(3*A-B)*sin(d*x+c)*se
c(d*x+c)^(1/2)/a/d-(3*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
-(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.28, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{(3A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(((3*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) - ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a*d) + ((3*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - ((A
- B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sqrt{\sec(c+dx)}\left(-\frac{1}{2}a(A-B)+\frac{1}{2}\right.}{a^2}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-B)\int\sqrt{\sec(c+dx)}\,dx}{2a} + \frac{(3A}{}$$

$$= \frac{(3A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(3A-B)\sqrt{\sec}}{}$$

$$= -\frac{(3A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A-B)\sqrt{\cos}}{}$$

Mathematica [C] time = 4.38, size = 400, normalized size = 2.52

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(6\sqrt{\sec(c+dx)}\left(2(B-A)\tan\left(\frac{1}{2}(c+dx)\right)+2(3A-B)\csc(c)\cos(dx)\right)+6\sqrt{2}\,A\csc(c)e^{-idx}\sqrt{\frac{e}{1+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))]))/E^(I*d*x) - (2*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((
2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I
)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4
, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - 12*A*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 12*B*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 6*Sqrt[Sec[c + d*x]]*(2*(3*A - B)*Cos[d*
x]*Csc[c] + 2*(-A + B)*Tan[(c + d*x)/2])))/(6*a*d*(1 + Cos[c + d*x]))
```

fricas [F] time = 2.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

maple [A] time = 3.36, size = 319, normalized size = 2.01

$$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\cdots\right.}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(cos(1/2*d*x+1
/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A-B)*sin(1/2*d*x+1/2*c)^4+(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A-B)*sin(1/2*d*x+1/2*c)^2
)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/cos(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{a+a\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x)),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.478 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=123

$$-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sqrt{\cos(c+dx)}}{}$$

```
[Out] -(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))+(A-B)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a/d
```

Rubi [A] time = 0.24, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4019, 3787, 3771, 2639, 2641}

$$-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a*d) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(a*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d
*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}\,(B+A\sec(c+dx))}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{1}{2}a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{(A-B)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{(A+$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\left((A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c-}{2a}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A+B)\sqrt{\cos(}{}$$

Mathematica [C] time = 1.12, size = 200, normalized size = 1.63

$$-\frac{\left(-1+e^{2ic}\right)e^{-\frac{1}{2}i(4c+dx)}\left(\csc\left(\frac{c}{2}\right)+i\sec\left(\frac{c}{2}\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left((A-B)\left(e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i}}\right.\right.}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x]),x]
```

```
[Out] -1/24*((-1 + E^((2*I)*c))*((3*I)*(A + B)*(1 + E^(I*(c + d*x)))*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2] + (A - B)*(-3*(1 + E^((2*I)*(c + d*x))) +
E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))*(Csc[c/2] + I*Sec[c/2])*S
ec[(c + d*x)/2]*Sqrt[Sec[c + d*x]])/(a*d*E^((I/2)*(4*c + d*x)))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

maple [A] time = 1.38, size = 243, normalized size = 1.98

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(A\,\mathrm{EllipticF}\left(\right.\right.\right.}{a\cos\left(\frac{dx}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{a+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x)),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\sec(c+dx)}}{\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sqrt{\sec(c+dx)}}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x) + Integral(B*cos(c +
d*x)*sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x))/a
```

3.479 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=125

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)\sqrt{\cos(c+dx)}\,\sqrt{}}{}$$

```
[Out] (A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))-(A-3*B)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/a/d
```

Rubi [A] time = 0.25, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4020, 3787, 3771, 2639, 2641}

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)\sqrt{\cos(c+dx)}\,\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -(((A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*d)) + ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[
c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
```

```
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx &= \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx \\
&= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \frac{-\frac{1}{2}a(A-3B)+\frac{1}{2}a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2} \\
&= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{(A-3B)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a}+\frac{(A-
\\
&= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\left((A-3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+
}{2a} \\
&= -\frac{(A-3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{(A-B)\sqrt{\co
\end{aligned}
$$

Mathematica [C] time = 2.59, size = 422, normalized size = 3.38

$$\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{6\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left((A-2B)\cos\left(\frac{1}{2}(c-dx)\right)-B\cos\left(\frac{1}{2}(3c+dx)\right)\right)}{\sqrt{\sec(c+dx)}}+2\sqrt{2}A\csc(c)e^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))]))/E^(I*d*x) - (6*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((
2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I
)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4
, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + (6*((A - 2*B)*Cos[(c - d*x)/2] -
 B*Cos[(3*c + d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/Sqrt[Sec[c + d*x
]] + 12*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] -
 12*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]))/(6*
a*d*(1 + Cos[c + d*x]))
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

maple [A] time = 1.48, size = 244, normalized size = 1.95

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(A\,\mathrm{EllipticF}\right.\right.}{a\cos\left(\frac{dx}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+a\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx+\int \frac{B\cos(c+dx)}{\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x) + In
tegral(B*cos(c + d*x)/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))
), x))/a
```

3.480 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$-\frac{(3A-5B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}+\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)}-\frac{(3A-5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right.}{3ad}$$

```
[Out] -1/3*(3*A-5*B)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)+(A-B)*sin(d*x+c)/d/(a+a*sec(
d*x+c))/sec(d*x+c)^(1/2)+3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/a/d-1/3*(3*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.28, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4020, 3787, 3769, 3771, 2641, 2639}

$$-\frac{(3A-5B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}+\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)}-\frac{(3A-5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) - ((3*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*a*d) - ((3*A - 5*B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
 + ((A - B)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(3A-5B)+\frac{3}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} - \frac{(3A-5B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{(3(A}{}$$

$$= -\frac{(3A-5B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} - \frac{(3A-}{}$$

$$= \frac{3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(3A-5B)\sin}{3ad\sqrt{\sec(c}}$$

$$= \frac{3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(3A-5B)\sqrt{\cos}}{}$$

Mathematica [C] time = 4.88, size = 444, normalized size = 2.72

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left((12A-13B)\cos\left(\frac{1}{2}(c-dx)\right)+(6A-5B)\cos\left(\frac{1}{2}(3c+dx)\right)-2B\sin(c)\sin\left(\frac{3}{2}(c+dx)\right)\right)}{\sqrt{\sec(c+dx)}} - 6\sqrt{2}\,A\,\mathrm{cs}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((-6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))]))/E^(I*d*x) + (6*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*
I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - 12*A*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 20*B*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - (Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]*((1
2*A - 13*B)*Cos[(c - d*x)/2] + (6*A - 5*B)*Cos[(3*c + d*x)/2] - 2*B*Sin[c]*
Sin[(3*(c + d*x))/2]))/Sqrt[Sec[c + d*x]]))/(6*a*d*(1 + Cos[c + d*x]))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

maple [A] time = 1.40, size = 262, normalized size = 1.61

$$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\;\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\;\left(3A\,\mathrm{EllipticF}\left(c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/
2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*A*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))+8*B*sin(1/2*d*x+1/2*c)^6+(6*A-18*B)*sin(1/2*d*x+1/2*c)^4+(-3*A+
7*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+a\,\cos(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{B\cos(c+dx)}{\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] (Integral(A/(cos(c + d*x)*sec(c + d*x)**(3/2) + sec(c + d*x)**(3/2)), x) +
Integral(B*cos(c + d*x)/(cos(c + d*x)*sec(c + d*x)**(3/2) + sec(c + d*x)**(
3/2)), x))/a
```

3.481 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=196

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{(5A-7B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}+\frac{5(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}}{3ad}$$

```
[Out] -1/5*(5*A-7*B)*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)+(A-B)*sin(d*x+c)/d/sec(d*x+c
)^(3/2)/(a+a*sec(d*x+c))+5/3*(A-B)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-3/5*(5*A
-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+5/3*(A-B)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.30, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4020, 3787, 3769, 3771, 2639, 2641}

$$\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{(5A-7B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}+\frac{5(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (-3*(5*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(5*a*d) + (5*(A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a*d) - ((5*A - 7*B)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(
3/2)) + (5*(A - B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) + ((A - B)*Sin[
c + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{B+A\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$=\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}+\frac{\int \frac{-\frac{1}{2}a(5A-7B)+\frac{5}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$=\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}-\frac{(5A-7B)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{2a}+\frac{(5(A-$$

$$=-\frac{(5A-7B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}+\frac{(A-B)\sin(c}{d\sec^{\frac{3}{2}}(c+dx)(a+a\,s}$$

$$=-\frac{(5A-7B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}+\frac{(A-B)\sin(c}{d\sec^{\frac{3}{2}}(c+dx)(a+a\,s}$$

$$=-\frac{3(5A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad}+\frac{5(A-B)\sqrt{c}}{}$$

Mathematica [C] time = 3.25, size = 518, normalized size = 2.64

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}\left(40(A-B)\sin(2c)\cos(2dx)-12(20A-33B)\cos(c)\sin(dx)+40(A-B)\cos(2c)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((60*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))]))/E^(I*d*x) - (84*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^
((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2
*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3
/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + 200*A*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 200*B*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + Sqrt[Sec[c + d*x]]*(3*(40*A - 51*B +
 (20*A - 33*B)*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2] + 40*(A - B)*Cos[2*d*x]
```

```
*Sin[2*c] + 12*B*Cos[3*d*x]*Sin[3*c] - 120*(A - B)*Sec[c/2]*Sec[(c + d*x)/2
]*Sin[(d*x)/2] - 12*(20*A - 33*B)*Cos[c]*Sin[d*x] + 40*(A - B)*Cos[2*c]*Sin
[2*d*x] + 12*B*Cos[3*c]*Sin[3*d*x] - 120*(A - B)*Tan[c/2])))/(60*a*d*(1 + C
os[c + d*x]))
```

fricas [F] time = 2.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

maple [A] time = 1.58, size = 281, normalized size = 1.43

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(25A\,\mathrm{Elliptic}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+
1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*A*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+45*A*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*EllipticE(cos(1/2*d*x
```

```
+1/2*c),2^(1/2)))+48*B*sin(1/2*d*x+1/2*c)^8+(-40*A-56*B)*sin(1/2*d*x+1/2*c)
^6+(90*A-30*B)*sin(1/2*d*x+1/2*c)^4+(-35*A+23*B)*sin(1/2*d*x+1/2*c)^2)/a/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\cos(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.482 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=208

$$-\frac{(5A-2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(4A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(5A-2B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}}{3a^2d}$$

```
[Out] -1/3*(5*A-2*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*(A-B)*s
ec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+(4*A-B)*sin(d*x+c)*sec(d*x+
c)^(1/2)/a^2/d-(4*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d-1
/3*(5*A-2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.43, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{(5A-2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(4A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(5A-2B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -(((4*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) - ((5*A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a^2*d) + ((4*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a
^2*d) - ((5*A - 2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c +
 d*x])) - ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x
])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}a(A-B)+\frac{1}{2}a(7A-B)\sec\right.}{a+a\sec(c+dx)}}{3a^2}$$

$$= -\frac{(5A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{(5A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{3d(a+a\sec(c+dx))^2}$$

$$= \frac{(4A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} - \frac{(5A-2B)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3a^2d(1+\sec(c+dx))}$$

$$= -\frac{(5A-2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{(4A-B)\sqrt{}}{}$$

$$= -\frac{(4A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{(5A-2B)\sqrt{}}{}$$

Mathematica [C] time = 3.21, size = 303, normalized size = 1.46

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-i(4A-B)e^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] -1/6*(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*((29*I)*A - (5*I)*B + (2*I)*(25*A
 - 7*B)*Cos[c + d*x] + (17*I)*A*Cos[2*(c + d*x)] - (5*I)*B*Cos[2*(c + d*x)]
 - (I*(4*A - B)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyper
geometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 8*(5*A
 - 2*B)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Co
s[(c + d*x)/2] - I*Sin[(c + d*x)/2]) - 12*A*Sin[c + d*x] - 7*A*Sin[2*(c + d
*x)] + B*Sin[2*(c + d*x)])*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(a^
2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

fricas [F] time = 1.34, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a^2*cos(d*x + c)^2 + 2*a^
2*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x
)
```

maple [B] time = 1.88, size = 494, normalized size = 2.38

$$-\frac{2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(5A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(5*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*A*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2
```

```
*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(4*A-B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*A-10*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(37*A-7*B)*sin(1/2*d*x+1/2*c)^2)/
a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^2, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.483 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=161

$$\frac{(2A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{A\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^2d}$$

```
[Out] -1/3*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-A*sin(d*x+c)*se
c(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))+A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/a^2/d+1/3*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/
d
```

Rubi [A] time = 0.39, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4019, 3787, 3771, 2639, 2641}

$$\frac{(2A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{A\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + ((2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*a^2*d) - (A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x
])) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2
)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}a(A-B)+\frac{1}{2}a(5A+B)\sec(}{a+a\sec(c+dx)}}{3a^2}$$

$$= -\frac{A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int}{}$$

$$= -\frac{A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{A}{}$$

$$= -\frac{A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(A}{}$$

$$= \frac{A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A+B)\sqrt{\cos(c+dx}}{}$$

Mathematica [C] time = 2.02, size = 256, normalized size = 1.59

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(2i\cos(c+dx)(i(A-B)\sin(c+dx)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^2,
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-I)*A*(1 + E^(I*(c + d*x)))^3*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^(I*(c + d*x)) + 8*(2*A + B)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*Co
s[c + d*x]*(7*A - B + (5*A + B)*Cos[c + d*x] + I*(A - B)*Sin[c + d*x]))*(Co
s[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d
*x])^2)
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a^2*cos(d*x + c)^2 + 2*a^
2*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x
)
```

maple [A] time = 1.62, size = 350, normalized size = 2.17

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(12A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*d
*x+1/2*c)^6-4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2*
d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^3-16*A*cos(1/2*d*x+1/2*c)^4-2*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1
/2*d*x+1/2*c)^2+3*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^2, x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sqrt{\sec{\left(c+dx \right)}}}{\cos^{2}{\left(c+dx \right)}+2 \cos{\left(c+dx \right)}+1}\, dx + \int \frac{B \cos{\left(c+dx \right)} \sqrt{\sec{\left(c+dx \right)}}}{\cos^{2}{\left(c+dx \right)}+2 \cos{\left(c+dx \right)}+1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x) +
 Integral(B*cos(c + d*x)*sqrt(sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*
x) + 1), x))/a**2
```

3.484 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=168

$$\frac{(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{B\sqrt{\cos(c+dx)}\sqrt{\mathrm{sec}}}{}$$

```
[Out] 1/3*(A+2*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))-1/3*(A-B)*sin(
d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^2-B*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/a^2/d+1/3*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a^2/d
```

Rubi [A] time = 0.39, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{B\sqrt{\cos(c+dx)}\sqrt{\mathrm{sec}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -((B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d)) + ((A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*a^2*d) + ((A + 2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1
+ Sec[c + d*x])) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Se
c[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}\,(B+A\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{3}{2}a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= \frac{(A+2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+d}{3d(a+a\sec(c+dx))^2}$$

$$= \frac{(A+2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+d}{3d(a+a\sec(c+dx))^2}$$

$$= \frac{(A+2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+d}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(A+2B)\sqrt{\cos(c+}{}$$

Mathematica [C] time = 2.39, size = 256, normalized size = 1.52

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(i\left(2\cos(c+dx)(-i(A-B)\sin(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(8*(A + 2*B)*Cos[(c + d*x)/2]^3*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/
2]) + I*((B*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 2*Cos[c +
d*x]*(-A - 5*B + (A - 7*B)*Cos[c + d*x] - I*(A - B)*Sin[c + d*x])))*(Cos[(c
 + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])
^2)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\left(a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2\right)\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a
^2)*sqrt(sec(d*x + c))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

maple [A] time = 1.62, size = 350, normalized size = 2.08

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+12*B*cos(1/2*d*x+1/2*c)^6+4*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4-20*B*cos(1/2*d*x+1/2*c)^4-3*A*cos(
1/2*d*x+1/2*c)^2+9*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos^2(c+dx)\sqrt{\sec(c+dx)}+2\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx+\int \frac{B\cos(c+dx)}{\cos^2(c+dx)\sqrt{\sec(c+dx)}+2\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(cos(c + d*x)**2*sqrt(sec(c + d*x)) + 2*cos(c + d*x)*sqrt(sec(c
 + d*x)) + sqrt(sec(c + d*x))), x) + Integral(B*cos(c + d*x)/(cos(c + d*x)*
*2*sqrt(sec(c + d*x)) + 2*cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*
x))), x))/a**2
```

3.485 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=176

$$\frac{(2A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-4B)\sqrt{\cos(c+a}}{}$$

```
[Out] 1/3*(2*A-5*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))+1/3*(A-B)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^2-(A-4*B)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+1/3*(2*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.41, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4020, 3787, 3771, 2639, 2641}

$$\frac{(2A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-4B)\sqrt{\cos(c+a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -(((A - 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d)) + ((2*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*a^2*d) + ((2*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(3*a^2*d*(1 + Sec[c + d*x])) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d*(a + a*Sec[c + d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(A-7B)+\frac{3}{2}a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= \frac{(2A-5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx}{3d(a+a\sec(c+dx))^2}$$

$$= \frac{(2A-5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx}{3d(a+a\sec(c+dx))^2}$$

$$= \frac{(2A-5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A-5B)\sqrt{c}}{}$$

Mathematica [C] time = 6.47, size = 475, normalized size = 2.70

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(5(A-4B)\cos\left(\frac{1}{2}(c-dx)\right)+4(A-4B)\cos\left(\frac{1}{2}(3c+dx)\right)+3A\cos\left(\frac{1}{2}(c+3dx)\right)-9B\cos\left(\frac{1}{2}(c+3dx)\right)-3B\cos\right.}{2\sqrt{\sec(c+dx)}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] (Cos[(c + d*x)/2]^4*((2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))]))/E^(I*d*x) - (8*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((
2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I
)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4
, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + ((5*(A - 4*B)*Cos[(c - d*x)/2] +
 4*(A - 4*B)*Cos[(3*c + d*x)/2] + 3*A*Cos[(c + 3*d*x)/2] - 9*B*Cos[(c + 3*d
*x)/2] - 3*B*Cos[(5*c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(2
*Sqrt[Sec[c + d*x]]) + 8*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]] - 20*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\left(a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2\right)\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a
^2)*sec(d*x + c)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

maple [A] time = 1.69, size = 421, normalized size = 2.39

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*B*cos(1/2*d*x+1/2*c)^6-10*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-24*B*cos(1/2*d*x+1/2*c)^3*(sin(1
```

```
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4+38*B*cos(1/2*d*x+1/2*c)^4+9*A
*cos(1/2*d*x+1/2*c)^2-15*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)
^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.486 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=206

$$-\frac{5(A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(\sec(c+dx)+1)}-\frac{5(A-2B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{3a^2d}$$

```
[Out] -5/3*(A-2*B)*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)+1/3*(4*A-7*B)*sin(d*x+c)/a^2
/d/(1+sec(d*x+c))/sec(d*x+c)^(1/2)+1/3*(A-B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^
2/sec(d*x+c)^(1/2)+(4*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a
^2/d-5/3*(A-2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.43, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4020, 3787, 3769, 3771, 2641, 2639}

$$-\frac{5(A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(\sec(c+dx)+1)}-\frac{5(A-2B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((4*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a^2*d) - (5*(A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*a^2*d) - (5*(A - 2*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c
+ d*x]]) + ((4*A - 7*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[
c + d*x])) + ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c +
d*x])^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$

$$=\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}+\frac{\int \frac{-\frac{3}{2}a(A-3B)+\frac{5}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}dx}{3a^2}$$

$$=\frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))}+\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\sec(c+$$

$$=\frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))}+\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\sec(c+$$

$$=-\frac{5(A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{(4A-7B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))}+\frac{}{3d\sqrt{}}$$

$$=\frac{(4A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5(A-2B)\,\mathrm{s}}{3a^2d\sqrt{\sec}}$$

$$=\frac{(4A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5(A-2B)\sqrt{}}{}$$

Mathematica [C] time = 6.91, size = 777, normalized size = 3.77

$$\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{8(A-2B)\cos(c)\sin(dx)}{d}-\frac{2\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{3d}-\frac{2(A-B)\tan\left(\frac{c}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}+\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] (-4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) + (7
*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
```

```
(2*I)*(c + d*x))])*Sec[c/2])/(3*d*E^(I*d*x)*(a + a*Cos[c + d*x])^2) - (10*A
*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]
*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2) + (20*B*C
os[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*S
ec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^2) + (Cos[c/2
+ (d*x)/2]^4*Sqrt[Sec[c + d*x]]*((-2*(3*A - 5*B + A*Cos[2*c] - 2*B*Cos[2*c]
)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (4*B*Cos[2*d*x]*Sin[2*c])/(3*d) + (4*Sec[
c/2]*Sec[c/2 + (d*x)/2]*(7*A*Sin[(d*x)/2] - 10*B*Sin[(d*x)/2]))/(3*d) - (2*
Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (8
*(A - 2*B)*Cos[c]*Sin[d*x])/d + (4*B*Cos[2*c]*Sin[2*d*x])/(3*d) + (4*(7*A -
 10*B)*Tan[c/2])/(3*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/
(a + a*Cos[c + d*x])^2
```

fricas [F] time = 2.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}\right)\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a
^2)*sec(d*x + c)^(5/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
 x)
```

maple [A] time = 1.85, size = 435, normalized size = 2.11

$$\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-16B\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24A\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-16*B*cos(1/2*
d*x+1/2*c)^8+24*A*cos(1/2*d*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1
/2*d*x+1/2*c)^3+24*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*co
s(1/2*d*x+1/2*c)^6-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-42*B
*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1/2*c)^4+4
8*B*cos(1/2*d*x+1/2*c)^4+15*A*cos(1/2*d*x+1/2*c)^2-21*B*cos(1/2*d*x+1/2*c)^
2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.487 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=261

$$-\frac{(13A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(49A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}{6a^3d}$$

```
[Out] -1/5*(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*(8*A-3*B)*
sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-1/6*(13*A-3*B)*sec(d*x+c
)^(3/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))+1/10*(49*A-9*B)*sin(d*x+c)*sec(d*
x+c)^(1/2)/a^3/d-1/10*(49*A-9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/a^3/d-1/6*(13*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.61, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{(13A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(49A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -((49*A - 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) - ((13*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((49*A - 9*B)*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(10*a^3*d) - ((A - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Se
c[c + d*x])^3) - ((8*A - 3*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(15*a*d*(a +
 a*Sec[c + d*x])^2) - ((13*A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(
a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx=\int \frac{\sec^{\frac{7}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$=-\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{5}{2}a(A-B)+\frac{1}{2}a(11A-B)\sec\right.}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$=-\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(8A-3B)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{15ad(a+a\sec(c+dx))^2}$$

$$=-\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(8A-3B)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{15ad(a+a\sec(c+dx))^2}$$

$$=-\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(8A-3B)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{15ad(a+a\sec(c+dx))^2}$$

$$=\frac{(49A-9B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{10a^3d}-\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+}{5d(a+a\sec(c+dx))^3}$$

$$=-\frac{(13A-3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d}+\frac{(49A-9}{}$$

$$=-\frac{(49A-9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-3}{}$$

Mathematica [C] time = 5.39, size = 358, normalized size = 1.37

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-i(49A-9B)e^{-2i(c+dx)}\sqrt{1+e^{2i(c+d}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] -1/120*(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-I)*(49*A - 9*B)*(1 + E^(I*(
c + d*x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + 160*(13*A - 3*B)*Cos[(c + d*x
)/2]^5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*S
```

```
in[(c + d*x)/2]) + (2*I)*(642*A - 102*B + (1082*A - 207*B)*Cos[c + d*x] + 6
*(87*A - 17*B)*Cos[2*(c + d*x)] + 106*A*Cos[3*(c + d*x)] - 21*B*Cos[3*(c +
d*x)] + (161*I)*A*Sin[c + d*x] - (6*I)*B*Sin[c + d*x] + (148*I)*A*Sin[2*(c
+ d*x)] - (18*I)*B*Sin[2*(c + d*x)] + (41*I)*A*Sin[3*(c + d*x)] - (6*I)*B*S
in[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(a^3*d*E^(I*
d*x)*(1 + Cos[c + d*x])^3)
```

fricas [F] time = 2.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a^3*cos(d*x + c)^3 + 3*a^
3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x
)
```

maple [B] time = 2.05, size = 685, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] -1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
```

```
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+27*B*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+27*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x
+1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(49*A-9*B)*
sin(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*(817*A-147*B)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(248*A-43*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(439*A-69*B)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*
d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^3, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.488 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=222

$$-\frac{(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B)\sqrt{\cos(c+d}}{}$$

```
[Out] -1/5*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*(6*A-B)*se
c(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-1/10*(9*A+B)*sin(d*x+c)*se
c(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))+1/10*(9*A+B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(3*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/a^3/d
```

Rubi [A] time = 0.58, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4019, 3787, 3771, 2639, 2641}

$$-\frac{(9A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A+B)\sqrt{\cos(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((9*A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(10*a^3*d) + ((3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(6*a^3*d) - ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(
a + a*Sec[c + d*x])^3) - ((6*A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*
d*(a + a*Sec[c + d*x])^2) - ((9*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10
*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}a(A-B)+\frac{1}{2}a(9A+B)\sec(c}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(6A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(6A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(6A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(6A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{15ad(a+a\sec(c+dx))^2}$$

$$= \frac{(9A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A+B)\sqrt{\cos}}{}$$

Mathematica [C] time = 6.98, size = 793, normalized size = 3.57

$$\frac{\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{2\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{5d} + \frac{2(A-B)\tan\left(\frac{c}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{5d} + \frac{4\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(3A\sin\left(\right.\right.}{15d}\right.}{(a\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^3,
x]
```

```
[Out] (-3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) - (S
qrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*
(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]
```

```
 + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A*C
os[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*S
ec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) + (2*B*Cos[c/
2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/
2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 + (d*
x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(9*A + B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d)
 + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d
) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(3*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(3*
d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(3*A*Sin[(d*x)/2] + 2*B*Sin[(d*x)/2])
)/(15*d) + (4*(3*A + B)*Tan[c/2])/(3*d) + (4*(3*A + 2*B)*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a
+ a*Cos[c + d*x])^3
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a^3*cos(d*x + c)^3 + 3*a^
3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3, x
)
```

maple [A] time = 1.65, size = 451, normalized size = 2.03

$$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(108A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-30A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/2
*d*x+1/2*c)^8-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos(
1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8-10*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*B*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-138*A*cos(1/2*d*x+1/2*c)^6-22*B*cos(1/2*d*x+1/2*c)^6+
24*A*cos(1/2*d*x+1/2*c)^4+6*B*cos(1/2*d*x+1/2*c)^4+3*A*cos(1/2*d*x+1/2*c)^2
+7*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+a\,\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^3, x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\sec(c+dx)}}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sqrt{\sec(c+dx)}}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos
(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*sqrt(sec(c + d*x))/(cos(c + d*
x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x))/a**3
```

3.489 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=216

$$\frac{(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{s}}{10}$$

```
[Out] -1/5*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*(4*A+B)*si
n(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*(A+B)*sin(d*x+c)*sec(d
*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))+1/10*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/a^3/d+1/6*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a^3/d
```

Rubi [A] time = 0.57, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{s}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
10*a^3*d) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(6*a^3*d) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a +
a*Sec[c + d*x])^3) - ((4*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a
 + a*Sec[c + d*x])^2) + ((A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3
 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}a(A-B)+\frac{1}{2}a(7A+3B)\sec\right.}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(4A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(4A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(4A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(4A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx}{15ad(a+a\sec(c+dx))^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+B)\sqrt{\cos(c}{}$$

Mathematica [C] time = 6.93, size = 792, normalized size = 3.67

$$\frac{\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(-\frac{2\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{5d} - \frac{2(A-B)\tan\left(\frac{c}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{5d} + \frac{4\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2A\sin\right.}{15d}\right.}{(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] -1/15*(Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c
+ d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4
, -E^((2*I)*(c + d*x))])*Sec[c/2])/(d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (
Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)
*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))
```

```
] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((
2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A*
Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*
Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (2*B*Cos
[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec
[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 +
(d*x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(A - B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d
) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(2*A*Sin[(d*x)/2] - 7*B*Sin[(d*x)/2]))
/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]
))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + B*Sin[(d*x)/2])
)/(3*d) + (4*(A + B)*Tan[c/2])/(3*d) + (4*(2*A - 7*B)*Sec[c/2 + (d*x)/2]^2*
Tan[c/2])/(15*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a
*Cos[c + d*x])^3
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}\right)\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 +
 3*a^3*cos(d*x + c) + a^3)*sqrt(sec(d*x + c))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{3}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

maple [A] time = 1.86, size = 451, normalized size = 2.09

$$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(12A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8-10*B*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*B*cos(1/2*d*x+1/2*c)^5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6+2*B*cos(1/2*d*x+1/2*c)^6+6*A*
cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/2*c)^2-17*
B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos(c+d\,x)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+a\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.490 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=222

$$\frac{(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B)\sqrt{\cos(c+dx}}{}$$

```
[Out] -1/5*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3+1/15*(2*A+3*B)*
sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*(A+3*B)*sin(d*x+c)*s
ec(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))-1/10*(A+9*B)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.58, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A+9B)\sqrt{\cos(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(10*a^3*d) + ((A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(6*a^3*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*
(a + a*Sec[c + d*x])^3) + ((2*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15
*a*d*(a + a*Sec[c + d*x])^2) + ((A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}\,(B+A\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(A-B)+\frac{5}{2}a(A+B)\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\sqrt{\sec(c+dx)}\,\sin(c+a}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\sqrt{\sec(c+dx)}\,\sin(c+a}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\sqrt{\sec(c+dx)}\,\sin(c+a}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\sqrt{\sec(c+dx)}\,\sin(c+a}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+3B)\sqrt{co}}{}$$

Mathematica [C] time = 7.04, size = 793, normalized size = 3.57

$$\frac{\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{2\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{5d} + \frac{2(A-B)\tan\left(\frac{c}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{5d} - \frac{4\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(7A\sin\left(\right.\right.}{15d}\right.}{(a\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (3*S
qrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*
(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))]
```

```
 + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2
*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A*Co
s[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Se
c[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (2*B*Cos[c
/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c
/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 + (d*x
)/2]^6*Sqrt[Sec[c + d*x]]*((2*(A + 9*B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) -
 (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(7*A*Sin[(d*x)/2] - 12*B*Sin[(d*x)/2]))/(
15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] - 9*B*Sin[(d*x)/2]))
/(3*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2])
)/(5*d) + (4*(A - 9*B)*Tan[c/2])/(3*d) - (4*(7*A - 12*B)*Sec[c/2 + (d*x)/2]
^2*Tan[c/2])/(15*d) + (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a
+ a*Cos[c + d*x])^3
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 +
 3*a^3*cos(d*x + c) + a^3)*sec(d*x + c)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

maple [A] time = 1.65, size = 451, normalized size = 2.03

$$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(12A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8+30*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*B*cos(1/2*d*x+1/2*c)^5*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6-198*B*cos(1/2*d*x+1/2*c)^6-
24*A*cos(1/2*d*x+1/2*c)^4+114*B*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c
)^2-27*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.491 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-49B)\sqrt{\cos(}}{}$$

```
[Out] 1/5*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3+1/15*(3*A-8*B)*s
in(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*(3*A-13*B)*sin(d*x+c)
*sec(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))-1/10*(9*A-49*B)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(3*A-13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.58, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4020, 3787, 3771, 2639, 2641}

$$\frac{(3A-13B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-49B)\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

```
[Out] -((9*A - 49*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(10*a^3*d) + ((3*A - 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(5*d*(a + a*Sec[c + d*x])^3) + ((3*A - 8*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A - 13*B)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{-\frac{1}{2}a(A-11B)+\frac{5}{2}a(A-B)\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sqrt{\sec(c+dx)}\,\sin(c+dx}{15ad(a+a\sec(c+dx))^2}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sqrt{\sec(c+dx)}\,\sin(c+dx}{15ad(a+a\sec(c+dx))^2}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sqrt{\sec(c+dx)}\,\sin(c+dx}{15ad(a+a\sec(c+dx))^2}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sqrt{\sec(c+dx)}\,\sin(c+dx}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(9A-49B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A-13B}{}$$

Mathematica [C] time = 7.15, size = 817, normalized size = 3.58

$$\frac{3\sqrt{2}\,Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\csc\left(\frac{c}{2}\right)\left(e^{2idx}\left(-1+e^{2ic}\right)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)\sec\left(\frac{c}{2}\right)\mathrm{co}}{5d(\cos(c+dx)a+a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] (3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2])/(5*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) - (49
*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
```

```
(2*I)*(c + d*x))])*Sec[c/2])/(15*d*E^(I*d*x)*(a + a*Cos[c + d*x])^3) + (2*A
*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]
*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(d*(a + a*Cos[c + d*x])^3) - (26*B*Cos
[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec
[c/2]*Sqrt[Sec[c + d*x]]*Sin[c])/(3*d*(a + a*Cos[c + d*x])^3) + (Cos[c/2 +
(d*x)/2]^6*Sqrt[Sec[c + d*x]]*((-2*(-9*A + 39*B + 10*B*Cos[2*c])*Cos[d*x]*C
sc[c/2]*Sec[c/2])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(9*A*Sin[(d*x)/2]
- 23*B*Sin[(d*x)/2]))/(3*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(12*A*Sin[(d
*x)/2] - 17*B*Sin[(d*x)/2]))/(15*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(A*S
in[(d*x)/2] - B*Sin[(d*x)/2]))/(5*d) + (16*B*Cos[c]*Sin[d*x])/d - (4*(9*A -
 23*B)*Tan[c/2])/(3*d) + (4*(12*A - 17*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(1
5*d) - (2*(A - B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x
])^3
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3\right)\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 +
 3*a^3*cos(d*x + c) + a^3)*sec(d*x + c)^(5/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)),
 x)
```

maple [A] time = 1.64, size = 451, normalized size = 1.98

$$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(108A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+30A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\ \sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-348*B*cos(1/2*d*x+1/2*c)^8-130*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-294*B*cos(1/2*d*x+1/2*c)^5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6+578*B*cos(1/2*d*x+1/2
*c)^6+114*A*cos(1/2*d*x+1/2*c)^4-264*B*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*
x+1/2*c)^2+37*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.492 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=259

$$-\frac{(13A-33B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}+\frac{7(7A-17B)\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{(13A-33B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}\right.}{6a^3d}$$

```
[Out] -1/6*(13*A-33*B)*sin(d*x+c)/a^3/d/sec(d*x+c)^(1/2)+1/5*(A-B)*sin(d*x+c)/d/(
a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2)+1/3*(A-2*B)*sin(d*x+c)/a/d/(a+a*sec(d*x+
c))^2/sec(d*x+c)^(1/2)+7/30*(7*A-17*B)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/se
c(d*x+c)^(1/2)+7/10*(7*A-17*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^3/d-1/6*(13*A-33*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] time = 0.62, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2960, 4020, 3787, 3769, 3771, 2641, 2639}

$$-\frac{(13A-33B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}+\frac{7(7A-17B)\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{(13A-33B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}\right.}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (7*(7*A - 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(10*a^3*d) - ((13*A - 33*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((13*A - 33*B)*Sin[c + d*x])/(6*a^3*d*S
qrt[Sec[c + d*x]]) + ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]*(a + a*
Sec[c + d*x])^3) + ((A - 2*B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]*(a +
a*Sec[c + d*x])^2) + (7*(7*A - 17*B)*Sin[c + d*x])/(30*d*Sqrt[Sec[c + d*x]]
*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} + \frac{\int \frac{-\frac{1}{2}a(3A-13B)+\frac{7}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} + \frac{(A-2B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}\,(a+a\sec(c+$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} + \frac{(A-2B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}\,(a+a\sec(c+$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} + \frac{(A-2B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}\,(a+a\sec(c+$$

$$= -\frac{(13A-33B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}} + \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} + \frac{}{3a}$$

$$= \frac{7(7A-17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(13A-33B}{6a^3d\sqrt{se}}$$

$$= \frac{7(7A-17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(13A-33B}{}$$

Mathematica [C] time = 4.72, size = 589, normalized size = 2.27

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left((806A-1961B)\cos\left(\frac{1}{2}(c-dx)\right)+(664A-1609B)\cos\left(\frac{1}{2}(3c+dx)\right)+470A\cos\left(\frac{1}{2}(c+3dx)\right)+265A\cos\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),
x]
```

```
[Out] (Cos[(c + d*x)/2]^6*((-98*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*
```

```
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))]))/E^(I*d*x) + (238*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^(
(2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - (((806*A - 1961*B)*Cos[(c -
d*x)/2] + (664*A - 1609*B)*Cos[(3*c + d*x)/2] + 470*A*Cos[(c + 3*d*x)/2] -
1165*B*Cos[(c + 3*d*x)/2] + 265*A*Cos[(5*c + 3*d*x)/2] - 620*B*Cos[(5*c + 3
*d*x)/2] + 117*A*Cos[(3*c + 5*d*x)/2] - 292*B*Cos[(3*c + 5*d*x)/2] + 30*A*C
os[(7*c + 5*d*x)/2] - 65*B*Cos[(7*c + 5*d*x)/2] - 5*B*Cos[(5*c + 7*d*x)/2]
+ 5*B*Cos[(9*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(8*Sqrt[S
ec[c + d*x]]) - 260*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]] + 660*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]]))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}\right)\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 +
 3*a^3*cos(d*x + c) + a^3)*sec(d*x + c)^(7/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)),
 x)
```

maple [A] time = 1.56, size = 465, normalized size = 1.80

$$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-160B\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+348A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130A\sqrt{\frac{1}{2}-\frac{\cos(d}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-160*B*cos(1/
2*d*x+1/2*c)^10+348*A*cos(1/2*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
cos(1/2*d*x+1/2*c)^5+294*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-4
68*B*cos(1/2*d*x+1/2*c)^8-330*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c
)^5-714*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+
1/2*c)^6+1058*B*cos(1/2*d*x+1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-474*B*cos(1
/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x+1/2*c)^2+47*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B
)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}(a+a\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.493 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=220

$$\frac{2a(8A+9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{4a(8A+9B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a(8A+9B)\sin(c+dx)\sec^{\frac{3}{2}}(c+$$

```
[Out] 16/315*a*(8*A+9*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+4/1
05*a*(8*A+9*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/63*a*
(8*A+9*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/9*a*A*sec(
d*x+c)^(9/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+32/315*a*(8*A+9*B)*sin(d*x
+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2980, 2772, 2771}

$$\frac{2a(8A+9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{4a(8A+9B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a(8A+9B)\sin(c+dx)\sec^{\frac{3}{2}}(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

```
[Out] (32*a*(8*A + 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c
+ d*x]]) + (16*a*(8*A + 9*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d*Sqrt[a
 + a*Cos[c + d*x]]) + (4*a*(8*A + 9*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(10
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*(8*A + 9*B)*Sec[c + d*x]^(7/2)*Sin[c +
 d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(9/2)*Sin[c +
d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx}}{\cos^{\frac{11}{2}}}$$

$$= \frac{2aA\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{1}{9}\left((8A+9B)\sqrt{\cos}\right.$$

$$= \frac{2a(8A+9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sec^{\frac{9}{2}}(}{9d\sqrt{a+}}$$

$$= \frac{4a(8A+9B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a(8A+9}{63d}$$

$$= \frac{16a(8A+9B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{4a(8A+}{10}$$

$$= \frac{32a(8A+9B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{16a(8A}{3}$$

Mathematica [A] time = 0.58, size = 124, normalized size = 0.56

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\,(11(8A+9B)\cos(c+dx)+11(8A+9B)\cos(2(c+dx))+16A}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2)
,x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(107*A + 81*B + 11*(8*A + 9*B)*Cos[c + d*x] +
 11*(8*A + 9*B)*Cos[2*(c + d*x)] + 16*A*Cos[3*(c + d*x)] + 18*B*Cos[3*(c +
d*x)] + 16*A*Cos[4*(c + d*x)] + 18*B*Cos[4*(c + d*x)])*Sec[c + d*x]^(9/2)*T
an[(c + d*x)/2])/(315*d)
```

fricas [A] time = 0.67, size = 121, normalized size = 0.55

$$\frac{2\left(16\,(8\,A+9\,B)\cos\left(dx+c\right)^{4}+8\,(8\,A+9\,B)\cos\left(dx+c\right)^{3}+6\,(8\,A+9\,B)\cos\left(dx+c\right)^{2}+5\,(8\,A+9\,B)\cos\left(dx+\right.\right.}{315\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(16*(8*A + 9*B)*cos(d*x + c)^4 + 8*(8*A + 9*B)*cos(d*x + c)^3 + 6*(8*
A + 9*B)*cos(d*x + c)^2 + 5*(8*A + 9*B)*cos(d*x + c) + 35*A)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*cos(d*x + c)^4)*sqrt(cos(d*x
 + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.42, size = 138, normalized size = 0.63

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(128A\left(\cos^{4}\left(dx+c\right)\right)+144B\left(\cos^{4}\left(dx+c\right)\right)+64A\left(\cos^{3}\left(dx+c\right)\right)+72B\left(\cos^{3}\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(128*A*cos(d*x+c)^4+144*B*cos(d*x+c)^4+64*A*cos(d*
x+c)^3+72*B*cos(d*x+c)^3+48*A*cos(d*x+c)^2+54*B*cos(d*x+c)^2+40*A*cos(d*x+c
)+45*B*cos(d*x+c)+35*A)*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))
^(1/2)/sin(d*x+c)
```

maxima [B] time = 0.76, size = 659, normalized size = 3.00

$$2\left(\frac{A\left(\frac{315\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{735\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{1302\sqrt{2}\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{1206\sqrt{2}\sqrt{a}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{431\sqrt{2}\sqrt{a}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{107\sqrt{2}\sqrt{a}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1}\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{5\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{10\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{5\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)*(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

```
[Out] 2/315*(A*(315*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 735*sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1302*sqrt(2)*sqrt(a)*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 1206*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 + 431*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 10
7*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(5*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(co
s(d*x + c) + 1)^10 + 1)) + 9*B*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x +
c) + 1) - 105*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 154*sqr
t(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 142*sqrt(2)*sqrt(a)*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7 + 67*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*
x + c) + 1)^9 - 9*sqrt(2)*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(5*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin
(d*x + c)^10/(cos(d*x + c) + 1)^10 + 1)))/d
```

mupad [B] time = 5.60, size = 479, normalized size = 2.18

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(256\,A+288\,B)\,1i}{315\,d}-\frac{e^{c\,9i+d\,x\,9i}\,\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(256\,A+288\,B)\,1i}{315\,d}+\cdots\right.}{e^{c\,1i+d\,x\,1i}+4\,e^{c\,2i\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + a*cos(c + d*x))^(1/2)
,x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(((a + a*(exp(-
c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(256*A + 288*B)*1i)/(315*d)
 - (exp(c*9i + d*x*9i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/
2))^(1/2)*(256*A + 288*B)*1i)/(315*d) + (exp(c*2i + d*x*2i)*(a + a*(exp(- c
*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(1152*A + 1296*B)*1i)/(315*d
) - (exp(c*7i + d*x*7i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)
/2))^(1/2)*(1152*A + 1296*B)*1i)/(315*d) + (exp(c*4i + d*x*4i)*(a + a*(exp(
- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(2016*A + 1008*B)*1i)/(31
5*d) - (exp(c*5i + d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*
1i)/2))^(1/2)*(2016*A + 1008*B)*1i)/(315*d)))/(exp(c*1i + d*x*1i) + 4*exp(c
*2i + d*x*2i) + 4*exp(c*3i + d*x*3i) + 6*exp(c*4i + d*x*4i) + 6*exp(c*5i +
d*x*5i) + 4*exp(c*6i + d*x*6i) + 4*exp(c*7i + d*x*7i) + exp(c*8i + d*x*8i)
+ exp(c*9i + d*x*9i) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(11/2)*(a+a*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.494 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=175

$$\frac{2a(6A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{8a(6A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 8/105*a*(6*A+7*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/35
*a*(6*A+7*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a*A*s
ec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+16/105*a*(6*A+7*B)*sin(
d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2980, 2772, 2771}

$$\frac{2a(6A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{8a(6A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (16*a*(6*A + 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c
+ d*x]]) + (8*a*(6*A + 7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a
+ a*Cos[c + d*x]]) + (2*a*(6*A + 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*
d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*
Sqrt[a + a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
```

```
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+d}}{\cos^{\frac{9}{2}}}$$

$$= \frac{2aA\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}\left((6A+7B)\sqrt{\cos}\right.$$

$$= \frac{2a(6A+7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sec^{\frac{7}{2}}}{7d\sqrt{a}}$$

$$= \frac{8a(6A+7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a(6A+}{35}$$

$$= \frac{16a(6A+7B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{8a(6A}{1}$$

Mathematica [A] time = 0.45, size = 102, normalized size = 0.58

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\,(9(6A+7B)\cos(c+dx)+2(6A+7B)\cos(2(c+dx))+12A\cos\cdots}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),
x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(27*A + 14*B + 9*(6*A + 7*B)*Cos[c + d*x] + 2
*(6*A + 7*B)*Cos[2*(c + d*x)] + 12*A*Cos[3*(c + d*x)] + 14*B*Cos[3*(c + d*x
)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(105*d)
```

fricas [A] time = 0.65, size = 104, normalized size = 0.59

$$\frac{2\left(8\,(6\,A+7\,B)\cos\left(dx+c\right)^{3}+4\,(6\,A+7\,B)\cos\left(dx+c\right)^{2}+3\,(6\,A+7\,B)\cos\left(dx+c\right)+15\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\cdots}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(8*(6*A + 7*B)*cos(d*x + c)^3 + 4*(6*A + 7*B)*cos(d*x + c)^2 + 3*(6*A
 + 7*B)*cos(d*x + c) + 15*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(
d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.38, size = 116, normalized size = 0.66

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(48A\left(\cos^{3}\left(dx+c\right)\right)+56B\left(\cos^{3}\left(dx+c\right)\right)+24A\left(\cos^{2}\left(dx+c\right)\right)+28B\left(\cos^{2}\left(dx+c\right)\right)+\cdots}{105d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(48*A*cos(d*x+c)^3+56*B*cos(d*x+c)^3+24*A*cos(d*x+
c)^2+28*B*cos(d*x+c)^2+18*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*cos(d*x+c)*(1/
cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

maxima [B] time = 0.49, size = 568, normalized size = 3.25

$$\frac{2\left(\frac{3\,A\left(\frac{35\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{70\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{84\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{58\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{9\,\sqrt{2}\,\sqrt{a}\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{4}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{4\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4\,\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+1\right)}+\frac{7\,B\left(\frac{15\,\sqrt{2}\,\sqrt{a}\,\sin(\cdots}{\cos(dx+c)+\cdots}\right.}{\left(\frac{\sin(d\cdots}{\cos(dx\cdots}\right.}\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/105*(3*A*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)
*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 7*B*(15*sq
rt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sqrt(2)*sqrt(a)*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 42*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 - 24*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sqrt
(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

mupad [B] time = 4.55, size = 441, normalized size = 2.52

$$\frac{\sqrt{\frac{1}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{\sqrt{a+a\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}\right)}\,(96\,A+112\,B)\,1\mathrm{i}}{105\,d}-\frac{\mathrm{e}^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,\sqrt{a+a\left(\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}\right)}\,(96\,A+112\,B)\,1\mathrm{i}}{105\,d}+\frac{\mathrm{e}^{c\,2\cdots}}{\cdots}\right.}{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+3\,\mathrm{e}^{c\,2\mathrm{i}\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(1/2),
x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(((a + a*(exp(-
c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(96*A + 112*B)*1i)/(105*d)
- (exp(c*7i + d*x*7i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2
))^(1/2)*(96*A + 112*B)*1i)/(105*d) + (exp(c*2i + d*x*2i)*(a + a*(exp(- c*1
i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(336*A + 392*B)*1i)/(105*d) -
(exp(c*5i + d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))
^(1/2)*(336*A + 392*B)*1i)/(105*d) - (B*exp(c*3i + d*x*3i)*(a + a*(exp(- c*
1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*8i)/(3*d) + (B*exp(c*4i + d*x
*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*8i)/(3*d
)))/(exp(c*1i + d*x*1i) + 3*exp(c*2i + d*x*2i) + 3*exp(c*3i + d*x*3i) + 3*e
xp(c*4i + d*x*4i) + 3*exp(c*5i + d*x*5i) + exp(c*6i + d*x*6i) + exp(c*7i +
d*x*7i) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.495 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=130

$$\frac{2a(4A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{4a(4A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/15*a*(4*A+5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*a
*A*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+4/15*a*(4*A+5*B)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2980, 2772, 2771}

$$\frac{2a(4A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{4a(4A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (4*a*(4*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*a*(4*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a
*Cos[c + d*x]]) + (2*a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*C
os[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(}$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{1}{5}\left((4A+5B)\sqrt{\cos(}\right.$$

$$= \frac{2a(4A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sec^{\frac{5}{2}}(c}{5d\sqrt{a+}}$$

$$= \frac{4a(4A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a(4A+5}{15d}$$

Mathematica [A] time = 0.28, size = 78, normalized size = 0.60

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\,((4A+5B)\cos(c+dx)+(4A+5B)\cos(2(c+dx))+7A+5B}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),
x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(7*A + 5*B + (4*A + 5*B)*Cos[c + d*x] + (4*A
+ 5*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

fricas [A] time = 0.63, size = 86, normalized size = 0.66

$$\frac{2\left(2\,(4\,A+5\,B)\cos\left(dx+c\right)^{2}+(4\,A+5\,B)\cos\left(dx+c\right)+3\,A\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(2*(4*A + 5*B)*cos(d*x + c)^2 + (4*A + 5*B)*cos(d*x + c) + 3*A)*sqrt(a
*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt
(cos(d*x + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.35, size = 94, normalized size = 0.72

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(8A\left(\cos^{2}\left(dx+c\right)\right)+10B\left(\cos^{2}\left(dx+c\right)\right)+4A\cos\left(dx+c\right)+5B\cos\left(dx+c\right)+3A\right)\cos}{15d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(8*A*cos(d*x+c)^2+10*B*cos(d*x+c)^2+4*A*cos(d*x+c)+
5*B*cos(d*x+c)+3*A)*cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2
)/sin(d*x+c)
```

maxima [B] time = 0.51, size = 475, normalized size = 3.65

$$2\left(\frac{A\left(\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{7\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}+\frac{5B\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\dots\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)\dots}\right. \over 15\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(A*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sq
rt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)) + 5*B*(3*sqrt(2)*sqrt(a)*sin(d*x + c)
/(cos(d*x + c) + 1) - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
 + 5*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - sqrt(2)*sqrt(a)*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

mupad [B] time = 2.69, size = 196, normalized size = 1.51

$$\frac{4\sqrt{a\,(\cos(c+d\,x)+1)}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,(14\,A\,\sin(c+d\,x)+10\,B\,\sin(c+d\,x)+8\,A\,\sin(2\,c+2\,d\,x)+18\,A\,\sin(3\,c\dots}{15\,d\,(10\,\cos(c+d\,x)+8\,\cos(2\,c+2\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(1/2),
x)
```

```
[Out] (4*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(14*A*sin(c + d*x) +
 10*B*sin(c + d*x) + 8*A*sin(2*c + 2*d*x) + 18*A*sin(3*c + 3*d*x) + 4*A*sin
(4*c + 4*d*x) + 4*A*sin(5*c + 5*d*x) + 10*B*sin(2*c + 2*d*x) + 15*B*sin(3*c
 + 3*d*x) + 5*B*sin(4*c + 4*d*x) + 5*B*sin(5*c + 5*d*x)))/(15*d*(10*cos(c +
 d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos(4*c + 4*d*x) + cos(
5*c + 5*d*x) + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.496 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=85

$$\frac{2a(2A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/3*a*A*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/3*a*(2*A+3*B
)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2961, 2980, 2771}

$$\frac{2a(2A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a*(2*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d
*x]]) + (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x
]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+d}}{\cos^{\frac{5}{2}}}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{3}\Big((2A+3B)\sqrt{\cos}$$

$$= \frac{2a(2A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sec^{\frac{3}{2}}}{3d\sqrt{a}}$$

Mathematica [A] time = 0.18, size = 57, normalized size = 0.67

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\,((2A+3B)\cos(c+dx)+A)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),
x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(A + (2*A + 3*B)*Cos[c + d*x])*Sec[c + d*x]^(
3/2)*Tan[(c + d*x)/2])/(3*d)
```

fricas [A] time = 0.66, size = 65, normalized size = 0.76

$$\frac{2\left((2A+3B)\cos(dx+c)+A\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

[Out] 2/3*((2*A + 3*B)*cos(d*x + c) + A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((
d*cos(d*x + c)^2 + d*cos(d*x + c))*sqrt(cos(d*x + c)))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")

[Out] Timed out

maple [A] time = 0.38, size = 70, normalized size = 0.82

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(2A\cos\left(dx+c\right)+3B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}}{3d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x)

[Out] -2/3/d*(-1+cos(d*x+c))*(2*A*cos(d*x+c)+3*B*cos(d*x+c)+A)*cos(d*x+c)*(1/cos(
d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)

maxima [B] time = 0.82, size = 380, normalized size = 4.47

$$\frac{2\left(\frac{A\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}+\frac{3B\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)\left(\frac{\sin(}{(\cos(d}\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx}{(\cos(dx+}\right.}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")

[Out] 2/3*(A*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/
2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) +
1)^4 + 1)) + 3*B*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sqrt(
2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x +

```
c)^5/(cos(d*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 1)))/d
```

mupad [B] time = 1.07, size = 114, normalized size = 1.34

$$\frac{2\sqrt{a}\,(\cos(c+d\,x)+1)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,(2\,A\,\sin(c+d\,x)+3\,B\,\sin(c+d\,x)+2\,A\,\sin(2\,c+2\,d\,x)+2\,A\,\sin(3\,c+}{3\,d\,(3\,\cos(c+d\,x)+2\,\cos(2\,c+2\,d\,x)+\cos(3\,c+3\,d\,x)+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(1/2),
x)
```

```
[Out] (2*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(2*A*sin(c + d*x) +
3*B*sin(c + d*x) + 2*A*sin(2*c + 2*d*x) + 2*A*sin(3*c + 3*d*x) + 3*B*sin(3*
c + 3*d*x)))/(3*d*(3*cos(c + d*x) + 2*cos(2*c + 2*d*x) + cos(3*c + 3*d*x) +
 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.497 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}\,B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d+2*a*A*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))
^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2980, 2774, 216}

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2\sqrt{a}\,B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a]*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+d}}{\cos^{\frac{3}{2}}}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \left(B\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2B\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{2\sqrt{a}\,B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{d}$$

Mathematica [A] time = 0.21, size = 86, normalized size = 0.90

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(2A\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\,B\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),
x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*B*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*A*Sin[(c + d*x)/2])
)/d
```

fricas [A] time = 0.57, size = 91, normalized size = 0.95

$$-\frac{2\left((B\cos(dx+c)+B)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{a\cos(dx+c)+a}A\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -2*((B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*A*sin(d*x + c)/
sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.39, size = 171, normalized size = 1.78

$$\frac{2\left(B\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+A\sin\right.}{d(1+\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+A*sin(d*x
+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c)
)
```

maxima [B] time = 1.36, size = 906, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(B*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)) + 4*A*(sqrt(2
)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3)/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*
x + c)/(cos(d*x + c) + 1) + 1)^(3/2)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.498 $\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\, d$

Optimal. Leaf size=98

$$\frac{\sqrt{a}\,(2A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

```
[Out] a*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(2*A+B)*arcsin(sin
(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d
```

Rubi [A] time = 0.27, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2981, 2774, 216}

$$\frac{\sqrt{a}\,(2A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(2*A + B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*B*Sin[c + d*x])/(d*Sqrt[a + a
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx}}{\sqrt{\cos}}$$

$$= \frac{aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{1}{2}\left((2A+B)\sqrt{}\right.$$

$$= \frac{aB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left((2A+B)\sqrt{\mathrm{co}}\right.}{}$$

$$= \frac{\sqrt{a}\,(2A+B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\mathrm{s}}}{d}$$

Mathematica [A] time = 0.21, size = 103, normalized size = 1.05

$$\frac{\sqrt{\cos(c+dx)}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\,(2A+B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\,\mathrm{s}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(Sqrt[2]*(2*A + B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*B*Sqrt[Cos[
c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

fricas [A] time = 0.65, size = 97, normalized size = 0.99

$$\frac{\sqrt{a\cos(dx+c)+a}\,B\sqrt{\cos(dx+c)}\,\sin(dx+c)-((2A+B)\cos(dx+c)+2A+B)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A + B)*co
s(d*x + c) + 2*A + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.46, size = 168, normalized size = 1.71

$$-\frac{\left(B\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+2A\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+B\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\right)\sqrt{a(1+\cos(dx+c))}\cdots}{d\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/d*(B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2*A*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+B*arctan(sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c
))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

maxima [B] time = 1.47, size = 939, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(4*A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{a+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\left(A+B\cos\left(c+dx\right)\right)\sqrt{\sec\left(c+dx\right)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))*sqrt(sec(c + d*x))
, x)
```

3.499 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{a}\,(4A+3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(4A+3B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{}{2d\sec^{\frac{3}{2}}(c}$$

```
[Out] 1/2*a*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/4*a*(4*A+3*B
)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/4*(4*A+3*B)*arcsin
(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/d
```

Rubi [A] time = 0.34, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2981, 2770, 2774, 216}

$$\frac{\sqrt{a}\,(4A+3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a(4A+3B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}+\frac{}{2d\sec^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(4*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*B*Sin[c + d*x])/(2*d*Sq
rt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*(4*A + 3*B)*Sin[c + d*x])/(
4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx}$$

$$= \frac{aB\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\left((4A+3B)\sqrt{\cos(c+dx}\right.$$

$$= \frac{aB\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{a(4A+3B)\sin(c+}{4d\sqrt{a+a\cos(c+dx)}\,\sqrt{se}}$$

$$= \frac{aB\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{a(4A+3B)\sin(c+}{4d\sqrt{a+a\cos(c+dx)}\,\sqrt{se}}$$

$$= \frac{\sqrt{a}\,(4A+3B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{4d}$$

Mathematica [A] time = 0.39, size = 120, normalized size = 0.79

$$\frac{\sqrt{\cos(c+dx)}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\,(4A+3B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\,\mathrm{s}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(Sqrt[2]*(4*A + 3*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[
c + d*x]]*(4*A + 3*B + 2*B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

fricas [A] time = 0.58, size = 127, normalized size = 0.84

$$-\frac{((4A+3B)\cos(dx+c)+4A+3B)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(2B\cos(dx+c)^2+(4A+3B)\cos(dx+c)\right)\sqrt{a\cos(}}{\sqrt{\cos(dx+c)}}}{4\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(((4*A + 3*B)*cos(d*x + c) + 4*A + 3*B)*sqrt(a)*arctan(sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*B*cos(d*x + c)^2
+ (4*A + 3*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(
d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.41, size = 238, normalized size = 1.58

$$\frac{(-1+\cos(dx+c))^2\left(2B\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+4A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+3B\sin(dx+c)\right.}{4d\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))^2*(2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+3*B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c))+3*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(
d*x+c)))^(3/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)^4
```

maxima [B] time = 0.82, size = 1851, normalized size = 12.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
```

```
+ c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*A + (2*(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2
*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x +
 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d
*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2
*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - cos(2*d*
x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqr
t(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
```

```
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\left(A+B\cos\left(c+dx\right)\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/sqrt(sec(c + d*x))
, x)
```

3.500 $\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=196

$$\frac{a(6A+5B)\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\,(6A+5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(6}{8d\sqrt{\sec(c}}$$

```
[Out] 1/3*a*B*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+1/12*a*(6*A+5*
B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/8*a*(6*A+5*B)*sin
(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/8*(6*A+5*B)*arcsin(sin(
d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/d
```

Rubi [A] time = 0.41, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2981, 2770, 2774, 216}

$$\frac{a(6A+5B)\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\,(6A+5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a(6}{8d\sqrt{\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*B*Sin[c + d*x])/(3*d*Sq
rt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a*(6*A + 5*B)*Sin[c + d*x])/(
12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*(6*A + 5*B)*Sin[c +
d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}\,(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}$$

$$= \frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{6}\Big((6A+5B)\sqrt{\cos(c+dx}$$

$$= \frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a(6A+5B)\sin(c+}{12d\sqrt{a+a\cos(c+dx)}\,\mathrm{se}}$$

$$= \frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a(6A+5B)\sin(c+}{12d\sqrt{a+a\cos(c+dx)}\,\mathrm{se}}$$

$$= \frac{aB\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a(6A+5B)\sin(c+}{12d\sqrt{a+a\cos(c+dx)}\,\mathrm{se}}$$

$$= \frac{\sqrt{a}\,(6A+5B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{8d}$$

Mathematica [A] time = 0.71, size = 138, normalized size = 0.70

$$\frac{\sqrt{\cos(c+dx)}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\sqrt{a(\cos(c+dx)+1)}\,\left(3\sqrt{2}\,(6A+5B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(3*Sqrt[2]*(6*A + 5*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Co
s[c + d*x]]*(18*A + 19*B + 2*(6*A + 5*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)
])*Sin[(c + d*x)/2]))/(48*d)
```

fricas [A] time = 0.83, size = 146, normalized size = 0.74

$$-\frac{3\,((6\,A+5\,B)\cos(dx+c)+6\,A+5\,B)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(8\,B\cos(dx+c)^{3}+2\,(6\,A+5\,B)\cos(dx+c)^{2}+3\,(\right.}{\sqrt{\cos}}}{24\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/24*(3*((6*A + 5*B)*cos(d*x + c) + 6*A + 5*B)*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*B*cos(d*x + c)
^3 + 2*(6*A + 5*B)*cos(d*x + c)^2 + 3*(6*A + 5*B)*cos(d*x + c))*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.44, size = 308, normalized size = 1.57

$$-\frac{(-1+\cos(dx+c))^3 \left(8B\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+12A\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^3*(8*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)+12*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+
10*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+18*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+15*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)+18*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x
+c))+15*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*
cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/co
s(d*x+c))^(3/2)/sin(d*x+c)^6
```

maxima [B] time = 1.00, size = 2981, normalized size = 15.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")

[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - 1)))*A + (4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin
(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 5*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
```

in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 - 4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sq
rt(a)*(arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2
(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(co
s(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*
sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co

```
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*co
s(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*B)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+a\,\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\cos(c+dx)+1)}\,(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*(A + B*cos(c + d*x))/sec(c + d*x)**(3/2
), x)
```

3.501 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=275

$$\frac{2a^2(12A+11B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(168A+187B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(168A+187B)\sin(}{1155d\sqrt{a\cos(}}$$

```
[Out] 16/3465*a^2*(168*A+187*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1
/2)+4/1155*a^2*(168*A+187*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))
^(1/2)+2/693*a^2*(168*A+187*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c
))^(1/2)+2/99*a^2*(12*A+11*B)*sec(d*x+c)^(9/2)*sin(d*x+c)/d/(a+a*cos(d*x+c)
)^(1/2)+2/11*a*A*sec(d*x+c)^(11/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+32/3
465*a^2*(168*A+187*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2772, 2771}

$$\frac{2a^2(12A+11B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(168A+187B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(168A+187B)\sin(}{1155d\sqrt{a\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/2),x]
```

```
[Out] (32*a^2*(168*A + 187*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a
*Cos[c + d*x]]) + (16*a^2*(168*A + 187*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(3465*d*Sqrt[a + a*Cos[c + d*x]]) + (4*a^2*(168*A + 187*B)*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(168*A + 187*B)
*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2
*(12*A + 11*B)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d*Sqrt[a + a*Cos[c + d*
x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(1
1*d)
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+}{\text{c}}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{11}{2}}(c+dx)\sin(c+dx}{11d}$$

$$= \frac{2a^2(12A+11B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}} + \frac{2aA}{}$$

$$= \frac{2a^2(168A+187B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

$$= \frac{4a^2(168A+187B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{1155d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

$$= \frac{16a^2(168A+187B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}} + $$

$$= \frac{32a^2(168A+187B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}} + $$

Mathematica [A] time = 0.78, size = 146, normalized size = 0.53

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\,((6342A+6193B)\cos(c+dx)+13(168A+187B)\cos(2(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/
2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(2478*A + 2057*B + (6342*A + 6193*B)*Cos[c +
d*x] + 13*(168*A + 187*B)*Cos[2*(c + d*x)] + 2184*A*Cos[3*(c + d*x)] + 2431
*B*Cos[3*(c + d*x)] + 336*A*Cos[4*(c + d*x)] + 374*B*Cos[4*(c + d*x)] + 336
*A*Cos[5*(c + d*x)] + 374*B*Cos[5*(c + d*x)])*Sec[c + d*x]^(11/2)*Tan[(c +
d*x)/2])/(3465*d)
```

fricas [A] time = 0.65, size = 144, normalized size = 0.52

$$\frac{2\left(16\,(168\,A+187\,B)a\cos\left(dx+c\right)^5+8\,(168\,A+187\,B)a\cos\left(dx+c\right)^4+6\,(168\,A+187\,B)a\cos\left(dx+c\right)^3+5\,(}{3465\left(d\cos\left(dx+c\right)^6+d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(16*(168*A + 187*B)*a*cos(d*x + c)^5 + 8*(168*A + 187*B)*a*cos(d*x +
 c)^4 + 6*(168*A + 187*B)*a*cos(d*x + c)^3 + 5*(168*A + 187*B)*a*cos(d*x +
c)^2 + 35*(21*A + 11*B)*a*cos(d*x + c) + 315*A*a)*sqrt(a*cos(d*x + c) + a)*
sin(d*x + c)/((d*cos(d*x + c)^6 + d*cos(d*x + c)^5)*sqrt(cos(d*x + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.41, size = 161, normalized size = 0.59

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(2688A\left(\cos^{5}\left(dx+c\right)\right)+2992B\left(\cos^{5}\left(dx+c\right)\right)+1344A\left(\cos^{4}\left(dx+c\right)\right)+1496B\left(\cos^{4}\left(d\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x)
```

```
[Out] -2/3465/d*(-1+cos(d*x+c))*(2688*A*cos(d*x+c)^5+2992*B*cos(d*x+c)^5+1344*A*c
os(d*x+c)^4+1496*B*cos(d*x+c)^4+1008*A*cos(d*x+c)^3+1122*B*cos(d*x+c)^3+840
*A*cos(d*x+c)^2+935*B*cos(d*x+c)^2+735*A*cos(d*x+c)+385*B*cos(d*x+c)+315*A)
*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(13/2)/sin(d*x+c)*a
```

maxima [B] time = 0.52, size = 712, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] 4/3465*(21*(165*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 495*sqrt(
2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1056*sqrt(2)*a^(3/2)*sin(d
```

```
*x + c)^5/(cos(d*x + c) + 1)^5 - 1254*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 781*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
299*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 46*sqrt(2)*a^(3
/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c) +
 1)^10 + 1)) + 11*(315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 11
55*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2184*sqrt(2)*a^(3/
2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2586*sqrt(2)*a^(3/2)*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 + 1759*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c)
+ 1)^9 - 611*sqrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 94*sqr
t(2)*a^(3/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*B*(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(
d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d
*x + c) + 1)^10 + 1)))/d
```

mupad [B] time = 5.14, size = 348, normalized size = 1.27

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(-\frac{32\,a\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A+3\,B\right)\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}{5\,d}+\frac{64\,a\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\left(21\,A+19\,B\right)\,\sqrt{a+a\,\mathrm{cc}}}{35\,d}\right.}{20\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+20\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+10\,e^{\frac{c\,11i}{2}+\frac{d\,x\,11i}{2}}\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(13/2)*(a + a*cos(c + d*x))^(3/2)
,x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((64*a*exp((c*11
i)/2 + (d*x*11i)/2)*sin((3*c)/2 + (3*d*x)/2)*(21*A + 19*B)*(a + a*cos(c + d
*x))^(1/2))/(35*d) - (32*a*exp((c*11i)/2 + (d*x*11i)/2)*sin(c/2 + (d*x)/2)*
(2*A + 3*B)*(a + a*cos(c + d*x))^(1/2))/(5*d) + (32*a*exp((c*11i)/2 + (d*x*
11i)/2)*sin((7*c)/2 + (7*d*x)/2)*(168*A + 187*B)*(a + a*cos(c + d*x))^(1/2)
)/(315*d) + (64*a*exp((c*11i)/2 + (d*x*11i)/2)*sin((11*c)/2 + (11*d*x)/2)*(
168*A + 187*B)*(a + a*cos(c + d*x))^(1/2))/(3465*d)))/(20*exp((c*11i)/2 + (
d*x*11i)/2)*cos(c/2 + (d*x)/2) + 20*exp((c*11i)/2 + (d*x*11i)/2)*cos((3*c)/
2 + (3*d*x)/2) + 10*exp((c*11i)/2 + (d*x*11i)/2)*cos((5*c)/2 + (5*d*x)/2) +
 10*exp((c*11i)/2 + (d*x*11i)/2)*cos((7*c)/2 + (7*d*x)/2) + 2*exp((c*11i)/2
 + (d*x*11i)/2)*cos((9*c)/2 + (9*d*x)/2) + 2*exp((c*11i)/2 + (d*x*11i)/2)*c
os((11*c)/2 + (11*d*x)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(13/2),x)
[Out] Timed out
```

3.502 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=228

$$\frac{2a^2(10A+9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(34A+39B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{8a^2(34A+39B)\sin(c+d}{315d\sqrt{a\cos(c+d}}$$

```
[Out] 8/315*a^2*(34*A+39*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+
2/105*a^2*(34*A+39*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+
2/63*a^2*(10*A+9*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/
9*a*A*sec(d*x+c)^(9/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+16/315*a^2*(34*A
+39*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2772, 2771}

$$\frac{2a^2(10A+9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(34A+39B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{8a^2(34A+39B)\sin(c+d}{315d\sqrt{a\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

```
[Out] (16*a^2*(34*A + 39*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Co
s[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d
*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(34*A + 39*B)*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(10*A + 9*B)*Sec[c + d*x]
^(7/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Co
s[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+a}{co}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{9}{2}}(c+dx)\sin(c+dx}{9d}$$

$$= \frac{2a^2(10A+9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2aA}{}$$

$$= \frac{2a^2(34A+39B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2}{}$$

$$= \frac{8a^2(34A+39B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2}{}$$

$$= \frac{16a^2(34A+39B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{8a}{}$$

Mathematica [A] time = 0.71, size = 124, normalized size = 0.54

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\,((374A+324B)\cos(c+dx)+11(34A+39B)\cos(2(c+dx))}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(376*A + 351*B + (374*A + 324*B)*Cos[c + d*x]
 + 11*(34*A + 39*B)*Cos[2*(c + d*x)] + 68*A*Cos[3*(c + d*x)] + 78*B*Cos[3*(
c + d*x)] + 68*A*Cos[4*(c + d*x)] + 78*B*Cos[4*(c + d*x)])*Sec[c + d*x]^(9/
2)*Tan[(c + d*x)/2])/(315*d)
```

fricas [A] time = 0.57, size = 126, normalized size = 0.55

$$\frac{2\left(8\,(34\,A+39\,B)a\cos\left(dx+c\right)^4+4\,(34\,A+39\,B)a\cos\left(dx+c\right)^3+3\,(34\,A+39\,B)a\cos\left(dx+c\right)^2+5\,(17\,A+9}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(8*(34*A + 39*B)*a*cos(d*x + c)^4 + 4*(34*A + 39*B)*a*cos(d*x + c)^3
+ 3*(34*A + 39*B)*a*cos(d*x + c)^2 + 5*(17*A + 9*B)*a*cos(d*x + c) + 35*A*a
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*cos(d*x + c)
^4)*sqrt(cos(d*x + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.38, size = 139, normalized size = 0.61

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(272A\left(\cos^{4}\left(dx+c\right)\right)+312B\left(\cos^{4}\left(dx+c\right)\right)+136A\left(\cos^{3}\left(dx+c\right)\right)+156B\left(\cos^{3}\left(dx+c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(272*A*cos(d*x+c)^4+312*B*cos(d*x+c)^4+136*A*cos(d
*x+c)^3+156*B*cos(d*x+c)^3+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+85*A*cos(d
*x+c)+45*B*cos(d*x+c)+35*A)*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+
c))^(11/2)/sin(d*x+c)*a
```

maxima [B] time = 0.51, size = 619, normalized size = 2.71

$$\frac{4\left(\frac{\left(\frac{315\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{840\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{1344\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{1242\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{517\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{94\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)A\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{4\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+1\right)}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 4/315*((315*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 840*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sqrt(2)*a^(3/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 1242*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 517*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 94*s
qrt(2)*a^(3/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(4*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 3*(105*sqrt(2)*a^(
3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 350*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + 518*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - 444*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 209*sqrt(2)*
a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 38*sqrt(2)*a^(3/2)*sin(d*x +
c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/
((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(11/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 + 1)))/d
```

mupad [B] time = 4.91, size = 316, normalized size = 1.39

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(\frac{96\,a\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\,\cos(c+d\,x)}\,(A+B)}{5\,d}-\frac{16\,B\,a\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,\sqrt{a+a\,\cos(c+d\,x)}}{3\,d}+\frac{16\,a\,e^{\frac{c}{\cdots}}\cdots}{\cdots}\right.}{12\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+8\,e^{\frac{c\,9i}{2}+\frac{d\,x\,9i}{2}}\,\cos\left(\frac{5\,c}{2}+\frac{5\,d}{2}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + a*cos(c + d*x))^(3/2)
,x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((96*a*exp((c*9i
)/2 + (d*x*9i)/2)*sin(c/2 + (d*x)/2)*(a + a*cos(c + d*x))^(1/2)*(A + B))/(5
*d) - (16*B*a*exp((c*9i)/2 + (d*x*9i)/2)*sin((3*c)/2 + (3*d*x)/2)*(a + a*co
s(c + d*x))^(1/2))/(3*d) + (16*a*exp((c*9i)/2 + (d*x*9i)/2)*sin((5*c)/2 + (
5*d*x)/2)*(34*A + 39*B)*(a + a*cos(c + d*x))^(1/2))/(35*d) + (32*a*exp((c*9
i)/2 + (d*x*9i)/2)*sin((9*c)/2 + (9*d*x)/2)*(34*A + 39*B)*(a + a*cos(c + d*
x))^(1/2))/(315*d)))/(12*exp((c*9i)/2 + (d*x*9i)/2)*cos(c/2 + (d*x)/2) + 8*
exp((c*9i)/2 + (d*x*9i)/2)*cos((3*c)/2 + (3*d*x)/2) + 8*exp((c*9i)/2 + (d*x
*9i)/2)*cos((5*c)/2 + (5*d*x)/2) + 2*exp((c*9i)/2 + (d*x*9i)/2)*cos((7*c)/2
 + (7*d*x)/2) + 2*exp((c*9i)/2 + (d*x*9i)/2)*cos((9*c)/2 + (9*d*x)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

3.503 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=181

$$\frac{2a^2(8A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(52A+63B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)}}$$

```
[Out] 2/105*a^2*(52*A+63*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+
2/35*a^2*(8*A+7*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7
*a*A*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+4/105*a^2*(52*A+6
3*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2772, 2771}

$$\frac{2a^2(8A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(52A+63B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (4*a^2*(52*A + 63*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos
[c + d*x]]) + (2*a^2*(52*A + 63*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*
Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(8*A + 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d
*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[
c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
```

```
f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+d}{\text{co}}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}$$

$$= \frac{2a^2(8A+7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a}}{}$$

$$= \frac{2a^2(52A+63B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(}{}$$

$$= \frac{4a^2(52A+63B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(}{}$$

Mathematica [A] time = 0.56, size = 102, normalized size = 0.56

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\,(3(78A+77B)\cos(c+dx)+(52A+63B)\cos(2(c+dx))+5}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(82*A + 63*B + 3*(78*A + 77*B)*Cos[c + d*x] +
 (52*A + 63*B)*Cos[2*(c + d*x)] + 52*A*Cos[3*(c + d*x)] + 63*B*Cos[3*(c + d
*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(105*d)
```

fricas [A] time = 0.52, size = 107, normalized size = 0.59

$$\frac{2\left(2\,(52\,A+63\,B)a\cos\left(dx+c\right)^{3}+(52\,A+63\,B)a\cos\left(dx+c\right)^{2}+3\,(13\,A+7\,B)a\cos\left(dx+c\right)+15\,Aa\right)\sqrt{a\cos(}}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(2*(52*A + 63*B)*a*cos(d*x + c)^3 + (52*A + 63*B)*a*cos(d*x + c)^2 +
3*(13*A + 7*B)*a*cos(d*x + c) + 15*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.36, size = 117, normalized size = 0.65

$$-\frac{2\left(-1+\cos \left(dx+c\right)\right)\left(104A\left(\cos^{3}\left(dx+c\right)\right)+126B\left(\cos^{3}\left(dx+c\right)\right)+52A\left(\cos^{2}\left(dx+c\right)\right)+63B\left(\cos^{2}\left(dx+c\right)\right)}{105d\sin \left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(104*A*cos(d*x+c)^3+126*B*cos(d*x+c)^3+52*A*cos(d*
x+c)^2+63*B*cos(d*x+c)^2+39*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*cos(d*x+c)*(
a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(9/2)/sin(d*x+c)*a
```

maxima [B] time = 0.51, size = 527, normalized size = 2.91

$$\frac{4\left(\frac{\left(\frac{105\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{245\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{273\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{171\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{38\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)A\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{3}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}+\frac{21\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}\right.}{\left(\right.}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 4/105*((105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1
)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 1)) + 21*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c
```

```
) + 1) - 15*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2
)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 9*sqrt(2)*a^(3/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1)))/d
```

mupad [B] time = 4.80, size = 259, normalized size = 1.43

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(-\frac{8\,a\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,A+3\,B)\,\sqrt{a+a\,\cos(c+d\,x)}}{3\,d}+\frac{16\,a\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\,(13\,A+12\,B)\,\sqrt{a+a\,\cos(c+}}{15\,d}\right.}{6\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)+2\,e^{\frac{c\,7i}{2}+\frac{d\,x\,7i}{2}}\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(3/2),
x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((16*a*exp((c*7i
)/2 + (d*x*7i)/2)*sin((3*c)/2 + (3*d*x)/2)*(13*A + 12*B)*(a + a*cos(c + d*x
))^(1/2))/(15*d) - (8*a*exp((c*7i)/2 + (d*x*7i)/2)*sin(c/2 + (d*x)/2)*(2*A
+ 3*B)*(a + a*cos(c + d*x))^(1/2))/(3*d) + (8*a*exp((c*7i)/2 + (d*x*7i)/2)*
sin((7*c)/2 + (7*d*x)/2)*(52*A + 63*B)*(a + a*cos(c + d*x))^(1/2))/(105*d))
)/(6*exp((c*7i)/2 + (d*x*7i)/2)*cos(c/2 + (d*x)/2) + 6*exp((c*7i)/2 + (d*x*
7i)/2)*cos((3*c)/2 + (3*d*x)/2) + 2*exp((c*7i)/2 + (d*x*7i)/2)*cos((5*c)/2
+ (5*d*x)/2) + 2*exp((c*7i)/2 + (d*x*7i)/2)*cos((7*c)/2 + (7*d*x)/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.504 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=134

$$\frac{2a^2(6A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(18A+25B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

```
[Out] 2/15*a^2*(6*A+5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5
*a*A*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+2/15*a^2*(18*A+25
*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2975, 2980, 2771}

$$\frac{2a^2(6A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(18A+25B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a^2*(18*A + 25*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[
c + d*x]]) + (2*a^2*(6*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt
[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*
Sin[c + d*x])/(5*d)
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+d}{\text{co}}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

$$= \frac{2a^2(6A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{a}}{}$$

$$= \frac{2a^2(18A+25B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2(}{}$$

Mathematica [A] time = 0.33, size = 80, normalized size = 0.60

$$\frac{a\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\,(2(9A+5B)\cos(c+dx)+(18A+25B)\cos(2(c+dx))+24A}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*(24*A + 25*B + 2*(9*A + 5*B)*Cos[c + d*x] + (
18*A + 25*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

fricas [A] time = 0.80, size = 88, normalized size = 0.66

$$\frac{2\left((18\,A+25\,B)a\cos\left(dx+c\right)^{2}+(9\,A+5\,B)a\cos\left(dx+c\right)+3\,Aa\right)\sqrt{a\cos\left(dx+c\right)+a}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*((18*A + 25*B)*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) + 3*A*a)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2
)*sqrt(cos(d*x + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.45, size = 95, normalized size = 0.71

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(18A\left(\cos^{2}\left(dx+c\right)\right)+25B\left(\cos^{2}\left(dx+c\right)\right)+9A\cos\left(dx+c\right)+5B\cos\left(dx+c\right)+3A\right)\cos}{15d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(18*A*cos(d*x+c)^2+25*B*cos(d*x+c)^2+9*A*cos(d*x+c)
+5*B*cos(d*x+c)+3*A)*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(7/
2)/sin(d*x+c)*a
```

maxima [B] time = 0.51, size = 436, normalized size = 3.25

$$\frac{4\left(\frac{3\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)A\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}+\frac{5\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{8\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+7\cdots}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx\cdots}{\cos(dx+\cdots}\right.}\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 4/15*(3*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(
3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7)*A*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) +
5*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 8*sqrt(2)*a^(3/2)*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*B*(
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1
) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

mupad [B] time = 2.44, size = 197, normalized size = 1.47

$$\frac{2\,a\,\sqrt{a}\,\left(\cos\left(c+d\,x\right)+1\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(48\,A\,\sin\left(c+d\,x\right)+50\,B\,\sin\left(c+d\,x\right)+36\,A\,\sin\left(2\,c+2\,d\,x\right)+66\,A\,\sin\cdots\right.}{15\,d\,\left(10\,\cos\left(c+d\,x\right)+8\,\cos\left(2\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(3/2),
x)
```

```
[Out] (2*a*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(48*A*sin(c + d*x)
 + 50*B*sin(c + d*x) + 36*A*sin(2*c + 2*d*x) + 66*A*sin(3*c + 3*d*x) + 18*A
*sin(4*c + 4*d*x) + 18*A*sin(5*c + 5*d*x) + 20*B*sin(2*c + 2*d*x) + 75*B*si
n(3*c + 3*d*x) + 10*B*sin(4*c + 4*d*x) + 25*B*sin(5*c + 5*d*x)))/(15*d*(10*
cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos(4*c + 4*d*x)
 + cos(5*c + 5*d*x) + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
[Out] Timed out
```

3.505 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=145

$$\frac{2a^{3/2}B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c+$$

```
[Out] 2/3*a*A*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+2*a^(3/2)*B*ar
csin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d+2/3*a^2*(4*A+3*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(
1/2)
```

Rubi [A] time = 0.45, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2774, 216}

$$\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{3/2}B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2aA\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a^(3/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(4*A + 3*B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c + d*
x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+d}{\text{co}}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

$$= \frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{}}{}$$

$$= \frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2aA\sqrt{}}{}$$

$$= \frac{2a^{3/2}B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{d}$$

Mathematica [A] time = 0.40, size = 106, normalized size = 0.73

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\,\left(2\sin\left(\frac{1}{2}(c+dx)\right)((5A+3B)\cos(c+dx)+A)+3\sqrt{2}B\sin^{}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(3*Sqrt[2
]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(A + (5*A + 3*B
)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(3*d)
```

fricas [A] time = 0.61, size = 130, normalized size = 0.90

$$-\frac{2\left(3\left(Ba\cos(dx+c)^2+Ba\cos(dx+c)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{((5A+3B)a\cos(dx+c)+Aa)\sqrt{a\cos(dx+c}}{\sqrt{\cos(dx+c)}}\right.}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -2/3*(3*(B*a*cos(d*x + c)^2 + B*a*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - ((5*A + 3*B)*a*co
s(d*x + c) + A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.47, size = 287, normalized size = 1.98

$$-\frac{2\left(3B\left(\cos^2\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+6B\cos\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(3*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+6*B*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c))+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+5*A*cos(d*x+c)*sin(d*x+c)+3*B*cos(d*x
+c)*sin(d*x+c)+A*sin(d*x+c))*cos(d*x+c)*sin(d*x+c)^2*(1/cos(d*x+c))^(5/2)*(
a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2*a
```

maxima [B] time = 0.71, size = 1462, normalized size = 10.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/6*(3*(6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*a^(3/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
```

```
((2*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c)
 - a*sin(2*d*x + 2*c) - 2*(a*cos(2*d*x + 2*c) + a)*sin(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + (2*a*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + a*cos(2*d*x + 2*c) + 2*(a*cos(2*d*x + 2*c) + a)*cos(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a)*sin(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + ((a*cos(2*d*x + 2*c)^2 + a*sin(2*d*
x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a*cos(2*d*x + 2*c)^2 + a*
sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - (a*cos(2*d*x + 2*c
)^2 + a*sin(2*d*x + 2*c)^2 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + 1) + (a*cos(2*d*x + 2*c)^2 + a*sin(2*d*x + 2*c)^2
 + 2*a*cos(2*d*x + 2*c) + a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
1))*sqrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1) + 8*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a
^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}(a+a\,\cos(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.506 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=146

$$\frac{a^{3/2}(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c$$

```
[Out] -a^2*(2*A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+a^(3/2)*(
2*A+3*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+2*a*A*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A] time = 0.47, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2981, 2774, 216}

$$\frac{a^{3/2}(2A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2aA\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^(3/2)*(2*A + 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^2*(2*A - B)*Sin[c + d*x])/(
d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Cos[c +
d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+d}{co}$$

$$= \frac{2aA\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

$$= -\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{a+}}{}$$

$$= -\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2aA\sqrt{a+}}{}$$

$$= \frac{a^{3/2}(2A+3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx}}{d}$$

Mathematica [A] time = 0.32, size = 107, normalized size = 0.73

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}(2A+3B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*
(2*A + 3*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*A +
B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(2*d)
```

fricas [A] time = 0.61, size = 119, normalized size = 0.82

$$-\frac{((2A+3B)a\cos(dx+c)+(2A+3B)a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(Ba\cos(dx+c)+2Aa)\sqrt{a\cos(dx+c)+a}\,s}{\sqrt{\cos(dx+c)}}}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -(((2*A + 3*B)*a*cos(d*x + c) + (2*A + 3*B)*a)*sqrt(a)*arctan(sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (B*a*cos(d*x + c)
+ 2*A*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d
*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.46, size = 308, normalized size = 2.11

$$\frac{\left(2A\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+3B\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(}{1+\cos}}}{\cos(dx+c)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] 1/d*(2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*B*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c))+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)/cos(d*x+c))+B*cos(d*x+c)*sin(d*x+c)+3*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos
(d*x+c))+2*A*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))
^(1/2)/(1+cos(d*x+c))*a
```

maxima [B] time = 0.85, size = 1801, normalized size = 12.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
```

```
2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*B + 2*(
(a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2
```

```
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*s
qrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+dx)) \left(\frac{1}{\cos(c+dx)}\right)^{3/2} (a+a\cos(c+dx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.507 $\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{a^{3/2}(12A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2(4A+5B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(}{}$$

```
[Out] 1/4*a^2*(4*A+5*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/2*
a*B*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*a^(3/2)*(12*A+
7*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.46, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2976, 2981, 2774, 216}

$$\frac{a^{3/2}(12A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2(4A+5B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{aB\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*(4*A + 5*B)*Sin[c +
d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a*B*Sqrt[a + a*C
os[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+a}{\sqrt{}}$$

$$= \frac{aB\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c\right.$$

$$= \frac{a^2(4A+5B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{aB\sqrt{a+a}}{2}$$

$$= \frac{a^2(4A+5B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{aB\sqrt{a+a}}{2}$$

$$= \frac{a^{3/2}(12A+7B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+d}}{4d}$$

Mathematica [A] time = 0.45, size = 121, normalized size = 0.79

$$\frac{a\sqrt{\cos(c+dx)}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\,(12A+7B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[
c + d*x]]*(Sqrt[2]*(12*A + 7*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[C
os[c + d*x]]*(4*A + 7*B + 2*B*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

fricas [A] time = 0.60, size = 133, normalized size = 0.87

$$-\frac{((12\,A+7\,B)a\cos(dx+c)+(12\,A+7\,B)a)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(2\,Ba\cos(dx+c)^2+(4\,A+7\,B)a\cos(dx\right.}{\sqrt{\cos(dx}}}{4\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(((12*A + 7*B)*a*cos(d*x + c) + (12*A + 7*B)*a)*sqrt(a)*arctan(sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*B*a*cos(d
*x + c)^2 + (4*A + 7*B)*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.39, size = 233, normalized size = 1.52

$$-\frac{\left(2B\sin\left(dx+c\right)\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+4A\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin\left(dx+c\right)+7B\sin\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+12A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+7*B*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)+12*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c))+7*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c
)))*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*a
```

maxima [B] time = 0.86, size = 1884, normalized size = 12.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
```

^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A +
(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c)
+ a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*arctan2
((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^

```
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - 1))*sqrt(a))*B)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}\,(a+a\cos(c+dx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.508 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=200

$$\frac{a^{3/2}(14A+11B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2(6A+7B)\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a}{8d\sqrt{\mathrm{se}}}$$

```
[Out] 1/12*a^2*(6*A+7*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/3
*a*B*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+1/8*a^2*(14*A+11*
B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/8*a^(3/2)*(14*A+1
1*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.54, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2976, 2981, 2770, 2774, 216}

$$\frac{a^2(6A+7B)\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(14A+11B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a}{8d\sqrt{\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(6*A + 7*B)*Sin[c +
 d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a*B*Sqrt[a + a
*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (a^2*(14*A + 11*B)*
Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+$$

$$= \frac{aB\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}}{3d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}}{3d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}}{3d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{a^{3/2}(14A+11B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{8d}$$

Mathematica [A] time = 0.50, size = 141, normalized size = 0.70

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\,(14A+11B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt[2
]*(14*A + 11*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (42*A
 + 37*B + 2*(6*A + 11*B)*Cos[c + d*x] + 4*B*Cos[2*(c + d*x)])*(-Sin[(c + d*
x)/2] + Sin[(3*(c + d*x))/2])))/(48*d)
```

fricas [A] time = 0.69, size = 153, normalized size = 0.76

$$-\frac{3\left((14A+11B)a\cos(dx+c)+(14A+11B)a\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right) - \frac{\left(8Ba\cos(dx+c)^3+2(6A+11B)\right.}{}}{24\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/24*(3*((14*A + 11*B)*a*cos(d*x + c) + (14*A + 11*B)*a)*sqrt(a)*arctan(sq
rt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*B*a*
cos(d*x + c)^3 + 2*(6*A + 11*B)*a*cos(d*x + c)^2 + 3*(14*A + 11*B)*a*cos(d*
x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*
x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.45, size = 309, normalized size = 1.54

$$\frac{(-1+\cos(dx+c))^2\left(8B\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+12A\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/24/d*(-1+cos(d*x+c))^2*(8*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)+12*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2
2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+42*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+33*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)+42*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+
c))+33*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*c
os(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/cos
(d*x+c))^(1/2)/sin(d*x+c)^4*a
```

maxima [B] time = 1.03, size = 3023, normalized size = 15.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")

[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
 + 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (4*(a*cos(3/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(
3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(3/4)*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c),
```

```
cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) -
 (3*a*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1))) + 1) - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/
4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))) - 1) - a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/
2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(
1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2
((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(
```

```
sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)) - 1))*sqrt(a))*B)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\ (a+a\cos(c+dx))^{3/2}}{\sqrt{\frac{1}{\cos(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.509 $\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=247

$$\frac{a^{3/2}(88A+75B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^2(88A+75B)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a}{24d\sec\cdots}$$

```
[Out] 1/24*a^2*(8*A+9*B)*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+1/9
6*a^2*(88*A+75*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/4*
a*B*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+1/64*a^2*(88*A+75*
B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/64*a^(3/2)*(88*A+
75*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.64, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2976, 2981, 2770, 2774, 216}

$$\frac{a^2(88A+75B)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(8A+9B)\sin(c+dx)}{24d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^{3/2}(88A+75B)\sqrt{\cos(c+dx)}\sqrt{\sec(\cdots}}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(8*A + 9*B)*Sin[c
+ d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a*B*Sqrt[a +
a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sec[c + d*x]^(5/2)) + (a^2*(88*A + 75*B)
*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*(8
8*A + 75*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
```

```
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx$$

$$= \frac{aB\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}\,\mathrm{s}}{4d\sec^{\frac{5}{2}}(c+d}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}\,\mathrm{s}}{4d\sec^{\frac{5}{2}}(c+d}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}\,\mathrm{s}}{4d\sec^{\frac{5}{2}}(c+d}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{aB\sqrt{a+a\cos(c+dx)}\,\mathrm{s}}{4d\sec^{\frac{5}{2}}(c+d}$$

$$= \frac{a^{3/2}(88A+75B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}{64d}$$

Mathematica [**A**] time = 0.79, size = 158, normalized size = 0.64

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\sqrt{a(\cos(c+dx)+1)}\,\left(3\sqrt{2}\,(88A+75B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt[2
]*(88*A + 75*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (296*
A + 285*B + 2*(88*A + 93*B)*Cos[c + d*x] + 4*(8*A + 15*B)*Cos[2*(c + d*x)]
+ 12*B*Cos[3*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(384*
d)
```

fricas [A] time = 0.69, size = 171, normalized size = 0.69

$$-\frac{3\left((88A+75B)a\cos(dx+c)+(88A+75B)a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(48\,Ba\cos(dx+c)^4+8\,(8\,A+15\,}{}}{192\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/192*(3*((88*A + 75*B)*a*cos(d*x + c) + (88*A + 75*B)*a)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (48*B*
a*cos(d*x + c)^4 + 8*(8*A + 15*B)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(
d*x + c)^2 + 3*(88*A + 75*B)*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.36, size = 381, normalized size = 1.54

$$-\frac{(-1+\cos(dx+c))^3\left(48B\sin(dx+c)\left(\cos^3(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+64A\left(\cos^2(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/192/d*(-1+cos(d*x+c))^3*(48*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)+64*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)+120*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+176*A*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+150*B*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+264*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)+225*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+264*A*
```

```
arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+225*B*arcta
n(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)*(a*(
1+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+c))^(3/2)
/sin(d*x+c)^6*a
```

maxima [B] time = 1.47, size = 8901, normalized size = 36.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*
d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 11*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqr
t(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/
4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(si
n(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1)
 - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
```

```
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*arc
tan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqrt(
a))*A + 3*(2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((5*a*cos(4*d*x + 4*c)^2*sin(4*
d*x + 4*c) + 5*a*sin(4*d*x + 4*c)^3 + 20*(a*sin(4*d*x + 4*c)^3 + (a*cos(4*d
*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(4*d*x + 4*c))*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 20*(a*sin(4*d*x + 4*c)^3 + (a*cos(4
*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(4*d*x + 4*c))*sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 5*(2*a*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c) - 2*(a*cos
(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*co
s(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 20*(a*sin(4*d*x + 4*c)
^3 + (a*cos(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (8*a*cos(4*d*x + 4*c)^2 + 32
*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*c
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os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*a*sin(4*d*x + 4*c
)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c
) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 5*a*cos(4*d
*x + 4*c) + 2*(16*a*cos(4*d*x + 4*c)^2 + 16*a*sin(4*d*x + 4*c)^2 - 21*a*cos
(4*d*x + 4*c) + 5*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) -
 2*(64*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4
*c) + 21*a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 20*(4*a*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + a*sin
(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(
3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (5*a*cos(4*d*x + 4*c)^3
 - 8*a*cos(4*d*x + 4*c)^2 + 4*(5*a*cos(4*d*x + 4*c)^3 - 18*a*cos(4*d*x + 4*
c)^2 + (5*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 21*a*cos(4*d*x + 4
*c) - 8*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (5*a*co
s(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 4*(5*a*cos(4*d*x + 4*c)^3 + 2*a*
cos(4*d*x + 4*c)^2 + (5*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 - 11*a
*cos(4*d*x + 4*c) - 8*a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))^2 + (8*a*cos(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4
*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^2 + 8*a*sin(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4
*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 - 5*a*cos(4*d*x + 4*c) + 2*(16*a*cos(4*d*x + 4*c)^2 +
 16*a*sin(4*d*x + 4*c)^2 - 21*a*cos(4*d*x + 4*c) + 5*a)*cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 21*a*sin(4*d*x + 4*c))*sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 4*(5*a*cos(4*d*x + 4*c)^3 - 13*a*cos(4*d*x + 4*c)^2
+ (5*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c)^2 + 8*a*cos(4*d*x + 4*c))*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 5*(2*a*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c
) - 2*(a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(5*a*
cos(4*d*x + 4*c) - 8*a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)*sin(4*d*x + 4*c) + (5*a*cos(4*d*x + 4*c) - 8*a)*sin(4*d*x + 4*c))*sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((3*a
*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a*sin(4*d*x + 4*c)^3 - 64*(a*cos(4
*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 12*(a*sin(4*d*x + 4*c)^3 + (
a*cos(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(4*d*x + 4*c) - 24*(a*c
os(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*sin(1/
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4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + 3*a*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))*sin(4*d*x + 4*c) + 4*(3*a*sin(4*d*x + 4*c)^3 + 64*a*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (3*a*cos(4*d*x
 + 4*c)^2 + 6*a*cos(4*d*x + 4*c) + 19*a)*sin(4*d*x + 4*c) - 72*(a*cos(4*d*x
 + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + 6*(2*a*sin(4*d*x + 4*c)^3 + a*cos(1/4*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 2*(a*cos(4*d*x + 4*c)^2 -
a*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) - (48*a*cos(4*d*x + 4*c)^2 + 48*a*sin(
4*d*x + 4*c)^2 - 47*a*cos(4*d*x + 4*c) - a)*sin(1/4*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 - 2*(8*a*cos(4*d*x + 4*c)^2 + 32*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c
)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + 14*a*sin(4*d*x + 4*c)^2 - 141*a*sin(4*d*x + 4*c)*sin(1/4*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*(4*a*cos(4*d*x + 4*c)^2 + 7*a*
sin(4*d*x + 4*c)^2 - 72*a*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) - 4*a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + 3*(a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) - 3*(24*a*cos(4*d*x + 4*c)^2 + 24*a*sin(4*d*x + 4*c)^2 + a*cos(4*d
*x + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*ar
ctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (3*a*cos(4*d*x + 4*c)^3 - 64*
(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 56*a*cos(4*d*x + 4*c
)^2 + 4*(3*a*cos(4*d*x + 4*c)^3 + 34*a*cos(4*d*x + 4*c)^2 + (3*a*cos(4*d*x
+ 4*c) + 40*a)*sin(4*d*x + 4*c)^2 - 93*a*cos(4*d*x + 4*c) - 40*(a*cos(4*d*x
 + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 56*a)*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + (3*a*cos(4*d*x + 4*c) + 56*a)*sin(4*d*x + 4*c)
^2 + 4*(3*a*cos(4*d*x + 4*c)^3 + 62*a*cos(4*d*x + 4*c)^2 + (3*a*cos(4*d*x +
 4*c) + 56*a)*sin(4*d*x + 4*c)^2 + 115*a*cos(4*d*x + 4*c) - 16*(a*cos(4*d*x
 + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 40*(a*cos(4*d*x + 4*c)^2 + a*sin(4
*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*cos(1/4*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 56*a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 - 3*a*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 2*(6*a*cos(4*d*x + 4*c)^3 + 98*a*cos(4*d*x + 4*c)^2 + 2*(3*a*cos
(4*d*x + 4*c) + 52*a)*sin(4*d*x + 4*c)^2 - 3*a*sin(4*d*x + 4*c)*sin(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 112*a*cos(4*d*x + 4*c) - (80*a*
cos(4*d*x + 4*c)^2 + 80*a*sin(4*d*x + 4*c)^2 - 77*a*cos(4*d*x + 4*c) - 3*a)
*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) - (40*a*cos(4*d*x + 4*c)^2 + 40*a*sin(4*d*
x + 4*c)^2 + 3*a*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*
```

d*x + 4*c))) + 2*(128*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2*sin(4*d*x + 4*c) + 77*a*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(40*a*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - (3*a*cos(4*d*x + 4*c) + 52*a)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(3*a*cos(4*d*x + 4*c) + 56*a)*sin(4*d*x + 4*c) + 3*(a*cos(4*d*x + 4*c) + a)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 75*((a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (a*cos(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4

*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a*cos(4*d*x + 4*c)^2 + 4*(a*co
s(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4*d*x + 4*c) + a)*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a*sin(4*d*x + 4*c)^2 + 4*
(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*cos(4*d*x + 4*c) + a)*si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a*cos(4*d*x + 4*c
)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(
1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
, cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + (a*cos(
4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - 2*a*cos(4
*d*x + 4*c) + a)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + a
*sin(4*d*x + 4*c)^2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 + 2*a*
cos(4*d*x + 4*c) + a)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 4*(a*cos(4*d*x + 4*c)^2 + a*sin(4*d*x + 4*c)^2 - a*cos(4*d*x + 4*c))*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a*sin(4*d*x + 4*c)
)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)),
(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)) - 1))*sqrt(a))*B/(4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*
cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*si

```
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 +
 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*
x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))))/d
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2
), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.510 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{15}{2}}(c+dx)\,dx$

Optimal. Leaf size=322

$$\frac{2a^3(280A+299B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{1287d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(4184A+4615B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{9009d\sqrt{a\cos(c+dx)+a}}+\frac{4a^3(4184A+461}{15015d}$$

```
[Out] 2/13*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(13/2)*sin(d*x+c)/d+16/45045*a^3
*(4184*A+4615*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+4/150
15*a^3*(4184*A+4615*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
+2/9009*a^3*(4184*A+4615*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^
(1/2)+2/1287*a^3*(280*A+299*B)*sec(d*x+c)^(9/2)*sin(d*x+c)/d/(a+a*cos(d*x+c
))^(1/2)+2/143*a^2*(16*A+13*B)*sec(d*x+c)^(11/2)*sin(d*x+c)*(a+a*cos(d*x+c)
)^(1/2)/d+32/45045*a^3*(4184*A+4615*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*c
os(d*x+c))^(1/2)
```

Rubi [A] time = 0.94, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2772, 2771}

$$\frac{2a^2(16A+13B)\sin(c+dx)\sec^{\frac{11}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{143d}+\frac{2a^3(280A+299B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{1287d\sqrt{a\cos(c+dx)+a}}+\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(15/2),x]
```

```
[Out] (32*a^3*(4184*A + 4615*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(45045*d*Sqrt[a
 + a*Cos[c + d*x]]) + (16*a^3*(4184*A + 4615*B)*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(45045*d*Sqrt[a + a*Cos[c + d*x]]) + (4*a^3*(4184*A + 4615*B)*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(15015*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(4184*
A + 4615*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(9009*d*Sqrt[a + a*Cos[c + d*x
]]) + (2*a^3*(280*A + 299*B)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(1287*d*Sqrt[
a + a*Cos[c + d*x]]) + (2*a^2*(16*A + 13*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c
+ d*x]^(11/2)*Sin[c + d*x])/(143*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec
[c + d*x]^(13/2)*Sin[c + d*x])/(13*d)
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{15}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+}{\text{co}}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{13}{2}}(c+dx)\sin(c+}{13d}$$

$$= \frac{2a^2(16A+13B)\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{11}{2}}(c+dx}{143d}$$

$$= \frac{2a^3(280A+299B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{1287d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

$$= \frac{2a^3(4184A+4615B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9009d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{4a^3(4184A+4615B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15015d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{16a^3(4184A+4615B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45045d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{32a^3(4184A+4615B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{45045d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.91, size = 171, normalized size = 0.53

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{13}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\,(35(5552A+5083B)\cos(c+dx)+14(15167A+15925B)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(15/
2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(171806*A + 162955*B + 35*(5552*A + 5083*B)
*Cos[c + d*x] + 14*(15167*A + 15925*B)*Cos[2*(c + d*x)] + 62760*A*Cos[3*(c
+ d*x)] + 69225*B*Cos[3*(c + d*x)] + 62760*A*Cos[4*(c + d*x)] + 69225*B*Cos
```

```
[4*(c + d*x)] + 8368*A*Cos[5*(c + d*x)] + 9230*B*Cos[5*(c + d*x)] + 8368*A*
Cos[6*(c + d*x)] + 9230*B*Cos[6*(c + d*x)])*Sec[c + d*x]^(13/2)*Tan[(c + d*
x)/2])/(90090*d)
```

fricas [A] time = 0.57, size = 176, normalized size = 0.55

$$\frac{2\left(16\,(4184\,A+4615\,B)a^2\cos\left(dx+c\right)^6+8\,(4184\,A+4615\,B)a^2\cos\left(dx+c\right)^5+6\,(4184\,A+4615\,B)a^2\cos\left(dx+\right.}{4504}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x, algo
rithm="fricas")
```

```
[Out] 2/45045*(16*(4184*A + 4615*B)*a^2*cos(d*x + c)^6 + 8*(4184*A + 4615*B)*a^2*
cos(d*x + c)^5 + 6*(4184*A + 4615*B)*a^2*cos(d*x + c)^4 + 5*(4184*A + 4615*
B)*a^2*cos(d*x + c)^3 + 35*(523*A + 416*B)*a^2*cos(d*x + c)^2 + 315*(38*A +
 13*B)*a^2*cos(d*x + c) + 3465*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
/((d*cos(d*x + c)^7 + d*cos(d*x + c)^6)*sqrt(cos(d*x + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.48, size = 185, normalized size = 0.57

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(66944A\left(\cos^6\left(dx+c\right)\right)+73840B\left(\cos^6\left(dx+c\right)\right)+33472A\left(\cos^5\left(dx+c\right)\right)+36920B\left(\mathrm{co}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x)
```

```
[Out] -2/45045/d*(-1+cos(d*x+c))*(66944*A*cos(d*x+c)^6+73840*B*cos(d*x+c)^6+33472
*A*cos(d*x+c)^5+36920*B*cos(d*x+c)^5+25104*A*cos(d*x+c)^4+27690*B*cos(d*x+c
)^4+20920*A*cos(d*x+c)^3+23075*B*cos(d*x+c)^3+18305*A*cos(d*x+c)^2+14560*B*
cos(d*x+c)^2+11970*A*cos(d*x+c)+4095*B*cos(d*x+c)+3465*A)*cos(d*x+c)*(a*(1+
cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(15/2)/sin(d*x+c)*a^2
```

maxima [B] time = 0.53, size = 763, normalized size = 2.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(15/2),x, algo
rithm="maxima")
```

```
[Out] 8/45045*((45045*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 165165*sq
rt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 414414*sqrt(2)*a^(5/2)*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 604890*sqrt(2)*a^(5/2)*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 + 522665*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 - 289185*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 88
980*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 11864*sqrt(2)*a
^(5/2)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*A*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
10*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c
) + 1)^10 + 1)) + 65*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) -
 3003*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6930*sqrt(2)*a^
(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 10098*sqrt(2)*a^(5/2)*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 9053*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x +
 c) + 1)^9 - 4875*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 1
500*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 200*sqrt(2)*a^(
5/2)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)*B*(sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 1)^5/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(15/2)*(-sin(d*x + c)
/(cos(d*x + c) + 1) + 1)^(15/2)*(5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + 5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + sin(d*x + c)^10/(cos(d*x + c)
+ 1)^10 + 1)))/d
```

mupad [B] time = 6.19, size = 789, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(15/2)*(a + a*cos(c + d*x))^(5/2)
,x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(a + a*(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(4184*A + 4615*B)*32i)/
(45045*d) - (a^2*exp(c*5i + d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*
1i + d*x*1i)/2))^(1/2)*(2*A + 5*B)*16i)/(5*d) + (a^2*exp(c*8i + d*x*8i)*(a
```

```
+ a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(2*A + 5*B)*16i)
/(5*d) + (a^2*exp(c*6i + d*x*6i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i
+ d*x*1i)/2))^(1/2)*(116*A + 115*B)*16i)/(35*d) - (a^2*exp(c*7i + d*x*7i)*(
a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(116*A + 115*B
)*16i)/(35*d) + (a^2*exp(c*4i + d*x*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 + ex
p(c*1i + d*x*1i)/2))^(1/2)*(1046*A + 1075*B)*16i)/(315*d) - (a^2*exp(c*9i +
 d*x*9i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(104
6*A + 1075*B)*16i)/(315*d) + (a^2*exp(c*2i + d*x*2i)*(a + a*(exp(- c*1i - d
*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(4184*A + 4615*B)*16i)/(3465*d) - (
a^2*exp(c*11i + d*x*11i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i
)/2))^(1/2)*(4184*A + 4615*B)*16i)/(3465*d) - (a^2*exp(c*13i + d*x*13i)*(a
+ a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(4184*A + 4615*B
)*32i)/(45045*d)))/(exp(c*1i + d*x*1i) + 6*exp(c*2i + d*x*2i) + 6*exp(c*3i
+ d*x*3i) + 15*exp(c*4i + d*x*4i) + 15*exp(c*5i + d*x*5i) + 20*exp(c*6i + d
*x*6i) + 20*exp(c*7i + d*x*7i) + 15*exp(c*8i + d*x*8i) + 15*exp(c*9i + d*x*
9i) + 6*exp(c*10i + d*x*10i) + 6*exp(c*11i + d*x*11i) + exp(c*12i + d*x*12i
) + exp(c*13i + d*x*13i) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(15/2),x)
[Out] Timed out
```

3.511 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=275

$$\frac{2a^3(194A+209B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(710A+803B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{1155d\sqrt{a\cos(c+dx)+a}}+\frac{8a^3(710A+803B)\text{s}}{3465d\sqrt{a\,\text{c}}}$$

```
[Out] 2/11*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(11/2)*sin(d*x+c)/d+8/3465*a^3*(
710*A+803*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/1155*a^
3*(710*A+803*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/693*
a^3*(194*A+209*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/99
*a^2*(14*A+11*B)*sec(d*x+c)^(9/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+16/34
65*a^3*(710*A+803*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2772, 2771}

$$\frac{2a^2(14A+11B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{99d}+\frac{2a^3(194A+209B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/2),x]
```

```
[Out] (16*a^3*(710*A + 803*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a
*Cos[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
3465*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(710*A + 803*B)*Sec[c + d*x]^(5/2
)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(194*A + 209*B)*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99
*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(
11*d)
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+\ldots}{\mathrm{co}\ldots}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{11}{2}}(c+dx)\sin(c+\ldots}{11d}$$

$$= \frac{2a^2(14A+11B)\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{9}{2}}(c+dx\ldots}{99d}$$

$$= \frac{2a^3(194A+209B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}} + \frac{2\ldots}{\ldots}$$

$$= \frac{2a^3(710A+803B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{1155d\sqrt{a+a\cos(c+dx)}} + \frac{2\ldots}{\ldots}$$

$$= \frac{8a^3(710A+803B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}} + \frac{2\ldots}{\ldots}$$

$$= \frac{16a^3(710A+803B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3465d\sqrt{a+a\cos(c+dx)}} + \ldots$$

Mathematica [A] time = 1.27, size = 147, normalized size = 0.53

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{11}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\,((25070A+24827B)\cos(c+dx)+(9230A+9284B)\cos(2\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/
2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(9070*A + 7678*B + (25070*A + 24827*B)*Cos[
c + d*x] + (9230*A + 9284*B)*Cos[2*(c + d*x)] + 9230*A*Cos[3*(c + d*x)] + 1
0439*B*Cos[3*(c + d*x)] + 1420*A*Cos[4*(c + d*x)] + 1606*B*Cos[4*(c + d*x)]
 + 1420*A*Cos[5*(c + d*x)] + 1606*B*Cos[5*(c + d*x)])*Sec[c + d*x]^(11/2)*T
an[(c + d*x)/2])/(6930*d)
```

fricas [A] time = 0.69, size = 156, normalized size = 0.57

$$\frac{2\left(8\,(710\,A+803\,B)a^2\cos\left(dx+c\right)^5+4\,(710\,A+803\,B)a^2\cos\left(dx+c\right)^4+3\,(710\,A+803\,B)a^2\cos\left(dx+c\right)^3+5\,(\right.}{3465\left(d\cos\left(dx+c\right)^6+d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(8*(710*A + 803*B)*a^2*cos(d*x + c)^5 + 4*(710*A + 803*B)*a^2*cos(d*
x + c)^4 + 3*(710*A + 803*B)*a^2*cos(d*x + c)^3 + 5*(355*A + 286*B)*a^2*cos
(d*x + c)^2 + 35*(32*A + 11*B)*a^2*cos(d*x + c) + 315*A*a^2)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^6 + d*cos(d*x + c)^5)*sqrt(cos(d*x
 + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.46, size = 163, normalized size = 0.59

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(5680A\left(\cos^5\left(dx+c\right)\right)+6424B\left(\cos^5\left(dx+c\right)\right)+2840A\left(\cos^4\left(dx+c\right)\right)+3212B\left(\cos^4\left(d\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x)
```

```
[Out] -2/3465/d*(-1+cos(d*x+c))*(5680*A*cos(d*x+c)^5+6424*B*cos(d*x+c)^5+2840*A*c
os(d*x+c)^4+3212*B*cos(d*x+c)^4+2130*A*cos(d*x+c)^3+2409*B*cos(d*x+c)^3+177
5*A*cos(d*x+c)^2+1430*B*cos(d*x+c)^2+1120*A*cos(d*x+c)+385*B*cos(d*x+c)+315
*A)*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(13/2)/sin(d*x+c)*a^
2
```

maxima [B] time = 0.52, size = 672, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] 8/3465*(5*(693*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 2310*sqrt(
2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4620*sqrt(2)*a^(5/2)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 5478*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 3575*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
 1300*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 200*sqrt(2)*a
^(5/2)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)*A*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^4/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x +
c)/(cos(d*x + c) + 1) + 1)^(13/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1)) + 11*(315*sqrt(2)*a^(5/2)*si
n(d*x + c)/(cos(d*x + c) + 1) - 1260*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 2394*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
2736*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1859*sqrt(2)*a^(
5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 676*sqrt(2)*a^(5/2)*sin(d*x + c)
^11/(cos(d*x + c) + 1)^11 + 104*sqrt(2)*a^(5/2)*sin(d*x + c)^13/(cos(d*x +
c) + 1)^13)*B*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/((sin(d*x + c)/(c
os(d*x + c) + 1) + 1)^(13/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(13/2)*
(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 + 1)))/d
```

mupad [B] time = 5.81, size = 751, normalized size = 2.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(13/2)*(a + a*cos(c + d*x))^(5/2)
,x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(a + a*(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(710*A + 803*B)*16i)/(3
465*d) - (B*a^2*exp(c*3i + d*x*3i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1
i + d*x*1i)/2))^(1/2)*8i)/(3*d) + (B*a^2*exp(c*8i + d*x*8i)*(a + a*(exp(- c
*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*8i)/(3*d) - (a^2*exp(c*5i +
d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(30*A
 + 41*B)*8i)/(15*d) + (a^2*exp(c*6i + d*x*6i)*(a + a*(exp(- c*1i - d*x*1i)/
2 + exp(c*1i + d*x*1i)/2))^(1/2)*(30*A + 41*B)*8i)/(15*d) + (a^2*exp(c*4i +
 d*x*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(160
*A + 157*B)*8i)/(35*d) - (a^2*exp(c*7i + d*x*7i)*(a + a*(exp(- c*1i - d*x*1
i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(160*A + 157*B)*8i)/(35*d) + (a^2*exp(c
```

```
*2i + d*x*2i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)
*(710*A + 803*B)*8i)/(315*d) - (a^2*exp(c*9i + d*x*9i)*(a + a*(exp(- c*1i -
 d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(710*A + 803*B)*8i)/(315*d) - (a^
2*exp(c*11i + d*x*11i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/
2))^(1/2)*(710*A + 803*B)*16i)/(3465*d)))/(exp(c*1i + d*x*1i) + 5*exp(c*2i
+ d*x*2i) + 5*exp(c*3i + d*x*3i) + 10*exp(c*4i + d*x*4i) + 10*exp(c*5i + d*
x*5i) + 10*exp(c*6i + d*x*6i) + 10*exp(c*7i + d*x*7i) + 5*exp(c*8i + d*x*8i
) + 5*exp(c*9i + d*x*9i) + exp(c*10i + d*x*10i) + exp(c*11i + d*x*11i) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

3.512 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=228

$$\frac{2a^3(124A+135B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(292A+345B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{4a^3(292A+345B)\text{s}}{315d\sqrt{a\,\text{co}}}$$

```
[Out] 2/9*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2)*sin(d*x+c)/d+2/315*a^3*(292
*A+345*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/315*a^3*(1
24*A+135*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/21*a^2*(
4*A+3*B)*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+4/315*a^3*(29
2*A+345*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2772, 2771}

$$\frac{2a^2(4A+3B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{21d}+\frac{2a^3(124A+135B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(2\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

```
[Out] (4*a^3*(292*A + 345*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^3*(124*A + 135*B)*Sec[c + d*x]^(5/2)*S
in[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(4*A + 3*B)*Sqrt[a +
 a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(21*d) + (2*a*A*(a + a*Co
s[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+}{\text{co}}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\sin(c+d}{9d}$$

$$= \frac{2a^2(4A+3B)\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{7}{2}}(c+dx)\,\text{s}}{21d}$$

$$= \frac{2a^3(124A+135B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

$$= \frac{2a^3(292A+345B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

$$= \frac{4a^3(292A+345B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2}{}$$

Mathematica [A] time = 0.95, size = 126, normalized size = 0.55

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\,((1396A+1215B)\cos(c+dx)+2(803A+870B)\cos(2(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(1454*A + 1395*B + (1396*A + 1215*B)*Cos[c
+ d*x] + 2*(803*A + 870*B)*Cos[2*(c + d*x)] + 292*A*Cos[3*(c + d*x)] + 345*
B*Cos[3*(c + d*x)] + 292*A*Cos[4*(c + d*x)] + 345*B*Cos[4*(c + d*x)])*Sec[c
 + d*x]^(9/2)*Tan[(c + d*x)/2])/(630*d)
```

fricas [A] time = 0.75, size = 135, normalized size = 0.59

$$\frac{2\left(2\,(292\,A+345\,B)a^2\cos\left(dx+c\right)^4+(292\,A+345\,B)a^2\cos\left(dx+c\right)^3+3\,(73\,A+60\,B)a^2\cos\left(dx+c\right)^2+5\,(2}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)\sqrt{\cos\left(dx\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 2/315*(2*(292*A + 345*B)*a^2*cos(d*x + c)^4 + (292*A + 345*B)*a^2*cos(d*x +
 c)^3 + 3*(73*A + 60*B)*a^2*cos(d*x + c)^2 + 5*(26*A + 9*B)*a^2*cos(d*x + c
) + 35*A*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*
cos(d*x + c)^4)*sqrt(cos(d*x + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.40, size = 141, normalized size = 0.62

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(584A\left(\cos^{4}\left(dx+c\right)\right)+690B\left(\cos^{4}\left(dx+c\right)\right)+292A\left(\cos^{3}\left(dx+c\right)\right)+345B\left(\cos^{3}\left(dx+c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(584*A*cos(d*x+c)^4+690*B*cos(d*x+c)^4+292*A*cos(d
*x+c)^3+345*B*cos(d*x+c)^3+219*A*cos(d*x+c)^2+180*B*cos(d*x+c)^2+130*A*cos(
d*x+c)+45*B*cos(d*x+c)+35*A)*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x
+c))^(11/2)/sin(d*x+c)*a^2
```

maxima [B] time = 0.50, size = 579, normalized size = 2.54

$$\frac{8\left(\frac{\left(\frac{315\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{945\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{1449\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{1287\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{572\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{104\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)A\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right.}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}\right.}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 8/315*((315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*
sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*A*(sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-s
in(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 1)) + 15*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) -
77*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 119*sqrt(2)*a^(5/2
)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 99*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 44*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*B*(sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 1)^3/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 1)))/d
```

mupad [B] time = 5.73, size = 617, normalized size = 2.71

$$\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(292\,A+345\,B)\,4i}{315\,d}-\frac{a^2\,e^{c\,3i+d\,x\,3i}\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(2\,A+5\,B)\,4i}{3\,d}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + a*cos(c + d*x))^(5/2)
,x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(a + a*(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(292*A + 345*B)*4i)/(31
5*d) - (a^2*exp(c*3i + d*x*3i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i +
d*x*1i)/2))^(1/2)*(2*A + 5*B)*4i)/(3*d) + (a^2*exp(c*6i + d*x*6i)*(a + a*(e
xp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(2*A + 5*B)*4i)/(3*d)
+ (a^2*exp(c*4i + d*x*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1
i)/2))^(1/2)*(24*A + 25*B)*4i)/(5*d) - (a^2*exp(c*5i + d*x*5i)*(a + a*(exp(
- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(24*A + 25*B)*4i)/(5*d) +
 (a^2*exp(c*2i + d*x*2i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i
)/2))^(1/2)*(146*A + 155*B)*4i)/(35*d) - (a^2*exp(c*7i + d*x*7i)*(a + a*(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(146*A + 155*B)*4i)/(35
*d) - (a^2*exp(c*9i + d*x*9i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d
*x*1i)/2))^(1/2)*(292*A + 345*B)*4i)/(315*d)))/(exp(c*1i + d*x*1i) + 4*exp(
c*2i + d*x*2i) + 4*exp(c*3i + d*x*3i) + 6*exp(c*4i + d*x*4i) + 6*exp(c*5i +
```

```
d*x*5i) + 4*exp(c*6i + d*x*6i) + 4*exp(c*7i + d*x*7i) + exp(c*8i + d*x*8i)
 + exp(c*9i + d*x*9i) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

3.513 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=181

$$\frac{2a^3(10A+11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(230A+301B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(10A+7B)\sin(c+$$

```
[Out] 2/7*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2/15*a^3*(10*A
+11*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/35*a^2*(10*A+
7*B)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+2/105*a^3*(230*A+
301*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2975, 2980, 2771}

$$\frac{2a^2(10A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{35d}+\frac{2a^3(10A+11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^3(230$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^3*(230*A + 301*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*C
os[c + d*x]]) + (2*a^3*(10*A + 11*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d
*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(10*A + 7*B)*Sqrt[a + a*Cos[c + d*x]]*S
ec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a*A*(a + a*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a + a\cos(c+dx))^{5/2}(A + B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a + a\cos(c + d}{\text{co}}$$

$$= \frac{2aA(a + a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx}{7d}$$

$$= \frac{2a^2(10A + 7B)\sqrt{a + a\cos(c+dx)}\ \sec^{\frac{5}{2}}(c+dx)\,\text{s}}{35d}$$

$$= \frac{2a^3(10A + 11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a + a\cos(c+dx)}} + \frac{2a^2(1}{}$$

$$= \frac{2a^3(230A + 301B)\sqrt{\sec(c+dx)}\ \sin(c+dx)}{105d\sqrt{a + a\cos(c+dx)}} + \frac{2a}{}$$

Mathematica [A] time = 0.70, size = 104, normalized size = 0.57

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\,((930A + 987B)\cos(c+dx) + 2(115A + 98B)\cos(2(c+dx)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(290*A + 196*B + (930*A + 987*B)*Cos[c + d*
x] + 2*(115*A + 98*B)*Cos[2*(c + d*x)] + 230*A*Cos[3*(c + d*x)] + 301*B*Cos
[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(210*d)
```

fricas [A] time = 0.57, size = 114, normalized size = 0.63

$$\frac{2\left((230\,A + 301\,B)a^2\cos(dx+c)^3 + (115\,A + 98\,B)a^2\cos(dx+c)^2 + 3\,(20\,A + 7\,B)a^2\cos(dx+c) + 15\,Aa^2\right)\sqrt{}}{105\left(d\cos(dx+c)^4 + d\cos(dx+c)^3\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*((230*A + 301*B)*a^2*cos(d*x + c)^3 + (115*A + 98*B)*a^2*cos(d*x + c)
^2 + 3*(20*A + 7*B)*a^2*cos(d*x + c) + 15*A*a^2)*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.35, size = 119, normalized size = 0.66

$$-\frac{2\left(-1+\cos\left(dx+c\right)\right)\left(230A\left(\cos^{3}\left(dx+c\right)\right)+301B\left(\cos^{3}\left(dx+c\right)\right)+115A\left(\cos^{2}\left(dx+c\right)\right)+98B\left(\cos^{2}\left(dx+c\right)\right)}{105d\sin\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(230*A*cos(d*x+c)^3+301*B*cos(d*x+c)^3+115*A*cos(d
*x+c)^2+98*B*cos(d*x+c)^2+60*A*cos(d*x+c)+21*B*cos(d*x+c)+15*A)*cos(d*x+c)*
(a*(1+cos(d*x+c)))^(1/2)*(1/cos(d*x+c))^(9/2)/sin(d*x+c)*a^2
```

maxima [B] time = 0.51, size = 488, normalized size = 2.70

$$\frac{8\left(\frac{5\left(\frac{21\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{56\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{63\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{36\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)A\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}+\frac{7\left(\frac{15\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\right.}{}\right.}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] 8/105*(5*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a
^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*A*(sin(d*x
```

```
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(
9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)) + 7*(15*sqrt(2)*a
^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 50*sqrt(2)*a^(5/2)*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 8*sqrt(2)*a^(
5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*B*(sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 1)^2/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(c
os(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + 1)))/d
```

mupad [B] time = 5.00, size = 579, normalized size = 3.20

$$\frac{\sqrt{\frac{1}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{a^2\,\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,(230\,A+301\,B)\,2i}{105\,d}-\frac{B\,a^2\,e^{c\,1i+d\,x\,1i}\,\sqrt{a+a\left(\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}\right)}\,2i}{d}+\frac{B\,a^2\,e^{c\,6i}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(5/2),
x)
```

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((a^2*(a + a*(ex
p(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(230*A + 301*B)*2i)/(10
5*d) - (B*a^2*exp(c*1i + d*x*1i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i
+ d*x*1i)/2))^(1/2)*2i)/d + (B*a^2*exp(c*6i + d*x*6i)*(a + a*(exp(- c*1i -
d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*2i)/d - (a^2*exp(c*3i + d*x*3i)*(a
 + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(10*A + 17*B)*2
i)/(3*d) + (a^2*exp(c*4i + d*x*4i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1
i + d*x*1i)/2))^(1/2)*(10*A + 17*B)*2i)/(3*d) + (a^2*exp(c*2i + d*x*2i)*(a
+ a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(100*A + 113*B)*
2i)/(15*d) - (a^2*exp(c*5i + d*x*5i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c
*1i + d*x*1i)/2))^(1/2)*(100*A + 113*B)*2i)/(15*d) - (a^2*exp(c*7i + d*x*7i
)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(230*A + 30
1*B)*2i)/(105*d)))/(exp(c*1i + d*x*1i) + 3*exp(c*2i + d*x*2i) + 3*exp(c*3i
+ d*x*3i) + 3*exp(c*4i + d*x*4i) + 3*exp(c*5i + d*x*5i) + exp(c*6i + d*x*6i
) + exp(c*7i + d*x*7i) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.514 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=192

$$\frac{2a^{5/2}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^3(32A+35B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(8A+$$

```
[Out] 2/5*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/15*a^2*(8*A+
5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+2*a^(5/2)*B*arcsi
n(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d+2/15*a^3*(32*A+35*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(
1/2)
```

Rubi [A] time = 0.62, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2980, 2774, 216}

$$\frac{2a^2(8A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a^3(32A+35B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2a^{5/2}B\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a^(5/2)*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(32*A + 35*B)*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a
 + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*(a + a*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int (a + a\cos(c + dx))^{5/2}(A + B\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx = \left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right)\int \frac{(a + a\cos(c + d}{\text{co}}$$

$$= \frac{2aA(a + a\cos(c + dx))^{3/2}\sec^{\frac{5}{2}}(c + dx)\sin(c + dx}{5d}$$

$$= \frac{2a^2(8A + 5B)\sqrt{a + a\cos(c + dx)}\,\sec^{\frac{3}{2}}(c + dx)\,\text{si}}{15d}$$

$$= \frac{2a^3(32A + 35B)\sqrt{\sec(c + dx)}\,\sin(c + dx)}{15d\sqrt{a + a\cos(c + dx)}} + \frac{2a^2(}{}$$

$$= \frac{2a^3(32A + 35B)\sqrt{\sec(c + dx)}\,\sin(c + dx)}{15d\sqrt{a + a\cos(c + dx)}} + \frac{2a^2(}{}$$

$$= \frac{2a^{5/2}B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c + dx)}\,\sqrt{\sec(c}}{d}$$

Mathematica [A] time = 0.84, size = 130, normalized size = 0.68

$$\frac{a^2\sec\left(\frac{1}{2}(c + dx)\right)\sec^{\frac{5}{2}}(c + dx)\sqrt{a(\cos(c + dx) + 1)}\,\left(2\sin\left(\frac{1}{2}(c + dx)\right)(2(14A + 5B)\cos(c + dx) + (43A + 40B)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(30*Sqr
t[2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(49*A + 40*B
 + 2*(14*A + 5*B)*Cos[c + d*x] + (43*A + 40*B)*Cos[2*(c + d*x)])*Sin[(c + d
*x)/2]))/(30*d)
```

fricas [A] time = 0.65, size = 162, normalized size = 0.84

$$-\frac{2\left(15\left(Ba^2\cos(dx + c)^3 + Ba^2\cos(dx + c)^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right) - \frac{\left((43\,A+40\,B)a^2\cos(dx+c)^2+(14\,A+}{}}{15\left(d\cos(dx + c)^3 + d\cos(dx + c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] -2/15*(15*(B*a^2*cos(d*x + c)^3 + B*a^2*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt
(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - ((43*A +
40*B)*a^2*cos(d*x + c)^2 + (14*A + 5*B)*a^2*cos(d*x + c) + 3*A*a^2)*sqrt(a*
cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*co
s(d*x + c)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.46, size = 389, normalized size = 2.03

$$2\left(15B\left(\cos^3(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+45B\left(\cos^2(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\arctan\left(\frac{\sin(\cdots}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] 2/15/d*(15*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+45*B*cos(d*x+c)^2*(cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)/cos(d*x+c))+45*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+15*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))+43*A*cos(d*x+c)^2*sin(d*x+c)+40*B*sin(d*x+c)*cos(d*x+c)^2+14*A*cos(d
*x+c)*sin(d*x+c)+5*B*cos(d*x+c)*sin(d*x+c)+3*A*sin(d*x+c))*cos(d*x+c)*(1/co
s(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^4/(-1+cos(d*x+c))^2/(1+
cos(d*x+c))^3*a^2
```

maxima [B] time = 0.76, size = 1713, normalized size = 8.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/30*(5*(10*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)*a^(5/2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c
) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2
*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x
+ 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*c
os(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt
(a) + 2*((3*a^2*sin(4*d*x + 4*c) + 7*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*sin(4*
d*x + 4*c) + 7*a^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 4*(3*a^2*cos(4*d*x + 4*c) + 7*a^2*cos(2*d*x + 2*c) + 4*a^2
)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (3*a^2*cos(4*d*x + 4*c) + 7*a^2*cos
(2*d*x + 2*c) + 4*a^2 + 4*(3*a^2*cos(4*d*x + 4*c) + 7*a^2*cos(2*d*x + 2*c)
+ 4*a^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(3*a^2*si
n(4*d*x + 4*c) + 7*a^2*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
```

```
) + 15*(a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x +
 2*c) + a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt
(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
5/4) + 16*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*
a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7)*A/((sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(7/2)))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{7/2}(a+a\cos(c+dx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.515 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=193

$$\frac{a^{5/2}(2A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(2A}{}$$

```
[Out] 2/3*a*A*(a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d-1/3*a^3*(14*A+
3*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+a^(5/2)*(2*A+5*B)
*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+2*a^2*(2*A+B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A] time = 0.65, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2975, 2981, 2774, 216}

$$\frac{a^{5/2}(2A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2(2A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^3*(14*A + 3*B)*Sin[c + d*x]
)/(3*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*(2*A + B)*Sqrt
[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*A*(a + a*Cos
[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+d}{\text{co}}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{3d}$$

$$= \frac{2a^2(2A+B)\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\ \text{sin}}{d}$$

$$= -\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2a^2(2A}{}$$

$$= -\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2a^2(2A}{}$$

$$= \frac{a^{5/2}(2A+5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx}}{d}$$

Mathematica [A] time = 0.74, size = 130, normalized size = 0.67

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\,(2A+5B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(3*Sqrt
[2]*(2*A + 5*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (4*A
+ 3*B + 4*(8*A + 3*B)*Cos[c + d*x] + 3*B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]
))/(6*d)
```

fricas [A] time = 0.58, size = 166, normalized size = 0.86

$$-\frac{3\left((2A+5B)a^2\cos(dx+c)^2+(2A+5B)a^2\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{(3Ba^2\cos(dx+c}{}}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*(3*((2*A + 5*B)*a^2*cos(d*x + c)^2 + (2*A + 5*B)*a^2*cos(d*x + c))*sqr
t(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
c))) - (3*B*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 2*A*a^2)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2
 + d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.43, size = 492, normalized size = 2.55

$$-\frac{\left(6A\left(\cos^{2}\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+15B\left(\cos^{2}\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arctan\left(\frac{\sin(d}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -1/3/d*(6*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+15*B*cos(d*x+c)^2*(cos(d*x+
c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)/cos(d*x+c))+12*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+30*B*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)/cos(d*x+c))+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+15*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*B
*sin(d*x+c)*cos(d*x+c)^2+16*A*cos(d*x+c)*sin(d*x+c)+6*B*cos(d*x+c)*sin(d*x+
c)+2*A*sin(d*x+c))*cos(d*x+c)*sin(d*x+c)^2*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d
*x+c)))^(1/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2*a^2
```

maxima [B] time = 0.88, size = 2780, normalized size = 14.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(2*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2
*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3
*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c)
 + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x +
 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*co
s(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
```

```
*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqr
t(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
+ 3*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*cos(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x + 2*c)
^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d*x + 2
*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*c) + 4*a
^2*cos(d*x + c) + 5*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - ((a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c)^2 + a^2*cos(d*x + c) +
(a^2*cos(d*x + c) - a^2)*sin(2*d*x + 2*c)^2 - a^2 + 2*(a^2*cos(d*x + c) - a
^2)*cos(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + 5*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 +
 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(
a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.516 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=198

$$\frac{a^{5/2}(20A+19B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}-\frac{a^2(4A-$$

```
[Out] -1/4*a^3*(4*A-9*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-1/2
*a^2*(4*A-B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+2*a*A*(a+
a*cos(d*x+c))^(3/2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+1/4*a^(5/2)*(20*A+19*B)*a
rcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d
```

Rubi [A] time = 0.66, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2975, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(20A+19B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}-\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}-\frac{a^2(4A-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) - (a^3*(4*A - 9*B)*Sin[c +
 d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (a^2*(4*A - B)*S
qrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a
+ a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx)}{\cos}$$

$$= \frac{2aA(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}$$

$$= -\frac{a^2(4A-B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{2aA}{}$$

$$= -\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{a^2(4A-B}{}$$

$$= -\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{a^2(4A-B}{}$$

$$= \frac{a^{5/2}(20A+19B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx}}{4d}$$

Mathematica [A] time = 0.78, size = 126, normalized size = 0.64

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\,(20A+19B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2
]*(20*A + 19*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(8*
A + B + (4*A + 11*B)*Cos[c + d*x] + B*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/
(8*d)
```

fricas [A] time = 0.62, size = 147, normalized size = 0.74

$$-\frac{\left((20\,A+19\,B)a^2\cos\left(dx+c\right)+(20\,A+19\,B)a^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(2\,Ba^2\cos(dx+c)^2+(4\,A+11\,B)a\right.}{}}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(((20*A + 19*B)*a^2*cos(d*x + c) + (20*A + 19*B)*a^2)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*B*a
^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 8*A*a^2)*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.44, size = 344, normalized size = 1.74

$$\left(20A\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+19B\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sin(dx+c)\sqrt{}}{\cos(d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] 1/4/d*(20*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+19*B*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/co
s(d*x+c))+2*B*sin(d*x+c)*cos(d*x+c)^2+20*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+4*A*cos(d
*x+c)*sin(d*x+c)+19*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+11*B*cos(d*x+c)*sin(d*x+c)+8*A
*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+co
s(d*x+c))*a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\,\cos(c + d\,x)) \left(\frac{1}{\cos(c + d\,x)}\right)^{3/2} (a + a\,\cos(c + d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.517 $\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=200

$$\frac{a^{5/2}(38A+25B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^3(54A+49B)\sin(c+dx)}{24d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^2(2A$$

```
[Out] 1/3*a*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+1/24*a^3*(54*A
+49*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/4*a^2*(2*A+3*
B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/8*a^(5/2)*(38*A+2
5*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.65, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2976, 2981, 2774, 216}

$$\frac{a^{5/2}(38A+25B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^3(54A+49B)\sin(c+dx)}{24d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^2(2A$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^3*(54*A + 49*B)*Sin[c
 + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*(2*A + 3
*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + (a*B*
(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+a}{\sqrt{}}$$

$$= \frac{aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(\sqrt{\cos(}\right.$$

$$= \frac{a^2(2A+3B)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{a}{}$$

$$= \frac{a^3(54A+49B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{a^2(2A+}{}$$

$$= \frac{a^3(54A+49B)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{a^2(2A+}{}$$

$$= \frac{a^{5/2}(38A+25B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+}}{8d}$$

Mathematica [A] time = 0.98, size = 141, normalized size = 0.70

$$\frac{a^2\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}(38A+25B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Se
c[c + d*x]]*(3*Sqrt[2]*(38*A + 25*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*S
qrt[Cos[c + d*x]]*(66*A + 79*B + 2*(6*A + 17*B)*Cos[c + d*x] + 4*B*Cos[2*(c
 + d*x)])*Sin[(c + d*x)/2]))/(48*d)
```

fricas [A] time = 0.80, size = 163, normalized size = 0.82

$$-\frac{3\left((38A+25B)a^2\cos(dx+c)+(38A+25B)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - \frac{\left(8Ba^2\cos(dx+c)^3+2(6A+\right.}{}}{24(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/24*(3*((38*A + 25*B)*a^2*cos(d*x + c) + (38*A + 25*B)*a^2)*sqrt(a)*arcta
n(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*
B*a^2*cos(d*x + c)^3 + 2*(6*A + 17*B)*a^2*cos(d*x + c)^2 + 3*(22*A + 25*B)*
a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.42, size = 305, normalized size = 1.52

$$-\frac{\left(8B\sin\left(dx+c\right)\left(\cos^{2}\left(dx+c\right)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+12A\sin\left(dx+c\right)\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+34B\sin\left(dx+c\right)\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/24/d*(8*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+12*A
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+34*B*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+66*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)+75*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+114*A*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+75*B*arctan(
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(1/cos(d*x+c))^(1
/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2
*(cos(d*x+c)^2-1)*a^2
```

maxima [B] time = 3.04, size = 3071, normalized size = 15.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

[Out] 1/96*(6*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + (4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin(3*d*x + 3*c

), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))

```
, cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1
))*sqrt(a))*B)/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\cos(c + d\,x))\,\sqrt{\frac{1}{\cos(c + d\,x)}}\,(a + a\cos(c + d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.518 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=247

$$\frac{a^{5/2}(200A+163B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{a^3(104A+95B)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a}{64d\cdots}$$

```
[Out] 1/4*a*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+1/96*a^3*(104*
A+95*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/24*a^2*(8*A+
11*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+1/64*a^3*(200*A+
163*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/64*a^(5/2)*(2
00*A+163*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.75, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2976, 2981, 2770, 2774, 216}

$$\frac{a^3(104A+95B)\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{24d\sec^{\frac{3}{2}}(c+dx)}+\frac{a^{5/2}(200A+163B)\sqrt{\cos(c+\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(104*A + 95*B)*S
in[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*(8*A
 + 11*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sec[c + d*x]^(3/2)) +
 (a*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/2)) + (
a^3*(200*A + 163*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
```

```
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,(a+a\cos(c+d$$

$$= \frac{aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{a^2(8A+11B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)} + \frac{aB(a+a\cos(c}{4d\,\mathrm{se}}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(8A+11B)\sqrt{a+a\,\mathrm{co}}}{24d\sec^{\frac{3}{2}}}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(8A+11B)\sqrt{a+a\,\mathrm{co}}}{24d\sec^{\frac{3}{2}}}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2(8A+11B)\sqrt{a+a\,\mathrm{co}}}{24d\sec^{\frac{3}{2}}}$$

$$= \frac{a^{5/2}(200A+163B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{64d}$$

Mathematica [A] time = 0.98, size = 159, normalized size = 0.64

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\,(200A+163B)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt
[2]*(200*A + 163*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + (
632*A + 581*B + (272*A + 362*B)*Cos[c + d*x] + 4*(8*A + 23*B)*Cos[2*(c + d*
x)] + 12*B*Cos[3*(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(
384*d)
```

fricas [A] time = 0.93, size = 183, normalized size = 0.74

$$-\frac{3\left((200A+163B)a^2\cos(dx+c)+(200A+163B)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(48Ba^2\cos(dx+c)^4+8\right.}{}}{192\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/192*(3*((200*A + 163*B)*a^2*cos(d*x + c) + (200*A + 163*B)*a^2)*sqrt(a)*
arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
- (48*B*a^2*cos(d*x + c)^4 + 8*(8*A + 23*B)*a^2*cos(d*x + c)^3 + 2*(136*A +
 163*B)*a^2*cos(d*x + c)^2 + 3*(200*A + 163*B)*a^2*cos(d*x + c))*sqrt(a*cos
(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.38, size = 383, normalized size = 1.55

$$\frac{(-1+\cos(dx+c))^2 \left(48B\sin(dx+c)\left(\cos^3(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 64A\left(\cos^2(dx+c)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/192/d*(-1+cos(d*x+c))^2*(48*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)+64*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)+184*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+272*A*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+326*B*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+600*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)+489*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+600*A*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+489*B*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1
+cos(d*x+c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/cos(d*x+c))^(1/2)/
sin(d*x+c)^4*a^2
```

maxima [B] time = 2.31, size = 9390, normalized size = 38.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(
3*d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(
1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos
```

(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
))) - 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
)^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2
*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) - 1))*sqrt(a))*A + (2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(4
*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^3 + 12*(a^2*sin(4*d
*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4
*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 12*(a
^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) +
a^2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
^2 + 3*(2*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*
x + 4*c) + a^2*sin(4*d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^
2 - a^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + (40*a^2*cos(4*d*x + 4*c)^2 + 40*a^2*sin(4*d*x + 4*c
)^2 - 3*a^2*cos(4*d*x + 4*c) + 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x
+ 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2
 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + 2*(80*a^2*cos(4*d*x + 4*c)^2 + 80*a^2*sin(4*d*x + 4*c)^2 - 8
3*a^2*cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) - 2*(320*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
*sin(4*d*x + 4*c) + 83*a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*

c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) - 12*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*
x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (3*
a^2*cos(4*d*x + 4*c)^3 - 40*a^2*cos(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4
*c)^3 - 46*a^2*cos(4*d*x + 4*c)^2 + 83*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*
d*x + 4*c) - 40*a^2)*sin(4*d*x + 4*c)^2 - 40*a^2)*cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c)))^2 + (3*a^2*cos(4*d*x + 4*c) - 40*a^2)*sin(4*d*x
 + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 34*a^2*cos(4*d*x + 4*c)^2 - 77*a^
2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 40*a^2)*sin(4*d*x + 4*c)^2 -
 40*a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (40*a^2*c
os(4*d*x + 4*c)^2 + 40*a^2*sin(4*d*x + 4*c)^2 - 3*a^2*cos(4*d*x + 4*c) + 16
0*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c)
 + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 160*(a^2*c
os(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(80*a^2*cos(4*d*
x + 4*c)^2 + 80*a^2*sin(4*d*x + 4*c)^2 - 83*a^2*cos(4*d*x + 4*c) + 3*a^2)*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(320*a^2*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 83*a^2*sin(4*
d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(3*a^2*cos(4*d*x + 4*c)^3 -
 43*a^2*cos(4*d*x + 4*c)^2 + 40*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4
*c) - 40*a^2)*sin(4*d*x + 4*c)^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) - 3*(2*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(3*a^2*cos(4*d*x + 4*c) - 40*a^2)*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (3*a^2
*cos(4*d*x + 4*c) - 40*a^2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)))*sqrt(a) + 6*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((a^2*cos(4*d*x + 4*c)^2*si
n(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c)^3 + a^2*cos(1/4*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 176*(a^2*cos(4*d*x + 4*c)^2 + a^
2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^3 + 4*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d
*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c) + 164*(a^2*cos
(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*si
n(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*sin(4*d*x + 4*c)^3 - 176*a^2*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (a^2*cos
(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) - 43*a^2)*sin(4*d*x + 4*c) + 164*(

a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) +
a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(2*a^2*sin(4*d*x + 4*c)^3 + a^2*
cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 2*(
a^2*cos(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) + (328*a^2*
cos(4*d*x + 4*c)^2 + 328*a^2*sin(4*d*x + 4*c)^2 - 329*a^2*cos(4*d*x + 4*c)
+ a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2*(22*a^2*cos(4*d*x + 4*c)^2 + 20*
a^2*sin(4*d*x + 4*c)^2 - 329*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + 88*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 8*(11*a^2*cos(4*d*x + 4*c)^2 + 10*a^2*sin(4*d*x + 4*c)
^2 - 164*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) - 11*a^2*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) - (a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + (164*a^2*cos(4*d*x + 4*c)^2 + 164*a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x
 + 4*c))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arct
an2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (a^2*cos(4*d*x + 4*c)^3 - 120*a
^2*cos(4*d*x + 4*c)^2 + 176*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^
2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^3 - a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 4*(a^2*cos(4*d*x + 4*c)^3 - 78*a^2*cos(4*d*x + 4*c)^2 + 19
7*a^2*cos(4*d*x + 4*c) + (a^2*cos(4*d*x + 4*c) - 76*a^2)*sin(4*d*x + 4*c)^2
 - 120*a^2 + 76*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*co
s(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))
*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (a^2*cos(4*d*x +
4*c) - 120*a^2)*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^3 - 118*a^2*co
s(4*d*x + 4*c)^2 - 239*a^2*cos(4*d*x + 4*c) + (a^2*cos(4*d*x + 4*c) - 120*a
^2)*sin(4*d*x + 4*c)^2 - 120*a^2 + 44*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d
*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 76*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2
 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(2*a
^2*cos(4*d*x + 4*c)^3 - 220*a^2*cos(4*d*x + 4*c)^2 - a^2*sin(4*d*x + 4*c)*s
in(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 240*a^2*cos(4*d*x + 4
c) + 2(a^2*cos(4*d*x + 4*c) - 109*a^2)*sin(4*d*x + 4*c)^2 + (152*a^2*cos(
4*d*x + 4*c)^2 + 152*a^2*sin(4*d*x + 4*c)^2 - 153*a^2*cos(4*d*x + 4*c) + a^
2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) + (76*a^2*cos(4*d*x + 4*c)^2 + 76*a^2*si
n(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) - 2*(352*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2*sin(4*d*x + 4*c) + 153*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(76*a^2*cos(1/4*arctan2(sin(4*d*x + 4

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*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (a^2*cos(4*d*x + 4*c) - 109*a^2)
*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 2
*(a^2*cos(4*d*x + 4*c) - 120*a^2)*sin(4*d*x + 4*c) - (a^2*cos(4*d*x + 4*c)
+ a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) + 1)))*sqrt(a) + 489*((a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x
+ 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4
*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c
) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*co
s(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*si
n(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))
 + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (a^2*cos(4*d*x + 4*c)^2 + a^2*si
n(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a
^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d
*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4
*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x +
 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
+ 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
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*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a^2*cos(4*d*x + 4*c)^2
+ a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)
^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2
*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x +
 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin
(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan
2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))) + 1)) + 1) + (a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c
)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x
+ 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a
^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a
^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d
*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
) - 1))*sqrt(a))*B/(4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*
x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(c
os(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(
```

```
4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c)
)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+a\cos(c+dx))^{5/2}}{\sqrt{\frac{1}{\cos(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.519 $\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{a^{5/2}(326A+283B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{128d}+\frac{a^3(326A+283B)\sin(c+dx)}{192d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\ldots$$

```
[Out] 1/5*a*B*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+1/240*a^3*(170
*A+157*B)*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+1/192*a^3*(3
26*A+283*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/40*a^2*(
10*A+13*B)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+1/128*a^3*(
326*A+283*B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+1/128*a^(
5/2)*(326*A+283*B)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.87, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2976, 2981, 2770, 2774, 216}

$$\frac{a^3(326A+283B)\sin(c+dx)}{192d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^3(170A+157B)\sin(c+dx)}{240d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2(10A+13B)\sin(c+dx)\sqrt{a\cos\ldots}}{40d\sec^{\frac{5}{2}}(c+dx)}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d
*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(170*A + 157*B)
*Sin[c + d*x])/(240*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a^2*(
10*A + 13*B)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(40*d*Sec[c + d*x]^(5/2
)) + (a*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(5/2))
 + (a^3*(326*A + 283*B)*Sin[c + d*x])/(192*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c
 + d*x]^(3/2)) + (a^3*(326*A + 283*B)*Sin[c + d*x])/(128*d*Sqrt[a + a*Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+$$

$$= \frac{aB(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{5}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec\right.$$

$$= \frac{a^2(10A+13B)\sqrt{a+a\cos(c+dx)}\,\sin(c+dx)}{40d\sec^{\frac{5}{2}}(c+dx)} + \frac{aB(a+a\cos}{5d}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(10A+13B)\sqrt{a+}}{40d\sec}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(10A+13B)\sqrt{a+}}{40d\sec}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(10A+13B)\sqrt{a+}}{40d\sec}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\sqrt{a+a\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2(10A+13B)\sqrt{a+}}{40d\sec}$$

$$= \frac{a^{5/2}(326A+283B)\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(}}{128d}$$

Mathematica [A] time = 1.44, size = 181, normalized size = 0.62

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}\,(326A+283B)\sin^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(15*Sqr
t[2]*(326*A + 283*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] +
(5810*A + 5521*B + (3620*A + 3874*B)*Cos[c + d*x] + 4*(230*A + 331*B)*Cos[2
```

```
*(c + d*x)] + 120*A*Cos[3*(c + d*x)] + 348*B*Cos[3*(c + d*x)] + 48*B*Cos[4*
(c + d*x)])*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(3840*d)
```

fricas [A] time = 0.96, size = 203, normalized size = 0.69

$$-\frac{15\left((326\,A+283\,B)a^2\cos\left(dx+c\right)+(326\,A+283\,B)a^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{\left(384\,Ba^2\cos(dx+c)^5\right.}{}}{1920\,(d\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/1920*(15*((326*A + 283*B)*a^2*cos(d*x + c) + (326*A + 283*B)*a^2)*sqrt(a
)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))
) - (384*B*a^2*cos(d*x + c)^5 + 48*(10*A + 29*B)*a^2*cos(d*x + c)^4 + 8*(23
0*A + 283*B)*a^2*cos(d*x + c)^3 + 10*(326*A + 283*B)*a^2*cos(d*x + c)^2 + 1
5*(326*A + 283*B)*a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.35, size = 455, normalized size = 1.55

$$-\frac{(-1+\cos\left(dx+c\right))^3\left(384B\sin\left(dx+c\right)\left(\cos^4\left(dx+c\right)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+480A\left(\cos^3\left(dx+c\right)\right)\sin\left(dx+c\right)\sqrt{\frac{\cos(}{1+\cos}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/1920/d*(-1+cos(d*x+c))^3*(384*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+480*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
```

```
^(1/2)+1392*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+184
0*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2264*B*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3260*A*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2830*B*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)+4890*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)+4245*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4890*A*arctan(sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+4245*B*arctan(sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+
c)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+c))^(3/2)/sin(d*x+c
)^6*a^2
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+a\,\cos(c+d\,x)\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2
), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.520 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=295

$$-\frac{2(A-9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2(19A-3B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(29A-93B)\sin(c+dx)\sec^{\frac{3}{2}}(c+}{315d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -2/315*(29*A-93*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/1
05*(19*A-3*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-2/63*(A-
9*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/9*A*sec(d*x+c)^
(9/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-(A-B)*arctan(1/2*sin(d*x+c)*a^(1/
2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d/a^(1/2)+2/315*(257*A-129*B)*sin(d*x+c)*sec(d*x+c)^(1/2
)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 1.06, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2984, 12, 2782, 205}

$$-\frac{2(A-9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{2(19A-3B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(29A-93B)\sin(c+dx)\sec^{\frac{3}{2}}(c+}{315d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*(257*A - 129*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a +
 a*Cos[c + d*x]]) - (2*(29*A - 93*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*
d*Sqrt[a + a*Cos[c + d*x]]) + (2*(19*A - 3*B)*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*(A - 9*B)*Sec[c + d*x]^(7/2)*Sin[
c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(9/2)*Sin[c +
 d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{11}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(A}{\cos^{\frac{9}{2}}(c}}{9a}$$

$$= -\frac{2(A-9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2(19A-3B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-9B)\sec^{\frac{7}{2}}(c+dx)\sin(c}{63d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{2(29A-93B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{2(19A-3B)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a+a\cos(c+}}$$

$$= \frac{2(257A-129B)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A-93B)\sec^{\frac{3}{2}}(c+d}{315d\sqrt{a+a\cos(c}}$$

$$= \frac{2(257A-129B)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A-93B)\sec^{\frac{3}{2}}(c+d}{315d\sqrt{a+a\cos(c}}$$

$$= \frac{2(257A-129B)\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} - \frac{2(29A-93B)\sec^{\frac{3}{2}}(c+d}{315d\sqrt{a+a\cos(c}}$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{\sqrt{a}\,d}$$

Mathematica [C] time = 9.33, size = 272, normalized size = 0.92

$$\frac{2e^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\left(-315i(A-B)\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{4}\sin\left(\frac{1}{2}(c+dx)\right)\sec}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2))/Sqrt[a + a*Cos[c + d*x
]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((-315*I)*(A - B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - ((-1279*A + 423*B + (214*A - 918*B)*
Cos[c + d*x] - 8*(157*A - 69*B)*Cos[2*(c + d*x)] + 58*A*Cos[3*(c + d*x)] -
186*B*Cos[3*(c + d*x)] - 257*A*Cos[4*(c + d*x)] + 129*B*Cos[4*(c + d*x)])*S
ec[c + d*x]^(9/2)*(Cos[(c + d*x)/2] + I*Sin[(c + d*x)/2])*Sin[(c + d*x)/2])
/4))/(315*d*E^((I/2)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 1.38, size = 198, normalized size = 0.67

$$\frac{\frac{315\sqrt{2}\left((A-B)a\cos(dx+c)^5+(A-B)a\cos(dx+c)^4\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((257A-129B)\cos(dx+c)^4-(29A-93B)\cos(dx+c)^3+3\right.}{}}{315\left(ad\cos\left(dx+c\right)^5+ad\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="fricas")
```

```
[Out] 1/315*(315*sqrt(2)*((A - B)*a*cos(d*x + c)^5 + (A - B)*a*cos(d*x + c)^4)*ar
ctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
 c)))/sqrt(a) + 2*((257*A - 129*B)*cos(d*x + c)^4 - (29*A - 93*B)*cos(d*x +
 c)^3 + 3*(19*A - 3*B)*cos(d*x + c)^2 - 5*(A - 9*B)*cos(d*x + c) + 35*A)*sq
rt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^5
 + a*d*cos(d*x + c)^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.37, size = 793, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/315/d*(315*A*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+co
s(d*x+c))/sin(d*x+c))-315*B*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*
```

```
arcsin((-1+cos(d*x+c))/sin(d*x+c))+1575*A*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d
*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-1575*B*cos(d*x+c)^4*(cos(d
*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+3150*A*cos(d
*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)-3150*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+
c))/sin(d*x+c))+3150*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))-3150*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)
))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+1575*A*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-1575*B*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+315*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-315*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+257*A*c
os(d*x+c)^4*2^(1/2)*sin(d*x+c)-129*B*cos(d*x+c)^4*2^(1/2)*sin(d*x+c)-29*A*c
os(d*x+c)^3*2^(1/2)*sin(d*x+c)+93*B*cos(d*x+c)^3*2^(1/2)*sin(d*x+c)+57*A*co
s(d*x+c)^2*2^(1/2)*sin(d*x+c)-9*B*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-5*A*cos(d
*x+c)*2^(1/2)*sin(d*x+c)+45*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)+35*A*2^(1/2)*si
n(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)^8/(-1+cos(d*x+c))^4/(1+cos(d*x+c))^5*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(11/2)/(a+a*cos(d*x+c))^(1/2),x, algo
rithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\cos(c + d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}{\sqrt{a + a\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2))/(a + a*cos(c + d*x))^(1/
2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2))/(a + a*cos(c + d*x))^(1/
2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(11/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.521 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=250

$$-\frac{2(A-7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{2(31A-7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(43A-91B)\sin(c+dx)\sqrt{\sec(c+}}{105d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] 2/105*(31*A-7*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-2/35*
(A-7*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*A*sec(d*x+
c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+(A-B)*arctan(1/2*sin(d*x+c)*a^
(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-2/105*(43*A-91*B)*sin(d*x+c)*sec(d*x+c)^(1/
2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2984, 12, 2782, 205}

$$-\frac{2(A-7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{2(31A-7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}-\frac{2(43A-91B)\sin(c+dx)\sqrt{\sec(c+}}{105d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*
d) - (2*(43*A - 91*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Co
s[c + d*x]]) + (2*(31*A - 7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt
[a + a*Cos[c + d*x]]) - (2*(A - 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d
*Sqrt[a + a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqr
t[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(A-}{\cos^{\frac{7}{2}}(c+}}{7a}$$

$$= -\frac{2(A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2(31A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{35d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{2(43A-91B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(31A-7B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{105d\sqrt{a+a\cos(c+}}$$

$$= -\frac{2(43A-91B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(31A-7B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{105d\sqrt{a+a\cos(c+}}$$

$$= -\frac{2(43A-91B)\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{2(31A-7B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{105d\sqrt{a+a\cos(c+}}$$

$$= \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{\sqrt{a}\,d}$$

Mathematica [C] time = 6.84, size = 250, normalized size = 1.00

$$\frac{2e^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\left(105i(A-B)\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{2}\sin\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*((105*I)*(A - B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt
```

```
[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - ((-122*A + 14*B + 3*(47*A - 119*B)*Co
s[c + d*x] + (-62*A + 14*B)*Cos[2*(c + d*x)] + 43*A*Cos[3*(c + d*x)] - 91*B
*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*(Cos[(c + d*x)/2] + I*Sin[(c + d*x)/2
])*Sin[(c + d*x)/2])/2))/(105*d*E^((I/2)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x
])])
```

fricas [A] time = 0.74, size = 181, normalized size = 0.72

$$-\frac{\frac{105\sqrt{2}\left((A-B)a\cos(dx+c)^4+(A-B)a\cos(dx+c)^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((43A-91B)\cos(dx+c)^3-(31A-7B)\cos(dx+c)^2+3(\ldots}{\sqrt{\cos(d\ldots}}}{105\left(ad\cos(dx+c)^4+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/105*(105*sqrt(2)*((A - B)*a*cos(d*x + c)^4 + (A - B)*a*cos(d*x + c)^3)*a
rctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x
+ c)))/sqrt(a) + 2*((43*A - 91*B)*cos(d*x + c)^3 - (31*A - 7*B)*cos(d*x + c
)^2 + 3*(A - 7*B)*cos(d*x + c) - 15*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c
)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.50, size = 657, normalized size = 2.63

$$\frac{\left(105A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^4(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}-105B\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^4(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+\ldots}\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/105/d*(105*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4*(cos(d*x+c)/
(1+cos(d*x+c)))^(7/2)-105*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4
```

```
*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+420*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-420*B*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+630*A*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-6
30*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(7/2)+420*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(7/2)-420*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+105*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-105*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+43*A*cos(d*x+c)^3*2^(1/2)*sin(d*x+c)-91*B
*cos(d*x+c)^3*2^(1/2)*sin(d*x+c)-31*A*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)+7*B*c
os(d*x+c)^2*2^(1/2)*sin(d*x+c)+3*A*cos(d*x+c)*2^(1/2)*sin(d*x+c)-21*B*cos(d
*x+c)*2^(1/2)*sin(d*x+c)-15*A*2^(1/2)*sin(d*x+c))*cos(d*x+c)*sin(d*x+c)^6*(
1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))^3/(1+cos(d*x+c
))^4*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a*cos(c + d*x))^(1/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.522 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=207

$$-\frac{2(A-5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2(13A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}}{}$$

```
[Out] -2/15*(A-5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*A*se
c(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-(A-B)*arctan(1/2*sin(d*x
+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2/15*(13*A-5*B)*sin(d*x+c)*sec(d*x+c)
^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2984, 12, 2782, 205}

$$-\frac{2(A-5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{2(13A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*(13*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*C
os[c + d*x]]) - (2*(A - 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a
+ a*Cos[c + d*x]]) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*
Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(A-}{\cos^{\frac{5}{2}}(c+}}{5a}$$

$$= -\frac{2(A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} +$$

$$= \frac{2(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{15d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{15d\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{2(13A-5B)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2(A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{15d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{\sqrt{a}\,d}$$

Mathematica [C] time = 7.80, size = 1718, normalized size = 8.30

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Si
n[c/2 + (d*x)/2]^2]*((2*B*Sin[c/2 + (d*x)/2])/(5*(1 - 2*Sin[c/2 + (d*x)/2]^
2)^(5/2)) + (8*B*(Sin[c/2 + (d*x)/2]/(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2) + (
2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]))/15 - ((A - B)*Csc[
c/2 + (d*x)/2]^7*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 +
210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2
+ (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*
HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 +
```

```
2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^
12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1
500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hype
rgeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (
d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2
+ (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2
*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-
1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricP
FQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^
2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2)))/(675*(1 - 2*Sin[c
/2 + (d*x)/2]^2)^(7/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2))))/(d*Sqrt[a*(1 + Cos[
c + d*x])])
```

fricas [A] time = 0.55, size = 164, normalized size = 0.79

$$\frac{\frac{15\sqrt{2}\left((A-B)a\cos(dx+c)^3+(A-B)a\cos(dx+c)^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\left((13A-5B)\cos(dx+c)^2-(A-5B)\cos(dx+c)+3A\right)\sqrt{a\cos}}{\sqrt{\cos(dx+c)}}}{15\left(ad\cos\left(dx+c\right)^3+ad\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(15*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*arct
an(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c
)))/sqrt(a) + 2*((13*A - 5*B)*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) + 3*A
```

```
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x +
c)^3 + a*d*cos(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(a*cos(d*x + c) + a),
 x)
```

maple [B] time = 0.43, size = 521, normalized size = 2.52

$$\frac{\left(15A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\left(\cos^3(dx+c)\right)-15B\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}\left(\cos^3(dx+c)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d*(15*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))
^(5/2)*cos(d*x+c)^3-15*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)*cos(d*x+c)^3+45*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(co
s(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2-45*B*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2+45*A*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)-45*B*arcs
in((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)
+15*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-
15*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+1
3*A*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-5*B*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-A*c
os(d*x+c)*2^(1/2)*sin(d*x+c)+5*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)+3*A*2^(1/2)*
sin(d*x+c))*cos(d*x+c)*sin(d*x+c)^4*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))
^(1/2)/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\cos(c + dx))\left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{\sqrt{a + a\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(1/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.523 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=162

$$-\frac{2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+$$

```
[Out] 2/3*A*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+(A-B)*arctan(1/2
*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-2/3*(A-3*B)*sin(d*x+c)*sec(d*
x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2984, 12, 2782, 205}

$$-\frac{2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*
Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*
d) - (2*(A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c +
d*x]]) + (2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{2}a(A}{\cos^{\frac{3}{2}}(c+}}{3a}$$

$$= -\frac{2(A-3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{2(A-3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{2(A-3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} -$$

$$= \frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d}}{\sqrt{a}\,d}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] $Aborted
```

fricas [A] time = 1.83, size = 143, normalized size = 0.88

$$-\frac{\frac{3\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)}{\sqrt{a}} + \frac{2\left((A-3B)\cos(dx+c)-A\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*(3*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*arctan(
sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))
/sqrt(a) + 2*((A - 3*B)*cos(d*x + c) - A)*sqrt(a*cos(d*x + c) + a)*sin(d*x
+ c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.40, size = 384, normalized size = 2.37

$$\frac{\left(3A\left(\cos^2\left(dx+c\right)\right)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}-3B\left(\cos^2\left(dx+c\right)\right)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/3/d*(3*A*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)-3*B*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(
d*x+c)/(1+cos(d*x+c)))^(3/2)+6*A*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+
c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-6*B*cos(d*x+c)*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+3*A*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-3*B*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+A*cos(d*x+c)*2^(1/2)*sin(d*x+c)-
3*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)-A*2^(1/2)*sin(d*x+c))*cos(d*x+c)*(1/cos(d
*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/(1+cos(d
*x+c))^2*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{\sqrt{a+a\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(1/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.524 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=119

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

```
[Out] -(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2*A*sin(d*x+
c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2984, 12, 2782, 205}

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a
]*d)) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -\frac{\cdots}{2\sqrt{\mathrm{co}\cdots}}}{a}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \left((A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cdots$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\cdots}{\cdots}$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+\cdots}}{\sqrt{a}\,d}$$

Mathematica [C] time = 1.58, size = 203, normalized size = 1.71

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(10B-(A-B)\sec(c+dx)\left(\frac{1}{2}\sin(c+dx)\tan(c+dx)\,{}_2F_1\left(2,\frac{5}{2};\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*Sin[(c + d*x)/2]*(10*B - (A - B)*Sec
[c + d*x]*((-5*(1 + 4*Cos[c + d*x] + Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^4*(
1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c
+ d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/4 + (Hypergeometric2F1[2, 5/2, 7/2, -(Sec
[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/2)))/(5*d*Sqrt[a*
(1 + Cos[c + d*x])])
```

fricas [A] time = 0.77, size = 110, normalized size = 0.92

$$\frac{\frac{\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\sqrt{a\cos(dx+c)+a}\,A\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x
 + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt(a*co
s(d*x + c) + a)*A*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a),
 x)
```

maple [B] time = 0.38, size = 231, normalized size = 1.94

$$\left(A\cos\left(dx+c\right)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-B\cos\left(dx+c\right)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+A\sqrt{2}\ \sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(A*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)-B*cos(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)+A*2^(1/2)*sin(d*x+c)+A*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(
d*x+c)))^(1/2)/(1+cos(d*x+c))*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\ \cos\left(c+d\,x\right)\right)\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+a\ \cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(1/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.525 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\right.}{\sqrt{a}\,d}$$

```
[Out] 2*B*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/d/a^(1/2)+(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+
c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d/a^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2982, 2782, 205, 2774, 216}

$$\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\right.}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sin
[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \left((A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+$$

$$= -\frac{\left(2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{}{\sqrt{\cos}}\right.}{d}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} + \frac{\sqrt{2}(A-B)\tan}{}$$

Mathematica [A] time = 0.21, size = 102, normalized size = 0.73

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left((A-B)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)+\sqrt{2}B\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + a*Cos[c + d*x]
],x]
```

```
[Out] (2*(Sqrt[2]*B*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + (A - B)*ArcTan[Sin[(c + d*
x)/2]/Sqrt[Cos[c + d*x]]])*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 3.10, size = 96, normalized size = 0.69

$$-\frac{\sqrt{2}(A-B)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2B\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -(sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*B*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(a*cos(d*x + c) + a),
 x)
```

maple [A] time = 0.38, size = 153, normalized size = 1.09

$$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{a\,(1+\cos(dx+c))}\,\left(-B\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)+A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B\arcsin\left(\frac{-1+}{\mathrm{si}}\right.\right.}{d\sin(dx+c)^2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-B*2^(1/2)*arctan(sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+A*arcsin((-1+cos(d*x+c))
/sin(d*x+c))-B*arcsin((-1+cos(d*x+c))/sin(d*x+c)))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a(\cos(c + dx) + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/sqrt(a*(cos(c + d*x) + 1))
, x)
```

3.526 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=181

$$\frac{(2A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{c}}\right.}{\sqrt{a}\,d}$$

```
[Out] B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(2*A-B)*arcsin(sin(d
*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a
^(1/2)-(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*co
s(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2961, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(2A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{c}}\right.}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((2*A - B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A - B)*ArcTan[(Sqrt[
a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqr
t[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(d*Sqrt[
a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
```

```
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{a}$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \left((A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(\right.}{}$$

$$= \frac{(2A-B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d} - \frac{\sqrt{2}\,(A-}{}$$

Mathematica [C] time = 1.37, size = 467, normalized size = 2.58

$$\frac{ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{\sec(c+dx)}\left(-(2A-B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)+2\sqrt{2}\,Ae^{i(c+dx)}\sqrt{1+e^{2i(c+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] ((I/4)*(1 + E^(I*(c + d*x)))*(B - B*E^(I*(c + d*x)) + B*E^((2*I)*(c + d*x))
 - B*E^((3*I)*(c + d*x)) - (2*A - B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*B*E^(I*(c + d*x))*Sqrt[1 + E^((2
*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c
 + d*x))])] + 2*Sqrt[2]*A*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Arc
Tanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - Sqrt
[2]*B*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(-1 + E^(I*(c +
 d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + 2*A*E^(I*(c + d*x))*Sqrt
[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - B*E^(I*(
c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x)
)]])*Sqrt[Sec[c + d*x]])/(d*E^((2*I)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 7.18, size = 168, normalized size = 0.93

$$\frac{\sqrt{a\cos(dx+c)+a}\,B\sqrt{\cos(dx+c)}\,\sin(dx+c)-((2A-B)\cos(dx+c)+2A-B)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos}}{\sqrt{a}\,\sin(dx+c)}\right.}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*sqrt(cos(d*x + c))*sin(d*x + c) - ((2*A - B)*co
s(d*x + c) + 2*A - B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) + sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a
)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\,\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

maple [A] time = 0.40, size = 232, normalized size = 1.28

$$\frac{\sqrt{a(1+\cos(dx+c))}\,\cos(dx+c)(-1+\cos(dx+c))^2\left(B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+2A\sqrt{2}\,\arctan\left(\frac{\sin(dx+c}{\mathrm{co}}\right.\right.}{2d\sqrt{\frac{1}{\cos(dx+c)}}\,\left(\frac{\cos(dx+c)}{1+\cos(dx+c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(B*2^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*A*2^(1/2)*arctan(sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-B*2^(1/2)*arctan(sin(d*x+c)*(cos(
```

```
d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*A*arcsin((-1+cos(d*x+c))/sin(d*x
+c))-2*B*arcsin((-1+cos(d*x+c))/sin(d*x+c)))/(1/cos(d*x+c))^(1/2)/(cos(d*x+
c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*sqrt(sec(c + d*x)
)), x)
```

3.527 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\ \sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=230

$$-\frac{(4A-7B)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}\ \sin^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}\,d}+\frac{(4A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\ \sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\sqrt{}}{}$$

```
[Out] 1/2*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/4*(4*A-B)*sin(
d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-1/4*(4*A-7*B)*arcsin(sin(d
*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a
^(1/2)+(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*co
s(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2961, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{(4A-7B)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}\ \sin^{-1}\left(\frac{\sqrt{a}\ \sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}\,d}+\frac{(4A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\ \sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((4*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(
Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(2*
d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + ((4*A - B)*Sin[c + d*x])/(
4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{B\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2}$$

$$= \frac{B\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{(4A-B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+d}}$$

$$= \frac{B\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{(4A-B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+d}}$$

$$= \frac{B\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{(4A-B)\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+d}}$$

$$= -\frac{(4A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}\,d} + \frac{\sqrt{2}(A}{}$$

Mathematica [C] time = 1.46, size = 412, normalized size = 1.79

$$-\frac{ie^{-3i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{\sec(c+dx)}\left(-(4A-7B)e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)-8\sqrt{2}(A-B)e^{2i(c+dx)}\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] ((-1/16*I)*(1 + E^(I*(c + d*x)))*(-B - 4*A*E^(I*(c + d*x)) + 2*B*E^(I*(c +
d*x)) + 4*A*E^((2*I)*(c + d*x)) - 3*B*E^((2*I)*(c + d*x)) - 4*A*E^((3*I)*(c
 + d*x)) + 3*B*E^((3*I)*(c + d*x)) + 4*A*E^((4*I)*(c + d*x)) - 2*B*E^((4*I)
*(c + d*x)) + B*E^((5*I)*(c + d*x)) - (4*A - 7*B)*E^((2*I)*(c + d*x))*Sqrt[
1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))] - 8*Sqrt[2]*(A - B)*E^((2
*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/
(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + 4*A*E^((2*I)*(c + d*x))*Sqrt[1 +
```

```
 E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 7*B*E^((2*I)
*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*
x))]]])*Sqrt[Sec[c + d*x]])/(d*E^((3*I)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])
])
```

fricas [A] time = 6.97, size = 194, normalized size = 0.84

$$\frac{((4\,A-7\,B)\cos(dx+c)+4\,A-7\,B)\sqrt{a}\,\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-\frac{4\,\sqrt{2}\,((A-B)a\cos(dx+c)+(A-B)a)\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{}\right.}{\sqrt{a}}}{4\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(((4*A - 7*B)*cos(d*x + c) + 4*A - 7*B)*sqrt(a)*arctan(sqrt(a*cos(d*x +
 c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 4*sqrt(2)*((A - B)*a*
cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + (2*B*cos(d*x + c)^2 + (4*A - B)
*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
*d*cos(d*x + c) + a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{a\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

maple [A] time = 0.42, size = 300, normalized size = 1.30

$$\frac{\sqrt{a\,(1+\cos(dx+c))}\,\cos(dx+c)\,(-1+\cos(dx+c))^{3}\left(-2B\sin(dx+c)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)-4A\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(-2*B*sin(d*x+c
)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-4*A*2^(1/2)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)+4*A*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/cos(d*x+c))-7*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)/cos(d*x+c))+8*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))-8*B*arcsin((-1+co
s(d*x+c))/sin(d*x+c)))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/
2)/sin(d*x+c)^6*2^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{a\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**(3/
2)), x)
```

3.528 $\int \frac{\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=192

$$\frac{(2aB+2Ab-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,(a-b)(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+a}}{\sqrt{a}\,d}$$

```
[Out] b*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(2*A*b+2*B*a-B*b)*
arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/d/a^(1/2)+(a-b)*(A-B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*
x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d/a^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4221, 3045, 2982, 2782, 205, 2774, 216}

$$\frac{(2aB+2Ab-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,(a-b)(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+a}}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^2)*Sqrt[Sec[c + d*x
]])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((2*A*b + 2*a*B - b*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(a - b)*(
A - B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (b*B*
Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3045

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n
 + 2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n
 + 2) + C*(a*c*m + b*d*(n + 1)) + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n +
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m, n}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[m
, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{\left(aA+(Ab+aB)\cos(c+dx)+bB\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{aA+(}{}$$

$$= \frac{bB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{(\surd}{}$$

$$= \frac{bB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \left((\right.$$

$$= \frac{bB\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{(2}{}$$

$$= \frac{(2Ab+2aB-bB)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [A] time = 0.47, size = 143, normalized size = 0.74

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\sqrt{2}(2aB+2Ab-bB)\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2(a-b)(A-B\right.}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*A + (A*b + a*B)*Cos[c + d*x] + b*B*Cos[c + d*x]^2)*Sqrt[Sec[c
 + d*x]])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*(2*A*b + 2
*a*B - b*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*(a - b)*(A - B)*ArcTan[Sin
[(c + d*x)/2]/Sqrt[Cos[c + d*x]]] + 2*b*B*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/
2]))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

fricas [A] time = 33.10, size = 208, normalized size = 1.08

$$\frac{\sqrt{a\cos(dx+c)+a}\,Bb\sqrt{\cos(dx+c)}\,\sin(dx+c)-(2\,Ba+(2\,A-B)b+(2\,Ba+(2\,A-B)b)\cos(dx+c))\sqrt{a}\,\mathrm{ar}}{ad\cos(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*B*b*sqrt(cos(d*x + c))*sin(d*x + c) - (2*B*a + (2
*A - B)*b + (2*B*a + (2*A - B)*b)*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(2)*((A - B)*
a^2 - (A - B)*a*b + ((A - B)*a^2 - (A - B)*a*b)*cos(d*x + c))*arctan(sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(
a))/(a*d*cos(d*x + c) + a*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)}}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(sec(d*
x + c))/sqrt(a*cos(d*x + c) + a), x)
```

maple [A] time = 0.48, size = 317, normalized size = 1.65

$$-\frac{\left(B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,b\sin\left(dx+c\right)+2A\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)b+2B\sqrt{2}\,\arctan\left(\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{\cos(dx+c)}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a*cos(d
*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b*sin(d*x+c)+2*A*2^(1/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*b+2*B*2^(
1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*a-B*2^
(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*b-2*A
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*a+2*A*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*b+2*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*a-2*B*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*b)*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*2^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)^2)*sec(d*x+c)^(1/2)/(a+a
*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(B\,b\cos\left(c+d\,x\right)^2+\left(A\,b+B\,a\right)\,\cos\left(c+d\,x\right)+A\,a\right)}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/cos(c + d*x))^(1/2)*(A*a + cos(c + d*x)*(A*b + B*a) + B*b*cos(c + d
*x)^2))/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(((1/cos(c + d*x))^(1/2)*(A*a + cos(c + d*x)*(A*b + B*a) + B*b*cos(c + d
*x)^2))/(a + a*cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))\,\sqrt{\sec(c+dx)}}{\sqrt{a\,(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*A+(A*b+B*a)*cos(d*x+c)+b*B*cos(d*x+c)**2)*sec(d*x+c)**(1/2)/(a
+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sqrt(sec(c + d*x))/sqrt(
a*(cos(c + d*x) + 1)), x)
```

3.529 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=317

$$\frac{(19A-15B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(11A-7B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{14ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -1/2*(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/210*(397*
A-273*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/70*(67*A-
63*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/14*(11*A-7*B
)*sec(d*x+c)^(7/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/4*(19*A-15*B)*ar
ctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)-1/210*(1201*A-1029*B)
*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 1.11, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(19A-15B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(11A-7B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{14ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((19*A - 15*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) - ((1201*A - 1029*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(210*a*d*S
qrt[a + a*Cos[c + d*x]]) + ((397*A - 273*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(210*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((67*A - 63*B)*Sec[c + d*x]^(5/2)*Si
n[c + d*x])/(70*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(7/2)
*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((11*A - 7*B)*Sec[c + d*x
]^(7/2)*Sin[c + d*x])/(14*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{2}\cdots}{2a^2}$$

$$= -\frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(11A-7B)\sec^{\frac{7}{2}}(c+dx)\sin(c+\cdots}{14ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(67A-63B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{70ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+\cdots}{2d(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(397A-273B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} - \frac{(67A-63B)\sec^{\frac{5}{2}}(c+dx)\,\mathrm{s}\cdots}{70ad\sqrt{a+a\cos(c+\cdots}}$$

$$= -\frac{(1201A-1029B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A-273B)\sec^{\frac{3}{2}}(c\cdots}{210ad\sqrt{a+a\,\mathrm{co}\cdots}}$$

$$= -\frac{(1201A-1029B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A-273B)\sec^{\frac{3}{2}}(c\cdots}{210ad\sqrt{a+a\,\mathrm{co}\cdots}}$$

$$= -\frac{(1201A-1029B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{210ad\sqrt{a+a\cos(c+dx)}} + \frac{(397A-273B)\sec^{\frac{3}{2}}(c\cdots}{210ad\sqrt{a+a\,\mathrm{co}\cdots}}$$

$$= \frac{(19A-15B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\cdots}}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] time = 10.18, size = 2966, normalized size = 9.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2))/(a + a*Cos[c + d*x])^(3
/2),x]
```

[Out] (2*Cos[c/2 + (d*x)/2]^3*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*
Sin[c/2 + (d*x)/2]^2]*(-1/28*((A - B)*(1 - 2*Sin[c/2 + (d*x)/2]))/((1 + Sin
[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)) + ((A - B)*(1 + 2*Sin[
c/2 + (d*x)/2]))/(28*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^
(7/2)) - ((A - B)*(315*ArcTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2
 + (d*x)/2]^2]] + (5 + 3*Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*(1 -
 2*Sin[c/2 + (d*x)/2]^2)^(5/2)) - (11 + 17*Sin[c/2 + (d*x)/2])/((1 - Sin[c/
2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (61 + 71*Sin[c/2 + (d*x
)/2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (193*Sq
rt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c/2 + (d*x)/2])))/70 + ((A - B)*(3
15*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]] + (5
 - 3*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2
]^2)^(5/2)) - (11 - 17*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/2])*(1 - 2
*Sin[c/2 + (d*x)/2]^2)^(3/2)) + (61 - 71*Sin[c/2 + (d*x)/2])/((1 + Sin[c/2
+ (d*x)/2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (193*Sqrt[1 - 2*Sin[c/2 + (
d*x)/2]^2])/(1 + Sin[c/2 + (d*x)/2])))/70 - ((-A - 3*B)*Csc[c/2 + (d*x)/2]^
9*(363825*Sin[c/2 + (d*x)/2]^2 - 4729725*Sin[c/2 + (d*x)/2]^4 + 26785605*Si
n[c/2 + (d*x)/2]^6 - 86790165*Sin[c/2 + (d*x)/2]^8 + 177677808*Sin[c/2 + (d
*x)/2]^10 - 239283044*Sin[c/2 + (d*x)/2]^12 + 52080*Hypergeometric2F1[2, 11
/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*
x)/2]^12 + 560*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 2131
20160*Sin[c/2 + (d*x)/2]^14 - 168280*Hypergeometric2F1[2, 11/2, 13/2, Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 2240
*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 121497024*Sin[c/2
+ (d*x)/2]^16 + 212520*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 3360*Hypergeometri
cPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 40125184*Sin[c/2 + (d*x)/2]^18 -
 124320*Hypergeometric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^18 - 2240*HypergeometricPFQ[{2, 2, 2,
2, 11/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^
2)]*Sin[c/2 + (d*x)/2]^18 - 5840384*Sin[c/2 + (d*x)/2]^20 + 28000*Hypergeom
etric2F1[2, 11/2, 13/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
*Sin[c/2 + (d*x)/2]^20 + 560*HypergeometricPFQ[{2, 2, 2, 2, 11/2}, {1, 1, 1
, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x
)/2]^20 + 363825*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 5336100*
ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 +
 (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 3463
6140*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[
c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] -
 131060160*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]

```
^2)] + 320535600*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)] - 530671680*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)] + 604296000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin
[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)] - 468948480*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 237726720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 70963200*ArcTanh[Sqrt[Sin[c/2 + (d
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^18*Sqrt[Sin[c/2
 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 9461760*ArcTanh[Sqrt[Sin[c/2
 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^20*Sqrt[Si
n[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1120*Cos[(c + d*x)/2]^6
*HypergeometricPFQ[{2, 2, 2, 11/2}, {1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6 + 5*Sin[c/2 + (d*x)/2]
^2) + 280*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 11/2}, {1, 13/2}, Sin
[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(103
 - 164*Sin[c/2 + (d*x)/2]^2 + 70*Sin[c/2 + (d*x)/2]^4)))/(80850*(1 - 2*Sin[
c/2 + (d*x)/2]^2)^(9/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2))))/(d*(a*(1 + Cos[c +
 d*x]))^(3/2))
```

fricas [A] time = 0.69, size = 237, normalized size = 0.75

$$-\frac{105\sqrt{2}\left((19A-15B)\cos(dx+c)^5+2(19A-15B)\cos(dx+c)^4+(19A-15B)\cos(dx+c)^3\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right)\cdots}{420\left(a^2d\cos(dx+c\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/420*(105*sqrt(2)*((19*A - 15*B)*cos(d*x + c)^5 + 2*(19*A - 15*B)*cos(d*x
 + c)^4 + (19*A - 15*B)*cos(d*x + c)^3)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((1201*A - 1029
*B)*cos(d*x + c)^4 + 12*(67*A - 63*B)*cos(d*x + c)^3 - 28*(7*A - 3*B)*cos(d
*x + c)^2 + 12*(3*A - 7*B)*cos(d*x + c) - 60*A)*sqrt(a*cos(d*x + c) + a)*si
n(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)
^4 + a^2*d*cos(d*x + c)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

maple [B] time = 0.55, size = 731, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/420/d*(-1995*A*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^
4*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+1575*B*sin(d*x+c)*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-7980*A*sin(d*x
+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+
c)))^(7/2)+6300*B*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^
3*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-11970*A*sin(d*x+c)*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+9450*B*sin(d*
x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(7/2)-7980*A*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+6300*B*sin(d*x+c)*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-1995*A*sin(d*x+c)
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+1575*
B*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))
^(7/2)+1201*A*2^(1/2)*cos(d*x+c)^5-1029*B*2^(1/2)*cos(d*x+c)^5-397*A*2^(1/2
)*cos(d*x+c)^4+273*B*2^(1/2)*cos(d*x+c)^4-1000*A*2^(1/2)*cos(d*x+c)^3+840*B
*2^(1/2)*cos(d*x+c)^3+232*A*2^(1/2)*cos(d*x+c)^2-168*B*2^(1/2)*cos(d*x+c)^2
-96*A*2^(1/2)*cos(d*x+c)+84*B*2^(1/2)*cos(d*x+c)+60*A*2^(1/2))*cos(d*x+c)*s
in(d*x+c)^5*(1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))^3
/(1+cos(d*x+c))^4*2^(1/2)/a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{9/2}}{(a+a\cos(c+dx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a*cos(c + d*x))^(3/2
),x)
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a*cos(c + d*x))^(3/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

3.530 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{(15A-11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx}{10ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -1/2*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-1/30*(39*A-
35*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/10*(9*A-5*B)
*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)-1/4*(15*A-11*B)*arc
tan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+1/30*(147*A-95*B)*sin(
d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$-\frac{(15A-11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(9A-5B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx}{10ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -((15*A - 11*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]
*a^(3/2)*d) + ((147*A - 95*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(30*a*d*Sqrt
[a + a*Cos[c + d*x]]) - ((39*A - 35*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(30
*a*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(2*d*(a + a*Cos[c + d*x])^(3/2)) + ((9*A - 5*B)*Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(10*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,d$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2a^2}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(9A-5B)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{10ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(39A-35B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c}{2d(a+a\cos(c+dx))^{3/}}$$

$$= \frac{(147A-95B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}} - \frac{(39A-35B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{30ad\sqrt{a+a\cos(c+}}$$

$$= \frac{(147A-95B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}} - \frac{(39A-35B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{30ad\sqrt{a+a\cos(c+}}$$

$$= \frac{(147A-95B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{30ad\sqrt{a+a\cos(c+dx)}} - \frac{(39A-35B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{30ad\sqrt{a+a\cos(c+}}$$

$$= -\frac{(15A-11B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [F] time = 0.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] $Aborted
```

fricas [A] time = 2.02, size = 220, normalized size = 0.81

$$\frac{15\sqrt{2}\left((15A-11B)\cos(dx+c)^4+2(15A-11B)\cos(dx+c)^3+(15A-11B)\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{60\left(a^2d\cos(dx+c)^4+2a^2d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/60*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^4 + 2*(15*A - 11*B)*cos(d*x +
c)^3 + (15*A - 11*B)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((147*A - 95*B)*co
s(d*x + c)^3 + 12*(9*A - 5*B)*cos(d*x + c)^2 - 4*(3*A - 5*B)*cos(d*x + c) +
 12*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos
(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

maple [B] time = 0.56, size = 595, normalized size = 2.20

$$\left(225A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)\left(\cos^3(dx+c)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}-165B\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/60/d*(225*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3*(c
os(d*x+c)/(1+cos(d*x+c)))^(5/2)-165*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*si
n(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+675*A*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(
5/2)-495*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2*(cos(
d*x+c)/(1+cos(d*x+c)))^(5/2)+675*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-495*B*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+22
5*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)-165*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1
```

```
+cos(d*x+c)))^(5/2)-147*A*2^(1/2)*cos(d*x+c)^4+95*B*2^(1/2)*cos(d*x+c)^4+39
*A*2^(1/2)*cos(d*x+c)^3-35*B*2^(1/2)*cos(d*x+c)^3+120*A*2^(1/2)*cos(d*x+c)^
2-80*B*2^(1/2)*cos(d*x+c)^2-24*A*2^(1/2)*cos(d*x+c)+20*B*2^(1/2)*cos(d*x+c)
+12*A*2^(1/2))*cos(d*x+c)*sin(d*x+c)^3*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c
)))^(1/2)/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3*2^(1/2)/a^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(3/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.531 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=223

$$\frac{(11A-7B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(7A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a\cos(c+dx)+a}}-\frac{(\ldots}{}$$

```
[Out] -1/2*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/6*(7*A-3*
B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+1/4*(11*A-7*B)*ar
ctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)-1/6*(19*A-15*B)*sin(d
*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(11A-7B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(7A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a\cos(c+dx)+a}}-\frac{(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((11*A - 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a
^(3/2)*d) - ((19*A - 15*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a +
 a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*C
os[c + d*x])^(3/2)) + ((7*A - 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*a*d*
Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int^{\frac{1}{2}}}{2a^2}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(7A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{6ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(19A-15B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{2d(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(19A-15B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{2d(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(19A-15B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{2d(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(11A-7B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] time = 6.83, size = 981, normalized size = 4.40

$$\frac{2\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\,\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\left(\frac{(A+3B)\left(-12\cos^4\left(\frac{1}{2}(c+dx)\right){}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};-\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)-12}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^3*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*
Sin[c/2 + (d*x)/2]^2]*(-1/12*((A - B)*(1 - 2*Sin[c/2 + (d*x)/2]))/((1 + Sin
[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)) + ((A - B)*(1 + 2*Sin[
c/2 + (d*x)/2]))/(12*(1 - Sin[c/2 + (d*x)/2])*(1 - 2*Sin[c/2 + (d*x)/2]^2)^
```

```
(3/2)) - ((A - B)*(5*ArcTan[(1 - 2*Sin[c/2 + (d*x)/2])/Sqrt[1 - 2*Sin[c/2 +
 (d*x)/2]^2]] + (1 + Sin[c/2 + (d*x)/2])/((1 - Sin[c/2 + (d*x)/2])*Sqrt[1 -
 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2])/(1 - Sin[c
/2 + (d*x)/2])))/2 + ((A - B)*(5*ArcTan[(1 + 2*Sin[c/2 + (d*x)/2])/Sqrt[1 -
 2*Sin[c/2 + (d*x)/2]^2]] + (1 - Sin[c/2 + (d*x)/2])/((1 + Sin[c/2 + (d*x)/
2])*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]) + (3*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]
)/(1 + Sin[c/2 + (d*x)/2])))/2 + ((A + 3*B)*Csc[c/2 + (d*x)/2]^5*(-12*Cos[(
c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, -(Sin[c/2 + (d*x)/2]
^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c/2 + (d*x)/2]^8 - 12*Hypergeometric2
F1[2, 7/2, 9/2, -(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*Sin[c
/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*S
qrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^2))]*(1 - 2*Sin[c/2 +
(d*x)/2]^2)^3*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*((3 -
 7*Sin[c/2 + (d*x)/2]^2)*Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)
/2]^2))] - 3*ArcTanh[Sqrt[-(Sin[c/2 + (d*x)/2]^2/(1 - 2*Sin[c/2 + (d*x)/2]^
2))]]*(1 - 2*Sin[c/2 + (d*x)/2]^2))))/(126*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/
2))))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 0.60, size = 197, normalized size = 0.88

$$-\frac{3\sqrt{2}\left((11A-7B)\cos(dx+c)^{3}+2(11A-7B)\cos(dx+c)^{2}+(11A-7B)\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos\cdots}}{\cdots}\right.}{12\left(a^{2}d\cos(dx+c)^{3}+2a^{2}d\cos(dx+c)^{2}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^3 + 2*(11*A - 7*B)*cos(d*x + c)
^2 + (11*A - 7*B)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c)
+ a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((19*A - 15*B)*cos(d*x
+ c)^2 + 12*(A - B)*cos(d*x + c) - 4*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2
*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

maple [B] time = 0.44, size = 457, normalized size = 2.05

$$-\frac{\left(-33A\sin\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{2}\left(dx+c\right)\right)+21B\sin\left(dx+c\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arcsin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/12/d*(-33*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*cos(d*x+c)^2+21*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-66*A*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c
)+42*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*cos(d*x+c)-33*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ar
csin((-1+cos(d*x+c))/sin(d*x+c))+21*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+19*A*2^(1/2)*cos(d*x+c)^3-15*B*2
^(1/2)*cos(d*x+c)^3-7*A*2^(1/2)*cos(d*x+c)^2+3*B*2^(1/2)*cos(d*x+c)^2-16*A*
2^(1/2)*cos(d*x+c)+12*B*2^(1/2)*cos(d*x+c)+4*A*2^(1/2))*cos(d*x+c)*sin(d*x+
c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/(1+cos(d*x
+c))^2*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(3/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.532 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{(7A-3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(5A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A$$

```
[Out] -1/2*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(3/2)-1/4*(7*A-3*
B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+1/2*(5*A-B)*sin(
d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$-\frac{(7A-3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(5A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{(A$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -((7*A - 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a
^(3/2)*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d
*x])^(3/2)) + ((5*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a
*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sqrt{\sec(c+dx)}\sin(c+dx}{2ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sqrt{\sec(c+dx)}\sin(c+dx}{2ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{(5A-B)\sqrt{\sec(c+dx)}\sin(c+dx}{2ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{(7A-3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+a}}{2\sqrt{2}a^{3/2}d}$$

Mathematica [C] time = 4.49, size = 443, normalized size = 2.52

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{(A+3B)\csc^3\left(\frac{1}{2}(c+dx)\right)\left(5(4\cos(c+dx)+\cos(2(c+dx))+1)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{2-2\operatorname{se}}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(30*(A - B)*ArcTa
n[(1 - 2*Sin[(c + d*x)/2])/Sqrt[Cos[c + d*x]]] - 30*(A - B)*ArcTan[(1 + 2*S
in[(c + d*x)/2])/Sqrt[Cos[c + d*x]]] - (20*(A - B)*Sqrt[Cos[c + d*x]])/(-1
+ Sin[(c + d*x)/2]) - (20*(A - B)*Sqrt[Cos[c + d*x]])/(1 + Sin[(c + d*x)/2]
) + (5*(A - B)*(-1 + 2*Sin[(c + d*x)/2]))/(Sqrt[Cos[c + d*x]]*(Cos[(c + d*x
)/4] + Sin[(c + d*x)/4])^2) - (5*(A - B)*(1 + 2*Sin[(c + d*x)/2]))/(Sqrt[Co
s[c + d*x]]*(-1 + Sin[(c + d*x)/2])) + ((A + 3*B)*Csc[(c + d*x)/2]^3*(5*(1
+ 4*Cos[c + d*x] + Cos[2*(c + d*x)])*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec
[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]) - 2*
```

```
Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[(c +
 d*x)/2]^4*Sin[c + d*x]*Tan[c + d*x]))/(2*Cos[c + d*x]^(3/2))))/(10*d*(a*(1
 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 0.76, size = 163, normalized size = 0.93

$$\frac{\sqrt{2}\left((7\,A-3\,B)\cos\left(dx+c\right)^2+2\,(7\,A-3\,B)\cos\left(dx+c\right)+7\,A-3\,B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)+}{4\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7*A
 - 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(sqrt(a)*sin(d*x + c))) + 2*((5*A - B)*cos(d*x + c) + 4*A)*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*c
os(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

maple [B] time = 0.41, size = 312, normalized size = 1.77

$$-\frac{\left(-7A\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sin\left(dx+c\right)+3B\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\arcsin\left(\frac{-1+\mathrm{c}}{\mathrm{sin}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(-7*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*
x+c))/sin(d*x+c))*sin(d*x+c)+3*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
```

```
2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+5*A*2^(1/2)*cos(d*x+c)^2-7
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin
(d*x+c)-B*2^(1/2)*cos(d*x+c)^2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-A*2^(1/2)*cos(d*x+c)+B*2^(1/2)*cos(
d*x+c)-4*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2
)/sin(d*x+c)/(1+cos(d*x+c))*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(3/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.533 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$\frac{(3A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] -1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/4*(3*A+B)
*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2978, 12, 2782, 205}

$$\frac{(3A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((3*A + B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3
/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c
+ d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{3/2}}\,d$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left((3A+B)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left((3A+B)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.}{}$$

$$= \frac{(3A+B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] time = 1.64, size = 196, normalized size = 1.54

$$\frac{i\cos^3\left(\frac{1}{2}(c+dx)\right)\left((3A+B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{2}i(A-B)\left(\sin\left(\frac{1}{2}(c+\right.\right.\right.}{d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (I*Cos[(c + d*x)/2]^3*(((3*A + B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]
*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) - (I/2)*(A - B)*Sec[(
c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))
)/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 0.71, size = 144, normalized size = 1.13

$$-\frac{\sqrt{2}\left((3A+B)\cos(dx+c)^2+2(3A+B)\cos(dx+c)+3A+B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2\sqrt{}}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A +
B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(A - B)*sqrt(cos(d*x + c))*
sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(3/2
), x)
```

maple [B] time = 0.37, size = 235, normalized size = 1.85

$$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos\left(dx+c\right)+B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos\left(dx+c\right)+3A\,\mathrm{arcs}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-A*2^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+A*2^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d
*x+c)-B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+B\,\cos\left(c+d\,x\right)\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(3/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\cos\left(c+dx\right)\right)\sqrt{\sec\left(c+dx\right)}}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a*(cos(c + d*x) + 1))**(3
/2), x)
```

3.534 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=185

$$\frac{(A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{a^{3/2}d}$$

```
[Out] 1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*B*arcsin(s
in(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^(3/2)/d+1/4*(A-5*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/
2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1
/2)
```

Rubi [A] time = 0.54, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2961, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{(A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A - 5*B)*ArcTan[(Sqrt[a]*Sin[c + d
*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2*d
*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
```

```
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,d$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{2}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left((A-5B)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left((A-5B)\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.}{}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{(A-5B)\tan}{}$$

Mathematica [C] time = 1.62, size = 243, normalized size = 1.31

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left((A-B)\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)} - i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e}{1+}}\right.}{2d(a(\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(((-I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(4*B*ArcSinh[E^(I*(c + d*x))] - Sqrt[
2]*(A - 5*B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c +
d*x))])] - 4*B*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(c + d*x))
 + (A - B)*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(-Sin[(c + d*x)/2] + Sin[(
3*(c + d*x))/2])))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

fricas [A] time = 8.00, size = 203, normalized size = 1.10

$$-\frac{\sqrt{2}\left((A-5B)\cos(dx+c)^2+2(A-5B)\cos(dx+c)+A-5B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a}}{4\left(a^2d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*
B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(A - B)*sqrt(cos(d*x + c))*
sin(d*x + c) + 8*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sqrt(a)*arctan(s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

maple [A] time = 0.38, size = 288, normalized size = 1.56

$$-\frac{\sqrt{a(1+\cos(dx+c))}\,\cos(dx+c)(-1+\cos(dx+c))^2\left(A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\cos(dx+c)-4B\sqrt{2}\,\arctan\left(\frac{\sin(d}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(A*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-4*B*2^(1/2)*arctan(sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)-B*2^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(
d*x+c)-A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*B*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*sin(d*x+c)+B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/c
os(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^5*2^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(sec(c + d
*x))), x)
```

3.535 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{(2A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(5A-9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{c}}\right.}{2\sqrt{2}a^{3/2}d}$$

```
[Out] 1/2*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)-1/2*(A-3*B)*
sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(2*A-3*B)*arcsin(sin
(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a
^(3/2)/d-1/4*(5*A-9*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/
2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1
/2)
```

Rubi [A] time = 0.74, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(2A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(5A-9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt{c}}\right.}{2\sqrt{2}a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] ((2*A - 3*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - ((5*A - 9*B)*ArcTan[(Sqrt[a]
*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c +
d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) - ((A - 3*B)*Sin[
c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}}\,d$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{(A-3B)\sin(c+dx}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{(A-3B)\sin(c+dx}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{(A-3B)\sin(c+dx}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec}}$$

$$= \frac{(2A-3B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} - \frac{(5A}{}$$

Mathematica [C] time = 6.71, size = 836, normalized size = 3.53

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sec\left(\frac{c}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{\sec\left(\frac{c}{2}\right)\left(A\sin\left(\frac{c}{2}\right)-B\sin\left(\frac{c}{2}\right)\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2A\cos\left(\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)}{d}+\frac{2B\cos\left(\frac{3dx}{2}\right)s}{d}\right.}{(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] ((-I)*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(
c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*
x))])]*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^
(3/2)) + ((3*I)*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 +
E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*
I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c
 + d*x]))^(3/2)) + ((2*I)*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]
*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] +
ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(
c + d*x))*(a*(1 + Cos[c + d*x]))^(3/2)) - ((3*I)*Sqrt[2]*B*Sqrt[E^(I*(c + d
*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(
I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^
((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d
*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(3/2)) + (Cos[c/2 +
 (d*x)/2]^3*Sqrt[Sec[c + d*x]]*((-2*A*Cos[(d*x)/2]*Sin[c/2])/d + (Sec[c/2]*
Sec[c/2 + (d*x)/2]*(A*Sin[c/2] - B*Sin[c/2]))/d + (2*B*Cos[(3*d*x)/2]*Sin[(
3*c)/2])/d - (2*A*Cos[c/2]*Sin[(d*x)/2])/d + (Sec[c/2]*Sec[c/2 + (d*x)/2]^2
*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2]))/d + (2*B*Cos[(3*c)/2]*Sin[(3*d*x)/2])/d
))/(a*(1 + Cos[c + d*x]))^(3/2)
```

fricas [A] time = 10.96, size = 246, normalized size = 1.04

$$\frac{\sqrt{2}\left((5\,A-9\,B)\cos\left(dx+c\right)^{2}+2\,(5\,A-9\,B)\cos\left(dx+c\right)+5\,A-9\,B\right)\sqrt{a}\;\arctan\left(\frac{\sqrt{2}\,\sqrt{a\cos(dx+c)+a}\,\sqrt{\cos(dx+c)}}{\sqrt{a}\,\sin(dx+c)}\right)-4}{4\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5*A
 - 9*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/
(sqrt(a)*sin(d*x + c))) - 4*((2*A - 3*B)*cos(d*x + c)^2 + 2*(2*A - 3*B)*cos
(d*x + c) + 2*A - 3*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))/(sqrt(a)*sin(d*x + c))) + 2*(2*B*cos(d*x + c)^2 - (A - 3*B)*cos(d*x
+ c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(
d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

maple [A] time = 0.44, size = 370, normalized size = 1.56

$$-\frac{\sqrt{a\left(1+\cos(dx+c)\right)}\,\cos(dx+c)\left(-1+\cos(dx+c)\right)^{3}\left(-2B\left(\cos^{2}(dx+c)\right)\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+A\sqrt{2}\,\sqrt{\frac{\cos(d}{1+\cos(}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(-2*B*cos(d*x+
c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+A*2^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)+4*A*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)-B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*cos(d*x+c)-6*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)/cos(d*x+c))*sin(d*x+c)+5*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d
*x+c)-A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-9*B*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*sin(d*x+c)+3*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/
cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^7*2^(1/2)/a^
2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\cos\left(c + d\,x\right)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\left(a + a\cos\left(c + d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.536 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{(283A-163B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(157A-85B)\sin(c+dx)\sec^{\frac{5}{2}}(}{80a^2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -1/4*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(21*A-
13*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)-1/240*(787*A-4
75*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)+1/80*(157*A-
85*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)-1/32*(283*A-
163*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+
c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+1/240*(2671*
A-1495*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 1.12, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(157A-85B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{80a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(787A-475B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{240a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(2671A-1495B)\sin(c+dx)}{240a^2d\sqrt{a\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] -((283*A - 163*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt
[2]*a^(5/2)*d) + ((2671*A - 1495*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(240*a
^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((787*A - 475*B)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(240*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(5/2)*
Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - ((21*A - 13*B)*Sec[c + d*x
]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((157*A - 85*B)
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(80*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,d$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int}{4a^2}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(21A-13B)\sec^{\frac{5}{2}}(c+dx)\sin(c-}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(21A-13B)\sec^{\frac{5}{2}}(c+dx)\sin(c-}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(787A-475B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin}{4d(a+a\cos(c+dx)}$$

$$= \frac{(2671A-1495B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A-475B)\sec^{\frac{3}{2}}(c-}{240a^2d\sqrt{a+a\,\mathrm{co}}}$$

$$= \frac{(2671A-1495B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A-475B)\sec^{\frac{3}{2}}(c-}{240a^2d\sqrt{a+a\,\mathrm{co}}}$$

$$= \frac{(2671A-1495B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{240a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(787A-475B)\sec^{\frac{3}{2}}(c-}{240a^2d\sqrt{a+a\,\mathrm{co}}}$$

$$= -\frac{(283A-163B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}}}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [C] time = 8.46, size = 261, normalized size = 0.82

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\Bigg(\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(10(2605A-1381B)\cos(c+dx)+108(157A-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-240*I)*(283*A - 163*B)*Sqrt[E^(I*(c + d*x))/(1 + E^
((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*
x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (15053
*A - 7685*B + 10*(2605*A - 1381*B)*Cos[c + d*x] + 108*(157*A - 85*B)*Cos[2*
(c + d*x)] + 9110*A*Cos[3*(c + d*x)] - 5030*B*Cos[3*(c + d*x)] + 2671*A*Cos
[4*(c + d*x)] - 1495*B*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^3*Sec[c + d*x]^(5
/2)*Tan[(c + d*x)/2]))/(960*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 0.61, size = 266, normalized size = 0.84

$$\frac{15\sqrt{2}\left((283\,A-163\,B)\cos\left(dx+c\right)^{5}+3\,(283\,A-163\,B)\cos\left(dx+c\right)^{4}+3\,(283\,A-163\,B)\cos\left(dx+c\right)^{3}+(283\,A\right.}{480\left(a^{3}d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/480*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^5 + 3*(283*A - 163*B)*cos(d
*x + c)^4 + 3*(283*A - 163*B)*cos(d*x + c)^3 + (283*A - 163*B)*cos(d*x + c)
^2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqr
t(a)*sin(d*x + c))) + 2*((2671*A - 1495*B)*cos(d*x + c)^4 + 5*(911*A - 503*
B)*cos(d*x + c)^3 + 32*(49*A - 25*B)*cos(d*x + c)^2 - 160*(A - B)*cos(d*x +
 c) + 96*A)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*
d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*
cos(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

maple [B] time = 0.52, size = 729, normalized size = 2.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x)

[Out] -1/480/d*(4245*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^4 *(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-2445*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+16980*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+ c)))^(5/2)-9780*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^ 3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+25470*A*arcsin((-1+cos(d*x+c))/sin(d*x+ c))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-14670*B*arcsi n((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d* x+c)))^(5/2)+16980*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+ c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-9780*B*arcsin((-1+cos(d*x+c))/sin(d*x+ c))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+4245*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-244 5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-2671*A*2^(1/2)*cos(d*x+c)^5+1495*B*2^(1/2)*cos(d*x+c)^5-1884*A*2^(1/2)*cos(d*x+c)^4+1020*B*2^(1/2)*cos(d*x+c)^4+2987*A*2^(1/2)*cos(d*x+c)^3-1 715*B*2^(1/2)*cos(d*x+c)^3+1728*A*2^(1/2)*cos(d*x+c)^2-960*B*2^(1/2)*cos(d* x+c)^2-256*A*2^(1/2)*cos(d*x+c)+160*B*2^(1/2)*cos(d*x+c)+96*A*2^(1/2))*cos(d*x+c)*sin(d*x+c)*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x +c))/(1+cos(d*x+c))^3*2^(1/2)/a^3

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(5/2),x)

[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a*cos(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.537 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=270

$$\frac{(163A-75B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(95A-39B)\sin(c+dx)\sec^{\frac{3}{2}}(c+$$

```
[Out] -1/4*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(17*A-
9*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+1/48*(95*A-39*B
)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)+1/32*(163*A-75*B
)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)-1/48*(299*A-147*B
)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(95A-39B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(299A-147B)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\cos(c+dx)+a}}+\frac{(163A-75B)\sqrt{\cos(c+dx)}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((163*A - 75*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2
]*a^(5/2)*d) - ((299*A - 147*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*
Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*
(a + a*Cos[c + d*x])^(5/2)) - ((17*A - 9*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + ((95*A - 39*B)*Sec[c + d*x]^(3/2)*S
in[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,d$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{4a^2}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(17A-9B)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(299A-147B)\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin}{4d(a+a\cos(c+dx}$$

$$= -\frac{(299A-147B)\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin}{4d(a+a\cos(c+dx}$$

$$= -\frac{(299A-147B)\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin}{4d(a+a\cos(c+dx}$$

$$= \frac{(163A-75B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{16\sqrt{2}a^{5/2}d}$$

Mathematica [C] time = 3.79, size = 243, normalized size = 0.90

$$\frac{i\cos^5\left(\frac{1}{2}(c+dx)\right)\left(3(163A-75B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\frac{1}{8}i\tan\left(\frac{1}{2}(c+dx)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] ((I/12)*Cos[(c + d*x)/2]^5*((3*(163*A - 75*B)*Sqrt[E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x
)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (I/8)*(
```

```
878*A - 510*B + (1537*A - 825*B)*Cos[c + d*x] + 2*(503*A - 255*B)*Cos[2*(c
+ d*x)] + 299*A*Cos[3*(c + d*x)] - 147*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]
^3*Sec[c + d*x]^(3/2)*Tan[(c + d*x)/2]))/(d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 0.87, size = 246, normalized size = 0.91

$$-\frac{3\sqrt{2}\left((163\,A-75\,B)\cos\left(dx+c\right)^4+3\,(163\,A-75\,B)\cos\left(dx+c\right)^3+3\,(163\,A-75\,B)\cos\left(dx+c\right)^2+(163\,A-\right.}{96\left(a^3d\cos\left(dx+c\right)^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/96*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^4 + 3*(163*A - 75*B)*cos(d*x
+ c)^3 + 3*(163*A - 75*B)*cos(d*x + c)^2 + (163*A - 75*B)*cos(d*x + c))*sqr
t(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*si
n(d*x + c))) + 2*((299*A - 147*B)*cos(d*x + c)^3 + (503*A - 255*B)*cos(d*x
+ c)^2 + 32*(5*A - 3*B)*cos(d*x + c) - 32*A)*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3
+ 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos\left(dx+c\right)+A)\sec\left(dx+c\right)^{\frac{5}{2}}}{(a\cos\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

maple [B] time = 0.51, size = 585, normalized size = 2.17

$$\left(-489A\sin\left(dx+c\right)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^3\left(dx+c\right)\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+225B\sin\left(dx+c\right)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/96/d*(-489*A*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+225*B*sin(d*x+c)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-1467*A*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*
x+c)^2+675*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*
x+c))/sin(d*x+c))*cos(d*x+c)^2-1467*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+675*B*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)
+299*A*2^(1/2)*cos(d*x+c)^4-489*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-147*B*2^(1/2)*cos(d*x+c)^4+225*B*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)+204*A*2^(1/2)*cos(d*x+c)^3-108*B*2^(1/2)*cos(d*x+c)^3-343*A*2^(1/2)*cos(d
*x+c)^2+159*B*2^(1/2)*cos(d*x+c)^2-192*A*2^(1/2)*cos(d*x+c)+96*B*2^(1/2)*co
s(d*x+c)+32*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(
1/2)/sin(d*x+c)/(1+cos(d*x+c))^2*2^(1/2)/a^3
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(5/2
), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
[Out] Timed out
```

3.538 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{(75A-19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(49A-9B)\sin(c+dx)\sqrt{\sec(c+a}}{16a^2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] -1/4*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(5/2)-1/16*(13*A-
5*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)-1/32*(75*A-19*B
)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+1/16*(49*A-9*B)*s
in(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(49A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{(75A-19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] -((75*A - 19*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2
]*a^(5/2)*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c
+ d*x])^(5/2)) - ((13*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a
+ a*Cos[c + d*x])^(3/2)) + ((49*A - 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,d$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{4a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}\,\sin(c}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}\,\sin(c}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}\,\sin(c}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+dx)}\,\sin(c}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{(75A-19B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [C] time = 2.30, size = 219, normalized size = 0.98

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{4}\tan\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,(2(85A-13B)\cos(c+dx)+(49A-9B)\cos(\right.}{4d(a(\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-I)*(75*A - 19*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(
Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + ((113*A - 9*
B + 2*(85*A - 13*B)*Cos[c + d*x] + (49*A - 9*B)*Cos[2*(c + d*x)])*Sec[(c +
d*x)/2]^3*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2])/4))/(4*d*(a*(1 + Cos[c + d*x
]))^(5/2))
```

fricas [A] time = 0.74, size = 210, normalized size = 0.94

$$\frac{\sqrt{2}\left((75\,A-19\,B)\cos\left(dx+c\right)^{3}+3\,(75\,A-19\,B)\cos\left(dx+c\right)^{2}+3\,(75\,A-19\,B)\cos\left(dx+c\right)+75\,A-19\,B\right)\sqrt{a}\,a}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(d*x + c)^
2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(a)*arctan(sqrt(2)*sqrt
(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((49*A
- 9*B)*cos(d*x + c)^2 + (85*A - 13*B)*cos(d*x + c) + 32*A)*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos\left(dx+c\right)+A)\sec\left(dx+c\right)^{\frac{3}{2}}}{(a\cos\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2
), x)
```

maple [B] time = 0.43, size = 457, normalized size = 2.05

$$-\frac{(-1+\cos\left(dx+c\right))\left(75A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-19B\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*(-1+cos(d*x+c))*(75*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)
^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-19*B*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+150*
A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d
```

```
*x+c))*sin(d*x+c)-49*A*2^(1/2)*cos(d*x+c)^3-38*B*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+9*B*2^(1/2
)*cos(d*x+c)^3+75*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*sin(d*x+c)-36*A*2^(1/2)*cos(d*x+c)^2-19*B*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+4*B*2^(1/2)*co
s(d*x+c)^2+53*A*2^(1/2)*cos(d*x+c)-13*B*2^(1/2)*cos(d*x+c)+32*A*2^(1/2))*co
s(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/(1+cos(
d*x+c))*2^(1/2)/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(5/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.539 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=176

$$\frac{(19A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3}}$$

```
[Out] -1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-1/16*(9*A-B
)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/32*(19*A+5*B)*ar
ctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2978, 12, 2782, 205}

$$\frac{(19A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(9A-B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((19*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*
a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Se
c[c + d*x]]) - ((9*A - B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*
Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{5/2}}\,d$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-B)\sin(c+d}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-B)\sin(c+d}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{(9A-B)\sin(c+d}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{}}$$

$$= \frac{(19A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [C] time = 1.84, size = 216, normalized size = 1.23

$$\frac{i\cos^5\left(\frac{1}{2}(c+dx)\right)\left((19A+5B)e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\frac{1}{4}i\left(\sin\left(\frac{1}{2}(c+dx)\right)-s\right.\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(5
/2),x]
```

```
[Out] ((I/4)*Cos[(c + d*x)/2]^5*(((19*A + 5*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(
Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])/E^((I/2)*(c + d*x)) - (I/4)*(13*A
- 5*B + (9*A - B)*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(Sin[
(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/(d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 0.72, size = 207, normalized size = 1.18

$$-\frac{\sqrt{2}\left((19\,A+5\,B)\cos\left(dx+c\right)^{3}+3\,(19\,A+5\,B)\cos\left(dx+c\right)^{2}+3\,(19\,A+5\,B)\cos\left(dx+c\right)+19\,A+5\,B\right)\sqrt{a}\,\mathrm{arc}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2
 + 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((9*A - B)
*cos(d*x + c)^2 + (13*A - 5*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
+ 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(5/2
), x)
```

maple [B] time = 0.39, size = 375, normalized size = 2.13

$$-\frac{\sqrt{\frac{1}{\cos(dx+c)}}\ \sqrt{a\left(1+\cos(dx+c)\right)}\ \cos(dx+c)\left(-1+\cos(dx+c)\right)^{2}\left(-9A\left(\cos^{2}(dx+c)\right)\sqrt{2}\ \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+B\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*
x+c))^2*(-9*A*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+B*cos(
d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+19*A*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-4*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*cos(d*x+c)+5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d
*x+c)+4*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+19*A*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+13*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)+5*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-5*B*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^5/(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(5/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.540 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=174

$$\frac{(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A+7B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] 1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+1/16*(A+7*B)
*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/32*(5*A+3*B)*arct
an(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2977, 2978, 12, 2782, 205}

$$\frac{(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A+7B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((5*A + 3*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a
^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec
[c + d*x]]) + ((A + 7*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*S
qrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
```

```
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{(A+7B)\sin(c+}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{(A+7B)\sin(c+}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{(A+7B)\sin(c+}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{(5A+3B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [C] time = 1.82, size = 213, normalized size = 1.22

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{4}\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\sec(c+dx)}\sec^4\left(\frac{1}{2}(c+dx)\right)((A+7B)\cos(c+dx)+5\right.}{4d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*((I*(5*A + 3*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[
2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + ((5*A + 3*B + (A
+ 7*B)*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(-Sin[(c + d*x)/
2] + Sin[(3*(c + d*x))/2]))/4))/(4*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

fricas [A] time = 0.57, size = 205, normalized size = 1.18

$$-\frac{\sqrt{2}\left((5A+3B)\cos(dx+c)^3+3(5A+3B)\cos(dx+c)^2+3(5A+3B)\cos(dx+c)+5A+3B\right)\sqrt{a}\,\mathrm{arctan}}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 +
 3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((A + 7*B)*cos
(d*x + c)^2 + (5*A + 3*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a
^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

maple [B] time = 0.47, size = 375, normalized size = 2.16

$$\frac{\sqrt{a(1+\cos(dx+c))}\ \cos(dx+c)(-1+\cos(dx+c))^3\left(A\left(\cos^2(dx+c)\right)\sqrt{2}\ \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+7B\left(\cos^2(dx+c)\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(A*cos(d*x+c)^
2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*B*cos(d*x+c)^2*2^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)+5*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+
c)*cos(d*x+c)+4*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+3*B*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-4*B*2^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+5*A*arcsin((-1+cos(d*x+c))/sin(d*x+c
))*sin(d*x+c)-5*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*B*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-3*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^7*
2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.541 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{(3A-43B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\right.}{a^{5/2}d}$$

```
[Out] 1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2)+1/16*(3*A-11
*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*B*arcsin(sin(d
*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(
5/2)/d+1/32*(3*A-43*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/
2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1
/2)
```

Rubi [A] time = 0.74, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2961, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{(3A-43B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\right.}{a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A - 43*B)*ArcTan[(Sqrt[a]*Sin[c
+ d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/
(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) + ((3*A - 11*B)*Sin[c +
 d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(3A-11B)\sin(c+d}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{s}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(3A-11B)\sin(c+d}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{s}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(3A-11B)\sin(c+d}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{s}}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{(3A-43B)\,\text{ta}}{}$$

Mathematica [C] time = 2.58, size = 264, normalized size = 1.13

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left((7A-15B)\cos(c+dx)+3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(((-I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(32*B*ArcSinh[E^(I*(c + d*x))] - Sqrt
[2]*(3*A - 43*B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(
c + d*x))])] - 32*B*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(c +
d*x)) + ((3*A - 11*B + (7*A - 15*B)*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[S
```

```
ec[c + d*x]]*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/2))/(8*d*(a*(1 + C
os[c + d*x]))^(5/2))
```

fricas [A] time = 16.16, size = 277, normalized size = 1.18

$$-\frac{\sqrt{2}\left((3\,A-43\,B)\cos\left(dx+c\right)^{3}+3\,(3\,A-43\,B)\cos\left(dx+c\right)^{2}+3\,(3\,A-43\,B)\cos\left(dx+c\right)+3\,A-43\,B\right)\sqrt{a}\ \mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^2
 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 64*(B*cos(d*
x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((7*A - 1
5*B)*cos(d*x + c)^2 + (3*A - 11*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c
)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

maple [B] time = 0.39, size = 476, normalized size = 2.03

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\ \left(-1+\cos\left(dx+c\right)\right)^{4}\cos\left(dx+c\right)\left(7A\left(\cos^{2}\left(dx+c\right)\right)\sqrt{2}\ \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-15B\left(\cos^{2}\left(dx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*cos(d*x+c)*(7*A*cos(d*x+
c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-15*B*cos(d*x+c)^2*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)-32*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)*arctan(s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*A*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-4*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)-43*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*
cos(d*x+c)+4*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-32*B*2^
(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d
*x+c)+3*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-3*A*2^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)-43*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+
c)+11*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(3/2)/(co
s(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^9*2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+a\,\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.542 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=286

$$\frac{(2A-5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(43A-115B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{}{\sqrt{2}}\right.}{16\sqrt{2}\,a^{5/2}d}$$

```
[Out] 1/4*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2)+1/16*(7*A-15
*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)-1/16*(11*A-35*B)
*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(2*A-5*B)*arcsin(
sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a^(5/2)/d-1/32*(43*A-115*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x
+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)
/d*2^(1/2)
```

Rubi [A] time = 0.98, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(2A-5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-35B)\sin(c+dx)}{16a^2d\sqrt{\sec(c+dx)}\,\sqrt{a\cos(c+dx)+a}}-\frac{(43A-115}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((2*A - 5*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - ((43*A - 115*B)*ArcTan[(Sqrt
[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sq
rt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[
c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((7*A - 15*
B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) - (
(11*A - 35*B)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
```

```
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}}\,d$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(7A-15B)\sin(c+}{16ad(a+a\cos(c+dx))^{3/2}\,\mathrm{s}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(7A-15B)\sin(c+}{16ad(a+a\cos(c+dx))^{3/2}\,\mathrm{s}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(7A-15B)\sin(c+}{16ad(a+a\cos(c+dx))^{3/2}\,\mathrm{s}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(7A-15B)\sin(c+}{16ad(a+a\cos(c+dx))^{3/2}\,\mathrm{s}}$$

$$= \frac{(2A-5B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} - \frac{(43A}{}$$

Mathematica [C] time = 7.21, size = 929, normalized size = 3.25

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sec\left(\frac{c}{2}\right)\left(B\sin\left(\frac{dx}{2}\right)-A\sin\left(\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} - \frac{(A-B)\tan\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} + \frac{\sec\left(\frac{c}{2}\right)\left(19A\sin\left(\frac{dx}{2}\right)-27B\sin\left(\frac{dx}{2}\right)\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d}\right.}{(a(\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/
2)),x]
```

```
[Out] (((-11*I)/4)*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*
```

```
(c + d*x))])]*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c +
d*x]))^(5/2)) + (((35*I)/4)*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
)]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d*x))*(
a*(1 + Cos[c + d*x]))^(5/2)) + ((4*I)*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x)
)] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c +
 d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^5)/(
d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2)) - ((10*I)*Sqrt[2]*B*Sqr
t[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*
(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])
*Cos[c/2 + (d*x)/2]^5)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2))
 + (Cos[c/2 + (d*x)/2]^5*Sqrt[Sec[c + d*x]]*((15*(-A + B)*Cos[(d*x)/2]*Sin[
c/2])/(2*d) + (4*B*Cos[(3*d*x)/2]*Sin[(3*c)/2])/d - (15*(A - B)*Cos[c/2]*Si
n[(d*x)/2])/(2*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^2*(19*A*Sin[(d*x)/2] - 27*
B*Sin[(d*x)/2]))/(4*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^4*(-(A*Sin[(d*x)/2])
+ B*Sin[(d*x)/2]))/(2*d) + (4*B*Cos[(3*c)/2]*Sin[(3*d*x)/2])/d + ((19*A - 2
7*B)*Sec[c/2 + (d*x)/2]*Tan[c/2])/(4*d) - ((A - B)*Sec[c/2 + (d*x)/2]^3*Tan
[c/2])/(2*d)))/(a*(1 + Cos[c + d*x]))^(5/2)
```

fricas [A] time = 29.03, size = 313, normalized size = 1.09

$$\frac{\sqrt{2}\left((43\,A-115\,B)\cos\left(dx+c\right)^{3}+3\,(43\,A-115\,B)\cos\left(dx+c\right)^{2}+3\,(43\,A-115\,B)\cos\left(dx+c\right)+43\,A-115\,B\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^3 + 3*(43*A - 115*B)*cos(d*x + c
)^2 + 3*(43*A - 115*B)*cos(d*x + c) + 43*A - 115*B)*sqrt(a)*arctan(sqrt(2)*
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 32*((
2*A - 5*B)*cos(d*x + c)^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + 3*(2*A - 5*B)*co
s(d*x + c) + 2*A - 5*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*
x + c))/(sqrt(a)*sin(d*x + c))) + 2*(16*B*cos(d*x + c)^3 - 5*(3*A - 11*B)*c
os(d*x + c)^2 - (11*A - 35*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*
x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

maple [B] time = 0.43, size = 609, normalized size = 2.13

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{5}\cos\left(dx+c\right)\left(-16B\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{3}\left(dx+c\right)\right)+15A\left(\cos^{2}\left(d\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*cos(d*x+c)*(-16*B*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+15*A*cos(d*x+c)^2*2^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+32*A*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)-39*B*cos(d*x+c)^2
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-80*B*cos(d*x+c)*2^(1/2)*sin(d*x+
c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+43*A*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-4*A*2^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+32*A*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)-115*B*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+20*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*cos(d*x+c)-80*B*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/cos(d*x+c))*sin(d*x+c)+43*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*si
n(d*x+c)-11*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-115*B*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*sin(d*x+c)+35*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2))/(1/cos(d*x+c))^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^11*
2^(1/2)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\cos(c + d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a + a\cos(c + d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.543 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=317

$$\frac{(1015A-363B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}+\frac{(579A-199B)\sin(c+dx)\sec^{\frac{3}{2}}}{192a^3d\sqrt{a\cos(c+dx)+}}$$

```
[Out] -1/6*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)-1/48*(23*A-
11*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)-1/64*(109*A-41
*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)+1/192*(579*A-1
99*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^(1/2)+1/128*(1015*
A-363*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*
x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)-1/192*(188
7*A-691*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 1.15, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(579A-199B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{192a^3d\sqrt{a\cos(c+dx)+a}}-\frac{(109A-41B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{(1887A-691B)\sin(c+dx)\sqrt{}}{192a^3d\sqrt{a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((1015*A - 363*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt
[2]*a^(7/2)*d) - ((1887*A - 691*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^
3*d*Sqrt[a + a*Cos[c + d*x]]) - ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) - ((23*A - 11*B)*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) - ((109*A - 41*B)*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2)) + ((579*A - 199*B)
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(192*a^3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,d$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int}{6a^2}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(23A-11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(1887A-691B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{6d(a+a\cos(c+d}$$

$$= -\frac{(1887A-691B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{6d(a+a\cos(c+d}$$

$$= -\frac{(1887A-691B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}} - \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{6d(a+a\cos(c+d}$$

$$= \frac{(1015A-363B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{sec}}}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] time = 5.77, size = 267, normalized size = 0.84

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(4(9415A-3579B)\cos(c+dx)+8(3069A-1145B)\cos(2(c+dx))+10164A\cos(3(c+d}{96d}\right.}{8(a(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*((I*(1015*A - 363*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(
Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])])/(d*E^((I/2)*(c + d*x))) - ((21641*
A - 8469*B + 4*(9415*A - 3579*B)*Cos[c + d*x] + 8*(3069*A - 1145*B)*Cos[2*(
c + d*x)] + 10164*A*Cos[3*(c + d*x)] - 3748*B*Cos[3*(c + d*x)] + 1887*A*Cos
[4*(c + d*x)] - 691*B*Cos[4*(c + d*x)])*Sec[(c + d*x)/2]^5*Sec[c + d*x]^(3/
2)*Tan[(c + d*x)/2])/(96*d)))/(8*(a*(1 + Cos[c + d*x]))^(7/2))
```

fricas [A] time = 0.84, size = 295, normalized size = 0.93

$$-\frac{3\sqrt{2}\left((1015\,A-363\,B)\cos\left(dx+c\right)^{5}+4\,(1015\,A-363\,B)\cos\left(dx+c\right)^{4}+6\,(1015\,A-363\,B)\cos\left(dx+c\right)^{3}+4\,(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((1015*A - 363*B)*cos(d*x + c)^5 + 4*(1015*A - 363*B)*cos
(d*x + c)^4 + 6*(1015*A - 363*B)*cos(d*x + c)^3 + 4*(1015*A - 363*B)*cos(d*
x + c)^2 + (1015*A - 363*B)*cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*((1887*A - 69
1*B)*cos(d*x + c)^4 + 2*(2541*A - 937*B)*cos(d*x + c)^3 + 39*(109*A - 41*B)
*cos(d*x + c)^2 + 128*(7*A - 3*B)*cos(d*x + c) - 128*A)*sqrt(a*cos(d*x + c)
 + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(
d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*
x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos\left(dx+c\right)+A)\sec\left(dx+c\right)^{\frac{5}{2}}}{(a\cos\left(dx+c\right)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

maple [B] time = 0.48, size = 729, normalized size = 2.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] -1/384/d*(-1+cos(d*x+c))*(-3045*A*cos(d*x+c)^4*sin(d*x+c)*arcsin((-1+cos(d*
x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1089*B*cos(d*x+c)^4*sin
(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)-12180*A*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)+4356*B*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-18270*A*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c
)^2+6534*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*cos(d*x+c)^2+1887*A*2^(1/2)*cos(d*x+c)^5-12180*A*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d
*x+c)-691*B*2^(1/2)*cos(d*x+c)^5+4356*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)+3195*A*2^(1/2)*cos(
d*x+c)^4-3045*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos
(d*x+c))/sin(d*x+c))-1183*B*2^(1/2)*cos(d*x+c)^4+1089*B*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-831*A*2^(1/2)*
cos(d*x+c)^3+275*B*2^(1/2)*cos(d*x+c)^3-3355*A*2^(1/2)*cos(d*x+c)^2+1215*B*
2^(1/2)*cos(d*x+c)^2-1024*A*2^(1/2)*cos(d*x+c)+384*B*2^(1/2)*cos(d*x+c)+128
*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*
x+c)^3/(1+cos(d*x+c))^2*2^(1/2)/a^4
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{(a+a\cos(c+dx))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(7/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a*cos(c + d*x))^(7/2
), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.544 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{3(121A-21B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{(691A-103B)\sin(c+dx)\sqrt{\sec}}{192a^3d\sqrt{a\cos(c+dx)+}}$$

```
[Out] -1/6*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(7/2)-1/48*(19*A-
7*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)-1/192*(199*A-43
*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)-3/128*(121*A-2
1*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)
)^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)+1/192*(691*A-1
03*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2978, 2984, 12, 2782, 205}

$$\frac{(691A-103B)\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^3d\sqrt{a\cos(c+dx)+a}}-\frac{(199A-43B)\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{3(121A-21B)\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-3*(121*A - 21*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqr
t[2]*a^(7/2)*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a + a*Cos
[c + d*x])^(7/2)) - ((19*A - 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a*d*
(a + a*Cos[c + d*x])^(5/2)) - ((199*A - 43*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)) + ((691*A - 103*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(192*a^3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,d$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{6a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}\,\sin(c}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}\,\sin(c}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}\,\sin(c}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}\,\sin(c}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{(19A-7B)\sqrt{\sec(c+dx)}\,\sin(c}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{3(121A-21B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec}}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] time = 3.28, size = 242, normalized size = 0.90

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{16}\tan\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,(9(941A-121B)\cos(c+dx)+4(937A-13\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + a*Cos[c + d*x])^(7
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((-9*I)*(121*A - 21*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2
*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x))
)/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + ((5284*A
```

```
- 532*B + 9*(941*A - 121*B)*Cos[c + d*x] + 4*(937*A - 133*B)*Cos[2*(c + d*x
)] + 691*A*Cos[3*(c + d*x)] - 103*B*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^5*Sq
rt[Sec[c + d*x]]*Tan[(c + d*x)/2])/16))/(24*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

fricas [A] time = 1.34, size = 260, normalized size = 0.96

$$\frac{9\sqrt{2}\left((121A-21B)\cos(dx+c)^4+4(121A-21B)\cos(dx+c)^3+6(121A-21B)\cos(dx+c)^2+4(121A-2\right.}{384\left(a^4d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(9*sqrt(2)*((121*A - 21*B)*cos(d*x + c)^4 + 4*(121*A - 21*B)*cos(d*x
+ c)^3 + 6*(121*A - 21*B)*cos(d*x + c)^2 + 4*(121*A - 21*B)*cos(d*x + c) +
121*A - 21*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) + 2*((691*A - 103*B)*cos(d*x + c)^3 + 2*(937*
A - 133*B)*cos(d*x + c)^2 + 39*(41*A - 5*B)*cos(d*x + c) + 384*A)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2
), x)
```

maple [B] time = 0.42, size = 595, normalized size = 2.20

$$\frac{(-1+\cos(dx+c))^2\left(1089A\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\left(\cos^3(dx+c)\right)-189B\arcsin\left(\frac{-1+\cos}{\sin(d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] 1/384/d*(-1+cos(d*x+c))^2*(1089*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3-189*B*arcsin((-1+cos(d*
x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3
+3267*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)-691*A*2^(1/2)*cos(d*x+c)^4-567*B*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+103*B*2^(1/2)*cos(d*x+c)^4+3267*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-1183*A*2^(1/2)*cos(d*x+
c)^3-567*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*sin(d*x+c)+163*B*2^(1/2)*cos(d*x+c)^3+1089*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+275*A*2^(
1/2)*cos(d*x+c)^2-189*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*
x+c))/sin(d*x+c))*sin(d*x+c)-71*B*2^(1/2)*cos(d*x+c)^2+1215*A*2^(1/2)*cos(d
*x+c)-195*B*2^(1/2)*cos(d*x+c)+384*A*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/
2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/(1+cos(d*x+c))*2^(1/2)/a^4
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}{{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(7/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a*cos(c + d*x))^(7/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
[Out] Timed out
```

3.545 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=223

$$\frac{(63A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}\,a^{7/2}d}-\frac{(103A+5B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)}$$

```
[Out] -1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)-1/16*(5*A-B
)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-1/192*(103*A+5*B)*
sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/128*(63*A+13*B)*
arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2978, 12, 2782, 205}

$$-\frac{(103A+5B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}\,(a\cos(c+dx)+a)^{3/2}}+\frac{(63A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos}}\right.}{64\sqrt{2}\,a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((63*A + 13*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sq
rt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]
*a^(7/2)*d) - ((A - B)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[S
ec[c + d*x]]) - ((5*A - B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)
*Sqrt[Sec[c + d*x]]) - ((103*A + 5*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c
 + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^{7/2}}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d\right.}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{(5A-B)\sin(c+}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{(5A-B)\sin(c+}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{(5A-B)\sin(c+}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{(5A-B)\sin(c+}{16ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{(63A+13B)\tan^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{\cos(c+dx)}\,\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] time = 3.07, size = 228, normalized size = 1.02

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\sqrt{\sec(c+dx)}\,\sec^6\left(\frac{1}{2}(c+dx)\right)((532A-4B)\cos(c+dx)+\right.}{384d(a(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + a*Cos[c + d*x])^(7
/2),x]

[Out] (Cos[(c + d*x)/2]^7*(((48*I)*(63*A + 13*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*
I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))
/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/E^((I/2)*(c + d*x)) + (493*A - 7
3*B + (532*A - 4*B)*Cos[c + d*x] + (103*A + 5*B)*Cos[2*(c + d*x)])*Sec[(c +
 d*x)/2]^6*Sqrt[Sec[c + d*x]]*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/(
384*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

fricas [A] time = 0.77, size = 257, normalized size = 1.15

$$-\frac{3\sqrt{2}\left((63A+13B)\cos(dx+c)^4+4(63A+13B)\cos(dx+c)^3+6(63A+13B)\cos(dx+c)^2+4(63A+13}{384\left(a^4d\cos(dx+c)^4+4a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((63*A + 13*B)*cos(d*x + c)^4 + 4*(63*A + 13*B)*cos(d*x +
 c)^3 + 6*(63*A + 13*B)*cos(d*x + c)^2 + 4*(63*A + 13*B)*cos(d*x + c) + 63*
A + 13*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)/(sqrt(a)*sin(d*x + c))) + 2*((103*A + 5*B)*cos(d*x + c)^3 + 2*(133*A - B)
*cos(d*x + c)^2 + 39*(5*A - B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3
+ 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(7/2
), x)
```

maple [B] time = 0.39, size = 512, normalized size = 2.30

$$-\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{a(1+\cos(dx+c))}\,\cos(dx+c)(-1+\cos(dx+c))^3\left(103A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^3(dx+c)\right)+5B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x)
```

```
[Out] -1/384/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d
*x+c))^3*(103*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+5*B*
```

```
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+163*A*cos(d*x+c)^2*2
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-189*A*cos(d*x+c)^2*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*sin(d*x+c)-7*B*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)-39*B*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x
+c)-71*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-378*A*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-37*B*2^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-78*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*si
n(d*x+c)*cos(d*x+c)-195*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-189*A*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+39*B*2^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)-39*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/sin(d*
x+c)^7/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+a\,\cos(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(7/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a*cos(c + d*x))^(7/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

3.546 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=221

$$\frac{(13A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{(5A-17B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a}$$

```
[Out] 1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)+1/16*(A+3*B)
*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-1/192*(5*A-17*B)*si
n(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/128*(13*A+7*B)*arc
tan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2977, 2978, 12, 2782, 205}

$$-\frac{(5A-17B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{(13A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+a}}\right.}{64\sqrt{2}a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((13*A + 7*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqr
t[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*
a^(7/2)*d) + ((A - B)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Se
c[c + d*x]]) + ((A + 3*B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*
Sqrt[Sec[c + d*x]]) - ((5*A - 17*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c +
 d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{(A+3B)\sin(c+d}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{(A+3B)\sin(c+d}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{(A+3B)\sin(c+d}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{(A+3B)\sin(c+d}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{(13A+7B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] time = 2.95, size = 233, normalized size = 1.05

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,(4(A+35B)\cos(c+dx)+(17B-5A)\cos(2(c+dx))+73A+59B)}{48d}+\frac{i(1}{}\right.}{8(a(\cos(c+dx)+1))^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*((I*(13*A + 7*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt
[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/(d*E^((I/2)*(c + d*x))) - ((73*A + 59*
B + 4*(A + 35*B)*Cos[c + d*x] + (-5*A + 17*B)*Cos[2*(c + d*x)])*Sec[(c + d*
x)/2]^6*Sqrt[Sec[c + d*x]]*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/(48*d
)))/(8*(a*(1 + Cos[c + d*x]))^(7/2))
```

fricas [A] time = 2.39, size = 255, normalized size = 1.15

$$-\frac{3\sqrt{2}\left((13A+7B)\cos(dx+c)^4+4(13A+7B)\cos(dx+c)^3+6(13A+7B)\cos(dx+c)^2+4(13A+7B)\cos\right.}{384\left(a^4d\cos(dx+c)^4+4a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((13*A + 7*B)*cos(d*x + c)^4 + 4*(13*A + 7*B)*cos(d*x + c
)^3 + 6*(13*A + 7*B)*cos(d*x + c)^2 + 4*(13*A + 7*B)*cos(d*x + c) + 13*A +
7*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sq
rt(a)*sin(d*x + c))) + 2*((5*A - 17*B)*cos(d*x + c)^3 - 2*(A + 35*B)*cos(d*
x + c)^2 - 3*(13*A + 7*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a
^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sqrt(sec(d*x + c
))), x)
```

maple [B] time = 0.39, size = 512, normalized size = 2.32

$$-\frac{\sqrt{a(1+\cos(dx+c))}\ \cos(dx+c)(-1+\cos(dx+c))^4\left(-5A\sqrt{2}\ \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \left(\cos^3(dx+c)\right)+17B\sqrt{2}\ \sqrt{\frac{\cos}{1+c}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^4*(-5*A*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+17*B*2^(1/2)*(cos(d*x+c)/(
```

```
1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+39*A*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*sin(d*x+c)+7*A*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)+21*B*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+53*B
*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+78*A*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+37*A*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)+42*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c
)*cos(d*x+c)-49*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+39*A
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-39*A*2^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)+21*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-21*B*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^9*2^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sqrt(sec(d*x + c
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(7/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(7/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.547 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=221

$$\frac{(7A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{(17A+67B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+}$$

```
[Out] 1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2)+1/48*(A-13*B
)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+1/192*(17*A+67*B)*
sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/128*(7*A+5*B)*ar
ctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2977, 2978, 12, 2782, 205}

$$\frac{(17A+67B)\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{(7A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx}}\right.}{64\sqrt{2}a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] ((7*A + 5*B)*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a
^(7/2)*d) + ((A - B)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c +
d*x]^(3/2)) + ((A - 13*B)*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)*
Sqrt[Sec[c + d*x]]) + ((17*A + 67*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c
+ d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,d$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-13B)\sin(c+}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-13B)\sin(c+}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-13B)\sin(c+}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{(A-13B)\sin(c+}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{(7A+5B)\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{64\sqrt{2}\,a^{7/2}d}$$

Mathematica [C] time = 7.23, size = 488, normalized size = 2.21

$$\frac{\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{(17A+67B)\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{12d} + \frac{(17A+67B)\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{12d} + \frac{\sec\left(\frac{c}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)}{3d} + \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] ((I/8)*(7*A + 5*B)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 +
 E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2
*I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[
c + d*x]))^(7/2)) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Sec[c + d*x]]*(((17*A + 67*B
)*Cos[(d*x)/2]*Sin[c/2])/(12*d) + ((17*A + 67*B)*Cos[c/2]*Sin[(d*x)/2])/(12
```

```
*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^2*(19*A*Sin[(d*x)/2] - 151*B*Sin[(d*x)/2
]))/(24*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^6*(A*Sin[(d*x)/2] - B*Sin[(d*x)/2
]))/(3*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^4*(-17*A*Sin[(d*x)/2] + 29*B*Sin[(
d*x)/2]))/(12*d) + ((19*A - 151*B)*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) - ((
17*A - 29*B)*Sec[c/2 + (d*x)/2]^3*Tan[c/2])/(12*d) + ((A - B)*Sec[c/2 + (d*
x)/2]^5*Tan[c/2])/(3*d)))/(a*(1 + Cos[c + d*x]))^(7/2)
```

fricas [A] time = 1.51, size = 257, normalized size = 1.16

$$-\frac{3\sqrt{2}\left((7A+5B)\cos(dx+c)^4+4(7A+5B)\cos(dx+c)^3+6(7A+5B)\cos(dx+c)^2+4(7A+5B)\cos(dx\right.}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((7*A + 5*B)*cos(d*x + c)^4 + 4*(7*A + 5*B)*cos(d*x + c)^
3 + 6*(7*A + 5*B)*cos(d*x + c)^2 + 4*(7*A + 5*B)*cos(d*x + c) + 7*A + 5*B)*
sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)
*sin(d*x + c))) - 2*((17*A + 67*B)*cos(d*x + c)^3 + 10*(7*A + 5*B)*cos(d*x
+ c)^2 + 3*(7*A + 5*B)*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*
d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(3/
2)), x)
```

maple [B] time = 0.43, size = 512, normalized size = 2.32

$$\frac{\sqrt{a(1+\cos(dx+c))}\,(-1+\cos(dx+c))^5\cos(dx+c)\left(17A\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos^3(dx+c)\right)+67B\sqrt{2}\sqrt{\frac{\cos(dx}{1+\cos(d}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*cos(d*x+c)*(17*A*2^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+67*B*2^(1/2)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+21*A*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*sin(d*x+c)+53*A*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)+15*B*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-17*B
*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+42*A*arcsin((-1+cos
(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-49*A*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)+30*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c
)*cos(d*x+c)-35*B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+21*A
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-21*A*2^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)+15*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-15*B*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)
/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^11*2^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(3/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos(c+d\,x)}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}\,{(a+a\cos(c+d\,x))}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(7/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(7/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.548 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=281

$$\frac{(5A-177B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}}{a^{7/2}d}$$

```
[Out] 1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2)+1/48*(5*A-17
*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2)+1/64*(5*A-49*B)*
sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*B*arcsin(sin(d*x
+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/
2)/d+1/128*(5*A-177*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/
2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1
/2)
```

Rubi [A] time = 0.92, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2961, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{(5A-49B)\sin(c+dx)}{64a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{(5A-177B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+a}}\right)}{64\sqrt{2}a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (2*B*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) + ((5*A - 177*B)*ArcTan[(Sqrt[a]*Sin[c
 + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Sin[c + d*x])
/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)) + ((5*A - 17*B)*Sin[c
+ d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) + ((5*A - 49
*B)*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
```

```
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,d$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A-17B)\sin(c+}{48ad(a+a\cos(c+dx))^{5/2}\,\mathrm{s}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A-17B)\sin(c+}{48ad(a+a\cos(c+dx))^{5/2}\,\mathrm{s}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A-17B)\sin(c+}{48ad(a+a\cos(c+dx))^{5/2}\,\mathrm{s}}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} + \frac{(5A-17B)\sin(c+}{48ad(a+a\cos(c+dx))^{5/2}\,\mathrm{s}}$$

$$= \frac{2B\sin^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d} + \frac{(5A-177B}{}$$

Mathematica [C] time = 3.96, size = 281, normalized size = 1.00

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{8}\left(\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,(4(25A-181B)\cos(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*(((-3*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(128*B*ArcSinh[E^(I*(c + d*x))] - S
qrt[2]*(5*A - 177*B)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*
I)*(c + d*x))])] - 128*B*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*
(c + d*x)) + ((97*A - 541*B + 4*(25*A - 181*B)*Cos[c + d*x] + (67*A - 247*B
)*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^6*Sqrt[Sec[c + d*x]]*(-Sin[(c + d*x)/2
] + Sin[(3*(c + d*x))/2]))/8))/(48*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

fricas [A] time = 27.17, size = 338, normalized size = 1.20

$$-\frac{3\sqrt{2}\left((5A-177B)\cos(dx+c)^4+4(5A-177B)\cos(dx+c)^3+6(5A-177B)\cos(dx+c)^2+4(5A-177B)\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((5*A - 177*B)*cos(d*x + c)^4 + 4*(5*A - 177*B)*cos(d*x +
 c)^3 + 6*(5*A - 177*B)*cos(d*x + c)^2 + 4*(5*A - 177*B)*cos(d*x + c) + 5*A
 - 177*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)
)/(sqrt(a)*sin(d*x + c))) + 768*(B*cos(d*x + c)^4 + 4*B*cos(d*x + c)^3 + 6*
B*cos(d*x + c)^2 + 4*B*cos(d*x + c) + B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c)
 + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*((67*A - 247*B)*cos(d*
x + c)^3 + 2*(25*A - 181*B)*cos(d*x + c)^2 + 3*(5*A - 49*B)*cos(d*x + c))*s
qrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c
)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c
) + a^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(5/
2)), x)
```

maple [B] time = 0.41, size = 667, normalized size = 2.37

$$-\frac{\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\,\left(-1+\cos\left(dx+c\right)\right)^{6}\cos\left(dx+c\right)\left(67A\sqrt{2}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\left(\cos^{3}\left(dx+c\right)\right)-247B\sqrt{2}\,\sqrt{\frac{c}{1+}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^6*cos(d*x+c)*(67*A*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-247*B*2^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-384*B*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+15*A*cos(d*x+
c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-17*A*cos(d*x+c)^2*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-531*B*cos(d*x+c)^2*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*sin(d*x+c)-115*B*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)-768*B*cos(d*x+c)*2^(1/2)*sin(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+30*A*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*sin(d*x+c)*cos(d*x+c)-35*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(
d*x+c)-1062*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+215*
B*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-384*B*2^(1/2)*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+15*A*a
rcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-15*A*2^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)-531*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+147*B*
2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(5/2)/(cos(d*x+c)
/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^13*2^(1/2)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(5/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\cos(c+d\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(7/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(7/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.549 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=333

$$\frac{(2A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{(177A-637B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\right.}{64\sqrt{2}\,a^{7/2}d}$$

```
[Out] 1/6*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2)+1/16*(3*A-7*
B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2)+1/192*(79*A-259*B
)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)-7/64*(7*A-27*B)*
sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(2*A-7*B)*arcsin(s
in(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^(7/2)/d-1/128*(177*A-637*B)*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*
x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2
)/d*2^(1/2)
```

Rubi [A] time = 1.20, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2961, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{(79A-259B)\sin(c+dx)}{192a^2d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}+\frac{(2A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{}{64a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)),x]
```

```
[Out] ((2*A - 7*B)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) - ((177*A - 637*B)*ArcTan[(Sqr
t[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*S
qrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) + ((A - B)*Sin
[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)) + ((3*A - 7*
B)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + (
(79*A - 259*B)*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c +
d*x]^(3/2)) - (7*(7*A - 27*B)*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
```

```
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin(c+dx}{16ad(a+a\cos(c+dx))^{5/2}\sec}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin(c+dx}{16ad(a+a\cos(c+dx))^{5/2}\sec}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin(c+dx}{16ad(a+a\cos(c+dx))^{5/2}\sec}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin(c+dx}{16ad(a+a\cos(c+dx))^{5/2}\sec}$$

$$= \frac{(A-B)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} + \frac{(3A-7B)\sin(c+dx}{16ad(a+a\cos(c+dx))^{5/2}\sec}$$

$$= \frac{(2A-7B)\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d} - \frac{(177A}{}$$

Mathematica [C] time = 7.65, size = 1017, normalized size = 3.05

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\sec\left(\frac{c}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)-B\sin\left(\frac{dx}{2}\right)\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d} + \frac{(A-B)\tan\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d} + \frac{\sec\left(\frac{c}{2}\right)\left(53B\sin\left(\frac{dx}{2}\right)-41A\sin\left(\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{12d} - \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/
2)),x]
```

```
[Out] (((-49*I)/8)*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*
(c + d*x))])]*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c +
d*x]))^(7/2)) + (((189*I)/8)*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))])]*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*
(a*(1 + Cos[c + d*x]))^(7/2)) + ((8*I)*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x
))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c
+ d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^7)/
(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(7/2)) - ((28*I)*Sqrt[2]*B*Sq
rt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]
*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[
2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]
)*Cos[c/2 + (d*x)/2]^7)/(d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(7/2)
) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Sec[c + d*x]]*(((-247*A + 427*B)*Cos[(d*x)/2
]*Sin[c/2])/(12*d) + (8*B*Cos[(3*d*x)/2]*Sin[(3*c)/2])/d - ((247*A - 427*B)
*Cos[c/2]*Sin[(d*x)/2])/(12*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^2*(379*A*Sin[
(d*x)/2] - 703*B*Sin[(d*x)/2]))/(24*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^6*(A*
Sin[(d*x)/2] - B*Sin[(d*x)/2]))/(3*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^4*(-41
*A*Sin[(d*x)/2] + 53*B*Sin[(d*x)/2]))/(12*d) + (8*B*Cos[(3*c)/2]*Sin[(3*d*x
)/2])/d + ((379*A - 703*B)*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) - ((41*A - 5
3*B)*Sec[c/2 + (d*x)/2]^3*Tan[c/2])/(12*d) + ((A - B)*Sec[c/2 + (d*x)/2]^5*
Tan[c/2])/(3*d)))/(a*(1 + Cos[c + d*x]))^(7/2)
```

fricas [A] time = 52.36, size = 379, normalized size = 1.14

$$\frac{3\sqrt{2}\left((177\,A-637\,B)\cos(dx+c)^4+4\,(177\,A-637\,B)\cos(dx+c)^3+6\,(177\,A-637\,B)\cos(dx+c)^2+4\,(177\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*((177*A - 637*B)*cos(d*x + c)^4 + 4*(177*A - 637*B)*cos(d*
x + c)^3 + 6*(177*A - 637*B)*cos(d*x + c)^2 + 4*(177*A - 637*B)*cos(d*x + c
) + 177*A - 637*B)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) - 384*((2*A - 7*B)*cos(d*x + c)^4 + 4*(2
*A - 7*B)*cos(d*x + c)^3 + 6*(2*A - 7*B)*cos(d*x + c)^2 + 4*(2*A - 7*B)*cos
(d*x + c) + 2*A - 7*B)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))/(sqrt(a)*sin(d*x + c))) + 2*(192*B*cos(d*x + c)^4 - (247*A - 1099*B)
*cos(d*x + c)^3 - 2*(181*A - 721*B)*cos(d*x + c)^2 - 21*(7*A - 27*B)*cos(d*
x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*co
```

```
s(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*co
s(d*x + c) + a^4*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.44, size = 855, normalized size = 2.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -1/384/d*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^7*cos(d*x+c)*(-192*B*cos(
d*x+c)^4*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+247*A*2^(1/2)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3+384*A*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*
arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-907*B*2^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3-1344*B*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+
c)+531*A*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+115*A*c
os(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+768*A*arctan(sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x
+c)-1911*B*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-343*B
*cos(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-2688*B*cos(d*x+c)*2
^(1/2)*sin(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))+1062*A*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)-215*
A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+384*A*2^(1/2)*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)-3822*B
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+875*B*2^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-1344*B*2^(1/2)*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+531*A*arcsin((-1+c
os(d*x+c))/sin(d*x+c))*sin(d*x+c)-147*A*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)-1911*B*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+567*B*2^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(7/2)/(cos(d*x+c)/(1+cos(d
*x+c)))^(9/2)/sin(d*x+c)^15*2^(1/2)/a^4
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+a\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(7/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(7/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.550 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=180

$$\frac{2(aB+Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(3aA+5bB)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}{3d}$$

```
[Out] 2/3*(A*b+B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*A*sec(d*x+c)^(5/2)*sin(d*
x+c)/d+2/5*(3*A*a+5*B*b)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*(3*A*a+5*B*b)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(A*b+B*a)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.22, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3997, 3787, 3768, 3771, 2639, 2641}

$$\frac{2(aB+Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(3aA+5bB)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(3*a*A + 5*b*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(5*d) + (2*(A*b + a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*
a*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))(B+A\sec(c+dx)$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sec^{\frac{3}{2}}(c+dx)\Big(\frac{1}{2}$$

$$= \frac{2aA\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + (Ab+aB)\int \sec^{\frac{5}{2}}(c$$

$$= \frac{2(3aA+5bB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(Ab+aB}{}$$

$$= \frac{2(3aA+5bB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(Ab+aB}{}$$

$$= -\frac{2(3aA+5bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}{5d}$$

Mathematica [A] time = 1.87, size = 132, normalized size = 0.73

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(20(aB+Ab)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-12(3aA+5bB)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(3*a*A + 5*b*B)*Cos[c + d*x]^(5/2)*EllipticE[(c +
d*x)/2, 2] + 20*(A*b + a*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
2*(15*(a*A + b*B) + 10*(A*b + a*B)*Cos[c + d*x] + 3*(3*a*A + 5*b*B)*Cos[2*(
c + d*x)])*Sin[c + d*x]))/(30*d)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sec(d*x + c)
^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

maple [B] time = 4.51, size = 663, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(-(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*a*A/(8
*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin
(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*(A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{7/2}(a+b\cos(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x)),x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
[Out] Timed out
```

3.551 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=143

$$\frac{2(aB+Ab)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2(aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aB+Ab)\sqrt{c}}{}$$

```
[Out] 2/3*a*A*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*(A*b+B*a)*sin(d*x+c)*sec(d*x+c)^(1/
2)/d-2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(A*a+3*
B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3997, 3787, 3771, 2641, 3768, 2639}

$$\frac{2(aB+Ab)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2(aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aB+Ab)\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*(a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*d) + (2*(A*b + a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
+ (2*a*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rubi steps

$$\int (a + b\cos(c + dx))(A + B\cos(c + dx))\sec^{\frac{5}{2}}(c + dx)\,dx = \int \sqrt{\sec(c + dx)}\,(b + a\sec(c + dx))(B + A\sec(c + d$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c + dx)\sin(c + dx)}{3d} + \frac{2}{3}\int \sqrt{\sec(c + dx)}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c + dx)\sin(c + dx)}{3d} + (Ab + aB)\int \sec^{\frac{3}{2}}($$

$$= \frac{2(Ab + aB)\sqrt{\sec(c + dx)}\,\sin(c + dx)}{d} + \frac{2aA\sec^{\frac{3}{2}}(c}{}$$

$$= \frac{2(aA + 3bB)\sqrt{\cos(c + dx)}\,F\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{\sec(c -}}{3d}$$

$$= -\frac{2(Ab + aB)\sqrt{\cos(c + dx)}\,E\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{\sec(c -}}{d}$$

Mathematica [A] time = 0.83, size = 104, normalized size = 0.73

$$\frac{2\sqrt{\cos(c + dx)}\,\sqrt{\sec(c + dx)}\left((aA + 3bB)F\left(\frac{1}{2}(c + dx)\middle|2\right) - 3(aB + Ab)E\left(\frac{1}{2}(c + dx)\middle|2\right) + \frac{\sin(c+dx)(3(aB+Ab)\cos(c+}{\cos^{\frac{3}{2}}(c+dx)}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(A*b + a*B)*EllipticE[(c + d*x
)/2, 2] + (a*A + 3*b*B)*EllipticF[(c + d*x)/2, 2] + ((a*A + 3*(A*b + a*B)*C
os[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

fricas [F] time = 4.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bb\cos\left(dx + c\right)^{2} + Aa + (Ba + Ab)\cos\left(dx + c\right)\right)\sec\left(dx + c\right)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sec(d*x + c)
^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

maple [B] time = 3.34, size = 428, normalized size = 2.99

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2Bb\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}+\frac{2(Ab+aB)(}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*(A*
b+B*a)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)
^2-1)+2*a*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,(a+b\,\cos\left(c+d\,x\right))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x)),x)
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.552 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=111

$$\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(aA-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+$$

```
[Out] 2*a*A*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*(A*a-B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A] time = 0.18, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2960, 3997, 3787, 3771, 2639, 2641}

$$\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2(aA-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/d + (2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + 2\int \frac{\frac{1}{2}(-aA+bB)$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + (Ab+aB)\int \sqrt{\sec$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \Big((Ab+aB)\sqrt{\cos(c$$

$$= -\frac{2(aA-bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c-}{d}$$

Mathematica [A] time = 0.27, size = 85, normalized size = 0.77

$$\frac{2\sqrt{\sec(c+dx)}\left((aB+Ab)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-\left((aA-bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+aA\sin(c}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-((a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]) + (A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*A*Si
n[c + d*x]))/d
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sec(d*x + c)
^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

maple [A] time = 1.39, size = 244, normalized size = 2.20

$$-\frac{2\left(Ab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -2*(A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-2*A*a*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}\,(a+b\,\cos(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x)),x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.553 $\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{2(3aA+bB)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2/3*b*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+2/3*(3*A*a+B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d
```

Rubi [A] time = 0.19, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2960, 3996, 3787, 3771, 2639, 2641}

$$\frac{2(3aA+bB)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*(3*a*A + b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*b*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}(Ab+aB)-\frac{1}{2}(3aA+b}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - (-Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2bB\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \left((-Ab-aB)\sqrt{\cos(c+dx)}\,\sqrt{\sec}\right.$$

$$= \frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{d}$$

Mathematica [A] time = 0.23, size = 90, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(2(3aA+bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(aB+Ab)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+bB\sin(\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2] + 2*(3*a*A + b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*B*S
in[2*(c + d*x)]))/(3*d)
```

fricas [F] time = 1.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(sec(d*x
 + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

maple [B] time = 1.35, size = 326, normalized size = 2.83

$$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4Bb\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3aA\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\right.\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*B*b*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
```

```
x+1/2*c),2^(1/2))*b+B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a-2*B*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\,(a+b\,\cos(c+d\,x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x)),x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\,(a+b\cos(c+dx))\,\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))*sqrt(sec(c + d*x)), x)
```

3.554 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=148

$$\frac{2(aB+Ab)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\sqrt{s}}{5d}$$

```
[Out] 2/5*b*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*(A*b+B*a)*sin(d*x+c)/d/sec(d*x+c)
^(1/2)+2/5*(5*A*a+3*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3
*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.21, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3996, 3787, 3769, 3771, 2641, 2639}

$$\frac{2(aB+Ab)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\sqrt{s}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(5*a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}(Ab+aB)-\frac{1}{2}(5aA+3bB)\sec(c+dx}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - (-Ab-aB)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx - \frac{1}{5}(-5aA-3b$$

$$= \frac{2bB\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}(-Ab-aB)\int \sqrt{se$$

$$= \frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2bB}{5d\,s}$$

$$= \frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2(A}{}$$

Mathematica [A] time = 0.54, size = 108, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(5aB+5Ab+3bB\cos(c+dx))+10(aB+Ab)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+6(5a\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(5*a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 10*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5
*A*b + 5*a*B + 3*b*B*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

fricas [F] time = 1.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)}{\sqrt{\sec\left(dx+c\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))/sqrt(sec(d*x
 + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

maple [B] time = 1.42, size = 371, normalized size = 2.51

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24Bb\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20Ab+20aB+24Bb)\left(\mathrm{si}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b+20*B*a+24*B*b)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b-10*B*a-6*B*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+5*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+
5*a*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\,\cos(c + d\,x))\,\,(a + b\,\cos(c + d\,x))}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/(1/cos(c + d*x))^(1/2),x)
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/(1/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\,(a + b\cos(c + dx))}{\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))/sqrt(sec(c + d*x)), x)
```

3.555 $\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{2(aB+Ab)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(7aA+5bB)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(7aA+5bB)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+$$

```
[Out] 2/7*b*B*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*(A*b+B*a)*sin(d*x+c)/d/sec(d*x+c)
^(3/2)+2/21*(7*A*a+5*B*b)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*(A*b+B*a)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(7*A*a+5*B*b)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.23, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2960, 3996, 3787, 3769, 3771, 2639, 2641}

$$\frac{2(aB+Ab)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(7aA+5bB)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(7aA+5bB)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*d) + (2*(7*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)
) + (2*(A*b + a*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(7*a*A + 5*b
*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3996

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*a*Cot[e +
 f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}(Ab+aB)-\frac{1}{2}(7aA+5bB)\sec(c+d$$

$$= \frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - (-Ab-aB)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx - \frac{1}{7}(-7aA-$$

$$= \frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(7aA+5bB)\sin(}{21d\sqrt{\sec(c+}$$

$$= \frac{2bB\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(7aA+5bB)\sin(}{21d\sqrt{\sec(c+}$$

$$= \frac{6(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2(7a}{}$$

Mathematica [A] time = 0.99, size = 125, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(42(aB+Ab)\cos(c+dx)+70aA+15bB\cos(2(c+dx))+65bB)+20(7aA+5bB\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(252*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2] + 20*(7*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
(70*a*A + 65*b*B + 42*(A*b + a*B)*Cos[c + d*x] + 15*b*B*Cos[2*(c + d*x)])*S
in[2*(c + d*x)]))/(210*d)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{Bb\cos\left(dx+c\right)^{2}+Aa+(Ba+Ab)\cos\left(dx+c\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))/sec(d*x + c)
^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

maple [A] time = 1.40, size = 413, normalized size = 2.29

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240Bb\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168Ab-168aB-360Bb)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b-168*B*a-360*B*b)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A*a+168*A*b+168*B*a+280*B*b)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A*a-42*A*b-42*B*a-80*B*b)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+35*a*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b+25*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\ \cos(c+d\,x))\ (a+b\ \cos(c+d\,x))}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x)))/(1/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\,(a+b\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))/sec(c + d*x)**(3/2), x)
```

3.556 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=221

$$\frac{2\left(a^2B+2aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(3a^2A+5b(2aB+Ab)\right)\sin(c+dx)\sqrt{\sec(}}{5d}$$

```
[Out] 2/15*a*(7*A*b+5*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*A*sec(d*x+c)^(3/2)
*(b+a*sec(d*x+c))*sin(d*x+c)/d+2/5*(3*a^2*A+5*b*(A*b+2*B*a))*sin(d*x+c)*sec
(d*x+c)^(1/2)/d-2/5*(3*a^2*A+5*b*(A*b+2*B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+2/3*(2*A*a*b+B*a^2+3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.38, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4026, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\left(a^2B+2aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(3a^2A+5b(2aB+Ab)\right)\sin(c+dx)\sqrt{\sec(}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a^2*A + 5*b*(A*b + 2*a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(2*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(3*a^2*A +
5*b*(A*b + 2*a*B))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*(7*A*b + 5
*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*Sec[c + d*x]^(3/2)*(
b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
```

```
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2(B+A\sec(c+$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}$$

$$= \frac{2\left(3a^2A+5b(Ab+2aB)\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}$$

$$= \frac{2\left(3a^2A+5b(Ab+2aB)\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}$$

$$= -\frac{2\left(3a^2A+5b(Ab+2aB)\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{5d}$$

Mathematica [A] time = 2.42, size = 171, normalized size = 0.77

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(20\left(a^2B+2aAb+3b^2B\right)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-12\left(3a^2A+10abB+5Ab^2\right)\cos^{\frac{5}{2}}(c+dx)E\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Cos[c + d*x]^(5/2)*
EllipticE[(c + d*x)/2, 2] + 20*(2*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x]^(5/
2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(a^2*A + A*b^2 + 2*a*b*B) + 10*a*(2*A*
b + a*B)*Cos[c + d*x] + 3*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Cos[2*(c + d*x)])*
Sin[c + d*x]))/(30*d)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x
)
```

maple [B] time = 4.51, size = 750, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
*(A*b+2*B*a)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1)-2/5*a^2*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin
(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a*(2*A*b+B*a)*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+
cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}(a+b\,\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.557 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=177

$$\frac{2\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{d}$$

```
[Out] 2/3*a*(5*A*b+3*B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/3*a*A*(b+a*sec(d*x+c))*
sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*(2*A*a*b+B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(A*a^2+3*A*b^2+6*B*a*b)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.35, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4026, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\left(a^2A+6abB+3Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*(2*a*A*b + a^2*B - b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(5*A*b + 3*a*B)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*Sqrt[Sec[c + d*x]]*(b + a*Se
c[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))\sin(c+dx)}{3d} +$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))\sin(c+dx)}{3d} +$$

$$= \frac{2a(5Ab+3aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aA\sqrt{s}}{}$$

$$= \frac{2\left(a^2A+3Ab^2+6abB\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right.}{3d}$$

$$= -\frac{2\left(2aAb+a^2B-b^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right.}{d}$$

Mathematica [A] time = 1.14, size = 125, normalized size = 0.71

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\left(\left(a^2A+6abB+3Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-3\left(a^2B+2aAb-b^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(2*a*A*b + a^2*B - b^2*B)*Elli
pticE[(c + d*x)/2, 2] + (a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2,
2] + (a*(a*A + 3*(2*A*b + a*B)*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/
2)))/(3*d)
```

fricas [F] time = 2.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sec(dx+c)^{\frac{5}{2}},x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x
)
```

maple [B] time = 3.47, size = 677, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*B*a*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^2*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
a*(2*A*b+B*a)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x
+1/2*c)^2-1)+2*a^2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}(a+b\cos(c+d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.558 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=161

$$\frac{2\left(3a^2B+6aAb+b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2A-b(2aB+Ab)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{d}$$

```
[Out] 2/3*b^2*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a^2*A*sin(d*x+c)*sec(d*x+c)^(1/2)
/d-2*(a^2*A-b*(A*b+2*B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2
/3*(6*A*a*b+3*B*a^2+B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.32, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4024, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\left(3a^2B+6aAb+b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(a^2A-b(2aB+Ab)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a^2*A - b*(A*b + 2*a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*(6*a*A*b + 3*a^2*B + b^2*B)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*B*Sin[c + d*x])
/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}b(Ab+2aB)+\left(-3aAb\right.}{}$$

$$= \frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}b(Ab+2aB)-\frac{3}{2}a^2A\sec}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2b^2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2a^2A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}$$

$$= \frac{2\left(6aAb+3a^2B+b^2B\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d}$$

$$= -\frac{2\left(a^2A-b(Ab+2aB)\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right|}{d}$$

Mathematica [A] time = 0.75, size = 124, normalized size = 0.77

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2\left(3a^2B+6aAb+b^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\left(-6a^2A+12abB+6Ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-6*a^2*A + 6*A*b^2 + 12*a*b*B)*Ell
ipticE[(c + d*x)/2, 2] + 2*(6*a*A*b + 3*a^2*B + b^2*B)*EllipticF[(c + d*x)/
2, 2] + (2*(3*a^2*A + b^2*B*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c + d*x]])
)/(3*d)
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos(dx+c)^2+\left(Ba^2+2\,Aab\right)\cos(dx+c)\right)\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

maple [B] time = 1.54, size = 404, normalized size = 2.51

$$-\frac{2\left(4B\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6Aab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6*A*a*b*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b
^2-6*A*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*a^2*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*
B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\,\cos(c + d\,x)) \left(\frac{1}{\cos(c + d\,x)}\right)^{3/2} (a + b\,\cos(c + d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.559 $\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=171

$$\frac{2\left(3a^2A+2abB+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(5a^2B+10aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{}}{5d}$$

```
[Out] 2/5*b^2*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*b*(A*b+2*B*a)*sin(d*x+c)/d/sec(
d*x+c)^(1/2)+2/5*(10*A*a*b+5*B*a^2+3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+2/3*(3*A*a^2+A*b^2+2*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.34, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4024, 4047, 3771, 2639, 4045, 2641}

$$\frac{2\left(3a^2A+2abB+Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(5a^2B+10aAb+3b^2B\right)\sqrt{\cos(c+dx)}\sqrt{}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A + A*b^2 + 2*a*b*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*B*Sin[c
 + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(3*d*S
qrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}b(Ab+2aB)+\left(-5aA\right.$$

$$= \frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}b(Ab+2aB)-\frac{5}{2}a^2A\,s}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b^2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}$$

$$= \frac{2\left(10aAb+5a^2B+3b^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{5d}$$

$$= \frac{2\left(10aAb+5a^2B+3b^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{5d}$$

Mathematica [A] time = 0.91, size = 128, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(10\left(3a^2A+2abB+Ab^2\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+6\left(5a^2B+10aAb+3b^2B\right)\sqrt{\cos(c+dx}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2] + 10*(3*a^2*A + A*b^2 + 2*a*b*B)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2] + b*(5*A*b + 10*a*B + 3*b*B*Cos[c + d*x])*Sin[2*
(c + d*x)]))/(15*d)
```

fricas [F] time = 1.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{\sec(dx+c)}, x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

maple [B] time = 1.40, size = 487, normalized size = 2.85

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\ \left(-24B\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(20A\,b^2+40Bab+24b^2B\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*b^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A*b^2+40*B*a*b+24*B*b^2)*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A*b^2-20*B*a*b-6*B*b^2)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+15*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*A*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+10*B*a*b*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b
^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B\,\cos(c + d\,x))\,\sqrt{\frac{1}{\cos(c + d\,x)}}\,(a + b\,\cos(c + d\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\cos(c + dx))\,(a + b\cos(c + dx))^2\,\sqrt{\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2*sqrt(sec(c + d*x)), x
)
```

3.560 $\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=213

$$\frac{2\left(7a^2B+14aAb+5b^2B\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(7a^2B+14aAb+5b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2}{-}$$

```
[Out] 2/7*b^2*B*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*b*(A*b+2*B*a)*sin(d*x+c)/d/sec(
d*x+c)^(3/2)+2/21*(14*A*a*b+7*B*a^2+5*B*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+
2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d+2/21*(14*A*a*b+7*B*a^2+5*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d
```

Rubi [A] time = 0.37, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4024, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\left(7a^2B+14aAb+5b^2B\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(7a^2B+14aAb+5b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2}{-}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b^2*B*Si
n[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(5*
d*Sec[c + d*x]^(3/2)) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sin[c + d*x])/(21
*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4024

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a^2*A*Cos
[e + f*x]*(d*Csc[e + f*x])^(n + 1))/(d*f*n), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b+a\sec(c+dx))^2(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}b(Ab+2aB)+\left(-7aAb+\left(-\frac{7a^2}{2}-\frac{5b}{2}\right.\right.}{\sec^{\frac{5}{2}}(c}$$

$$= \frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}b(Ab+2aB)-\frac{7}{2}a^2A\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(14aAb+7a^2\right.}{21d\sqrt{}}$$

$$= \frac{2b^2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(14aAb+7a^2\right.}{21d\sqrt{}}$$

$$= \frac{2\left(5a^2A+3Ab^2+6abB\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c-}}{5d}$$

Mathematica [A] time = 1.34, size = 161, normalized size = 0.76

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(5\left(14a^2B+28aAb+3b^2B\cos(2(c+dx))+13b^2B\right)+42b(2aB+Ab)\cos(c+dx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^2*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2] + 20*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + (42*b*(A*b + 2*a*B)*Cos[c + d*x] + 5*(28*a*
A*b + 14*a^2*B + 13*b^2*B + 3*b^2*B*Cos[2*(c + d*x)]))*Sin[2*(c + d*x)]))/(
210*d)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

maple [B] time = 1.40, size = 548, normalized size = 2.57

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\;\left(240B\,b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-168A\,b^2-336Bab-3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^2-336*B*a*b-360*B*b^2)*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a*b+168*A*b^2+140*B*a^2+336*B*a
*b+280*B*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-140*A*a*b-42*A*b^2-
70*B*a^2-84*B*a*b-80*B*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+70*A*a*
b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-63*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*b^2+35*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^2*B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-126*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+
```

```
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^2}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**2/sqrt(sec(c + d*x)), x
)
```

3.561 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=295

$$\frac{2a\left(5a^2A+21abB+18Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a^2(7aB+11Ab)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d}+\frac{2\left(3a^3B+9a\right.}{}$$

```
[Out] 2/21*a*(5*A*a^2+18*A*b^2+21*B*a*b)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/35*a^2*(
11*A*b+7*B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*a*A*sec(d*x+c)^(3/2)*(b+a*s
ec(d*x+c))^2*sin(d*x+c)/d+2/5*(9*A*a^2*b+5*A*b^3+3*B*a^3+15*B*a*b^2)*sin(d*
x+c)*sec(d*x+c)^(1/2)/d-2/5*(9*A*a^2*b+5*A*b^3+3*B*a^3+15*B*a*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(5*A*a^3+21*A*a*b^2+21*B*a^2*b+
21*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.60, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2960, 4026, 4076, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2a\left(5a^2A+21abB+18Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2\left(9a^2Ab+3a^3B+15ab^2B+5Ab^3\right)\sin(c+dx)\sqrt{\sec(c-}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (-2*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*a^3*A + 21*a*A*b^2 + 2
1*a^2*b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(21*d) + (2*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*(5*a^2*A + 18*A*b^2 + 21*a*b*B)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a^2*(11*A*b + 7*a*B)*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(35*d) + (2*a*A*Sec[c + d*x]^(3/2)*(b + a*Sec[c + d*x]
)^2*Sin[c + d*x])/(7*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^3(B+A\sec(c+$$

$$= \frac{2aA\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^2\sin(c+dx)}{7d} +$$

$$= \frac{2a^2(11Ab+7aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2aA\,s}{}$$

$$= \frac{2a^2(11Ab+7aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2aA\,s}{}$$

$$= \frac{2\left(9a^2Ab+5Ab^3+3a^3B+15ab^2B\right)\sqrt{\sec(c+dx)}}{5d}$$

$$= \frac{2\left(9a^2Ab+5Ab^3+3a^3B+15ab^2B\right)\sqrt{\sec(c+dx)}}{5d}$$

$$= -\frac{2\left(9a^2Ab+5Ab^3+3a^3B+15ab^2B\right)\sqrt{\cos(c+dx}}{5d}$$

Mathematica [A] time = 3.65, size = 225, normalized size = 0.76

$$\frac{2\sqrt{\sec(c+dx)}\left(15a^3A\tan(c+dx)\sec^2(c+dx)+5a\left(5a^2A+21abB+21Ab^2\right)\tan(c+dx)+21a^2(aB+3Ab)\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-21*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*a^3*A + 21*a*A*b^2 + 21*a^
2*b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 21*(9*a^2*
A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sin[c + d*x] + 5*a*(5*a^2*A + 21*A*b^
2 + 21*a*b*B)*Tan[c + d*x] + 21*a^2*(3*A*b + a*B)*Sec[c + d*x]*Tan[c + d*x]
 + 15*a^3*A*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d)
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^3\cos(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\cos(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\cos(dx+c)^2 + \left(Ba^3 + 3\,A\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
*sec(d*x + c)^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c) + A)(b\cos(dx+c) + a)^3 \sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x
)
```

maple [B] time = 6.09, size = 944, normalized size = 3.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
```

```
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/5
*a^2*(3*A*b+B*a)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d
*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2*(A*b+3*B*a)*(-(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*
d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+6*a*b*(A*b+B*
a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^3*(-1/56*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+
1/2*c)^2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{9/2}\,{(a+b\,\cos(c+d\,x))}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.562 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=244

$$\frac{2a\left(3a^2A+15abB+14Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a^2(5aB+9Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{2\left(a^3B+3a^2\right.}{}$$

```
[Out] 2/15*a^2*(9*A*b+5*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*(3*A*a^2+14*A*b^
2+15*B*a*b)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/5*a*A*(b+a*sec(d*x+c))^2*sin(d*
x+c)*sec(d*x+c)^(1/2)/d-2/5*(3*A*a^3+15*A*a*b^2+15*B*a^2*b-5*B*b^3)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(3*A*a^2*b+3*A*b^3+B*a^3+9*B*a*
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.58, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4026, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a\left(3a^2A+15abB+14Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2\left(3a^2Ab+a^3B+9ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A*b + 3*A*b^3 + a
^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*a*(3*a^2*A + 14*A*b^2 + 15*a*b*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(5*d) + (2*a^2*(9*A*b + 5*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(15*d) + (2*a*A*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*
d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
```

```
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2\sin(c+dx)}{5d}$$

$$= \frac{2a^2(9Ab+5aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2aA\sqrt{}}{}$$

$$= \frac{2a^2(9Ab+5aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2aA\sqrt{}}{}$$

$$= \frac{2a\left(3a^2A+14Ab^2+15abB\right)\sqrt{\sec(c+dx)}\,\sin(c+}{5d}$$

$$= \frac{2\left(3a^2Ab+3Ab^3+a^3B+9ab^2B\right)\sqrt{\cos(c+dx)}\,F\left(\right.}{3d}$$

$$= -\frac{2\left(3a^3A+15aAb^2+15a^2bB-5b^3B\right)\sqrt{\cos(c+d}}{5d}$$

Mathematica [A] time = 1.60, size = 192, normalized size = 0.79

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)\left(9\left(a^2A+5abB+5Ab^2\right)\cos(2(c+dx))+15\left(a^2A+3abB+3Ab^2\right)+10a(aB+3Ab)\cos(c+dx)\right)}{2\cos^{\frac{5}{2}}(c+dx)}+5\left(a^3\right.\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(3*a^3*A + 15*a*A*b^2 + 15*a^2
*b*B - 5*b^3*B)*EllipticE[(c + d*x)/2, 2] + 5*(3*a^2*A*b + 3*A*b^3 + a^3*B
+ 9*a*b^2*B)*EllipticF[(c + d*x)/2, 2] + (a*(15*(a^2*A + 3*A*b^2 + 3*a*b*B)
 + 10*a*(3*A*b + a*B)*Cos[c + d*x] + 9*(a^2*A + 5*A*b^2 + 5*a*b*B)*Cos[2*(c
 + d*x)])*Sin[c + d*x])/(2*Cos[c + d*x]^(5/2))))/(15*d)
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^3\cos\left(dx+c\right)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\cos\left(dx+c\right)^3 + 3\left(Ba^2b + Aab^2\right)\cos\left(dx+c\right)^2 + \left(Ba^3 + 3\,A\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
*sec(d*x + c)^(7/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x
)
```

maple [B] time = 4.75, size = 997, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*a*b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*b^3*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
6*a*b*(A*b+B*a)*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
```

```
*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d
*x+1/2*c)^2-1)-2/5*A*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*
sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*s
in(1/2*d*x+1/2*c)^4-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^2*(3*A*b+B*a)*(-1/
6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-1/2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}\,{\left(a+b\,\cos(c+d\,x)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.563 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=239

$$\frac{2a\left(3a^2B+9aAb-2b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2a^2(aA-bB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\left(a^3A+9a^2bB+9\right.}{}$$

```
[Out] 2/3*a^2*(A*a-B*b)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*b*B*(b+a*sec(d*x+c))^2*
sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/3*a*(9*A*a*b+3*B*a^2-2*B*b^2)*sin(d*x+c)*se
c(d*x+c)^(1/2)/d-2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+2/3*(A*a^3+9*A*a*b^2+9*B*a^2*b+B*b^3)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.57, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4025, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a\left(3a^2B+9aAb-2b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2\left(a^3A+9a^2bB+9aAb^2+b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b
^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d
) + (2*a*(9*a*A*b + 3*a^2*B - 2*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*
d) + (2*a^2*(a*A - b*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*b*B*(b
+ a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
```

```
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{(b+a\,\mathrm{se}}{}$$

$$= \frac{2a^2(aA-bB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2bB(b+a}{3}$$

$$= \frac{2a^2(aA-bB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2bB(b+a}{3}$$

$$= \frac{2a\left(9aAb+3a^2B-2b^2B\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d}$$

$$= \frac{2\left(a^3A+9aAb^2+9a^2bB+b^3B\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(\right.}{3d}$$

$$= -\frac{2\left(3a^2Ab-Ab^3+a^3B-3ab^2B\right)\sqrt{\cos(c+dx)}\,E\left(\right.}{d}$$

Mathematica [A] time = 1.94, size = 166, normalized size = 0.69

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{\sin(c+dx)\left(2a^3A+6a^2(aB+3Ab)\cos(c+dx)+b^3B\cos(2(c+dx))+b^3B\right)}{\cos^{\frac{3}{2}}(c+dx)}+2\left(a^3A+9a^2bB+9aAb^2+b^3\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-6*(3*a^2*A*b - A*b^3 + a^3*B - 3*a
*b^2*B)*EllipticE[(c + d*x)/2, 2] + 2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*
B)*EllipticF[(c + d*x)/2, 2] + ((2*a^3*A + b^3*B + 6*a^2*(3*A*b + a*B)*Cos[
c + d*x] + b^3*B*Cos[2*(c + d*x)])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^3\cos\left(dx+c\right)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\cos\left(dx+c\right)^3 + 3\left(Ba^2b + Aab^2\right)\cos\left(dx+c\right)^2 + \left(Ba^3 + 3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
*sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos\left(dx+c\right) + A)(b\cos\left(dx+c\right) + a)^3 \sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x
)
```

maple [B] time = 4.13, size = 1212, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(8*B*b^3*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*sin(1/2*d*
x+1/2*c)^2+18*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2+18*A*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2-6*A*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*b^3*sin(1/2*d*x+1/2*c)^2-36*A*a^2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*
c)^4+18*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+2*B*EllipticF
```

```
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*b^3*sin(1/2*d*x+1/2*c)^2+6*B*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*s
in(1/2*d*x+1/2*c)^2-18*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^
2-12*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-8*B*b^3*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^4-A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*a*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b+3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*b^3+2*A*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+18*A*a^2*b*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^3*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+9*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a*b^2+6*B*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*B*b^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}(a+b\cos(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
[Out] Timed out
```

3.564 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=237

$$\frac{2a^2(5aA-bB)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2\left(3a^3B+9a^2Ab+3ab^2B+Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d}$$

```
[Out] 2/5*b*B*(b+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/15*b^2*(5*A*b+9*
B*a)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/5*a^2*(5*A*a-B*b)*sin(d*x+c)*sec(d*x+c
)^(1/2)/d-2/5*(5*A*a^3-15*A*a*b^2-15*B*a^2*b-3*B*b^3)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(9*A*a^2*b+A*b^3+3*B*a^3+3*B*a*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.53, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4025, 4074, 4047, 3771, 2641, 4046, 2639}

$$\frac{2\left(9a^2Ab+3a^3B+3ab^2B+Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2\left(5a^3A-15a^2bB-15aAb^2-3b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(9*a^2*A*b + A*b^3 + 3*a
^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*b^2*(5*A*b + 9*a*B)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x
]]) + (2*a^2*(5*a*A - b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*B*
(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{(b+a\,\mathrm{se}}{}$$

$$= \frac{2b^2(5Ab+9aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2bB(b+a\sec(c+dx}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2b^2(5Ab+9aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2bB(b+a\sec(c+dx}{5d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2b^2(5Ab+9aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a^2(5aA-bB)\sqrt{\sec(}}{5d}$$

$$= \frac{2\left(9a^2Ab+Ab^3+3a^3B+3ab^2B\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{3d}$$

$$= -\frac{2\left(5a^3A-15aAb^2-15a^2bB-3b^3B\right)\sqrt{\cos(c+dx)}}{5d}$$

Mathematica [A] time = 1.44, size = 172, normalized size = 0.73

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)\left(3\left(10a^3A+b^3B\cos(2(c+dx))+b^3B\right)+10b^2(3aB+Ab)\cos(c+dx)\right)}{\sqrt{\cos(c+dx)}}+20\left(3a^3B+9a^2Ab+3ab^2\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(12*(-5*a^3*A + 15*a*A*b^2 + 15*a^2*
b*B + 3*b^3*B)*EllipticE[(c + d*x)/2, 2] + 20*(9*a^2*A*b + A*b^3 + 3*a^3*B
+ 3*a*b^2*B)*EllipticF[(c + d*x)/2, 2] + (2*(10*b^2*(A*b + 3*a*B)*Cos[c + d
*x] + 3*(10*a^3*A + b^3*B + b^3*B*Cos[2*(c + d*x)]))*Sin[c + d*x])/Sqrt[Cos
[c + d*x]]))/(30*d)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^3\cos(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\cos(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\cos(dx+c)^2 + \left(Ba^3 + 3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
*sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

maple [B] time = 1.66, size = 867, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/15*(-24*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*b^2*(5*A*b+15*B*a+6*B*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*a^3+5*A*b
^3+15*B*a*b^2+3*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+45*A*a^2*b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
```

```
)+5*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)+15*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a^3-45*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+15*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+15*B*a*b^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-45*B*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^
2*b-9*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}(a+b\,\cos(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.565 $\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{2b\left(18a^2B+21aAb+5b^2B\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}\right.}{21d}$$

```
[Out] 2/35*b^2*(7*A*b+11*B*a)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*b*B*(b+a*sec(d*x+
c))^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/21*b*(21*A*a*b+18*B*a^2+5*B*b^2)*sin(
d*x+c)/d/sec(d*x+c)^(1/2)+2/5*(15*A*a^2*b+3*A*b^3+5*B*a^3+9*B*a*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(21*A*a^3+21*A*a*b^2+21*B*a^2
*b+5*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.54, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4025, 4074, 4047, 3771, 2639, 4045, 2641}

$$\frac{2b\left(18a^2B+21aAb+5b^2B\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(21a^3A+21a^2bB+21aAb^2+5b^3B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^3*A + 21*a*A*b^2 + 2
1*a^2*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*b^2*(7*A*b + 11*a*B)*Sin[c + d*x])/(35*d*Sec[c + d*x
]^(3/2)) + (2*b*(21*a*A*b + 18*a^2*B + 5*b^2*B)*Sin[c + d*x])/(21*d*Sqrt[Se
c[c + d*x]]) + (2*b*B*(b + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c + d*x
]^(5/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
```

```
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{(b+a\,\mathrm{se}}{}$$

$$= \frac{2b^2(7Ab+11aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2bB(b+a\sec(c+d}{7d\sec^{\frac{5}{2}}(c}$$

$$= \frac{2b^2(7Ab+11aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2bB(b+a\sec(c+d}{7d\sec^{\frac{5}{2}}(c}$$

$$= \frac{2b^2(7Ab+11aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\left(21aAb+18a^2B\right.}{21d\sqrt{\mathrm{se}}}$$

$$= \frac{2\left(15a^2Ab+3Ab^3+5a^3B+9ab^2B\right)\sqrt{\cos(c+dx)}\,E}{5d}$$

$$= \frac{2\left(15a^2Ab+3Ab^3+5a^3B+9ab^2B\right)\sqrt{\cos(c+dx)}\,E}{5d}$$

Mathematica [A] time = 1.31, size = 180, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)}\left(b\sin(2(c+dx))\left(5\left(42a^2B+42aAb+3b^2B\cos(2(c+dx))+13b^2B\right)+42b(3aB+Ab)\cos(c+dx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(21*a^3*A + 21*a*A*b^2 + 21*a^2
*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*(42*b*(A*b
 + 3*a*B)*Cos[c + d*x] + 5*(42*a*A*b + 42*a^2*B + 13*b^2*B + 3*b^2*B*Cos[2*
(c + d*x)]))*Sin[2*(c + d*x)]))/(210*d)
```

fricas [F] time = 1.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^3\cos(dx+c)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\cos(dx+c)^3 + 3\left(Ba^2b + Aab^2\right)\cos(dx+c)^2 + \left(Ba^3 + 3\,A\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c) + A)(b\cos(dx+c) + a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

maple [B] time = 1.46, size = 664, normalized size = 2.71

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240B\,b^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-168A\,b^3-504Ba\,b^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*b^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A*b^3-504*B*a*b^2-360*B*b^3)*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*A*a*b^2+168*A*b^3+420*B*a^2*b+5
04*B*a*b^2+280*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*A*a*b^2
-42*A*b^3-210*B*a^2*b-126*B*a*b^2-80*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-63*A*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*b^3+105*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
```

```
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*A*a*b^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+1
05*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+b\cos(c+d\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3*sqrt(sec(c + d*x)), x
)
```

3.566 $\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=295

$$\frac{2b\left(22a^2B+27aAb+7b^2B\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(7a^3B+21a^2Ab+15ab^2B+5Ab^3\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(7a^3B+21a^2Ab\right.}{}$$

```
[Out] 2/63*b^2*(9*A*b+13*B*a)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/45*b*(27*A*a*b+22*B
*a^2+7*B*b^2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/9*b*B*(b+a*sec(d*x+c))^2*sin(
d*x+c)/d/sec(d*x+c)^(7/2)+2/21*(21*A*a^2*b+5*A*b^3+7*B*a^3+15*B*a*b^2)*sin(
d*x+c)/d/sec(d*x+c)^(1/2)+2/15*(15*A*a^3+27*A*a*b^2+27*B*a^2*b+7*B*b^3)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(21*A*a^2*b+5*A*b^3+7*B*a^
3+15*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.58, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2960, 4025, 4074, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2b\left(22a^2B+27aAb+7b^2B\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(21a^2Ab+7a^3B+15ab^2B+5Ab^3\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(21a^2Ab+7a^3B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(21*a^2*A*b + 5*A*b^3 +
 7*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(21*d) + (2*b^2*(9*A*b + 13*a*B)*Sin[c + d*x])/(63*d*Sec[c + d
*x]^(5/2)) + (2*b*(27*a*A*b + 22*a^2*B + 7*b^2*B)*Sin[c + d*x])/(45*d*Sec[c
 + d*x]^(3/2)) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B + 15*a*b^2*B)*Sin[c + d
*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*b*B*(b + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(9*d*Sec[c + d*x]^(7/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4025

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*A*Co
t[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*n), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b+a\sec(c+dx))^3(B+A\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2bB(b+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{(b+a\sec(c+dx))}{}$$

$$= \frac{2b^2(9Ab+13aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2bB(b+a\sec(c+dx))^2\sin(c-}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2b^2(9Ab+13aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2bB(b+a\sec(c+dx))^2\sin(c-}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2b^2(9Ab+13aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\left(27aAb+22a^2B+7b^2B\right)\mathrm{si}}{45d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b^2(9Ab+13aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\left(27aAb+22a^2B+7b^2B\right)\mathrm{si}}{45d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2\left(15a^3A+27aAb^2+27a^2bB+7b^3B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{15d}$$

Mathematica [A] time = 1.83, size = 219, normalized size = 0.74

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(7b\left(108a^2B+108aAb+43b^2B\right)\cos(c+dx)+5\left(84a^3B+252a^2Ab+18b^2(3aB+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^3*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 120*(21*a^2*A*b + 5*A*b^3 + 7*a
^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*b*(108
*a*A*b + 108*a^2*B + 43*b^2*B)*Cos[c + d*x] + 5*(252*a^2*A*b + 78*A*b^3 + 8
4*a^3*B + 234*a*b^2*B + 18*b^2*(A*b + 3*a*B)*Cos[2*(c + d*x)] + 7*b^3*B*Cos
[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(1260*d)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bb^3\cos(dx+c)^4+Aa^3+\left(3\,Bab^2+Ab^3\right)\cos(dx+c)^3+3\left(Ba^2b+Aab^2\right)\cos(dx+c)^2+\left(Ba^3+3\,Aa\right.}{\sqrt{\sec(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*cos(d*x + c))
/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

maple [B] time = 1.60, size = 745, normalized size = 2.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*B*b^3
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*A*b^3+2160*B*a*b^2+2240*B*b^
3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*A*a*b^2-1080*A*b^3-1512*B
*a^2*b-3240*B*a*b^2-2072*B*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(12
60*A*a^2*b+1512*A*a*b^2+840*A*b^3+420*B*a^3+1512*B*a^2*b+2520*B*a*b^2+952*B
*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-630*A*a^2*b-378*A*a*b^2-240
*A*b^3-210*B*a^3-378*B*a^2*b-720*B*a*b^2-168*B*b^3)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)-315*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*a*b^2+315*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+75*A*b^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-567*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-147*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*b^3+105*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*B*a*b^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^3}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\,(a + b\cos(c + dx))^3}{\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**3/sqrt(sec(c + d*x)), x
)
```

3.567 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=210

$$-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\sqrt{}}{}$$

```
[Out] 2/3*A*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-2*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/
2)/a^2/d+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+2/3
*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+2*b*(A*b-B*a)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a
+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a+b)/d
```

Rubi [A] time = 0.81, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4033, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*d) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(3*a*d) + (2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d) - (2*(A*b - a*B)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (2*A*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(3*a*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
```

```
0] &&  !IGtQ[m, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{b+a\sec(c+dx)}\,dx$$

$$= \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{Ab}{2}+\frac{1}{2}aA\sec(c+dx)-\frac{3}{2}(Ab-aB)\mathrm{se}}{b+a\sec(c+dx)}}{3a}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d} - \frac{2(Ab}{}$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2A\sqrt{\cos(c+}}{}$$

Mathematica [A] time = 3.43, size = 225, normalized size = 1.07

$$-\frac{\cot(c+dx)\left(-2\left(a^2(A-3B)+3ab(A-B)+3Ab^2\right)\sqrt{-\tan^2(c+dx)}\,F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-a^2A\sec^{\frac{5}{2}}(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] -1/3*(Cot[c + d*x]*(-(a^2*A*Sec[c + d*x]^(5/2)) + a^2*A*Cos[2*(c + d*x)]*Se
c[c + d*x]^(5/2) - 6*a*(-(A*b) + a*B)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[-Tan[c + d*x]^2] - 2*(3*A*b^2 + a^2*(A - 3*B) + 3*a*b*(A - B))*El
lipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*A*b^2*Ell
ipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*a
*b*B*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^
2]))/(a^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

maple [A] time = 4.12, size = 468, normalized size = 2.23

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4(Ab-aB)b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\right.}{a^2\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(A*b-B*a)*b^
2/a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b+B*a)/a^2*(-(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*A/a*(-1/6*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/2+cos(1/2
*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+b\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x)),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.568 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=126

$$-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2A\sqrt{\cos(c+dx)}}{}$$

```
[Out] 2*A*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a/d-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/a/(a+b)/d
```

Rubi [A] time = 0.46, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4033, 4106, 3849, 2805, 12, 3771, 2639}

$$-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}+\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2A\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d
) - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
```

```
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4033

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d^2
*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2))/(b*f*(
m + n)), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f
*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m + n)
 - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m
}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n,
0] &&  !IGtQ[m, 1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{b+a\sec(c+dx)}\,dx$$

$$= \frac{2A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} + \frac{2\int \frac{-\frac{Ab}{2}-\frac{1}{2}aA\sec(c+dx)-\frac{1}{2}(Ab-aB)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))}}{a}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} + \frac{2\int -\frac{Ab^2}{2\sqrt{\sec(c+dx)}}\,dx}{ab^2} + \frac{(-Ab+aB)\int}{a}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} - \frac{A\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a} + \frac{\left((-Ab+aB)\sqrt{\mathrm{co}}\right.}{}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a+b)d} + \frac{2A}{}$$

$$= -\frac{2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2(Ab-aB)\sqrt{\cos(}}{}$$

Mathematica [A] time = 1.29, size = 125, normalized size = 0.99

$$-\frac{2\cos(2(c+dx))\sqrt{-\tan^2(c+dx)}\,\csc(c+dx)\sec(c+dx)\left(-(aA-aB+Ab)F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+(A\right.}{a^2d\left(\sec^2(c+dx)-2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*Cos[2*(c + d*x)]*Csc[c + d*x]*(a*A*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]]
, -1] - (a*A + A*b - a*B)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (A*b
- a*B)*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sec[c + d*x]*Sqr
t[-Tan[c + d*x]^2])/(a^2*d*(-2 + Sec[c + d*x]^2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

maple [A] time = 2.86, size = 327, normalized size = 2.60

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4(-Ab+aB)b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{\cdots}\right.}{a\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}\cdots\right.}{\sin\left(\frac{dx}{2}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(-A*b+B*a)/a
/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*A/a*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d
*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x)),x)
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
[Out] Timed out
```

3.569 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d+2*(A*b-B*a)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+
b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a+b)/d
```

Rubi [A] time = 0.28, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4038, 3771, 2641, 3849, 2805}

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
 + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4038

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
 NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx=\int \frac{\sqrt{\sec(c+dx)}\,(B+A\sec(c+dx))}{b+a\sec(c+dx)}\,dx$$

$$=\frac{B\int \sqrt{\sec(c+dx)}\,dx}{b}-\frac{(-Ab+aB)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx}{b}$$

$$=\frac{\left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b}-\frac{\left((-Ab+aB)\sqrt{\cos(\right.}{}$$

$$=\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd}+\frac{2(Ab-aB)\sqrt{\cos(c}}{}$$

Mathematica [A] time = 0.54, size = 76, normalized size = 0.75

$$\frac{2\sqrt{-\tan^2(c+dx)}\,\cot(c+dx)\left((aB-Ab)\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+AbF\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(A*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (-(A*b) +
a*B)*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]
^2])/(a*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

maple [A] time = 1.50, size = 217, normalized size = 2.15

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{(a-b)\,b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\mathrm{s}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))*b+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))*a)/(a-b)/b/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x)),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x)), x)
```

3.570 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=149

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\right.}{b^2d(a+b)}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d+2*(A*b-B*a)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/d-2*a*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a+b)/d
```

Rubi [A] time = 0.35, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4038, 3771, 2639, 3848, 2803, 2641, 2805}

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\right.}{b^2d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
 + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(b^2*d) - (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
```

```
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3848

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[(Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]])/d, Int[S
qrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
x] && NeQ[a^2 - b^2, 0]
```

Rule 4038

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
 NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))}\,dx$$
$$= \frac{B\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b} - \frac{(-Ab+aB)\int \frac{\sqrt{\sec(c+dx)}}{b+a\sec(c+dx)}\,dx}{b}$$
$$= \frac{\left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b} - \frac{\left((-Ab+aB)\sqrt{\cos\right.}{}$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{\left((-Ab+aB)\sqrt{\cos(c\right.}{}$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} + \frac{2(Ab-aB)\sqrt{\cos(c+}{}$$

Mathematica [A] time = 6.36, size = 220, normalized size = 1.48

$$\frac{\cot(c+dx)\left(2Ab\sqrt{-\tan^2(c+dx)}\,\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right) - 2aB\sqrt{-\tan^2(c+dx)}\,\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Cot[c + d*x]*(-(b*B*Sec[c + d*x]^(3/2)) - b*B*Cos[2*(c + d*x)]*Sec[c + d*x
]^(3/2) + b*B*Sec[c + d*x]^(7/2) + b*B*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2)
- 2*b*B*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2
*b*B*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*A*
b*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
- 2*a*B*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*
x]^2]))/(b^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

maple [A] time = 1.59, size = 295, normalized size = 1.98

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+A*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-B*EllipticPi(cos(1/2*d*x+1/2*c),-2
*b/(a-b),2^(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-B*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a*b-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/b^2/(a-b)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))),x)
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos\left(c+dx\right)}{\left(a+b\cos\left(c+dx\right)\right)\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(1/2),x)
[Out] Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.571 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{2a^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2\left(-3a^2B+3aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{3b^3d}$$

```
[Out] 2/3*B*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/b^2/d-2/3*(3*A*a*b-3*B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/b^3/d+2*a^2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/b^3/(a+b)/d
```

Rubi [A] time = 0.56, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2960, 4034, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{2\left(-3a^2B+3aAb-b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}+\frac{2a^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}{b^3d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(b^2*d) - (2*(3*a*A*b - 3*a^2*B - b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^3*d) + (2*a^2*(A*b - a*B)*Sqrt[Cos
[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b
^3*(a + b)*d) + (2*B*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4034

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))}\,dx$$

$$= \frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} - \frac{2\int \frac{-\frac{3}{2}(Ab-aB)-\frac{1}{2}bB\sec(c+dx)-\frac{1}{2}aB\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))}\,dx}{3b}$$

$$= \frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} - \frac{2\int \frac{-\frac{3}{2}b(Ab-aB)-\left(\frac{b^2B}{2}-\frac{3}{2}a(Ab-aB)\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3b^3} + \frac{(a^2(Ab\ldots}{\ldots}$$

$$= \frac{2B\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b^2} - \frac{\left(3aAb-3a^2B-b^2B\right)\ldots}{3b\ldots}$$

$$= \frac{2a^2(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3(a+b)d} + \frac{2B\ldots}{3bd\ldots}$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} - \frac{2\left(3aAb-3\ldots\right.}{\ldots}$$

Mathematica [A] time = 6.66, size = 278, normalized size = 1.41

$$\frac{2\csc(c+dx)\left(3a^2B\sqrt{-\tan^2(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+b(-3aB+3Ab+bB)\sqrt{-\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*Csc[c + d*x]*(-3*A*b^2 + 3*a*b*B + 3*A*b^2*Sec[c + d*x]^2 - 3*a*b*B*Sec[
c + d*x]^2 + b^2*B*Sin[c + d*x]*Tan[c + d*x] - 3*b*(A*b - a*B)*EllipticE[Ar
```

```
cSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + b*
(3*A*b - 3*a*B + b*B)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[-Tan[c + d*x]^2] - 3*a*A*b*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + 3*a^2*B*Elliptic
Pi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c +
 d*x]^2]))/(3*b^3*d*Sec[c + d*x]^(3/2))
```

fricas [F] time = 160.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

maple [B] time = 1.57, size = 786, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-4*B*a*b^2+4*
B*b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(2*B*a*b^2-2*B*b^3)*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*a*b^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
```

```
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*b^3-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*a^2*b-3*a^3*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+3*a^2*b*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*a*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+b^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a^2*b+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))*a^3)/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+b\cos(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))*sec(c + d*x)**(3/2)), x
)
```

3.572 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=405

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2A+3abB-5Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)}+\frac{\left(2a^2A+3abB-5Ab^2\right)}{}$$

```
[Out] 1/3*(2*A*a^2-5*A*b^2+3*B*a*b)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/(a^2-b^2)/d+b
*(A*b-B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))-(4*A*
a^2*b-5*A*b^3-2*B*a^3+3*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/
d+(4*A*a^2*b-5*A*b^3-2*B*a^3+3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a^3/(a^2-b^2)/d+1/3*(2*A*a^2-5*A*b^2+3*B*a*b)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d+b*(7*A*a^2*b-5*A*b^3-5*B*a^3+3*B*
a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d
*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)/(a
+b)^2/d
```

Rubi [A] time = 1.29, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4029, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2A+3abB-5Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)}-\frac{\left(4a^2Ab-2a^3B+3ab^2B\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d) + ((2*a^2*A - 5*A*b^
2 + 3*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(3*a^2*(a^2 - b^2)*d) + (b*(7*a^2*A*b - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*S
qrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^3*(a - b)*(a + b)^2*d) - ((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*
B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((2*a^2*A - 5*A*b
^2 + 3*a*b*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b*(
A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c +
 d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}b(Ab-aB)+a(Ab-aB)\text{se}}{b+a}}{a\left(\right.}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sec^{\frac{5}{2}}(c-}{a\left(a^2-b^2\right)d(b+}$$

$$= -\frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A\right.}{}$$

$$= -\frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A\right.}{}$$

$$= -\frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A\right.}{}$$

$$= \frac{b\left(7a^2Ab-5Ab^3-5a^3B+3ab^2B\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3(a-b)(a+b)^2d}$$

$$= \frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{a^3\left(a^2-b^2\right)d}$$

Mathematica [A] time = 7.11, size = 735, normalized size = 1.81

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{ab^2B\sin(c+dx)-Ab^3\sin(c+dx)}{a^2\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{2A\tan(c+dx)}{3a^2}+\frac{\left(2a^3B-4a^2Ab-3ab^2B+5Ab^3\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)}\right)}{d}+\frac{\left(6a^3bB-12a^2Ab^2-9ab^3B+15Ab^4\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] ((2*(-4*a^4*A - 44*a^2*A*b^2 + 45*A*b^4 + 30*a^3*b*B - 27*a*b^3*B)*Cos[c +
d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcS
```

```
in[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*
Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-28*a^3*A
*b + 40*a*A*b^3 + 12*a^4*B - 24*a^2*b^2*B)*Cos[c + d*x]^2*EllipticPi[-(a/b)
, ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x
]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-12*a^
2*A*b^2 + 15*A*b^4 + 6*a^3*b*B - 9*a*b^3*B)*Cos[2*(c + d*x)]*(b + a*Sec[c +
 d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*El
lipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
 d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Se
c[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt
[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d
*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2
 - Sec[c + d*x]^2)))/(12*a^3*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(((-
4*a^2*A*b + 5*A*b^3 + 2*a^3*B - 3*a*b^2*B)*Sin[c + d*x])/(a^3*(a^2 - b^2))
+ (-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x])/(a^2*(a^2 - b^2)*(a + b*Co
s[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^2)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x
)
```

maple [B] time = 6.90, size = 1031, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-1/
2+cos(1/2*d*x+1/2*c)^2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4*b^2*(2*A*b-B*a)/a^3/(-2*a*b+2*b^2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a
-b),2^(1/2))+2*(A*b-B*a)*b/a^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b
)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)
/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2
^(1/2)))+2*(-2*A*b+B*a)/a^3*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/
(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{(a+b\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^2, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.573 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=316

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2A+abB-3Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\cdots}{ad\left(\cdots\right.}$$

```
[Out] b*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))+(2*A
*a^2-3*A*b^2+B*a*b)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-(2*A*a^2-3*
A*b^2+B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d+(
A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d-(5*A*a^
2*b-3*A*b^3-3*B*a^3+B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a^2/(a-b)/(a+b)^2/d
```

Rubi [A] time = 0.94, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4029, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2A+abB-3Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\cdots}{ad\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -(((2*a^2*A - 3*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d)) + ((A*b - a*B)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*d) - ((5*a^2*
A*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)*(a + b)^2*d) + ((2*a^
2*A - 3*A*b^2 + a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*(a^2 - b^2)*d)
 + (b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*
Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
```

```
2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}b(Ab-aB)+a(Ab-aB\right.}{b+}$$

$$= \frac{\left(2a^2A-3Ab^2+abB\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c}{a\left(a^2-b^2\right)d(b}$$

$$= \frac{\left(2a^2A-3Ab^2+abB\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c}{a\left(a^2-b^2\right)d(b}$$

$$= \frac{\left(2a^2A-3Ab^2+abB\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c}{a\left(a^2-b^2\right)d(b}$$

$$= -\frac{\left(5a^2Ab-3Ab^3-3a^3B+ab^2B\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a-b)(a+b)^2d}$$

$$= -\frac{\left(2a^2A-3Ab^2+abB\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2\left(a^2-b^2\right)d}$$

Mathematica [B] time = 6.90, size = 681, normalized size = 2.16

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(2a^2A+abB-3Ab^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)} + \frac{Ab^2\sin(c+dx)-abB\sin(c+dx)}{a\left(a^2-b^2\right)(a+b\cos(c+dx))}\right)}{d} - \frac{\left(2a^2Ab+ab^2B-3Ab^3\right)\sin(c+dx)\cos(2(c+dx))(a\sec(c+dx)+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] -1/4*((2*(10*a^2*A*b - 9*A*b^3 - 4*a^3*B + 3*a*b^2*B)*Cos[c + d*x]^2*(Ellip
ticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c
 + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])
/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(4*a^3*A - 8*a*A*b^2 +
4*a^2*b*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1
]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos
```

```
[c + d*x])*(1 - Cos[c + d*x]^2)) + ((2*a^2*A*b - 3*A*b^3 + a*b^2*B)*Cos[2*(
c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*Ellip
ticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*
x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*Ellipt
icPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Se
c[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^
2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(a^2*(a - b)*(a + b)*d) + (Sqr
t[Sec[c + d*x]]*(((2*a^2*A - 3*A*b^2 + a*b*B)*Sin[c + d*x])/(a^2*(a^2 - b^2
)) + (A*b^2*Sin[c + d*x] - a*b*B*Sin[c + d*x])/(a*(a^2 - b^2)*(a + b*Cos[c
+ d*x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx + c) + A)\sec(dx + c)^{\frac{3}{2}}}{(b\cos(dx + c) + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x
)
```

maple [B] time = 4.26, size = 883, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*b^2/a^2/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
```

```
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2*(-A*b+B*a)/a*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3
*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*
b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2)))+2*A/a^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2
/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{(a+b\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^2, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
[Out] Timed out
```

3.574 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=260

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{(Ab-aB)\sqrt{\cos(c}{}$$

```
[Out] b*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))-(A*b
-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d-(A*b-B*a)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d+(3*A*a^2*b-A*b^3
-B*a^3-B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(
a-b)/b/(a+b)^2/d
```

Rubi [A] time = 0.61, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2960, 4029, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{(Ab-aB)\sqrt{\cos(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -(((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a*(a^2 - b^2)*d)) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) + ((3*a^2*A*b - A*b^3 - a^3*
B - a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a*(a - b)*b*(a + b)^2*d) + (b*(A*b - a*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
```

```
, 1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}b(Ab-aB)+a(Ab-aB)\sec(c+dx)-}{\sqrt{\sec(c+dx)}\,(b+a}}{a\left(a^2-\right.}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}b^2(Ab-aB)+\frac{1}{2}ab(Ab-aB)\sec(c+d}{\sqrt{\sec(c+dx)}}}{ab^2\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a\left(a^2-b^2\right)} - $$

$$= \frac{\left(3a^2Ab-Ab^3-a^3B-ab^2B\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{a(a-b)b(a+b)^2d}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d} - \frac{(Ab-aB)}{}$$

Mathematica [B] time = 6.84, size = 639, normalized size = 2.46

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{aB\sin(c+dx)-Ab\sin(c+dx)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{(aB-Ab)\sin(c+dx)}{a\left(a^2-b^2\right)}\right)}{d}+\frac{2\left(-4a^2A+abB+3Ab^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}\,(a\sec(c+dx)}{a\left(1-\cos^2(c+dx)\right)(a+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^2,
x]
```

```
[Out] ((2*(-4*a^2*A + 3*A*b^2 + a*b*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[
c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b +
a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*
x])*(1 - Cos[c + d*x]^2)) + (2*(4*a*A*b - 4*a^2*B)*Cos[c + d*x]^2*EllipticP
i[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec
[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) +
((A*b^2 - a*b*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[
c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi
[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
+ d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c +
d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*a*(
-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-(((-(A*b) + a*B)*Sin[c + d*x])/(
a*(a^2 - b^2))) + (-(A*b*Sin[c + d*x]) + a*B*Sin[c + d*x])/((a^2 - b^2)*(a
+ b*Cos[c + d*x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2, x
)
```

maple [B] time = 3.65, size = 721, normalized size = 2.77

$$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{4B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{\left(-2ab+2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B/(-2*a*b+2*
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+2*(A*b-B*a)/b*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b
+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-
b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos (c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+b\,\cos (c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^2, x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos (c+dx))\sqrt{\sec (c+dx)}}{(a+b\cos (c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**2, x
)
```

3.575 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=258

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(a^2B+aAb-2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{(Ab-}{}$$

```
[Out] -(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))+(A*b-B*
a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d+(A*a*b+B*a^2-
2*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-(A*a^
2*b+A*b^3+B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/(a-b)/b^2/(a+b)^2/d
```

Rubi [A] time = 0.61, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2960, 4027, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(a^2B+aAb-2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{(Ab-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(b*(a^2 - b^2)*d) + ((a*A*b + a^2*B - 2*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) - ((a^2*A*b + A
*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^2*(a + b)^2*d) - ((A*b - a*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(d*(A*
b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)
)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d
*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^
2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && Ne
```

```
Q[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx=\int \frac{\sqrt{\sec(c+dx)}\,(B+A\sec(c+dx))}{(b+a\sec(c+dx))^2}\,dx$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))}+\frac{\int \frac{\frac{1}{2}(Ab-aB)+(aA-bB)\sec(c+dx)-\frac{1}{2}(}{\sqrt{\sec(c+dx)}\,(b+a\sec(}}{a^2-b^2}$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))}+\frac{\int \frac{\frac{1}{2}b(Ab-aB)-\left(\frac{1}{2}a(Ab-aB)-b(aA-b\right.}{\sqrt{\sec(c+dx)}}}{b^2\left(a^2-b^2\right)}$$

$$=-\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))}+\frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2b\left(a^2-b^2\right)}+$$

$$=-\frac{\left(a^2Ab+Ab^3+a^3B-3ab^2B\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{(a-b)b^2(a+b)^2d}$$

$$=\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b\left(a^2-b^2\right)d}+\frac{\left(aAb+a^2B\right.}{}$$

Mathematica [B] time = 6.82, size = 626, normalized size = 2.43

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{(Ab-aB)\sin(c+dx)}{b\left(b^2-a^2\right)}+\frac{a^2B\sin(c+dx)-aAb\sin(c+dx)}{b\left(b^2-a^2\right)(a+b\cos(c+dx))}\right)}{d}+\frac{(Ab-aB)\sin(c+dx)\cos(2(c+dx))(a\sec(c+dx)+b)\left(-4a^2\sqrt{\sec(c+dx)}\,\sqrt{1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),
x]
```

```
[Out] ((2*(-(A*b) + a*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1
] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x]
)*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c
 + d*x]^2)) + (2*(4*a*A - 4*b*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[S
qrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c
 + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((A*b - a*B)*Cos[2
*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*Ell
ipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[
c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[S
ec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*Elli
pticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x
]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*(a + b)*d) + (Sqr
t[Sec[c + d*x]]*(((A*b - a*B)*Sin[c + d*x])/(b*(-a^2 + b^2)) + (-(a*A*b*Sin
[c + d*x]) + a^2*B*Sin[c + d*x])/(b*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

maple [B] time = 3.80, size = 808, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/b
*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a*(A*b-B*a)/b^2*(-b^2/a/(a^2-
b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*
b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b
^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos(c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2), x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^2 \sqrt{\sec(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))**2*sqrt(sec(c + d*x))),
 x)
```

3.576 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=284

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\left(-3a^2B+aAb+2b^2B\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{\left(-3\right.}{}$$

```
[Out] a*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b+a*sec(d*x+c))-(A*a
*b-3*B*a^2+2*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b
^2)/d+(A*a^2*b-2*A*b^3-3*B*a^3+4*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/b^3/(a^2-b^2)/d-a*(A*a^2*b-3*A*b^3-3*B*a^3+5*B*a*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a
+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^3/(a+b)^2/d
```

Rubi [A] time = 0.66, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2960, 4030, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(a^2Ab-3a^3B+4ab^2B-2Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right|}{b^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -(((a*A*b - 3*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d)) + ((a^2*A*b - 2*A*b^3 - 3*a^3*B +
 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(b^3*(a^2 - b^2)*d) - (a*(a^2*A*b - 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*Sqrt[C
os[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
((a - b)*b^3*(a + b)^2*d) + (a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(b*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
```

```
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2}\,dx$$

$$=\frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))}+\frac{\int \frac{\frac{1}{2}\left(-aAb+3a^2B-2b^2B\right)-b(Ab-aB)\,\mathrm{se}}{\sqrt{\sec(c+dx)}\,(b+}}{b\left(a^2-\right.}$$

$$=\frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))}+\frac{\int \frac{\frac{1}{2}b\left(-aAb+3a^2B-2b^2B\right)-\left(b^2(Ab-aB\right.}{\sqrt{\sec}}}{b^3\left(a\right.}$$

$$=\frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))}-\frac{\left(aAb-3a^2B+2b^2B\right)\int \frac{1}{\sqrt{\sec(c}}}{2b^2\left(a^2-b^2\right)}$$

$$=-\frac{a\left(a^2Ab-3Ab^3-3a^3B+5ab^2B\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right.}{(a-b)b^3(a+b)^2d}$$

$$=-\frac{\left(aAb-3a^2B+2b^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2\left(a^2-b^2\right)d}+$$

Mathematica [B] time = 6.89, size = 655, normalized size = 2.31

$$\frac{2\left(a^2(-B)-aAb+2b^2B\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}\,(a\sec(c+dx)+b)\left(F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{a\left(1-\cos^2(c+dx)\right)(a+b\cos(c+dx))}+\frac{\left(-3a^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] ((2*(-(a*A*b) - a^2*B + 2*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[
c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b +
a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*
x])*(1 - Cos[c + d*x]^2)) + (2*(4*A*b^2 - 4*a*b*B)*Cos[c + d*x]^2*EllipticP
i[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec
[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) +
((a*A*b - 3*a^2*B + 2*b^2*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b
+ 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
rt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^
2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a
+ b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]
^2)))/(4*b*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-((a*(-(A*b) + a*B)*S
in[c + d*x])/(b^2*(a^2 - b^2))) + (a^2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x
])/(b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

maple [B] time = 4.75, size = 849, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-2
*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*b)+4*a/b^2*(2*A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2*(A*b-
B*a)/b^3*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*
b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos(c+dx)}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}(a+b\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.577 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=363

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+b)}-\frac{\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{\left(-5a^3B+3a^2Ab+4ab^2B\right.}{}$$

```
[Out] -1/3*(3*A*a*b-5*B*a^2+2*B*b^2)*sin(d*x+c)/b^2/(a^2-b^2)/d/sec(d*x+c)^(1/2)+
a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(b+a*sec(d*x+c))/sec(d*x+c)^(1/2)+(3*A
*a^2*b-2*A*b^3-5*B*a^3+4*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/b^3/(a^2-b^2)/d-1/3*(9*A*a^3*b-12*A*a*b^3-15*B*a^4+16*B*a^2*b^2+2*B*b^4
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^4/(a^2-b^2)/d+a^2*(3*A*a^2
*b-5*A*b^3-5*B*a^3+7*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/(a-b)/b^4/(a+b)^2/d
```

Rubi [A] time = 0.96, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4030, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+b)}-\frac{\left(-5a^2B+3aAb+2b^2B\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}-\frac{\left(9a^3Ab+16a^2b^2B-15a^4B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d) - ((9*a^3*A*b - 12*a
*A*b^3 - 15*a^4*B + 16*a^2*b^2*B + 2*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^4*(a^2 - b^2)*d) + (a^2*(3*a^2*A*b -
 5*A*b^3 - 5*a^3*B + 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^4*(a + b)^2*d) - ((3*a*A*b
 - 5*a^2*B + 2*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]
) + (a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(b + a
*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(-3aAb+5a^2B-2b^2B\right)-b(\ldots}{\sec^{\frac{3}{2}}(\ldots}}{\ldots}$$

$$= -\frac{\left(3aAb-5a^2B+2b^2B\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{a(Ab-aB)\sin(c+d\ldots}{b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(b+a\ldots}$$

$$= -\frac{\left(3aAb-5a^2B+2b^2B\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{a(Ab-aB)\sin(c+d\ldots}{b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(b+a\ldots}$$

$$= -\frac{\left(3aAb-5a^2B+2b^2B\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{a(Ab-aB)\sin(c+d\ldots}{b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(b+a\ldots}$$

$$= \frac{a^2\left(3a^2Ab-5Ab^3-5a^3B+7ab^2B\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\ldots}{(a-b)b^4(a+b)^2d}$$

$$= \frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c\ldots}}{b^3\left(a^2-b^2\right)d}$$

Mathematica [A] time = 6.99, size = 701, normalized size = 1.93

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{a^2(aB-Ab)\sin(c+dx)}{b^3\left(a^2-b^2\right)} - \frac{a^3Ab\sin(c+dx)-a^4B\sin(c+dx)}{b^3\left(b^2-a^2\right)(a+b\cos(c+dx))} + \frac{B\sin(2(c+dx))}{3b^2}\right)}{d} - \frac{2\left(8a^2bB-12aAb^2+4b^3B\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1\ldots}}{b\left(1-\cos^2(c\ldots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] -1/12*((2*(-3*a^2*A*b + 6*A*b^3 + 5*a^3*B - 8*a*b^2*B)*Cos[c + d*x]^2*(Elli
pticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[
c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]
)/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-12*a*A*b^2 + 8*a^2*b
*B + 4*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]],
```

```
-1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*
Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-9*a^2*A*b + 6*A*b^3 + 15*a^3*B - 1
2*a*b^2*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*
x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*S
qrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b
), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]
^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
 d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*
(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/((a - b)*b^2
*(a + b)*d) + (Sqrt[Sec[c + d*x]]*((a^2*(-(A*b) + a*B)*Sin[c + d*x])/(b^3*(
a^2 - b^2)) - (a^3*A*b*Sin[c + d*x] - a^4*B*Sin[c + d*x])/(b^3*(-a^2 + b^2)
*(a + b*Cos[c + d*x])) + (B*Sin[2*(c + d*x)])/(3*b^2)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
 x)
```

maple [B] time = 5.32, size = 1066, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^4/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*b^2*cos(1/2*d*x+1/2*c)
```

```
*sin(1/2*d*x+1/2*c)^4+6*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^2-9*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^2*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+2*B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1
/2*c)^2)-4*a^2/b^3*(3*A*b-4*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*a^3*(A*b-
B*a)/b^4*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*
b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,{\left(a+b\,\cos(c+d\,x)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.578 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=480

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b\left(-7a^3B+11a^2Ab+ab^2B-5Ab^3\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{\left(-7a^3B+11\right.}{}$$

```
[Out] 1/2*b*(A*b-B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^
2+1/4*b*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^
2/(a^2-b^2)^2/d/(b+a*sec(d*x+c))+1/4*(8*A*a^4-29*A*a^2*b^2+15*A*b^4+9*B*a^3
*b-3*B*a*b^3)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)^2/d-1/4*(8*A*a^4-29
*A*a^2*b^2+15*A*b^4+9*B*a^3*b-3*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/a^3/(a^2-b^2)^2/d+1/4*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a*b^2)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d-1/4*(35*A*a^4*b-3
8*A*a^2*b^3+15*A*b^5-15*B*a^5+6*B*a^3*b^2-3*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)^2/(a+b)^3/d
```

Rubi [A] time = 1.45, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2960, 4029, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b\left(11a^2Ab-7a^3B+ab^2B-5Ab^3\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{\left(-29a^2Ab^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d)
 + ((11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) - ((35*a^4*A*b
 - 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c
 + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a
^3*(a - b)^2*(a + b)^3*d) + ((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B
 - 3*a*b^3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2*d) + (b
*(A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec
```

```
[c + d*x])^2) + (b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
```

```
C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{3}{2}b(Ab-aB)+2a(Ab-a\right.}{(b+}}{2a}$$

$$= \frac{b(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b\left(11a^2Ab-5Ab^3-7a^3B+a\right.}{4a^2\left(a^2-b^2\right)^2d(b}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\sec(c+dx)}\,\sin(c+d}{4a^3\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\sec(c+dx)}\,\sin(c+d}{4a^3\left(a^2-b^2\right)^2d}$$

$$= \frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\sec(c+dx)}\,\sin(c+d}{4a^3\left(a^2-b^2\right)^2d}$$

$$= -\frac{\left(35a^4Ab-38a^2Ab^3+15Ab^5-15a^5B+6a^3b^2B-3ab^4B\right)\sqrt{\cos(c+}}{4a^3(a-b)^2(a+b)^3d}$$

$$= -\frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c\right.}{4a^3\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.21, size = 844, normalized size = 1.76

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(8Aa^4+9bBa^3-29Ab^2a^2-3b^3Ba+15Ab^4\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2}+\frac{Ab^2\sin(c+dx)-abB\sin(c+dx)}{2a\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{-5A\sin(c+dx)b^4+aB\sin(c+dx)b^3+11a}{4a^2\left(a^2-b^2\right)^2(a+b}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] -1/16*((2*(56*a^4*A*b - 95*a^2*A*b^3 + 45*A*b^5 - 16*a^5*B + 19*a^3*b^2*B -
 9*a*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - Ell
ipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[
1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]
^2)) + (2*(16*a^5*A - 80*a^3*A*b^2 + 40*a*A*b^4 + 32*a^4*b*B - 8*a^2*b^3*B)
*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*S
ec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])
*(1 - Cos[c + d*x]^2)) + ((8*a^4*A*b - 29*a^2*A*b^3 + 15*A*b^5 + 9*a^3*b^2*
B - 3*a*b^4*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c
+ d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x
]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-
(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*
x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(a^3*(a
- b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((8*a^4*A - 29*a^2*A*b^2 + 15*A*
b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2) + (A*b^2*S
in[c + d*x] - a*b*B*Sin[c + d*x])/(2*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2)
+ (11*a^2*A*b^2*Sin[c + d*x] - 5*A*b^4*Sin[c + d*x] - 7*a^3*b*B*Sin[c + d*x
] + a*b^3*B*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x
)
```

maple [B] time = 7.64, size = 2002, normalized size = 4.17

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b+B*a)/a*
(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2
/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/
(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8
*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-
15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a
*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2)))+4*A*b^2/a^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2*A*b/a^2*(-b^2/a/(a^
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
```

```
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/
2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2
-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2/a^3*A*(-(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{(a+b\cos(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^3, x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.579 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=405

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{\left(-3a^3B+7a\right.}{}$$

```
[Out] 1/2*b*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^
2+1/4*b*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2
/(a^2-b^2)^2/d/(b+a*sec(d*x+c))-1/4*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d-1/4*(7*A*a^2*b-
A*b^3-3*B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/b/(
a^2-b^2)^2/d+1/4*(15*A*a^4*b-6*A*a^2*b^3+3*A*b^5-3*B*a^5-10*B*a^3*b^2+B*a*b
^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a-b)^2/b/(
a+b)^3/d
```

Rubi [A] time = 1.04, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4029, 4098, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b\left(9a^2Ab-5a^3B-ab^2B-3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{\left(7a^2Ab-3a^3B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A
*b^3 - 3*a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(4*a*b*(a^2 - b^2)^2*d) + ((15*a^4*A*b - 6*a^2*A*b^3 + 3*
A*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*
b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*b*(a + b)^
3*d) + (b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(
b + a*Sec[c + d*x])^2) + (b*(9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}b(Ab-aB)+2a(Ab-\ldots\right.}{(b\ldots}}{2\ldots}$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b\left(9a^2Ab-3Ab^3-5a^3B-ab\ldots\right.}{4a^2\left(a^2-b^2\right)^2 d(b\ldots}$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b\left(9a^2Ab-3Ab^3-5a^3B-ab\ldots\right.}{4a^2\left(a^2-b^2\right)^2 d(b\ldots}$$

$$= \frac{b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b\left(9a^2Ab-3Ab^3-5a^3B-ab\ldots\right.}{4a^2\left(a^2-b^2\right)^2 d(b\ldots}$$

$$= \frac{\left(15a^4Ab-6a^2Ab^3+3Ab^5-3a^5B-10a^3b^2B+ab^4B\right)\sqrt{\cos(c+dx)}\ldots}{4a^2(a-b)^2b(a+b)^3d}$$

$$= -\frac{\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec\ldots}}{4a^2\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 7.11, size = 797, normalized size = 1.97

$$\frac{2\left(16Aa^4-9bBa^3-19Ab^2a^2+3b^3Ba+9Ab^4\right)\left(F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\,\sin(c+dx)\cos\ldots}{a(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^3,
x]
```

```
[Out] ((2*(16*a^4*A - 19*a^2*A*b^2 + 9*A*b^4 - 9*a^3*b*B + 3*a*b^3*B)*Cos[c + d*x
]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[
Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin
[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-32*a^3*A*b
```

```
+ 8*a*A*b^3 + 16*a^4*B + 8*a^2*b^2*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*
Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-9*a^2*A*b^
2 + 3*A*b^4 + 5*a^3*b*B + a*b^3*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-
4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
- 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x
]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b
^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c
+ d*x]^2)))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(-1/4*((-9
*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Sin[c + d*x])/(a^2*(a^2 - b^2)^2) +
 (-(A*b*Sin[c + d*x]) + a*B*Sin[c + d*x])/(2*(a^2 - b^2)*(a + b*Cos[c + d*x
])^2) + (-7*a^2*A*b*Sin[c + d*x] + A*b^3*Sin[c + d*x] + 3*a^3*B*Sin[c + d*x
] + 3*a*b^2*B*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^3, x
)
```

maple [B] time = 6.01, size = 1744, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)/b*(
-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/
(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(
a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*
b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1
5/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*
b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),
2^(1/2)))+2*B/b*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)
*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1
```

```
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^3, x
)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**3, x
)
```

3.580 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=402

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{\left(a^3B+3a^2A\right.}{}$$

```
[Out] 1/2*b*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^
2-1/4*(7*A*a^2*b-A*b^3-3*B*a^3-3*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^
2-b^2)^2/d/(b+a*sec(d*x+c))+1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/b/(a^2-b^2)^2/d+1/4*(3*A*a^2*b+3*A*b^
3+B*a^3-7*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^
2)^2/d-1/4*(3*A*a^4*b+10*A*a^2*b^3-A*b^5+B*a^5-10*B*a^3*b^2-3*B*a*b^4)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2
*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a-b)^2/b^2/(a+b)^3/d
```

Rubi [A] time = 1.09, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4029, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{\left(7a^2Ab-3a^3B-3ab^2B-Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{\left(3a^2Ab+a^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*b*(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*A*
b^3 + a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) - ((3*a^4*A*b + 10*a^2*A*b^3 - A*b^5
 + a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^2*b^2*(a + b)^3*d)
+ (b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a
*Sec[c + d*x])^2) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4029

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(a*d^2*
(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n -
 2))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx=\int \frac{\sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))}{(b+a\sec(c+dx))^3}\,dx$$

$$=\frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}-\frac{\int \frac{\frac{1}{2}b(Ab-aB)+2a(Ab-aB)\sec(c+dx)-\frac{1}{2}}{\sqrt{\sec(c+dx)}\,(b+a\,s}}{2a\left(a^2-\right.}$$

$$=\frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}-\frac{\left(7a^2Ab-Ab^3-3a^3B-3ab^2B\right.}{4a\left(a^2-b^2\right)^2d(b+}$$

$$=\frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}-\frac{\left(7a^2Ab-Ab^3-3a^3B-3ab^2B\right.}{4a\left(a^2-b^2\right)^2d(b+}$$

$$=\frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2}-\frac{\left(7a^2Ab-Ab^3-3a^3B-3ab^2B\right.}{4a\left(a^2-b^2\right)^2d(b+}$$

$$=-\frac{\left(3a^4Ab+10a^2Ab^3-Ab^5+a^5B-10a^3b^2B-3ab^4B\right)\sqrt{\cos(c+dx)}\,\Pi}{4a(a-b)^2b^2(a+b)^3d}$$

$$=\frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{4ab\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 6.93, size = 784, normalized size = 1.95

$$\frac{\sqrt{\sec(c+dx)}\left(-\frac{aAb\sin(c+dx)-a^2B\sin(c+dx)}{2b\left(b^2-a^2\right)(a+b\cos(c+dx))^2}+\frac{\left(a^3B-5a^2Ab+5ab^2B-Ab^3\right)\sin(c+dx)}{4ab\left(a^2-b^2\right)^2}+\frac{a^3B\sin(c+dx)+3a^2Ab\sin(c+dx)-7ab^2B\sin(c+dx)+3}{4b\left(b^2-a^2\right)^2(a+b\cos(c+dx))}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),
x]

[Out] ((2*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Cos[c + d*x]^2*(EllipticF[Ar
cSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]
]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a
+ b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16*a^3*A + 8*a*A*b^2 - 24*a^2
```

```
*b*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b
+ a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c +
d*x])*(1 - Cos[c + d*x]^2)) + ((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*Cos[
2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*El
lipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
 d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec
[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[
Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*Ell
ipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
 Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*
x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(16*a*(a - b)^2*(a + b)^2*d
) + (Sqrt[Sec[c + d*x]]*(((-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Sin[c +
d*x])/(4*a*b*(a^2 - b^2)^2) - (a*A*b*Sin[c + d*x] - a^2*B*Sin[c + d*x])/(2*
b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (3*a^2*A*b*Sin[c + d*x] + 3*A*b^3*
Sin[c + d*x] + a^3*B*Sin[c + d*x] - 7*a*b^2*B*Sin[c + d*x])/(4*b*(-a^2 + b^
2)^2*(a + b*Cos[c + d*x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

maple [B] time = 6.47, size = 1850, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(A*b-B*a)/b^2*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-4*B/b/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*(A*b-2*B*a)/b^2*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/

```
a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos(c+d\,x)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.581 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=400

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{\left(3a^3B+a^2Ab\right.}{}$$

```
[Out] -1/2*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+1
/4*(3*A*a^2*b+3*A*b^3+B*a^3-7*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b
^2)^2/d/(b+a*sec(d*x+c))-1/4*(A*a^2*b+5*A*b^3+3*B*a^3-9*B*a*b^2)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d+1/4*(A*a^3*b-7*A*a*b^3
+3*B*a^4-5*B*a^2*b^2+8*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
b^3/(a^2-b^2)^2/d-1/4*(A*a^4*b-10*A*a^2*b^3-3*A*b^5+3*B*a^5-6*B*a^3*b^2+15*
B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2
*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^
3/(a+b)^3/d
```

Rubi [A] time = 0.96, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {2960, 4027, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\left(3a^2Ab+a^3B-7ab^2B+3Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{\left(a^3Ab-5a^2b^2B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((a^3*A*b - 7*a
*A*b^3 + 3*a^4*B - 5*a^2*b^2*B + 8*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d) - ((a^4*A*b - 10*a^
2*A*b^3 - 3*A*b^5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*Sqrt[Cos[c + d*x]]*
EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*
b^3*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 -
b^2)*d*(b + a*Sec[c + d*x])^2) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x])
)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4027

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(d*(A*
b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)
)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) + d
*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x]^
2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && Ne
Q[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}\,(B+A\sec(c+dx))}{(b+a\sec(c+dx))^3}\,dx$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\int \frac{\frac{1}{2}(Ab-aB)+2(aA-bB)\sec(c+dx)-\frac{3}{2}}{\sqrt{\sec(c+dx)}\,(b+a\sec(c}}{2\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\left(3a^2Ab+3Ab^3+a^3B-7ab^2\right.}{4b\left(a^2-b^2\right)^2d(b}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\left(3a^2Ab+3Ab^3+a^3B-7ab^2\right.}{4b\left(a^2-b^2\right)^2d(b}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\left(3a^2Ab+3Ab^3+a^3B-7ab^2\right.}{4b\left(a^2-b^2\right)^2d(b}$$

$$= -\frac{\left(a^4Ab-10a^2Ab^3-3Ab^5+3a^5B-6a^3b^2B+15ab^4B\right)\sqrt{\cos(c+dx)}}{4(a-b)^2b^3(a+b)^3d}$$

$$= -\frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{4b^2\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 6.93, size = 786, normalized size = 1.96

$$\frac{\sqrt{\sec(c+dx)}\left(-\frac{a^3B\sin(c+dx)-a^2Ab\sin(c+dx)}{2b^2\left(b^2-a^2\right)(a+b\cos(c+dx))^2}+\frac{\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2}+\frac{-5a^4B\sin(c+dx)+a^3Ab\sin(c+dx)+11a^2b^2B\sin(}{4b^2\left(b^2-a^2\right)^2(a+b\cos(c+d}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),
x]
```

```
[Out] -1/16*((2*(-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[c + d*x]^2*(Elliptic
F[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c +
d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a
*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(24*a*A*b^2 - 8*a^2*b*B -
```

```
16*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]
*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[
c + d*x])*(1 - Cos[c + d*x]^2)) + ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B
)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*
a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - S
ec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
rt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b
^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sq
rt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[
c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/((a - b)^2*b*(a + b)^
2*d) + (Sqrt[Sec[c + d*x]]*(((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sin[
c + d*x])/(4*b^2*(a^2 - b^2)^2) - (-(a^2*A*b*Sin[c + d*x]) + a^3*B*Sin[c +
d*x])/(2*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (a^3*A*b*Sin[c + d*x] -
 7*a*A*b^3*Sin[c + d*x] - 5*a^4*B*Sin[c + d*x] + 11*a^2*b^2*B*Sin[c + d*x])
/(4*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

maple [B] time = 6.60, size = 1937, normalized size = 4.84

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a
^2*(A*b-B*a)/b^3*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^
2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a
^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/
(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+
2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2)))-4/b^2*(A*b-3*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)
)-2*a/b^3*(2*A*b-3*B*a)*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(
a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*
```

```
b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.582 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=427

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{a\left(-5a^3B+a^2Ab+11ab^2B-7Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{\left(-15a^4B\right.}{}$$

```
[Out] 1/2*a*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b+a*sec(d*x+c))^
2+1/4*a*(A*a^2*b-7*A*b^3-5*B*a^3+11*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^
2/(a^2-b^2)^2/d/(b+a*sec(d*x+c))-1/4*(3*A*a^3*b-9*A*a*b^3-15*B*a^4+29*B*a^2
*b^2-8*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/
d+1/4*(3*A*a^4*b-5*A*a^2*b^3+8*A*b^5-15*B*a^5+33*B*a^3*b^2-24*B*a*b^4)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^4/(a^2-b^2)^2/d-1/4*a*(3*A*a^4*b
-6*A*a^2*b^3+15*A*b^5-15*B*a^5+38*B*a^3*b^2-35*B*a*b^4)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^4/(a+b)^3/d
```

Rubi [A] time = 1.06, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {2960, 4030, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a\left(a^2Ab-5a^3B+11ab^2B-7Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{\left(-5a^2Ab^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

```
[Out] -((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*Sqrt[Cos[c +
d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d)
 + ((3*a^4*A*b - 5*a^2*A*b^3 + 8*A*b^5 - 15*a^5*B + 33*a^3*b^2*B - 24*a*b^4
*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^4
*(a^2 - b^2)^2*d) - (a*(3*a^4*A*b - 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*
a^3*b^2*B - 35*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^4*(a + b)^3*d) + (a*(A*b - a*B
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^
2) + (a*(a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(4*b^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(-aAb+5a^2B-4b^2B\right)-2b(Ab-aB)\sec}{\sqrt{\sec(c+dx)}\,(b+a}}{2b\left(a^2-\right.}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{a\left(a^2Ab-7Ab^3-5a^3B+11ab\right.}{4b^2\left(a^2-b^2\right)^2d(b}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{a\left(a^2Ab-7Ab^3-5a^3B+11ab\right.}{4b^2\left(a^2-b^2\right)^2d(b}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{a\left(a^2Ab-7Ab^3-5a^3B+11ab\right.}{4b^2\left(a^2-b^2\right)^2d(b}$$

$$= -\frac{a\left(3a^4Ab-6a^2Ab^3+15Ab^5-15a^5B+38a^3b^2B-35ab^4B\right)\sqrt{\cos(c+}}{4(a-b)^2b^4(a+b)^3d}$$

$$= -\frac{\left(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{4b^3\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.08, size = 820, normalized size = 1.92

$$\frac{2\left(5Ba^4-Aba^3-7b^2Ba^2-5Ab^3a+8b^4B\right)\left(F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-\Pi\left(-\frac{a}{b};\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\,\sin(c+dx)\cos^2(c+}{a(a+b\cos(c+dx))\left(1-\cos^2(c+dx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),
x]
```

```
[Out] ((2*(-(a^3*A*b) - 5*a*A*b^3 + 5*a^4*B - 7*a^2*b^2*B + 8*b^4*B)*Cos[c + d*x]
^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[S
qrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[
c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(8*a^2*A*b^2 +
 16*A*b^4 + 8*a^3*b*B - 32*a*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSi
```

```
n[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Si
n[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-3*a^3*A*b +
9*a*A*b^3 + 15*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*Cos[2*(c + d*x)]*(b + a*Sec[
c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[
c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b
*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[
c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt
[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c
+ d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]
*(2 - Sec[c + d*x]^2)))/(16*(a - b)^2*b^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]
]*(-1/4*(a*(-3*a^2*A*b + 9*A*b^3 + 7*a^3*B - 13*a*b^2*B)*Sin[c + d*x])/(b^3
*(a^2 - b^2)^2) - (a^3*A*b*Sin[c + d*x] - a^4*B*Sin[c + d*x])/(2*b^3*(-a^2
+ b^2)*(a + b*Cos[c + d*x])^2) + (-5*a^4*A*b*Sin[c + d*x] + 11*a^2*A*b^3*Si
n[c + d*x] + 9*a^5*B*Sin[c + d*x] - 15*a^3*b^2*B*Sin[c + d*x])/(4*b^3*(-a^2
 + b^2)^2*(a + b*Cos[c + d*x]))))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)),
 x)
```

maple [B] time = 7.44, size = 1977, normalized size = 4.63

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^4/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)-2*a^3*(A*b-B*a)/b^4*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+12/b^3*a*(A*b-2*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2/b^4*(3*A*b-4*B*a)*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1

```
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3
*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*
b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.583 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=521

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+b)^2}+\frac{a\left(-7a^3B+3a^2Ab+13ab^2B-9Ab^3\right)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+b)}-\frac{\left(-35a^4B+15\right.}{}$$

```
[Out] -1/12*(15*A*a^3*b-33*A*a*b^3-35*B*a^4+61*B*a^2*b^2-8*B*b^4)*sin(d*x+c)/b^3/
(a^2-b^2)^2/d/sec(d*x+c)^(1/2)+1/2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(b+
a*sec(d*x+c))^2/sec(d*x+c)^(1/2)+1/4*a*(3*A*a^2*b-9*A*b^3-7*B*a^3+13*B*a*b^
2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(b+a*sec(d*x+c))/sec(d*x+c)^(1/2)+1/4*(15*A
*a^4*b-29*A*a^2*b^3+8*A*b^5-35*B*a^5+65*B*a^3*b^2-24*B*a*b^4)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^4/(a^2-b^2)^2/d-1/12*(45*A*a^5*b-99*A*a^3
*b^3+72*A*a*b^5-105*B*a^6+223*B*a^4*b^2-128*B*a^2*b^4-8*B*b^6)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^5/(a^2-b^2)^2/d+1/4*a^2*(15*A*a^4*b-38*A
*a^2*b^3+35*A*b^5-35*B*a^5+86*B*a^3*b^2-63*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^5/(a+b)^3/d
```

Rubi [A] time = 1.53, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2960, 4030, 4100, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a\left(3a^2Ab-7a^3B+13ab^2B-9Ab^3\right)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+b)}+\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+b)^2}-\frac{\left(15a^3Ab+61a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

```
[Out] ((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4*
B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^4*
(a^2 - b^2)^2*d) - ((45*a^5*A*b - 99*a^3*A*b^3 + 72*a*A*b^5 - 105*a^6*B + 2
23*a^4*b^2*B - 128*a^2*b^4*B - 8*b^6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*b^5*(a^2 - b^2)^2*d) + (a^2*(15*a^4*A*b -
 38*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B + 86*a^3*b^2*B - 63*a*b^4*B)*Sqrt[Cos[c
 + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(
a - b)^2*b^5*(a + b)^3*d) - ((15*a^3*A*b - 33*a*A*b^3 - 35*a^4*B + 61*a^2*b
```

```
^2*B - 8*b^4*B)*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]) +
 (a*(A*b - a*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(b + a*
Sec[c + d*x])^2) + (a*(3*a^2*A*b - 9*A*b^3 - 7*a^3*B + 13*a*b^2*B)*Sin[c +
d*x])/(4*b^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x]))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4030

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(b*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*
(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
```

```
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(-3aAb+7a^2B-4b^2B\right.}{\text{s}}}{}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(b+a\sec(c+dx))^2} + \frac{a\left(3a^2Ab-9Ab^3-\right.}{4b^2\left(a^2-b^2\right)^2d\sqrt{\text{se}}}$$

$$= -\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}} + \frac{}{2b\left(a\right.}$$

$$= -\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}} + \frac{}{2b\left(a\right.}$$

$$= -\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}} + \frac{}{2b\left(a\right.}$$

$$= \frac{a^2\left(15a^4Ab-38a^2Ab^3+35Ab^5-35a^5B+86a^3b^2B-63ab^4B\right)\sqrt{\cos(}}{4(a-b)^2b^5(a+b)^3d}$$

$$= \frac{\left(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B\right)\sqrt{\cos(c+}}{4b^4\left(a^2-b^2\right)^2d}$$

Mathematica [A] time = 7.32, size = 865, normalized size = 1.66

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(11Ba^3-7Aba^2-17b^2Ba+13Ab^3\right)\sin(c+dx)a^2}{4b^4\left(a^2-b^2\right)^2} - \frac{a^5B\sin(c+dx)-a^4Ab\sin(c+dx)}{2b^4\left(b^2-a^2\right)(a+b\cos(c+dx))^2} + \frac{-13B\sin(c+dx)a^6+9Ab\sin(c+dx)a^5+19b^2}{4b^4\left(b^2-a^2\right)^2(a+b\,\text{c}}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),
x]
```

```
[Out] -1/48*((2*(-15*a^4*A*b + 21*a^2*A*b^3 - 24*A*b^5 + 35*a^5*B - 73*a^3*b^2*B
+ 56*a*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - E
llipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqr
t[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*
x]^2)) + (2*(-24*a^3*A*b^2 + 96*a*A*b^4 + 56*a^4*b*B - 112*a^2*b^3*B - 16*b
^5*B)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b
+ a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c +
d*x])*(1 - Cos[c + d*x]^2)) + ((-45*a^4*A*b + 87*a^2*A*b^3 - 24*A*b^5 + 105
*a^5*B - 195*a^3*b^2*B + 72*a*b^4*B)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*
(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF
[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d
*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a
*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[
c + d*x]^2)))/((a - b)^2*b^3*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*((a^2*(-7*a
^2*A*b + 13*A*b^3 + 11*a^3*B - 17*a*b^2*B)*Sin[c + d*x])/(4*b^4*(a^2 - b^2)
^2) - (-(a^4*A*b*Sin[c + d*x]) + a^5*B*Sin[c + d*x])/(2*b^4*(-a^2 + b^2)*(a
 + b*Cos[c + d*x])^2) + (9*a^5*A*b*Sin[c + d*x] - 15*a^3*A*b^3*Sin[c + d*x]
 - 13*a^6*B*Sin[c + d*x] + 19*a^4*b^2*B*Sin[c + d*x])/(4*b^4*(-a^2 + b^2)^2
*(a + b*Cos[c + d*x])) + (B*Sin[2*(c + d*x)])/(3*b^3)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)),
 x)
```

maple [B] time = 8.18, size = 2195, normalized size = 4.21

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3/b^5/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*b^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4+9*A*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*b^2-18*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b^2*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+2*B*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^2)-8*a^2/b^4*(3*A*b-5*B*a)/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^4*(A*b
-B*a)/b^5*(-1/2*b^2/a/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)^2-3/4*b^2*(3*a^
2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*b+a-b)-7/8/(a+b)/(a^2-b^2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
```

```
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^
2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*
b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
-2*b/(a-b),2^(1/2)))-2/b^5*a^3*(4*A*b-5*B*a)*(-b^2/a/(a^2-b^2)*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2*b+a-b)-1/2/(a+b)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/a/(a^2-b^2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2
)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3), x
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.584 $\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{2B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] 2/3*B*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 3768, 3771, 2641}

$$\frac{2B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = B\int \sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}B\int \sqrt{\sec(c+dx)}\,dx$$
$$= \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2B\sec^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{3d}$$

Mathematica [A] time = 0.07, size = 47, normalized size = 0.73

$$\frac{2B\sec^{\frac{3}{2}}(c+dx)\left(\sin(c+dx)+\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*Sec[c + d*x]^(3/2)*(Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + Sin
[c + d*x]))/(3*d)
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(B\sec\left(dx+c\right)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B*sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c) + Ba)\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a),
 x)
```

maple [B] time = 1.30, size = 214, normalized size = 3.34

$$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*B*((2*cos(1/2*d*x+1/2*c)^2-
1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c) + Ba)\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/cos(c + d*x))^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),
x)
```

```
[Out] int(((1/cos(c + d*x))^(5/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.585 $\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*B*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 3768, 3771, 2639}

$$\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = B\int \sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} - B\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} - \left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{}$$
$$= -\frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2B\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.05, size = 46, normalized size = 0.77

$$\frac{2B\sqrt{\sec(c+dx)}\left(\sin(c+dx)-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*Sqrt[Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
Sin[c + d*x]))/d
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(B\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B*sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a),
 x)
```

maple [A] time = 1.51, size = 102, normalized size = 1.70

$$-\frac{2B\left(\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*B*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(B\,a+B\,b\,\cos(c+d\,x))}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/cos(c + d*x))^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),
x)
```

```
[Out] int(((1/cos(c + d*x))^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

3.586 $\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 3771, 2641}

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{(aB + bB\cos(c+dx))\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = B\int \sqrt{\sec(c+dx)}\,dx$$
$$= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 1.00

$$\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]
),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

fricas [F] time = 1.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(B\sqrt{\sec\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] integral(B*sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bb\cos\left(dx+c\right) + Ba\right)\sqrt{\sec\left(dx+c\right)}}{b\cos\left(dx+c\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a),
 x)
```

maple [B] time = 1.07, size = 134, normalized size = 3.62

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\sec(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}\ (B\,a+B\,b\,\cos(c+d\,x))}{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/cos(c + d*x))^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),
x)
```

```
[Out] int(((1/cos(c + d*x))^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int \sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
[Out] B*Integral(sqrt(sec(c + d*x)), x)
```

3.587 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=37

$$\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {21, 3771, 2639}

$$\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = B\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.04, size = 37, normalized size = 1.00

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(B/sqrt(sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

maple [B] time = 0.87, size = 134, normalized size = 3.62

$$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\operatorname{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{\right.}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{B\,a+B\,b\,\cos(c+d\,x)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+b\,\cos(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))),
x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{1}{\sqrt{\sec \left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(1/2),x)
[Out] B*Integral(1/sqrt(sec(c + d*x)), x)
```

3.588 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] 2/3*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 3769, 3771, 2641}

$$\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx &= B\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}B\int \sqrt{\sec(c+dx)}\,dx\\ &= \frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\\ &= \frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.78

$$\frac{B\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))+2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] (B*Sqrt[Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin
[2*(c + d*x)]))/(3*d)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{B}{\sec\left(dx+c\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(B/sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos{(dx+c)} + Ba}{(b\cos{(dx+c)} + a)\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

maple [B] time = 1.43, size = 180, normalized size = 2.81

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(c\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos{(dx+c)} + Ba}{(b\cos{(dx+c)} + a)\sec{(dx+c)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))),
x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{1}{\sec^{\frac{3}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] B*Integral(sec(c + d*x)**(-3/2), x)
```

3.589 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

```
[Out] 2/5*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)+6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 3769, 3771, 2639}

$$\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = B\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3B)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(3B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}$$
$$= \frac{6B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.07, size = 56, normalized size = 0.88

$$\frac{B\sqrt{\sec(c+dx)}\left(\sin(c+dx)+\sin(3(c+dx))+12\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)
),x]
```

```
[Out] (B*Sqrt[Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

fricas [F] time = 1.37, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(B/sec(d*x + c)^(5/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c) + Ba}{(b\cos(dx+c) + a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)
), x)
```

maple [B] time = 1.22, size = 203, normalized size = 3.17

$$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,B\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\;\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(-8*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c) + Ba}{(b\cos(dx+c) + a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,\left(a+b\,\cos\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))),
x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.590 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=473

$$\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d}+\frac{2(a-b)\sqrt{a+b}\,\left(a^2(25A-63B)+2ab(3\right.}{}$$

```
[Out] 2/105*(25*A*a^2-4*A*b^2+7*B*a*b)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c
))^(1/2)/a^2/d+2/35*(A*b+7*B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c)
)^(1/2)/a/d+2/7*A*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/10
5*(a-b)*(19*A*a^2*b+8*A*b^3+63*B*a^3-14*B*a*b^2)*csc(d*x+c)*EllipticE((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^
(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-
b))^(1/2)/a^4/d/sec(d*x+c)^(1/2)+2/105*(a-b)*(8*A*b^2+a^2*(25*A-63*B)+2*a*b
*(3*A-7*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x
+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.44, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2999, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d}+\frac{2(a-b)\sqrt{a+b}\,\left(a^2(25A-63B)+2ab(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^4*d*Sqrt[Sec[c + d*
x]]) + (2*(a - b)*Sqrt[a + b]*(8*A*b^2 + a^2*(25*A - 63*B) + 2*a*b*(3*A - 7
*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[
Sec[c + d*x]]) + (2*(25*a^2*A - 4*A*b^2 + 7*a*b*B)*Sqrt[a + b*Cos[c + d*x]]
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a^2*d) + (2*(A*b + 7*a*B)*Sqrt[a + b
*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*a*d) + (2*A*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
```

```
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+d}}{\cos^{\frac{9}{2}}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} +$$

$$= \frac{2(Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(}{35ad}$$

$$= \frac{2\left(25a^2A-4Ab^2+7abB\right)\sqrt{a+b\cos(c+dx)}\,\sec}{105a^2d}$$

$$= \frac{2\left(25a^2A-4Ab^2+7abB\right)\sqrt{a+b\cos(c+dx)}\,\sec}{105a^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(19a^2Ab+8Ab^3+63a^3B-14ab^2\right.}{}$$

Mathematica [B] time = 23.76, size = 3321, normalized size = 7.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),
x]

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(19*a^2*A*b + 8*A*b^3 + 63
*a^3*B - 14*a*b^2*B)*Sin[c + d*x])/(105*a^3) + (2*Sec[c + d*x]^2*(A*b*Sin[c
 + d*x] + 7*a*B*Sin[c + d*x]))/(35*a) + (2*Sec[c + d*x]*(25*a^2*A*Sin[c + d
*x] - 4*A*b^2*Sin[c + d*x] + 7*a*b*B*Sin[c + d*x]))/(105*a^2) + (2*A*Sec[c
+ d*x]^2*Tan[c + d*x])/7))/d + (2*((-19*A*b)/(105*Sqrt[a + b*Cos[c + d*x]]*
Sqrt[Sec[c + d*x]]) - (8*A*b^3)/(105*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) - (3*a*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b
^2*B)/(15*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a*A*Sqrt[Sec[
c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (17*A*b^2*Sqrt[Sec[c + d*x]])/(1
05*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^4*Sqrt[Sec[c + d*x]])/(105*a^3*Sqrt
[a + b*Cos[c + d*x]]) - (2*b*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d
*x]]) + (2*b^3*B*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a + b*Cos[c + d*x]]) - (1
9*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[a + b*Cos[c + d*x]
```

]) - (8*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a^3*Sqrt[a + b*Cos[
c + d*x]]) - (3*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[
c + d*x]]) + (2*b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*a^2*Sqrt[a +
 b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(19*a^
2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A
+ 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)] - (19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*
Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(10
5*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(19*a^2*A*b + 8*A*b^3 + 6
3*a^3*B - 14*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A
+ 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] - (19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b
*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^3*(a + b*Cos[c
+ d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*
b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)] + 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (19*a^
2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1/2*((1
9*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*
x])*Sec[(c + d*x)/2]^4) - ((a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*
b^2*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 +
 Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])] + (a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*
B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Co
s[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])] - ((a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Co
s[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(8*A*b^2 - 2*a*b*(3*A
+ 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF

```
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos
[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b
*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/
2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*
a*b^2*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)
/2] - (19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(8*A*b^2 - 2
*a*b*(3*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2
)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a +
 b)]) - ((a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])
/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(105*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
(c + d*x)/2]^2]) + ((-2*(a + b)*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2
*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
] + 2*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (19*a^2*A
*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c +
 d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*a^3*Sqrt[a
+ b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]])))
```

fricas [F] time = 1.64, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2),
x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.62, size = 3435, normalized size = 7.26

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105/d*(63*B*cos(d*x+c)^4*a^4-42*B*cos(d*x+c)^3*a^4-21*B*cos(d*x+c)*a^4+2
5*A*cos(d*x+c)^4*a^4-10*A*cos(d*x+c)^2*a^4+8*A*cos(d*x+c)^5*b^4-8*A*cos(d*x
+c)^4*b^4-28*B*cos(d*x+c)^2*a^3*b+25*A*cos(d*x+c)^5*a^3*b+19*A*cos(d*x+c)^5
*a^2*b^2-4*A*cos(d*x+c)^5*a*b^3+19*A*cos(d*x+c)^4*a^3*b-20*A*cos(d*x+c)^4*a
^2*b^2+8*A*cos(d*x+c)^4*a*b^3-26*A*cos(d*x+c)^3*a^3*b-4*A*cos(d*x+c)^3*a*b^
3+A*cos(d*x+c)^2*a^2*b^2-18*A*cos(d*x+c)*a^3*b+63*B*cos(d*x+c)^5*a^3*b+7*B*
cos(d*x+c)^5*a^2*b^2-14*B*cos(d*x+c)^5*a*b^3-35*B*cos(d*x+c)^4*a^3*b-14*B*c
os(d*x+c)^4*a^2*b^2+14*B*cos(d*x+c)^4*a*b^3+7*B*cos(d*x+c)^3*a^2*b^2+8*A*si
n(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b^3-8*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*sin(d*x+c)
*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^4+63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^4-8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*
B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^4+63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-15*A*a^4-63*B*sin(d*x+c)*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+1
4*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
```

b)/(a+b))^(1/2))*a^2*b^2+14*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+49*B*sin(d*x+c)*cos(d*x+c)
^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-14
*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b^2-19*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-19*A*sin(d*x+c)*cos(d*x+c)^
3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8
*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^3+19*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*A*sin(d*x+c)*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8*A*
sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^3-63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+14*B*sin(d*x+c)*cos(d*x+c)^3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+14*B*s
in(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^3+49*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-14*B*sin(d*x+c)*cos(d*x+c)^3*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-19*A*si
n(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^3*b-19*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)^4*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+19*A*sin(
d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^3*b+2*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2)*cos(d*x+c)*(1/cos(d*x+c))^(9/2)/(a

```
+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(9/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(1/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(9/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.591 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=390

$$-\frac{2(a-b)\sqrt{a+b}\,(9aA-5aB+2Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c-}}{\sqrt{a+b}\,\sqrt{\cos(c}}\right.\right.}{15a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/15*(A*b+5*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d+2/5
*A*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/15*(a-b)*(9*A*a^2
-2*A*b^2+5*B*a*b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2
)-2/15*(a-b)*(9*A*a+2*A*b-5*B*a)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/
sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.06, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2999, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\,\left(9a^2A+5abB-2Ab^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,\mathrm{cc}}}{\sqrt{a+b}\,\sqrt{\mathrm{c}}}\right.\right.}{15a^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sqrt[Cos[c + d*x]]*Csc
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*
Sqrt[a + b]*(9*a*A + 2*A*b - 5*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(A*b + 5*a*B)*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d) + (2*A*Sqrt[a + b*C
os[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} +$$

$$= \frac{2(Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c}{15ad}$$

$$= \frac{2(Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c}{15ad}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2A-2Ab^2+5abB\right)\sqrt{\cos(c+}}{}$$

Mathematica [A] time = 17.47, size = 423, normalized size = 1.08

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2(9a^2A+5abB-2Ab^2)\sin(c+dx)}{15a^2}+\frac{2\sec(c+dx)(5aB\sin(c+dx)+Ab\sin(c+dx))}{15a}+\frac{2}{5}A\tan(c+dx)\sec(c+dx)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),
x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(9*a^2*A - 2*A*b^2 + 5
*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)] + 2*a*(a + b)*(9*a*A - 2*A*b + 5*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2*A - 2*A*b^2 + 5*a*b*B)*C
os[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*
a^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sin[c + d*x]
)/(15*a^2) + (2*Sec[c + d*x]*(A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x]))/(15*a
) + (2*A*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2),
x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.46, size = 2489, normalized size = 6.38

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d*(3*A*a^3-9*A*cos(d*x+c)^3*a^3-2*A*cos(d*x+c)^3*b^3+6*A*cos(d*x+c)^2*
a^3-5*B*cos(d*x+c)^3*a^3-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*cos(d*x+c)^4*b^3+5*B*co
s(d*x+c)*a^3+2*A*cos(d*x+c)^3*a*b^2-A*cos(d*x+c)^2*a*b^2+4*A*cos(d*x+c)*a^2
*b-5*B*cos(d*x+c)^4*a*b^2-5*B*cos(d*x+c)^3*a^2*b+10*B*cos(d*x+c)^2*a^2*b+9*
A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-7*A*sin(d*x+c)*cos(d*x+c)^2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(
d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b^2+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-5*B*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d
*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^2*b+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2+9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*
cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
^2-7*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+5*B*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*
x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+5*B
*cos(d*x+c)^3*a*b^2-9*A*cos(d*x+c)^4*a^2*b-A*cos(d*x+c)^4*a*b^2+5*A*cos(d*x
```

```
+c)^3*a^2*b-5*B*cos(d*x+c)^4*a^2*b-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)
*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*b^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-5*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^
3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+9*A*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3-2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^3-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3)*cos(d*x+c)*(1/cos(d*x
+c))^(7/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(1/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)*(a+b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

3.592 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=324

$$\frac{2(a-b)\sqrt{a+b}\,(3aB+Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{3a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*A*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*(a-b)*(A*b+3
*B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a
+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/3*(a-b)*
(A-3*B)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2999, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\,(3aB+Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{3a^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(A -
3*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[
c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(3*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(Ab+3aB)\sqrt{\cos(c+dx)}\,\csc(c+dx}{3a^2}$$

Mathematica [A] time = 14.45, size = 346, normalized size = 1.07

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2(3aB+Ab)\sin(c+dx)}{3a}+\frac{2}{3}A\tan(c+dx)\right)}{d} + \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-\left((3a\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),
x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(A*b + 3*a*B)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a +
 b)*(A + 3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] - (A*b + 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2]))/(3*a*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)
/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(A*b + 3*a*B)*Si
n[c + d*x])/(3*a) + (2*A*Tan[c + d*x])/3))/d
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2),
x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.40, size = 1735, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(3*B*cos(d*x+c)^2*a^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*a*b-3*B*cos(d*x+c)*a^2+A*cos(
d*x+c)^3*b^2-A*cos(d*x+c)^2*b^2-a^2*A-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+A*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^2
-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)^2*b^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*a^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+A*sin(d*x+c)*
cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
a^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*cos(d*x+c)*a^2+A*cos(d*x+c)^2*a^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+3*B*sin(d*x+c)
```

```
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
*a*b-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+A*cos(d*x+c)^3*a*b-3*B*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*
a*b+A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*a*b+A*cos(d*x+c)^2*a*b-2*A*cos(d*x+c)*a*b+3*B*cos(d*x+c)
^3*a*b-3*B*cos(d*x+c)^2*a*b-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2)*cos(d*x+c)
*(1/cos(d*x+c))^(5/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.593 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=411

$$\frac{2\sqrt{a+b}\,(Ab-a(A-B))\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2*A*(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+2*(A*b-a
*(A-B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-2*B*csc(d*
x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b
,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b)
)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2991, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\,(Ab-a(A-B))\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(A*b - a*(A - B))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (2*Sq
rt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(d*Sqrt[Sec[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2991

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)]])/((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(B*d
)/b^2, Int[Sqrt[b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Int[(A*c
 + (B*c + A*d)*Sin[e + f*x])/((b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(}$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{aA+(Ab+aB)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+}}$$

$$= -\frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\right(}{d\sqrt{se}}$$

$$= \frac{2A(a-b)\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\right.}{ad\sqrt{se}}$$

Mathematica [A] time = 17.28, size = 635, normalized size = 1.55

$$\frac{2\left(-(a(A+B)+b(A-B))\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)+a-b\tan^2\left(\frac{1}{2}(c+dx)\right)+b}{a+b}}\,F\left(\sin\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),
x]
```

```
[Out] (2*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*(a*A*
Tan[(c + d*x)/2] + A*b*Tan[(c + d*x)/2] - 2*A*b*Tan[(c + d*x)/2]^3 - a*A*Ta
n[(c + d*x)/2]^5 + A*b*Tan[(c + d*x)/2]^5 - 2*b*B*EllipticPi[-1, ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
```

```
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b*B*EllipticPi[
-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)] + A*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (b*(A - B) + a*(A + B
))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*
Tan[(c + d*x)/2]^2)/(a + b)]))/(d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 +
 Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c
+ d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2),
x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.46, size = 1361, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
```

```
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+B*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*a-B*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*b+2*B*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+A*sin(d*x+c)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-B*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+2*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)
+A*cos(d*x+c)^2*b+A*cos(d*x+c)*a-A*cos(d*x+c)*b-a*A)*cos(d*x+c)*(1/cos(d*x+
c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.594 $\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=445

$$\frac{\sqrt{a+b}\,(2A+B)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}-\sqrt{a+$$

```
[Out] B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-(a-b)*B*csc(d*x+c)*E
llipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+(2*A+B)*csc(d*x+c)*EllipticF((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-(2*A*b+B*a)*csc(d*x+c)*EllipticPi((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(
a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2961, 3003, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\,(2A+B)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}-\sqrt{a+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*(2*A + B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*
A*b + a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(b*d*Sqrt[Sec[c + d*x]]) + (B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/d
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx}}{\sqrt{\cos}}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}$$

$$= -\frac{\sqrt{a+b}\,(2Ab+aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a-}{b}\right.}{b}$$

$$= -\frac{(a-b)\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\right.}{ad\sqrt{s}}$$

Mathematica [A] time = 17.55, size = 787, normalized size = 1.77

$$2(a(B-A)+Ab)\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)+a-b\tan^2\left(\frac{1}{2}(c+dx)\right)+b}{a+b}}F\left(\sin^{-1}\left(\tan\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (-(a*B*Tan[(c + d*x)/2]) - b*B*Tan[(c + d*x)/2] + 2*b*B*Tan[(c + d*x)/2]^3
+ a*B*Tan[(c + d*x)/2]^5 - b*B*Tan[(c + d*x)/2]^5 - 4*A*b*EllipticPi[-1, Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*B*Ell
ipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b
)] - 4*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(
c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*B*EllipticPi[-1, ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a +
 b)*B*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*(A*b + a*(-A + B))*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
+ d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
+ b)])/(d*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]
*(-1 + Tan[(c + d*x)/2]^4))
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

maple [B] time = 0.49, size = 1369, normalized size = 3.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(2*A*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-2*A*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*b+4*A*sin(d*x+c)*cos(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*b-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+2*B*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+B*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*sin(d*x+c)-2*A*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+4*A*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b-2*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+2*B*sin(d*x+c)*EllipticP
```

```
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a+B*sin(d*x+c)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+B*sin(d*x+
c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+B*cos(
d*x+c)^3*b+B*cos(d*x+c)^2*a-b*B*cos(d*x+c)^2-B*cos(d*x+c)*a)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{a+b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x)),
x)
```

3.595 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=533

$$-\frac{\sqrt{a+b}\,\left(a^2(-B)+4aAb+4b^2B\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\,\sqrt{\cos}}\right.\right.}{4b^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 1/2*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*(4*A*b+B*a)*
sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b/d-1/4*(a-b)*(4*A*b+B*a
)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/sec(d*x+c)^(1/2)+1/4*(4*A*b+(a+
2*b)*B)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)-1/4*(4*A*a
*b-B*a^2+4*B*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)
*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*
x+c)^(1/2)
```

Rubi [A] time = 1.27, antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 3003, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\,\left(a^2(-B)+4aAb+4b^2B\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\,\sqrt{\cos}}\right.\right.}{4b^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(4*a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(4*A*b + (a + 2*
b)*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec
[c + d*x]]) - (Sqrt[a + b]*(4*a*A*b - a^2*B + 4*b^2*B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*Sqrt[Sec[c + d*x]])
```

```
+ (B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + ((4*
A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*d
)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+a$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+a$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4Ab+aB)\sqrt{a+b\cos(c+}{4$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4Ab+aB)\sqrt{a+b\cos(c+}{4$$

$$= -\frac{\sqrt{a+b}\,\left(4aAb-a^2B+4b^2B\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+}{b}\right.}{4b^2d\sqrt{se}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(4Ab+aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\right.\right.}{4abd\sqrt{\sec(c-}$$

Mathematica [B] time = 18.62, size = 1121, normalized size = 2.10

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (S
qrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(4*a*A*b*Tan[(c + d*x)/2] + 4*A*b^2*Tan[
(c + d*x)/2] + a^2*B*Tan[(c + d*x)/2] + a*b*B*Tan[(c + d*x)/2] - 8*A*b^2*Ta
n[(c + d*x)/2]^3 - 2*a*b*B*Tan[(c + d*x)/2]^3 - 4*a*A*b*Tan[(c + d*x)/2]^5
+ 4*A*b^2*Tan[(c + d*x)/2]^5 - a^2*B*Tan[(c + d*x)/2]^5 + a*b*B*Tan[(c + d*
x)/2]^5 + 8*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(a + b)] - 2*a^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x
)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*B*EllipticPi[-1, ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*a*A*b*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
```

```
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] - 2*a^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*B*EllipticPi[-1,
 ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2
)/(a + b)] + (a + b)*(4*A*b + a*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b*(4*a*A -
 a*B + 2*b*B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x
)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*b*d*(1 + Tan[(c + d*x)/2]^2)^(
3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c
 + d*x)/2]^2)])
```

fricas [F] time = 2.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

maple [B] time = 0.40, size = 2054, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*b^2+B*cos(d*x+c)^2*a^2+2*B*cos(d*x+c)^4*b^2-2*B*cos(d*x+c)^2*b^
2-B*cos(d*x+c)*a^2+4*A*cos(d*x+c)^3*b^2-4*A*cos(d*x+c)^2*b^2+8*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+4*A*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-8*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)*a*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*a^2+4*A*cos(d*x+c)^2*a*b-4*A*cos(d*x+c)*a*b+3*B*cos(d*x+c)^
3*a*b-B*cos(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2+8*A*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
a*b+4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*a*b-8*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*a*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+8*B*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2
+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)*a^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2+B*sin(d*x+c)*(cos(d*x+c)/(
```

```
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2)*(1/cos(d*x+
c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/sqrt(sec(c + d*x)),
x)
```

3.596 $\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=620

$$\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{24b^2d}-\frac{(a-b)\sqrt{a+b}\,\left(-3a^2B+6aAb+16b^2B\right.}{}$$

```
[Out] 1/3*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1/4*(2*A*b-B*a
)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/24*(6*A*a*b-3*B*
a^2+16*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b^2/d-1/24
*(a-b)*(6*A*a*b-3*B*a^2+16*B*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c
)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2
/d/sec(d*x+c)^(1/2)+1/24*(a+2*b)*(6*A*b-3*B*a+8*B*b)*csc(d*x+c)*EllipticF((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)+1/8*(2*A*a^2*b-8*A*b^3-B*a^3-4*B*a*b^2
)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c
))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.74, antiderivative size = 620, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(-3a^2B+6aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{24b^2d}-\frac{(a-b)\sqrt{a+b}\,\left(-3a^2B+6aAb+16b^2B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(6*a*A*b - 3*a^2*B + 16*b^2*B)*Sqrt[Cos[c + d*x]]*Csc
[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a +
 b]*(a + 2*b)*(6*A*b - 3*a*B + 8*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(24*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(2*a^2*A*b -
8*A*b^3 - a^3*B - 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a
```

```
+ b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(8*b^3*d*Sqrt[Sec[c + d*x]]) + ((2*A*b - a*B)*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Sec[c + d*x]]) + (B*(a + b*Cos[c
+ d*x])^(3/2)*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]]) + ((6*a*A*b - 3*a^2*
B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24
*b^2*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
```

```
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}\,(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}$$

$$= \frac{B(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d}\right.}{}$$

$$= \frac{(2Ab-aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{B(a+b\cos(c+dx}{3bd\sqrt{\sec(}}$$

$$= \frac{(2Ab-aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{B(a+b\cos(c+dx}{3bd\sqrt{\sec(}}$$

$$= \frac{(2Ab-aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{B(a+b\cos(c+dx}{3bd\sqrt{\sec(}}$$

$$= \frac{\sqrt{a+b}\,\left(2a^2Ab-8Ab^3-a^3B-4ab^2B\right)\sqrt{\cos(c+dx)}\,\csc(c+dx}{8b^3d\sqrt{}}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\left(6aAb-3a^2B+16b^2B\right)\sqrt{\cos(c+dx)}\,\csc(c+dx}{24ab^2d\sqrt{}}$$

Mathematica [B] time = 14.65, size = 1533, normalized size = 2.47

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((B*Sin[c + d*x])/12 + ((6*A*b
 + a*B)*Sin[2*(c + d*x)])/(24*b) + (B*Sin[3*(c + d*x)])/12))/d + (Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]
*(6*a^2*A*b*Tan[(c + d*x)/2] + 6*a*A*b^2*Tan[(c + d*x)/2] - 3*a^3*B*Tan[(c
+ d*x)/2] - 3*a^2*b*B*Tan[(c + d*x)/2] + 16*a*b^2*B*Tan[(c + d*x)/2] + 16*b
^3*B*Tan[(c + d*x)/2] - 12*a*A*b^2*Tan[(c + d*x)/2]^3 + 6*a^2*b*B*Tan[(c +
d*x)/2]^3 - 32*b^3*B*Tan[(c + d*x)/2]^3 - 6*a^2*A*b*Tan[(c + d*x)/2]^5 + 6*
a*A*b^2*Tan[(c + d*x)/2]^5 + 3*a^3*B*Tan[(c + d*x)/2]^5 - 3*a^2*b*B*Tan[(c
+ d*x)/2]^5 - 16*a*b^2*B*Tan[(c + d*x)/2]^5 + 16*b^3*B*Tan[(c + d*x)/2]^5 -
 12*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(a + b)] + 48*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^
2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*B*EllipticPi[-1, ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a*b^2*B*EllipticPi[
-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*
a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c +
 d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*A*b^3*EllipticPi[-1, ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^
3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x
)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*
Tan[(c + d*x)/2]^2)/(a + b)] + 24*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b
)*(-6*a*A*b + 3*a^2*B - 16*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b*(-12*A*b^
2 + 2*a*b*(3*A - 7*B) + a^2*B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*b^2*d*(-1 +
Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]
*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.47, size = 2956, normalized size = 4.77

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/24/d*(12*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b-6*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-6*A*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2
-2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b+28*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-24*B*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+3*B*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b-12*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-12*A*cos(d*x+c)
^4*b^3+12*A*cos(d*x+c)^2*b^3-8*B*cos(d*x+c)^5*b^3-8*B*cos(d*x+c)^3*b^3+3*B*
cos(d*x+c)^2*a^3+16*B*cos(d*x+c)^2*b^3-3*B*cos(d*x+c)*a^3-18*A*cos(d*x+c)^3
```

```
*a*b^2-6*A*cos(d*x+c)^2*a^2*b+6*A*cos(d*x+c)^2*a*b^2+6*A*cos(d*x+c)*a^2*b+1
2*A*cos(d*x+c)*a*b^2-10*B*cos(d*x+c)^4*a*b^2+B*cos(d*x+c)^3*a^2*b-3*B*cos(d
*x+c)^2*a^2*b-6*B*cos(d*x+c)^2*a*b^2+2*B*cos(d*x+c)*a^2*b+16*B*cos(d*x+c)*a
*b^2-48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*b^3-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+3*B*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-16*B
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*b^3-12*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^2+12*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b-6*A*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-6*A*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-2*B*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+28*
B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^2-24*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*a*b^2+3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*A*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+24*A*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-48
*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^
(1/2))*b^3-6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*a^3+3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^3-16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/si
```

```
n(d*x+c)/(a+b*cos(d*x+c))^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{a+b\,\cos(c+d\,x)}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(a + b*cos(c + d*x))/sec(c + d*x)**(3/2),
 x)
```

3.597 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=562

$$\frac{2\left(49a^2A+72abB+3Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad}+\frac{2\left(75a^3B+88a^2Ab+9ab^2B-4Ab^3\right)}{3}$$

```
[Out] 2/315*(88*A*a^2*b-4*A*b^3+75*B*a^3+9*B*a*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(
a+b*cos(d*x+c))^(1/2)/a^2/d+2/315*(49*A*a^2+3*A*b^2+72*B*a*b)*sec(d*x+c)^(5
/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d+2/63*(10*A*b+9*B*a)*sec(d*x+c)^(7
/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/9*a*A*sec(d*x+c)^(9/2)*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/d+2/315*(a-b)*(147*A*a^4+33*A*a^2*b^2+8*A*b^4+246*B
*a^3*b-18*B*a*b^3)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/sec(d*x+c)^(1/
2)+2/315*(a-b)*(8*A*b^3-a^3*(147*A-75*B)+3*a^2*b*(13*A-57*B)+6*a*b^2*(A-3*B
))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b)
)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 2.08, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2989, 3055, 2998, 2816, 2994}

$$\frac{2\left(49a^2A+72abB+3Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad}+\frac{2\left(88a^2Ab+75a^3B+9ab^2B-4Ab^3\right)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B -
18*a*b^3*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^4*
d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(8*A*b^3 - a^3*(147*A - 75*B
) + 3*a^2*b*(13*A - 57*B) + 6*a*b^2*(A - 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(315*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(88*a^2*A*b -
 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2
```

```
)*Sin[c + d*x])/(315*a^2*d) + (2*(49*a^2*A + 3*A*b^2 + 72*a*b*B)*Sqrt[a + b
*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*a*d) + (2*(10*A*b + 9*
a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*
a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
&& PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx}{\cos}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d}$$

$$= \frac{2(10Ab+9aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,\mathrm{si}}{63d}$$

$$= \frac{2\left(49a^2A+3Ab^2+72abB\right)\sqrt{a+b\cos(c+dx)}\,\mathrm{se}}{315ad}$$

$$= \frac{2\left(88a^2Ab-4Ab^3+75a^3B+9ab^2B\right)\sqrt{a+b\cos(}}{315a^2d}$$

$$= \frac{2\left(88a^2Ab-4Ab^3+75a^3B+9ab^2B\right)\sqrt{a+b\cos(}}{315a^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(147a^4A+33a^2Ab^2+8Ab^4+24\right.}{}$$

Mathematica [**B**] time = 25.99, size = 3739, normalized size = 6.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4*A + 33*a^2*A*b^2
+ 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sin[c + d*x])/(315*a^3) + (2*Sec[c +
d*x]^3*(10*A*b*Sin[c + d*x] + 9*a*B*Sin[c + d*x]))/63 + (2*Sec[c + d*x]^2*(
49*a^2*A*Sin[c + d*x] + 3*A*b^2*Sin[c + d*x] + 72*a*b*B*Sin[c + d*x]))/(315
*a) + (2*Sec[c + d*x]*(88*a^2*A*b*Sin[c + d*x] - 4*A*b^3*Sin[c + d*x] + 75*
a^3*B*Sin[c + d*x] + 9*a*b^2*B*Sin[c + d*x]))/(315*a^2) + (2*a*A*Sec[c + d*
x]^3*Tan[c + d*x])/9))/d + (2*((-7*a^2*A)/(15*Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) - (11*A*b^2)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (8*A*b^4)/(315*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (82
```

```
*a*b*B)/(105*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b^3*B)/(35*a
*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (13*a*A*b*Sqrt[Sec[c + d*x]
])/(105*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^3*Sqrt[Sec[c + d*x]])/(315*a*Sq
rt[a + b*Cos[c + d*x]]) - (8*A*b^5*Sqrt[Sec[c + d*x]])/(315*a^3*Sqrt[a + b*
Cos[c + d*x]]) + (5*a^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]])
 - (31*b^2*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) + (2*b^4*B*
Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos[c + d*x]]) - (7*a*A*b*Cos[2*(c +
 d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (11*A*b^3*Cos[2*
(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^5*
Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*a^3*Sqrt[a + b*Cos[c + d*x]]) - (
82*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]
) + (2*b^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos[c
+ d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(147*a^4*A + 33
*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 - 6*a
*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (147*a^4*A +
33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[
c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^3*d*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*Sin[c + d*x]*(-2*(a + b)*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B
 - 18*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*
B) + 3*a^2*b*(13*A + 57*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)] - (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b
*B - 18*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]))/(315*a^3*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2])
 - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(147
*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*
A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B))*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (1
47*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Cos[c + d*x]*
(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^3*Sqrt[a
+ b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Se
c[c + d*x]]*(-1/2*((147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a
*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4) - ((a + b)*(1
47*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2
```

```
]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - S
in[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a
*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A
 + 57*B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(
1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])] - ((a + b)*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3
*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]
)^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)
*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B)
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a
+ b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a +
b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(147*a^4*A + 33*a^2*A*b^2
 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[
c + d*x]*Tan[(c + d*x)/2] + (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b
*B - 18*a*b^3*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(
c + d*x)/2] - (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*
B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2
+ (a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(
13*A + 57*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(147*a^
4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1
 - Tan[(c + d*x)/2]^2]))/(315*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*
x)/2]^2]) + ((-2*(a + b)*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B
- 18*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B
) + 3*a^2*b*(13*A + 57*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)] - (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*
B - 18*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d
*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(315*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

fricas [F] time = 2.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{11}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^(11/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.72, size = 4400, normalized size = 7.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

```
[Out] 2/315/d*(-33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3-8*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)
^4*a*b^4+246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^4*a^4*b+246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b^2-18*B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^4*a^2*b^3-18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^4-246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^4*b-153*B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
```

ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+
c)^4*a^3*b^2+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3+147*A*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b+33*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^5*a^3*b^2+33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^2*b^3+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a*b^4-186*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)^5*a^4*b-33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^3*b^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^2*b^3-8*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^5*a*b^4+246*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b+246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^3*b^2-18*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)^5*a^2*b^3-18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a*b^4-246*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^4*b-153*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^5*a^3*b^2+18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^2*b^3+147*A*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^4*b-186*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)^4*a^4*b+117*B*cos(d*x+c)^2*a^4*b-75*B*cos(d*x+c)^6*a^4*b-246*B*co
s(d*x+c)^6*a^3*b^2-9*B*cos(d*x+c)^6*a^2*b^3+18*B*cos(d*x+c)^6*a*b^4-246*B*c

```
os(d*x+c)^5*a^4*b+165*B*cos(d*x+c)^5*a^3*b^2+18*B*cos(d*x+c)^5*a^2*b^3-18*B
*cos(d*x+c)^5*a*b^4+204*B*cos(d*x+c)^4*a^4*b+35*A*a^5-147*A*cos(d*x+c)^6*a^
4*b-88*A*cos(d*x+c)^6*a^3*b^2-33*A*cos(d*x+c)^6*a^2*b^3+4*A*cos(d*x+c)^6*a*
b^4+10*A*cos(d*x+c)^5*a^4*b-33*A*cos(d*x+c)^5*a^3*b^2+34*A*cos(d*x+c)^5*a^2
*b^3-8*A*cos(d*x+c)^5*a*b^4+68*A*cos(d*x+c)^4*a^3*b^2+4*A*cos(d*x+c)^4*a*b^
4+52*A*cos(d*x+c)^3*a^4*b-A*cos(d*x+c)^3*a^2*b^3+53*A*cos(d*x+c)^2*a^3*b^2+
85*A*cos(d*x+c)*a^4*b-9*B*cos(d*x+c)^4*a^2*b^3+81*B*cos(d*x+c)^3*a^3*b^2-8*
A*cos(d*x+c)^6*b^5-147*A*cos(d*x+c)^5*a^5+8*A*cos(d*x+c)^5*b^5+98*A*cos(d*x
+c)^4*a^5+14*A*cos(d*x+c)^2*a^5-75*B*cos(d*x+c)^5*a^5+30*B*cos(d*x+c)^3*a^5
+45*B*cos(d*x+c)*a^5+147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5+8*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*b^5-147*A*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*a^5-75*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5+147*A*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5+8*A*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
b^5-147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)^4*a^5-75*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^5+33*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b^2+
33*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)^4*a^2*b^3+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^4)*cos(d*x+c)/(a+b*cos(d*x+c
))^(1/2)*(1/cos(d*x+c))^(11/2)/sin(d*x+c)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(11/
2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B\cos(c + d\,x)) \left(\frac{1}{\cos(c + d\,x)}\right)^{11/2} (a + b\cos(c + d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(3/2)
,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(3/2)
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

3.598 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=473

$$\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad}-\frac{2(a-b)\sqrt{a+b}\,\left(-\left(a^2(25A-63B)\right)+\right.}{}$$

```
[Out] 2/105*(25*A*a^2+3*A*b^2+42*B*a*b)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+
c))^(1/2)/a/d+2/35*(8*A*b+7*B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c
))^(1/2)/d+2/7*a*A*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/1
05*(a-b)*(82*A*a^2*b-6*A*b^3+63*B*a^3+21*B*a*b^2)*csc(d*x+c)*EllipticE((a+b
*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)
^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a
-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)-2/105*(a-b)*(6*A*b^2-a^2*(25*A-63*B)+3*a*
b*(19*A-7*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d
*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.53, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2989, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2A+42abB+3Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad}-\frac{2(a-b)\sqrt{a+b}\,\left(a^2(-(25A-63B))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[
Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[Sec[c + d*
x]]) - (2*(a - b)*Sqrt[a + b]*(6*A*b^2 - a^2*(25*A - 63*B) + 3*a*b*(19*A -
7*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d*Sqrt
[Sec[c + d*x]]) + (2*(25*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b*Cos[c + d*x
]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a*d) + (2*(8*A*b + 7*a*B)*Sqrt[a +
 b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a*A*Sqrt[a +
b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
```

```
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx)}{\cos}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} +$$

$$= \frac{2(8Ab+7aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin}{35d}$$

$$= \frac{2\left(25a^2A+3Ab^2+42abB\right)\sqrt{a+b\cos(c+dx)}\,\sec}{105ad}$$

$$= \frac{2\left(25a^2A+3Ab^2+42abB\right)\sqrt{a+b\cos(c+dx)}\,\sec}{105ad}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(82a^2Ab-6Ab^3+63a^3B+21ab^2\right.}{}$$

Mathematica [B] time = 23.90, size = 3318, normalized size = 7.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(-82*a^2*A*b + 6*A*b^3 -
63*a^3*B - 21*a*b^2*B)*Sin[c + d*x])/(105*a^2) + (2*Sec[c + d*x]^2*(8*A*b*S
in[c + d*x] + 7*a*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(25*a^2*A*Sin[c + d
*x] + 3*A*b^2*Sin[c + d*x] + 42*a*b*B*Sin[c + d*x]))/(105*a) + (2*a*A*Sec[c
 + d*x]^2*Tan[c + d*x])/7))/d + (2*((-82*a*A*b)/(105*Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]) + (2*A*b^3)/(35*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) - (3*a^2*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b
^2*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a^2*A*Sqrt[Sec[c
 + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^2*Sqrt[Sec[c + d*x]])/(10
5*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a +
 b*Cos[c + d*x]]) + (a*b*B*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]])
 - (b^3*B*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) - (82*A*b^2*Co
s[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^
```

```
4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*a^2*Sqrt[a + b*Cos[c + d*x]]) -
(3*a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x]])
- (b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]
))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(82*a^2*A*b - 6*A*b^3
+ 63*a^3*B + 21*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(
25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] - (82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(
a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^2*d*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec
[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a
*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)] + 2*a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(25*A + 63*B))*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (82
*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x
])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^2*(a + b*Cos[c + d*x])^(3/2
)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c
 + d*x)/2]*(-2*(a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(
a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (82*a^2*A*b - 6*
A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c +
 d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1/2*((82*a^2*A*b
- 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c
 + d*x)/2]^4) - ((a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d
*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])] + (a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(25*A + 63*B))*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*
x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])] - ((a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*
x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B)
 + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSi
```

```
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d
*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(82*a
^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*S
in[c + d*x]*Tan[(c + d*x)/2] + (82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*
B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] -
(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c +
d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(-6*A*b^2 + 3*a*b*
(19*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(S
qrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]
) - ((a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqr
t[1 - Tan[(c + d*x)/2]^2]))/(105*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c +
 d*x)/2]^2]) + ((-2*(a + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] +
2*a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (82*a^2*A*b
 - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d
*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(105*a^2*Sqrt[a +
b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]])))
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^(9/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.51, size = 3421, normalized size = 7.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(63*B*cos(d*x+c)^4*a^4-42*B*cos(d*x+c)^3*a^4-21*B*cos(d*x+c)*a^4+2
5*A*cos(d*x+c)^4*a^4-10*A*cos(d*x+c)^2*a^4-6*A*cos(d*x+c)^5*b^4+6*A*cos(d*x
+c)^4*b^4-63*B*cos(d*x+c)^2*a^3*b+25*A*cos(d*x+c)^5*a^3*b+82*A*cos(d*x+c)^5
*a^2*b^2+3*A*cos(d*x+c)^5*a*b^3+82*A*cos(d*x+c)^4*a^3*b-55*A*cos(d*x+c)^4*a
^2*b^2-6*A*cos(d*x+c)^4*a*b^3-68*A*cos(d*x+c)^3*a^3*b+3*A*cos(d*x+c)^3*a*b^
3-27*A*cos(d*x+c)^2*a^2*b^2-39*A*cos(d*x+c)*a^3*b+63*B*cos(d*x+c)^5*a^3*b+4
2*B*cos(d*x+c)^5*a^2*b^2+21*B*cos(d*x+c)^5*a*b^3+21*B*cos(d*x+c)^4*a^2*b^2-
21*B*cos(d*x+c)^4*a*b^3-63*B*cos(d*x+c)^3*a^2*b^2-6*A*sin(d*x+c)*cos(d*x+c)
^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+6*
A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*b^4+25*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+63*B*sin(d*x
+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^4+6*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*sin(d*x+c)*cos(d*
x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+
63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^4-15*A*a^4-63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-21*B*sin(d*x+c)*cos(
d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
```

2*b^2-21*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^3+84*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+21*B*sin(d*x+c)*cos(d
x+c)^4(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b^2-82*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3*b-82*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*A*sin(d*x+c)*cos(d
x+c)^3(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
^3+82*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3*b+51*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-6*A*sin(d*x+c)*cos(d*x
+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3
-63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^3*b-21*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-21*B*sin(d*x+c)*cos(d*x+
c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+
84*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3*b+21*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-82*A*sin(d*x+c)*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-8
2*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b^2+6*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+82*A*sin(d*x+c)*cos(d*x+c)^
4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+51*
A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b^2)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(9

```
/2)/sin(d*x+c)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}\,(a+b\,\cos(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.599 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=393

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2A+20abB+3Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{c}}\right.\right.}{15a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/15*(6*A*b+5*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/5
*a*A*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/15*(a-b)*(9*A*a
^2+3*A*b^2+20*B*a*b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2
)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(
1/2)-2/15*(a-b)*(9*A*a-3*A*b-5*B*a+15*B*b)*csc(d*x+c)*EllipticF((a+b*cos(d*
x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*
cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.09, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2989, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2A+20abB+3Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{c}}\right.\right.}{15a^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)
*Sqrt[a + b]*(9*a*A - 3*A*b - 5*a*B + 15*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) + (2*(6*A*b + 5*a*B)*S
qrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*Sq
rt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx)}{\cos}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\ \sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} +$$

$$= \frac{2(6Ab+5aB)\sqrt{a+b\cos(c+dx)}\ \sec^{\frac{3}{2}}(c+dx)\sin}{15d}$$

$$= \frac{2(6Ab+5aB)\sqrt{a+b\cos(c+dx)}\ \sec^{\frac{3}{2}}(c+dx)\sin}{15d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2A+3Ab^2+20abB\right)\sqrt{\cos(c+}}{}$$

Mathematica [A] time = 18.75, size = 427, normalized size = 1.09

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2(9a^2A+20abB+3Ab^2)\sin(c+dx)}{15a}+\frac{2}{15}\sec(c+dx)(5aB\sin(c+dx)+6Ab\sin(c+dx\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2
),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(9*a^2*A + 3*A*b^2 + 2
0*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + 2*a*(a + b)*(3*b*(A + 5*B) + a*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2*A + 3*A*b^2 + 2
0*a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]))/(15*a*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a
+ b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sin
[c + d*x])/(15*a) + (2*Sec[c + d*x]*(6*A*b*Sin[c + d*x] + 5*a*B*Sin[c + d*x
]))/15 + (2*a*A*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Aa+(Ba+Ab)\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^(7/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.41, size = 2674, normalized size = 6.80

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-3*A*a^3+9*A*cos(d*x+c)^3*a^3-3*A*cos(d*x+c)^3*b^3-6*A*cos(d*x+c)^
2*a^3+5*B*cos(d*x+c)^3*a^3+20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+15*B*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos
(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+
3*A*cos(d*x+c)^4*b^3-5*B*cos(d*x+c)*a^3+3*A*cos(d*x+c)^3*a*b^2-9*A*cos(d*x+
c)^2*a*b^2-9*A*cos(d*x+c)*a^2*b+20*B*cos(d*x+c)^4*a*b^2+20*B*cos(d*x+c)^3*a
^2*b-25*B*cos(d*x+c)^2*a^2*b-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-3*A*sin(d*x+c)*cos(d*x+
c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+
12*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-20*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x
+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-20*B
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^2+20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-20*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)
^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin
(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2*b-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+12*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+3*A*sin(d*x+
c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b^2-20*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
```

```
d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+15*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-20*B*cos(d*x+c)^
3*a*b^2+9*A*cos(d*x+c)^4*a^2*b+6*A*cos(d*x+c)^4*a*b^2+5*B*cos(d*x+c)^4*a^2*
b+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^
3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*s
in(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-3*A*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9
*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(7/2)
/sin(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}\,{(a+b\,\cos(c+d\,x))}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.600 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=479

$$\frac{2\sqrt{a+b}\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\right.\right.}{3ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*a*A*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*(a-b)*(4*A
*b+3*B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+2/3*(3*A
*b^2+a^2*(A-3*B)-a*(4*A*b-6*B*b))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/
2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c
)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/s
ec(d*x+c)^(1/2)-2*b*B*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(
1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*
x+c)^(1/2)
```

Rubi [A] time = 1.08, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2961, 2989, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(a^2(A-3B)-a(4Ab-6bB)+3Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\right.\right.}{3ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(4*A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(3*A*b^2 + a^
2*(A - 3*B) - a*(4*A*b - 6*b*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[a + b]*B*Sqrt[Cos[c + d*
x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
```

```
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) +
 (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx)}{\cos}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} +$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} +$$

$$= -\frac{2b\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-}\right.}{d\sqrt{s}}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(4Ab+3aB)\sqrt{\cos(c+dx)}\,\csc(c+}{3}$$

Mathematica [B] time = 24.55, size = 5981, normalized size = 12.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2
),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^(5/2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.36, size = 2326, normalized size = 4.86

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(3*B*cos(d*x+c)^2*a^2+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+6*B*sin(d*x+c)*cos(d*x+c)
^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^2-
3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^2+6*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*a*b-3*B*cos(d*x+c)*a^2+4*A*cos(d*x+c)
^3*b^2-4*A*cos(d*x+c)^2*b^2-a^2*A-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+A*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^2-4
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)^2*b^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*cos(d*x+c)^2*a^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+A*sin(d*x+c)*
cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
a^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*cos(d*x+c)*a^2+A*cos(d*x+c)^2*a^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+6*B*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
```

```
*a*b-4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*a*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+A*cos(d*x+c)^3*a*b-3*B*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)^2*a*b+4*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*a*b+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2+4*A*cos(d*x+c)^2*a*b-5*A*co
s(d*x+c)*a*b+3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*b-4*A*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+6
*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2)*cos(d*x+c)/(a+
b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}\,{(a+b\,\cos(c+d\,x))}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.601 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=509

$$-\frac{(2aA-bB)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}-\frac{\sqrt{a+b}\,(2a(A-B)-b(4A+B))\sqrt{\cos(c+dx)}\,\csc(c+$$

```
[Out] 2*a*A*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-(2*A*a-B*b)*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d+(a-b)*(2*A*a-B*b)*csc(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-(2*a*(A-B)-b*(4*A+B))*csc(
d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-(2*A*b+3*B*a)*csc(d*x+c)
*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.38, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2989, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{(2aA-bB)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}-\frac{\sqrt{a+b}\,(2a(A-B)-b(4A+B))\sqrt{\cos(c+dx)}\,\csc(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(2*a*A - b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*a*(A - B) - b*(4*A
 + B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c
+ d*x]]) - (Sqrt[a + b]*(2*A*b + 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + b*C
```

```
os[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d - ((2*a*A - b*B)*Sqrt[a + b
*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+d}{co}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

$$= \frac{2aA\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

$$= -\frac{\sqrt{a+b}\,(2Ab+3aB)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,(2aA-bB)\sqrt{\cos(c+dx)}\csc(c+d}{}$$

Mathematica [A] time = 16.72, size = 927, normalized size = 1.82

$$\frac{2aA\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(2a^2A\tan^5\left(\frac{1}{2}(c+dx)\right)-2aAb\tan^5\left(\frac{1}{2}(c+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (2*a*A*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (Sqrt[
(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-2*a^2*A*Tan[(c + d*x)/2] - 2*a*A*b*Tan[(c
+ d*x)/2] + a*b*B*Tan[(c + d*x)/2] + b^2*B*Tan[(c + d*x)/2] + 4*a*A*b*Tan[(
c + d*x)/2]^3 - 2*b^2*B*Tan[(c + d*x)/2]^3 + 2*a^2*A*Tan[(c + d*x)/2]^5 - 2
*a*A*b*Tan[(c + d*x)/2]^5 - a*b*B*Tan[(c + d*x)/2]^5 + b^2*B*Tan[(c + d*x)/
2]^5 + 4*A*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*S
qrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
```

```
d*x)/2]^2)/(a + b)] + 6*a*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2
]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*A*b^2*EllipticPi[-1, ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6
*a*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(2*a*A - b*B)*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
 + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)] + 2*(-(A*b^2) + 2*a*b*(A - B) + a^2*(A + B))*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d
*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b
)]))/(d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 1.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Aa+\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sec(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2
), x)
```

maple [B] time = 0.36, size = 2196, normalized size = 4.31

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)

[Out] 1/d*(-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^2-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2-B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)*b^2-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*a*b+B*cos(d*x+c)^2*b^2-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+2*a^2*A-2*A*sin
(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*a^2-2*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*cos(d*x+c)*a^2-2*A*cos(d*x+c)*a^2-B*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+4*B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)*a*b+2*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*a*b-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+2*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2-B*cos(d
*x+c)^3*b^2-2*A*cos(d*x+c)^2*a*b+2*A*cos(d*x+c)*a*b-B*cos(d*x+c)^2*a*b+B*co
s(d*x+c)*a*b+2*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*a*b-4*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*a*b-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
```

```
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+2*A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-4*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-B*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^
2)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,(a+b\,\cos(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.602 $\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=532

$$-\frac{\sqrt{a+b}\left(3a^2B+12aAb+4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\ldots\right)}{4bd\sqrt{\sec(c+dx)}}$$

```
[Out] 1/2*b*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*(4*A*b+5*B
*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-1/4*(a-b)*(4*A*b+5
*B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a
+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+1/4*(8*A*a+4
*A*b+5*B*a+2*B*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-1/
4*(12*A*a*b+3*B*a^2+4*B*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(
a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d
*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b
/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.37, antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2990, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(3a^2B+12aAb+4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\ldots\right)}{4bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(4*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*a*A + 4*A*b +
 5*a*B + 2*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d*
Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(12*a*A*b + 3*a^2*B + 4*b^2*B)*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec[c
+ d*x]]) + (b*B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*
```

```
x]]) + ((4*A*b + 5*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(4*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx}{\sqrt{\text{c}}}$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+\right.$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4Ab+5aB)}{}$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{(4Ab+5aB)}{}$$

$$= -\frac{\sqrt{a+b}\left(12aAb+3a^2B+4b^2B\right)\sqrt{\cos(c+dx)}\ \text{cs}}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(4Ab+5aB)\sqrt{\cos(c+dx)}\ \csc(c+}{}$$

Mathematica [B] time = 18.56, size = 1134, normalized size = 2.13

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (b*B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) +
(Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(4*a*A*b*Tan[(c + d*x)/2] + 4*A*b^2*Ta
n[(c + d*x)/2] + 5*a^2*B*Tan[(c + d*x)/2] + 5*a*b*B*Tan[(c + d*x)/2] - 8*A*
b^2*Tan[(c + d*x)/2]^3 - 10*a*b*B*Tan[(c + d*x)/2]^3 - 4*a*A*b*Tan[(c + d*x
)/2]^5 + 4*A*b^2*Tan[(c + d*x)/2]^5 - 5*a^2*B*Tan[(c + d*x)/2]^5 + 5*a*b*B*
Tan[(c + d*x)/2]^5 + 24*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*B*EllipticPi[-1, ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*B*EllipticPi[-1
, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a*
```

```
A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x
)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*
Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*B*E
llipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^
2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(a + b)] + (a + b)*(4*A*b + 5*a*B)*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x
)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
 + 2*(4*a^2*(A - B) - 2*b^2*B + a*b*(-8*A + B))*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/
2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))
/(4*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 72.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(dx+c\right)^2 + Aa + \left(Ba + Ab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\sec\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*a + (B*a + A*b)*cos(d*x + c))*sqrt(b*cos(d
*x + c) + a)*sqrt(sec(d*x + c)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.37, size = 2432, normalized size = 4.57

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(8*A*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+4*A*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2+5*B*cos
(d*x+c)^2*a^2-8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2+2*B*cos(d*x+c)^4*b^2-2*B*cos(d*x+c)^2*b^2-5*B*cos(d*x
+c)*a^2+4*A*cos(d*x+c)^3*b^2-4*A*cos(d*x+c)^2*b^2+24*A*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+8
*A*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*a^2-8*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*a^2+5*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+2*B*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+
4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*a*b-16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2+4*A*cos(d*x+c)^2*a*b-4*A
*cos(d*x+c)*a*b+7*B*cos(d*x+c)^3*a*b-5*B*cos(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*
b+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)*a^2+24*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+4*A*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-16*A*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+5*B*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*B*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+4*A*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*b^2+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
```

```
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2+5*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2-4*B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2
+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*b^2+5*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*b^2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+b\cos(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.603 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=626

$$\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{24bd}+\frac{\sqrt{a+b}\,\left(3a^2B+30aAb+14abB+12Ab\right.}{}$$

```
[Out] 1/3*b*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+1/12*(6*A*b+7*
B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/24*(30*A*a*b+3*
B*a^2+16*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b/d-1/24
*(a-b)*(30*A*a*b+3*B*a^2+16*B*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+
c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/
d/sec(d*x+c)^(1/2)+1/24*(30*A*a*b+12*A*b^2+3*B*a^2+14*B*a*b+16*B*b^2)*csc(d
*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)
/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)-1/8*(6*A*a^2*b+8*A*b^3-
B*a^3+12*B*a*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)
*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*
x+c)^(1/2)
```

Rubi [A] time = 1.97, antiderivative size = 626, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(3a^2B+30aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{24bd}+\frac{\sqrt{a+b}\,\left(3a^2B+30aAb+14abB+12Ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a +
b]*(30*a*A*b + 12*A*b^2 + 3*a^2*B + 14*a*b*B + 16*b^2*B)*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[C
os[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a +
```

```
b]*(6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^2*d*Sqrt[Sec[c + d*x]]) + (b*B*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + ((6*A*b +
7*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[Sec[c + d*x]]) + (
(30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(24*b*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
```

```
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+b\cos(}{12d\sqrt{\sec(c}}$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+b\cos(}{12d\sqrt{\sec(c}}$$

$$= \frac{bB\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+b\cos(}{12d\sqrt{\sec(c}}$$

$$= -\frac{\sqrt{a+b}\,\left(6a^2Ab+8Ab^3-a^3B+12ab^2B\right)\sqrt{\cos(c+dx)}\,\csc(}{8}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\left(30aAb+3a^2B+16b^2B\right)\sqrt{\cos(c+dx)}\,\csc(c}{24ab}$$

Mathematica [B] time = 19.35, size = 1489, normalized size = 2.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*B*Sin[c + d*x])/12 + ((6*A
*b + 7*a*B)*Sin[2*(c + d*x)])/24 + (b*B*Sin[3*(c + d*x)])/12))/d + (Sqrt[(1
 - Tan[(c + d*x)/2]^2)^(-1)]*(30*a^2*A*b*Tan[(c + d*x)/2] + 30*a*A*b^2*Tan[
(c + d*x)/2] + 3*a^3*B*Tan[(c + d*x)/2] + 3*a^2*b*B*Tan[(c + d*x)/2] + 16*a
*b^2*B*Tan[(c + d*x)/2] + 16*b^3*B*Tan[(c + d*x)/2] - 60*a*A*b^2*Tan[(c + d
*x)/2]^3 - 6*a^2*b*B*Tan[(c + d*x)/2]^3 - 32*b^3*B*Tan[(c + d*x)/2]^3 - 30*
a^2*A*b*Tan[(c + d*x)/2]^5 + 30*a*A*b^2*Tan[(c + d*x)/2]^5 - 3*a^3*B*Tan[(c
 + d*x)/2]^5 + 3*a^2*b*B*Tan[(c + d*x)/2]^5 - 16*a*b^2*B*Tan[(c + d*x)/2]^5
 + 16*b^3*B*Tan[(c + d*x)/2]^5 + 36*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Ta
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*A*b^3*EllipticPi[-1,
 ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^3*
B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(
a + b)] + 72*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Ta
n[(c + d*x)/2]^2)/(a + b)] + 36*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*A*b^3
*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2
]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan
[(c + d*x)/2]^2)/(a + b)] - 6*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 72*a*b^2*B*E
llipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^
2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(
c + d*x)/2]^2)/(a + b)] + (a + b)*(30*a*A*b + 3*a^2*B + 16*b^2*B)*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(
1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x
)/2]^2)/(a + b)] - 2*b*(12*A*b^2 + a^2*(24*A - 7*B) + a*(-6*A*b + 26*b*B))*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan
[(c + d*x)/2]^2)/(a + b)]))/(24*b*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)
])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

maple [B] time = 0.48, size = 3141, normalized size = 5.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/24/d*(36*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+30*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+30*A*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^2+14*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b-52*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+72*B*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+3
```

```
*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b+12*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+16*B*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-48*A*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b+12*A*cos(d*x+c)^4*b^3-12*A*cos(d*x+c)^2*b^3+8*B*cos(d*x+c)^5*b^3
+8*B*cos(d*x+c)^3*b^3+3*B*cos(d*x+c)^2*a^3-16*B*cos(d*x+c)^2*b^3-3*B*cos(d*
x+c)*a^3+42*A*cos(d*x+c)^3*a*b^2+30*A*cos(d*x+c)^2*a^2*b-30*A*cos(d*x+c)^2*
a*b^2-30*A*cos(d*x+c)*a^2*b-12*A*cos(d*x+c)*a*b^2+22*B*cos(d*x+c)^4*a*b^2+1
7*B*cos(d*x+c)^3*a^2*b-3*B*cos(d*x+c)^2*a^2*b-6*B*cos(d*x+c)^2*a*b^2-14*B*c
os(d*x+c)*a^2*b-16*B*cos(d*x+c)*a*b^2+48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-6*B*sin(d*x+c
)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*a^3+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^3+16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+12*A*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+36*A*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+
30*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b+30*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^2+14*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-52*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+72*B*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+3*B*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+16*B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b^2-24*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
```

```
-(a-b)/(a+b))^(1/2))*b^3-48*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+48*A*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3-6*B*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+3*B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3+16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*b^3)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,(a+b\,\cos(c+d\,x))^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.604 $\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=730

$$\frac{\left(-3a^2B+8aAb+12b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{32bd\sqrt{\sec(c+dx)}}+\frac{\left(-9a^3B+24a^2Ab+156ab^2B+128Ab^3\right)\sin(c+dx)}{192b^2d}$$

```
[Out] 1/24*(8*A*b-3*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1
/4*B*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)+1/32*(8*A*a*b-3
*B*a^2+12*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/1
92*(24*A*a^2*b+128*A*b^3-9*B*a^3+156*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(
1/2)*sec(d*x+c)^(1/2)/b^2/d-1/192*(a-b)*(24*A*a^2*b+128*A*b^3-9*B*a^3+156*B
*a*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d/sec(d*x+c)^(1/2)-1/192*
(9*a^3*B-6*a^2*b*(4*A+B)-8*b^3*(16*A+9*B)-4*a*b^2*(28*A+39*B))*csc(d*x+c)*E
llipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)+1/64*(8*A*a^3*b-96*A*a*b^3-3
*B*a^4-24*B*a^2*b^2-48*B*b^4)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(
d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/
b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 2.44, antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(24a^2Ab-9a^3B+156ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{192b^2d}+\frac{\left(-3a^2B+8aAb+12b^2B\right)}{32bd\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Sqrt
[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b^2*d*Sqrt[Sec[c +
 d*x]]) - (Sqrt[a + b]*(9*a^3*B - 6*a^2*b*(4*A + B) - 8*b^3*(16*A + 9*B) -
4*a*b^2*(28*A + 39*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
```

```
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(192*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*a^3*A*b - 96*a*A*b^3 - 3
*a^4*B - 24*a^2*b^2*B - 48*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticP
i[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(64*b^3*d*Sqrt[Sec[c + d*x]]) + ((8*a*A*b - 3*a^2*
B + 12*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*b*d*Sqrt[Sec[c + d
*x]]) + ((8*A*b - 3*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*b*d*S
qrt[Sec[c + d*x]]) + (B*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(4*b*d*Sqr
t[Sec[c + d*x]]) + ((24*a^2*A*b + 128*A*b^3 - 9*a^3*B + 156*a*b^2*B)*Sqrt[a
 + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b^2*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
```

```
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
```

```
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+d$$

$$= \frac{B(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{4bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= \frac{(8Ab-3aB)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{24bd\sqrt{\sec(c+dx)}} + \frac{B(a+b\cos(}{4bd}$$

$$= \frac{\left(8aAb-3a^2B+12b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{32bd\sqrt{\sec(c+dx)}} + \frac{(8}{}$$

$$= \frac{\left(8aAb-3a^2B+12b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{32bd\sqrt{\sec(c+dx)}} + \frac{(8}{}$$

$$= \frac{\left(8aAb-3a^2B+12b^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{32bd\sqrt{\sec(c+dx)}} + \frac{(8}{}$$

$$= \frac{\sqrt{a+b}\,\left(8a^3Ab-96aAb^3-3a^4B-24a^2b^2B-48b^4B\right)\sqrt{\cos(}}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,\left(24a^2Ab+128Ab^3-9a^3B+156ab^2B\right)\sqrt{\cos}}{}$$

Mathematica [B] time = 21.40, size = 1888, normalized size = 2.59

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((8*A*b + 9*a*B)*Sin[c + d*x]
)/96 + ((56*a*A*b + 3*a^2*B + 48*b^2*B)*Sin[2*(c + d*x)])/(192*b) + ((8*A*b
 + 9*a*B)*Sin[3*(c + d*x)])/96 + (b*B*Sin[4*(c + d*x)])/32))/d - (Sqrt[(1 -
 Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
```

```
d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(24*a^3*A*b*Tan[(c + d*x)/2] + 24*a^2*
A*b^2*Tan[(c + d*x)/2] + 128*a*A*b^3*Tan[(c + d*x)/2] + 128*A*b^4*Tan[(c +
d*x)/2] - 9*a^4*B*Tan[(c + d*x)/2] - 9*a^3*b*B*Tan[(c + d*x)/2] + 156*a^2*b
^2*B*Tan[(c + d*x)/2] + 156*a*b^3*B*Tan[(c + d*x)/2] - 48*a^2*A*b^2*Tan[(c
+ d*x)/2]^3 - 256*A*b^4*Tan[(c + d*x)/2]^3 + 18*a^3*b*B*Tan[(c + d*x)/2]^3
- 312*a*b^3*B*Tan[(c + d*x)/2]^3 - 24*a^3*A*b*Tan[(c + d*x)/2]^5 + 24*a^2*A
*b^2*Tan[(c + d*x)/2]^5 - 128*a*A*b^3*Tan[(c + d*x)/2]^5 + 128*A*b^4*Tan[(c
 + d*x)/2]^5 + 9*a^4*B*Tan[(c + d*x)/2]^5 - 9*a^3*b*B*Tan[(c + d*x)/2]^5 -
156*a^2*b^2*B*Tan[(c + d*x)/2]^5 + 156*a*b^3*B*Tan[(c + d*x)/2]^5 - 48*a^3*
A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)] + 576*a*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(a + b)] + 18*a^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(
c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 144*a^2*b^2*B*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 288*
b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^
2)/(a + b)] - 48*a^3*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Ta
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 576*a*A*b^3*EllipticPi[
-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)] + 18*a^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Ta
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 144*a^2*b^2*B*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] + 288*b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(-24*a^2*A*b
 - 128*A*b^3 + 9*a^3*B - 156*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b*(2*a^
2*b*(28*A - 57*B) - 4*a*b^2*(52*A - 9*B) + 3*a^3*B - 72*b^3*B)*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 +
 Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)]))/(192*b^2*d*Sqrt[1 + Tan[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d
*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

maple [B] time = 0.62, size = 4056, normalized size = 5.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/192/d*(136*A*cos(d*x+c)^3*a^2*b^2-3*B*cos(d*x+c)^3*a^3*b+108*B*cos(d*x+c
)^3*a*b^3+78*B*cos(d*x+c)^2*a^2*b^2-156*B*cos(d*x+c)^2*a*b^3-6*B*cos(d*x+c)
*a^3*b-156*B*cos(d*x+c)*a^2*b^2-72*B*cos(d*x+c)*a*b^3+24*A*cos(d*x+c)^2*a^3
*b-48*A*cos(d*x+c)^2*a*b^3-112*A*cos(d*x+c)*a^2*b^2-128*A*cos(d*x+c)*a*b^3+
128*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^4-9*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^4+18*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*a^4+288*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-144*B*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+48*B*cos(d*x+c)^6*b^
4+64*A*cos(d*x+c)^3*b^4-128*A*cos(d*x+c)^2*b^4+24*B*cos(d*x+c)^4*b^4-72*B*c
os(d*x+c)^2*b^4-9*B*cos(d*x+c)^2*a^4+9*B*cos(d*x+c)*a^4+64*A*cos(d*x+c)^5*b
^4+9*B*cos(d*x+c)^2*a^3*b+176*A*cos(d*x+c)^4*a*b^3-24*A*cos(d*x+c)^2*a^2*b^
```

```
2-24*A*cos(d*x+c)*a^3*b+120*B*cos(d*x+c)^5*a*b^3+78*B*cos(d*x+c)^4*a^2*b^2+
24*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*a^3*b-9*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4+18*B*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4+288*B*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+
b))^(1/2))*b^4-144*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+24*A*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+128*A*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-48*
A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
1/2))*a^3*b+576*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a*b^3+112*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-416*A*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-9*B*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+156*B*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
^2+156*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b^3+144*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-228*B*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+72*B*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+128*A*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
```

```
))^(1/2))*b^4+24*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^2+128*A*sin(d*x+c)*cos(d*x+c)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^3-48*A*sin(d*x+c
)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*a^3*b+576*A*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^3+112*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-416*A*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b^3-9*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+156*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+156*B*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^3+144*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-228*B*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b^2+72*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a*b^3)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(a+
b*cos(d*x+c))^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\ (a+b\cos(c+dx))^{3/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.605 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx$

Optimal. Leaf size=662

$$\frac{2\left(81a^2A+209abB+113Ab^2\right)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{693d}+\frac{2\left(539a^3B+1145a^2Ab+825ab^2B\right.}{}$$

```
[Out] 2/11*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(11/2)*sin(d*x+c)/d+2/3465*(675*
A*a^4+1025*A*a^2*b^2-20*A*b^4+1793*B*a^3*b+55*B*a*b^3)*sec(d*x+c)^(3/2)*sin
(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/d+2/3465*(1145*A*a^2*b+15*A*b^3+539*B*a^
3+825*B*a*b^2)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d+2/693
*(81*A*a^2+113*A*b^2+209*B*a*b)*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c)
)^(1/2)/d+2/99*a*(14*A*b+11*B*a)*sec(d*x+c)^(9/2)*sin(d*x+c)*(a+b*cos(d*x+c
))^(1/2)/d+2/3465*(a-b)*(3705*A*a^4*b+255*A*a^2*b^3+40*A*b^5+1617*B*a^5+306
9*B*a^3*b^2-110*B*a*b^4)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/sec(d*x+
c)^(1/2)+2/3465*(a-b)*(40*A*b^4+3*a^4*(225*A-539*B)-6*a^3*b*(505*A-209*B)+1
5*a^2*b^2*(19*A-121*B)+10*a*b^3*(3*A-11*B))*csc(d*x+c)*EllipticF((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)
*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/a^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 2.91, antiderivative size = 662, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2961, 2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(81a^2A+209abB+113Ab^2\right)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{693d}+\frac{2\left(1145a^2Ab+539a^3B+825ab^2B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*
B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(3465*a^4*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(40*A
*b^4 + 3*a^4*(225*A - 539*B) - 6*a^3*b*(505*A - 209*B) + 15*a^2*b^2*(19*A -
 121*B) + 10*a*b^3*(3*A - 11*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[
```

```
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(3465*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(675*a^4*A + 1025*a^2*A*b^
2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(3465*a^2*d) + (2*(1145*a^2*A*b + 15*A*b^3 + 539*a
^3*B + 825*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(3465*a*d) + (2*(81*a^2*A + 113*A*b^2 + 209*a*b*B)*Sqrt[a + b*Cos[c + d*
x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(693*d) + (2*a*(14*A*b + 11*a*B)*Sqrt[
a + b*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(99*d) + (2*a*A*(a + b
*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(11*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
```

```
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0])))
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{13}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx}{\cos}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{11}{2}}(c+dx)\sin(c+dx}{11d}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{9}{2}}(c+dx)}{99d}$$

$$= \frac{2\left(81a^2A+113Ab^2+209abB\right)\sqrt{a+b\cos(c+dx}}{693d}$$

$$= \frac{2\left(1145a^2Ab+15Ab^3+539a^3B+825ab^2B\right)\sqrt{a+}}{3465ad}$$

$$= \frac{2\left(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB+5\right.}{346}$$

$$= \frac{2\left(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3bB+5\right.}{346}$$

$$= \frac{2(a-b)\sqrt{a+b}\,\left(3705a^4Ab+255a^2Ab^3+40Ab^5\right.}{}$$

Mathematica [B] time = 27.24, size = 4198, normalized size = 6.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(13/
2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3705*a^4*A*b + 255*a^2*A*
b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sin[c + d*x])/(
```

3465*a^3) + (2*Sec[c + d*x]^4*(23*a*A*b*Sin[c + d*x] + 11*a^2*B*Sin[c + d*x
]))/99 + (2*Sec[c + d*x]^3*(81*a^2*A*Sin[c + d*x] + 113*A*b^2*Sin[c + d*x]
+ 209*a*b*B*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2*(1145*a^2*A*b*Sin[c + d*
x] + 15*A*b^3*Sin[c + d*x] + 539*a^3*B*Sin[c + d*x] + 825*a*b^2*B*Sin[c + d
*x]))/(3465*a) + (2*Sec[c + d*x]*(675*a^4*A*Sin[c + d*x] + 1025*a^2*A*b^2*S
in[c + d*x] - 20*A*b^4*Sin[c + d*x] + 1793*a^3*b*B*Sin[c + d*x] + 55*a*b^3*
B*Sin[c + d*x]))/(3465*a^2) + (2*a^2*A*Sec[c + d*x]^4*Tan[c + d*x])/11))/d
+ (2*((-247*a^2*A*b)/(231*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (1
7*A*b^3)/(231*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^5)/(693
*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (7*a^3*B)/(15*Sqrt[a +
b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (31*a*b^2*B)/(35*Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]) + (2*b^4*B)/(63*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]) + (15*a^3*A*Sqrt[Sec[c + d*x]])/(77*Sqrt[a + b*Cos[c + d*x]])
- (26*a*A*b^2*Sqrt[Sec[c + d*x]])/(231*Sqrt[a + b*Cos[c + d*x]]) - (7*A*b^4
*Sqrt[Sec[c + d*x]])/(99*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^6*Sqrt[Sec[c
+ d*x]])/(693*a^3*Sqrt[a + b*Cos[c + d*x]]) + (38*a^2*b*B*Sqrt[Sec[c + d*x]
])/(105*Sqrt[a + b*Cos[c + d*x]]) - (124*b^3*B*Sqrt[Sec[c + d*x]])/(315*Sqr
t[a + b*Cos[c + d*x]]) + (2*b^5*B*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Co
s[c + d*x]]) - (247*a*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*Sqrt[
a + b*Cos[c + d*x]]) - (17*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*
a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^6*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])
/(693*a^3*Sqrt[a + b*Cos[c + d*x]]) - (7*a^2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[
c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (31*b^3*B*Cos[2*(c + d*x)]*Sqrt[
Sec[c + d*x]])/(35*Sqrt[a + b*Cos[c + d*x]]) + (2*b^5*B*Cos[2*(c + d*x)]*Sq
rt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*(-2*(a + b)*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617
*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*
A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3*a^4*(22
5*A + 539*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)] - (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*
a^3*b^2*B - 110*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(3465*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*
x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)
*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 1
10*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] + 2*a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*
A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3*a^4*(225*A + 539*B))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3705*a^4*A*b
 + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Co
s[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3465

a^3(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3*a^4*(225*A + 539*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3465*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1/2*((3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4) - ((a + b)*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3*a^4*(225*A + 539*B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3*a^4*(225*A + 539*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3*a^4*(225*A + 539*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 11

```
0*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c
+ d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3465*a^3*Sqrt[a + b*
Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(3705*a^4*A*b + 255*
a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a +
b)*(40*A*b^4 - 10*a*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*
(505*A + 209*B) + 3*a^4*(225*A + 539*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3705*a^4*A*b + 255*a^2*A*b^3 + 40
*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*Cos[c + d*x]*(a + b*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c +
 d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(3
465*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x
)/2]^2*Sec[c + d*x]])))
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(13/
2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.94, size = 5381, normalized size = 8.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{13}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(13/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(13/
2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{13/2}(a+b\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(13/2)*(a + b*cos(c + d*x))^(5/2)
,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(13/2)*(a + b*cos(c + d*x))^(5/2)
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

3.606 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=562

$$\frac{2\left(49a^2A+135abB+75Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315d}+\frac{2\left(75a^3B+163a^2Ab+135ab^2B+5\right.}{}$$

```
[Out] 2/9*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2)*sin(d*x+c)/d+2/315*(163*A*a
^2*b+5*A*b^3+75*B*a^3+135*B*a*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x
+c))^(1/2)/a/d+2/315*(49*A*a^2+75*A*b^2+135*B*a*b)*sec(d*x+c)^(5/2)*sin(d*x
+c)*(a+b*cos(d*x+c))^(1/2)/d+2/21*a*(4*A*b+3*B*a)*sec(d*x+c)^(7/2)*sin(d*x+
c)*(a+b*cos(d*x+c))^(1/2)/d+2/315*(a-b)*(147*A*a^4+279*A*a^2*b^2-10*A*b^4+4
35*B*a^3*b+45*B*a*b^3)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1
/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)
^(1/2)-2/315*(a-b)*(10*A*b^3-6*a^2*b*(19*A-60*B)+3*a^3*(49*A-25*B)+15*a*b^2
*(11*A-3*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x
+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 2.07, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2961, 2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(49a^2A+135abB+75Ab^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315d}+\frac{2\left(163a^2Ab+75a^3B+135ab^2B+5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B
+ 45*a*b^3*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^
3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(10*A*b^3 - 6*a^2*b*(19*A
- 60*B) + 3*a^3*(49*A - 25*B) + 15*a*b^2*(11*A - 3*B))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(163*
a^2*A*b + 5*A*b^3 + 75*a^3*B + 135*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c
```

```
+ d*x]^(3/2)*Sin[c + d*x])/(315*a*d) + (2*(49*a^2*A + 75*A*b^2 + 135*a*b*B)
*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (2*a*(
4*A*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(2
1*d) + (2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(
9*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx}{\cos}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx}{9d}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,\mathrm{si}}{21d}$$

$$= \frac{2\left(49a^2A+75Ab^2+135abB\right)\sqrt{a+b\cos(c+dx)}}{315d}$$

$$= \frac{2\left(163a^2Ab+5Ab^3+75a^3B+135ab^2B\right)\sqrt{a+b\,\mathrm{c}}}{315ad}$$

$$= \frac{2\left(163a^2Ab+5Ab^3+75a^3B+135ab^2B\right)\sqrt{a+b\,\mathrm{c}}}{315ad}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(147a^4A+279a^2Ab^2-10Ab^4+\right.}{}$$

Mathematica [B] time = 26.36, size = 3755, normalized size = 6.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(11/
2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4*A + 279*a^2*A*b^2
 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sin[c + d*x])/(315*a^2) + (2*Sec[c
+ d*x]^3*(19*a*A*b*Sin[c + d*x] + 9*a^2*B*Sin[c + d*x]))/63 + (2*Sec[c + d*
x]^2*(49*a^2*A*Sin[c + d*x] + 75*A*b^2*Sin[c + d*x] + 135*a*b*B*Sin[c + d*x
]))/315 + (2*Sec[c + d*x]*(163*a^2*A*b*Sin[c + d*x] + 5*A*b^3*Sin[c + d*x]
+ 75*a^3*B*Sin[c + d*x] + 135*a*b^2*B*Sin[c + d*x]))/(315*a) + (2*a^2*A*Sec
[c + d*x]^3*Tan[c + d*x])/9))/d + (2*((-7*a^3*A)/(15*Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]) - (31*a*A*b^2)/(35*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) + (2*A*b^4)/(63*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

- (29*a^2*b*B)/(21*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b^3*B)/(
7*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (38*a^2*A*b*Sqrt[Sec[c + d
*x]])/(105*Sqrt[a + b*Cos[c + d*x]]) - (124*A*b^3*Sqrt[Sec[c + d*x]])/(315*
Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^5*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b
*Cos[c + d*x]]) + (5*a^3*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]
) - (2*a*b^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (b^4*B*S
qrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Cos[c + d*x]]) - (7*a^2*A*b*Cos[2*(c + d
*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (31*A*b^3*Cos[2*(c
 + d*x)]*Sqrt[Sec[c + d*x]])/(35*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^5*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(63*a^2*Sqrt[a + b*Cos[c + d*x]]) - (29*a*b
^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (
b^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Cos[c + d*x]]))*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(147*a^4*A + 279*a^2*A*b^
2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-10*A*b^3 + 15*a*b^2*
(11*A + 3*B) + 3*a^3*(49*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (147*a^4*A + 27
9*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[
c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^2*d*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*Sin[c + d*x]*(-2*(a + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b
*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)] + 2*a*(a + b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3*(49
*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (-a + b)/(a + b)] - (147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 +
 435*a^3*b*B + 45*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(315*a^2*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d
*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a
 + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a
(a + b)(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3*(49*A + 25*B) + 6*a^2*b
*(19*A + 60*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d
x])/((a + b)(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)] - (147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*
b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
]))/(315*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*(-1/2*((147*a^4*A + 279*a^2*A*b^2 - 10*A*b^
4 + 435*a^3*b*B + 45*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*
x)/2]^4) - ((a + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 4
5*a*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic

```
E[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/
(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])] + (a*(a + b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3
*(49*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c
 + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(147*a^4*A + 27
9*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*S
in[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c +
d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))] + (a*(a + b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3
*(49*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a +
 b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))] + b*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3
*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (147*a^
4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*(a + b*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (147*a^4*A + 279*a^
2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c +
d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(-10*A*b^3 + 15*a*
b^2*(11*A + 3*B) + 3*a^3*(49*A + 25*B) + 6*a^2*b*(19*A + 60*B))*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b
)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*
A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqr
t[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2])
)/(315*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b
)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a
+ b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3*(49*A + 25*B) + 6*a^2*b*(19
*A + 60*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] - (147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*
B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(
-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[
c + d*x]*Tan[c + d*x]))/(315*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x
)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(11/
2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.70, size = 4400, normalized size = 7.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x)
```

```
[Out] 2/315/d*(-279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b^2-155*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3+10*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^4*a*b^4+435*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^4*b+435*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3*b^2+45*B*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d
*x+c)^4*a^2*b^3+45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^4-435*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^4*b-405*B*(cos
```

(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^4*a^3*b^2-45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3+147*A*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b+279*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)^5*a^3*b^2+279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^2*b^3-10*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a*b^4-261*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^5*a^4*b-279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^3*b^2-155*A*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^2*b^3+
10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)^5*a*b^4+435*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b+435*B*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^3*b^2
+45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)^5*a^2*b^3+45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a*b^4-435*B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^4*b
-405*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)^5*a^3*b^2-45*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^2*b^3+147*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^
4*b-261*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)*cos(d*x+c)^4*a^4*b+180*B*cos(d*x+c)^2*a^4*b-75*B*cos(d*x+c)^6*a^4

```
*b-435*B*cos(d*x+c)^6*a^3*b^2-135*B*cos(d*x+c)^6*a^2*b^3-45*B*cos(d*x+c)^6*
a*b^4-435*B*cos(d*x+c)^5*a^4*b+165*B*cos(d*x+c)^5*a^3*b^2-45*B*cos(d*x+c)^5
*a^2*b^3+45*B*cos(d*x+c)^5*a*b^4+330*B*cos(d*x+c)^4*a^4*b+35*A*a^5-147*A*co
s(d*x+c)^6*a^4*b-163*A*cos(d*x+c)^6*a^3*b^2-279*A*cos(d*x+c)^6*a^2*b^3-5*A*
cos(d*x+c)^6*a*b^4-65*A*cos(d*x+c)^5*a^4*b-279*A*cos(d*x+c)^5*a^3*b^2+199*A
*cos(d*x+c)^5*a^2*b^3+10*A*cos(d*x+c)^5*a*b^4+272*A*cos(d*x+c)^4*a^3*b^2-5*
A*cos(d*x+c)^4*a*b^4+82*A*cos(d*x+c)^3*a^4*b+80*A*cos(d*x+c)^3*a^2*b^3+170*
A*cos(d*x+c)^2*a^3*b^2+130*A*cos(d*x+c)*a^4*b+180*B*cos(d*x+c)^4*a^2*b^3+27
0*B*cos(d*x+c)^3*a^3*b^2+10*A*cos(d*x+c)^6*b^5-147*A*cos(d*x+c)^5*a^5-10*A*
cos(d*x+c)^5*b^5+98*A*cos(d*x+c)^4*a^5+14*A*cos(d*x+c)^2*a^5-75*B*cos(d*x+c
)^5*a^5+30*B*cos(d*x+c)^3*a^5+45*B*cos(d*x+c)*a^5+147*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5-10
*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)^5*b^5-147*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^5-75*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5+147*A*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*a^5-10*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*b^5-147*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^5-75*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4
*a^5+279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*cos(d*x+c)^4*a^3*b^2+279*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3-10*A*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^
4*a*b^4)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(11/2)/sin(d*x+c)
/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(11/
2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}\,(a+b\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(5/2)
,x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(5/2)
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

3.607 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=474

$$\frac{2\left(25a^2A+77abB+45Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}+\frac{2(a-b)\sqrt{a+b}\,\left(a^2(25A-63B)-8a\right.}{}$$

```
[Out] 2/7*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2/105*(25*A*a^
2+45*A*b^2+77*B*a*b)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2
/35*a*(10*A*b+7*B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2
/105*(a-b)*(145*A*a^2*b+15*A*b^3+63*B*a^3+161*B*a*b^2)*csc(d*x+c)*EllipticE
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/105*(a-b)*(a^2*(25*A-63*B)+15*b^2*
(A-7*B)-8*a*b*(15*A-7*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c
)^(1/2)
```

Rubi [A] time = 1.50, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2961, 2989, 3047, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2A+77abB+45Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}+\frac{2(a-b)\sqrt{a+b}\,\left(a^2(25A-63B)-8a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d*Sqrt[Sec[c +
 d*x]]) + (2*(a - b)*Sqrt[a + b]*(a^2*(25*A - 63*B) + 15*b^2*(A - 7*B) - 8*
a*b*(15*A - 7*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(10
5*a*d*Sqrt[Sec[c + d*x]]) + (2*(25*a^2*A + 45*A*b^2 + 77*a*b*B)*Sqrt[a + b*
Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d) + (2*a*(10*A*b + 7*a
```

```
*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a
*A*(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx)}{\cos}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,\mathrm{s}}{35d}$$

$$= \frac{2\left(25a^2A+45Ab^2+77abB\right)\sqrt{a+b\cos(c+dx)}\,\mathrm{s}}{105d}$$

$$= \frac{2\left(25a^2A+45Ab^2+77abB\right)\sqrt{a+b\cos(c+dx)}\,\mathrm{s}}{105d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(145a^2Ab+15Ab^3+63a^3B+161\right.}{}$$

Mathematica [B] time = 24.67, size = 3348, normalized size = 7.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(9/2
),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(145*a^2*A*b + 15*A*b^3 +
63*a^3*B + 161*a*b^2*B)*Sin[c + d*x])/(105*a) + (2*Sec[c + d*x]^2*(15*a*A*b
*Sin[c + d*x] + 7*a^2*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(25*a^2*A*Sin[c
 + d*x] + 45*A*b^2*Sin[c + d*x] + 77*a*b*B*Sin[c + d*x]))/105 + (2*a^2*A*Se
c[c + d*x]^2*Tan[c + d*x])/7))/d + (2*((-29*a^2*A*b)/(21*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) - (A*b^3)/(7*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]]) - (3*a^3*B)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (23
*a*b^2*B)/(15*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a^3*A*Sqrt[
Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) - (2*a*A*b^2*Sqrt[Sec[c + d*x]
])/(21*Sqrt[a + b*Cos[c + d*x]]) - (A*b^4*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a +
 b*Cos[c + d*x]]) + (8*a^2*b*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d
*x]]) - (8*b^3*B*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c + d*x]]) - (29*a*
A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[a + b*Cos[c + d*x]]) -
```

```
(A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*a*Sqrt[a + b*Cos[c + d*x]])
- (3*a^2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[a + b*Cos[c + d*x
]]) - (23*b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[a + b*Cos[c +
 d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(145*a^2*A*b + 1
5*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15
*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)] - (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b
^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
))/(105*a*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(145*a^2*A*b + 15*A*b
^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15*A +
7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)] - (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1
05*a*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c +
d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(145*a^2*A*b + 15*A*b^
3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
 b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15*A + 7
*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] - (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*
Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(10
5*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(-1/2*((145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^
2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4) - ((a + b)*(145*
a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 +
Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(15*b^2*
(A + 7*B) + 8*a*b*(15*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[(a + b*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x
]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(14
5*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x]
)/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))] + (a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15*A + 7*B) + a^2*(
```

```
25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2))
)/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(145*a^2*A*b
+ 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c
+ d*x]*Tan[(c + d*x)/2] + (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)
*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (1
45*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(15*b^2*(A + 7*B)
 + 8*a*b*(15*A + 7*B) + a^2*(25*A + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)
/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)
/(a + b)]) - ((a + b)*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(
a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(105*a*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B +
161*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)] + 2*a*(a + b)*(15*b^2*(A + 7*B) + 8*a*b*(15*A + 7*B) + a^2*(25*A
 + 63*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] - (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*Cos[c + d*x]*(a
 + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]
*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d
*x]))/(105*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c
+ d*x)/2]^2*Sec[c + d*x]])))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2
), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.54, size = 3636, normalized size = 7.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*(105*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+63*B*cos(d*x+c)^4*a^4-42*B*cos(d*x+c)^3*
a^4-21*B*cos(d*x+c)*a^4+25*A*cos(d*x+c)^4*a^4-10*A*cos(d*x+c)^2*a^4+15*A*co
s(d*x+c)^5*b^4-15*A*cos(d*x+c)^4*b^4-98*B*cos(d*x+c)^2*a^3*b+25*A*cos(d*x+c
)^5*a^3*b+145*A*cos(d*x+c)^5*a^2*b^2+45*A*cos(d*x+c)^5*a*b^3+145*A*cos(d*x+
c)^4*a^3*b-55*A*cos(d*x+c)^4*a^2*b^2+15*A*cos(d*x+c)^4*a*b^3-110*A*cos(d*x+
c)^3*a^3*b-60*A*cos(d*x+c)^3*a*b^3-90*A*cos(d*x+c)^2*a^2*b^2-60*A*cos(d*x+c
)*a^3*b+63*B*cos(d*x+c)^5*a^3*b+77*B*cos(d*x+c)^5*a^2*b^2+161*B*cos(d*x+c)^
5*a*b^3+35*B*cos(d*x+c)^4*a^3*b+161*B*cos(d*x+c)^4*a^2*b^2-161*B*cos(d*x+c)
^4*a*b^3-238*B*cos(d*x+c)^3*a^2*b^2+15*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+105*B*sin(d*x+c
)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^3-15*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*sin(d*x+c)*cos(d
*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4
+63*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^4-15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*A*sin(d*x+c)*cos(d*x+c)^3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-63*B*sin
(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^4+63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
```

in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-15*A*a^4-63*B*sin(d*x+c)*cos(d*x+c)^4*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-161*B*
sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b^2-161*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+119*B*sin(d*x+c)*cos(d*x+c)^4
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+161*
B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b^2-145*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-145*A*sin(d*x+c)*cos(d*x+c)
^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-
15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^3+145*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+135*A*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2
+15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a*b^3-63*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-161*B*sin(d*x+c)*cos(d*x+c
)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2
-161*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^3+119*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+161*B*sin(d*x+c)*cos(d*x
+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
^2-145*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3*b-145*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-15*A*sin(d*x+c)*cos(
d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*

```
b^3+145*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3*b+135*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2)*cos(d*x+c)/(a+b*co
s(d*x+c))^(1/2)*(1/cos(d*x+c))^(9/2)/sin(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}\,(a+b\,\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.608 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=553

$$\frac{2(a-b)\sqrt{a+b}\,\left(9a^2A+35abB+23Ab^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cdots}}{\sqrt{a+b}\,\sqrt{\cdots}}\right.\right.}{15ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2/5*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/15*a*(8*A*b+
5*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/15*(a-b)*(9*A
*a^2+23*A*b^2+35*B*a*b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a
*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^
(1/2)+2/15*(15*A*b^3-a*b^2*(23*A-45*B)+a^2*b*(17*A-35*B)-a^3*(9*A-5*B))*csc
(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-2*b^2*B*csc(d*x+c)*El
lipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.47, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2989, 3047, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\,\left(a^2b(17A-35B)+a^3(-(9A-5B))-ab^2(23A-45B)+15Ab^3\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\cdots}{15ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a +
 b]*(15*A*b^3 - a*b^2*(23*A - 45*B) + a^2*b*(17*A - 35*B) - a^3*(9*A - 5*B)
)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c
+ d*x]]) - (2*b^2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[
(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
```

```
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a*(8*A*b + 5*a*B)*Sqrt[a
 + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a*A*(a + b*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+d\ldots}{\text{co}\ldots}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\sin(c+d\ldots}{5d}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\,\text{s}\ldots}{15d}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\,\text{s}\ldots}{15d}$$

$$= -\frac{2b^2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\text{si}\ldots\right.}{d\ldots}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2A+23Ab^2+35abB\right)\sqrt{\cos(\ldots}}{\ldots}$$

Mathematica [B] time = 25.57, size = 7032, normalized size = 12.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2
),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 27.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2
), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.45, size = 3282, normalized size = 5.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-3*A*a^3+9*A*cos(d*x+c)^3*a^3-23*A*cos(d*x+c)^3*b^3-6*A*cos(d*x+c)
^2*a^3+5*B*cos(d*x+c)^3*a^3+35*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+45*B*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*co
s(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2
+23*A*cos(d*x+c)^4*b^3-5*B*cos(d*x+c)*a^3+23*A*cos(d*x+c)^3*a*b^2-34*A*cos(
d*x+c)^2*a*b^2-14*A*cos(d*x+c)*a^2*b+35*B*cos(d*x+c)^4*a*b^2+35*B*cos(d*x+c
)^3*a^2*b-40*B*cos(d*x+c)^2*a^2*b-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-23*A*sin(d*x+c)*co
s(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^2+17*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b+23*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-35*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*
cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b-35*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^2+35*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
```

(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-35*B*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos
(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-
9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b-23*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+17*A*sin(d*x+c)*cos(d*x+c)^3
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+23*A
*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^2-35*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+45*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)
^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-35*B*co
s(d*x+c)^3*a*b^2+9*A*cos(d*x+c)^4*a^2*b+11*A*cos(d*x+c)^4*a*b^2+5*A*cos(d*x
+c)^3*a^2*b+5*B*cos(d*x+c)^4*a^2*b+15*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-15*B*sin(d*x+c)*co
s(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
b^3+30*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*b^3-15*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+30*B*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b
^3+15*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*b^3+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c
)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-23*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*b^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*sin(d*x+c)*cos(

```
d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3-23*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*b^3+9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(
1/2)*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}(a+b\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

3.609 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=596

$$-\frac{\left(6a^2B+14aAb-3b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{3d}-\frac{\sqrt{a+b}\left(-2a^2(A-3B)+2ab(7A-9B)\right.}{}$$

```
[Out] 2/3*a*A*(a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*(2*A*b+B*a
)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-1/3*(14*A*a*b+6*B*a^
2-3*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d+1/3*(a-b)*(
14*A*a*b+6*B*a^2-3*B*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c
)^(1/2)-1/3*(2*a*b*(7*A-9*B)-2*a^2*(A-3*B)-3*b^2*(6*A+B))*csc(d*x+c)*Ellipt
icF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2
))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*
x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-b*(2*A*b+5*B*a)*csc(d*x+c)*EllipticPi
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.90, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2989, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(6a^2B+14aAb-3b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{3d}-\frac{\sqrt{a+b}\left(-2a^2(A-3B)+2ab(7A-9B)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2
*a*b*(7*A - 9*B) - 2*a^2*(A - 3*B) - 3*b^2*(6*A + B))*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d*Sqrt[Sec[c + d*x]]) - (b*Sqrt[a + b]*
(2*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSi
```

```
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a*(2*A*b + a*B)*Sqrt[a + b*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d - ((14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + b*
Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx)}{\cos}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin}{d}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin}{d}$$

$$= \frac{2a(2Ab+aB)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin}{d}$$

$$= -\frac{b\sqrt{a+b}(2Ab+5aB)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(14aAb+6a^2B-3b^2B\right)\sqrt{\cos(c+d}}{}$$

Mathematica [B] time = 26.07, size = 7700, normalized size = 12.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2
),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 65.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2
), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.41, size = 3212, normalized size = 5.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] -1/3/d*(-2*A*a^3+2*A*cos(d*x+c)^2*a^3-14*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-14*A*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b^2+18*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2*b-18*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
```

d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+30*B*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^
2-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^2*b+18*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+3*B*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-18*B*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a*b^2+14*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-3*B*cos(d*x+c)^3*b^3+6*B*cos(d*x+c)^2*
a^3-6*B*cos(d*x+c)*a^3+14*A*cos(d*x+c)^3*a*b^2+14*A*cos(d*x+c)^2*a^2*b-14*A
*cos(d*x+c)^2*a*b^2-16*A*cos(d*x+c)*a^2*b+6*B*cos(d*x+c)^3*a^2*b-6*B*cos(d*
x+c)^2*a^2*b-3*B*cos(d*x+c)^2*a*b^2+3*B*cos(d*x+c)^4*b^3+12*A*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
)*b^3-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3+3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-6*A*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-14*A*sin(d*x
+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^2*b-14*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+14*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+18*A*sin(d*x+c)
*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b^2-6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+18*B*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)
*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b+30*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co

```
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*a*b^2+3*B*cos(d*x+c)^3*a*b^2+2*A*cos(d*x+c)^3*a^2
*b+2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+3*B*sin(d*x+c
)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^3-6*A*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*b^3+12*A*sin(d*x+c)*cos(d*x+c)^2*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^3+6*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*
cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
+2*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/
2)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\,\cos(c+d\,x))\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}\,{(a+b\,\cos(c+d\,x))}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.610 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=607

$$-\frac{\left(8a^2A-9abB-4Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{4d}-\frac{\sqrt{a+b}\,\left(8a^2(A-B)-3ab(8A+3B)-2\right.}{}$$

```
[Out] -1/2*b*(4*A*a-B*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+2*a
*A*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-1/4*(8*A*a^2-4*A*b^
2-9*B*a*b)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d+1/4*(a-b)*(
8*A*a^2-4*A*b^2-9*B*a*b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)
^(1/2)-1/4*(8*a^2*(A-B)-2*b^2*(2*A+B)-3*a*b*(8*A+3*B))*csc(d*x+c)*EllipticF
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-1/4*(20*A*a*b+15*B*a^2+4*B*b^2)*csc(d*x+
c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,(
(-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.88, antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2989, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(8a^2A-9abB-4Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{4d}-\frac{\sqrt{a+b}\,\left(8a^2(A-B)-3ab(8A+3B)-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(8*a^2*A - 4*A*b^2 - 9*a*b*B)*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(4*a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(8*
a^2*(A - B) - 2*b^2*(2*A + B) - 3*a*b*(8*A + 3*B))*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(20*a
*A*b + 15*a^2*B + 4*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a +
```

```
b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (b*(4*a*A - b*B)*Sqrt[a + b*Co
s[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) - ((8*a^2*A - 4*A*b^2 -
9*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d) +
(2*a*A*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx)}{\cos}$$

$$= \frac{2aA(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx}{d}$$

$$= -\frac{b(4aA-bB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{2a}{}$$

$$= -\frac{b(4aA-bB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} - \frac{(8}{}$$

$$= -\frac{b(4aA-bB)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} - \frac{(8}{}$$

$$= -\frac{\sqrt{a+b}\left(20aAb+15a^2B+4b^2B\right)\sqrt{\cos(c+dx)}\ c}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(8a^2A-4Ab^2-9abB\right)\sqrt{\cos(c+dx}}{}$$

Mathematica [B] time = 19.52, size = 1278, normalized size = 2.11

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(2*a^2*A*Sin[c + d*x] + (b^2*B
*Sin[2*(c + d*x)])/4))/d + (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-8*a^3*A*T
an[(c + d*x)/2] - 8*a^2*A*b*Tan[(c + d*x)/2] + 4*a*A*b^2*Tan[(c + d*x)/2] +
 4*A*b^3*Tan[(c + d*x)/2] + 9*a^2*b*B*Tan[(c + d*x)/2] + 9*a*b^2*B*Tan[(c +
 d*x)/2] + 16*a^2*A*b*Tan[(c + d*x)/2]^3 - 8*A*b^3*Tan[(c + d*x)/2]^3 - 18*
a*b^2*B*Tan[(c + d*x)/2]^3 + 8*a^3*A*Tan[(c + d*x)/2]^5 - 8*a^2*A*b*Tan[(c
+ d*x)/2]^5 - 4*a*A*b^2*Tan[(c + d*x)/2]^5 + 4*A*b^3*Tan[(c + d*x)/2]^5 - 9
*a^2*b*B*Tan[(c + d*x)/2]^5 + 9*a*b^2*B*Tan[(c + d*x)/2]^5 + 40*a*A*b^2*Ell
ipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b
)] + 30*a^2*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)] + 8*b^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/
2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 40*a*A*b^2*EllipticPi[-1, ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
 + 30*a^2*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)
/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^3*B*EllipticPi[-1, ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] -
 (a + b)*(8*a^2*A - 4*A*b^2 - 9*a*b*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*(12*a^
2*b*(A - B) - 2*b^3*B + a*b^2*(-12*A + B) + 4*a^3*(A + B))*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)]))/(4*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*
x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 2.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos\left(dx+c\right)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos\left(dx+c\right)^2 + \left(Ba^2 + 2\,Aab\right)\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2
), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.44, size = 3278, normalized size = 5.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*(-8*A*a^3+8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3+30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+4*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b
^3-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)*b^3+8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3+40*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+40*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)*a*b^2+30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+4*A*cos(d*x+c)^3*b^3+8*A*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-8*A*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^2*b+4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
```

x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*cos(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2
+9*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b-24*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+9*B*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*A*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^2*b-4*A*cos(d*x+c)^2*b^3-2*B*cos(d*x+c)^2*b^3+8*A*cos(d*x+c)^2*a^2*
b+4*A*cos(d*x+c)^2*a*b^2-8*A*cos(d*x+c)*a^2*b-4*A*cos(d*x+c)*a*b^2+9*B*cos(
d*x+c)^2*a^2*b-9*B*cos(d*x+c)^2*a*b^2-9*B*cos(d*x+c)*a^2*b-2*B*cos(d*x+c)*a
*b^2+8*A*cos(d*x+c)*a^3+2*B*cos(d*x+c)^4*b^3-24*A*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-8*A*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+4*A*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2
-24*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a^2*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^2+9*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b+9*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+24*A*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+11*B*cos(d*x+c)^3*a*b^
2+8*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a^3+8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+4*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-4*B*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/

```
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b^3+8*B*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3
)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2
), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}(a+b\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2),
 x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.611 $\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=624

$$\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\sqrt{a+b}\left(a^2(48A+33B)+a(54Ab+2\right.}{}$$

```
[Out] 1/3*b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+1/4*b*(2*A*b+3
*B*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/24*(54*A*a*b+3
3*B*a^2+16*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-1/24
*(a-b)*(54*A*a*b+33*B*a^2+16*B*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x
+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
/sec(d*x+c)^(1/2)+1/24*(4*b^2*(3*A+4*B)+a^2*(48*A+33*B)+a*(54*A*b+26*B*b))*
csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((
-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-1/8*(30*A*a^2*b+8*A*
b^3+5*B*a^3+20*B*a*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^
(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/se
c(d*x+c)^(1/2)
```

Rubi [A] time = 1.95, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(33a^2B+54aAb+16b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\sqrt{a+b}\left(a^2(48A+33B)+a(54Ab+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b
]*(4*b^2*(3*A + 4*B) + a^2*(48*A + 33*B) + a*(54*A*b + 26*b*B))*Sqrt[Cos[c
+ d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d*Sqrt[Sec[c + d*x]]) - (Sqr
t[a + b]*(30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
```

```
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b*d*Sqrt[Sec[c + d*x]]) +
(b*(2*A*b + 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c +
 d*x]]) + (b*B*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d
*x]]) + ((54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(24*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
```

```
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx}{\sqrt{c}}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(\sqrt{\cos(c}\right.$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{bl}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{bl}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{bl}{}$$

$$= -\frac{\sqrt{a+b}\left(30a^2Ab+8Ab^3+5a^3B+20ab^2B\right)\sqrt{co}}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(54aAb+33a^2B+16b^2B\right)\sqrt{\cos(}}{}$$

Mathematica [B] time = 19.60, size = 1504, normalized size = 2.41

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*B*Sin[c + d*x])/12 + (b*
(6*A*b + 13*a*B)*Sin[2*(c + d*x)])/24 + (b^2*B*Sin[3*(c + d*x)])/12))/d + (
Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(54*a^2*A*b*Tan[(c + d*x)/2] + 54*a*A*b
^2*Tan[(c + d*x)/2] + 33*a^3*B*Tan[(c + d*x)/2] + 33*a^2*b*B*Tan[(c + d*x)/
2] + 16*a*b^2*B*Tan[(c + d*x)/2] + 16*b^3*B*Tan[(c + d*x)/2] - 108*a*A*b^2*
Tan[(c + d*x)/2]^3 - 66*a^2*b*B*Tan[(c + d*x)/2]^3 - 32*b^3*B*Tan[(c + d*x)
/2]^3 - 54*a^2*A*b*Tan[(c + d*x)/2]^5 + 54*a*A*b^2*Tan[(c + d*x)/2]^5 - 33*
a^3*B*Tan[(c + d*x)/2]^5 + 33*a^2*b*B*Tan[(c + d*x)/2]^5 - 16*a*b^2*B*Tan[(
c + d*x)/2]^5 + 16*b^3*B*Tan[(c + d*x)/2]^5 + 180*a^2*A*b*EllipticPi[-1, Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*A*b^3*
EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
+ b)] + 30*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
 + d*x)/2]^2)/(a + b)] + 120*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 180*a^2*A*b*EllipticPi[-1, Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)] + 48*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^3*B*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
b)] + 120*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(54*a*A*b + 33*a^2*B +
 16*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2
]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*(-12*A*b^3 + 2*a*b^2*(3*A - 19*B)
+ 24*a^3*(A - B) + a^2*(-72*A*b + 13*b*B))*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*
d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan
[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

fricas [F] time = 94.33, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)
), x)
```

maple [B] time = 0.47, size = 3514, normalized size = 5.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/24/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(-48*B*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+48*A*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+180
*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a
-b)/(a+b))^(1/2))*a^2*b+54*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+54*A*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+26*B*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^2*b-76*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+120*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+33*B*sin(d*
```

```
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b+12*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-144*A*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2
*b+12*A*cos(d*x+c)^4*b^3-12*A*cos(d*x+c)^2*b^3+8*B*cos(d*x+c)^5*b^3+8*B*cos
(d*x+c)^3*b^3+33*B*cos(d*x+c)^2*a^3-16*B*cos(d*x+c)^2*b^3-33*B*cos(d*x+c)*a
^3+66*A*cos(d*x+c)^3*a*b^2+54*A*cos(d*x+c)^2*a^2*b-54*A*cos(d*x+c)^2*a*b^2-
54*A*cos(d*x+c)*a^2*b-12*A*cos(d*x+c)*a*b^2+34*B*cos(d*x+c)^4*a*b^2+59*B*co
s(d*x+c)^3*a^2*b-33*B*cos(d*x+c)^2*a^2*b-18*B*cos(d*x+c)^2*a*b^2-26*B*cos(d
*x+c)*a^2*b-16*B*cos(d*x+c)*a*b^2+48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+30*B*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*a^3+33*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^3+16*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+12*A*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+180*A*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+5
4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b+54*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b^2+26*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b-76*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+120*B*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2+33*B*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+16*B*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^2-24*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
```

```
(-(a-b)/(a+b))^(1/2))*b^3-144*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-24*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+48*A*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+30*B*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3+33
*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^3+16*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*b^3+48*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-48*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3)/sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,(a+b\cos(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.612 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=724

$$\frac{\left(5a^2B+24aAb+12b^2B\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{32d\sqrt{\sec(c+dx)}}+\frac{\left(15a^3B+264a^2Ab+284ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{}}{192bd}$$

```
[Out] 1/4*b*B*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+1/24*(8*A*b+11
*B*a)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+1/32*(24*A*a*b+5
*B*a^2+12*B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/192
*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(
1/2)*sec(d*x+c)^(1/2)/b/d-1/192*(a-b)*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B
*a*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^
(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/
(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/sec(d*x+c)^(1/2)+1/192*(1
5*a^3*B+8*b^3*(16*A+9*B)+2*a^2*b*(132*A+59*B)+4*a*b^2*(52*A+71*B))*csc(d*x+
c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*
(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)-1/64*(40*A*a^3*b+160*A*a*b
^3-5*B*a^4+120*B*a^2*b^2+48*B*b^4)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(
1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)
*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/b^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 2.52, antiderivative size = 724, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {2961, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(264a^2Ab+15a^3B+284ab^2B+128Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}{192bd}+\frac{\left(5a^2B+24aAb+12b^2B\right.}{32d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192*a*b*d*Sqrt[Sec[c +
 d*x]]) + (Sqrt[a + b]*(15*a^3*B + 8*b^3*(16*A + 9*B) + 2*a^2*b*(132*A + 59
*B) + 4*a*b^2*(52*A + 71*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcS
```

```
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(40*a^3*A*b + 160*a*A*b
^3 - 5*a^4*B + 120*a^2*b^2*B + 48*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*El
lipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(64*b^2*d*Sqrt[Sec[c + d*x]]) + (b*B*(a + b
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Sec[c + d*x]^(3/2)) + ((24*a*A*b +
5*a^2*B + 12*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(32*d*Sqrt[Sec[c
 + d*x]]) + ((8*A*b + 11*a*B)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(24*
d*Sqrt[Sec[c + d*x]]) + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)
*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
```

```
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
```

```
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,(a+b\cos(c+d$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{(8Ab+11aB)(a+b\cos}{24d\sqrt{\sec}}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(24aAb+5a^2B+12b^2B\right.}{32d}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(24aAb+5a^2B+12b^2B\right.}{32d}$$

$$= \frac{bB(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(24aAb+5a^2B+12b^2B\right.}{32d}$$

$$= -\frac{\sqrt{a+b}\left(40a^3Ab+160aAb^3-5a^4B+120a^2b^2B+48b^4B\right)\sqrt{c}}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(264a^2Ab+128Ab^3+15a^3B+284ab^2B\right)\sqrt{co}}{}$$

Mathematica [B] time = 20.04, size = 1857, normalized size = 2.56

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*(8*A*b + 17*a*B)*Sin[c + d
*x])/96 + ((104*a*A*b + 59*a^2*B + 48*b^2*B)*Sin[2*(c + d*x)])/192 + (b*(8*
A*b + 17*a*B)*Sin[3*(c + d*x)])/96 + (b^2*B*Sin[4*(c + d*x)])/32))/d + (Sqr
```

```
t[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(264*a^3*A*b*Tan[(c + d*x)/2] + 264*a^2*A*
b^2*Tan[(c + d*x)/2] + 128*a*A*b^3*Tan[(c + d*x)/2] + 128*A*b^4*Tan[(c + d*
x)/2] + 15*a^4*B*Tan[(c + d*x)/2] + 15*a^3*b*B*Tan[(c + d*x)/2] + 284*a^2*b
^2*B*Tan[(c + d*x)/2] + 284*a*b^3*B*Tan[(c + d*x)/2] - 528*a^2*A*b^2*Tan[(c
 + d*x)/2]^3 - 256*A*b^4*Tan[(c + d*x)/2]^3 - 30*a^3*b*B*Tan[(c + d*x)/2]^3
 - 568*a*b^3*B*Tan[(c + d*x)/2]^3 - 264*a^3*A*b*Tan[(c + d*x)/2]^5 + 264*a^
2*A*b^2*Tan[(c + d*x)/2]^5 - 128*a*A*b^3*Tan[(c + d*x)/2]^5 + 128*A*b^4*Tan
[(c + d*x)/2]^5 - 15*a^4*B*Tan[(c + d*x)/2]^5 + 15*a^3*b*B*Tan[(c + d*x)/2]
^5 - 284*a^2*b^2*B*Tan[(c + d*x)/2]^5 + 284*a*b^3*B*Tan[(c + d*x)/2]^5 + 24
0*a^3*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] + 960*a*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^4*B*EllipticPi[-1, ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 720*a^2*b^2*B*Ellipt
icPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
+ 288*b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(a + b)] + 240*a^3*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 960*a*A*b^3*Elli
pticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*S
qrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
 d*x)/2]^2)/(a + b)] - 30*a^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 720*a^2*b^2*B*El
lipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
 + d*x)/2]^2)/(a + b)] + 288*b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(264*
a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]
^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2
*b*(a^3*(192*A - 59*B) + 4*a*b^2*(76*A - 9*B) + 72*b^3*B + a^2*(-104*A*b +
322*b*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(192*b*d*(1 + Tan[(c + d*x)/2]^2)^(3/
2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
 d*x)/2]^2)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

maple [B] time = 0.62, size = 4240, normalized size = 5.86

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/192/d*(472*A*cos(d*x+c)^3*a^2*b^2+133*B*cos(d*x+c)^3*a^3*b+172*B*cos(d*x
+c)^3*a*b^3+30*B*cos(d*x+c)^2*a^2*b^2-284*B*cos(d*x+c)^2*a*b^3-118*B*cos(d*
x+c)*a^3*b-284*B*cos(d*x+c)*a^2*b^2-72*B*cos(d*x+c)*a*b^3+264*A*cos(d*x+c)^
2*a^3*b-144*A*cos(d*x+c)^2*a*b^3-208*A*cos(d*x+c)*a^2*b^2-128*A*cos(d*x+c)*
a*b^3+128*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*b^4+15*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^4-30*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4+288*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^4-144*B*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+48*B*cos(d*x+
c)^6*b^4+64*A*cos(d*x+c)^3*b^4-128*A*cos(d*x+c)^2*b^4+24*B*cos(d*x+c)^4*b^4
-72*B*cos(d*x+c)^2*b^4+15*B*cos(d*x+c)^2*a^4-15*B*cos(d*x+c)*a^4+64*A*cos(d
*x+c)^5*b^4-15*B*cos(d*x+c)^2*a^3*b+272*A*cos(d*x+c)^4*a*b^3-264*A*cos(d*x+
```

c)^2*a^2*b^2-264*A*cos(d*x+c)*a^3*b+184*B*cos(d*x+c)^5*a*b^3+254*B*cos(d*x+
c)^4*a^2*b^2+264*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b+15*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-30*B*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*a^4+288*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*b^4-144*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+264*A*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+264*A*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^
2+128*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^3+240*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*a^3*b+960*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^3+208*A*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-608*A*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+15*
B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^3*b+284*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b^2+284*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+720*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+118*B*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-644*B*sin(d
x+c)(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
^2+72*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^3+128*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si

```
n(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+264*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^2+128*A*sin(d*x+c
)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*a*b^3+240*A*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b+960*A*sin(d*x+c)*cos(d*x+c)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^3+208*A*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*b^2-608*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+15*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+284*B*sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2*b^2+284*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^3+720*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b^2+118*B*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3*b-644*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b^2+72*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-384*A*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-384*A*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3*b)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algor
```

```
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\cos(c + d\,x))\,(a + b\cos(c + d\,x))^{5/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.613 $\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=839

$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{5bd\sqrt{\sec(c+dx)}}+\frac{(10Ab-3aB)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{40bd\sqrt{\sec(c+dx)}}+\frac{\left(-15Ba^2+50Aba+64b^2B\right)}{240bd}$$

```
[Out] 1/240*(50*A*a*b-15*B*a^2+64*B*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d/se
c(d*x+c)^(1/2)+1/40*(10*A*b-3*B*a)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d/se
c(d*x+c)^(1/2)+1/5*B*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)
+1/320*(50*A*a^2*b+120*A*b^3-15*B*a^3+172*B*a*b^2)*sin(d*x+c)*(a+b*cos(d*x+
c))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/1920*(150*A*a^3*b+2840*A*a*b^3-45*B*a^4+16
92*B*a^2*b^2+1024*B*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)
/b^2/d-1/1920*(a-b)*(150*A*a^3*b+2840*A*a*b^3-45*B*a^4+1692*B*a^2*b^2+1024*
B*b^4)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(
1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d/sec(d*x+c)^(1/2)-1/1920*
(45*a^4*B-30*a^3*b*(5*A+B)-16*b^4*(45*A+64*B)-8*a*b^3*(355*A+193*B)-4*a^2*b
^2*(295*A+423*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2
)+1/128*(10*A*a^4*b-240*A*a^2*b^3-96*A*b^5-3*B*a^5-40*B*a^3*b^2-240*B*a*b^4
)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c
))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 3.60, antiderivative size = 839, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2990, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{B\sin(c+dx)(a+b\cos(c+dx))^{7/2}}{5bd\sqrt{\sec(c+dx)}}+\frac{(10Ab-3aB)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{40bd\sqrt{\sec(c+dx)}}+\frac{\left(-15Ba^2+50Aba+64b^2B\right)}{240bd}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2
*B + 1024*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(192
```

```
0*a*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(45*a^4*B - 30*a^3*b*(5*A + B)
 - 16*b^4*(45*A + 64*B) - 8*a*b^3*(355*A + 193*B) - 4*a^2*b^2*(295*A + 423*
B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(1920*b^2*d*Sqrt[
Sec[c + d*x]]) + (Sqrt[a + b]*(10*a^4*A*b - 240*a^2*A*b^3 - 96*A*b^5 - 3*a^
5*B - 40*a^3*b^2*B - 240*a*b^4*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticP
i[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(128*b^3*d*Sqrt[Sec[c + d*x]]) + ((50*a^2*A*b + 12
0*A*b^3 - 15*a^3*B + 172*a*b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3
20*b*d*Sqrt[Sec[c + d*x]]) + ((50*a*A*b - 15*a^2*B + 64*b^2*B)*(a + b*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(240*b*d*Sqrt[Sec[c + d*x]]) + ((10*A*b - 3*a*
B)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(40*b*d*Sqrt[Sec[c + d*x]]) + (
B*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(5*b*d*Sqrt[Sec[c + d*x]]) + ((1
50*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^2*b^2*B + 1024*b^4*B)*Sqrt[a
+ b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1920*b^2*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx$$

$$= \frac{B(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{5bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+\right.}{}$$

$$= \frac{(10Ab-3aB)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{40bd\sqrt{\sec(c+dx)}} + \frac{B(a+b\cos(}{5bd}$$

$$= \frac{\left(50aAb-15a^2B+64b^2B\right)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{240bd\sqrt{\sec(c+dx)}} +$$

$$= \frac{\left(50a^2Ab+120Ab^3-15a^3B+172ab^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin}{320bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(50a^2Ab+120Ab^3-15a^3B+172ab^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin}{320bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(50a^2Ab+120Ab^3-15a^3B+172ab^2B\right)\sqrt{a+b\cos(c+dx)}\,\sin}{320bd\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{a+b}\left(10a^4Ab-240a^2Ab^3-96Ab^5-3a^5B-40a^3b^2B-240\right.}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(150a^3Ab+2840aAb^3-45a^4B+1692a^2b^2B\right.}{}$$

Mathematica [A] time = 16.07, size = 703, normalized size = 0.84

$$\frac{\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}\left(\frac{1}{960}\left(93a^2B+170aAb+88b^2B\right)\sin(c+dx)+\frac{1}{960}\left(93a^2B+170aAb+100b^2B\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(((170*a*A*b + 93*a^2*B + 88*b
^2*B)*Sin[c + d*x])/960 + ((590*a^2*A*b + 480*A*b^3 + 15*a^3*B + 1024*a*b^2
*B)*Sin[2*(c + d*x)])/(1920*b) + ((170*a*A*b + 93*a^2*B + 100*b^2*B)*Sin[3*
(c + d*x)])/960 + (b*(10*A*b + 21*a*B)*Sin[4*(c + d*x)])/320 + (b^2*B*Sin[5
*(c + d*x)])/80))/d - (-(b*(a + b)*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B +
 1692*a^2*b^2*B + 1024*b^4*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)]) + a*(a + b)*(45*a^4*B - 30*a^3*b*(5*A + 3*B) + 60*a^2*b^2*(5*A +
11*B) + 16*b^4*(45*A + 64*B) + 8*a*b^3*(265*A + 129*B))*EllipticF[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c +
d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 15*(10*a^4*A*b - 240*a^2*A*b^3 - 96*A*
b^5 - 3*a^5*B - 40*a^3*b^2*B - 240*a*b^4*B)*((a - b)*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] + 2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)] - b*(150*a^3*A*b + 2840*a*A*b^3 - 45*a^4*B + 1692*a^
2*b^2*B + 1024*b^4*B)*(a + b*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2
)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2])/(1920*b^3*d*Sqrt[a + b*Cos[c + d*x]]
*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]^(3/2))
```

fricas [F] time = 5.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bb^2 \cos(dx+c)^3 + Aa^2 + \left(2\,Bab + Ab^2\right)\cos(dx+c)^2 + \left(Ba^2 + 2\,Aab\right)\cos(dx+c)\right)\sqrt{b\cos(dx+c)}}{\sec(dx+c)^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c) + A)(b\cos(dx+c) + a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

maple [B] time = 0.84, size = 5172, normalized size = 6.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.614 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=403

$$-\frac{2(4Ab-5aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{15a^2d}+\frac{2(a-b)\sqrt{a+b}\left(9a^2A-10abB+8Ab^2\right)\sqrt{\cos(c+}}{}$$

```
[Out] -2/15*(4*A*b-5*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/
d+2/5*A*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d+2/15*(a-b)*(
9*A*a^2+8*A*b^2-10*B*a*b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*
(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/sec(d*x
+c)^(1/2)-2/15*(8*A*b^2+a^2*(9*A-5*B)-2*a*b*(A+5*B))*csc(d*x+c)*EllipticF((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.03, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 3000, 3055, 2998, 2816, 2994}

$$-\frac{2\sqrt{a+b}\left(a^2(9A-5B)-2ab(A+5B)+8Ab^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\right.}{15a^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c + d*x]]*Cs
c[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[
c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^4*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a
+ b]*(8*A*b^2 + a^2*(9*A - 5*B) - 2*a*b*(A + 5*B))*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(4*A*b - 5
*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d)
+ (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
```

```
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{s\right.}{}$$

$$= -\frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} + \frac{2A\sqrt{a}}{}$$

$$= -\frac{2(4Ab-5aB)\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} + \frac{2A\sqrt{a}}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2A+8Ab^2-10abB\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\Big(}{15a^4d\sqrt{\sec(c-}}$$

Mathematica [B] time = 22.26, size = 2987, normalized size = 7.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2))/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(9*a^2*A + 8*A*b^2 - 10*a*
b*B)*Sin[c + d*x])/(15*a^3) + (2*Sec[c + d*x]*(-4*A*b*Sin[c + d*x] + 5*a*B*
Sin[c + d*x]))/(15*a^2) + (2*A*Sec[c + d*x]*Tan[c + d*x])/(5*a)))/d + (2*((
-3*A)/(5*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^2)/(15*a^2*S
qrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*b*B)/(3*a*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]]) - (7*A*b*Sqrt[Sec[c + d*x]])/(15*a*Sqrt[a + b*
Cos[c + d*x]]) - (8*A*b^3*Sqrt[Sec[c + d*x]])/(15*a^3*Sqrt[a + b*Cos[c + d*
x]]) + (B*Sqrt[Sec[c + d*x]])/(3*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*B*Sqrt[
Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Cos[c + d*x]]) - (3*A*b*Cos[2*(c + d*x)]*S
qrt[Sec[c + d*x]])/(5*a*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^3*Cos[2*(c + d*x
)]*Sqrt[Sec[c + d*x]])/(15*a^3*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*B*Cos[2*(
c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(
8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2*A + 8*A*b^2 - 10*a*b*B)
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1
5*a^3*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(a + b)*(9*a^2*A + 8*A*b^2 - 10*a
*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)] + 2*a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2*A + 8*A*b^2
- 10*a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2]))/(15*a^3*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(9*a^2*A +
 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)] + 2*a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a
^2*A + 8*A*b^2 - 10*a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(15*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)
/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1/2*((9*a^2*A + 8*A*b^
2 - 10*a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4) - ((a +
 b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos
```

```
[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*
x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(8*A*b^2 + 2*a*b*(A - 5*B)
 + a^2*(9*A + 5*B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c
 + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Co
s[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(8*A*b^2 + 2*a*b*(A - 5*B) + a
^2*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2
)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(9*a^2*A +
8*A*b^2 - 10*a*b*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d
*x)/2] + (9*a^2*A + 8*A*b^2 - 10*a*b*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (9*a^2*A + 8*A*b^2 - 10*a*b*B)*Cos[c +
 d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(8*A*
b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2
]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(
a + b)]) - ((a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(
c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan
[(c + d*x)/2]^2]))/(15*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2
]) + ((-2*(a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(8*A*b^2 + 2*a*b*(A -
5*B) + a^2*(9*A + 5*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] - (9*a^2*A + 8*A*b^2 - 10*a*b*B)*Cos[c + d*x]*(a + b*
Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[
c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))
/(15*a^3*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*
x)/2]^2*Sec[c + d*x]])))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

maple [B] time = 0.41, size = 2488, normalized size = 6.17

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(-3*A*a^3+9*A*cos(d*x+c)^3*a^3-8*A*cos(d*x+c)^3*b^3-6*A*cos(d*x+c)^
2*a^3+5*B*cos(d*x+c)^3*a^3-10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*cos(d*x+c)^4*b^3-5*B
*cos(d*x+c)*a^3+8*A*cos(d*x+c)^3*a*b^2-4*A*cos(d*x+c)^2*a*b^2+A*cos(d*x+c)*
a^2*b-10*B*cos(d*x+c)^4*a*b^2-10*B*cos(d*x+c)^3*a^2*b+5*B*cos(d*x+c)^2*a^2*
b-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*A*sin(d*x+c)*cos(d*x+c)^2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^2+10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+10*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^
2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-10*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
```

```
))^(1/2))*a^2*b+10*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-9*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-8*A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin
(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b^2+2*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+8*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+10*B*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+
c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b+10*B*cos(d*x+c)^3*a*b^2+9*A*cos(d*x+c)^4*a^2*b-4*A*cos(d*x+c)^4*a*b^2
-10*A*cos(d*x+c)^3*a^2*b+5*B*cos(d*x+c)^4*a^2*b+5*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x
+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-9*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^3-8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+5*B*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c
)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^3-9*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+9*A*sin(d*x+c)*cos(d*x+c)^2*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3)*cos(d*x+
c)*(1/cos(d*x+c))^(7/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\cos(c + d x))\left(\frac{1}{\cos(c+d x)}\right)^{7/2}}{\sqrt{a + b\cos(c + d x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b*cos(c + d*x))^(1/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.615 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=330

$$-\frac{2(a-b)\sqrt{a+b}\,(2Ab-3aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|}{3a^3d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*A*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d-2/3*(a-b)*(2*A
*b-3*B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)+2/3*(2
*A*b+a*(A-3*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos
(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 3000, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\,(a(A-3B)+2Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-}{3a^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(3*a^3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(2*A*b + a
*(A - 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d*
Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[
c + d*x])/(3*a*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
```

```
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
```

```
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{s\right.}{}$$

$$= \frac{2A\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\left((2Ab+a(A-3B)\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}(2Ab-3aB)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a+b}}\right.\right.}{3a^3d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 15.82, size = 355, normalized size = 1.08

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2(3aB-2Ab)\sin(c+dx)}{3a^2}+\frac{2A\tan(c+dx)}{3a}\right)}{d}+\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left((2Ab-3\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-2*A*b + 3*a*B)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(
-2*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)] + (2*A*b - 3*a*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-2*
A*b + 3*a*B)*Sin[c + d*x])/(3*a^2) + (2*A*Tan[c + d*x])/(3*a)))/d
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

maple [B] time = 0.47, size = 1544, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(3*B*cos(d*x+c)^2*a^2-3*B*cos(d*x+c)*a^2-2*A*cos(d*x+c)^3*b^2+2*A*co
s(d*x+c)^2*b^2-a^2*A+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+A*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*a^2+2*A*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)
^2*b^2+3*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*cos(d*x+c)^2*a^2-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(
```

```
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+A*sin(d*x+c)*cos(d*x+c)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2+3*B*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d
*x+c)*a^2+A*cos(d*x+c)^2*a^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+2*A*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-2*A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)*a*b+A*cos(d*x+c)^3*a*b-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a*b-2*A*sin(d*x+c)*cos(d*x+c
)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-2*A
*cos(d*x+c)^2*a*b+A*cos(d*x+c)*a*b+3*B*cos(d*x+c)^3*a*b-3*B*cos(d*x+c)^2*a*
b+2*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(a
+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(1/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.616 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=270

$$\frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-2\sqrt{a}$$

```
[Out] 2*A*(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)-2*(A-B
)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 2998, 2816, 2994}

$$\frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-2\sqrt{a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*A*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*(A - B)*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx \\
&= \left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx \\
&= \frac{2A(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{a^2d\sqrt{\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 14.30, size = 279, normalized size = 1.03

$$\frac{2\left(A\sin(c+dx)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))-\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-2a(A+B)\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin\cdots\right.\right.}{\cdots}\right.}{ad\sqrt{a\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] (2*(A*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] - (Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*(2*A*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sq
rt[(1 + Sec[c + d*x])^(-1)] - 2*a*(A + B)*Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Sqrt[(1 + Sec[c + d*x])^(-1)] + A*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)/2]^2]))/(a*d*Sqrt[a + b
*Cos[c + d*x]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

maple [B] time = 0.40, size = 812, normalized size = 3.01

$$-\frac{2\left(A\sin\left(dx+c\right)\cos\left(dx+c\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\ \mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)a-A\sin\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+B*sin(d*x+c)*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-A*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+
c)+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(
d*x+c)+A*cos(d*x+c)^2*b+A*cos(d*x+c)*a-A*cos(d*x+c)*b-a*A)*cos(d*x+c)*(1/co
s(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(1/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

3.617 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=268

$$\frac{2A\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}-\frac{2B\sqrt{a+b}\,\sqrt{}}{}$$

```
[Out] 2*A*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)-2*B*csc(d*x+c)
*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2961, 3006, 2809, 2816}

$$\frac{2A\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}-\frac{2B\sqrt{a+b}\,\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*A*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d
*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3006

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[B/d, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Dist
[(B*c - A*d)/d, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \left(A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\,\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2A\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 2.58, size = 157, normalized size = 0.59

$$\frac{2\sqrt{\sec(c+dx)+1}\,\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\,\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\,\left((A-B)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+\right.}{d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\,\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/Sqrt[a + b*Cos[c + d*x]
],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((A - B)*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*B*EllipticPi[-1, ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]
*Sqrt[1 + Sec[c + d*x]])/(d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^
2])
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

maple [A] time = 0.38, size = 199, normalized size = 0.74

$$\frac{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\left(A\,\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)-B\,\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)+2B\,\text{E}\right.}{d\sqrt{a+b\cos(dx+c)}\,(-1+\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-B*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))+2*B*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))/(a+b*cos(d*x+c))^(1/2)*(1
/cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{\sqrt{a+b\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(1/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/sqrt(a + b*cos(c + d*x)),
x)
```

3.618 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=487

$$-\frac{\sqrt{a+b}\,(2Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{b^2d\sqrt{\sec(c+dx)}}$$

```
[Out] B*sin(d*x+c)/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+a*B*sin(d*x+c)*sec(d
*x+c)^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)-(a-b)*B*csc(d*x+c)*EllipticE((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1
/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b)
)^(1/2)/a/b/d/sec(d*x+c)^(1/2)+B*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/se
c(d*x+c)^(1/2)-(2*A*b-B*a)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x
+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2
/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.27, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 3003, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\,(2Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{b^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*A*b
- a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b
^2*d*Sqrt[Sec[c + d*x]]) + (B*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) + (a*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[a + b*Co
s[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)$$

$$= \frac{B\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}\,\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int}{2}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\,\sqrt{\cos}}\right.\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\,\sqrt{\cos}}\right.\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\right.}{abd\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 18.43, size = 1091, normalized size = 2.24

$$\frac{\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)-b\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}}\left(-a\sqrt{\frac{a-b}{a+b}}\,B\tan^5\left(\frac{1}{2}(c+dx)\right)+b\sqrt{\frac{a-b}{a+b}}\,B\tan^5\left(\frac{1}{2}(c+dx)\right)-2b\sqrt{\frac{a-b}{a+b}}\,B\tan^3\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] (Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d
*x)/2]^2)]*(a*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2] + b*Sqrt[(a - b)/(a
+ b)]*B*Tan[(c + d*x)/2] - 2*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2]^3 -
 a*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + b*Sqrt[(a - b)/(a + b)]*B*T
an[(c + d*x)/2]^5 - (4*I)*A*b*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a
 - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
 + (2*I)*a*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Ta
```

```
n[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*A*b*Elli
pticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]],
-((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*a*B*El
lipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]]
, -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*
B*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a
 - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b +
 a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(A*b - a*B)*
EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a -
 b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b*Sqrt[(a - b)/(a +
 b)]*d*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c
+ d*x)/2]^2)]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\,\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

maple [B] time = 0.46, size = 1004, normalized size = 2.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(4*A*sin(d*x+c)*cos(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*b-2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*B*sin(d*x+c)*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+B*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*b+4*A*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*b-2*A*sin(d*x+c)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b-2*B*sin(d*x+c)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a+B*sin(d*x+c)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a+B*sin(d*x+c)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+B*cos(d*x+c)^3*b+B*co
s(d*x+c)^2*a-b*B*cos(d*x+c)^2-B*cos(d*x+c)*a)*(1/cos(d*x+c))^(1/2)/(a+b*cos
(d*x+c))^(1/2)/sin(d*x+c)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{\sqrt{a + b\cos(c + dx)}\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x)))
, x)
```

3.619 $\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=539

$$\frac{\sqrt{a+b}\left(-3a^2B+4aAb-4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{4b^3d\sqrt{\sec(c+dx)}}$$

```
[Out] 1/2*B*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/4*(4*A*b-3*B
*a)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b^2/d-1/4*(a-b)*(4*A
*b-3*B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d/sec(d*x+c)^(1/2)+1/4*
(4*A*b-3*B*a+2*B*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-
sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1
/2)+1/4*(4*A*a*b-3*B*a^2-4*B*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1
/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*
cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.25, antiderivative size = 539, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2990, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(-3a^2B+4aAb-4b^2B\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{4b^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(4*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((
a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
d*x]))/(a - b)])/(4*a*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(4*A*b - 3*a
*B + 2*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*
Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(4*a*A*b - 3*a^2*B - 4*b^2*B)*Sqrt[Cos[c
 + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
```

```
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^3*d*Sqrt[Sec[c
 + d*x]]) + (B*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d*Sqrt[Sec[c + d
*x]]) + ((4*A*b - 3*a*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(4*b^2*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2990

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
+ 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*
x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m -
 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m + n
)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ[n
, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\frac{aB}{2}+}{}}{2b}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{(4Ab-3aB)\sqrt{a+b\cos(c+dx)}\,\sqrt{\mathrm{se}}}{4b^2d}$$

$$= \frac{B\sqrt{a+b\cos(c+dx)}\,\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{(4Ab-3aB)\sqrt{a+b\cos(c+dx)}\,\sqrt{\mathrm{se}}}{4b^2d}$$

$$= \frac{\sqrt{a+b}\,\left(4aAb-3a^2B-4b^2B\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\right(}{4b^3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(4Ab-3aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,\mathrm{co}}}{\sqrt{a+b}\,\sqrt{\mathrm{c}}}\right.\right.}{4ab^2d\sqrt{\sec(c+dx)}}$$

Mathematica [B] time = 19.73, size = 1157, normalized size = 2.15

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] (B*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*b*d) +
(Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-4*a*A*b*Tan[(c + d*x)/2] -
 4*A*b^2*Tan[(c + d*x)/2] + 3*a^2*B*Tan[(c + d*x)/2] + 3*a*b*B*Tan[(c + d*x
)/2] + 8*A*b^2*Tan[(c + d*x)/2]^3 - 6*a*b*B*Tan[(c + d*x)/2]^3 + 4*a*A*b*Ta
n[(c + d*x)/2]^5 - 4*A*b^2*Tan[(c + d*x)/2]^5 - 3*a^2*B*Tan[(c + d*x)/2]^5
+ 3*a*b*B*Tan[(c + d*x)/2]^5 + 8*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c
+ d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^2*B*EllipticPi[-1, ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*b^2*B*Elli
```

```
pticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)
] + 8*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[
(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2
]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^2*B*EllipticPi[-1, ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8
*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
 b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(-4*A*b + 3*a*B)*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)] - 2*(a - 2*b)*b*B*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*b^2*d*Sqrt[1 + T
an[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2
)))
```

fricas [F] time = 1.84, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2))
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

maple [B] time = 0.39, size = 1878, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)

[Out] 1/4/d*(-4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*b^2+3*B*cos(d*x+c)^2*a^2-2*B*cos(d*x+c)^4*b^2+2*B*cos(d*x+c)^2*
b^2-3*B*cos(d*x+c)*a^2-4*A*cos(d*x+c)^3*b^2+4*A*cos(d*x+c)^2*b^2+8*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)*a*b+3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)*a*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b-4*A*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-6*B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*a^2-4*A*cos(d*x+c)^2*a*b+4*A*cos(d*x+c)*a*b+B*cos(d*x+c)^3*a*b-3*B*cos(d
*x+c)^2*a*b+2*B*cos(d*x+c)*a*b-6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2+8*A*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-4*A*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+3
*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b-4*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2+3*B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2
+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)*b^2-8*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*b^2+3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d

```
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2)*cos(d*x+c)*(1/cos(d*x+c))^
(3/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\sqrt{a+b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**(3/2)
), x)
```

3.620 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=433

$$\frac{2(a+2b)(a(A-3B)+4Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\right.}{3a^3d\sqrt{a+b}\,\sqrt{\sec(c+dx)}}$$

```
[Out] 2*b*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1
/2)+2/3*(A*a^2-4*A*b^2+3*B*a*b)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c)
)^(1/2)/a^2/(a^2-b^2)/d-2/3*(5*A*a^2*b-8*A*b^3-3*B*a^3+6*B*a*b^2)*csc(d*x+c
)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-
b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/a^4/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+2/3*(a+2*b)*(4*A*b+a*(A-3
*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/
2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.16, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 3000, 3055, 2998, 2816, 2994}

$$\frac{2\left(a^2A+3abB-4Ab^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(a
 + 2*b)*(4*A*b + a*(A - 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(3*a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*
(a^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
```

```
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{a+}}{3a^2}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2A-4Ab^2+3abB\right)\sqrt{a+}}{3a^2}$$

$$= -\frac{2\left(5a^2Ab-8Ab^3-3a^3B+6ab^2B\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\right.}{3a^4\sqrt{a+b}\,d\sqrt{\sec(c}}$$

Mathematica [B] time = 24.41, size = 3433, normalized size = 7.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-5*a^2*A*b + 8*A*b^3 + 3*
a^3*B - 6*a*b^2*B)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)) + (2*(-(A*b^3*Sin[c +
d*x]) + a*b^2*B*Sin[c + d*x]))/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*
A*Tan[c + d*x])/(3*a^2)))/d + (2*((5*A*b)/(3*(a^2 - b^2)*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) - (8*A*b^3)/(3*a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) - (a*B)/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]) + (2*b^2*B)/(a*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) + (a*A*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]) + (7*A*b^2*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[a + b*Cos[c +
 d*x]]) - (8*A*b^4*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]) - (2*b*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]
) + (2*b^3*B*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]])
 + (5*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*Sqrt[a +
b*Cos[c + d*x]]) - (8*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^
2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) - (b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x
]])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]) + (2*b^3*B*Cos[2*(c + d*x)]*Sqrt
[Sec[c + d*x]])/(a^2*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^
2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] + 2*a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (-5*a^2*A*b + 8*A*b^3 +
 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(
c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(-2*(
a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a^2 - a*b - 2*b^2)
*(-4*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] - (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Cos[c
+ d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a
^2 - b^2)*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[
(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(-5*a^2*A*b + 8*A
*b^3 + 3*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
```

b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1/2*((-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4) - ((a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a^2 - a*b - 2*b^2)*(-4*A*b + a*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*

```
x]))/(3*a^3*(a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

maple [B] time = 0.40, size = 3343, normalized size = 7.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(-5*A*cos(d*x+c)^3*a^2*b^2+3*B*cos(d*x+c)^3*a^3*b-6*B*cos(d*x+c)^3*a
*b^3-6*B*cos(d*x+c)^2*a^2*b^2+6*B*cos(d*x+c)^2*a*b^3+3*B*cos(d*x+c)*a^2*b^2
-5*A*cos(d*x+c)^2*a^3*b+8*A*cos(d*x+c)^2*a*b^3-4*A*cos(d*x+c)*a*b^3+A*a^2*b
^2+8*A*cos(d*x+c)^3*b^4-8*A*cos(d*x+c)^2*b^4+3*B*cos(d*x+c)^2*a^4-3*B*cos(d
*x+c)*a^4+A*cos(d*x+c)^2*a^4-3*B*cos(d*x+c)^2*a^3*b+A*cos(d*x+c)^3*a^3*b-4*
A*cos(d*x+c)^3*a*b^3+4*A*cos(d*x+c)^2*a^2*b^2+4*A*cos(d*x+c)*a^3*b+3*B*cos(
d*x+c)^3*a^2*b^2+5*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
```

c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-8*A*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4
-A*a^4+5*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*a^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^3+2*A*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+8
*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^3-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+6*B*sin(d*x
+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^3-3*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^3*b-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+5*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*
b+5*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-5*A*sin(d*x+c)*cos(d*x+c)
^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*a^2*b^2+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b^3-3*B*sin(d*x+c)*cos(d*x+c)^2*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+6*B*si
n(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b^2+6*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^

```
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-3*B*sin(d*x+c)*cos(d*x+c)^2*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-6*B*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^2*b^2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4-8*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+A*sin(d*x+c)*cos(d*x
+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-3
*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)^2*a^4+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^4+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4-5*A*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3*b)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b
)/(a-b)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(3/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.621 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=345

$$\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2(a(A-B)+2Ab)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+}{a-b}}}{a^2d\sqrt{a+b}\,\sqrt{\sec(c+dx)}}$$

```
[Out] 2*b*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1
/2)+2*(A*a^2-2*A*b^2+B*a*b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+
b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)/sec(d
*x+c)^(1/2)-2*(2*A*b+a*(A-B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(
a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-se
c(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)/sec
(d*x+c)^(1/2)
```

Rubi [A] time = 0.79, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 3000, 2998, 2816, 2994}

$$\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2A+abB-2Ab^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+}{a-}}}{a^3d\sqrt{a+b}\,\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2*A - 2*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(2*A*b + a*(A - B))*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqr
t[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d*Sqrt[
Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2
- b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
```

```
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,d$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left((a-b)(2Ab+a(A-B))\sqrt{c}\right.}{}$$

$$= \frac{2\left(a^2A-2Ab^2+abB\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c-}}\right.\right.}{a^3\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 18.94, size = 433, normalized size = 1.26

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\left(a^2A+abB-2Ab^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)} - \frac{2\left(abB\sin(c+dx)-Ab^2\sin(c+dx)\right)}{a\left(a^2-b^2\right)(a+b\cos(c+dx))}\right)}{d} + \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\operatorname{se}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(a^2*A - 2*A*b^2 + a*b*B)*
Sin[c + d*x])/(a^2*(a^2 - b^2)) - (2*(-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c +
 d*x]))/(a*(a^2 - b^2)*(a + b*Cos[c + d*x]))))/d + (2*Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*(-2*(a + b)*(a^2*A - 2*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b
+ a*(A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)] - (a^2*A - 2*A*b^2 + a*b*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

maple [B] time = 0.44, size = 2291, normalized size = 6.64

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d*(-A*a^3+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3+A*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+A*a*b^2-A*sin(
d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
```

```
(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-B*sin(d*x+c)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*A
*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^2-B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-2*A*cos(d*x+c)^2*b^
3+A*cos(d*x+c)^2*a^2*b+A*cos(d*x+c)^2*a*b^2-A*cos(d*x+c)*a^2*b-2*A*cos(d*x+
c)*a*b^2-B*cos(d*x+c)^2*a^2*b+B*cos(d*x+c)^2*a*b^2+B*cos(d*x+c)*a^2*b-B*cos
(d*x+c)*a*b^2+A*cos(d*x+c)*a^3-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+B*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-B*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-
B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^2+2*A*cos(d*x+c)*b^3-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+B*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3-A*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+
2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*b^3)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/
a^2/(a-b)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(3/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.622 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=324

$$-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\mathrm{si}\right.}{a^2d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

```
[Out] -2*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
+2*(A*b-B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^
(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+2*(
A+B)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/
2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2961, 2993, 2998, 2816, 2994}

$$-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\mathrm{si}\right.}{a^2d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*S
qrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A + B)*Sqrt[Cos[c + d*x]]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(A*b - a
*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]
])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{3/2}}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx\right.}{a^2}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left((a-b)(A+B)\sqrt{\cos(c+d\right.}{}$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)\mid -\frac{a+}{a-}\right.}{a^2\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 13.78, size = 305, normalized size = 0.94

$$\frac{2\left(\frac{b(Ab-aB)\sin(c+dx)}{\sqrt{\sec(c+dx)}} + \frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-\left((Ab-aB)\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))\right)+2a(a+b)(A-B)\sqrt{\frac{}{\sec(}}\right.}{}\right.}{d\left(a^3-ab^2\right)\sqrt{a+b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(3
/2),x]
```

```
[Out] (2*((b*(A*b - a*B)*Sin[c + d*x])/Sqrt[Sec[c + d*x]] + (Sqrt[Cos[(c + d*x)/2
]^2*Sec[c + d*x]]*(2*(a + b)*(-(A*b) + a*B)*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*
x])/((a + b)*(1 + Sec[c + d*x]))] + 2*a*(a + b)*(A - B)*EllipticF[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b +
a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (A*b - a*B)*Cos[c + d*x]*(a
 + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)
/2]^2]))/((a^3 - a*b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sqrt{\sec(dx+c)}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2
), x)
```

maple [B] time = 0.45, size = 1636, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(A*sin(d*x+c)*cos(d*x+c)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2+A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)*a*b-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*a*b-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-B*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*cos(d*x+c)*a^2-B*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*
b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*a^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
```

```
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+A*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-A*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b-A*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-B*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-B*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+B*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*b-A*cos(d*x+c)^2*a*b+A*cos(d*x+c)^2*b^2+B*cos(d*x+c)^2*a^2-B*cos(d*x
+c)^2*a*b+A*cos(d*x+c)*a*b-A*cos(d*x+c)*b^2-B*cos(d*x+c)*a^2+B*cos(d*x+c)*a
*b)/sin(d*x+c)/(a+b)/(a-b)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos(c+d\,x))\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{(a+b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(3/2
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\cos(c + dx))\sqrt{\sec(c + dx)}}{(a + b\cos(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**(3/2
), x)
```

3.623 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=476

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\Big(\sin}{abd\sqrt{a+b}\,\sqrt{\sec(c+dx)}}$$

```
[Out] 2*a*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1
/2)-2*(A*b-B*a)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos
(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+
2*(A*b-B*a)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x
+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)-2*B*
csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(
a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))
/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2961, 2992, 2809, 2794, 2795, 2816, 2994}

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\Big(\sin}{abd\sqrt{a+b}\,\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*
Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(a*b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*S
qrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(b^2*d*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2794

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[(-2*a*d*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2992

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> D
ist[B/b, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Di
st[(A*b - a*B)/b, Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}$$

$$= \frac{\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} + \frac{\left((Ab-aB)\sqrt{\text{c}\right.}{}$$

$$= -\frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\right.}{ab\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 13.98, size = 1403, normalized size = 2.95

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(A*b - a*B)*Sin[c + d*x])/
(b*(-a^2 + b^2)) - (2*(a*A*b*Sin[c + d*x] - a^2*B*Sin[c + d*x]))/(b*(-a^2 +
 b^2)*(a + b*Cos[c + d*x]))))/d + (2*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b
*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a*A*b*Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2] + A*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] - a^2*Sqrt
[(a - b)/(a + b)]*B*Tan[(c + d*x)/2] - a*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c +
 d*x)/2] - 2*A*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 + 2*a*b*Sqrt[(a
 - b)/(a + b)]*B*Tan[(c + d*x)/2]^3 - a*A*b*Sqrt[(a - b)/(a + b)]*Tan[(c +
d*x)/2]^5 + A*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 + a^2*Sqrt[(a -
b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - a*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x
)/2]^5 - (2*I)*a^2*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a
+ b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*
b^2*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c +
d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*B*EllipticP
i[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a
+ b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*b^2*B*Ellip
ticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -
((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*(-(
A*b) + a*B)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -(
(a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b
)*(-(A*b) + (2*a + b)*B)*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c +
 d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d
*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b
)]))/(b*Sqrt[(a - b)/(a + b)]*(a^2 - b^2)*d*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[
(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b*(-1 + Tan[(c + d*x)/2
]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + a^2)*sqrt(sec(d*x + c))), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.42, size = 2016, normalized size = 4.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/d*(-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*b^2+B*cos(d*x+c)^2*a^2-B*cos(d*x+c)*a^2+A*cos(d*x+c)^2*b^2-A*cos(d*
x+c)*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*a*b-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a*b-A*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+A*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*
b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*a^2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*sin(d*x+c)*cos(d*x+c)*b^2-A*cos(d*x+c)^2*a*b+A*cos(d*x+c)*a*b-B*cos(d*
x+c)^2*a*b+B*cos(d*x+c)*a*b-2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2-A*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+A*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+B*sin(d*x+c)
```

```
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-B*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+
A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*b^2-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*b^2+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a
+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*a^2-B*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
*b^2+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(
a+b))^(1/2))*b^2+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2-B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*b^2)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+
c))^(1/2)/(a+b)/(a-b)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2)
), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{\frac{3}{2}}\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((a + b*cos(c + d*x))**(3/2)*sqrt(sec(c + d*x
))), x)
```

3.624 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=560

$$-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+}}$$

```
[Out] 2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1
/2)-(2*A*a*b-3*B*a^2+B*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1
/2)/b^2/(a^2-b^2)/d+(2*A*a*b-3*B*a^2+B*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^
(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b^2/d
/(a+b)^(1/2)/sec(d*x+c)^(1/2)-(2*A*b-(3*a+b)*B)*csc(d*x+c)*EllipticF((a+b*c
os(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x
+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2
/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)-(2*A*b-3*B*a)*csc(d*x+c)*EllipticPi((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.51, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2989, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)}+\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
 - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
 - b)])/(a*b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - ((2*A*b - (3*a + b)*B)*S
qrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*Sqrt[a + b]*d*Sqr
t[Sec[c + d*x]]) - (Sqrt[a + b]*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
```

```
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) + (2*a*(A
*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\mathrm{s}\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(2aAb-3a^2B+b^2\right.}{}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-3aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right.\right.}{b^3d\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(2aAb-3a^2B+b^2B\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{ab^2\sqrt{a+b}\,d\sqrt{\sec(c+dx)}}$$

Mathematica [B] time = 19.56, size = 1551, normalized size = 2.77

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a*(-(A*b) + a*B)*Sin[c +
d*x])/(b^2*(a^2 - b^2)) + (2*(a^2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x]))/(
b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d - (Sqrt[(1 - Tan[(c + d*x)/2]^2)
^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[
(c + d*x)/2]^2)]*(2*a^2*A*b*Tan[(c + d*x)/2] + 2*a*A*b^2*Tan[(c + d*x)/2] -
 3*a^3*B*Tan[(c + d*x)/2] - 3*a^2*b*B*Tan[(c + d*x)/2] + a*b^2*B*Tan[(c + d
*x)/2] + b^3*B*Tan[(c + d*x)/2] - 4*a*A*b^2*Tan[(c + d*x)/2]^3 + 6*a^2*b*B*
Tan[(c + d*x)/2]^3 - 2*b^3*B*Tan[(c + d*x)/2]^3 - 2*a^2*A*b*Tan[(c + d*x)/2
]^5 + 2*a*A*b^2*Tan[(c + d*x)/2]^5 + 3*a^3*B*Tan[(c + d*x)/2]^5 - 3*a^2*b*B
*Tan[(c + d*x)/2]^5 - a*b^2*B*Tan[(c + d*x)/2]^5 + b^3*B*Tan[(c + d*x)/2]^5
```

```
 - 4*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d
*x)/2]^2)/(a + b)] + 4*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^
2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*B*EllipticPi[-1, ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a*b^2*B*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a^
2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d
*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*B
*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2
]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan
[(c + d*x)/2]^2)/(a + b)] - 6*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(-
2*a*A*b + 3*a^2*B - b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a
*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b*(a + b)*(-(A*b)
+ a*B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b^2*(-a^2 + b^2)*d*Sqrt[1 + Tan[(c + d*
x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F] time = 53.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^(3/2)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.38, size = 2890, normalized size = 5.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/d*(-4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*
B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b+2*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2-3*B*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*b-2*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a*b^2+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+B*cos(d*x+c)^3*b^3-3*B*c
os(d*x+c)^2*a^3-B*cos(d*x+c)^2*b^3+3*B*cos(d*x+c)*a^3+2*A*cos(d*x+c)^2*a^2*
b-2*A*cos(d*x+c)^2*a*b^2-2*A*cos(d*x+c)*a^2*b+2*A*cos(d*x+c)*a*b^2-B*cos(d*
x+c)^3*a^2*b+3*B*cos(d*x+c)^2*a^2*b+B*cos(d*x+c)^2*a*b^2-2*B*cos(d*x+c)*a^2
*b-B*cos(d*x+c)*a*b^2+4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3+6*B*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3-3*B*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^3+B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
```

```
c),(-(a-b)/(a+b))^(1/2))*b^3-2*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+2*A*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+2*
B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b+2*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^2-6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*a*b^2-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2-2*A*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3-2*A*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^
3+4*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*b^3+6*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a^3-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3+B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/si
n(d*x+c)/(a+b*cos(d*x+c))^(1/2)/(a+b)/(a-b)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.625 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=607

$$\frac{2b(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2b\left(-7a^3B+10a^2Ab+3ab^2B-6Ab^3\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^4A-\right.}{}$$

```
[Out] 2/3*b*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^
(3/2)+2/3*b*(10*A*a^2*b-6*A*b^3-7*B*a^3+3*B*a*b^2)*sec(d*x+c)^(3/2)*sin(d*x
+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(A*a^4-13*A*a^2*b^2+8*A*b^
4+8*B*a^3*b-4*B*a*b^3)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a
^3/(a^2-b^2)^2/d-2/3*(8*A*a^4*b-28*A*a^2*b^3+16*A*b^5-3*B*a^5+15*B*a^3*b^2-
8*B*a*b^4)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1
/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^5/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)
-2/3*(16*A*b^4-a^4*(A-3*B)+4*a*b^3*(3*A-2*B)-9*a^3*b*(A-B)-2*a^2*b^2*(8*A+3
*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/
2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a
*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a^2-b^2)/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 2.20, antiderivative size = 607, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 3000, 3055, 2998, 2816, 2994}

$$\frac{2\left(-13a^2Ab^2+a^4A+8a^3bB-4ab^3B+8Ab^4\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d\left(a^2-b^2\right)^2}+\frac{2b\left(10a^2Ab-7a^3B-\right.}{3a^2d\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(8*a^4*A*b - 28*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4
*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5*(a - b)*(a
 + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(16*A*b^4 - a^4*(A - 3*B) + 4*a*b^3*
(3*A - 2*B) - 9*a^3*b*(A - B) - 2*a^2*b^2*(8*A + 3*B))*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec
```

```
[c + d*x]]) + (2*b*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 -
 b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B
+ 3*a*b^2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a
 + b*Cos[c + d*x]]) + (2*(a^4*A - 13*a^2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*
b^3*B)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^3*(a^
2 - b^2)^2*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,d$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B+\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B+\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(10a^2Ab-6Ab^3-7a^3B+\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{2\left(8a^4Ab-28a^2Ab^3+16Ab^5-3a^5B+15a^3b^2B-8ab^4B\right)\sqrt{\cos(c+}}{3a^5(a-b)(}$$

Mathematica [B] time = 27.14, size = 4316, normalized size = 7.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(-8*a^4*A*b + 28*a^2*A*b^3
 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sin[c + d*x])/(3*a^4*(a^2
 - b^2)^2) + (2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x]))/(3*a^2*(a^2
 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-11*a^2*A*b^3*Sin[c + d*x] + 7*A*b^5*
Sin[c + d*x] + 8*a^3*b^2*B*Sin[c + d*x] - 4*a*b^4*B*Sin[c + d*x]))/(3*a^3*(
a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*A*Tan[c + d*x])/(3*a^3)))/d + (2*((
8*a*A*b)/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2
8*A*b^3)/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) +
(16*A*b^5)/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
```

) - (a^2*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (
5*b^2*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*b
^4*B)/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (
a^2*A*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (5*A
*b^2*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (32*A*b
^4*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (16
*A*b^6*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) -
 (3*a*b*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (1
7*b^3*B*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) -
(8*b^5*B*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]])
 + (8*A*b^2*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a +
b*Cos[c + d*x]]) - (28*A*b^4*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a
^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (16*A*b^6*Cos[2*(c + d*x)]*Sqrt[Sec
[c + d*x]])/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (a*b*B*Cos[2*(
c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (5
*b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)^2*Sqrt[a + b*Cos
[c + d*x]]) - (8*b^5*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b
^2)^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*
(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*
a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + 2*a*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) +
 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)] - (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5
 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c
 + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*Sin[c + d*x]*(-2*(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B -
 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) -
 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B))*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (-8*a^4*A*b + 28*a^
2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(a +
b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*
(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3
 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-16*A*b^4 + 2
*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B
))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]

- (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4
*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))
/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) +
(2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1/2*((-8*a^4*A*b + 28*a^2*A*b^3
- 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(a + b*Cos[c
+ d*x])*Sec[(c + d*x)/2]^4) - ((a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^
5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c
+ d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(-16*A*b^4 +
2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*
B))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Co
s[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])] - ((a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 1
5*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c
+ d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (a*(
a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A
+ 2*B) + a^4*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c +
d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + b*(-8
*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Co
s[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-8*a^4*A*b +
28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*(a + b*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-8*a^4*A*b + 2
8*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(
a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (a*(a + b)*(-16
*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^
4*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*(-8*a^4*
A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a +
b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c +
d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3
- 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-16*A*b^4 + 2
*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a*b^3*(3*A + 2*B) + a^4*(A + 3*B
))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]

```
 - (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*b^2*B + 8*a*b^4
*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*
(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec
[c + d*x]*Tan[c + d*x]))/(3*a^4*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

maple [B] time = 0.60, size = 8101, normalized size = 13.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,\cos\left(c+d\,x\right))\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b*cos(c + d*x))^(5/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.626 $\int \frac{(A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=496

$$\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2b\left(-5a^3B+8a^2Ab+ab^2B-4Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-3a^3(A\right.}{}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^
(3/2)+2/3*b*(8*A*a^2*b-4*A*b^3-5*B*a^3+B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)
/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(3*A*a^4-15*A*a^2*b^2+8*A*b^4
+6*B*a^3*b-2*B*a*b^3)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/
2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d/sec(d
*x+c)^(1/2)+2/3*(8*A*b^3-3*a^3*(A-B)+2*a*b^2*(3*A-B)-3*a^2*b*(3*A+B))*csc(d
*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)
/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*
x+c))/(a-b))^(1/2)/a^3/(a^2-b^2)/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.38, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 3000, 3055, 2998, 2816, 2994}

$$\frac{2b\left(8a^2Ab-5a^3B+ab^2B-4Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-3a^2b(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d*Sqr
t[Sec[c + d*x]]) + (2*(8*A*b^3 - 3*a^3*(A - B) + 2*a*b^2*(3*A - B) - 3*a^2*
b*(3*A + B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*S
qrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b
*(8*a^2*A*b - 4*A*b^3 - 5*a^3*B + a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
```

```
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rubi steps

$$\int \frac{(A + B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A + B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,d$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(8a^2Ab-4Ab^3-5a^3B+\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b\left(8a^2Ab-4Ab^3-5a^3B+\right.}{3a^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{2\left(3a^4A-15a^2Ab^2+8Ab^4+6a^3bB-2ab^3B\right)\sqrt{\cos(c+dx)}\,\csc(c+}{3a^4(a-b)(a+b)^{3/}}$$

Mathematica [B] time = 26.75, size = 3891, normalized size = 7.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^4*A - 15*a^2*A*b^2 +
8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2) - (2*(
-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(a + b*Cos[c
+ d*x])^2) - (2*(-8*a^2*A*b^2*Sin[c + d*x] + 4*A*b^4*Sin[c + d*x] + 5*a^3*b
*B*Sin[c + d*x] - a*b^3*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c
+ d*x]))))/d + (2*(-((a^2*A)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]])) + (5*A*b^2)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) - (8*A*b^4)/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[
Sec[c + d*x]]) - (2*a*b*B)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) + (2*b^3*B)/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]) - (3*a*A*b*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c
 + d*x]]) + (17*A*b^3*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos
[c + d*x]]) - (8*A*b^5*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*
```

```
Cos[c + d*x]]) + (a^2*B*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c
 + d*x]]) - (5*b^2*B*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c
+ d*x]]) + (2*b^4*B*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos
[c + d*x]]) - (a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sq
rt[a + b*Cos[c + d*x]]) + (5*A*b^3*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(a*
(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (8*A*b^5*Cos[2*(c + d*x)]*Sqrt[Se
c[c + d*x]])/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (2*b^2*B*Cos[
2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) +
 (2*b^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a
+ b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(a + b)*(3*a^
4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^3
+ 3*a^2*b*(-3*A + B) + 3*a^3*(A + B) - 2*a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^4*A - 15*a^2
*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Co
s[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*Sin[c + d*x]*(-2*(a + b)*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B
- 2*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b
)/(a + b)] + 2*a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*a^3*(A + B) - 2*
a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)] - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3
*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(-2*(a + b)*(3*a^4*A
 - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 + 3
*a^2*b*(-3*A + B) + 3*a^3*(A + B) - 2*a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^4*A - 15*a^2*A*
b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c +
 d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]
*(-1/2*((3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cos[c +
d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^4) - ((a + b)*(3*a^4*A - 15*a^2*
A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[(a + b*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[
c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (a*(a + b)*(8*A*b^3 + 3
*a^2*b*(-3*A + B) + 3*a^3*(A + B) - 2*a*b^2*(3*A + B))*Sqrt[(a + b*Cos[c +
```

```
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c +
d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((a + b)*
(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*
Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))] + (a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*
a^3*(A + B) - 2*a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1
+ Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
] + b*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Cos[c + d*
x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (3*a^4*A - 15*a^2*A*b
^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^
3*b*B - 2*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2]^2 + (a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*a^3*(A + B)
- 2*a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - ((a + b)*
(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt
[1 - Tan[(c + d*x)/2]^2]))/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sq
rt[Sec[(c + d*x)/2]^2]) + ((-2*(a + b)*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 +
6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)] + 2*a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*a^3*(A
 + B) - 2*a*b^2*(3*A + B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a +
b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (-a + b)/(a + b)] - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B -
2*a*b^3*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2
]^2*Sec[c + d*x]*Tan[c + d*x]))/(3*a^3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
```

```
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.63, size = 6506, normalized size = 13.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+d\,x))\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{(a+b\cos(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b*cos(c + d*x))^(5/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.627 $\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=469

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}-\frac{2\left(-3a^2(A+B)+ab(3A+B)+2Ab^2\right)\sqrt{\cos(c+dx)}\,\csc(c+d}{3a^2d\sqrt{a+b}\,\left(a^2\right.}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^
(1/2)-2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a
/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^
2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-2/3*(2*
A*b^2-3*a^2*(A+B)+a*b*(3*A+B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a^2-b^2)/d/(a+b
)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.19, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 3000, 2993, 2998, 2816, 2994}

$$-\frac{2\left(6a^2Ab-3a^3B-ab^2B-2Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]])
- (2*(2*A*b^2 - 3*a^2*(A + B) + a*b*(3*A + B))*Sqrt[Cos[c + d*x]]*Csc[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x
]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
 Sec[c + d*x]))/(a - b)])/(3*a^2*Sqrt[a + b]*(a^2 - b^2)*d*Sqrt[Sec[c + d*x
]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^(3/2)*Sqrt[Sec[c + d*x]]) - (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*
x]])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))^{5/2}}\,d$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\right.}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{2\left(6a^2Ab-2Ab^3\right.}{3a\left(a^2\right.}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{2\left(6a^2Ab-2Ab^3\right.}{3a\left(a^2\right.}$$

$$= \frac{2\left(6a^2Ab-2Ab^3-3a^3B-ab^2B\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\right.\right.}{3a^3(a-b)(a+b)^{3/2}d\sqrt{\sec}}$$

Mathematica [B] time = 24.52, size = 3493, normalized size = 7.45

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(5
/2),x]

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(-6*a^2*A*b + 2*A*b^3 + 3
*a^3*B + a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2) + (2*(-(A*b*Sin[c + d
*x]) + a*B*Sin[c + d*x]))/(3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (2*(-5*a
^2*A*b*Sin[c + d*x] + A*b^3*Sin[c + d*x] + 2*a^3*B*Sin[c + d*x] + 2*a*b^2*B
*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + (2*((-2*a*A*
b)/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*A*b^3)/
(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*B)/(
(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (b^2*B)/(3*(a^
2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*A*Sqrt[Sec[c
 + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (5*A*b^2*Sqrt[Sec[c +
d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*Sqrt[Sec[c + d
*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (a*b*B*Sqrt[Sec[c +
d*x]])/(3*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (b^3*B*Sqrt[Sec[c + d*x
]])/(3*a*(a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) - (2*A*b^2*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]) + (2*A*b^4*C
os[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*Sqrt[a + b*Cos[c +
 d*x]]) + (a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)^2*Sqrt[a
 + b*Cos[c + d*x]]) + (b^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2
 - b^2)^2*Sqrt[a + b*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*
(2*(a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b^2
 + 3*a^2*(A - B) + a*b*(3*A - B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] + (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B
)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(
3*(a^3 - a*b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*((b*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-6*a^2*A*b +
2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a
+ b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3
*A - B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x]*(a + b*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^2*(a + b*
Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Se
c[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a
*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)] + 2*a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3*A - B))*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (-6*a^2*A*b

```
+ 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-6*a^2
*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(
c + d*x)/2]^4)/2 + ((a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*
x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])] + (a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3*A - B))*Sqrt[(a + b*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Si
n[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a
 + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*(-((b*Sin
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b*Cos[c + d*x])*Sin[c + d*
x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))] + (a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3*A - B))*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*(-((b*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((a + b
*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - b*(-6*a^2*A*b + 2*A*b^3 + 3*a^
3*B + a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2
] - (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*(a + b*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B
 + a*b^2*B)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2]^2 + (a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b*(3*A - B))*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((-a +
b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B
 + a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((-a + b)*Tan[(c
 + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(3*(a^3 - a*b^2)^2*S
qrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + ((2*(a + b)*(-6*a^2*A*b
 + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(-2*A*b^2 + 3*a^2*(A - B) + a*b
*(3*A - B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)] + (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x]*(a + b*
Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[
c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))
/(3*(a^3 - a*b^2)^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[
Cos[(c + d*x)/2]^2*Sec[c + d*x]])))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(
b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3),
x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.50, size = 5202, normalized size = 11.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/2
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\cos(c+dx))\sqrt{\frac{1}{\cos(c+dx)}}}{(a+b\cos(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b*cos(c + d*x))^(5/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.628 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=431

$$\frac{2\left(3a^2A-4abB+Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2A-4\right.}{}$$

```
[Out] -2/3*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/
2)+2/3*(3*A*a^2+A*b^2-4*B*a*b)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a
+b*cos(d*x+c))^(1/2)-2/3*(3*A*a^2+A*b^2-4*B*a*b)*csc(d*x+c)*EllipticE((a+b*
cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*
x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^
2/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)+2/3*(a*(3*A+B)-b*(A+3*B))*csc(d*x+c)
*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b
))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 1.08, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2961, 2999, 2993, 2998, 2816, 2994}

$$\frac{2\left(3a^2A-4abB+Ab^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2(Ab-aB)\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2A-4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*(3*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(3*a^2*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(a*(3*A
+ B) - b*(A + 3*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
3*a*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(A*b - a*B)*Sin[c + d*
x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) + (2*(3
*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)^2
*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2999

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[((B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*
x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{2\left(3a^2A+Ab^2\right.}{3\left(a^2-\right.}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{2\left(3a^2A+Ab^2\right.}{3\left(a^2-\right.}$$

$$= -\frac{2\left(3a^2A+Ab^2-4abB\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,c}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{3a^2(a-b)(a+b)^{3/2}d\sqrt{\sec(}}$$

Mathematica [A] time = 18.80, size = 528, normalized size = 1.23

$$\frac{\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}\left(-\frac{2\left(3a^2A-4abB+Ab^2\right)\sin(c+dx)}{3a\left(a^2-b^2\right)^2}+\frac{2\left(a^2B\sin(c+dx)-aAb\sin(c+dx)\right)}{3b\left(b^2-a^2\right)(a+b\cos(c+dx))^2}+\frac{2\left(a^3B\sin(c+dx)+2a^2Ab\sin\right.}{3b\left(b^2-\right.}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x
]]),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(3*a^2*A + A*b^2 - 4*a*b*
B)*Sin[c + d*x])/(3*a*(a^2 - b^2)^2) + (2*(-(a*A*b*Sin[c + d*x]) + a^2*B*Si
n[c + d*x]))/(3*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(2*a^2*A*b*Sin[
c + d*x] + 2*A*b^3*Sin[c + d*x] + a^3*B*Sin[c + d*x] - 5*a*b^2*B*Sin[c + d*
x]))/(3*b*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d - (2*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*(-2*(a + b)*(3*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(a + b)*(3*a
*A + A*b - a*B - 3*b*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] - (3*a^2*A + A*b^2 - 4*a*b*B)*Cos[c + d*x]*(a + b*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a*(a^2 - b^2)^2*d*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3
 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sqrt(sec(d*x + c)))
, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.47, size = 4243, normalized size = 9.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)

[Out] 2/3/d*(-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4+3*A*cos(d*x+c)^3*a^2*b^2-4*B*cos(d*x+c)^3*a*b^3-8*B*cos(d*x+c)^2*a^2*b^2+4*B*cos(d*x+c)^2*a*b^3-4*B*cos(d*x+c)*a^3*b+3*B*cos(d*x+c)*a^2*b^2+6*A*cos(d*x+c)^2*a^3*b+2*A*cos(d*x+c)^2*a*b^3+A*cos(d*x+c)*a^2*b^2-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4+A*cos(d*x+c)^3*b^4-A*cos(d*x+c)^2*b^4-B*cos(d*x+c)^3*a^4+B*cos(d*x+c)*a^4-3*A*cos(d*x+c)^2*a^4+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4+4*B*cos(d*x+c)^2*a^3*b-2*A*cos(d*x+c)^3*a^3*b-2*A*cos(d*x+c)^3*a*b^3-4*A*cos(d*x+c)^2*a^2*b^2-4*A*cos(d*x+c)*a^3*b+5*B*cos(d*x+c)^3*a^2*b^2-6*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*a*b^3-3*A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-4*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-4*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^2-2*A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^3+5*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a

^2*b^2+A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^3+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+8*B*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+4*B*s
in(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^3-5*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-7*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-3*B*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a*b^3+3*A*cos(d*x+c)*a^4-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^3*b-3*A*sin(d*x+c)*cos(d*x+c)^2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-A*
sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a*b^3+3*A*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2*b^2+A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^2*a*b^3+4*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+4*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-B*sin(d*x+c)*c
os(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^3*b-4*B*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2*b^2-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4+4*A*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-A*sin(d*x+c)*cos
(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c)

```
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^4+3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)*a^4+7*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4)*(1/cos(d*x+c
))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(3/2)/a/(a+b)^2/(a-b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos(c+d\,x)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.629 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=602

$$\frac{2\left(3a^3B-7ab^2B+4Ab^3\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right.}{3ab^2d(a-b)(a+b)^{3/2}\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^
(1/2)-2/3*a*(4*A*b^3+3*B*a^3-7*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^
2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(4*A*b^3+3*B*a^3-7*B*a*b^2)*csc(d*x+c
)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-
b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/a/(a-b)/b^2/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-2/3*(3*A*b^3+3*a^
3*B+a^2*b*B-a*b^2*(A+6*B))*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b
)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d
*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/b^2/(a+b)^(3/2)/
d/sec(d*x+c)^(1/2)-2*B*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^
(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d/s
ec(d*x+c)^(1/2)
```

Rubi [A] time = 1.65, antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2961, 2989, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\cos(c+dx))^{3/2}}-\frac{2\left(a^2bB\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(3*
A*b^3 + 3*a^3*B + a^2*b*B - a*b^2*(A + 6*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]
) - (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b,
```

```
ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b
)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x])
)/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) - (2*a*(4*A*
b^3 + 3*a^3*B - 7*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b
^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A + B\cos(c + dx)}{(a + b\cos(c + dx))^{5/2}\sec^{\frac{3}{2}}(c + dx)}\,dx = \left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c + dx)(A + B\cos(c + dx))}{(a + b\cos(c + dx))^{5/2}}\,dx$$

$$= \frac{2a(Ab - aB)\sin(c + dx)}{3b\left(a^2 - b^2\right)d(a + b\cos(c + dx))^{3/2}\sqrt{\sec(c + dx)}} - \frac{\left(2\sqrt{\cos(c + dx)}\right.}{}$$

$$= \frac{2a(Ab - aB)\sin(c + dx)}{3b\left(a^2 - b^2\right)d(a + b\cos(c + dx))^{3/2}\sqrt{\sec(c + dx)}} - \frac{\left(2\sqrt{\cos(c + dx)}\right.}{}$$

$$= -\frac{2\sqrt{a + b}\,B\sqrt{\cos(c + dx)}\csc(c + dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{b^3 d\sqrt{\sec(c + dx)}}$$

$$= -\frac{2\sqrt{a + b}\,B\sqrt{\cos(c + dx)}\csc(c + dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{b^3 d\sqrt{\sec(c + dx)}}$$

$$= \frac{2\left(4Ab^3 + 3a^3B - 7ab^2B\right)\sqrt{\cos(c + dx)}\csc(c + dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\,c}}{\sqrt{a+b}\,\sqrt{}}\right.\right.}{3a(a - b)b^2(a + b)^{3/2}d\sqrt{\sec(}}$$

Mathematica [C] time = 16.21, size = 1994, normalized size = 3.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/
2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(4*A*b^3 + 3*a^3*B - 7*a*b
^2*B)*Sin[c + d*x])/(3*b^2*(-a^2 + b^2)^2) - (2*(-(a^2*A*b*Sin[c + d*x]) +
a^3*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) - (2*(-(a^
3*A*b*Sin[c + d*x]) + 5*a*A*b^3*Sin[c + d*x] + 4*a^4*B*Sin[c + d*x] - 8*a^2
*b^2*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d - (2*
Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*
x)/2]^2)]*(4*a*A*b^3*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] + 4*A*b^4*Sqrt[
(a - b)/(a + b)]*Tan[(c + d*x)/2] + 3*a^4*Sqrt[(a - b)/(a + b)]*B*Tan[(c +
d*x)/2] + 3*a^3*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2] - 7*a^2*b^2*Sqrt
```

```
[(a - b)/(a + b)]*B*Tan[(c + d*x)/2] - 7*a*b^3*Sqrt[(a - b)/(a + b)]*B*Tan[
(c + d*x)/2] - 8*A*b^4*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 - 6*a^3*b*S
qrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + 14*a*b^3*Sqrt[(a - b)/(a + b)]*
B*Tan[(c + d*x)/2]^3 - 4*a*A*b^3*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 +
 4*A*b^4*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 - 3*a^4*Sqrt[(a - b)/(a +
 b)]*B*Tan[(c + d*x)/2]^5 + 3*a^3*b*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2
]^5 + 7*a^2*b^2*Sqrt[(a - b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - 7*a*b^3*Sqrt[(
a - b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + (6*I)*a^4*B*EllipticPi[(a + b)/(a -
b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)] - (12*I)*a^2*b^2*B*EllipticPi[(a + b)/(a - b), I*ArcS
inh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^
2)/(a + b)] + (6*I)*b^4*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b
)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (
6*I)*a^4*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[
(c + d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]
 - (12*I)*a^2*b^2*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a +
 b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)] + (6*I)*b^4*B*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/
(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)] + I*(a - b)*(4*A*b^3 + 3*a^3*B - 7*a*b^2*B)*EllipticE[I*ArcSi
nh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*(3*b^3*(A - B) + 6*a^3*B +
4*a^2*b*B - a*b^2*(A + 9*B))*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[
(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
 + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)]))/(3*b^2*Sqrt[(a - b)/(a + b)]*(a^2 - b^2)^2*d*(-1 + Tan[(c + d*x)/2
]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b*(-1 + Tan[(
c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F] time = 25.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3
 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(3/2))
, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.43, size = 5757, normalized size = 9.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos\left(dx+c\right)+A}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.630 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=733

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}+\frac{2a\left(-5a^3B+2a^2Ab+9ab^2B-6Ab^3\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{\left(15a^3B-a\right.}{}$$

```
[Out] 2/3*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^
(3/2)+2/3*a*(2*A*a^2*b-6*A*b^3-5*B*a^3+9*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^
2/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-1/3*(6*A*a^3*b-14*A*a*b^3-15*B*
a^4+26*B*a^2*b^2-3*B*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2
)/b^3/(a^2-b^2)^2/d+1/3*(6*A*a^3*b-14*A*a*b^3-15*B*a^4+26*B*a^2*b^2-3*B*b^4
)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),
((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1
+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/b^3/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)+1/3*(3
*b^3*(4*A-B)+15*a^3*B-a*b^2*(2*A+21*B)-a^2*(6*A*b-5*B*b))*csc(d*x+c)*Ellipt
icF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2
))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/(a-b)/b^3/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-(2*A*b-5*B*a)*csc(d*x+c)*El
lipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)
*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 2.49, antiderivative size = 733, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2961, 2989, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}-\frac{\left(6a^3Ab+26a^2b^2B-15a^4B-14aAb^3-3b^4B\right)\sin(c+dx)\sqrt{\text{se}}}{3b^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*S
qrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*b^3*(a + b)^(3/2)*d*S
qrt[Sec[c + d*x]]) + ((3*b^3*(4*A - B) + 15*a^3*B - a*b^2*(2*A + 21*B) - a^
2*(6*A*b - 5*b*B))*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a
```

```
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*(a - b)*b^3*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*A*b - 5*
a*B)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^4
*d*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3
- 5*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]) - ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^
2*b^2*B - 3*b^4*B)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(3*b^3*(a^2 - b^2)^2*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2961

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[((a + b*Sin[e + f*x])^m*(c + d
*Sin[e + f*x])^n)/(g*Sin[e + f*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 2989

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
```

```
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A + B\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(a+b\cos(c+dx))^{5/2}}\,d$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\left(2a^2Ab-6A\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\left(2a^2Ab-6A\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\left(2a^2Ab-6A\right.}{3b^2\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{\sqrt{a+b}\,(2Ab-5aB)\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b}}\right.\right.}{b^4d\sqrt{\sec(c+dx)}}$$

$$= \frac{\left(6a^3Ab-14aAb^3-15a^4B+26a^2b^2B-3b^4B\right)\sqrt{\cos(c+dx)}\,\csc(c+}{3a(a-b)b^3(a+b)}$$

Mathematica [B] time = 22.37, size = 2318, normalized size = 3.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/
2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a*(-3*a^2*A*b + 7*A*b^3 +
 6*a^3*B - 10*a*b^2*B)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2) + (2*(-(a^3*A*b*
Sin[c + d*x]) + a^4*B*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)*(a + b*Cos[c + d*x
])^2) + (2*(-4*a^4*A*b*Sin[c + d*x] + 8*a^2*A*b^3*Sin[c + d*x] + 7*a^5*B*Si
```

n[c + d*x] - 11*a^3*b^2*B*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d + (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(6*a^4*A*b*Tan[(c + d*x)/2] + 6*a^3*A*b^2*Tan[(c + d*x)/2] - 14*a^2*A*b^3*Tan[(c + d*x)/2] - 14*a*A*b^4*Tan[(c + d*x)/2] - 15*a^5*B*Tan[(c + d*x)/2] - 15*a^4*b*B*Tan[(c + d*x)/2] + 26*a^3*b^2*B*Tan[(c + d*x)/2] + 26*a^2*b^3*B*Tan[(c + d*x)/2] - 3*a*b^4*B*Tan[(c + d*x)/2] - 3*b^5*B*Tan[(c + d*x)/2] - 12*a^3*A*b^2*Tan[(c + d*x)/2]^3 + 28*a*A*b^4*Tan[(c + d*x)/2]^3 + 30*a^4*b*B*Tan[(c + d*x)/2]^3 - 52*a^2*b^3*B*Tan[(c + d*x)/2]^3 + 6*b^5*B*Tan[(c + d*x)/2]^3 - 6*a^4*A*b*Tan[(c + d*x)/2]^5 + 6*a^3*A*b^2*Tan[(c + d*x)/2]^5 + 14*a^2*A*b^3*Tan[(c + d*x)/2]^5 - 14*a*A*b^4*Tan[(c + d*x)/2]^5 + 15*a^5*B*Tan[(c + d*x)/2]^5 - 15*a^4*b*B*Tan[(c + d*x)/2]^5 - 26*a^3*b^2*B*Tan[(c + d*x)/2]^5 + 26*a^2*b^3*B*Tan[(c + d*x)/2]^5 + 3*a*b^4*B*Tan[(c + d*x)/2]^5 - 3*b^5*B*Tan[(c + d*x)/2]^5 - 12*a^4*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a^2*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*A*b^5*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^5*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 60*a^3*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a*b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^4*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a^2*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*A*b^5*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^5*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 60*a^3*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a*b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*(-6*a^3*A*b + 14*a*A*b^3 + 15*a^4*B - 26*a^2*b^2*B + 3*b^4*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(

```
a + b)] + 2*b*(a + b)*(3*A*b^3 + 3*a*b^2*(A - 2*B) + 5*a^3*B - a^2*b*(2*A +
 3*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
- b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*b^3*(a^2 - b^2)^2*d*Sqrt[1 + Tan[(c +
 d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

fricas [F] time = 1.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)+a}}{\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3
 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(5/2))
, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.56, size = 8621, normalized size = 11.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B*cos(c + d*x))/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2)
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.631 $\int \frac{(aB+bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=266

$$\frac{2B(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-2B\sqrt{}$$

```
[Out] 2*(a-b)*B*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c
)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)-2*B*cs
c(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/
2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {21, 4222, 2801, 2816, 2994}

$$\frac{2B(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-2B\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(aB + bB\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$= -\left(\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}\right.$$

$$= \frac{2(a-b)\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{a^2d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 6.11, size = 298, normalized size = 1.12

$$B\left(\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{ad} - \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(\cos(c+dx)\tan\left(\frac{1}{2}(c+dx\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sec[c + d*x]^(3/2))/(a + b*Cos[c + d*x]
)^(3/2),x]
```

```
[Out] B*((2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (2*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + Cos[c + d*x]*(a + b*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a*d*Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[(c + d*x)/2]^2]))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B*sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
(3/2), x)
```

maple [B] time = 0.33, size = 621, normalized size = 2.33

$$-\frac{2B\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos(dx+c)\sin(dx+c)\,a-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*B/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+cos(d*x+c)^2*b+a*cos(d
*x+c)-b*cos(d*x+c)-a)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2
)/sin(d*x+c)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^
(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/cos(c + d*x))^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^
(3/2),x)
```

```
[Out] int(((1/cos(c + d*x))^(3/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^
(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.632 $\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{2B\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2*B*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2
),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b
))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {21, 4222, 2816}

$$\frac{2B\sqrt{a+b}\,\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x])^(3/2
),x]
```

```
[Out] (2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d
*Sqrt[Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{(aB+bB\cos(c+dx))\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = B\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx}}$$
$$= \frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 0.15, size = 104, normalized size = 0.80

$$\frac{2B\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\,F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)}{d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\,\sqrt{\sec(c+dx)}\,\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B + b*B*Cos[c + d*x])*Sqrt[Sec[c + d*x]])/(a + b*Cos[c + d*x]
)^(3/2),x]
```

```
[Out] (2*B*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcS
in[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B*sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^
(3/2), x)
```

maple [A] time = 0.32, size = 126, normalized size = 0.97

$$\frac{2B\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{1}{\cos(dx+c)}}\,\left(\sin^2(dx+c)\right)}{d\sqrt{a+b\cos(dx+c)}\,\left(-1+\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2*B/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))/(a+b*
cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bb\cos(dx+c)+Ba)\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)*sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^
(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(B\,a+B\,b\,\cos\left(c+d\,x\right)\right)}{{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/cos(c + d*x))^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^
(3/2),x)
```

```
[Out] int(((1/cos(c + d*x))^(1/2)*(B*a + B*b*cos(c + d*x)))/(a + b*cos(c + d*x))^
(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B \int \frac{\sqrt{\sec{\left(c + dx \right)}}}{\sqrt{a + b \cos{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))*sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] B*Integral(sqrt(sec(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

3.633 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=137

$$-\frac{2B\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}$$

```
[Out] -2*B*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {21, 4222, 2809}

$$-\frac{2B\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]
),x]
```

```
[Out] (-2*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, Arc
Sin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(
a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(
a - b)])/(b*d*Sqrt[Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx &= B\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx\\ &= \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx\\ &= -\frac{2\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right.}{bd\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.19, size = 147, normalized size = 1.07

$$-\frac{2B\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\sec(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-2\Pi\left(-1;\sin^{-1}\left(\tan\left(\frac{1}{2}(\right.\right.\right.\right.}{d\sqrt{\frac{1}{\cos(c+dx)+1}}\sqrt{a+b\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c +
 d*x]]),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sqrt[1
 + Sec[c + d*x]])/(d*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Cos[c + d*x]]
)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B}{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, a
lgorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.30, size = 144, normalized size = 1.05

$$\frac{2B\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,\sqrt{-\frac{a-b}{a+b}}\right)\right)\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}}{d\sqrt{a+b\cos\left(dx+c\right)}\,\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 2*B/d/(a+b*cos(d*x+c))^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))-2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2)))*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)/(1/cos(d*x+c))^(1/2)/(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos\left(dx+c\right)+Ba}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x
 + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{\sqrt{\frac{1}{\cos(c+d\,x)}}\,\left(a+b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(
3/2)),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(
3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$B\int \frac{1}{\sqrt{a+b\cos\left(c+dx\right)}\,\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] B*Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)
```

3.634 $\int \frac{aB+bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=479

$$\frac{aB\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}+\frac{aB\sin(}{bd\sqrt{}}$$

```
[Out] B*sin(d*x+c)/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+a*B*sin(d*x+c)*sec(d
*x+c)^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)-(a-b)*B*csc(d*x+c)*EllipticE((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1
/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b)
)^(1/2)/a/b/d/sec(d*x+c)^(1/2)+B*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)
^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/se
c(d*x+c)^(1/2)+a*B*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)
/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2
)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d
*x+c)^(1/2)
```

Rubi [A] time = 0.92, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {21, 4222, 2820, 2809, 3003, 2993, 12, 2801, 2816, 2994}

$$\frac{aB\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}+\frac{aB\sin(}{bd\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)
),x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*B*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b]*B*Sqr
t[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d*Sqrt[
```

```
Sec[c + d*x]]) + (B*Sin[c + d*x])/(d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c +
d*x]]) + (a*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[a + b*Cos[c + d*x]
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2820

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> -Dist[(a*d)/(2*b), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a
 + b*Sin[e + f*x]], x], x] + Dist[d/(2*b), Int[(Sqrt[d*Sin[e + f*x]]*(a + 2
*b*Sin[e + f*x]))/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = B\int \frac{1}{\sqrt{a+b\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{\left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx}{2b} - \frac{(aB\sqrt{}}{}$$

$$= \frac{a\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{a\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{a\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{a\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right)|\right.}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\,B\sqrt{\cos(c+dx)}\,\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\,\sqrt{\cos(c+dx)}}\right.\right.}{abd\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 3.33, size = 508, normalized size = 1.06

$$\frac{B\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\,\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)+1}\,\left(2a\sqrt{\frac{a-b}{a+b}}\,\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\,\tan\left(\frac{1}{2}(c+dx)\right) - b\sqrt{\frac{a-b}{a+b}}\,\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]
^(3/2)),x]
```

```
[Out] (B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^2*Sqrt[1 + Sec[c
+ d*x]]*((2*I)*(a - b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/
```

```
(a - b))] - (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a
- b))] + (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]]
, -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a - b)/(a + b)
]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sqrt[(a - b)/(
a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(4*b*Sqrt[
(a - b)/(a + b)]*d*((1 + Cos[c + d*x])^(-1))^(3/2)*Sqrt[a + b*Cos[c + d*x]]
)
```

fricas [F] time = 25.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{B}{\sqrt{b\cos\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, a
lgorithm="fricas")
```

```
[Out] integral(B/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos\left(dx+c\right)+Ba}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, a
lgorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)
^(3/2)), x)
```

maple [A] time = 0.35, size = 631, normalized size = 1.32

$$-\frac{B\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{a+b\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\text{EllipticE}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)a+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -B/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+b*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*a*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+b*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)+cos(d*x+c)^3
*b+a*cos(d*x+c)^2-cos(d*x+c)^2*b-a*cos(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3
/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bb\cos(dx+c)+Ba}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)
^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,a+B\,b\,\cos\left(c+d\,x\right)}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(
3/2)),x)
```

```
[Out] int((B*a + B*b*cos(c + d*x))/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(
3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.635 $\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=59

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Int}\left((A+B\cos(e+fx))(c\cos(e+fx))^{-m}(a+b\cos(e+fx))^n,x\right)$$

```
[Out] (c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Unintegrable((a+b*cos(f*x+e))^n*(A+B*cos(
f*x+e))/((c*cos(f*x+e))^m),x)
```

Rubi [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Defer[Int][((a + b*Cos[e + f*x])^n*(A
 + B*Cos[e + f*x]))/(c*Cos[e + f*x])^m, x]
```

Rubi steps

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \left((c\cos(e+fx))^m(c\sec(e+fx))^m\right)\int (c\cos(e+$$

Mathematica [A] time = 9.52, size = 0, normalized size = 0.00

$$\int (a+b\cos(e+fx))^n(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[(a + b*Cos[e + f*x])^n*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m, x
]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^n\left(c\sec\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*sec(f*x + e))^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{n}\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*sec(f*x + e))^m, x
)
```

maple [A] time = 3.11, size = 0, normalized size = 0.00

$$\int \left(a+b\cos\left(fx+e\right)\right)^{n}\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{n}\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^n*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^n*(c*sec(f*x + e))^m, x
)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{c}{\cos\left(e+f\,x\right)}\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^n,x)
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**n*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
[Out] Timed out
```

3.636 $\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=644

$$-\frac{a^2c^5\tan(e+fx)\sec(e+fx)\left(a^2A(2-m)^2+2abB(1-m)^2+Ab^2\left(m^2-m+6\right)\right)(c\sec(e+fx))^{m-5}}{f(1-m)(2-m)(3-m)}-\frac{ac^5\tan(e+$$

```
[Out] -c^6*(4*a^3*A*b*(m^2-8*m+15)+a^4*B*(m^2-8*m+15)+4*a*A*b^3*(m^2-7*m+10)+6*a^
2*b^2*B*(m^2-7*m+10)+b^4*B*(m^2-6*m+8))*hypergeom([1/2, 3-1/2*m],[4-1/2*m],
cos(f*x+e)^2)*(c*sec(f*x+e))^(-6+m)*sin(f*x+e)/f/(-m^3+12*m^2-44*m+48)/(sin
(f*x+e)^2)^(1/2)-c^5*(a^4*A*(m^2-6*m+8)+6*a^2*A*b^2*(m^2-5*m+4)+4*a^3*b*B*(
m^2-5*m+4)+A*b^4*(m^2-4*m+3)+4*a*b^3*B*(m^2-4*m+3))*hypergeom([1/2, 5/2-1/2
*m],[7/2-1/2*m],cos(f*x+e)^2)*(c*sec(f*x+e))^(-5+m)*sin(f*x+e)/f/(1-m)/(m^2
-8*m+15)/(sin(f*x+e)^2)^(1/2)-a*c^5*(4*a^2*A*b*(m^2-4*m+3)+a^3*B*(m^2-4*m+3
)+2*A*b^3*(m^2-2*m+4)+a*b^2*B*(5*m^2-13*m+8))*(c*sec(f*x+e))^(-5+m)*tan(f*x
+e)/f/(1-m)/(m^2-6*m+8)-a^2*c^5*(2*a*b*B*(1-m)^2+a^2*A*(2-m)^2+A*b^2*(m^2-m
+6))*sec(f*x+e)*(c*sec(f*x+e))^(-5+m)*tan(f*x+e)/f/(-m^3+6*m^2-11*m+6)-a*c^
5*(a*B*(1-m)-A*b*(2+m))*(c*sec(f*x+e))^(-5+m)*(b+a*sec(f*x+e))^2*tan(f*x+e)
/f/(m^2-3*m+2)-a*A*c^5*(c*sec(f*x+e))^(-5+m)*(b+a*sec(f*x+e))^3*tan(f*x+e)/
f/(1-m)
```

Rubi [A] time = 2.04, antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2960, 4026, 4096, 4076, 4047, 3772, 2643, 4046}

$$-\frac{c^6\sin(e+fx)\left(4a^3Ab\left(m^2-8m+15\right)+6a^2b^2B\left(m^2-7m+10\right)+a^4B\left(m^2-8m+15\right)+4aAb^3\left(m^2-7m+10\right.\right.}{f(2-m)(4-m)(6-m)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] -((c^6*(4*a^3*A*b*(15 - 8*m + m^2) + a^4*B*(15 - 8*m + m^2) + 4*a*A*b^3*(10
 - 7*m + m^2) + 6*a^2*b^2*B*(10 - 7*m + m^2) + b^4*B*(8 - 6*m + m^2))*Hyper
geometric2F1[1/2, (6 - m)/2, (8 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-
6 + m)*Sin[e + f*x])/(f*(2 - m)*(4 - m)*(6 - m)*Sqrt[Sin[e + f*x]^2])) - (c
^5*(a^4*A*(8 - 6*m + m^2) + 6*a^2*A*b^2*(4 - 5*m + m^2) + 4*a^3*b*B*(4 - 5*
m + m^2) + A*b^4*(3 - 4*m + m^2) + 4*a*b^3*B*(3 - 4*m + m^2))*Hypergeometri
c2F1[1/2, (5 - m)/2, (7 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-5 + m)*S
in[e + f*x])/(f*(1 - m)*(3 - m)*(5 - m)*Sqrt[Sin[e + f*x]^2]) - (a*c^5*(4*a
```

```
^2*A*b*(3 - 4*m + m^2) + a^3*B*(3 - 4*m + m^2) + 2*A*b^3*(4 - 2*m + m^2) +
a*b^2*B*(8 - 13*m + 5*m^2))*(c*Sec[e + f*x])^(-5 + m)*Tan[e + f*x])/(f*(1 -
 m)*(2 - m)*(4 - m)) - (a^2*c^5*(2*a*b*B*(1 - m)^2 + a^2*A*(2 - m)^2 + A*b^
2*(6 - m + m^2))*Sec[e + f*x]*(c*Sec[e + f*x])^(-5 + m)*Tan[e + f*x])/(f*(1
 - m)*(2 - m)*(3 - m)) - (a*c^5*(a*B*(1 - m) - A*b*(2 + m))*(c*Sec[e + f*x]
)^(-5 + m)*(b + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(1 - m)*(2 - m)) - (a*A*
c^5*(c*Sec[e + f*x])^(-5 + m)*(b + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(1 -
m))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
```

```
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4096

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[
e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e + f
*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*B)
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 -
b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(e+fx))^4(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^5\int (c\sec(e+fx))^{-5+m}(b+a\sec(e+fx))^4(B$$

$$= -\frac{aAc^5(c\sec(e+fx))^{-5+m}(b+a\sec(e+fx))^3\,\mathrm{t}}{f(1-m)}$$

$$= -\frac{ac^5(aB(1-m)-Ab(2+m))(c\sec(e+fx))^{-5}}{f(1-m)(2-}$$

$$= -\frac{a^2c^5\left(2abB(1-m)^2+a^2A(2-m)^2+Ab^2\left(6-\right.\right.}{f(3-m}$$

$$= -\frac{a^2c^5\left(2abB(1-m)^2+a^2A(2-m)^2+Ab^2\left(6-\right.\right.}{f(3-m}$$

$$= -\frac{ac^5\left(4a^2Ab\left(3-4m+m^2\right)+a^3B\left(3-4m+m\right.\right.}{}$$

$$= -\frac{\left(a^4A\left(8-6m+m^2\right)+6a^2Ab^2\left(4-5m+m^2\right)\right.}{}$$

$$= -\frac{\left(a^4A\left(8-6m+m^2\right)+6a^2Ab^2\left(4-5m+m^2\right)\right.}{}$$

Mathematica [A] time = 4.37, size = 317, normalized size = 0.49

$$\frac{\sqrt{-\tan^2(e+fx)}\cot(e+fx)(c\sec(e+fx))^m\left(\frac{b^3(4aB+Ab)\cos^4(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{m-4}{2};\frac{m-2}{2};\sec^2(e+fx)\right)}{m-4}+a\left(\frac{2b^2(3aB+2Ab)\cos^3(e+f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[e + f*x])^4*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[e + f*x]*((b^4*B*Cos[e + f*x]^5*Hypergeometric2F1[1/2, (-5 + m)/2, (-3
 + m)/2, Sec[e + f*x]^2])/(-5 + m) + (b^3*(A*b + 4*a*B)*Cos[e + f*x]^4*Hype
rgeometric2F1[1/2, (-4 + m)/2, (-2 + m)/2, Sec[e + f*x]^2])/(-4 + m) + a*((
2*b^2*(2*A*b + 3*a*B)*Cos[e + f*x]^3*Hypergeometric2F1[1/2, (-3 + m)/2, (-1
 + m)/2, Sec[e + f*x]^2])/(-3 + m) + a*((2*b*(3*A*b + 2*a*B)*Cos[e + f*x]^2
*Hypergeometric2F1[1/2, (-2 + m)/2, m/2, Sec[e + f*x]^2])/(-2 + m) + a*(((4
```

```
*A*b + a*B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[
e + f*x]^2])/(-1 + m) + (a*A*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[e +
 f*x]^2])/m))))*(c*Sec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/f
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^4\cos\left(fx+e\right)^5 + Aa^4 + \left(4\,Bab^3 + Ab^4\right)\cos\left(fx+e\right)^4 + 2\left(3\,Ba^2b^2 + 2\,Aab^3\right)\cos\left(fx+e\right)^3 + 2\left(2\,B\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((B*b^4*cos(f*x + e)^5 + A*a^4 + (4*B*a*b^3 + A*b^4)*cos(f*x + e)^4
 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*cos(f*x + e)^3 + 2*(2*B*a^3*b + 3*A*a^2*b^2)
*cos(f*x + e)^2 + (B*a^4 + 4*A*a^3*b)*cos(f*x + e))*(c*sec(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^4\left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*sec(f*x + e))^m, x
)
```

maple [F] time = 2.79, size = 0, normalized size = 0.00

$$\int \left(a+b\cos\left(fx+e\right)\right)^4\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^4\left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^4*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^4*(c*sec(f*x + e))^m, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{c}{\cos\left(e+f\,x\right)}\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^4,x)
```

```
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**4*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.637 $\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=455

$$-\frac{ac^4\tan(e+fx)\left(a^2A(2-m)+3abB(1-m)-2Ab^2m\right)(c\sec(e+fx))^{m-4}}{f(1-m)(3-m)}-\frac{a^2c^4\tan(e+fx)\sec(e+fx)(aB(1-}{f(1-m)}$$

```
[Out] -c^5*(a^3*A*(m^2-6*m+8)+3*a*A*b^2*(m^2-5*m+4)+3*a^2*b*B*(m^2-5*m+4)+b^3*B*(
m^2-4*m+3))*hypergeom([1/2, 5/2-1/2*m],[7/2-1/2*m],cos(f*x+e)^2)*(c*sec(f*x
+e))^(-5+m)*sin(f*x+e)/f/(1-m)/(m^2-8*m+15)/(sin(f*x+e)^2)^(1/2)-c^4*(A*b^3
*(2-m)+3*a*b^2*B*(2-m)+3*a^2*A*b*(3-m)+a^3*B*(3-m))*hypergeom([1/2, 2-1/2*m
],[3-1/2*m],cos(f*x+e)^2)*(c*sec(f*x+e))^(-4+m)*sin(f*x+e)/f/(m^2-6*m+8)/(s
in(f*x+e)^2)^(1/2)-a*c^4*(3*a*b*B*(1-m)+a^2*A*(2-m)-2*A*b^2*m)*(c*sec(f*x+e
))^(-4+m)*tan(f*x+e)/f/(m^2-4*m+3)-a^2*c^4*(a*B*(1-m)-A*b*(1+m))*sec(f*x+e)
*(c*sec(f*x+e))^(-4+m)*tan(f*x+e)/f/(m^2-3*m+2)-a*A*c^4*(c*sec(f*x+e))^(-4+
m)*(b+a*sec(f*x+e))^2*tan(f*x+e)/f/(1-m)
```

Rubi [A] time = 1.15, antiderivative size = 455, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2960, 4026, 4076, 4047, 3772, 2643, 4046}

$$-\frac{c^5\sin(e+fx)\left(a^3A\left(m^2-6m+8\right)+3a^2bB\left(m^2-5m+4\right)+3aAb^2\left(m^2-5m+4\right)+b^3B\left(m^2-4m+3\right)\right)(c\sec}{f(1-m)(3-m)(5-m)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] -((c^5*(a^3*A*(8 - 6*m + m^2) + 3*a*A*b^2*(4 - 5*m + m^2) + 3*a^2*b*B*(4 -
5*m + m^2) + b^3*B*(3 - 4*m + m^2))*Hypergeometric2F1[1/2, (5 - m)/2, (7 -
m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-5 + m)*Sin[e + f*x])/(f*(1 - m)*(3
 - m)*(5 - m)*Sqrt[Sin[e + f*x]^2])) - (c^4*(A*b^3*(2 - m) + 3*a*b^2*B*(2 -
 m) + 3*a^2*A*b*(3 - m) + a^3*B*(3 - m))*Hypergeometric2F1[1/2, (4 - m)/2,
(6 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-4 + m)*Sin[e + f*x])/(f*(2 -
m)*(4 - m)*Sqrt[Sin[e + f*x]^2]) - (a*c^4*(3*a*b*B*(1 - m) + a^2*A*(2 - m)
- 2*A*b^2*m)*(c*Sec[e + f*x])^(-4 + m)*Tan[e + f*x])/(f*(1 - m)*(3 - m)) -
(a^2*c^4*(a*B*(1 - m) - A*b*(1 + m))*Sec[e + f*x]*(c*Sec[e + f*x])^(-4 + m)
*Tan[e + f*x])/(f*(1 - m)*(2 - m)) - (a*A*c^4*(c*Sec[e + f*x])^(-4 + m)*(b
+ a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(1 - m))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rubi steps

$$\int (a+b\cos(e+fx))^3(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^4\int (c\sec(e+fx))^{-4+m}(b+a\sec(e+fx))^3(B+$$

$$= -\frac{aAc^4(c\sec(e+fx))^{-4+m}(b+a\sec(e+fx))^2\tan}{f(1-m)}$$

$$= -\frac{a^2c^4(aB(1-m)-Ab(1+m))\sec(e+fx)(c\sec}{f(1-m)(2-m)}$$

$$= -\frac{a^2c^4(aB(1-m)-Ab(1+m))\sec(e+fx)(c\sec}{f(1-m)(2-m)}$$

$$= -\frac{ac^4\left(3abB(1-m)+a^2A(2-m)-2Ab^2m\right)(c\sec}{f(1-m)(3-m)}$$

$$= -\frac{\left(Ab^3(2-m)+3ab^2B(2-m)+3a^2Ab(3-m)\right.}{}$$

$$= -\frac{\left(Ab^3(2-m)+3ab^2B(2-m)+3a^2Ab(3-m)\right.}{}$$

Mathematica [A] time = 2.52, size = 259, normalized size = 0.57

$$\frac{\sqrt{-\tan^2(e+fx)}\cot(e+fx)(c\sec(e+fx))^m\left(\frac{b^2(3aB+Ab)\cos^3(e+fx)\,_2F_1\left(\frac{1}{2},\frac{m-3}{2};\frac{m-1}{2};\sec^2(e+fx)\right)}{m-3}+a\left(\frac{3b(aB+Ab)\cos^2(e+fx)\,_2F_1}{m-}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[e + f*x])^3*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[e + f*x]*((b^3*B*Cos[e + f*x]^4*Hypergeometric2F1[1/2, (-4 + m)/2, (-2
 + m)/2, Sec[e + f*x]^2])/(-4 + m) + (b^2*(A*b + 3*a*B)*Cos[e + f*x]^3*Hype
rgeometric2F1[1/2, (-3 + m)/2, (-1 + m)/2, Sec[e + f*x]^2])/(-3 + m) + a*((
3*b*(A*b + a*B)*Cos[e + f*x]^2*Hypergeometric2F1[1/2, (-2 + m)/2, m/2, Sec[
e + f*x]^2])/(-2 + m) + a*(((3*A*b + a*B)*Cos[e + f*x]*Hypergeometric2F1[1/
2, (-1 + m)/2, (1 + m)/2, Sec[e + f*x]^2])/(-1 + m) + (a*A*Hypergeometric2F
1[1/2, m/2, (2 + m)/2, Sec[e + f*x]^2])/m)))*(c*Sec[e + f*x])^m*Sqrt[-Tan[e
 + f*x]^2])/f
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^3\cos\left(fx+e\right)^4 + Aa^3 + \left(3\,Bab^2 + Ab^3\right)\cos\left(fx+e\right)^3 + 3\left(Ba^2b + Aab^2\right)\cos\left(fx+e\right)^2 + \left(Ba^3 + 3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((B*b^3*cos(f*x + e)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*cos(f*x + e)^3
 + 3*(B*a^2*b + A*a*b^2)*cos(f*x + e)^2 + (B*a^3 + 3*A*a^2*b)*cos(f*x + e))
*(c*sec(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right) + A\right)\left(b\cos\left(fx+e\right) + a\right)^3 \left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*sec(f*x + e))^m, x
)
```

maple [F] time = 2.23, size = 0, normalized size = 0.00

$$\int \left(a + b\cos\left(fx+e\right)\right)^3 \left(A + B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^3\left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^3*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^3*(c*sec(f*x + e))^m, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{c}{\cos\left(e+f\,x\right)}\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^3,x)
```

```
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sec\left(e+fx\right)\right)^m\left(A+B\cos\left(e+fx\right)\right)\left(a+b\cos\left(e+fx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**3*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))**3,
x)
```

3.638 $\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=327

$$-\frac{c^4\sin(e+fx)\left(a^2B(3-m)+2aAb(3-m)+b^2B(2-m)\right)(c\sec(e+fx))^{m-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-m}{2};\frac{6-m}{2};\cos^2(e+fx)\right)}{f(2-m)(4-m)\sqrt{\sin^2(e+fx)}}-\cdots$$

```
[Out] -c^4*(b^2*B*(2-m)+2*a*A*b*(3-m)+a^2*B*(3-m))*hypergeom([1/2, 2-1/2*m],[3-1/
2*m],cos(f*x+e)^2)*(c*sec(f*x+e))^(-4+m)*sin(f*x+e)/f/(m^2-6*m+8)/(sin(f*x+
e)^2)^(1/2)-c^3*(A*b^2*(1-m)+2*a*b*B*(1-m)+a^2*A*(2-m))*hypergeom([1/2, 3/2
-1/2*m],[5/2-1/2*m],cos(f*x+e)^2)*(c*sec(f*x+e))^(-3+m)*sin(f*x+e)/f/(m^2-4
*m+3)/(sin(f*x+e)^2)^(1/2)-a*c^3*(a*B*(1-m)-A*b*m)*(c*sec(f*x+e))^(-3+m)*ta
n(f*x+e)/f/(m^2-3*m+2)-a*A*c^3*(c*sec(f*x+e))^(-3+m)*(b+a*sec(f*x+e))*tan(f
*x+e)/f/(1-m)
```

Rubi [A] time = 0.64, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2960, 4026, 4047, 3772, 2643, 4046}

$$-\frac{c^4\sin(e+fx)\left(a^2B(3-m)+2aAb(3-m)+b^2B(2-m)\right)(c\sec(e+fx))^{m-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-m}{2};\frac{6-m}{2};\cos^2(e+fx)\right)}{f(2-m)(4-m)\sqrt{\sin^2(e+fx)}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] -((c^4*(b^2*B*(2 - m) + 2*a*A*b*(3 - m) + a^2*B*(3 - m))*Hypergeometric2F1[
1/2, (4 - m)/2, (6 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-4 + m)*Sin[e
+ f*x])/(f*(2 - m)*(4 - m)*Sqrt[Sin[e + f*x]^2])) - (c^3*(A*b^2*(1 - m) + 2
*a*b*B*(1 - m) + a^2*A*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m)/2
, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-3 + m)*Sin[e + f*x])/(f*(1 - m)*(3 - m
)*Sqrt[Sin[e + f*x]^2]) - (a*c^3*(a*B*(1 - m) - A*b*m)*(c*Sec[e + f*x])^(-3
 + m)*Tan[e + f*x])/(f*(1 - m)*(2 - m)) - (a*A*c^3*(c*Sec[e + f*x])^(-3 + m
)*(b + a*Sec[e + f*x])*Tan[e + f*x])/(f*(1 - m))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 4026

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*Sim
p[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))*C
sc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b
^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int (a+b\cos(e+fx))^2(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^3\int (c\sec(e+fx))^{-3+m}(b+a\sec(e+fx))^2(B\cdots$$

$$= -\frac{aAc^3(c\sec(e+fx))^{-3+m}(b+a\sec(e+fx))\,\mathrm{ta}\cdots}{f(1-m)}$$

$$= -\frac{aAc^3(c\sec(e+fx))^{-3+m}(b+a\sec(e+fx))\,\mathrm{ta}\cdots}{f(1-m)}$$

$$= -\frac{ac^3(aB(1-m)-Abm)(c\sec(e+fx))^{-3+m}\tan\cdots}{f(1-m)(2-m)}$$

$$= -\frac{\left(Ab^2(1-m)+2abB(1-m)+a^2A(2-m)\right)\mathrm{co}\cdots}{f(1\cdots}$$

$$= -\frac{\left(Ab^2(1-m)+2abB(1-m)+a^2A(2-m)\right)\mathrm{co}\cdots}{f(1\cdots}$$

Mathematica [A] time = 0.99, size = 205, normalized size = 0.63

$$\frac{\sqrt{-\tan^2(e+fx)}\cot(e+fx)(c\sec(e+fx))^m\left(\frac{b(2aB+Ab)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{m-2}{2};\frac{m}{2};\sec^2(e+fx)\right)}{m-2}+a\left(\frac{(aB+2Ab)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\cdots\right.}{m-1}\cdots\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[e + f*x])^2*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[e + f*x]*((b^2*B*Cos[e + f*x]^3*Hypergeometric2F1[1/2, (-3 + m)/2, (-1
 + m)/2, Sec[e + f*x]^2])/(-3 + m) + (b*(A*b + 2*a*B)*Cos[e + f*x]^2*Hyperg
eometric2F1[1/2, (-2 + m)/2, m/2, Sec[e + f*x]^2])/(-2 + m) + a*(((2*A*b +
a*B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[e + f*x
]^2])/(-1 + m) + (a*A*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[e + f*x]^2
])/m))*(c*Sec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/f
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bb^2\cos\left(fx+e\right)^3+Aa^2+\left(2\,Bab+Ab^2\right)\cos\left(fx+e\right)^2+\left(Ba^2+2\,Aab\right)\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(f*x + e)^3 + A*a^2 + (2*B*a*b + A*b^2)*cos(f*x + e)^2 +
 (B*a^2 + 2*A*a*b)*cos(f*x + e))*(c*sec(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{2}\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*sec(f*x + e))^m, x
)
```

maple [F] time = 1.80, size = 0, normalized size = 0.00

$$\int \left(a+b\cos\left(fx+e\right)\right)^{2}\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{2}\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^2*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^2*(c*sec(f*x + e))^m, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{c}{\cos\left(e+f\,x\right)}\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^2,x)
```

```
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sec\left(e+fx\right)\right)^{m}\left(A+B\cos\left(e+fx\right)\right)\left(a+b\cos\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**2*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))**2,
x)
```

3.639 $\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=217

$$-\frac{c^3\sin(e+fx)(aA(2-m)+bB(1-m))(c\sec(e+fx))^{m-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-m}{2};\frac{5-m}{2};\cos^2(e+fx)\right)}{f(1-m)(3-m)\sqrt{\sin^2(e+fx)}}-\frac{c^2(aB+Ab)\sin(e+\ldots}{}$$

```
[Out] -c^3*(b*B*(1-m)+a*A*(2-m))*hypergeom([1/2, 3/2-1/2*m],[5/2-1/2*m],cos(f*x+e
)^2)*(c*sec(f*x+e))^(-3+m)*sin(f*x+e)/f/(m^2-4*m+3)/(sin(f*x+e)^2)^(1/2)-(A
*b+B*a)*c^2*hypergeom([1/2, 1-1/2*m],[2-1/2*m],cos(f*x+e)^2)*(c*sec(f*x+e))
^(-2+m)*sin(f*x+e)/f/(2-m)/(sin(f*x+e)^2)^(1/2)-a*A*c^2*(c*sec(f*x+e))^(-2+
m)*tan(f*x+e)/f/(1-m)
```

Rubi [A] time = 0.36, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2960, 3997, 3787, 3772, 2643}

$$-\frac{c^3\sin(e+fx)(aA(2-m)+bB(1-m))(c\sec(e+fx))^{m-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-m}{2};\frac{5-m}{2};\cos^2(e+fx)\right)}{f(1-m)(3-m)\sqrt{\sin^2(e+fx)}}-\frac{c^2(aB+Ab)\sin(e+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] -((c^3*(b*B*(1 - m) + a*A*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m
)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-3 + m)*Sin[e + f*x])/(f*(1 - m)*(3
- m)*Sqrt[Sin[e + f*x]^2])) - ((A*b + a*B)*c^2*Hypergeometric2F1[1/2, (2 -
m)/2, (4 - m)/2, Cos[e + f*x]^2]*(c*Sec[e + f*x])^(-2 + m)*Sin[e + f*x])/(f
*(2 - m)*Sqrt[Sin[e + f*x]^2]) - (a*A*c^2*(c*Sec[e + f*x])^(-2 + m)*Tan[e +
 f*x])/(f*(1 - m))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rubi steps

$$\int (a+b\cos(e+fx))(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = c^2\int (c\sec(e+fx))^{-2+m}(b+a\sec(e+fx))(B+$$

$$= -\frac{aAc^2(c\sec(e+fx))^{-2+m}\tan(e+fx)}{f(1-m)} - \frac{c^2\int(c}{}$$

$$= -\frac{aAc^2(c\sec(e+fx))^{-2+m}\tan(e+fx)}{f(1-m)} + ((Ab+$$

$$= -\frac{aAc^2(c\sec(e+fx))^{-2+m}\tan(e+fx)}{f(1-m)} + \Big((Ab+$$

$$= -\frac{(Ab+aB)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{2};\frac{4-m}{2};\cos^2(\right.}{f(2-m)\sqrt{\sin^2(e}}$$

Mathematica [A] time = 0.38, size = 163, normalized size = 0.75

$$\frac{\sqrt{-\tan^2(e+fx)}\cot(e+fx)(c\sec(e+fx))^m\left((m-2)\left(m(aB+Ab)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\sec^2(e+fx)\right)+\right.\right.}{f(m-2)(m-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[e + f*x])*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[e + f*x]*(b*B*(-1 + m)*m*Cos[e + f*x]^2*Hypergeometric2F1[1/2, (-2 + m
)/2, m/2, Sec[e + f*x]^2] + (-2 + m)*((A*b + a*B)*m*Cos[e + f*x]*Hypergeome
tric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[e + f*x]^2] + a*A*(-1 + m)*Hypergeo
metric2F1[1/2, m/2, (2 + m)/2, Sec[e + f*x]^2]))*(c*Sec[e + f*x])^m*Sqrt[-T
an[e + f*x]^2])/(f*(-2 + m)*(-1 + m)*m)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb\cos\left(fx+e\right)^2+Aa+(Ba+Ab)\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*(c*sec(f*x +
 e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)\left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m, x)
```

maple [F] time = 1.94, size = 0, normalized size = 0.00

$$\int\left(a+b\cos\left(fx+e\right)\right)\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(\frac{c}{\cos\left(e+f\,x\right)}\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)),x)
```

```
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sec\left(e+fx\right)\right)^{m}\left(A+B\cos\left(e+fx\right)\right)\left(a+b\cos\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x)), x)
```

3.640 $\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx$

Optimal. Leaf size=299

$$-\frac{(Ab-aB)\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{m/2}(c\sec(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)}{cf\left(a^2-b^2\right)}+\cdots$$

```
[Out] -(A*b-B*a)*AppellF1(1/2,1/2*m,1,3/2,sin(f*x+e)^2,-b^2*sin(f*x+e)^2/(a^2-b^2
))*cos(f*x+e)*(cos(f*x+e)^2)^(1/2*m)*(c*sec(f*x+e))^(1+m)*sin(f*x+e)/(a^2-b
^2)/c/f+a*(A*b-B*a)*AppellF1(1/2,1/2+1/2*m,1,3/2,sin(f*x+e)^2,-b^2*sin(f*x+
e)^2/(a^2-b^2))*(cos(f*x+e)^2)^(1/2+1/2*m)*(c*sec(f*x+e))^(1+m)*sin(f*x+e)/
b/(a^2-b^2)/c/f-B*c*hypergeom([1/2, 1/2-1/2*m],[3/2-1/2*m],cos(f*x+e)^2)*(c
*sec(f*x+e))^(-1+m)*sin(f*x+e)/b/f/(1-m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2960, 4038, 3772, 2643, 3869, 2823, 3189, 429}

$$-\frac{(Ab-aB)\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{m/2}(c\sec(e+fx))^{m+1}F_1\left(\frac{1}{2};\frac{m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)}{cf\left(a^2-b^2\right)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x]),x]
```

```
[Out] -(((A*b - a*B)*AppellF1[1/2, m/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*
x]^2)/(a^2 - b^2))]*Cos[e + f*x]*(Cos[e + f*x]^2)^(m/2)*(c*Sec[e + f*x])^(1
 + m)*Sin[e + f*x])/((a^2 - b^2)*c*f)) + (a*(A*b - a*B)*AppellF1[1/2, (1 +
m)/2, 1, 3/2, Sin[e + f*x]^2, -((b^2*Sin[e + f*x]^2)/(a^2 - b^2))]*(Cos[e +
 f*x]^2)^((1 + m)/2)*(c*Sec[e + f*x])^(1 + m)*Sin[e + f*x])/(b*(a^2 - b^2)*
c*f) - (B*c*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[e + f*x]^2]*(c
*Sec[e + f*x])^(-1 + m)*Sin[e + f*x])/(b*f*(1 - m)*Sqrt[Sin[e + f*x]^2])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Csc[e + f*x])^(p - m - n)*(b + a*Csc[e + f*x])^m*(d + c
*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] && !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] && !IntegerQ[m]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 4038

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
 NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{a+b\cos(e+fx)}\,dx = \int \frac{(c\sec(e+fx))^m(B+A\sec(e+fx))}{b+a\sec(e+fx)}\,dx$$

$$= \frac{B\int (c\sec(e+fx))^m\,dx}{b} + \frac{(Ab-aB)\int \frac{(c\sec(e+fx))^{1+m}}{b+a\sec(e+fx)}\,dx}{bc}$$

$$= \frac{\left(B\left(\frac{\cos(e+fx)}{c}\right)^m (c\sec(e+fx))^m\right)\int \left(\frac{\cos(e+fx)}{c}\right)^{-m}\,dx}{b} + \frac{\left((Ab-aB)\,c\right.}{}$$

$$= -\frac{B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)(c\sec(e+fx))^m\sin(}{bf(1-m)\sqrt{\sin^2(e+fx)}}$$

$$= -\frac{B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)(c\sec(e+fx))^m\sin(}{bf(1-m)\sqrt{\sin^2(e+fx)}}$$

$$= -\frac{(Ab-aB)F_1\left(\frac{1}{2};\frac{m}{2},1;\frac{3}{2};\sin^2(e+fx),-\frac{b^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos}{\left(a^2-b^2\right)cf}$$

Mathematica [B] time = 26.44, size = 10630, normalized size = 35.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x]),x]
[Out] Result too large to show
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^m}{b\cos\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/
2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_noste
p/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/
t_nostep/2)Unable to divide, perhaps due to rounding error%%%{-1,[0,1,0,0]%
%%} / %%%{1,[0,0,1,0]%%%}+%%%{-1,[0,0,0,1]%%%} Error: Bad Argument Value
```

maple [F] time = 1.27, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}}{a+b\cos\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x)
```

```
[Out] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{b\cos\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)}{a+b\,\cos\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x)),x)
```

```
[Out] int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c \sec \left(e + f x\right)\right)^{m} \left(A + B \cos \left(e + f x\right)\right)}{a + b \cos \left(e + f x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))**m/(a+b*cos(f*x+e)),x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))/(a + b*cos(e + f*x)), x)
```

3.641 $\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=210

$$\frac{2(c\cos(e+fx))^m(c\sec(e+fx))^m\mathrm{Int}\left(\frac{(c\cos(e+fx))^{-m}\left(\frac{1}{2}c\cos(e+fx)\left(a(5-2m)(aB+2Ab)+b^2B(3-2m)\right)+\frac{1}{2}bc\cos^2(e+fx)(2aB(3-m)+Ab(\ldots}{\sqrt{a+b\cos(e+fx)}}\right.}{c(5-2m)}$$

```
[Out] 2*b*B*cos(f*x+e)*(c*sec(f*x+e))^m*sin(f*x+e)*(a+b*cos(f*x+e))^(1/2)/f/(5-2*
m)+2*(c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Unintegrable((1/2*a*c*(2*b*B*(1-m)+2
*a*A*(5/2-m))+1/2*c*(b^2*B*(3-2*m)+a*(2*A*b+B*a)*(5-2*m))*cos(f*x+e)+1/2*b*
c*(A*b*(5-2*m)+2*a*B*(3-m))*cos(f*x+e)^2)/((c*cos(f*x+e))^m)/(a+b*cos(f*x+e
))^(1/2),x)/c/(5-2*m)
```

Rubi [A] time = 0.72, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (2*b*B*Cos[e + f*x]*Sqrt[a + b*Cos[e + f*x]]*(c*Sec[e + f*x])^m*Sin[e + f*x
])/(f*(5 - 2*m)) + (2*(c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Defer[Int][((a*
c*(2*b*B*(1 - m) + 2*a*A*(5/2 - m)))/2 + (c*(b^2*B*(3 - 2*m) + a*(2*A*b + a
*B)*(5 - 2*m))*Cos[e + f*x])/2 + (b*c*(A*b*(5 - 2*m) + 2*a*B*(3 - m))*Cos[e
 + f*x]^2)/2)/((c*Cos[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]), x])/(c*(5 - 2*
m))
```

Rubi steps

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \left((c\cos(e+fx))^m(c\sec(e+fx))^m\right)\int (c\cos(e\ldots$$

$$= \frac{2bB\cos(e+fx)\sqrt{a+b\cos(e+fx)}\,(c\sec(e+\ldots}{f(5-2m)}$$

Mathematica [A] time = 61.59, size = 0, normalized size = 0.00

$$\int (a+b\cos(e+fx))^{3/2}(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^
m,x]
```

```
[Out] Integrate[(a + b*Cos[e + f*x])^(3/2)*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^
m, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bb\cos\left(fx+e\right)^2+Aa+(Ba+Ab)\cos\left(fx+e\right)\right)\sqrt{b\cos\left(fx+e\right)+a}\,\left(c\sec\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="fricas")
```

```
[Out] integral((B*b*cos(f*x + e)^2 + A*a + (B*a + A*b)*cos(f*x + e))*sqrt(b*cos(f
*x + e) + a)*(c*sec(f*x + e))^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*sec(f*x + e))^
m, x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \left(a+b\cos\left(fx+e\right)\right)^{\frac{3}{2}}\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^(3/2)*(c*sec(f*x + e))^
m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{c}{\cos\left(e+f\,x\right)}\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\left(a+b\,\cos\left(e+f\,x\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(3/2),x)
```

```
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**(3/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.642 $\int \sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$

Optimal. Leaf size=61

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Int}\left(\sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))(c\cos(e+fx))^{-m},x\right)$$

```
[Out] (c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Unintegrable((A+B*cos(f*x+e))*(a+b*cos(f*
x+e))^(1/2)/((c*cos(f*x+e))^m),x)
```

Rubi [A] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,x]
```

```
[Out] (c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Defer[Int][(Sqrt[a + b*Cos[e + f*x]]*
(A + B*Cos[e + f*x]))/(c*Cos[e + f*x])^m, x]
```

Rubi steps

$$\int \sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx = \left((c\cos(e+fx))^m(c\sec(e+fx))^m\right)\int (c\cos(e+f$$

Mathematica [A] time = 13.20, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos(e+fx)}\,(A+B\cos(e+fx))(c\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,
x]
```

```
[Out] Integrate[Sqrt[a + b*Cos[e + f*x]]*(A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m,
 x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\,\left(c\sec\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m,
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\,\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m,
 x)
```

maple [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos\left(fx+e\right)}\,\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\,\left(c\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))^m,x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m,
 x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{c}{\cos\left(e+f\,x\right)}\right)}^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)\,\sqrt{a+b\,\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(1/2),x)
```

```
[Out] int((c/cos(e + f*x))^m*(A + B*cos(e + f*x))*(a + b*cos(e + f*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sec\left(e+fx\right)\right)^{m}\left(A+B\cos\left(e+fx\right)\right)\sqrt{a+b\cos\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**(1/2)*(A+B*cos(f*x+e))*(c*sec(f*x+e))**m,x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))*sqrt(a + b*cos(e + f*x)),
 x)
```

3.643 $\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx$

Optimal. Leaf size=61

$$(c\cos(e+fx))^m(c\sec(e+fx))^m\text{Int}\left(\frac{(A+B\cos(e+fx))(c\cos(e+fx))^{-m}}{\sqrt{a+b\cos(e+fx)}},x\right)$$

```
[Out] (c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Unintegrable((A+B*cos(f*x+e))/((c*cos(f*x
+e))^m)/(a+b*cos(f*x+e))^(1/2),x)
```

Rubi [A] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/Sqrt[a + b*Cos[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*Defer[Int][(A + B*Cos[e + f*x])/((c*C
os[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]), x]
```

Rubi steps

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx = \left((c\cos(e+fx))^m(c\sec(e+fx))^m\right)\int \frac{(c\cos(e+fx))^{-m}(A+B\cos}{\sqrt{a+b\cos(e+fx)}}$$

Mathematica [A] time = 8.87, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{\sqrt{a+b\cos(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/Sqrt[a + b*Cos[e + f*x]
],x]
```

```
[Out] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/Sqrt[a + b*Cos[e + f*x]
], x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{\sqrt{b\cos\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{\sqrt{b\cos\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

maple [A] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}}{\sqrt{a+b\cos\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x)
```

```
[Out] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{\sqrt{b\cos\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/sqrt(b*cos(f*x + e) + a),
 x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^m\,\left(A+B\,\cos\left(e+f\,x\right)\right)}{\sqrt{a+b\,\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(1/2),x)
```

```
[Out] int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(1/2), x
)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sec\left(e+fx\right)\right)^m\left(A+B\cos\left(e+fx\right)\right)}{\sqrt{a+b\cos\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))**m/(a+b*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))/sqrt(a + b*cos(e + f*x)),
 x)
```

3.644 $\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=213

$$\frac{2(c\cos(e+fx))^m(c\sec(e+fx))^m\mathrm{Int}\left(\frac{(c\cos(e+fx))^{-m}\left(\frac{1}{2}c\left(a^2A-2abB(1-m)+Ab^2(1-2m)\right)-\frac{1}{2}bc(3-2m)(Ab-aB)\cos^2(e+fx)-\frac{1}{2}ac(Ab-aB)\right.}{\sqrt{a+b\cos(e+fx)}}\right.}{ac\left(a^2-b^2\right)}$$

```
[Out] 2*b*(A*b-B*a)*cos(f*x+e)*(c*sec(f*x+e))^m*sin(f*x+e)/a/(a^2-b^2)/f/(a+b*cos
(f*x+e))^(1/2)+2*(c*cos(f*x+e))^m*(c*sec(f*x+e))^m*Unintegrable((1/2*c*(a^2
*A+A*b^2*(1-2*m)-2*a*b*B*(1-m))-1/2*a*(A*b-B*a)*c*cos(f*x+e)-1/2*b*(A*b-B*a
)*c*(3-2*m)*cos(f*x+e)^2)/((c*cos(f*x+e))^m)/(a+b*cos(f*x+e))^(1/2),x)/a/(a
^2-b^2)/c
```

Rubi [A] time = 0.69, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x])^(3/2),x]
```

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x]*(c*Sec[e + f*x])^m*Sin[e + f*x])/(a*(a^2 - b^
2)*f*Sqrt[a + b*Cos[e + f*x]]) + (2*(c*Cos[e + f*x])^m*(c*Sec[e + f*x])^m*D
efer[Int][((c*(a^2*A + A*b^2*(1 - 2*m) - 2*a*b*B*(1 - m)))/2 - (a*(A*b - a*
B)*c*Cos[e + f*x])/2 - (b*(A*b - a*B)*c*(3 - 2*m)*Cos[e + f*x]^2)/2)/((c*Co
s[e + f*x])^m*Sqrt[a + b*Cos[e + f*x]]), x])/(a*(a^2 - b^2)*c)
```

Rubi steps

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx = \left((c\cos(e+fx))^m(c\sec(e+fx))^m\right)\int \frac{(c\cos(e+fx))^{-m}(A+B\cos(e}{(a+b\cos(e+fx))^{3/2}}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)(c\sec(e+fx))^m\sin(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\cos(e+fx)}} + \frac{(2(c\cos(e+}{}$$

Mathematica [A] time = 11.21, size = 0, normalized size = 0.00

$$\int \frac{(A+B\cos(e+fx))(c\sec(e+fx))^m}{(a+b\cos(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x])^(3
/2),x]
```

```
[Out] Integrate[((A + B*Cos[e + f*x])*(c*Sec[e + f*x])^m)/(a + b*Cos[e + f*x])^(3
/2), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(B\cos\left(fx+e\right)+A\right)\sqrt{b\cos\left(fx+e\right)+a}\,\left(c\sec\left(fx+e\right)\right)^{m}}{b^{2}\cos\left(fx+e\right)^{2}+2\,ab\cos\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*cos(f*x + e) + A)*sqrt(b*cos(f*x + e) + a)*(c*sec(f*x + e))^m/(
b^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a)^(3/2
), x)
```

maple [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\cos\left(fx+e\right)\right)\left(c\sec\left(fx+e\right)\right)^{m}}{\left(a+b\cos\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x)
```

```
[Out] int((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(B\cos\left(fx+e\right)+A\right)\left(c\sec\left(fx+e\right)\right)^{m}}{\left(b\cos\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))^m/(a+b*cos(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*cos(f*x + e) + A)*(c*sec(f*x + e))^m/(b*cos(f*x + e) + a)^(3/2
), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{c}{\cos\left(e+f\,x\right)}\right)^{m}\,\left(A+B\,\cos\left(e+f\,x\right)\right)}{\left(a+b\,\cos\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(3/2),x)
```

```
[Out] int(((c/cos(e + f*x))^m*(A + B*cos(e + f*x)))/(a + b*cos(e + f*x))^(3/2), x
)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sec\left(e+fx\right)\right)^{m}\left(A+B\cos\left(e+fx\right)\right)}{\left(a+b\cos\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(f*x+e))*(c*sec(f*x+e))**m/(a+b*cos(f*x+e))**(3/2),x)
```

```
[Out] Integral((c*sec(e + f*x))**m*(A + B*cos(e + f*x))/(a + b*cos(e + f*x))**(3/
2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
     as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#               Port of original Maple grading function by
#               Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#               added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```